Extremals of the Kähler Action

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Contents

Abstract

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the "world of classical worlds" (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [**?**] It is not clear whether criticality and associativeity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

   Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute "preferred" is not relevant for them.

3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.

4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.

5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

1 Introduction

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable. The progress made during next five years led to a detailed understanding of quantum TGD at the fundamental parton level and this provides considerable additional insights concerning the interpretation of field equations.

1.1 About The Notion Of Preferred Extremal

The notion of preferred extremal has been central in classical TGD although the known solutions could be preferred or not: the main challenge has been to understand what "preferred" could mean.

In zero energy ontology (ZEO) one can also consider the releaving possibility that all extremals are preferred ones! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. The non-determinism of Kähler action suggests that "preferred" could be obsolete in given length scale resolution.

Actually all the discussions of this chapter are about known extremals in general so that the attribute "preferred" is not relevant for them.

1.2 Beltrami Fields And Extremals

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that preferred extremals satisfy the condition. The vanishing of the Lorentz 4-force in turn implies a local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. If Kähler action is defined by CP_2 Kähler form alone, the condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

An important outcome is the notion of Hamilton-Jacobi structure meaning dual slicings of M^4 projection of preferred extremals to string world sheets and partonic 2-surfaces. The necessity of this slicing was discovered years later from number theoretic compactification and is now a key element of quantum TGD allowing to deduce Equivalence Principle in its stringy form from quantum TGD and formulate and understand quantum TGD in terms of Kähler-Dirac action assignable to Kähler action. The conservation of Noether charges associated with Kähler-Dirac action requires the vanishing of the second second variation of Kähler action for preferred extremals. Preferred extremals would thus define space-time representation for quantum criticality. Infinite-dimensional variant for the hierarchy of criticalities analogous to the hierarchy assigned to the extrema of potential function with levels labeled by the rank of the matrix defined by the second

derivatives of the potential function in catastrophe theory would suggest itself.

A natural interpretation for deformations would be as conformal gauge symmetries due to the non-determinism of Kähler action. They would transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They would preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

1.3   In What Sense Field Equations Could Mimic Dissipative Dynamics?

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. The nontrivial question is what this means in TGD framework.

1. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property of Kähler action abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. The general solution ansatz discussed in the last section of the chapter assumes that all conserved isometry currents are proportional to instanton current so that various charges are conserved separately for all flow lines: this means essentially the integrability of the theory. This ansatz is forced by the hypothesis that TGD reduces to almost topological QFT and this idea. The basic consequence is that dissipation is impossible classically.

2. A more radical view inspired by zero energy ontology is that the light-like 3-surfaces and corresponding space-time regions with Euclidian signature defining generalized Feynman diagrams provide a space-time representation of dissipative dynamics just as they provide this representation in quantum field theory. Minkowskian regions would represent empty space so that the vanishing of Lorentz 4-force and absence of dissipation would be natural. This would mean very precise particle field duality and the topological pattern associated with the generalized Feynman diagram would represent dissipation. One could also interpret dissipation as transfer of energy between sheets of the many-sheeted space time and thus as an essentially topological phenomenon. This option seems to be the only viable one.

1.4   The Dimension Of CP_2 Projection As Classifier For The Fundamental Phases Of Matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. The natural guess would be that this phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. It seems however that preferred extremals can correspond only to small perturbations of these extremals resulting by topological condensation of CP_2 type vacuum extremals and through topological condensation to larger space-time sheets. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase could be seen as the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

The original proposal was that $D(CP_2) = 4$ phase is completely chaotic. This is not true if the reduction to almost topological QFT takes place. This phase must correspond to Maxwellian phase with a vanishing Kähler current as concluded already earlier. Various isometry currents are however proportional to the instanton current and conserved along the flow lines of the instanton current whose flow parameter extends to a global coordinate. Hence a completely chaotic phase is not in question even in this case.

1.5   Specific Extremals Of Kähler Action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2 \# CP_2 \# ... CP_2$ are identified as the generalized Feynman diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.

   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

   (b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

   (c) In the so called Maxwell phase, ordinary Maxwell equations for the induced Kähler field would be satisfied in an excellent approximation. It is however far from clear whether this kind of extremals exist. Their non-existence would actually simplify the theory enormously since all extremals would have quantal character. The recent view indeed is that Maxwell phase makes sense only as as genuinely many-sheeted structure and solutions of Maxwell's equation appear only at the level of effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Gauge potentials in effective space-time are determined in the same way. Since the gauge potentials sum up, it is possible to understand how field configurations of Maxwell's theory emerge at this limit.

1.6   The Weak Form Of Electric-Magnetic Duality And Modification Of Kähler Action

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

Generalized Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers

are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector and there are reasons to hope that TGD is completely integrable theory.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L1].

2 General Considerations

The solution families of field equations studied in this chapter were found already during eighties. The physical interpretation turned out to be the really tough problem. What is the principle selecting preferred extremals of Kähler action as analogs of Bohr orbits assigning to 3-surface X^3 a unique space-time surface $X^4(X^3)$? Does Equivalence Principle hold true and if so, in what sense? These have been the key questions. The realization that light-like 3-surfaces X_l^3 associated with the light-like wormhole throats at which the signature of the induced metric changes from Minkowskian to Euclidian led to the formulation of quantum TGD in terms of second quantized induced spinor fields at these surfaces. Together with the notion of number theoretical compactification this approach allowed to identify the conditions characterizing the preferred extremals. What is remarkable that these conditions are consistent with what is known about extremals.

Also a connection with string models emerges and partial understanding of the space-time realization of Equivalence Principle suggests itself. However, much more general argument allows to understand how GRT space-time appears from the many-sheeted space-time of TGD (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) as effective concept [K13]: this more general view is not in conflict with the much earlier proposal discussed below.

In this section the theoretical background behind field equations is briefly summarized. I will not repeat the discussion of previous two chapters [?, K7] summarizing the general vision about many-sheeted space-time, and consideration will be restricted to those aspects of vision leading to direct predictions about the properties of preferred extremals of Kähler action.

2.1 Number Theoretical Compactification And $M^8 - H$ Duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4 \subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naïve and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

1. The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X_l^3 (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X_l^3 is 1-dimensional. The surfaces $X^4(X_l^3) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.

2. One can also consider a more local map of $X^4(X_l^3) \subset H$ to $X^4(X_l^3) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes

the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X_l^3) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X_l^3 and represented by $X^4(X_l^3) \subset H$. Space-time surfaces $X^4(X_l^3) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of WCW . The possibility to assign $M^2(x) \subset M^4$ to each point of M^4 projection $P_{M^4}(X^4(X_l^3))$ is consistent with what is known about extremals of Kähler action with only one exception: CP_2 type vacuum extremals. In this case M^2 can be assigned to the normal space.

3. Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X_l^3) \subset M^8$ and $X^4(X_l^3) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.

4. The map of $X_l^3 \subset H \to X_l^3 \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of embedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.

5. The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the $M^8 - H$ duality.

Number theoretical compactification has quite deep implications for quantum TGD and is actually responsible for most of the progress in the understanding of the mathematical structure of quantum TGD. A very powerful prediction is that preferred extremals should allow slicings to either stringy world sheets or dual partonic 2-surfaces as well as slicing by light-like 3-surfaces. Both predictions are consistent with what is known about extremals.

1. If the distribution of planes $M^2(x)$ is integrable, it is possible to slice $X^4(X^3)$ to a union of 2-dimensional surfaces having interpretation as string world sheets and dual 2-dimensional copies of partonic surfaces X^2. This decomposition defining 2+2 Kaluza-Klein type structure could realize quantum gravitational holography and might allow to understand Equivalence Principle at space-time level in the sense that dimensional reduction defined by the integral of Kähler action over the 2-dimensional space labeling stringy world sheets gives rise to the analog of stringy action and one obtains string model like description of quantum TGD as dual for a description based on light-like partonic 3-surfaces. String tension is not however equal to the inverse of gravitational constant as one might naïvely expect but the connection is more delicate. As already mentioned, TGD-GRT connection and EP can be understood at general level only from very general arguments [K13].

2. Second implication is the slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 "parallel" to X_l^3. Also this slicing realizes quantum gravitational holography if one requires General Coordinate Invariance in the sense that the Dirac determinant differs for two 3-surfaces Y_l^3 in the slicing only by an exponent of a real part of a holomorphic function of WCW complex coordinates giving no contribution to the Kähler metric.

3. The square of the Dirac determinant would be equal to the modulus squared for the exponent of vacuum functional and would be formally defined as the product of conformal weights assignable to the modes of the Dirac operator at string world sheets at the ends of strings at partonic 2-surfaces defining the ends of Y_l^3. The detailed definition requires to specify

what one means with the conformal weights assignable with the modes of the Kähler-Dirac operator.

4. The localization of the modes of Kähler-Dirac operator to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) [K16] following from the condition that electromagnetic charges of the modes is well-defined is very strong restriction and reduces Dirac determinant to a product of Dirac determinants assignable with these 2-surfaces.

2.2 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries. The natural identification would be as conformal symmetries. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K / \partial h_\alpha^k \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K / \partial h_\alpha^k \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

For instance, the natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig. http://tgdtheory.fi/appfigures/ planckhierarchy.jpg** or **Fig. ??**in the appendix of this book).

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X_l^3)$ vanishing at the intersections of $X^4(X_l^3)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X_l^3 with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This

would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \to X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X_l^3)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X_l^3 involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

The basic question is whether number theoretic view about preferred extremals imply absolute minimization or something analogous to it.

1. The number theoretic conditions defining preferred extremals are purely algebraic and make sense also p-adically and this is enough since p-adic variants of field equations make sense although the notion of Kähler action does not make sense as integral. Despite this the identification of the vacuum functional as exponent of Kähler function as Dirac determinant allows to define the exponent of Kähler function as a p-adic number [K16].

2. The general objection against all extremization principles is that they do not make sense p-adically since p-adic numbers are not well-ordered.

3. These observations do not encourage the idea about equivalence of the two approaches. On the other hand, real and p-adic sectors are related by algebraic continuation and it could be quite enough if the equivalence were true in real context alone.

The finite-dimensional analogy allows to compare absolute minimization and criticality with each other.

1. Absolute minimization would select the branch of Thom's catastrophe surface with the smallest value of potential function for given values of control variables. In general this value would not correspond to criticality since absolute minimization says nothing about the values of control variables (zero modes).

2. Criticality forces the space-time surface to belong to the bifurcation set and thus fixes the values of control variables, that is the interior of 3-surface assignable to the partonic 2-surface, and realized holography. If the catastrophe has more than $N = 3$ sheets, several preferred extremals are possible for given values of control variables fixing $X^3(X^2)$ unless one assumes that absolute minimization or some other criterion is applied in the bifurcation set. In this sense absolute minimization might make sense in the real context and if the selection is between finite number of alternatives is in question, it should be possible carry out the selection in number theoretically universal manner.

It must be emphasized that there are several proposals for what preferred extremal property could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of embedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of embedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K3] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K3].

2.3 Can One Determine Experimentally The Shape Of The Space-Time Surface?

The question "Can one determine experimentally the shape of the space-time surface?" does not relate directly to the topic of this chapter in technical sense, and the only excuse for its inclusion is the title of this section plus the fact that the general conceptual framework behind quantum TGD assumes an affirmative answer to this question. If physics were purely classical physics, operationalism in the strong sense of the word would require that one can experimentally determine the shape of the space-time as a surface of the embedding space with arbitrary accuracy by measuring suitable classical observables. In quantum physics situation is considerably more complex and quantum effects are both a blessing and a curse.

2.3.1 Measuring classically the shape of the space-time surface

Consider first the purely classical situation to see what is involved.

1. All classical gauge fields are expressible in terms of CP_2 coordinates and their space-time gradients so that the measurement of four field quantities with some finite resolution in some space-time volume could in principle give enough information to deduce the remaining field quantities. The requirement that space-time surface corresponds to an extremal of Kähler action gives a further strong consistency constraint and one can in principle test whether this constraint is satisfied. A highly over-determined system is in question.

2. The freedom to choose the space-time coordinates freely causes complications and it seems that one must be able to determine also the distances between the points at which the field quantities are determined. At purely classical Riemannian level this boils down to the measurement of the induced metric defining classical gravitational field. In macroscopic length scales one could base the approach to iterative procedure in which one starts from the assumption that the coordinates used are Minkowski coordinates and gravitational corrections are very weak.

3. The measurement of induced Kähler form in some space-time volume determines space-time surface only modulo canonical transformations of CP_2 and isometries of the embedding space. If one measures classical electromagnetic field, which is not canonical invariant in general case, with some precision, one can determine to what kind of surface space-time region corresponds apart from the action of the isometries of H.

2.3.2 Quantum measurement of the shape of the space-time surface

In practice the measurement of the shape of the space-time surface is necessarily a bootstrap procedure based on the model for space-time region and on the requirement of internal consistency. Many-sheeted space-time and quantum phenomena produce considerable complications but also provide universal measurement standards.

Consider first how quantum effects could help to measure classical fields and distances.

1. The measurement of distances by measuring first induced metric at each point of space-time sheet is rather unpractical procedure. Many-sheeted space-time however comes in rescue here. p-Adic length scale hypothesis provides a hierarchy of natural length scales and one can use p-adic length and time scales as natural units of length and time: space-time sheets serve as meter sticks. For instance, length measurement reduces in principle to a finite number of operations using various space-time sheets with standardized lengths given by p-adic length scales. Also various transition frequencies and corresponding wavelengths provide universal time and length units. Atomic clock provides a standard example of this kind of time unit. A highly nontrivial implication is the possibility to deduce the composition of distant star from its spectral lines. Without p-adic length scale hypothesis the scales for the mass spectra of the elementary particles would be variable and atomic spectra would vary from point to point in TGD universe.

   Do the p-adic length scales correspond to the length units of the induced metric or of M_+^4 metric? If the topological condensation a meter stick space-time sheet at a larger space-time

sheet does not stretch the meter stick but only bends it, the length topologically condensed meter stick in the induced metric equals to its original length measured using M_+^4 metric.

2. If superconducting order parameters are expressible in terms of the CP_2 coordinates (there is evidence for this, see the chapter "Macroscopic quantum phenomena and CP_2 geometry"), one might determine directly the CP_2 coordinates as functions of Minkowski coordinates and this would allow to estimate all classical fields directly and thus to deduce strong consistency constraints.

3. At quantum level only the fluxes of the classical fields through surface areas with some minimum size determined by the length scale resolution can be measured. In case of magnetic fields the quantization of the magnetic flux simplifies the situation dramatically. Topological field quantization quite generally modifies the measurement of continuous field variables to the measurement of fluxes. Interestingly, the construction of WCW geometry uses as WCW coordinates various electric and magnetic fluxes over 2-dimensional cross sections of 3-surface.

Quantum effects introduce also difficulties and restrictions.

