The work with TGD inspired model for topological quantum computation led to the realization that von Neumann algebras, in particular so called hyper-finite factors of type II_1, seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD. The discussions of this chapter have been restricted to the basic notions are discussed and only short mention is made to TGD applications discussed in second chapter.

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id)=1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a\ density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1.

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_{∞} associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD. It however seems that in TGD framework based on Zero Energy Ontology identifiable as \blockquote{square root} of thermodynamics a square root of thermodynamical state is needed.

The inclusions of hyper-finite factors define an excellent candidate for the description of finite measurement resolution with included factor representing the degrees of freedom below measurement resolution. The would also give connection to the notion of quantum group whose physical interpretation has remained unclear. This idea is central to the proposed applications to quantum TGD discussed in separate chapter.