In this chapter the classical field equations associated with the K\"ahler
action are studied.

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{\it 1. Are all extremals actually \blockquote{preferred}?} \vm

The notion of preferred extremal has been central concept in TGD but is
there really compelling need to pose any condition to select preferred
extremals in zero energy ontology (ZEO) as there would be in positive
energy ontology? In ZEO the union of the space-like ends of space-time
surfaces at the boundaries of causal diamond (CD) are the first guess
for 3-surface. If one includes to this 3-surface also the light-like
partonic orbits at which the signature of the induced metric changes to
get analog of Wilson loop, one has good reasons to expect that the
preferred extremal is highly unique without any additional conditions
apart from non-determinism of K\"ahler action proposed to correspond to
sub-algebra of conformal algebra acting on the light-like 3-surface and
respecting light-likeness. One expects that there are finite number n
of conformal equivalence classes and n corresponds to n in
$h_{eff}=nh$. These objects would allow also to understand the
assignment of discrete physical degrees of freedom to the partonic
orbits as required by the assignment of hierarchy of Planck constants to
the non-determinism of K\"ahler action.

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{\it 2. Preferred extremals and quantum criticality} \vm

The identification of preferred extremals of K\"ahler action defining
counterparts of Bohr orbits has been one of the basic challenges of quantum
TGD.   By quantum classical correspondence the non-deterministic space-time
dynamics should mimic the dissipative dynamics of the quantum jump sequence.

The space-time representation for dissipation comes from the interpretation of regions of space-time surface with Euclidian signature of induced metric as generalized Feynman diagrams (or equivalently the light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions). Dissipation would be represented in terms of Feynman graphs representing irreversible dynamics and expressed in the structure of zero energy state in which positive energy part corresponds to the initial state and negative energy part to the final state. Outside Euclidian regions classical dissipation should be absent and this indeed the case for the known extremals.

The non-determinism should also give rose to space-time correlate for quantum criticality. The study of K\"ahler-Dirac equations suggests how to define quantum criticality. Noether currents assignable to the K\"ahler-Dirac equation are conserved only if the first variation of K\"ahler-Dirac operator D_K defined by K\"ahler action vanishes. This is equivalent with the vanishing of the second variation of K\"ahler action - at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

It became later clear that the well-definedness of em charge forces in the generic case the localization of the spinor modes to 2-D surfaces - string world sheets. This would suggest that the equations stating the vanishing of the second variation of K\"ahler action hold true only at string world sheets.

The vanishing of second variations of preferred extremals suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In zero energy ontology (ZEO) catastrophe theory would be generalized to

infinite-dimensional context. Finite number of sheets for catastrophe
would be replaced with finite number of conformal equivalence classes of
space-time surfaces connecting given space-like 3-surfaces at the
boundaries causal diamond (CD).

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{\it 3. Hamilton-Jacobi structure} \vm

Most known extremals share very general properties. One of them is
Hamilton-Jacobi structure meaning the possibility to assign to the extremal
so called Hamilton-Jacobi coordinates. This means dual slicings of M^4 by
string world sheets and partonic 2-surfaces. Number theoretic
compactification led years later to the same condition. This slicing allows
a dimensional reduction of quantum TGD to Minkowskian and Euclidian
variants of string model. Also holography in the sense that the
dynamics of 3-dimensional space-time surfaces reduces to that for 2-D
partonic surfaces in a given measurement resolution follows. The
construction of quantum TGD relies in essential manner to this property.
CP_2 type vacuum extremals do not possess Hamilton-Jaboci structure
but have holomorphic structure.

\vm {\it 4. Specific extremals of K\"ahler action}

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The study of extremals of K\"ahler action represents more than decade old
layer in the development of TGD.

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\item The huge vacuum degeneracy is the most characteristic feature of
K\"ahler action (any 4-surface having CP_2 projection which is Legendre
sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in
general 2-dimensional). This vacuum degeneracy is behind the spin glass

analogy and leads to the p-adic TGD. As found in the second part of the
book, various particle like vacuum extremals also play an important role in
the understanding of the quantum TGD.

\item The so called CP_2 type vacuum extremals have finite, negative
action and are therefore an excellent candidate for real particles whereas
vacuum extremals with vanishing K\"ahler action are candidates for the
virtual particles. These extremals have one dimensional M^4 projection,
which is light like curve but not necessarily geodesic and locally the
metric of the extremal is that of CP_2: the quantization of this motion
leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$
are identified as the generalized Feynmann diagrams with lines thickened to
4-manifolds of \blockquote{thickness} of the order of CP_2 radius. The quantization
of the random motion with light velocity associated with the CP_2 type
extremals in fact led to the discovery of Super Virasoro invariance, which
through the construction of the WCW geometry, becomes a
basic symmetry of quantum TGD.

\item There are also various non-vacuum extremals.

\begin{enumerate}

\item String like objects, with   string tension of same order of
magnitude as possessed by the cosmic strings of GUTs, have a crucial role
in TGD inspired model for the galaxy formation and in the TGD based cosmology.

\item The so called massless extremals describe non-linear plane waves
propagating with the velocity of light such that the polarization is fixed
in given point of the space-time surface. The purely TGD:eish feature is
the light like K\"ahler current: in the ordinary Maxwell theory vacuum
gauge currents are not possible. This current serves as a source of
coherent photons, which might play an important role in the quantum model
of bio-system as a macroscopic quantum system.

\end{enumerate}

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