

An updated view about M^8-H duality is discussed. M^8-H duality allows to deduce $M^4 \times CP_2$ via number theoretical compactification. One important correction is that octonionic spinor structure makes sense only for M^8 whereas for $M^4 \times CP_2$ complexified quaternions characterized the spinor structure.

Octonions, quaternions associative and co-associative space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized.

There is a beautiful pattern present suggesting that $H=M^4 \times CP_2$ is completely unique on number theoretical grounds. Consider only the following facts. M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. Octonionic projective space OP_2 appears as octonionic twistor space (there are no higher-dimensional octonionic projective spaces). Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing OP_2 to 12-D $G_2/U(1) \times U(1)$ having same dimension as the the twistor space $CP_3 \times SU(3)/U(1) \times U(1)$ of H assignable to complexified quaternionic representation of gamma matrices.

A further fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically six-sphere. Also the analogy of quaternionicity of preferred extremals in TGD with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional

conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. M^8-H correspondence in turn would map the space-time surface in M^8 to $M^4 \times CP_2$.

A long-standing question has been the origin of preferred p-adic primes characterizing elementary particles. I have proposed several explanations and the most convincing hitherto is related to the algebraic extensions of rationals and p-adic numbers selecting naturally preferred primes as those which are ramified for the extension in question.