1. Canonical transformations localized with respect to the boundary of the light cone or more general light like surfaces act as isometries of WCW and one can determine the space-time surface only modulo these isometries. Even more, only the values of the non-quantum fluctuating zero modes characterizing the shape and size of the space-time surface are measurable with arbitrary precision in quantum theory. At the level of conscious experience quantum fluctuating degrees of freedom correspond to sensory qualia like color having no classical geometric content.

2. Space-time surface is replaced by a new one in each quantum jump (or rather the superposition of perceptively equivalent space-time surfaces). Only in the approximation that the change of the space-time region in single quantum jump is negligible, the measurement of the shape of space-time surface makes sense. The physical criterion for this is that dissipation is negligible. The change of the space-time region in single quantum jump can indeed be negligible if the measurement is performed with a finite resolution.

3. Conscious experience of self is an average over quantum jumps defining moments of consciousness. In particular, only the average increment of the zero modes is experienced and this means that one cannot fix the space-time surface apart from canonical transformation affecting the zero modes. Again the notion of measurement resolution comes in rescue.

4. The possibility of coherent states of photons and gravitons brings in a further quantum complication since the effective classical em and gravitational fields are superpositions of classical field and the order parameter describing the coherent state. In principle the extremely strong constraints between the classical field quantities allow to measure both the order parameters of the coherent phases and classical fields.

2.3.3 Quantum holography and the shape of the space-time surface

If the Dirac determinant asssognable to the mass squared eigenvalue spectrum of the Kähler-Dirac operator $D_K(X^2)$ equals to the exponent of Kähler action of a preferred extremal, it is fair to say that a lot of information about the shape of the space-time surface is coded to physical observables, which eigenvalues indeed represent. Quantum gravitational holography due to the Bohr orbit like character of space-time surface reduces the amount of information needed. Only a finite number of eigenvalues is involved and the eigen modes are associated with the 3-D light-like wormhole throats rather than with the space-time surface itself. If the eigenvalues were known or could be measured with infinite accuracy, one could in principle fix the boundary conditions at X_l^3 and solve field equations determining the preferred extremal of Kähler action.

What is of course needed is the complete knowledge of the light-like 3-surfaces X_l^3. Needless to say, in practice a complete knowledge of X_l^3 is impossible since measurement resolution is finite. The notion number theoretic braid provides a precise realization for the finite measurement accuracy at space-time level. At the level of WCW spinors fields (world of classical worlds) just

the fact that the number of eigenvalues is finite is correlate for the finite measurement accuracy. Furthermore, quantum states are actually quantum superpositions of 3-surfaces, which means that one can only speak about quantum average space-time surface for which the phase factors coding for the quantum numbers of elementary particles assigned to the strands of number theoretic braids are stationary so that correlation of classical gauge charges with quantum gauge charges is obtained.

3 The Vanishing Of Super-Conformal Charges As Gauge Conditions Selecting Preferred Extremals Of Kähler Action

Classical TGD [K3] involves several key questions waiting for clearcut answers.

1. The notion of preferred extremal emerges naturally in positive energy ontology, where Kähler metric assigns a unique (apart from gauge symmetries) preferred extremal to given 3-surface at M^4 time= constant section of embedding space $H = M^4 \times CP_2$. This would quantize the initial values of the time derivatives of embedding coordinates and this could correspond to the Bohr orbitology in quantum mechanics.

2. In zero energy ontology (ZEO) initial conditions are replaced by boundary conditions. One fixes only the 3-surfaces at the opposite boundaries of CD and in an ideal situation there would exist a unique space-time surface connecting them. One must however notice that the existence of light-like wormhole throat orbits at which the signature of the induced metric changes $(det(g_4) = 0)$ its signature might change the situation. Does the attribute "preferred" become obsolete and does one lose the beautiful Bohr orbitology, which looks intuitively compelling and would realize quantum classical correspondence?

3. Intuitively it has become clear that the generalization of super-conformal symmetries by replacing 2-D manifold with metrically 2-D but topologically 3-D light-like boundary of causal diamond makes sense. Generalized super-conformal symmetries should apply also to the wormhole throat orbits which are also metrically 2-D and for which conformal symmetries respect $detg(g_4) = 0$ condition. Quantum classical correspondence demands that the generalized super-conformal invariance has a classical counterpart. How could this classical counterpart be realized?

4. Holography is one key aspect of TGD and mean that 3-surfaces dictate everything. In positive energy ontology the content of this statement would be rather obvious and reduce to Bohr orbitology but in ZEO situation is different. On the other hand, TGD strongly suggests strong form of holography based stating that partonic 2-surfaces (the ends of wormhole throat orbits at boundaries of CD) and tangent space data at them code for quantum physics of TGD. General coordinate invariance would be realized in strong sense: one could formulate the theory either in terms of space-like 3-surfaces at the ends of CD or in terms of light-like wormhole throat orbits. This would realize Bohr orbitology also in ZEO by reducing the boundary conditions to those at partonic 2-surfaces. How to realize this explicitly at the level of field equations? This has been the challenge.

Answering questions is extremely useful activity. During last years my friend Hamed has posed continually questions related to the basic TGD. At this time Hamed asked about the derivation of field equations of TGD. In "simple" field theories involving some polynomial non-linearities the deduction of field equations is of course totally trivial process but in the extremely non-linear geometric framework of TGD situation is quite different.

While answering the questions I ended up with the following question. Could one assume that the variations at the light-like boundaries of CD vanish for all conformal variations, which are not isometries. For isometries the contributions from the ends of CD cancel each other automatically so that the corresponding variations need not vanish separately at boundaries of CD! This is extremely simple and profound fact. This would be nothing but the realisation of the analogs of

conformal symmetries classically and give precise content for the notion of preferred external, Bohr orbitology, and strong form of holography. And the condition makes sense only in ZEO!

I attach below the answers to the questions of Hamed almost as such apart from slight editing and little additions, re-organization, and correction of typos.

3.1   Field Equations For Kähler Action

My friend Hamed made some questions relating to the derivation of field equations for the extremals of Kähler action, which led to the recent progress. I comment first these questions since they lead naturally to the basic new idea.

The addition of the volume term implied by the twistor lift of TGD [L6, L7], having interpretation in terms of cosmological constant, adds to the field equations only a term proportional to $D_\beta g^{alpha\beta}\partial_\beta h^k$. There are excellent reasons to believe that solutions of field equations representing preferred extremals as analogs of Bohr orbits are actually minimal surfaces except at singular surfaces of dimension $D > 4$. One might speak of 4-D analogies of soap films spanned by frames.

3.1.1   The physical interpretation of the canonical momentum current

Hamed asked about the physical meaning of $T_k^n \equiv \partial L/\partial(\partial_n h^k)$ - normal components of canonical momentum labelled by the label k of embedding space coordinates - it is good to start from the physical meaning of a more general vector field

$$T_k^\alpha \equiv \frac{\partial L}{\partial(\partial_\alpha h^k)}$$

with both embedding space indices k and space-time indices α - canonical momentum currents. L refers to Kähler action.

1. One can start from the analogy with Newton's equations derived from action principle (Lagrangian). Now the analogs are the partial derivatives $\partial L/\partial(dx^k/dt)$. For a particle in potential one obtains just the momentum. Therefore the term canonical momentum current/density: one has kind of momentum current for each embedding space coordinate.

2. By contracting with Killing vector fields of the embedding space isometries j_A^k (Poincare and color) one indeed obtains conserved currents associated with isometries by Noether's theorem:

$$j^{A\alpha} = T_k^\alpha j^{Ak} .$$

   By field equations the divergences of these currents vanish and one obtains conserved charged-classical four-momentum and color charges:

$$D_\alpha T^{A\alpha} = 0 .$$

   The field equations are essentially hydrodynamical and replace Einstein's equations $T^{alpha\beta} = kG^{\alpha\beta}$. The conditions $D_\beta G^{\alpha\beta} = 0$, stating the vanishing of the covariant divergence of Einstein's tensor $G^{\alpha\beta}$, is the counterpart for the conservation of the isometry currents, but do not give rise to conserved charges.

3. The normal component of the conserved current must vanish at boundaries with one time-like direction if one has such:

$$T^{An} = 0.$$

   Now one has wormhole throat orbits which are not genuine boundaries albeit analogous to them and one must be very careful. The quantity T_k^n determines the values of normal components of currents and must vanish at possible space-like boundaries.

Note that in TGD field equations reduce to the conservation of isometry currents as in hydrodynamics where basic equations are just conservation laws.

3.1.2   The basic steps in the derivation of field equations

First a general recipe for deriving field equations from Kähler action - or any action as a matter of fact.

1. At the first step one writes an expression of the variation of the Kähler action as sum of variations with respect to the induced metric g and induced Kähler form J. The partial derivatives in question are energy momentum tensor and contravariant Kähler form.

2. After this the variations of g and J are expressed in terms of variations of embedding space coordinates, which are the primary dynamical variables.

3. The integral defining the variation can be decomposed to a total divergence plus a term vanishing for extremals for all variations: this gives the field equations. Total divergence term gives a boundary term and it vanishes by boundary conditions if the boundaries in question have time-like direction.

   If the boundary is space-like, the situation is more delicate in TGD framework: this will be considered in the sequel. In TGD situation is also delicate also because the light-like 3-surfaces which are common boundaries of regions with Minkowskian or Euclidian signature of the induced metric are not ordinary topological boundaries. Therefore a careful treatment of both cases is required in order to not to miss important physics.

Expressing this summary more explicitly, the variation of the Kahler action with respect to the gradients of the embedding space coordinates reduces to the integral of

$$T_k^\alpha \partial_\alpha \delta h^k + \frac{\partial K}{\partial h^k} \delta h^k \ .$$

The latter term comes only from the dependence of the embedding space metric and Kähler form on embedding space coordinates. One can use a simple trick. Assume that they do not depend at all on embedding space coordinates, derive field equations, and replaced partial derivatives by covariant derivatives at the end. Covariant derivative means covariance with respect to both space-time and embedding space vector indices for the tensorial quantities involved. The trick works because embedding space metric and Kähler form are covariantly constant quantities.

The integral of the first term $T_k^\alpha \partial_\alpha \delta h^k$ decomposes to two parts.

1. The first term, whose vanishing gives rise to field equations, is integral of

$$D_\alpha T_k^\alpha \delta h^k \ .$$

2. The second term is integral of

$$\partial_\alpha (T_k^\alpha \delta h^k) \ .$$

   This term reduces as a total divergence to a 3-D surface integral over the boundary of the region of fixed signature of the induced metric consisting of the ends of CD and wormhole throat orbits (boundary of region with fixed signature of induced metric). This term vanishes if the normal components T_k^n of canonical momentum currents vanishes at the boundary like region.

In the sequel the boundary terms are discussed explicitly and it will be found that their treatment indeed involves highly non-trivial physics.

3.1.3   Complex isometry charges and twistorialization

TGD space-time contains regions of both Minkowskian and Euclidian signature of metric. This has some highly non-trivial consequences.

1. Should one assume that $\sqrt{det(g_4)}$ is imaginary in Minkowskian and real in Euclidian region? For Kähler action this is sensible and Euclidian region would give a real negative contribution giving rise to exponent of Kähler function of WCW ("world of classical worlds") making the functional integral convergent. Minkowskian regions would give imaginary contribution to the exponent causing interference effects absolutely essential in quantum field theory. This contribution would correspond to Morse function for WCW .

   The implication would be that the classical four-momenta in Euclidian/Minkowskian regions are imaginary/real. What could the interpretation be? Should one accept as a fact that four-momenta are complex.

2. Twistor approach to TGD is now in quite good shape [K11]. $M^4 \times CP_2$ is the unique choice is one requires that the Cartesian factors allow twistor space with Kähler structure [A5] and classical TGD allows twistor formulation.

   In the recent formulation the fundamental fermions are assumed to propagate with light-like momenta along wormhole throats. At gauge theory limit particles must have massless or massive four-momenta. One can however also consider the possibility of complex massless momenta and in the standard twistor approach on mass shell massless particles appearing in graphs indeed have complex momenta. These complex momenta should by quantum classical correspondence correspond directly to classical complex momenta.

3. A funny question popping in mind is whether the massivation of particles could be such that the momenta remain massless in complex sense! The complex variant of light-likeness condition would be

$$p_{re}^2 = p_{Im}^2 \ , \ \ p_{re} \cdot p_{Im} = 0 \ .$$

   Could one interpret p_{Im}^2 as the mass squared of the particle? Or could p_{Im}^2 code for the decay width of an unstable particle? This option does not look feasible.

4. The complex momenta could provide an elegant 4-D space-time level representation for the isometry quantum numbers at the level of embedding space. The ground states of the super-conformal representations have as building bricks the spinor harmonics of the embedding space which correspond to the analogs of massless particles in 8-D sense [K8]. Indeed, the condition giving mass squared eigenvalues for the spinor harmonics is just massless condition in $M^4 \times CP_2$.

   At the space-time level these conditions must be replaced by 4-D conditions and complex masslessness would be the elegant manner to realizes this. Also the massivation of massless states by p-adic thermodynamics could have similar description.

   This interpretation would also conform with $M^8 - M^4 \times CP_2$ duality [K14] at the level of momentum space.

3.2   Boundary Conditions At Boundaries Of CD

In positive energy ontology one would formulate boundary conditions as initial conditions by fixing both the 3-surface and associated canonical momentum densities at either end of CD (positions and momenta of particles in mechanics). This would bring asymmetry between boundaries of CD. In ZEO the basic boundary condition is that space-time surfaces have as their ends the members of pairs of surfaces at the ends of CD. Besides this one can have additional boundary conditions and the notion of preferred extremal suggests this.

3.2.1   Do boundary conditions realize quantum classical correspondence?

In TGD framework one must carefully consider the boundary conditions at the boundaries of CDs. What is clear that the time-like boundary contributions from the boundaries of CD to the variation must vanish.

1. This is true if the variations are assumed to vanish at the ends of CD. This might be however too strong a condition.

2. One cannot demand the vanishing of T^t_k (t refers to time coordinate as normal coordinate) since this would give only vacuum extremals. One could however require quantum classical correspondence for any Cartan sub-algebra of isometries whose elements define maximal set of isometry generators. The eigenvalues of quantal variants of isometry charge assignable to second quantized induced spinors at the ends of space-time surface are equal to the classical charges. Is this actually a formulation of Equivalence Principle, is not quite clear to me.

3.2.2 Do boundary conditions realize preferred extremal property as a choice of conformal gauge?

While writing this a completely new idea popped to my mind. What if one poses the vanishing of the boundary terms at boundaries of CDs as additional boundary conditions for *all* variations *except isometries*? Of perhaps for all conformal variations (conformal in TGD sense)? This would *not* imply vanishing of isometry charges since the variations coming from the opposite ends of CD cancel each other! It soon became clear that this would allow to meet all the challenges listed in the beginning!

