Does the Modified Dirac Equation Define the Fundamental Action Principle?

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Contents

Abstract

In this chapter the recent view about the Kähler-Dirac action (or Kähler-Dirac action) is explained in more detail.

1. Identification of the Kähler-Dirac action

The most general form of the Kähler-Dirac (or Kähler-Dirac) action involves several terms. The first one is 4-dimensional assignable to Kähler action. Second term is instanton term reducible to an expression restricted to wormhole throats or any light-like 3-surfaces parallel to them in the slicing of space-time surface by light-like 3-surfaces. The third term is a measurement interaction term linear in Cartan algebra of the isometry group of the imbedding space in order to obtain stringy propagators and also to realize coupling between the quantum numbers associated with super-conformal representations and space-time geometry required by quantum classical correspondence.

This means that 3-D light-like wormhole throats carry induced spinor field which can be regarded as independent degrees of freedom having the spinor fields at partonic 2-surfaces as sources and acting as 3-D sources for the 4-D induced spinor field. The most general measurement interaction would involve the corresponding coupling also for Kähler action but is not physically motivated. There are good arguments in favor of Chern-Simons Dirac action and corresponding measurement interaction.

1. Cartan algebra plays a key role not only at quantum level but also at the level of space-time geometry since quantum critical conserved currents vanish for Cartan algebra of isometries and the measurement interaction terms giving rise to conserved currents are possible only for Cartan algebras. Furthermore, Kähler-Dirac equation makes sense only for eigen states of Cartan algebra generators. The hierarchy of Planck constants realized in terms of the book like structure of the generalized imbedding space assigns to each CD (causal diamond) preferred Cartan algebra: in case of Poincare algebra there are two of them corresponding to linear and cylindrical M^4 coordinates.
2. Quantum holography and dimensional reduction hierarchy in which partonic 2-surface defined fermionic sources for 3-D fermionic fields at light-like 3-surfaces Y_l^3 in turn defining fermionic sources for 4-D spinors find an elegant realization. Effective 2-dimensionality is achieved if the replacement of light-like wormhole throat X_l^3 with light-like 3-surface Y_l^3 "parallel" with it in the definition of Dirac determinant corresponds to the $U(1)$ gauge transformation $K \rightarrow K+f+\overline{f}$ for Kähler function of WCW so that WCW Kähler metric is not affected. Here f is holomorphic function of WCW ("world of classical worlds") complex coordinates and arbitrary function of zero mode coordinates.
3. An elegant description of the interaction between super-conformal representations realized at partonic 2-surfaces and dynamics of space-time surfaces is achieved since the values of Cartan charges are feeded to the 3-D Dirac equation which also receives mass term at the same time. Almost topological QFT at wormhole throats results at the limit when four-momenta vanish: this is in accordance with the original vision about TGD as almost topological QFT.
4. A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

   Criticality is accompanied by conformal invariance and this leads to the proposal that critical deformations correspond to Kac-Moody type conformal algebra respecting the light-likeness of the partonic orbits and acting trivially at partonic 2-surfaces. Sub-algebras of conformal algebras with conformal weights divisible by integer n would act as gauge symmetries and these algebras would form an inclusion hierarchy defining hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.
5. The condition that em charge is well-defined for the modes of Kähler-Dirac action implies that they are in the generic case localized at 2-dimensional surfaces: string world sheets and perhaps also partonic 2-surfaces. This implies that string model in 4-D space-time becomes part of TGD.

6. The inclusion of imaginary instanton term to the definition of the Kähler-Dirac gamma matrices is not consistent with the conjugation of the induced spinor fields. Measurement interaction can be however assigned to both Kähler action and possible instanton term. The CP and T oddness of the instanton part of the measurement interaction term could provide first level description for CP breaking. Arrow of time means T-breaking too but the description of dissipation relates to the quantum jumps and to the possibility of state function reductions take place to either boundary of causal diamond (CD) so that breaking of the arrow of time and dissipation are the outcome.

   Instanton term reduces to Chern-Simons term as does also Kähler action if weak form of electric-magnetic duality and the condition $j \cdot A = 0$ is assumed for Kähler action. Does this imply that CP breaking results spontaneously as a kind of boundary effect and no instanton term is needed?

2. Dirac determinant as the exponent of Kähler function?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. An obvious guess is that vacuum functional identified as exponent of Kähler function from Euclidian space-time regions and its its imaginary counterpart from Minkowskian space-time regions can be regarded as Dirac determinant assignable to Kähler-Dirac action.

1. By the localization of the spinor modes to string worlds sheets and possibly also partonic 2-surfaces Dirac determinant should reduce to a product of determinants associated with these. Thus one would have a product of stringy determinants and there would exist a machinery for defining them.

   Since the Kähler-Dirac operator annihilates the induced spinor fields, it seems that Dirac determinants must reduce to contributions coming from the "boundaries" of string world sheets at given space-time sheet. They are closed curves containing light-like curves at the light-like orbits of partonic 2-surfaces and space-like curves at the ends of space-time surface at light-like boundaries of causal diamond.

2. The boundary term associated with Kähler-Dirac operator equals to $p^k\gamma_k + \Gamma^n$, where p^k is the four-momentum assignable to the space-like 3-surface and Γ^n is Kähler-Dirac gamma matrix defined as contribution of normal components of the canonical momentum densities contracted with imbedding space gamma matrices. This operator annihilates the spinor modes. At the light-like partonic 2-surface Γ^n annihilate the modes. There is a strong temptation to define the Dirac determinant as the product of the eigenvalues of the square of the operator $p^k\gamma_k$ just as in the case of massless fields. In this case the determinant should reduce to the square root of the product of the eigenvalues of the mass squared operator given by stringy mass formula containing only the ground state conformal weight or rather, its deviation from the required negative half-integer value required by p-adic mass calculations and made possible by the properties of Γ^n which is expected to be time-like as a vector. If this deviation depends on space-time surface, it can code for the value of Dirac determinant.

