Quantum arithmetics provides a possible resolution of a long-lasting challenge of finding a mathematical justification for the canonical identification mapping p-adics to reals playing a key role in TGD - in particular in p-adic mass calculations. p-Adic numbers have p-adic pinary expansions $\sum a_np^n$ satisfying $a_n<p$. of powers p^n to be products of primes $p_1<p$ satisfying $a_n<p$ for ordinary p-adic numbers. One could map this expansion to its quantum counterpart by replacing a_n with their counterpart and by canonical identification map $p\rightarrow 1/p$ the expansion to real number. This definition might be criticized as being essentially equivalent with ordinary p-adic numbers since one can argue that the map of coefficients a_n to their quantum counterparts takes place only in the canonical identification map to reals.

One could however modify this recipe. Represent integer n as a product of primes l and allow for l all expansions for which the coefficients a_n consist of primes $p_1<p$ but give up the condition $a_n<p$. This would give 1-to-many correspondence between ordinary p-adic numbers and their quantum counterparts.

It took time to realize that $l<p$ condition might be necessary in which case the quantization in this sense - if present at all - could be associated with the canonical identification map to reals. It would correspond only to the process taking into account finite measurement resolution rather than replacement of p-adic number field with something new, hopefully a field. At this step one might perhaps allow $l>p$ so that one would obtain several real images under canonical identification.

One can however imagine a third generalization of number concept. One can replace integer n with n-dimensional Hilbert space and sum $+$ and product $\times$ with direct sum $\oplus$ and tensor product $

\otimes$ and introduce their co-operations, the definition of which is highly non-trivial. This procedure yields also Hilbert space variants of rationals, algebraic numbers, p-adic number fields, and even complex, quaternionic and octonionic algebraics. Also adeles can be replaced with their Hilbert space counterparts. Even more, one can replace the points of Hilbert spaces with Hilbert spaces and repeat this process, which is very similar to the construction of infinite primes having interpretation in terms of repeated second quantization. This process could be the counterpart for construction of n^{th} order logics and one might speak of Hilbert or quantum mathematics. The construction would also generalize the notion of algebraic holography and provide self-referential cognitive representation of mathematics.

This vision emerged from the connections with generalized Feynman diagrams, braids, and with the hierarchy of Planck constants realized in terms of coverings of the imbedding space. Hilbert space generalization of number concept seems to be extremely well suited for the purposes of TGD. For instance, generalized Feynman diagrams could be identifiable as arithmetic Feynman diagrams describing sequences of arithmetic operations and their co-operations. One could interpret $\times_q$ and $+_q$ and their co-algebra operations as 3-vertices for number theoretical Feynman diagrams describing algebraic identities $X=Y$ having natural interpretation in zero energy ontology. The two vertices have direct counterparts as two kinds of basic topological vertices in quantum TGD (stringy vertices and vertices of Feynman diagrams). The definition of co-operations would characterize quantum dynamics. Physical states would correspond to the Hilbert space states assignable to numbers. One prediction is that all loops can be eliminated from generalized Feynman diagrams and diagrams are in projective sense invariant under permutations of incoming (outgoing legs).