1. These conditions would realize Bohr orbitology also to ZEO approach and define what "preferred extremal" means.

2. The conditions would be very much like super-Virasoro conditions stating that the superconformal generators with non-vanishing conformal weight annihilate states or create zero norm states but no conditions are posed on generators with vanishing conformal weight (now isometries). One could indeed assume only deformations, which are local isometries assignable to the generalised conformal algebra of the $\delta M^4_+/ - \times CP_2$. For arbitrary variations one would not require the vanishing. This could be the long sought for precise formulation of super-conformal invariance at the level of classical field equations!

   It is enough co consider the weaker conditions that the conformal charges defined as integrals of corresponding Noether currents vanish. These conditions would be direct equivalents of quantal conditions.

3. The natural interpretation would be as a fixing of conformal gauge. This fixing would be motivated by the fact that WCW Kähler metric must possess isometries associated with the conformal algebra and can depend only on the tangent data at partonic 2-surfaces as became clear already for more than two decades ago. An alternative, non-practical option would be to allow all 3-surfaces at the ends of CD: this would lead to the problem of eliminating the analog of the volume of gauge group from the functional integral.

4. The conditions would also define precisely the notion of holography and its reduction to strong form of holography in which partonic 2-surfaces and their tangent space data code for the dynamics.

   Needless to say, the modification of this approach could make sense also at partonic orbits.

3.3 Boundary Conditions At Parton Orbits

The contributions from the orbits of wormhole throats are singular since the contravariant form of the induced metric develops components which are infinite ($det(g_4) = 0$). The contributions are real at Euclidian side of throat orbit and imaginary at the Minkowskian side so that they must be treated as independently.

3.3.1 Conformal gauge choice, preferred extremal property, hierarchy of Planck constants, and TGD as almost topological QFT

The generalization of the boundary conditions as a classical realization conformal gauge invariance is natural.

1. One can consider the possibility that under rather general conditions the normal components $T_k^n \sqrt{det(g_4)}$ approach to zero at partonic orbits since $det(g_4)$ is vanishing. Note however the appearance of contravariant appearing twice as index raising operator in Kähler action. If so, the vanishing of $T_k^n \sqrt{det(g_4)}$ need not fix completely the "boundary" conditions. In fact, I assign to the wormhole throat orbits conformal gauge symmetries so that just this is expected on physical grounds.

2. Generalized conformal invariance would suggest that the variations defined as integrals of $T_k^n \sqrt{det(g_4)} \delta h^k$ vanish in a non-trivial manner for the conformal algebra associated with the light-like wormhole throats with deformations respecting $det(g_4) = 0$ condition. Also the variations defined by infinitesimal isometries (zero conformal weight sector) should vanish since otherwise one would lose the conservation laws for isometry charges. The conditions for isometries might reduce to $T_k^n \sqrt{det(g_4)} \to 0$ at partonic orbits. Also now the interpretation would be in terms of fixing of conformal gauge.

3. Even $T_k^n \sqrt{g} = 0$ condition need not fix the partonic orbit completely. The Gribov ambiguity meaning that gauge conditions do not fix uniquely the gauge potential could have counterpart in TGD framework. It could be that there are several conformally non-equivalent space-time surfaces connecting 3-surfaces at the opposite ends of CD.

   If so, the boundary values at wormhole throats orbits could matter to some degree: very natural in boundary value problem thinking but new in initial value thinking. This would conform with the non-determinism of Kähler action implying criticality and the possibility that the 3-surfaces at the ends of CD are connected by several space-time surfaces which are physically non-equivalent.

4. The hierarchy of Planck [K4] constants assigned to dark matter, quantum criticality and even criticality indeed relies on the assumption that $h_{eff} = n \times h$ corresponds to n-fold coverings having n space-time sheets which coincide at the ends of CD and that conformal symmetries act on the sheets as gauge symmetries. One would have as Gribov copies n conformal equivalence classes of wormhole throat orbits and corresponding space-time surfaces. Depending on whether one fixes the conformal gauge one has n equivalence classes of space-time surfaces or just one representative from each conformal equivalent class.

5. There is also the question about the correspondence with the weak form of electric magnetic duality [K3]. This duality plus the condition that $j^\alpha A_\alpha = 0$ in the interior of space-time surface imply the reduction of Kähler action to Chern-Simons terms. This would suggest that the boundary variation of the Kähler action reduces to that for Chern-Simons action which is indeed well-defined for light-like 3-surfaces.

   If so, the gauge fixing would reduce to variational equations for Chern-Simons action! A weaker condition is that classical conformal charges vanish. This would give a nice connection to the vision about TGD as almost topological QFT. In TGD framework these conditions do not imply the vanishing of Kähler form at boundaries. The conditions are satisfied if the CP_2 projection of the partonic orbit is 2-D: the reason is that Chern-Simons term vanishes identically in this case.

3.3.2 Fractal hierarchy of conformal symmetry breakings

A further intuitively natural hypothesis is that there is a fractal hierarchy of breakings of conformal symmetry.

1. Only the generators of conformal sub-algebra with conformal weight multiple of n act as gauge symmetries. This would give infinite hierarchies of breakings of conformal symmetry interpreted in terms of criticality: in the hierarchy n_i divides n_{i+1}.

   Similar degeneracy would be associated with both the parton orbits and the space-like ends at CD boundaries and I have considered the possibility that the integer n appearing in h_{eff} has decomposition $n = n_1 n_2$ corresponding to the degeneracies associated with the two kinds of boundaries. Alternatively, one could have just $n = n_1 = n_2$ from the condition that the two conformal symmetries are 3-dimensional manifestations of single 4-D analog of conformal symmetry.

2. In the symmetry breaking $n_i \to n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Could one require that they are conserved that is the contributions of the boundary terms at the ends of CD cancel each other? If so, one would have dynamical conformal symmetry.

What could the proper interpretation of the conformal hierarchies $n_i \to n_{i+1}$?

1. Could one interpret the hierarchy in terms of increasing measurement resolution? Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and the conformal hierarchies would correspond to an inclusion hierarchies for hyper-finite factors of type II_1 [K15]. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

    This interpretation might make sense for the symplectic algebra of $\delta M_\pm^4 \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

2. Suppose that the Kähler action has vanishing variation under deformations defined by the broken conformal symmetries so that the corresponding conformal charges As a consequence, Kähler function would be critical with respect to the corresponding variations. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories.

    In this kind of situation one could consider the interpretation in terms of criticality: the lower the criticality, the larger then value of h_{eff} and h and the higher the resolution.

3. n gives also the number of space-time sheets in the singular covering. Could the interpretation be in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

As should have become clear, the derivation of field equations in TGD framework is not just an application of a formal recipe as in field theories and a lot of non-trivial physics is involved!

4 General View About Field Equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. This condition implies that covariant divergence of energy momentum tensor vanishes and in General Relativity context this leads to Einstein's equations. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics. There are however could reasons to keep the identification of preferred extremely property open.

4.1 Field Equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$D_\beta(T^{\alpha\beta} h_\alpha^k) \quad - \quad j^\alpha J_l^k \partial_\alpha h^l = 0 \ ,$$
$$T^{\alpha\beta} \quad = \quad J^{\nu\alpha} J_\nu^{\ \beta} - \frac{1}{4} g^{\alpha\beta} J^{\mu\nu} J_{\mu\nu} \ . \tag{4.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$T^{\alpha\beta} H_{\alpha\beta}^k \quad - \quad j^\alpha (J_\alpha^{\ \beta} h_{\ \beta}^k + J_l^k \partial_\alpha h^l) = 0 \ .$$
$$H_{\alpha\beta}^k \quad = \quad D_\beta \partial_\alpha h^k \ . \tag{4.2}$$

$H_{\alpha\beta}^k$ denotes the components of the
second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} \quad = \quad T^{n\beta} \partial_\beta h^k - J^{n\alpha}(J_\alpha^{\ \beta} \partial_\beta h^k + J_l^k) \partial_\alpha h^k) = 0 \ . \tag{4.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} \quad = \quad 0 \tag{4.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K7]. Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K16] led to the conditions

$$g_{ni} = 0 \ , \quad J_{ni} = 0 \ . \tag{4.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X_l^3). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$J_{ni} = 0 \ , \qquad g_{ni} = 0 \ , \qquad J_{ir} = 0 \ , \qquad g_{ir} = 0 \ ,$$
$$J^{nk} = 0 \ k \neq r \ , \quad g^{nk} = 0 \ k \neq r \quad , \quad J^{nr}\sqrt{g_4} \neq 0 \ , \quad g^{nr}\sqrt{g_4} \neq 0 \ . \tag{4.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X_l^3 and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.

2. Third and fourth condition state that the induced Kähler field at X_l^3 is purely magnetic and that the metric of x_l^3 reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the Kähler-Dirac operator is considered [K16].

3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X_l^3.

4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X_l^3)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X_l^3)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y_l^3.

5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

4.2 Topologization And Light-Likeness Of The Kähler Current As Alternative ways To Guarantee Vanishing Of Lorentz 4-Force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

4.2.1 Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j_I^\alpha = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \ . \tag{4.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla \psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j_I^\alpha J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{4.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}_{\ \ l} \partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M_+^4 and CP_2 degrees of freedom to

$$T^{\alpha\beta} H^k_{\alpha\beta} \;\; = \;\; 0 \;\; . \tag{4.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric
tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance
in a suitable sense might provide a general solution of the field equations could be seen as a
generalization of the ordinary conformal invariance of string models. If the topologization of the
Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent
with quaternion conformal invariance, the quaternion conformal structures must differ for the
different dimensions of CP_2 projection.

4.2.2   Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these
arguments using the 3-dimensional notation. The components of the instanton 4-current read in
three-dimensional notation as

$$\overline{j}_I = \overline{E} \times \overline{A} + \phi\overline{B} \;\; , \quad \rho_I = \overline{B} \cdot \overline{A} \;\; . \tag{4.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation
and read

$$\begin{aligned}
\nabla \times \overline{B} - \partial_t \overline{E} &= \overline{j} = \psi\overline{j}_I = \psi\left(\phi\overline{B} + \overline{E} \times \overline{A}\right) \;\; , \\
\nabla \cdot E &= \rho = \psi\rho_I \;\; .
\end{aligned} \tag{4.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami
condition

$$\nabla \times \overline{B} = \alpha\overline{B} \;\; , \quad \alpha = \psi\phi \;\; . \tag{4.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines
of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami
condition with $\alpha = \phi = constant$, which allows an explicit solution [B1].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz
3-force can be written as

$$\rho_I\overline{E} + \overline{j} \times \overline{B} = \psi\overline{B} \cdot \overline{AE} + \psi\left(\overline{E} \times \overline{A} + \phi\overline{B}\right) \times \overline{B} = 0 \;\; . \tag{4.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j} \cdot \overline{E} = \psi\overline{B} \cdot \overline{E} + \psi\left(\overline{E} \times \overline{A} + \phi\overline{B}\right) \cdot \overline{E} = 0 \;\; . \tag{4.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by
expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity
density of the so called helicity charge $\rho = \psi\rho_I = \psi B \cdot A$. This charge is topological charge in the
sense that it does not depend on the induced metric at all. Note the presence of arbitrary function
ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 inde-
pendent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality
condition for the Kähler current allows to understand the general features of the solution ansatz
to very high degree without any detailed knowledge about the detailed solution. The question
whether field equations allow solutions consistent with the self duality conditions of the current
will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler
current relate to each other very intimately.

4.2.3 Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

4.2.4 Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} \;=\; \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{4.15}$$

   This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinatesand $\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} \;=\; \beta \overline{B} \tag{4.16}$$

   The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla \times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

4.2.5 Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the embedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B4].

In hydrodynamics the role of the magnetic field is taken by the velocity field. This raises the idea that the incompressible flow could occur along the field lines of some natural vector field. The considerations of the last section show that the instanton current defines a universal candidate as far as the general solution of the field equations is considered. All conserved currents defined by the isometry charges would be parallel to the instanton current: one can say each flow line of instanton current is a carrier of conserved quantum numbers. Perhaps even the flow lines of an incompressible hydrodynamic flow could in reasonable approximation correspond to those of instanton current.

The conservation laws are satisfied for each flow line separately and therefore it seems that one cannot have the analog of viscous hydrodynamic flow in this framework. One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Does something go badly wrong?

The following argument suggests a way out of the problem. Dissipation is certainly due to the quantum jumps at scales below that associated with causal diamond (CD) associated with the observer and is thus assignable to sub-CDs. The quantum jumps for sub-CDs would eventually lead to a thermal ensemble of sub-CDs.

The usual description of dissipation in terms of viscocity and similar parameters emerges at the GRT-QFT limit of TGD replacing in long length scales the many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) with a piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. This lumping of space-time sheets means that induced gauge fields and gravitational fields from various space-time sheet sum up and become random (by central limit theorems). Thus locally the dynamics is dissipation free for individual space-time sheets and dissipation emerges at the level of GRT space-time carrying effective metric and effective gauge fields.

4.2.6 The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds contact. Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta}A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta}J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.

2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler

current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;\,) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B2], and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B2] !

Beltrami flows associated with Euler equations are known to be unstable [B2]. Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

4.3 How To Satisfy Field Equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^k_l \partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta} H^k_{\alpha\beta} \;\; = \;\; 0 \;\; . \tag{4.17}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

4.3.1 String model as a starting point

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u, v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.

2. For Euclidian minimal surfaces (u,v) is replaced by complex coordinates $(w,\overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{ww}}$. The mechanism should generalize to the recent case.

4.3.2 The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$
T = \begin{pmatrix}
\frac{E^2+B^2}{2} & 0 & 0 & EB \\
0 & \frac{E^2+B^2}{2} & 0 & 0 \\
0 & 0 & \frac{-E^2+B^2}{2} & 0 \\
EB & 0 & 0 & \frac{E^2-B^2}{2}
\end{pmatrix}
\tag{4.18}
$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla \times X \cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E}\times\overline{B}$ such that the coordinates $u = t - z$ and $v = t + z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

4.3.3 Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t + z, S^- \equiv v = t - z, w = x + iy, \overline{w} = x - iy)$.

2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$
(\nabla S_\pm)^2 = 0 \quad .
$$

The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with

the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \rightarrow v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t - z, t + z, x, y) \rightarrow S^-(t - z, t + z, x, y) = S^+(t + z, t - z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$
\begin{aligned}
ds^2 \;=\; & g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\
+\; & g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\
+\; & g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \; .
\end{aligned}
\tag{4.19}
$$

Conformal transformations of M_+^4 leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$
\begin{aligned}
g_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \; , & \quad g_{+-} = \partial_{S^+} \partial_{S^-} K \; , \\
g_{w\pm} = \partial_w \partial_{S\pm} K \; , & \quad g_{\overline{w}\pm} = \partial_{\overline{w}} \partial_{S\pm} K \; .
\end{aligned}
\tag{4.20}
$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$
K = w_0 \overline{w}_0 + uv \; , \quad w_0 = x + iy \; , \quad u = t - z \; , \quad v = t + z \; .
\tag{4.21}
$$

The Christoffel symbols satisfy the conditions

$$
\{ {}^{\;k}_{w\,\overline{w}} \} = 0 \; , \quad \{ {}^{\;k}_{+-} \} = 0 \; .
\tag{4.22}
$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M_+^4 degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M_+^4. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +$ or $= -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y_l^3 labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y_l^3 is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K7, K10].