3. One can worry about the finiteness of the Dirac determinant: it seems that regularization is required and for vanishing ground state conformal weight one obtains zeta regularization. In the more general case Taylor expansion as function of deviation is suggestive.

1 Introduction

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the Kähler-Dirac action is an excellent candidate in this respect.

The original working hypothesis was that Dirac determinant defines the vacuum functional of the theory having interpretation as product of the exponent of Kähler function of world of classical worlds (WCW) identified as Kähler action coming from Euclidian space-time regions and the exponent of imaginary contribution identified as Kähler action from Minkowskian regions. It seems however that the most one can demand is that Dirac determinant equals to the exponent of

Kähler action. The reason is that Kähler-Dirac gamma matrices involving canonical momentum densities for Kähler action appear in modified (Kähler-Dirac) action.

1.1 What Are The Basic Equations Of Quantum TGD?

A good place to start is to as what might the basic equations of quantum TGD. There are two kinds of equations at the level of space-time surfaces.

1. Purely classical equations define the dynamics of the space-time sheets as preferred extremals of Kähler action. Preferred extremals are quantum critical in the sense that second variation vanishes for critical deformations representing zero modes. This condition guarantees that corresponding fermionic currents linear in deformations are conserved. There is infinite hierarchy of these currents and they define fermionic counterparts for zero modes.

   Zero energy ontology (ZEO) was motivated by the non-determinism of Kähler action suggesting that it difficult to assign unique preferred extremal to given 3-surface in positive energy ontology. In ZEO one can consider the possibility that the attribute "preferred" is not needed in given measurement resolution since the basic objects are now either pairs of space-like 3-surfaces at the ends of CD or these plus parton orbits (light-like 3-surfaces at which the signature of the induced metric changes).

2. The purely quantal equations are associated with the representations of various super-conformal algebras and with the Kähler-Dirac equation. The requirement that there are deformations of the space-time surface - actually infinite number of them - giving rise to conserved fermionic charges implies quantum criticality at the level of Kähler action in the sense of critical deformations.

3. The precise forms of Kähler action and Kähler Dirac equation at effective and real boundaries (boundary conditions) are not completely fixed without further input. For Kähler action the inputs are Lagrange multiplier terms at boundary like 3-surfaces expressing weak form of electric-magnetic duality and the equality of quantal and classical charges in Cartan algebra required by quantum classical correspondence (QCC). These states with well-defined classical charges might correspond to outcomes of state function reduction implying localization in WCW.

   The condition that fermionic propagator is non-trivial forces the addition of Chern-Simons Dirac term at the partonic orbits at which the signature of the induced metric changes. Supersymmetry requires the addition of Chern-Simons term at partonic orbits to Kähler action. This means explicit breaking of CP and T. The effective reduction of both Kähler and Kähler-Dirac equation to boundary terms means enormous calculational simplification and is consistent with the vision inspired by twistor approach [K13].

4. At the level of WCW spinor fields describing zero energy states quantal equations involve also generalized Feynman rules for M-matrix generalizing S-matrix to a "complex square root" of density matrix and defined by time-like entanglement coefficients between positive and negative energy parts of zero energy states is certainly the basic goal of quantum TGD.

5. The notion of weak electric-magnetic duality leads to a detailed understanding of how TGD reduces to almost topological quantum field theory. If Kähler current defines 4-D Beltrami flow, it is possible to find a gauge in which Coulomb contribution to Kähler action vanishes so that it reduces to Chern-Simons term. If light-like 3-surfaces and ends of space-time surface are extremals of Chern-Simons action also effective 2-dimensionality is realized. The condition that the theory reduces to almost topological QFT and the hydrodynamical character of field equations leads to a detailed ansatz for the general solution of field equations and also for the solutions of the modified Dirac equation relying on the notion of Beltrami flow for which the flow parameter associated with the flow lines defined by a conserved current extends to a global coordinate. This makes the theory is in well-defined sense completely integrable. Direct connection with massless theories emerges: every conserved Beltrami currents corresponds to a pair of scalar functions with the first one satisfying massless d'Alembert equation

in the induced metric. The orthogonality of the gradients of these functions allows interpretation in terms of polarization and momentum directions. The Beltrami flow property can be also seen as one aspect of quantum criticality since the conserved currents associated with critical deformations define this kind of pairs.

6. The hierarchy of Planck constants provides also a fresh view to the quantum criticality. The original justification for the hierarchy of Planck constants came from the indications that Planck constant could have large values in both astrophysical systems involving dark matter and also in biology. The realization of the hierarchy in terms of the singular coverings and possibly also factor spaces of CD and CP_2 emerged from consistency conditions. It however seems that TGD actually predicts this hierarchy of covering spaces. The extreme non-linearity of the field equations defined by Kähler action means that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many. This leads naturally to the introduction of the covering space of $CD \times CP_2$, where CD denotes causal diamond defined as intersection of future and past directed light-cones.

At the level of WCW there is the generalization of the Dirac equation, which can be regarded as a purely classical Dirac equation. The modified Dirac operators associated with quarks and leptons carry fermion number but the Dirac equations are well-defined. An orthogonal basis of solutions of these Dirac operators define in zero energy ontology a basis of zero energy states. The M-matrices defining entanglement between positive and negative energy parts of the zero energy state define what can be regarded as analogs of thermal S-matrices. The M-matrices associated with the solution basis of the WCW Dirac equation define by their orthogonality unitary U-matrix between zero energy states. This matrix finds the proper interpretation in TGD inspired theory of consciousness. WCW Dirac equation as the analog of super-Virasoro conditions for the "gamma fields" of superstring models defining super counterparts of Virasoro generators was the main focus during earlier period of quantum TGD but has not received so much attention lately and will not be discussed in this chapter.

