There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional K\"ahler geometry for the \blockquote{world of classical worlds} identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein's geometrization of physics program is in question.

The second vision identifies physics as a generalized number theory and involves three threads: various p-adic physics and their fusion together with real number based physics to a larger structure, the attempt to understand basic physics in terms of classical number fields (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory.

\vm{\it 1. p-Adic physics and their fusion with real physics}\vm

The basic technical problems of the fusion of real physics and various p-adic physics to single coherent whole relate to the notion of definite integral both at space-time level, imbedding space level and the level of WCW (the \blockquote{world of classical worlds}). The expressibility of WCW as a union of symmetric spacesleads to a proposal that harmonic analysis of symmetric spaces can be used to define various integrals as sums over Fourier components. This leads to the proposal the p-adic variant of symmetric space is obtained by a algebraic continuation through a common intersection of these spaces, which basically reduces to an algebraic variant of coset space involving algebraic extension of rationals by roots of unity. This brings in the notion of angle measurement resolution coming as $\Delta \phi= 2\pi/p^n$ for given p-adic prime p.   Also a proposal how one can complete the discrete version of symmetric space to a continuous p-adic versions emerges and means that each point is effectively replaced with the p-adic variant of the symmetric space identifiable as a p-adic counterpart of the real discretization volume so that a fractal p-adic variant of symmetric space results.

If the K\"ahler geometry of WCW is expressible in terms of rational or algebraic functions, it can in principle be continued the p-adic context. One can however consider the possibility that that the integrals over partonic 2-surfaces defining flux Hamiltonians exist p-adically as Riemann sums. This requires that the geometries of the partonic 2-surfaces effectively reduce to finite sub-manifold geometries in the discretized version of $\delta M^4_+\times CP_2$. If K\"ahler action is required to exist p-adically same kind of condition applies to the space-time surfaces themselves. These strong conditions might make sense in the intersection of the real and p-adic worlds assumed to characterized living matter.

\vm{\it 2. TGD and classical number fields}\vm

The basis vision is that the geometry of the infinite-dimensional WCW (\blockquote{world of classical worlds}) is unique from its mere existence. This leads to its identification as union of symmetric spaces whose K\"ahler geometries are fixed by generalized conformal symmetries. This fixes space-time dimension and the decomposition $M^4\times S$ and the idea is that the symmetries of the K\"ahler manifold S make it somehow unique. The motivating observations are that the dimensions of classical number fields are the dimensions of partonic 2-surfaces, space-time surfaces, and imbedding space and M^8 can be identified as hyper-octonions- a sub-space of complexified octonions obtained by adding a commuting imaginary unit. This stimulates some questions.

Could one understand $S=CP_2$ number theoretically in the sense that M^8 and $H=M^4\times CP_2$ be in some deep sense equivalent (\blockquote{number theoretical compactification} or M^8-H duality)? Could associativity

define the fundamental dynamical principle so that space-time surfaces
could be regarded as associative or co-associative (defined properly)
sub-manifolds of M^8 or equivalently of H.

One can indeed define the associative (co-associative) 4-surfaces using
octonionic representation of gamma matrices of 8-D spaces as surfaces for
which the K\"ahler-Dirac gamma matrices span an associate (co-associative)
sub-space at each point of space-time surface. In fact, only
octonionic structure is needed. Also M^8-H duality holds
true if one assumes that this associative sub-space at each point contains
preferred plane of M^8 identifiable as a preferred commutative or
co-commutative plane (this condition generalizes to an integral
distribution of commutative planes in M^8). These planes are parametrized
by CP_2 and this leads to M^8-H duality.

WCW itself can be identified as the space of 4-D local sub-algebras of the
local Clifford algebra of M^8 or H which are associative or
co-associative. An open conjecture is that this characterization of the
space-time surfaces is equivalent with the preferred extremal property of
K\"ahler action with preferred extremal identified as a critical extremal
allowing infinite-dimensional algebra of vanishing second variations.

\vm {\it 3. Infinite primes}\vm

The construction of infinite primes is formally analogous to a repeated
second quantization of an arithmetic quantum field theory by taking the
many particle states of previous level elementary particles at the new
level.   Besides free many particle states also the analogs of bound
states appear. In the representation in terms of polynomials the free
states correspond to products of first order polynomials with rational
zeros. Bound states correspond to n^{th} order polynomials with
non-rational but algebraic zeros at the lowest level. At higher levels
polynomials depend on several variables.

The construction might allow a generalization to algebraic extensions of rational numbers, and also to classical number fields and their complexifications obtained by adding a commuting imaginary unit. Special class corresponds to hyper-octonionic primes for which the imaginary part of ordinary octonion is multiplied by the commuting imaginary unit so that one obtains a sub-space M^8 with Minkowski signature of metric. Also in this case the basic construction reduces to that for rational or complex rational primes and more complex primes are obtained by acting using elements of the octonionic automorphism group which preserve the complex octonionic integer property.

Can one map infinite primes/integers/rationals to quantum states? Do they have space-time surfaces as correlates? Quantum classical correspondence suggests that if infinite rationals can be mapped to quantum states then the mapping of quantum states to space-time surfaces automatically gives the map to space-time surfaces. The question is therefore whether the mapping to quantum states defined by WCW spinor fields is possible. A natural hypothesis is that number theoretic fermions can be mapped to real fermions and number theoretic bosons to WCW (\blockquote{world of classical worlds}) Hamiltonians.

The crucial observation is that one can construct infinite hierarchy of rational units by forming ratios of infinite integers such that their ratio equals to one in real sense: the integers have interpretation as positive and negative energy parts of zero energy states. One can generalize the construction to quaternionic and octonionic units. One can construct also sums of these units with complex coefficients using commuting imaginary unit and these sums can be normalized to unity and have interpretation as states in Hilbert space. These units can be assumed to possess well defined standard model quantum numbers. It is possible to map the quantum number combinations of WCW spinor fields to these states. Hence the points of M^8 can be said to have infinitely complex number theoretic anatomy so that quantum states of the

universe
can be mapped to this anatomy. One could talk about algebraic holography or number theoretic Brahman=Atman identity.

Also the question how infinite primes might relate to the p-adicization program
and to the hierarchy of Planck constants is discussed.