4.3.4 Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \bar{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M_+^4 case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \quad , \quad s_{ws} = \partial_w \partial_s K \quad , \quad s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \quad . \tag{4.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{_{\xi\,\overline{\xi}}^{\;k\;}\}$.

$$\{_{\xi\,\overline{\xi}}^{\;k\;}\} \;\; = \;\; 0 \quad . \tag{4.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \bar{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \bar{\xi}, s)$ and some coordinate of M_+^4, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta (J^{\alpha\beta} \sqrt{g}) \;\; &= \;\; 0 \text{ for } \alpha \in \{\xi, \bar{\xi}, s\} \quad , \\ g^{4i} &\neq 0 \quad . \end{aligned} \tag{4.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

4.3.5   A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M_+^4 respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \bar{\xi}, s, S^-)$.

1. The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \tag{4.26}$$

   Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\bar{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M_+^4 Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \bar{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\bar{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\bar{\xi} s} = 0 \ . \tag{4.27}$$

   Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned}
s_{ss} &= m_{+w}\partial_s w(\xi, s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi, s)\partial_s S^+(s) \ , \\
s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\
s_{s\bar{\xi}} &= m_{+w}\partial_{\bar{\xi}} w(\bar{\xi})\partial_s S^+(s) \ .
\end{aligned} \tag{4.28}$$

   Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$j^\xi \sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\bar{\xi}}\sqrt{g} = \partial_\beta(J^{\bar{\xi}\beta}\sqrt{g}) = 0 \ ,$$

$$j^+ \sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \tag{4.29}$$

   Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^+ = \partial_\beta(J^{+\beta}\sqrt{g}) \quad = \quad 0 \tag{4.30}$$

   is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 type extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\bar{\xi}}, J^{\xi-}, J^{\bar{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If preferred extremals correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \bar{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

4.3.6 Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y_l^3 of $X^4(X_l^3)$ "parallel" to X_l^3 requires only that gauge currents are parallel to Y_l^3 and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \bar{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \ , \quad w = w(\xi, s) \ . \tag{4.31}$$

If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

3. The components of the induced metric are

$$g_{ZZ} = m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , \quad g_{Zs} = m_{TT}\partial_Z T \partial_s T \ ,$$

$$g_{ss} = s_{ss} + m_{TT}\partial_s T \partial_s T \ , \qquad g_{w\overline{w}} = s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}}\overline{w} \ , \qquad (4.32)$$

$$g_{s\xi} = s_{s\xi} \ , \qquad\qquad g_{s\overline{\xi}} = s_{s\overline{\xi}} \ .$$

Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} \ = \ 0 \qquad\qquad (4.33)$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) \ = \ 0 \ ,$$
$$\partial_\xi(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi}Z}\sqrt{g}) \ = \ 0 \ . \qquad (4.34)$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M_+^4 Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\{^{\ k}_{T\ w}\} = 0 \ , \quad \{^{\ k}_{T\ \overline{w}}\} = 0 \ ,$$
$$\{^{\ k}_{Z\ w}\} = 0 \ , \quad \{^{\ k}_{Z\ \overline{w}}\} = 0 \qquad (4.35)$$

hold true. The conditions are equivalent with the conditions

$$\{^{\ k}_{\pm\ w}\} = 0 \ , \quad \{^{\ k}_{\pm\ \overline{w}}\} = 0 \ . \qquad (4.36)$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s,Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

4.3.7 $D_{CP_2} = 4$ case

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M_+^4 is expected to be crucial also now.

2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M_+^4 projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \tag{4.37}$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{4.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M_+^4 degrees of freedom one must pose the conditions

$$\{ {}^{k}_{w+} \} = 0 \ , \quad \{ {}^{k}_{\overline{w}+} \} = 0 \ , \quad \{ {}^{k}_{++} \} = 0 \ . \tag{4.39}$$

on Christoffel symbols. These conditions are satisfied if the the M_+^4 metric does not depend on S^+:

$$\partial_+ m_{kl} \ = \ 0 \ . \tag{4.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \ = \ a_k \xi^k + \overline{a}_k \overline{\xi}^k \ . \tag{4.41}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M_+^4 coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j(J^{j\bar{i}}\sqrt{g}) = 0 \ , \quad \partial_{\bar{j}}(J^{\bar{j}i}\sqrt{g}) = 0 \ , \tag{4.42}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M_+^4 projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M_+^4 projection is a light like curve correspond to a special case of this solution ansatz: transversal M_+^4 coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M_+^4 projection is 1-dimensional.

2. Are solutions with a 4-dimensional M_+^4 projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M_+^4 projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i}S^+\partial_{\xi^j}S^- + m_{+-}\partial_{\xi^i}S^-\partial_{\xi^j}S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M_+^4 and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \ . \tag{4.43}$$

This ansatz would describe ordinary Maxwell field in M_+^4 since the roles of M_+^4 coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M_+^4 projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time. Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

4.3.8 $D_{CP_2} = 2$ case

Hamilton Jacobi structure for M_+^4 is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M_+^4 \times CP_2$ of minimal surfaces Y^2 of M_+^4 with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M_+^4. $w = constant$ surfaces define minimal surfaces X^2 of M_+^4. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the embedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD

counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s, r) = (s(u, v), r(u, v)) \quad , \quad \xi = constant \quad , \tag{4.44}$$

and corresponds to a vanishing Kähler current.

3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B} \cdot \overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s, r) = (s, r)(u, v, w, \overline{w}) \quad , \quad \xi = constant \quad . \tag{4.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

  1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w, \overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \quad , \quad w = w(\xi) \quad , \quad S^- = s^- \quad , \quad S^+ = s^+ + f(\xi, \overline{\xi}) \quad .$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi \partial_+ h^k$ and $\partial_{\overline{\xi}} \partial_+ h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

4.3.9   Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the embeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K12].

  Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta), \Phi)$ and let (t, ρ, ϕ, z) denote cylindrical coordinates for a cylindrical space-time

sheet. The simplest possible space-time surfaces $X^4 \subset M_+^4 \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$
\begin{aligned}
U = U(\rho) \ , & \qquad \Phi = n\phi + kz \ , \\
J_{\rho\phi} = n\partial_\rho U \ , & \qquad J_{\rho z} = k\partial_\rho U \ .
\end{aligned}
\tag{4.46}
$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$ \Phi = \omega t + kz + n\phi $$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \to 3$. This could help to understand various strange effects related to the rotating magnetic systems [K12]. For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts/flux tubes and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

4.4 $D_{CP_2} = 3$ Phase Allows Infinite Number Of Topological Charges Characterizing The Linking Of Magnetic Field Lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B3]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla \times$ as $A = (1/\nabla \times)B$. The inverse is non-local operator expressible as

$$ \frac{1}{\nabla \times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ , $$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$ \int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r - r')}{|r - r'|^3} \times B(r') \right) \ , $$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M_+^4 is deformed in M_+^4 degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

4.5 Preferred Extremal Property And The Topologization/Light-Likeness Of Kähler Current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. Zero energy ontology challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.

2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic

configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X_l^3. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y_l^3 varies. This conforms nicely with the notion of quantum gravitational holography.

3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\bar{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X_l^3)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X_l^3 so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM_-^4 of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM_+^4? An identification consistent with the fractal structure of zero energy ontology and

TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [K1].

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

4.6   Generalized Beltrami Fields And Biological Systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y_l^3 associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

4.6.1   Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B4] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

4.6.2 $D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ corresponds to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A} \cdot \overline{B} \neq 0$ and Kähler current $\overline{E} \times \overline{A} + \phi \overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [**?**]. For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the

covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

4.6.3   Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A1] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.

2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.

2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.

3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing by a continuous Schrödinger time evolution.

4. The condition that the modes of the induced spinor field have well-defined em charge implies that CP_2 projection for the region of space-time in which induced spinor field is non-vanishing is 2-dimensional. In the generic case a localization to 2-surfaces - string world sheets and possibly partonic 2-surface. At light-like 3-surfaces this implies that modes are localized at 1-D curves so that the hydrodynamic picture is realized [K16].

3. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.

2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.

3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D embedding space which defines one sector of the generalized embedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

4.7   About Small Perturbations Of Field Equations

The study of small perturbations of the known solutions of field equations is a standard manner to get information about the properties of the solutions, their stability in particular. Fourier expansion is the standard manner to do the perturbation theory. In the recent case an appropriate modification of this ansatz might make sense if the solution in question is representable as a map $M_+^4 \to CP_2$, and the perturbations are rapidly varying when compared to the components of the induced metric and Kähler form so that one can make adiabatic approximation and approximate them as being effectively constant. Presumably also restrictions on directions of wave 4-vectors $k_\mu = (\omega, \overline{k})$ are necessary so that the direction of wave vector adapts to the slowly varying background as in ray optics. Also Hamilton Jacobi structure is expected to modify the most straightforward approach. The four CP_2 coordinates are the dynamical variables so that the situation is relatively simple.

A completely different approach is inspired by the physical picture. In this approach one glues CP_2 type vacuum extremal to a known extremal and tries to deduce the behavior of the deformed extremal in the vicinity of wormhole throat by posing the general conditions on the slicing by light-like 3-surfaces Y_l^3. This approach is not followed now.

4.7.1   Generalized plane waves

Individual plane waves are geometrically very special since they represent a deformation of the space-time surface depending on single coordinate only. Despite this one might hope that plane waves or their appropriate modifications allowing to algebraize the treatment of small perturbations could give useful information also now.

1. Lorentz invariance plus the translational invariance due to the assumption that the induced metric and Kähler form are approximately constant encourage to think that the coordinates reduce Minkowski coordinates locally with the orientation of the local Minkowski frame depending slowly on space-time position. Hamilton Jacobi $(S^+, S^-, w, \overline{w})$ are a good candidate for this kind of coordinates. The properties of the Hamilton Jacobi structure and of the solution ansatz suggest that excitations are generalized plane waves in longitudinal degrees of freedom only so that four-momentum would be replaced by the longitudinal momentum. In transverse degrees of freedom one might expect that holomorphic plane-waves $exp(ik_T w)$, where k_T is transverse momentum, make algebraization possible.

   For time-like longitudinal momenta one can choose the local M^4 coordinates in such a way that longitudinal momentum reduces to $(\omega_0, 0)$, where ω_0 plays the role of rest mass and is analogous to the plasma frequency serving as an infrared cutoff for plasma waves. In these coordinates the simplest candidates for excitations with time-like momentum would be of form $\Delta s^k = \epsilon a^k exp(i\omega_0 u)$, where s^k are some real coordinates for CP_2, a^k are Fourier coefficients, and time-like coordinate is defined as $u = S^+ + S^-$. The excitations moving with light velocity correspond to $\omega_0 = 0$, and one must treat this case separately using plane wave $exp(i\omega S^\pm)$, where ω has continuum of values.

2. It is possible that only some preferred CP_2 coordinates are excited in longitudinal degrees of freedom. For $D_{CP_2} = 3$ ansatz the simplest option is that the complex CP_2 coordinate ξ depends analytically on w and the longitudinal CP_2 coordinate s obeys the plane wave ansatz. $\xi(w) = a \times exp(ik_T w)$, where k_T is transverse momentum allows the algebraization of the solution ansatz also in the transversal degrees of freedom so that a dispersion relation results. For imaginary values of k_T and ω the equations are real.

2. General form for the second variation of the field equations

For time-like four-momentum the second variation of field equations contains three kinds of terms. There are terms quadratic in ω_0 and coming from the second derivatives of the deformation, terms proportional to $i\omega_0$ coming from the variation with respect to the derivatives of CP_2 coordinates, and terms which do not depend on ω_0 and come from the variations of metric and Kähler form with respect to the CP_2 coordinates.

In standard perturbation theory the terms proportional to $i\omega_0$ would have interpretation as analogs of dissipative terms. This forces to assume that ω_0 is complex: note that in purely imaginary ω_0 the equations are real. The basic assumption is that Kähler action is able to mimic dissipation despite the fact that energy and momentum are conserved quantities. The vanishing of the Lorentz force has an interpretation as the vanishing of the dissipative effects. This would suggest that the terms proportional to $i\omega_0$ vanish for the perturbations of the solution preserving the non-dissipative character of the asymptotic solutions. This might quite well result from the vanishing of the contractions with the deformation of the energy momentum tensor with the second fundamental form and of energy momentum tensor with the deformation of the second fundamental form coming from first derivatives.

Physical intuition would suggest that dissipation-less propagation is possible only along special directions. Thus the vanishing of the linear terms should occur only for special directions of the longitudinal momentum vector, say for light-like four-momenta in the direction of coordinate lines of S^+ or S^-. Quite generally, the sub-space of allowed four-momenta is expected to depend on position since the components of metric and Kähler form are slowly varying. This dependence is completely analogous with that appearing in the Hamilton Jacobi (ray-optics) approach to the approximate treatment of wave equations and makes sense if the phase of the plane wave varies rapidly as compared to the variation of CP_2 coordinates for the unperturbed solution.

Complex values of ω_0 are also possible, and would allow to deduce important information about the rate at which small deviations from asymptotia vanish as well as about instabilities of the asymptotic solutions. In particular, for imaginary values of ω_0 one obtains completely well-defined solution ansatz representing exponentially decaying or increasing perturbation.

4.7.2 High energy limit

One can gain valuable information by studying the perturbations at the limit of very large four-momentum. At this limit the terms which are quadratic in the components of momentum dominate and come from the second derivatives of the CP_2 coordinates appearing in the second fundamental form. The resulting equations reduce for all CP_2 coordinates to the same condition

$$T^{\alpha\beta}k_\alpha k_\beta = 0 \ .$$

This condition is generalization of masslessness condition with metric replaced by the energy momentum tensor, which means that light velocity is replaced by an effective light velocity. In fact, energy momentum tensor effectively replaces metric also in the modified Dirac equation whose form is dictated by super symmetry. Light-like four momentum is a rather general solution to the condition and corresponds to $\omega_0 = 0$ case.

4.7.3 Reduction of the dispersion relation to the graph of swallowtail catastrophe

Also the general structure of the equations for small perturbations allows to deduce highly non-trivial conclusions about the character of perturbations.

1. The equations for four CP_2 coordinates are simultaneously satisfied if the determinant associated with the equations vanishes. This condition defines a 3-dimensional surface in the 4-dimensional space defined by ω_0 and coordinates of 3-space playing the role of slowly varying control parameters. 4×4 determinant results and corresponds to a polynomial which is of order $d = 8$ in ω_0. If the determinant is real, the polynomial can depend on ω_0^2 only so that a fourth order polynomial in $w = \omega_0^2$ results.