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question mark is how to realize this coupling.

1. The proposal discussed in this chapter is that the addition of a measurement interaction term to the Kähler-Dirac action could do the job, solve a handful of problems of quantum TGD and unify various visions about the physics predicted by quantum TGD. This proposal implies QCC at the level of Kähler-Dirac action and Kähler action.

2. Another possibility is that QCC is realized at the level of WCW Dirac operator and Kähler-Dirac operator contains only interior term. The vanishing of the normal component of fermion current replaces Chern-Simons Dirac operator at various boundary like surfaces. I have proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

   QCC could be realized at the level of WCW by putting it in by hand. One can of course consider also the possibility that the equality of quantal and classical Cartan charges is realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system with Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD in zero energy ontology (ZEO) can be regarded as square root of thermodynamics, the procedure looks logically sound.

1.2 Kähler-Dirac Equation For Induced Classical Spinor Fields

The basic vision is that WCW geometry reduces to the second quantization of induced spinor fields. This means that WCW gamma matrices are linear combinations of fermionic oscillator operators and the Dirac determinant equals to vacuum functional of the theory. An unproven conjecture is that this determinant equals to the exponent of Kähler action for its preferred extremal.

The motivation for the Kähler-Dirac action came from the observation that the counterpart of the ordinary Dirac equation is internally consistent only if the space-time surfaces are minimal surfaces. One can however assign to any general coordinate invariant action principle for space-time surfaces a unique Kähler-Dirac action, which is internally consistent and super-symmetric. By quantum-classical correspondence space-time geometry must carry information about conserved quantum charges assignable to partonic 2-surfaces and it took considerable to to realize that this is achieved via measurement interaction terms realized as Lagrangian multiplier terms stating that classical conserved charges belonging to Cartan algebra are equal to their quantum counterparts for the space-time surfaces in quantum suerposition.

Second key idea [K16, K17] is that the well-definedness of em charge eigenvalue for spinor modes requires their localization to 2-D string world sheets and possibly also partonic 2-surfaces at which induced W boson field and possibly also Z^0 field vanish. Due to the presence of classical W boson fields this is possible only if localization takes plce at 2-D string world sheets and partonic 2-surfaces. Therefore string theory like structure emerges as part of TGD. The super Hamiltoanians defined in terms fluxes of Hammiltonians over partonic 2-surfaces are modified: a super-Hamiltonian at point of partonic 2-surface is replaced with an integral over stringy curve connecting points of two partonic 2-surfaces. Boundary conditions for the modes of induced spinor field can be interpreted as classical correlate for the stringy mass formula.

1.2.1 Preferred extremals as critical extremals

The study of the Kähler-Dirac equation leads to a detailed view about criticality. Quantum criticality [D2] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The discovery that the hierarchy of Planck constants realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [K6].

Criticality is accompanied by conformal invariance and this leads to the proposal that critical deformations correspond to Kac-Moody type conformal algebra respecting the light-likeness of the partonic orbits and acting trivially at partonic 2-surfaces. Sub-algebras of conformal algebras with conformal weights divisible by integer n would act as gauge symmetries and these algebras would form an inclusion hierarchy defining hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

1.2.2 Inclusion of the Chern-Simons Dirac term

Kähler action contains Chern-Simons term cancelling the Chern-Simons contribution of Kähler action at space-time interior at partonic orbit reducing to Chern-Simons terms so that only the contribution at space-like ends of space-time surface at the boundaries of causal diamond (CD) remains.

By supersymmetry also Kähler-Dirac action contains Chern-Simons Dirac term at partonic orbits implying non-trivial fermionic propagator at the boundaries of string world sheets at which the spinor modes are localized. The generalized eigenvalues $ip^k\gamma_k$ of C-S-D operator correspond to virtual four-momenta.

The inclusion of Chern-Simons term localized at partonic orbits to the definition of Kähler action and Chern-Simons-Dirac term to the definition Kähler-Dirac action at partonic orbits implies explicit breaking of CP and T. This term should explain the CP breaking associated with the CKM matrix of quarks.

1.3 Dirac Determinant As Exponent Of Kähler Action?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. An obvious guess is that Dirac determinant equals to the vacuum functional identified as exponent of Kähler function from Euclidian space-time regions and its its imaginary counterpart from Minkowskian space-time regions. This does not mean that Kähler-Dirac action would be alone enough as the original dream was. The reason is simple: Kähler-Dirac gamma matrices are defined in terms of canonical momentum densities of Kähler action.

1. The natural definition of Dirac determinant is as the product of the generalized eigenvalues. This product makes sense in Clifford algebra and by symmetries must be equal proportional to unit matrix. One can defined the product also as product of hyper-quaternionic numbers. The product contains natural IR cutoff posed by the size of the CD involved and UV cutoff defined by the size of the smalles sub-CD. The hypohtesis that the determinant equals to exponent of Kähler action forces its finiteness. Dirac determinant depends on string world sheet. For instance, if one poses periodic boundary conditions the generalized eigenvalues of C-S-D operator depend on the length of the fermion line measured using the metric defined by the anticommutators of C-S-D gamma matrices.

2. One can also add to Kähler action 3-D boundary terms defining measurement interaction. In particular, fixing the classical conserved charges of the space-time surfaces in the quantum superposition. Also Kähler-Dirac action contains measurement interaction term coming from these terms. In absence of measurement interaction terms Kähler-Dirac equation gives boundary term $\Gamma^n\Psi = 0$. This equation is satisfied if one has $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$ where p^k is light-like incoming four-momentum. Space-like boundaries correspond to on-mass-shell states and do not contribute to Dirac determinant.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found here [L2]. Another glossary type representation involving both pdf and html files can be found at `http://tgdtheory.fi/tgdglossary.pdf`. The topics relevant to this chapter are given by the following list.

