%\begin{abstract}

In this chapter the general TGD inspired mathematical ideas related to p-adic numbers are discussed. The extensions of the p-adic numbers including extensions containing transcendentals, the correspondences between p-adic and real numbers, p-adic differential and integral calculus, and p-adic symmetries and Fourier analysis belong the topics of the chapter.

The basic hypothesis is that p-adic space-time regions correspond to cognitive representations for the real physics appearing already at the elementary particle level. The interpretation of the p-adic physics as a physics of cognition is justified by the inherent p-adic non-determinism of the p-adic differential equations making possible the extreme flexibility of imagination.

p-Adic canonical identification and the identification of reals and p-adics by common rationals are the two basic identification maps between p-adics and reals and can be interpreted as two basic types of cognitive maps. The concept of p-adic fractality is defined and p-adic fractality is the basic property of the cognitive maps mapping real world to the p-adic internal world. Canonical identification is not general coordinate invariant and at the fundamental level it is applied only to map p-adic probabilities and predictions of p-adic thermodynamics to real numbers. The correspondence via common rationals is general coordinate invariant correspondence when general coordinate transformations are restricted to rational or extended rational maps: this has interpretation in terms of fundamental length scale unit provided by CP_2 length.

A natural outcome is the generalization of the notion of number. Different number fields form a book like structure with number fields and their extensions representing the pages of the book glued together along common rationals representing the rim of the book. This generalization

forces also
the generalization of the manifold concept: both imbedding space and WCW are obtained as union of copies corresponding to
various number fields glued together along common points, in particular
rational ones. Space-time surfaces decompose naturally to real and p-adic
space-time sheets. In this framework the fusion of real and various p-adic
physics reduces more or less to to an algebraic continuation of rational
number based physics to various number fields and their extensions.

The definition of p-adic manifold is not discussed although it has turned
out to be highly non-trivial. The feasible definition of p-adic
sub-manifold emerged two decades after the emergence of the notion of of
p-adic space-time sheet. The definition relies on the idea that p-adic
space-time surfaces serve as p-adic charts - cognitive maps - for real
space-time surfaces and vice versa and that both real and p-adic space-time
sheets are preferred extremals of K\"ahler action and defined only modulo
finite measurement/cognitive resolution.

p-Adic differential calculus obeys the same rules as real one and an
interesting outcome are p-adic fractals involving canonical identification.
Perhaps the most crucial ingredient concerning the practical formulation
of the p-adic physics is the concept of the p-adic valued definite
integral. Quite generally, all general coordinate invariant definitions are
based on algebraic continuation by common rationals. Integral functions
can be defined using just the rules of ordinary calculus and the ordering
of the integration limits is provided by the correspondence via common
rationals. Residy calculus generalizes to p-adic context and also free
Gaussian functional integral generalizes to p-adic context and is expected
to play key role in quantum TGD at WCW level.

The special features of p-adic Lie-groups are briefly discussed: the most
important of them being an infinite fractal hierarchy of nested groups.

Various versions of the p-adic Fourier analysis are proposed: ordinary Fourier analysis generalizes naturally only if finite-dimensional extensions of p-adic numbers are allowed and this has interpretation in terms of p-adic length scale cutoff. Also p-adic Fourier analysis provides a possible definition of the definite integral in the p-adic context by using algebraic continuation.

%\end{abstract}