2. Only complex roots are possible in the case that the terms linear in $i\omega_0$ are non-vanishing. One might hope that the linear term vanishes for certain choices of the direction of slowly varying four-momentum vector $k^\mu(x)$ at least. For purely imaginary values of ω_0 the equations determinant are real always. Hence catastrophe theoretic description applies in this case at least, and the so called swallow tail [A2] with three control parameters applies to the situation.

3. The general form of the vanishing determinant is

$$D(w, a, b, c) = w^4 - ew^3 - cw^2 - bw - a \ .$$

The transition from the oscillatory to purely dissipative case changes only the sign of w. By the shift $w = \hat{w} + e/4$ the determinant reduces to the canonical form

$$D(\hat{w}, a, b, c) = \hat{w}^4 - c\hat{w}^2 - b\hat{w} - a$$

of the swallowtail catastrophe. This catastrophe has three control variables, which basically correspond to the spatial 3-coordinates on which the induced metric and Kähler form depend. The variation of these coefficients at the space-time sheet of course covers only a finite region of the parameter space of the swallowtail catastrophe. The number of real roots for $w = \omega_0^2$ is four, two, or none since complex roots appear in complex conjugate pairs for a real polynomial. The general shape of the region of 3-space is that for a portion of swallow tail catastrophe (see **Fig. 1**).

4. The dispersion relation for the "rest mass" ω_0 (decay rate for the imaginary value of ω_0) has at most four real branches, which conforms with the fact that there are four dynamical variables. In real case ω_0 is analogous to plasma frequency acting as an infrared cutoff for the frequencies of plasma excitations. To get some grasp on the situation notice that for $a = 0$ the swallowtail reduces to $\hat{w} = 0$ and

$$\hat{w}^3 - c\hat{w} - b = 0 \ ,$$
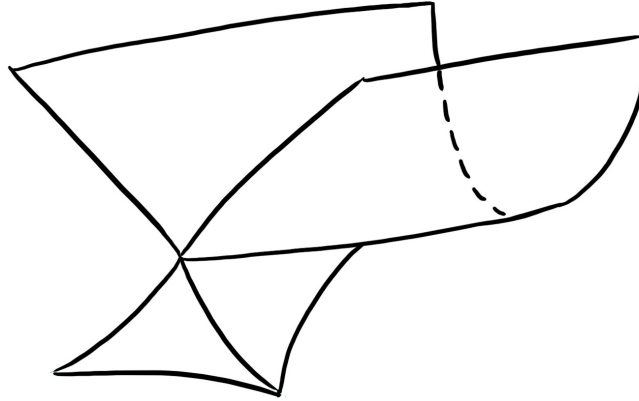
Figure 1: The projection of the bifurcation set of the swallowtail catastrophe to the 3-dimensional space of control variables. The potential function has four extrema in the interior of the swallowtail bounded by the triangles, no extrema in the valley above the swallowtail, and 2 extrema elsewhere.

which represents the cusp catastrophe easy to illustrate in 3-dimensional space. Cusp (see **Fig. 2**) in turn reduces for $b = 0$ to $\hat{w} = 0$ and fold catastrophe $\hat{w} = \pm\sqrt{c}$. Thus the catastrophe surface becomes 4-sheeted for $c \geq 0$ for sufficiently small values of the parameters a and b. The possibility of negative values of $\hat{w}$ in principle allows $\omega^2 = \hat{w} + e/4 < 0$ solutions identifiable as exponentially decaying or amplified perturbations. At the high frequency limit the 4 branches degenerate to a single branch $T^{\alpha\beta} k_\alpha k_\beta = 0$, which as a special case gives light-like four-momenta corresponding to $\omega_0 = 0$ and the origin of the swallowtail catastrophe.

Figure 2: Cusp catastrophe. Vertical direction corresponds to the behavior variable and orthogonal directions to control variables.

5. It is quite possible that the imaginary terms proportional to $i\omega_0$ cannot be neglected in the time-like case. The interpretation would be as dissipative effects. If these effects are not too large, an approximate description in terms of butterfly catastrophe makes still sense. Note however that the second variation contains besides gravitational terms potentially large dissipative terms coming from the variation of the induced Kähler form and from the variation of CP_2 Christoffel symbols.

6. Additional complications are encountered at the points, where the induced Kähler field van-

ishes since the second variation vanishes identically at these points. By the arguments represented earlier, these points quite generally represent instabilities.

5 Vacuum Extremals

Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe. Vacuum extremals are not gravitational vacua and they are indeed fundamental in TGD inspired cosmology.

5.1 CP_2 Type Extremals

5.1.1 CP_2 type vacuum extremals

These extremals correspond to various isometric embeddings of CP_2 to $M_+^4 \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the embedding is given by the formula

$$
\begin{aligned}
m^k &= m^k(u) \ , \\
m_{kl}\dot{m}^k\dot{m}^l &= 0 \ ,
\end{aligned}
\tag{5.1}
$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1, 2, 3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the p-adic QFT limit of TGD. Furthermore, it has turned out that Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD and appears both at the level of embedding space and space-time surfaces.

The action for all extremals is same and given by the Kähler action for the embedding of CP_2. The value of the action is given by

$$
S = -\frac{\pi}{8\alpha_K} \ .
\tag{5.2}
$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is

negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action.

The absolute minimization of Kähler action was the original suggestion for what preferred extremal property could mean, and suggested that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. The same conclusion however follows also from the mere vacuum degeneracy of Käbler action. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. This identification seems reasonable: the 4-D lines of generalized Feynman graphs [K5] would be regions with Euclidian signature of induced metric and identifiable as deformations of CP_2 type vacuum extremals, and even TGD counterparts of blackholes would be analogous to lines of Feynman diagrams. Their M^4 projection would be of course arbitrarily of macroscopic size. The nice generalization of the area law for the entropy of black hole [K6] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere zitterbewegung so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A4]. A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

5.1.2 Are CP_2 type non-vacuum extremals possible?

The isometric embeddings of CP_2 are all vacuum extremals so that these extremals as such cannot correspond to physical particles. One obtains however non-vacuum extremals as deformations of these solutions. There are several types of deformations leading to non-vacuum solutions. In order to describe some of them, recall the expressions of metric and Kähler form of CP_2 in the coordinates (r, Θ, Ψ, Φ) [A3] are given by

$$
\begin{aligned}
\frac{ds^2}{R^2} &= \frac{dr^2}{(1+r^2)^2} + \frac{r}{2(1+r^2))^2}(d\Psi + cos(\Theta)d\Phi)^2 \\
&+ \frac{r^2}{(4(1+r^2)}(d\Theta^2 + sin^2\Theta d\Phi^2) \ , \\
J &= \frac{r}{(1+r^2)}dr \wedge (d\Psi + cos(\Theta)d\Phi) \\
&- \frac{r^2}{(2(1+r^2)}sin(\Theta)d\Theta \wedge d\Phi \ .
\end{aligned}
\tag{5.3}
$$

The scaling of the line element is defined so that πR is the length of the CP_2 geodesic line. Note that Φ and Ψ appear as "cyclic" coordinates in metric and Kähler form: this feature plays important role in the solution ansatze to be described.

Let $M^4 = M^2 \times E^2$ denote the decomposition of M^4 to a product of 2-dimensional Minkowski space and 2-dimensional Euclidian plane. This decomposition corresponds physically to the decomposition of momentum degrees of freedom for massless particle: E^2 corresponds to polarization degrees of freedom.

There are several types of non-vacuum extremals.

"Virtual particle" extremals: the mass spectrum is continuous (also Euclidian momenta are allowed) but these extremals reduce to vacuum extremals in the massless limit.

2. Massless extremals.

Consider first an example of virtual particle extremal. The simplest extremal of this type is obtained in the following form

$$m^k = a^k \Psi + b^k \Phi \ . \tag{5.4}$$

Here a^k and b^k are some constant quantities. Field equations are equivalent to the conditions expressing four-momentum conservation and are identically satisfied the reason being that induced metric and Kähler form do not depend on the coordinates Ψ and Φ.

Extremal describes 3-surface, which moves with constant velocity in M^4. Four-momentum of the solution can be both space and time like. In the massless limit solution however reduces to a vacuum extremal. Therefore the interpretation as an off mass shell massless particle seems appropriate.

Massless extremals are obtained from the following solution ansatz.

$$
\begin{aligned}
m^0 &= m^3 = a\Psi + b\Phi \ , \\
(m^1, m^2) &= (m^1(r,\Theta), m^2(r,\Theta)) \ .
\end{aligned}
\tag{5.5}
$$

Only E^2 degrees of freedom contribute to the induced metric and the line element is obtained from

$$ds^2 = ds^2_{CP_2} - (dm^1)^2 - (dm^2)^2 \ . \tag{5.6}$$

Field equations reduce to conservation condition for the componenets of four-momentum in E^2 plane. By their cyclicity the coordinates Ψ and Φ disappear from field equations and one obtains essentially current conservation condition for two-dimensional field theory defined in space spanned by the coordinates r and Θ.

$$
\begin{aligned}
(J^i_a)_{,i} &= 0 \ , \\
J^i_a &= T^{ij} f^a_{,j} \sqrt{g} \ .
\end{aligned}
\tag{5.7}
$$

Here the index i and a refer to r and Θ and to E^2 coordinates m^1 and m^2 respectively. T^{ij} denotes the canonical energy momentum tensor associated with Kähler action. One can express the components of T^{ij} in terms of induced metric and CP_2 metric in the following form

$$T^{ij} = (-g^{ik}g^{jl} + g^{ij}g^{kl}/2)s_{kl} \ . \tag{5.8}$$

This expression holds true for all components of the energy momentum tensor.

Since field equations are essentially two-dimensional conservation conditions they imply that components of momentum currents can be regarded as vector fields of some canonical transformations

$$J^i_a = \varepsilon^{ij} H^a_{,j} \ , \tag{5.9}$$

where ε^{ij} denotes two-dimensional constant symplectic form. An open problem is whether one could solve field equations exactly and whether there exists some nonlinear superposition principle for the solutions of these equations. Solutions are massless since transversal momentum densities vanish identically.

Consider as a special case the solution obtained by assuming that one E^2 coordinate is constant and second coordinate is function $f(r)$ of the variable r only. Field equations reduce to the following form

$$f_{,r} = \pm \frac{k}{(1+r^2)^{1/3}} \sqrt{r^2 - k^2(1+r^2)^{4/3}} \ . \tag{5.10}$$

The solution is well defined only for sufficiently small values of the parameter k appearing as integration constant and becomes ill defined at two singular values of the variable r. Boundary conditions are identically satisfied at the singular values of r since the radial component of induced metric diverges at these values of r. The result leads to suspect that the generation of boundary components dynamically is a general phenomenon so that all non-vacuum solutions have boundary components in accordance with basic ideas of TGD.

5.1.3 $CP_2\#CP_2\#...\#CP_2$: s as generalized Feynman graphs

There are reasons to believe that point like particles might be identified as CP_2 type extremals in TGD approach. Also the geometric counterparts of the massless on mass shell particles and virtual particles have been identified. It is natural to extend this idea to the level of particle interactions: the lines of Feynman diagrams of quantum field theory are thickened to four-manifolds, which are in a good approximation CP_2 type vacuum extremals. This would mean that generalized Feynman graphs are essentially connected sums of CP_2: s (see **Fig. 3**): $X^4 = CP_2\#CP_2....\#CP_2$).
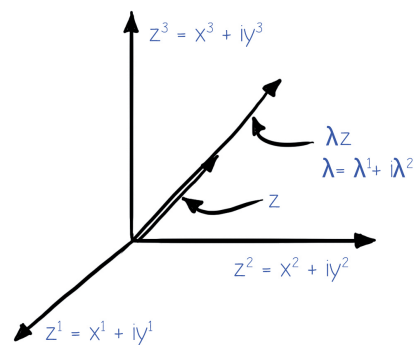

Figure 3: Topological sum of CP_2: s as Feynman graph with lines thickened to four-manifolds

Unfortunately, this picture seems to be oversimplified. First, it is questionable whether the cross sections for the scattering of CP_2 type extremals have anything to do with the cross sections associated with the standard gauge interactions. A naïve geometric argument suggests that the cross section should reflect the geometric size of the scattered objects and therefore be of the order of CP_2 radius for topologically non-condensed CP_2 type extremals. The observed cross sections would result at the first level of condensation, where particles are effectively replaced by surfaces with size of order Compton length. Secondly, the $h_{vac} = -D$ rule, considered in the previous chapter, suggests that only real particles correspond to the CP_2 type extremals whereas virtual particles in general correspond to the vacuum extremals with a vanishing Kähler action. The reason is that the negative exponent of the Kähler action reduces the contribution of the CP_2 type extremals to the functional integral very effectively. Therefore the exchanges of CP_2 type extremals are suppressed by the negative exponent of the Kähler action very effectively so that geometric scattering cross section is obtained.

5.2   Vacuum Extremals With Vanishing Kähler Field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Legendre manifold. The condition expressing Legendre manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A \quad = \quad \sum_k P_k dQ^k \ . \tag{5.11}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{5.12}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M_+^4 diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the configuration space metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D_{CP_2} = 3$ for CP_2 type extremals, $D_{CP_2} = 2$ for string like objects, $D_{CP_2} = 1$ for membranes and $D_{CP_2} = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M_+^4 \to CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M_+^4 \to D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

6 Non-Vacuum Extremals

6.1 Cosmic Strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M_+^4 (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the absolute minimization of the Kähler action is global rather than a local principle. A more general approach gives up absolute minimization as definition of preferred extremal property and there are indeed several proposals for what preferred extremal property could mean. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T \;=\; \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \;\;, \tag{6.1}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

6.2   Massless Extremals

Massless extremals (or topological light rays) are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$
\begin{aligned}
&s^k = f^k(u, v) \;\;, \\
&u = p \cdot m \;\;, \qquad v = \varepsilon \cdot m \;\;, \\
&p \cdot \varepsilon = 0 \;\;, \qquad p^2 = 0 \;\;.
\end{aligned}
$$

CP_2 coordinates are arbitrary functions of $p \cdot m$ and $\varepsilon \cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2 + m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$
\begin{aligned}
H(p \cdot m, \varepsilon \cdot m, \varepsilon_1 \cdot m) = 0 \;\;, \\
\varepsilon \cdot p = 0 \;\;, \qquad \varepsilon_1 \cdot p = 0 \;\;, \quad \varepsilon \cdot \varepsilon_1 = 0 \;\;.
\end{aligned}
\tag{6.2}
$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many ways to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta} J^k_l \partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a way to satisfy all boundary conditions but it is not clear whether there are any other ways to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamcis this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are bounded this is in accordance with the general ideas about topological condensation. The fact that massless

plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

6.3 Does GRT really allow gravitational radiation: could cosmological constant save the situation?

In Facebook discussion Nils Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see `http://tinyurl.com/y7fsnzk8`) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R. Weyl tensor C is vanishing for conformally flat space-times. In dimensions D=2,3 Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g: $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product $\cdot$ is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2}\left[Ric - \frac{s}{2(D-1)}g \right] \quad .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [K3, K2, K9] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2. CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.