- TGD as infinite-dimensional geometry [L5]
- WCW spinor fields [L6]
- KD equation [L4]
- Kaehler-Dirac action [L3]

2 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B2] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K3] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current. Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

2.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on Kähler-Dirac gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

2.1.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{2.1}$$

A more general form of this duality is suggested by the considerations of [K6] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{2.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} \ , \tag{2.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

2.1.2 Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1], [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{2.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{2.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{2.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{2.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

2.1.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K10] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{2.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

2.1.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{2.10}$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K11]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and

CP_2 are allowed as simplest possible solutions of field equations [K14]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

2.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

2.2.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

2.2.2 Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

2.2.3 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D1].

2.2.4 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K5]. The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies zero energy ontology. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^\pm$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^\pm$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy.

In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K7]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K8].

2.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.

2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $\hbar_0 \rightarrow r\hbar_0$ would effectively describe this. Boundary conditions would however give $1/r$ factor so that $\hbar$ would disappear from the Kähler function! The original attempt to realize quantum TGD as an almost topological QFT was in terms of Chern-Simons action but was given up. It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K2]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed

variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \quad . \tag{2.11}$$

The (1, 1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K (J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j^\alpha_K \partial_\alpha \phi = -j^\alpha A_\alpha \quad . \tag{2.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j^\alpha_K$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} = 0 \quad . \tag{2.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B4] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K2]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j^\alpha_I \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j^\alpha_K \phi$ and $j^\alpha_I \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations

$A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 . \quad (2.14)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with 1-D Dirac action in induced metric at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

9. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCW s associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

3 Some Attempts To Understand Preferred Extremals Of Kähler action

Preferred extremal of Kähler action has remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K13].

Preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states.

4 Handful Of Problems With A Common Resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose Kähler-Dirac action (or Kähler Dirac action as solution). After that I will describe the general structures of Kähler action and Kähler Dirac action. The non-trivial terms are associated to 3-D boundary like surfaces - that is ends of space-time surface inside CD and light-like 3-surfaces at which the signature of the induced metric changes. These terms are induced as Lagrange multiplier terms guaranteeing weak form of E-M duality and quantum classical correspondence (QCC) between classical and quantal Cartan charges. The condition guaranteeing that Chern-Simons Dirac propagator reduces to ordinary massless Dirac propagator must be however assumed as a property of the modes of Kähler Dirac equation rather than forced by a separate term in the Kähler-Dirac action as thought originally.

4.1 Why Kähler-Dirac Action?

4.1.1 Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of Kähler-Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K3, K12].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

4.1.2 Super-symmetry forces Kähler-Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T_k^\alpha &= 0 \ , \\ T_k^\alpha &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{4.1}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T_l^\alpha \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{4.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T_l^\alpha \Gamma^l \Psi \tag{4.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T_l^\alpha \Gamma^l D_\alpha \Psi \ . \tag{4.4}$$

The requirement that this current vanishes is guaranteed if one assumes that Kähler-Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T_l^\alpha \Gamma^l \ . \end{aligned} \tag{4.5}$$

This equation must be derivable from a Kähler-Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi} \hat{\Gamma}^\alpha D_\alpha \Psi \ . \tag{4.6}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \quad (4.7)$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

4.1.3 How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T_\mu^k T_{\nu k} \ . \quad (4.8)$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

4.1.4 Does the Kähler-Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [K12]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the Kähler-Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant equals to the exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X_l^3, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the symplectic currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-symplectic algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the Kähler-Dirac action so that there is no need to introduce this mixing by hand.

4.2 Overall View About Kähler Action And Kähler Dirac Action

In the following the most recent view about Kähler action and the Kähler-Dirac action (Kähler-Dirac action) is explained in more detail. The proposal is one of the many that I have considered.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action. The action could contain also Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term could be chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD). For Euclidian wormhole contacts Chern-Simons term need not reduce to a mere boundary terms since the gauge potential is not globally defined. One can also consider the possibility that only Minkowskian regions involve the Chern-Simons boundary term. One can also argue that Chern-Simons term is actually an un-necessary complication not needed in the recent interpretation of TGD.

   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

   The vanishing of conformal Noether charges for sub-algebras of various conformal algebras are also posed. They could be also realized as Lagrange multiplied terms at the ends of 3-surface.

2. By supersymmetry requirement the Kähler-Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with imbedding space gamma matrices to obtain K-D gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. As explained, it is assumed that localiztion to 2-D string world sheets occurs. At the light-like boundaries the limit of K-D equation gives K-D equation at the ferminonic liness expressing 8-D light-likeness or 4-D light-likeness in effective metric.

4.2.1 Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

4.2.2 Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess has been that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This is however a mere guess and need not be correct.

One should try to make first clear what one really wants.

1. What one wants are generalized Feynman diagrams demanding massless Dirac propagators in 8-D sense at the light-like boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that 8-D generalization of the twistor Grassmannian approach works. The localization of spinors at string world sheets is crucial for achieving this.

   In ordinary QFT fermionic propagator results from the kinetic term in Dirac action. Could the situation be same also now at the boundary of string world sheet associated with parton orbit? One can consider the Dirac action

   $$L_{ind} = \int \overline{\Psi}\Gamma^t_{ind}\partial_t\Psi\sqrt{g_1}dt$$

   defined by the induced gamma matrix Γ^t_{ind} and induced 1-metric. This action need to be associated only to the Minkowskian side of the space-surface. By supersymmetry Dirac action must be accompanied by a bosonic action $\int \sqrt{g_1}dt$. It forces the boundary line to be a geodesic line. Dirac equation gives

   $$\Gamma^t_{ind}D_t\Psi = ip^k(M^8)\gamma_k\Psi = 0 \ .$$

   The square of the Dirac operator gives $(\Gamma^t_{ind})^2 = 0$ for geodesic lines (the components of the second fundamental form vanish) so that one obtains 8-D light-likeness.

