Langlands correspondence is for mathematics what unified theories are for physics. The number theoretic vision about TGD has intriguing resemblances with number theoretic Langlands program. There is also geometric variant of Langlands program. I am of course amateur and do not have grasp about the mathematical technicalities and can only try to understand the general ideas and related them to those behind TGD. Physics as geometry of WCW ("world of classical worlds") and physics as generalized number theory are the two visions about quantum TGD: this division brings in mind   geometric and number theoretic Langlands programs. This motivates re-consideration of Langlands program from TGD point of view. I have written years ago a chapter about this earlier but TGD has evolved considerably since then so that it is time for a second attempt to understand what Langlands is about.

By Langlands correspondence the representations of $G\rtimes Gal$ and G should correspond to each other. The analogy with the representations of Lorentz group suggests that the representations of G should have \blockquote{spin} for some compact subgroup acting from left or right   such that the dimension of this representation is same as the representation of non-commutative Galois group.

Automorphic functions are indeed typically functions in G, which reduce to a function invariant under left and/or right action of a compact or even discrete subgroups H_1 and H_2 or more generally, belong to a finite-dimensional unitary representation of $H_1\times H_2$ in $H_1\backslash G/H_2$. Therefore they can be said to have $H_1\times H_2$ quantum numbers analogous to spin if interpreted as \blockquote{field modes} in the space of double cosets H_1gH_2. This would conform with the vision about physics as generalized number theory. If I have understood correctly, the question is whether a finite-dimensional representation of H_1 or H_2 could correspond to a finite-dimensional representation of Galois group at the number theory side.