2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistor lift of Kähler action [K5] and identifiable in terms of cosmological constant is added to Kähler action.

3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) \ ,$$

where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

6.4   Gravitational memory effect and quantum criticality of TGD

Gary Ehlenberg sent an interesting post about the gravitational memory effect (see this and this).

Classical gravitational waves would leave a memory of its propagation to the metric of space-time affecting distances between mass points. The computations are done by treating Einstein's theory as a field theory in the background defined by the energy momentum tensor of matter and calculations are carried out only in the lowest non-trivial order.

There are two kinds of effects: the linear memory effect occurs for instance when a planet moves along non-closed hyperbolic orbit around a star and involves only the energy momentum tensor of the system. The non-linear memory effect also involves the energy momentum tensor of gravitational radiation as a source added to the energy momentum tensor of matter.

The effect is accumulative and involves integration over the history of the matter source over the entire past. The reason why the memory effect is non-vanishing is basically that the source of the gravitational radiation is quadratic in metric. In Maxwellian electrodynamics the source does not have this property.

I have never thought of the memory effect. The formula used to estimate the effect is however highly interesting.

1. In the formula for the non-linear memory effect, that is for the action of d'Alembert operator acting on the radiation contribution to the metric, the source term is obtained by adding to the energy momentum tensor of the matter, the energy momentum tensor of the gravitational radiation.

2. This formula can be iterated and if the limit as a fixed point exists, the energy momentum tensor of the gravitational radiation produced by the total energy momentum tensor,

including also the radiative contribution, should vanish. This brings in mind fractals and criticality.

One of the basic facts about iteration for polynomials is that it need not always converge. Limit cycles typically emerge. In more complex situations also objects known as strange attractors can appear. Does the same problem occur now, when the situation is much much more complex?

3. What is interesting is that gravitational wave solutions indeed have vanishing energy momentum tensors. This is problematic if one considers them as radiation in empty space. In the presence of matter, this might be true only for very special background metrics as a sum of matter part and radiation part: just these gravitationally critical fixed point metrics.

   Could the fixed point property of these metrics (matter plus gravitational radiation) be used to gain information of the total metric as sum of matter and gravitational parts?

4. As a matter of fact, all solutions of non-linear field theories are constructed by similar iteration and the radiative contribution in a given order is determined by the contribution in lower orders.

   Under what conditions can one assume convergence of the perturbation series, that is fixed point property? Are limit cycles and chaotic attractors, and only a specialist knows what, unavoidable? Could this fixed point property have some physical relevance? Could the fixed points correspond in quantum field theory context to fixed points of the renormalization group and lead to quantization of coupling constants?

Does the fixed point property have a TGD counterpart?

1. In the TGD, framework Einstein's equations are expected only at the QFT limit at which space-time sheets are replaced with a single region of M^4 carrying gauge fields and gravitational fields, which are sums of the induced fields associated with space-time sheets. What happens at the level of the basic TGD?

   What is intriguing, is that quantum criticality is the basic principle of TGD and fixes discrete coupling constant evolution: could the quantum criticality realize itself also as gravitational criticality in the above sense? And even the idea that perturbation series can converge only at critical points and becomes actually trivial?

2. What does the year 2023 version of classical TGD say about the situation? In TGD, space-time surfaces obey almost deterministic holography required by general coordinate invariance [L9, L4]. Holography follows from the general coordinate invariance and implies that path integral trivializes to sum over the analogs of Bohr orbits of particles represented as 3-D surfaces. This states quantum criticality and fixed point property: radiative contributions vanish. This also implies a number theoretic view of coupling constant evolution based on number theoretic vision about physics.

   $M^8 - H$ duality [L2, L3, L8] implies that the space-time regions defined by Bohr orbits are extremely simple and form an evolutionary hierarchy characterized by extensions of rationals associated with polynomials characterizing the counterparts of space-time surfaces as 4-surfaces in complexified M^8 mapped to space-time surfaces in $M^4 \times CP_2$ by $M^8 - H$ duality.

   There is also universality: the Bohr orbits in H are minimal surfaces [L5], which satisfy a 4-D generalization of 2-D holomorphy and are independent of the action principle as long as it is general coordinate invariant and constructible in terms of the induced geometry. The only dependence on coupling constants comes from singularities at which minimal surface property fails. Also classical conserved quantities depend on coupling constants.

3. The so called "massless extremals" (MEs) represent radiation with very special properties such as precisely targeted propagation with light velocity, absence of dispersion of wave packed, and restricted linear superposition for massless modes propagating in the direction of ME. They are analogous to laser beams, Bohr orbits for radiation fields. The gauge currents associated with MEs are light-like and Lorentz 4-force vanishes.

4. Could the Einstein tensor of ME vanish? The energy momentum tensor expressed in terms of Einstein tensor involves a dimensional parameter and measures the breaking of scale invariance. MEs are conformally invariant objects: does this imply the vanishing of the Einstein tensor? Note however that the energy momentum tensor assignable to the induced gauge fields is non-vanishing: however, its scale covariance is an inherent property of gauge fields so that it need not vanish.

6.5   Generalization Of The Solution Ansatz Defining Massless Extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

6.5.1   Local light cone coordinates

The solution involves a decomposition of M_+^4 tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M_+^4 defining a *local* decomposition of the tangent space M^4 of M_+^4 into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

2. With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 \;\; = \;\; 2g_{+-}dS^+ dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \;\; . \tag{6.3}$$

If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

6.5.2   A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k \cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \;\; , \;\; (\nabla_3 f)^2 = 1 \;\; ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \;\; .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from

this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many ways.

6.5.3   Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \quad ,$$

   where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$

   one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+)$$

   is satisfiedin this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \quad , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \quad ,$$

$$\nabla S^+ \cdot \nabla E^i = 0 \quad , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \quad .$$

   The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M_+^4 must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

6.5.4   General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M_+^4 tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$

   where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then "free" solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

5. One can try to generalize the solution ansatz further by allowing the metric of M_+^4 to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the embedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 \;=\; 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \qquad (6.4)$$

   $g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} \;=\; s_{kl}\partial_1 s^k \partial_+ s^k \ . \qquad (6.5)$$

   The diagonalization of the metric seems to be a general aspect of preferred extremals. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

6.5.5   Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M_+^4 can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.

2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the embedding space $M_+^4 \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the absolute minimization of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of embedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

6.6   Maxwell Phase

"Maxwell phase" corresponds to small deformations of the M^4 type vacuum extremals. Since energy momentum tensor is quadratic in Kähler field the term proportional to the contraction of the energy momentum tensor with second fundamental form drops from field equations and one obtains in lowest order the following field equations

$$j^\alpha J_l^k s_{,\alpha}^l \;\; = \;\; 0 \;\; . \tag{6.6}$$

These equations are satisfied if Maxwell's equations

$$j^\alpha \;\; = \;\; 0 \tag{6.7}$$

hold true. Massless extremals and Maxwell phase clearly exclude each other and it seems that they must corresponds to different space-time sheets.

The explicit construction of these extremals reduces to the task of finding an embedding for an arbitrary free Maxwell field to H. One can also allow source terms corresponding to the presence of point like charges: these should correspond to the regions of the space-time, where the flat space-time approximation of the space-time fails. The regions where the approximation defining the Maxwell phase fails might correspond to a topologically condensed CP_2 type extremals, for example. As a consequence, Kähler field is superposition of radiation type Kähler field and of Coulomb term. A second possibility is the generation of "hole" with similar Coulombic Kähler field.

An important property of the Maxwell phase (also of massless extremals) is its approximate canonical invariance. Canonical transformations do not spoil the extremal property of the four-surface in the approximation used, since it corresponds to a mere $U(1)$ gauge transformation. This

implies the counterpart of the vacuum degeneracy, that is, the existence of an enormous number of four-surfaces with very nearly the same action. Also there is an approximate $Diff(M_+^4)$ invariance.

The canonical degeneracy has some very interesting consequences concerning the understanding of the electro-weak symmetry breaking and color confinement. Kähler field is canonical invariant and satisfies Maxwells equations. This is in accordance with the identification of Kähler field as $U(1)$ part of the electro-weak gauge field. Electromagnetic gauge field is a superposition of Kähler field and Z^0 field $\gamma = 3J - sin^2(\theta_W)Z^0/2$ so that also electromagnetic gauge field is long ranged assuming that Z^0 and W^+ fields are short ranged. These fields are not canonical invariants and their behavior seems to be essentially random, which implies short range correlations and the consequent massivation.

There is an objection against this argument. For the known $D < 4$ solutions of field equations weak fields are not random at all. These situations could represent asymptotic configurations assignable to space-time sheets. This conforms with the interpretation that weak gauge fields are essentially massless within the asymptotic space-time sheets representing weak bosons. Gauge fields are however transferred between space-time sheets through # contacts modellable as pieces of CP_2 type extremals having $D_{CP_2} = 4$. In contrast to Kähler and color gauge fluxes, weak gauge fluxes are not conserved in the Euclidian time evolution between the 3-D causal horizons separating the Euclidian # contact from space-time sheets with Minkowskian signature. This non-conservation implying the loss of coherence in the transfer of fields between space-time sheets is a plausible mechanism for the loss of correlations and massivation of the weak gauge fields.

Classical gluon fields are proportional to Kähler field and to the Hamiltonians associated with the color isometry generators.

$$g_{\alpha\beta}^A \quad = \quad kH^A J_{\alpha\beta} \ . \tag{6.8}$$

This implies that the direction of gluon fields in color algebra is random. One can always perform a canonical transformation, which reduces to a global color rotation in some arbitrary small region of space-time and reduces to identity outside this region. The proportionality of a gluon field to Kähler form implies that there is a classical long range correlation in X^4 degrees of freedom: in this sense classical gluon fields differ from massive electro-weak fields in Maxwell phase.

6.7   Stationary, Spherically Symmetric Extremals

The stationary, spherically symmetric extremals of the Kähler action imbeddable in $M^4 \times S^2$, where S^2 is geodesic sphere, are the simplest extremals, which one can study as models for the space-time surrounding a topologically condensed particle, say CP_2 type vacuum extremal. In the region near the particle the spherical symmetry is an unrealistic assumption since it excludes the presence of magnetic fields needed to cancel the total Kähler action. The stationarity is also unrealistic assumption since zitterbewegung seems to provide a necessary mechanism for generating Kähler magnetic field and for satisfying boundary conditions. Also the imbeddability to $M^4 \times S^2$ implies unrealistic relationship between Z^0 and photon charges.

According to the general wisdom, the generation of a Kähler electric field must take place in order to minimize the action and it indeed turns out that the extremal is characterized by essentially $1/r^2$ Kähler electric field. The necessary presence of a hole or of a topologically condensed object is also demonstrated: it is impossible to find extremals well defined in the region surrounding the origin. It is impossible to satisfy boundary conditions at a hole: this is in accordance with the idea that Euclidian region corresponding to a CP_2 type extremal performing zitterbewegung is generated. In case of CP_2 type extremal radius is of the order of the Compton length of the particle and in case of a "hole" of the order of Planck length. The value of the vacuum frequency ω is of order of particle mass whereas for macroscopic vacuum extremals it must be of the order of $1/R$. This does not lead to a contradiction if the concept of a many-sheeted space-time is accepted.

The Poincare energy of the exterior region is considerably smaller than the gravitational mass; this conforms with the interpretation that gravitational mass is sum of absolute values of positive and negative inertial masses associated with matter and negative energy antimatter. It is quite possible that classical considerations cannot provide much understanding concerning the inertial masses of topologically condensed particles. Electro-weak gauge forces are considerably weaker

than the gravitational force at large distances, when the value of the frequency parameter ω is of order $1/R$. Both these desirable properties fail to be true if CP_2 radius is of order Planck length as believed earlier.

In light of the general ideas about topological condensation it is clear that in planetary length scales these kind of extremals cannot provide a realistic description of space-time. Indeed, spherically symmetric extremals predict a wrong rate for the precession of the perihelion of Mercury. Scwhartschild and Reissner-Nordström metric do this and indeed allow embedding as vacuum extremals for which the inertial masses of positive energy matter and negative energy antimatter sum up to zero.

This does not yet resolve the interpretational challenge due to the unavoidable long range color and weak gauge fields. A dark matter hierarchy giving rise to a hierarchy of color and electro-weak physics characterized by increasing values of weak and confinement scales explains these fields. # contacts involve a pair of causal horizons at which the Euclidian metric signature of # contact transforms to Minkowskian one. These causal horizons have interpretation as partons so that # contact can be regarded as a bound state of partons bound together by a gravitational instanton (CP_2 type extremal). # contacts provide basic example of dark matter creating long ranged weak fields.

An important result is the correlation between the sign of the vacuum frequency ω and that of the Kähler charge, which is of opposite sign for fermions and anti-fermions. This suggests an explanation for matter-antimatter asymmetry. Matter and antimatter condense stably on disjoint regions of the space-time surface at different space-time sheets. Stable antimatter could correspond to negative time orientation and negative energy. This leads to a model for the primordial generation of matter as spontaneous generation of zero energy # contacts between space-time sheets of opposite time orientations. If CP conjugation is not exact symmetry, # contacts and their CP conjugates are created with slightly different rates and this gives rise to CP asymmetry at each of the two space-time sheets involved. After the splitting of # contacts and subsequent annihilation of particles and antiparticles at each space-time sheet, the two space-time sheets contain only positive energy matter and negative energy antimatter.

6.7.1 General solution ansatz

The general form of the solution ansatz is obtained by assuming that the space-time surface in question is a sub-manifold of $M^4 \times S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2. S^2 is most conveniently realized as $r = \infty$ surface of CP_2, for which all values of the coordinate Ψ correspond to same point of CP_2 so that one can use Θ and Φ as the coordinates of S^2.

The solution ansatz is given by the expression

$$
\begin{aligned}
cos(\Theta) &= u(r) \ , \\
\Phi &= \omega t \ , \\
m^0 &= \lambda t \ , \\
r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ .
\end{aligned}
\tag{6.9}
$$

The induced metric is given by the expression

$$
ds^2 = \left[\lambda^2 - \frac{R^2}{4} \omega^2 (1 - u^2) \right] dt^2 - (1 + \frac{R^2}{4} \theta_{,r}^2) dr^2 - r^2 d\Omega^2 \ .
\tag{6.10}
$$

The value of the parameter λ is fixed by the condition $g_{tt}(\infty) = 1$:

$$
\lambda^2 - \frac{R^2}{4} \omega^2 (1 - u(\infty)^2) = 1 \ .
\tag{6.11}
$$

From the condition $e^0 \wedge e^3 = 0$ the non-vanishing components of the induced Kähler field are given by the expression

$$J_{tr} \quad = \quad \frac{\omega}{4} u_{,r} \ . \tag{6.12}$$

Geodesic sphere property implies that Z^0 and photon fields are proportional to Kähler field:

$$
\begin{aligned}
\gamma &= (3 - p/2)J \ , \\
Z^0 &= J \ .
\end{aligned}
\tag{6.13}
$$

From this formula one obtains the expressions

$$
\begin{aligned}
Q_{em} &= \frac{(3 - p/2)}{4\pi\alpha_{em}} Q_K \ , \quad Q_Z = \frac{1}{4\pi\alpha_Z} Q \ , \\
Q &\equiv \frac{J_{tr} 4\pi r^2}{\sqrt{-g_{rr}g_{tt}}} \ .
\end{aligned}
\tag{6.14}
$$

for the electromagnetic and Z^0 charges of the solution using e and g_Z as unit.