   Boundary line would behave like point-like elementary particle for which conserved 8-momentum is conserved and light-like: just as twistor diagrammatics suggests. 8-momentum must be real since otherwise the particle orbit would belong to the complexification of H. These conditions can be regarded as boundary conditions on the string world sheet and spinor modes. There would be no additional contribution to the Kähler action.

2. The special points are the ends of the fermion lines at incoming and outgoing partonic 2-surfaces and at these points M^4 mass squared is assigned to the imbedding space spinor harmonic associated with the incoming fermion. CP_2 mass squared corresponds to the eigenvalue of CP_2 spinor d'Alembertian for the spinor harmonic.

   At the end of the fermion line $p(M^4)^k$ corresponds to the incoming fermionic four-momentum. The direction of $p(E^4)^k$ is not fixed and one has $SO(4)$ harmonic at the mass shell $p(E^4)^2 = m^2$, m the mass of the incoming particle. At imbedding space level color partial waves correspond to $SO(4)$ partial waves ($SO(4)$ could be seen as the symmetry group of low energy hadron physics giving rise to vectorial and axial isospin).

4.2.3 Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

4.3 A Connection With Quantum Measurement Theory

It is encouraging that isometry charges and also other charges could make themselves visible in the geometry of space-time surface as they should by quantum classical correspondence. This suggests an interpretation in terms of quantum measurement theory.

1. The interpretation resolves the problem caused by the fact that the choice of the commuting isometry charges is not unique. Cartan algebra corresponds naturally to the measured observables. For instance, one could choose the Cartan algebra of Poincare group to consist of energy and momentum, angular momentum and boost (velocity) in particular direction as generators of the Cartan algebra of Poincare group. In fact, the choices of a preferred plane $M^2 \subset M^4$ and geodesic sphere $S^2 \subset CP_2$ allowing to fix the measurement sub-algebra to a high degree are implied by the replacement of the imbedding space with a book like structure forced by the hierarchy of Planck constants. Therefore the hierarchy of Planck constants seems to be required by quantum measurement theory. One cannot overemphasize the importance of this connection.

2. One can add similar couplings of the net values of the measured observables to the currents whose existence and conservation is guaranteed by quantum criticality. It is essential that one maps the observables to Cartan algebra coupled to critical current characterizing the observable in question. The coupling should have interpretation as a replacement of the induced Kähler gauge potential with its gauge transform. Quantum classical correspondence encourages the identification of the classical charges associated with Kähler action with quantal Cartan charges. This would support the interpretation in terms of a measurement interaction feeding information to classical space-time physics about the eigenvalues of the observables of the measured system. The resulting field equations remain second order partial differential equations since the second order partial derivatives appear only linearly in the added terms.

3. What about the space-time correlates of electro-weak charges? The earlier proposal explains this correlation in terms of the properties of quantum states: the coupling of electro-weak charges to Chern-Simons term could give the correlation in stationary phase approximation. It would be however very strange if the coupling of electro-weak charges with the geometry of the space-time sheet would not have the same universal description based on quantum measurement theory as isometry charges have.

   (a) The hint as how this description could be achieved comes from a long standing unanswered question motivated by the fact that electro-weak gauge group identifiable as the holonomy group of CP_2 can be identified as $U(2)$ subgroup of color group. Could the electro-weak charges be identified as classical color charges? This might make sense since the color charges have also identification as fermionic charges implied by quantum criticality. Or could electro-weak charges be only represented as classical color charges by mapping them to classical color currents in the measurement interaction term in the Kähler-Dirac action? At least this question might make sense.

   (b) It does not make sense to couple both electro-weak and color charges to the same fermion current. There are also other fundamental fermion currents which are conserved. All the following currents are conserved.

$$\begin{aligned} J^{\alpha} &= \overline{\Psi} O \hat{\Gamma}^{\alpha} \Psi \\ O &\in \{1 \ , \ J \equiv J_{kl}\Sigma^{kl} \ , \ \Sigma_{AB} \ , \ \Sigma_{AB} J\} \ . \end{aligned} \tag{4.9}$$

   Here J_{kl} is the covariantly constant CP_2 Kähler form and Σ_{AB} is the (also covariantly) constant sigma matrix of M^4 (flatness is absolutely essential).

   (c) Electromagnetic charge can be expressed as a linear combination of currents corresponding to $O = 1$ and $O = J$ and vectorial isospin current corresponds to J. It is natural to couple of electromagnetic charge to the the projection of Killing vector field of color hyper charge and coupling it to the current defined by $O_{em} = a + bJ$. This allows to interpret the puzzling finding that electromagnetic charge can be identified as anomalous color hyper-charge for induced spinor fields made already during the first years of TGD. There exist no conserved axial isospin currents in accordance with CVC and PCAC hypothesis which belong to the basic stuff of the hadron physics of old days.

(d) Color charges would couple naturally to lepton and quark number current and the $U(1)$ part of electro-weak charges to the $n = 1$ multiple of quark current and $n = 3$ multiple of the lepton current (note that leptons *resp.* quarks correspond to $t = 0$ *resp.* $t = \pm 1$ color partial waves). If electro-weak *resp.* couplings to H-chirality are proportional to 1 *resp.* Γ_9, the fermionic currents assigned to color and electro-weak charges can be regarded as independent. This explains why the possibility of both vectorial and axial couplings in 8-D sense does not imply the doubling of gauge bosons.

(e) There is also an infinite variety of conserved currents obtained as the quantum critical deformations of the basic fermion currents identified above. This would allow in principle to couple an arbitrary number of observables to the geometry of the space-time sheet by mapping them to Cartan algebras of Poincare and color group for a particular conserved quantum critical current. Quantum criticality would therefore make possible classical space-time correlates of observables necessary for quantum measurement theory.

(f) The coupling constants associated with the deformations would appear in the couplings. Quantum criticality ($K \rightarrow K + f + \overline{f}$ condition) should predict the spectrum of these couplings. In the case of momentum the coupling would be proportional to $\sqrt{G/\hbar_0} = kR/\hbar_0$ and $k \sim 2^{11}$ should follow from quantum criticality. p-Adic coupling constant evolution should follow from the dependence on the scale of CD coming as powers of 2.

4. Quantum criticality implies fluctuations in long length and time scales and it is not surprising that quantum criticality is needed to produce a correlation between quantal degrees of freedom and macroscopic degrees of freedom. Note that quantum classical correspondence can be regarded as an abstract form of entanglement induced by the entanglement between quantum charges Q_A and fermion number type charges assignable to zero modes.

5. Space-time sheets can have an arbitrary number of wormhole contacts so that the interpretation in terms of measurement theory coupling short and long length scales suggests that the measurement interaction terms are localizable at the wormhole throats. This would favor Chern-Simons term or possibly instanton term if reducible to Chern-Simons terms. The breaking of CP and T might relate to the fact that state function reductions performed in quantum measurements indeed induce dissipation and breaking of time reversal invariance.

   The formulation of quantum TGD in terms of the Kähler-Dirac action requires the addition of CP and T breaking Chern-Simons term and corresponding Chern-Simons Dirac term to partonic orbits such that it cancels the similar contribution coming from Kähler action. Chern-Simons Dirac term fixed by superconformal symmetry and gives rise to massless fermionic propagators at the boundaries of string world sheets. This seems to be a natural first principle explanation for the CP breaking as it manifests at the level of CKM matrix and perhaps also in breaking of matter antimatter asymmetry.

6. The experimental arrangement quite concretely splits the quantum state to a quantum superposition of space-time sheets such that each eigenstate of the measured observables in the superposition corresponds to different space-time sheet already before the realization of state function reduction. This relates interestingly to the question whether state function reduction really occurs or whether only a branching of wave function defined by WCW spinor field takes place as in multiverse interpretation in which different branches correspond to different observers. TGD inspired theory consciousness requires that state function reduction takes place. Maybe multiversalist might be able to find from this picture support for his own beliefs.

7. One can argue that "free will" appears not only at the level of quantum jumps but also as the possibility to select the observables appearing in the Kähler-Dirac action dictating in turn the Kähler function defining the Kähler metric of WCW representing the "laws of physics". This need not to be the case. The choice of CD fixes M^2 and the geodesic sphere S^2: this does

not fix completely the choice of the quantization axis but by isometry invariance rotations and color rotations do not affect Kähler function for given CD and for a given type of Cartan algebra. In M^4 degrees of freedom the possibility to select the observables in two manners corresponding to linear and cylindrical Minkowski coordinates could imply that the resulting Kähler functions are different. The corresponding Kähler metrics do not differ if the real parts of the Kähler functions associated with the two choices differ by a term $f(Z) + \overline{f(Z)}$, where Z denotes complex coordinates of WCW , the Kähler metric remains the same. The function f can depend also on zero modes. If this is the case then one can allow in given CD superpositions of WCW spinor fields for which the measurement interactions are different. This condition is expected to pose non-trivial constraints on the measurement action and quantize coupling parameters appearing in it.

4.4 How To Calculate Dirac Determinant?

If the modes of the Kähler-Dirac equation (or Kähler-Dirac equation) are localized to 2-D string world sheets as the well-definedness of em charge eigenvalue for the modes of induced spinor field strongly suggests, the definition of Dirac determinant could be rather simple as following argument shows.

The modes of Kähler-Dirac operator (Kähler-Dirac operator) are localized at string world sheets and are holomorphic spinors. K-D operator annihilates these modes so that Dirac determinant must be assigned with the 1-D Dirac operator associated with the induced metric at the light-like partonic orbits with vanishing metric determinant g_4.

The spectrum of light-like 8-momenta p^k is determined by the boundary conditions for 1-D Dirac operator at the ends of CD and periodic boundary conditions is one natural possibility. As in massless QFTs Dirac determinant could be identified as a square root of the product of - now 8-D - mass squared eigenvalues p^2. If the spectrum is unbounded, a regularization must be used. Finite measurement resolution means UV and IR cutoffs and would make Dirac determinant finite. Finite IR resolution would be due to the fact that only space-time surfaces within CD and thus having finite size scale are considered. UV resolution would be due to the lower limit on the size of sub-CDs.

One can however define Dirac determinant directly as the product of the generalized eigenvalues $p^k\gamma_k$ or as product of octonions in quaternionic sub-algebra defined by p^k. For

The full Dirac determinant would be product of Dirac determinants associated with various string world sheets. Needless to say that this is an enormous calculational advantage. If Dirac determinant identified in this manner reduces to exponent of Kähler action for preferred extremal this definition of Dirac determinant should give exponent of Kähler function reducing by weak form of electric-magnetic duality to exponent of Chern-Simons terms associated with the space-like ends of the space-time surface. Euclidian and Minkowskian regions would give contributions different by a phase factor $\sqrt{-1}$. The reduction of determinant to exponent of Chern-Simons terms would guarantee its finiteness.

Before trying to calculate Dirac determinant it is good to try to guess what the reduction to Chern Simons action could give as a result. This kind of guesses are of course highly speculative but nothing prevents from trying.