Field equations can be written as conditions for energy momentum conservation (two equations is in principle all what is needed in the case of geodesic sphere). Energy conservation holds identically true and conservation of momentum, say, in z-direction gives the equation

$$(T^{rr} z_{,r})_{,r} + (T^{\theta\theta} z_{,\theta})_{,\theta} \quad = \quad 0 \ . \tag{6.15}$$

Using the explicit expressions for the components of the energy momentum tensor

$$
\begin{aligned}
T^{rr} &= g^{rr} L/2 \ , \\
T^{\theta\theta} &= -g^{\theta\theta} L/2 \ , \\
L &= g^{tt} g^{rr} (J_{tr})^2 \sqrt{g}/2 \ ,
\end{aligned}
\tag{6.16}
$$

and the following notations

$$
\begin{aligned}
A &= g^{tt} g^{rr} r^2 \sqrt{-g_{tt}g_{rr}} \ , \\
X &\equiv (J_{tr})^2 \ ,
\end{aligned}
\tag{6.17}
$$

the field equations reduce to the following form

$$(g^{rr} AX)_{,r} - \frac{2AX}{r} \quad = \quad 0 \ . \tag{6.18}$$

In the approximation $g^{rr} = 1$ this equation can be readily integrated to give $AX = C/r^2$. Integrating Eq. (6.18), one obtains integral equation for X

$$J_{tr} \quad = \quad \frac{q}{r_c} (|g_{rr}|^3 g_{tt})^{1/4} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r}) \frac{1}{r} \ , \tag{6.19}$$

where q is integration constant, which is related to the charge parameter of the long range Kähler electric field associated with the solution. r_c denotes the critical radius at which the solution ceases to be well defined.

The inspection of this formula shows that J_{tr} behaves essentially as $1/r^2$ Coulomb field. This behavior doesn't depend on the detailed properties of the solution ansatz (for example the imbeddability to $M^4 \times S^2$): stationarity synd spherical symmetry is what matters only. The compactness of CP_2 means that stationary, spherically symmetric solution is not possible in the region containing origin. This is in concordance with the idea that either a hole surrounds the origin or there is a

topologically condensed CP_2 type extremal performing zitterbewegung near the origin and making the solution non-stationary and breaking spherical symmetry.

Second integration gives the following integral equation for CP_2 coordinate $u = cos(\Theta)$

$$u(r) \quad = \quad u_0 + \frac{4q}{\omega} \int_{r_c}^{r} (-g_{rr}^3 g_{tt})^{1/4} \frac{1}{r} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r}) \quad . \tag{6.20}$$

Here u_0 denotes the value of the coordinate u at $r = r_0$.

The form of the field equation suggests a natural iterative procedure for the numerical construction of the solution for large values of r.

$$u_n(r) \quad = \quad T_{n-1} \quad , \tag{6.21}$$

where T_{n-1} is evaluated using the induced metric associated with u_{n-1}. The physical content of the approximation procedure is clear: estimate the gravitational effects using lower order solution since these are expected to be small.

A more convenient manner to solve u is based on Taylor expansion around the point $V \equiv 1/r = 0$. The coefficients appearing in the power series expansion $u = \sum_n u_n A^n V^n$: $A = q/\omega$ can be solved by calculating successive derivatives of the integral equation for u.

The lowest order solution is simply

$$u_0 \quad = \quad u_\infty \quad , \tag{6.22}$$

and the corresponding metric is flat metric. In the first order one obtains for $u(r)$ the expression

$$u \quad = \quad u_\infty - \frac{4q}{\omega r} \quad , \tag{6.23}$$

which expresses the fact that Kähler field behaves essentially as $1/r^2$ Coulomb field. The behavior of u as a function of r is identical with that obtained for the embedding of the Reissner-Nordström solution.

To study the properties of the solution we fix the signs of the parameters in the following manner:

$$u_\infty < 0 \quad , \quad q < 0 \quad , \quad \omega > 0 \tag{6.24}$$

(reasons become clear later).

Concerning the behavior of the solution one can consider two different cases.

1. The condition $g_{tt} > 0$ hold true for all values of Θ. In this case u decreases and the rate of decrease gets faster for small values of r. This means that in the lowest order the solution becomes certainly ill defined at a critical radius $r = r_c$ given by the the condition $u = 1$: the reason is that u cannot get values large than one. The expression of the critical radius is given by

$$\begin{aligned} r_c \quad &\geq \quad \frac{4q}{(|u_\infty| + 1)\omega} \\ &= \quad \frac{4\alpha Q_{em}}{(3 - p/2)} \frac{1}{(|u_\infty| + 1)\omega} \quad . \end{aligned} \tag{6.25}$$

The presence of the critical radius for the actual solution is also a necessity as the inspection of the expression for J_{tr} shows: $\partial_r \theta$ grows near the origin without bound and $u = 1$ is reached at some finite value of r. Boundary conditions require that the quantity $X = T^{rr}\sqrt{g}$ vanishes at critical radius (no momentum flows through the boundary). Substituting the expression

of J_{tr} from the field equation to T^{rr} the expression for X reduces to a form, from which it is clear that X cannot vanish. The cautious conclusion is that boundary conditions cannot be satisfied and the underlying reason is probably the stationarity and spherical symmetry of the solution. Physical intuition suggests that that CP_2 type extremal performing zitterbewegung is needed to satisfy the boundary conditions.

2. g_{tt} vanishes for some value of Θ. In this case the radial derivative of u together with g_{tt} can become zero for some value of $r = r_c$. Boundary conditions can be satisfied only provided $r_c = 0$. Thus it seems that for the values of ω satisfying the condition $\omega^2 = \frac{4\lambda^2}{R^2 sin^2(\Theta_0)}$ it might be possible to find a globally defined solution. The study of differential equation for u however shows that the ansatz doesn't work. The conclusion is that although the boundary is generated it is not possible to satisfy boundary conditions.

A direct calculation of the coefficients u_n from power series expansion gives the following third order polynomial approximation for u ($V = 1/r$)

$$
\begin{aligned}
u &= \sum_n u_n A^n V^n \ , \\
u_0 &= u_\infty (< 0) \ , \quad u_1 = 1 \ , \\
u_2 &= K|u_\infty| \ , \quad u_3 = K(1 + 4K|u_\infty|) \ , \\
A &\equiv \frac{4q}{\omega} \ , \quad K \equiv \omega^2 \frac{R^2}{4} \ .
\end{aligned}
$$

$$(6.26)$$

The coefficients u_2 and u_3 are indeed positive which means that the value of the critical radius gets larger at least in these orders.

Solution contains three parameters: Kähler electric flux $Q = 4\pi q$, parameter ωR and parameter u_∞. The latter parameters can be regarded as parameters describing the properties of a flat vacuum extremal (lowest order solution) to which particle like solution is glued and are analogous to the parameters describing symmetry broken vacuum in gauge theories.

6.7.2   Solution is not a realistic model for topological condensation

The solution does not provide realistic model for topological condensation although it gives indirect support for some essential assumptions of TGD based description of Higgs mechanism.

1. When the value of ω is of the order of CP_2 mass the solution could be interpreted as the "exterior metric" of a "hole".
   i) The radius of the hole is of the order of CP_2 length and its mass is of the order of CP_2 mass.
   ii) Kähler electric field is generated and charge renormalization takes place classically at CP_2 length scales as is clear from the expression of $Q(r)$: $Q(r) \propto (\frac{-g_{rr}}{g_{tt}})^{1/4}$ and charge increases at short distances.
   iii) The existence of the critical radius is unavoidable but boundary conditions cannot be satisfied. The failure to satisfy boundary conditions might be related to stationarity or to the absence of magnetic field. The motion of the boundary component with velocity of light might be the only manner to satisfy boundary conditions. Second possibility is the breaking of spherical symmetry by the generation of a static magnetic field.
   iv) The absence of the Kähler magnetic field implies that the Kähler action has an infinite magnitude and the probability of the configuration is zero. A more realistic solution ansatz would break spherical symmetry containing dipole type magnetic field in the nearby region of the hole. The motion of the boundary with a velocity of light could serves as an alternative mechanism for the generation of magnetic field. The third possibility, supported by physical intuition, is that one must give up "hole" type extremal totally.

2. For sufficiently large values of r and for small values of ω (of the order of elementary particle mass scale), the solution might provide an approximate description for the region surrounding

elementary particle. Although it is not possible to satisfy boundary conditions the order of magnitude estimate for the size of critical radius ($r_c \simeq \alpha/\omega$) should hold true for more realistic solutions, too. The order of magnitude for the critical radius is smaller than Compton length or larger if the vacuum parameter ω is larger than the mass of the particle. In macroscopic length scales the value of ω is of order $1/R$. This does not lead to a contradiction if the many-sheeted space-time concept is accepted so that $\omega < m$ corresponds to elementary particle space-time sheet. An unrealistic feature of the solution is that the relationship between Z^0 and em charges is not correct: Z^0 charge should be very small in these length scales.

6.7.3 Exterior solution cannot be identified as a counterpart of Schwartshild solution

The first thing, which comes into mind is to ask whether one might identify exterior solution as the TGD counterpart of the Schwartshild solution. The identification of gravitational mass as absolute value of inertial mass which is negative for antimatter implies that vacuum extremals are vacua only with respect to the inertial four-momentum and have a non-vanishing gravitational four-momentum. Hence, in the approximation that the net density of inertial mass vanishes, vacuum extremals provide the proper manner to model matter, and the identification of the ansatz for a spherically symmetric extremal as the counterpart of Scwhartschild metric is certainly not possible. It is however useful to show explicitly that the identification is indeed unrealistic. The solution is consistent with Equivalence Principle but the electro-weak gauge forces are considerably weaker than gravitational forces. A wrong perihelion shift is also predicted so that the identification as an exterior metric of macroscopic objects is out of question.

1. Is Equivalence Principle respected?

The following calculation demonstrates that Equivalence Principle might not be satisfied for the solution ansatz (which need not actually define a preferred extremal!).

The gravitational mass of the solution is determined from the asymptotic behavior of g_{tt} and is given by

$$ M_{gr} \quad = \quad \frac{R^2}{G}\omega q u_\infty \ , \tag{6.27} $$

and is proportional to the Kähler charge q of the solution.

One can estimate the gravitational mass density also by applying Newtonian approximation to the time component of the metric $g_{tt} = 1 - 2\Phi_{gr}$. One obtains Φ_{gr} corresponds in the lowest order approximation to a solution of Einstein's equations with the source consisting of a mass point at origin and the energy density of the Kähler electric field. The effective value of gravitational constant is however $G_{eg} = 8R^2\alpha_K$. Thus the only sensible interpretation is that the density of Kähler (inertial) energy is only a fraction $G/G_{eq} \equiv \epsilon \simeq .22 \times 10^{-6}$ of the density of gravitational mass. Hence the densities of positive energy matter and negative energy antimatter cancel each other in a good approximation.

The work with cosmic strings lead to a possible interpretation of the solution as a space-time sheet containing topologically condensed magnetic flux tube idealizable as a point. The negative Kähler electric action must cancel the positive Kähler magnetic action. The resulting structure in turn can condense to a vacuum extremal and Schwartshild metric is a good approximation for the metric.

One can estimate the contribution of the exterior region ($r > r_c$) to the inertial mass of the system and Equivalence principle requires this to be a fraction of order ϵ about the gravitational mass unless the region $r < r_c$ contains negative inertial mass density, which is of course quite possible. Approximating the metric with a flat metric and using first order approximation for $u(r)$ the energy reduces just to the standard Coulomb energy of charged sphere with radius r_c

$$
\begin{aligned}
M_I(ext) &= \frac{1}{32\pi\alpha_K}\int_{r>r_c} E^2\sqrt{g}d^3x \\
&\simeq \frac{\lambda q^2}{2\alpha_K r_c} \ , \\
\lambda &= \sqrt{1 + \frac{R^2}{4}\omega^2(1-u_\infty^2)} \ (>1) \ .
\end{aligned}
\tag{6.28}
$$

Approximating the metric with flat metric the contribution of the region $r > r_c$ to the energy of the solution is given by

$$
M_I(ext) = \frac{1}{8\alpha_K}\lambda q\omega(1+|u_\infty|) \ .
\tag{6.29}
$$

The contribution is proportional to Kähler charge as expected. The ratio of external inertial and gravitational masses is given by the expression

$$
\begin{aligned}
\frac{M_I(ext)}{M_{gr}} &= \frac{G}{4R^2\alpha_K}x \ , \\
x &= \frac{(1+|u_\infty|)}{|u_\infty|} > 1 \ .
\end{aligned}
\tag{6.30}
$$

In the approximation used the ratio of external inertial and gravitational masses is of order 10^{-6} for $R \sim 10^4\sqrt{G}$ implied by the p-adic length scale hypothesis and for $x \sim 1$. The result conforms with the above discussed interpretation.

The result forces to challenge the underlying implicit assumptions behind the calculation.

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K13]. This effective metric should correspond to that of General Relativity and Einstein's equations reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.

2. The systems considered need not be preferred extremals of Kähler action so that one cannot take the results obtained too seriously. For vacuum extremals one does not encounter this problem at all and it could be that vacuum extremals with induced metric identified as GRT metric are a good approximation in astrophysical systems. This requires that single-sheetedness is a good approximation. TGD based single-sheeted models for astrophysical and cosmological systems rely on this assumption.

2. Z^0 and electromagnetic forces are much weaker than gravitational force

The extremal in question carries Kähler charge and therefore also Z^0 and electromagnetic charge. This implies long range gauge interactions, which ought to be weaker than gravitational interaction in the astrophysical scales. This is indeed the case as the following argument shows.

Expressing the Kähler charge using Planck mass as unit and using the relationships between gauge fields one obtains a direct measure for the strength of the Z^0 force as compared with the strength of gravitational force.

$$
Q_Z \equiv \varepsilon_Z M_{gr}\sqrt{G} \ .
\tag{6.31}
$$

The value of the parameter ε_Z should be smaller than one. A transparent form for this condition is obtained, when one writes $\Phi = \omega t = \Omega m^0 : \Omega = \lambda\omega$:

$$\varepsilon_Z \quad = \quad \frac{\alpha_K}{\alpha_Z} \frac{1}{\pi(1 + |u_\infty|)\Omega R} \sqrt{\frac{G}{R}} \quad . \tag{6.32}$$

The order of magnitude is determined by the values of the parameters $\sqrt{\frac{G}{R^2}} \sim 10^{-4}$ and ΩR. Global Minkowskian signature of the induced metric implies the condition $\Omega R < 2$ for the allowed values of the parameter ΩR. In macroscopic length scales one has $\Omega R \sim 1$ so that Z^0 force is by a factor of order 10^{-4} weaker than gravitational force. In elementary particle length scales with $\omega \sim m$ situation is completely different as expected.