1. Chern Simons action to which Kähler action is expected to reduce for the preferred extremals should be expressible in terms of invariants associated with string world sheets. By the generalizaton of AdS/CFT duality the action in question should be proportional to the area of the string world sheet in the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices at string world sheet.

2. The arguent about finite measurement resolution can be of course criticized. An alternative argument relies on idea that the sum over logariths of eigenvalues reduces to integral using as measure the transversal induced Kähler form J_T and the magnetic flux J over string world sheet. This conforms with the existence of slicing by string world sheets labelled by points of partonic 2-surface.

5 Quantum Criticality And Kähler-Dirac Action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The belief has been that the existence of conserved current for Kähler-Dirac equation are possible if Kähler action is critical for the 3-surface in question in the sense that the deformation in question corresponds to vanishing of second variation of Kähler action. The vanishing of the second variation states that the deformation of the Kähler-Dirac gamma matrix is divergence free just like the Kähler-Dirac gamma matrix itself and is therefore very natural.

2-D conformal invariance accompanies 2-D criticality and allows to satisfy these conditions for spinor modes localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This localization is in the generic case forced by the conditions that em charge is well-defined for the spinor modes: this requires that classical W fields vanish and also the vanishing of classical Z^0 field is natural -at least above weak scale. Only 2 Kähler-Dirac gamma matrices can be non-vanishing and this is possible only for Kähler-Dirac action.

5.1 What Quantum Criticality Could Mean?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to correspond to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

   (b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A2]. Cusp catastrophe [A1] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

3. Quantum criticality makes sense also for Kähler action.

   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K4] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

(b) Also now one expects a hierarchy of criticalities and and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

(c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. I have discussed what criticality could mean for Kähler-Dirac action [K15].

(a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

(b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in Kähler-Dirac action. Kähler-Dirac equation is satisfied if the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. One obtains conserved fermion current associated with deformations only if the deformation of the Kähler-Dirac gamma matrix is divergenceless just like the Kähler-Dirac gamma matrix itself. This conditions requires the vanishing of the second variation of Kähler action.

(c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the Kähler-Dirac gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor.

This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as a matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteeing vanishing of classical W fields can be satisfied without the restriction to 2-surface.

The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

5.2 Quantum Criticality And Fermionic Representation Of Conserved Charges Associated With Second Variations Of Kähler Action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been however slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. The problem is that the imbedding space coordinates are in the role of classical external fields and induces spinor fields are second quantized so that it is not at all clear whether one obtains conserved charges.

5.2.1 What does the conservation of the fermionic Noether current require?

The obvious anser to the question of the title is that the conservation of the fermionic current requires the vanishing of the first variation of Kähler-Dirac action with respect to imbedding space coordinates. This is certainly true but need not mean vanishing of the second variation of Kähler action as thought first. Hence fermionic conserved currents might be obtained for much more general variations than critical ones.

1. The Kähler-Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the Kähler-Dirac action under this deformation vanishes.

   The vanishing of the first variation for the Kähler-Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the Kähler-Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{5.1}$$

   Here h^k_β denote partial derivative of the imbedding space coordinates with respect to space-time coordinates. ΔS_D vanishes if this term vanishes:

$$D_\alpha J^\alpha_k = 0 \ .$$

   The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J^α_k does not define conserved classical charge in the general case.

2. This condition is however un-necessarily strong. It is enough that that the deformation of Dirac operator anihilates the spinor mode, which can also change in the deformation. It must be possible to compensate the change of the covariant derivative in the deformation by a gauge transformation which requires that deformations act as gauge transformations on induce gauge potentials. This gives additional constraint and strongly suggests Kac-Moody type algebra for the deformations. Conformal transformations would satisfy this constraint and are suggested by quantum criticality.

3. It is essential that the Kähler-Dirac equation holds true so that the Kähler-Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the Kähler-Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{5.2}$$

Here $1/D$ is the inverse of the Kähler-Dirac operator defining the counterpart of the fermionic propagator.

4. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{5.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the Kähler-Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing Kähler-Dirac gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \tag{5.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra.

5. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the Kähler-Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

6. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

It is far from obvious that the criticality conditions or even the weaker conditions guaranteing the existence of (say) isometry charges can be satisfied. It seems that the restriction of spinor modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - implied by the condition that em charge is well-define for them, is the manner to achieve this. The reason is that conformal invariance allows complexification of the Kähler-Dirac gamma matrices and allows to construct spinor modes as holomorphic modes and their conjugates. Holomorphy reduces K-D equation to algebraic condition that Γ^z annihilates the spinor mode. If this is true also the deformation of Γ^z then the existince of conserved current follows. It is essential that only two Kähler-Dirac gamma matrices are non-vanishing and this is possible only for Kähler-Dirac action.

5.2.2 About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding second order charges for Kähler action are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates.

2. Contrary to the original conclusion, the corresponding fermionic charges expressible as fermionic bilinears are first order in deformation and do not vanish! Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities and for Kähler-Dirac action as quantal charges.

5.2.3 Critical manifold is infinite-dimensional for Kähler action

Some examples might help to understand what is involved.

1. The action defined by four-volume gives a first glimpse about what one can expect. In this case Kähler-Dirac gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

2. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical imbedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a non-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs_k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

3. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

4. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical imbedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

This leads to the conjecture that the critical deformations correspond to sub-algebras of super-conformal algebras with conformal weights coming as integer multiples of fixed integer m. One would have infinite hierarchy of breakings of conformal symmetry labelled by m. The super-conformal algebras would be effectively m-dimensional. Since all commutators with the critical sub-algebra would create zero energy states. In ordinary conformal field theory one have maximal criticality corresponding to $m = 1$.

5.2.4 Critical super-algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the WCW metric.

   The original expectation was that critical deformations correspond to zero modes but this interpretation need not be correct since critical deformations can leave 3-surface invariant but affect corresponding preferred extremal: this would conform with the non-deterministic character of the dynamics which is indeed the basic signature of criticality. Rather, critical deformations are limiting cases of ordinary deformations acting in quantum fluctuating degrees of freedom.

   This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4 and that Kähler action has fourth order terms as first non-vanishing terms in perturbative expansion (for Kähler-Dirac the expansion is quadratic in deformation).

   Therefore the super-conformal algebra associated with the critical deformations has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

5.2.5 Connection with quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question how to make TGD unique if Kähler function for WCW is defined by the Kähler action for a preferred extremal assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obviou idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength is analogous to critical temperature so that TGD would be more or less uniquely defined.

To understand the delicacies it is convenient to consider various variations of Kähler action first.

1. The variation can leave 3-surface invariant but modify space-time surface such that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface and perhaps also at light-like 3-surfaces. In this case the correspondence between X^3 and $X^4(X^3)$ would not be unique and one would have non-deterministic dynamics characteristic for critical systems. This criticality would correspond to criticality of Kähler action at X^3. Note that the original working hypothesis was that $X^4(X^3)$ is unique. The failure of the strict classical determinism implying spin glass type vacuum degeneracy indeed suggets that this is the case.

2. The variation could act on zero modes which do not affect Kähler metric which corresponds to (1, 1) part of Hessian in complex coordinates for WCW . Only the zero modes characterizing 3-surface appearing as parameters in the metric WCW would be affected and the result would be a generalization of conformal transformation. Kähler function would change but only due to the change in zero modes. These transformations do not seem to correspond to critical transformations since Kähler function changes.

3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would of course affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that (1, 1) part of Hessian is degenerate. This could mean that in the vicinity of X^3 the Kähler form has non-definite signature: physically this is unacceptable since inner product in Hilbert space would not be positive definite.

Critical transformations might relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

1. The critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical transformations then critical transformation would extend WCW to a bundle for which 3-surfaces would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.

2. Critical deformations could correspond to H or sub-group of H (which dependes on X^3). For other 3-surfaces than X^3 the action of H is non-trivial as the case of $CP_2 = SU(3)/U(2)$ makes easy to understand.

3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies of a given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1. For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. Quantum criticality realized as the vanishing of the second variation gives hopes about a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to deformations of four-surface and second for the Kähler function itself with respect to deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

1. The criticality for preferred extremals would make 4-D criticality a property of all physical systems.

2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1, 1) part of Hessian in complex coordinates must be positive definite. Thus criticality might imply problems.

   This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no zero modes causing troubles in Gaussian approximation to functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function.

3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW . These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interaction seems to be that these degrees of freedom correspond to deformations in zero modes.

Both the super-symmetry of D_K and conservation Dirac Noether currents for Kähler-Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$.

   The formulation of quantal version of Equivalence Principle (EP) in string picture demonstrates that the conservation of of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires that the deformation of the Kähler-Dirac equation obtained by replacing Kähler-Dirac gamma matrices with their deformations is also satisfied. Holomorphy can guarantee this. The original wrong conclusion was that this condition is equivalent with much stronger condition stating the vanishing of the second variation of Kähler action, which it is not. There is analogy for this: massless Dirac equation does not imply the vanishing of four-momentum.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the imbedding space define conserved currents represented in terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the Kähler-Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K4] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere S^12_{II} is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

5.3 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [B3] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that

classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

5.4 Quantum Criticality And Electroweak Symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

5.4.1 What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the imbedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Khler function or Kähler action.

   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

   (b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A2]. Cusp catastrophe [A1] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

3. Quantum criticality makes sense also for Kähler action.

   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K4] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

(b) Also now one expects a hierarchy of criticalities and and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

(c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. I have discussed what criticality could mean for Kähler-Dirac action [K15].

   (a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

   (b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since imbedding space coordinates appear as parameters in Kähler-Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the imbedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

   (c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

5.4.2 The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed Kähler-Dirac operator D annihilates the modified mode. By writing explicitly the variation of the Kähler-Dirac action (the action vanishes by Kähler-Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi . \quad (5.5)$$

D^{-1} is the inverse of the Kähler-Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k \delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence [K9] these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

5.4.3 What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the Kähler-Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: Kähler-Dirac gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^\mu = \Lambda(x)\Gamma^\mu . \quad (5.6)$$

This guarantees that the Kähler-Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_μ is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^μ at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^\mu, \Gamma^\nu\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Kähler-Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2)\times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^\mu = \overline{\Psi}\Gamma^\mu \delta_i \Psi + \delta_i \overline{\Psi}\Gamma^\mu \Psi \ . \tag{5.7}$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ. As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

5.4.4 What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

1. Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing

conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

2. The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

3. The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

   The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n \ mod \ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n \ mod \ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

   A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

   Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

5.5 The Emergence Of Yangian Symmetry And Gauge Potentials As Duals Of Kac-Moody Currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B5]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \ . \tag{5.8}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative manners to obtain the conservation.

1. The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{5.9}$$

2. The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC} \int_{-\infty}^{\infty} dx \int_x^{\infty} dy j^{B0}(x,t) j^{C0}(y,t) - 2 \int_{-\infty}^{\infty} j^A_x dx \ . \tag{5.10}$$

   These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

1. The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Kähler-Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

2. An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential. This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

   This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [K5].

3. The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \tag{5.11}$$

   so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

4. It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC} \sum_{i<j} J_i^B J_j^C \ . \tag{5.12}$$

   What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

5. This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the Kähler-Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f^{AB}_C J^C \ , \ \left[J^A, Q^B\right] = f^{AB}_C Q^C \ . \tag{5.13}$$

plus the rather complex Serre relations described in [B5].

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