3. The shift of the perihelion is predicted incorrectly

The g_{rr} component of Reissner-Nordström and TGD metrics are given by the expressions

$$g_{rr} \quad = \quad -\frac{1}{\left(1 - \frac{2GM}{r}\right)} \quad , \tag{6.33}$$

and

$$g_{rr} \quad \simeq \quad 1 - \frac{\frac{Rq}{\omega^2}}{\left[1 - \left(u_\infty - \frac{4q}{\omega r}\right)^2\right] r^4} \quad , \tag{6.34}$$

respectively. For reasonable values of q, ω and u_∞ the this terms is extremely small as compared with $1/r$ term so that these expressions differ by $1/r$ term.

The absence of the $1/r$ term from g_{rr}-component of the metric predicts that the shift of the perihelion for elliptic plane orbits is about $2/3$ times that predicted by GRT so that the identification as a metric associated with objects of a planetary scale leads to an experimental contradiction. Reissner-Nordström solutions are obtained as vacuum extremals so that standard predictions of GRT are obtained for the planetary motion.

One might hope that the generalization of the form of the spherically symmetric ansatz by introducing the same modification as needed for the embedding of Reissner-Nordstrm̈ metric might help. The modification would read as

$$\begin{aligned}
cos(\Theta) \quad &= \quad u(r) \quad , \\
\Phi \quad &= \quad \omega t + f(r) \quad , \\
m^0 \quad &= \quad \lambda t + h(r) \quad , \\
r_M \quad &= \quad r \quad , \quad \theta_M = \theta \quad , \quad \phi_M = \phi \quad .
\end{aligned} \tag{6.35}$$

The vanishing of the g_{tr} component of the metric gives the condition

$$\lambda \partial_r h - \frac{R^2}{4} sin^2(\Theta) \omega \partial_r f \quad = \quad 0 \quad . \tag{6.36}$$

The expression for the radial component of the metric transforms to

$$g_{rr} \quad \simeq \quad \partial_r h^2 - 1 - \frac{R^2}{4}(\partial_r \Theta)^2 - \frac{R^2}{4} sin^2(\Theta) \partial_r f^2 \quad , \tag{6.37}$$

Essentially the same perihelion shift as for Schwartschild metric is obtained if g_{rr} approaches asymptotically to its expression for Schwartschild metric. This is guaranteed if the following conditions hold true:

$$f(r)_{r\to\infty} \to \omega r \quad , \quad \Lambda^2 - 1 = \frac{R^2\omega^2}{4} sin^2(\Theta_\infty) \ll \frac{2GM}{\langle r \rangle} \quad . \tag{6.38}$$

In the second equation $\langle r \rangle$ corresponds to the average radius of the planetary orbit.

The field equations for this ansatz can be written as conditions for energy momentum and color charge conservation. Two equations are enough to determine the functions $\Theta(r)$ and $f(r)$. The equation for momentum conservation is same as before. Second field equation corresponds to the conserved isometry current associated with the color isometry $\Phi \to \Phi + \epsilon$ and gives equation for f.

$$[T^{rr} f_{,r} s_{\Phi\Phi} \sqrt{g}]_{,r} \quad = \quad 0 \quad . \tag{6.39}$$

The conservation laws associated with other infinitesimal $SU(2)$ rotations of S_I^2 should be satisfied identically. This equation can be readily integrated to give

$$T^{rr} f_{,r} s_{\Phi\Phi} \sqrt{g_{tt} g_{rr}} \quad = \quad \frac{C}{r^2} \quad . \tag{6.40}$$

Unfortunately, the result is inconsistent with the $1/r^4$ behavior of T^{rr} and $f \to \omega r$ implies by correct red shift.

It seems that the only possible way out of the difficulty is to replace spherical symmetry with a symmetry with respect to the rotations around z-axis. The simplest modification of the solution ansatz is as follows:

$$m^0 = \lambda t + h(\rho) \ , \quad \Phi = \omega t + k\rho \quad .$$

Thanks to the linear dependence of Φ on ρ, the conservation laws for momentum and color isospin reduce to the same condition. The ansatz induces a small breaking of spherical symmetry by adding to $g_{\rho\rho}$ the term

$$(\partial_\rho h)^2 - \frac{R^2}{4} sin^2(\Theta) k^2 \quad .$$

One might hope that in the plane $\theta = \pi/2$, where $r = \rho$ holds true, the ansatz could behave like Schwartschild metric if the conditions discussed above are posed (including the condition $k = \omega$). The breaking of the spherical symmetry in the planetary system would be coded already to the gravitational field of Sun.

Also the study of the embeddings of Reissner-Nordström metric as vacuum extremals and the investigation of spherically symmetric (inertial) vacuum extremals for which gravitational four-momentum is conserved [K13] leads to the conclusion that the loss of spherical symmetry due to rotation is inevitable characteristic of realistic solutions.

6.8   Maxwell Hydrodynamics As A Toy Model For TGD

The field equations of TGD are extremely non-linear and all known solutions have been discovered by symmetry arguments. Chern-Simons term plays essential role also in the construction of solutions of field equations and at partonic level defines braiding for light-like partonic 3-surfaces expected to play key role in the construction of S-matrix. The inspiration for this section came from Terence Tao's blog posting *2006 ICM: Etienne Ghys, "Knots and dynamics"* [A6] giving an elegant summary about amazing mathematical results related to knots, links, braids and hydrodynamical flows in dimension $D = 3$. Posting tells about really amazing mathematical results related to knots.

6.8.1   Chern-Simons term as helicity invariant

Tao mentions helicity as an invariant of fluid flow. Chern-Simons action defined by the induced Kähler gauge potential for light-like 3-surfaces has interpretation as helicity when Kähler gauge potential is identified as fluid velocity. This flow can be continued to the interior of space-time sheet. Also the dual of the induced Kähler form defines a flow at the light-like partonic surfaces but not in the interior of space-time sheet. The lines of this flow can be interpreted as magnetic field lines. This flow is incompressible and represents a conserved charge (Kähler magnetic flux).

The question is which of these flows should define number theoretical braids. Perhaps both of them can appear in the definition of S-matrix and correspond to different kinds of partonic matter (electric/magnetic charges, quarks/leptons?, ...). Second kind of matter could not flow in the interior of space-time sheet. Or could interpretation in terms of electric magnetic duality make sense?

Helicity is not gauge invariant and this is as it must be in TGD framework since CP_2 symplectic transformations induce $U(1)$ gauge transformation, which deforms space-time surface an modifies induced metric as well as classical electroweak fields defined by induced spinor connection. Gauge degeneracy is transformed to spin glass degeneracy.

6.8.2   Maxwell hydrodynamics

In TGD Maxwell's equations are replaced with field equations which express conservation laws and are thus hydrodynamical in character. With this background the idea that the analogy between gauge theory and hydrodynamics might be applied also in the reverse direction is natural. Hence one might ask what kind of relativistic hydrodynamics results if assumes that the action principle is Maxwell action for the four-velocity u^α with the constraint term saying that light velocity is maximal signal velocity.

1. For massive particles the length of four-velocity equals to 1: $u^\alpha u_\alpha = 1$. In massless case one has $u^\alpha u_\alpha = 0$. Geometrically this means that one has sigma model with target space which is 3-D Lobatschevski space or at light-cone boundary. This condition means the addition of constraint term

$$\lambda(u^\alpha u_\alpha - \epsilon) \tag{6.41}$$

   to the Maxwell action. $\epsilon = 1/0$ holds for massive/massless flow. In the following the notation of electrodynamics is used to make easier the comparison with electrodynamics.

2. The constraint term destroys gauge invariance by allowing to express A^0 in terms of A^i but in general the constraint is not equivalent to a choice of gauge in electrodynamics since the solutions to the field equations with constraint term are not solutions of field equations without it. One obtains field equations for an effectively massive em field with Lagrange multiplier λ having interpretation as photon mass depending on space-time point:

$$\begin{aligned}
j^\alpha &= \partial_\beta F^{\alpha\beta} = \lambda A^\alpha \ , \\
A^\alpha &\equiv u^\alpha \ , \ F^{\alpha\beta} = \partial^\beta A^\alpha - \partial^\alpha A^\beta \ .
\end{aligned} \tag{6.42}$$

3. In electrodynamic context the natural interpretation would be in terms of spontaneous massivation of photon and seems to occur for both values of ϵ. The analog of em current given by λA^α is in general non-vanishing and conserved. This conservation law is quite strong additional constraint on the hydrodynamics. What is interesting is that breaking of gauge invariance does not lead to a loss of charge conservation.

4. One can solve λ by contracting the equations with A_α to obtain

$$\lambda = j^\alpha A_\alpha$$

   for $\epsilon = 1$. For $\epsilon = 0$ one obtains

$$j^\alpha A_\alpha = 0$$

   stating that the field does not dissipate energy: λ can be however non-vanishing unless field equations imply $j^\alpha = 0$. One can say that for $\epsilon = 0$ spontaneous massivation can occur. For $\epsilon = 1$ massivation is present from the beginning and dissipation rate determines photon

mass: a natural interpretation for $\epsilon = 1$ would be in terms of thermal massivation of photon. Non-tachyonicity fixes the sign of the dissipation term so that the thermodynamical arrow of time is fixed by causality.

5. For $\epsilon = 0$ massless plane wave solutions are possible and one has

$$\partial_\alpha \partial_\beta A^\beta = \lambda A_\alpha \ .$$

$\lambda = 0$ is obtained in Lorentz gauge which is consistent with the condition $\epsilon = 0$. Also superpositions of plane waves with same polarization and direction of propagation are solutions of field equations: these solutions represent dispersionless precisely targeted pulses. For superpositions of plane waves λ with 4-momenta, which are not all parallel λ is non-vanishing so that non-linear self interactions due to the constraint can be said to induce massivation. In asymptotic states for which gauge symmetry is not broken one expects a decomposition of solutions to regions of space-time carrying this kind of pulses, which brings in mind final states of particle reactions containing free photons with fixed polarizations.

6. Gradient flows satisfying the conditions

$$A_\alpha \ \ = \ \ \partial_\alpha \Phi \ \ , \ \ A^\alpha A_\alpha = \epsilon \tag{6.43}$$

give rise to identically vanishing hydrodynamical gauge fields and $\lambda = 0$ holds true. These solutions are vacua since energy momentum tensor vanishes identically. There is huge number of this kind of solutions and spin glass degeneracy suggests itself. Small deformations of these vacuum flows are expected to give rise to non-vacuum flows.

7. The counterparts of charged solutions are of special interest. For $\epsilon = 0$ the solution $(u^0, u^r) = (Q/r)(1,1)$ is a solution of field equations outside origin and corresponds to electric field of a point charge Q. In fact, for $\epsilon = 0$ any ansatz $(u^0, u^r) = f(r)(1,1)$ satisfies field equations for a suitable choice of $\lambda(r)$ since the ratio of equations associate with j^0 and j^r gives an equation which is trivially satisfied. For $\epsilon = 1$ the ansatz $(u^0, u^r) = (cosh(u), sinh(u))$ expressing solution in terms of hyperbolic angle linearizes the field equation obtained by dividing the equations for j^0 and j^r to eliminate λ. The resulting equation is

$$\partial_r^2 u + \frac{2\partial_r u}{r} = 0$$

for ordinary Coulomb potential and one obtains $(u^0, u^r) = (cosh(u_0 + k/r), sinh(u_0 + k/r))$. The charge of the solution at the limit $r \to \infty$ approaches to the value $Q = sinh(u_0)k$ and diverges at the limit $r \to 0$. The charge increases exponentially as a function of $1/r$ near origin rather than logarithmically as in QED and the interpretation in terms of thermal screening suggests itself. Hyperbolic ansatz might simplify considerably the field equations also in the general case.

6.8.3   Similarities with TGD

There are strong similarities with TGD which suggests that the proposed model might provide a toy model for the dynamics defined by Kähler action.

1. Also in TGD field equations are essentially hydrodynamical equations stating the conservation of various isometry charges. Gauge invariance is broken for the induced Kähler field although Kähler charge is conserved. There is huge vacuum degeneracy corresponding to vanishing of induced Kähler field and the interpretation is in terms of spin glass degeneracy.

2. Also in TGD dissipation rate vanishes for the known solutions of field equations and a possible interpretation is as space-time correlates for asymptotic non-dissipating self organization patterns.

3. In TGD framework massless extremals represent the analogs for superpositions of plane waves with fixed polarization and propagation direction and representing targeted and dispersion-less propagation of signal. Gauge currents are light-like and non-vanishing for these solutions. The decomposition of space-time surface to space-time sheets representing particles is much more general counterpart for the asymptotic solutions of Maxwell hydrodynamics with vanishing λ.

4. In TGD framework one can consider the possibility that the four-velocity assignable to a macroscopic quantum phase is proportional to the induced Kähler gauge potential. In this kind of situation one could speak of a quantal variant of Maxwell hydrodynamics, at least for light-like partonic 3-surfaces. For instance, the condition

$$D^\alpha D_\alpha \Psi = 0 \ , \ \ D_\alpha \Psi = (\partial_\alpha - iq_K A_\alpha)\Psi$$

for the order parameter of the quantum phase corresponds at classical level to the condition $p^\alpha = q_K Q^\alpha + l^\alpha$, where q_K is Kähler charge of fermion and l^α is a light-like vector field naturally assignable to the partonic boundary component. This gives $u^\alpha = (q_K Q^\alpha + l^\alpha)/m$, $m^2 = p^\alpha p_\alpha$, which is somewhat more general condition. The expressibility of u^α in terms of the vector fields provided by the induced geometry is very natural.

The value ϵ depends on space-time region and it would seem that also $\epsilon = -1$ is possible meaning tachyonicity and breaking of causality. Kähler gauge potential could however have a time-like pure gauge component in M^4 possibly saving the situation. The construction of quantum TGD at parton level indeed forces to assume that Kähler gauge potential has Lorentz invariant M^4 component $A_a = constant$ in the direction of the light-cone proper time coordinate axis a. Note that the decomposition of WCW to sectors consisting of space-time sheets inside future or past light-cone of M^4 is an essential element of the construction of WCW geometry and does not imply breaking of Poincare invariance. Without this component $u_\alpha u^\alpha$ could certainly be negative. The contribution of M^4 component could prevent this for preferred extremals.

If TGD is taken seriously, these similarities force to ask whether Maxwell hydrodynamics might be interpreted as a nonlinear variant of electrodynamics. Probably not: in TGD em field is proportional to the induced Kähler form only in special cases and is in general non-vanishing also for vacuum extremals.

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