WCW Spinor Structure

M. Pitkänen,

February 2, 2024
Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland. ORCID: 0000-0002-8051-4364.

Contents

Abstract

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.
2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.
3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.
4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

   The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

awcwspin

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the WCW spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the WCW and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the WCW are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

2. Kähler-Dirac equation for induced spinor fields

Super-symmetry between fermionic and and WCW degrees of freedom dictates that Kähler-Dirac action is the unique choice for the Dirac action

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vahish at these surfaces.

   The condition that also spinor dynamics is associative suggests strongly that the localization to 2-D surface occurs always (for right-handed neutrino the above conditions does not apply this). The induced gauge potentials are the possible source of trouble but the holomorphy of spinor modes completely analogous to that encountered in string models saves the situation. Whether holomorphy could be replaced with its quaternionic counterpart in Euclidian regions is not clear (this if W fields vanish at the entire space-time surface so that 4-D modes are possible). Neither it is clear whether the localization to 2-D surfaces occurs also in Euclidian regions with 4-D CP_2 projection.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry. This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

1 Introduction

Quantum TGD should be reducible to the classical spinor geometry of the configuration space ("world of classical worlds" (WCW)). The possibility to express the components of WCW Kähler metric as anti-commutators of WCW gamma matrices becomes a practical tool if one assumes that WCW gamma matrices correspond to Noether super charges for super-symplectic algebra of WCW. The possibility to express the Kähler metric also in terms of Kähler function identified as Kähler for Euclidian space-time regions leads to a duality analogous to AdS/CFT duality.

1.1 Basic Principles

Physical states should correspond to the modes of the WCW spinor fields and the identification of the fermionic oscillator operators as super-symplectic charges is highly attractive. WCW spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the WCW spinor structure there are some important clues.

1.1.1 Geometrization of fermionic statistics in terms of WCW spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. [B7] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   As a consequence, WCW spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field.

2. The classical theory for the bosonic fields is an essential part of the WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. Contrary to the long held belief it seems that covariantly constant right handed neutrino does not generate . The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the embedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \ .$$

where J_{AB} denotes the matrix elements of the Kähler form of the WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

5. TGD as a generalized number theory vision leads to the understanding of how the second quantization of the induced spinor fields should be carried out and space-time conformal symmetries allow to explicitly solve the Dirac equation associated with the Kähler-Dirac action in the interior and at the 3-D light like causal determinants. An essentially new element is the notion of number theoretic braid forced by the fact that the Kähler-Dirac operator allows only finite number of generalized eigen modes so that the number of fermionic oscillator operators is finite. As a consequence, anti-commutation relations can be satisfied only for a finite set of points defined by the number theoretic braid, which is uniquely identifiable. The interpretation is in terms of finite measurement resolution. The finite Clifford algebra spanned by the fermionic oscillator operators is interpreted as the factor space $\mathcal{M}/\mathcal{N}$ of infinite hyper-finite factors of type II_1 defined by WCW Clifford algebra $\mathcal{N}$ and included Clifford algebra $\mathcal{M} \subset \mathcal{N}$ interpreted as the characterizer of the finite measurement resolution. Note that the finite number of eigenvalues guarantees that Dirac determinant identified as the exponent of Kähler function is finite. Finite number of eigenvalues is also essential for number theoretic universality.

1.1.2 Identification of WCW gamma matrices as super Hamiltonians and expression of WCW Kähler metric

The basic super-algebra corresponds to the fermionic oscillator operators and can be regarded as a generalization $\mathcal{N}$ super algebras by replacing $\mathcal{N}$ with the number of solutions of the Kähler-Dirac equation which can be infinite. This leads to QFT SUSY limit of TGD different in many respects crucially from standard SUSYs.

WCW gamma matrices are identified as super generators of super-symplectic and are expressible in terms of these oscillator operators. In the original proposal super-symplectic and super charges were assumed to be expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory.

It took quite long time to realize that it is possible to second quantize induced spinor fields by using just the standard canonical quantization. The only new element is the replacement of the ordinary gamma matrices with K-D gamma matrices identified as canonical momentum currents contracted with the embedding space gamma matrices. This allows to deduce super-generators of super-symplectic algebra as Noether supercharges assignable to the fermionic strings connecting partonic 2-surfaces. Their anti-commutators giving the matrix elements of WCW Kähler metric can be deduced explicitly. This is a decisive calculational advantage since the formal expression of the matrix elements in terms of second derivatives of Kähler function is not possible to calculate with the recent understanding. WCW gamma matrices provide also a natural identification for the counterparts of fermionic oscillator operators creating physical states.

One can also deduce the fermionic Hamiltonians as conserved Noether charges. The expressions for Hamiltonians generalized the earlier expressions as Hamiltonian fluxes in the sense that the embedding space Hamiltonian is replaced with the corresponding fermionic Noether charge. This replacement is analogous to a transition from field theory to string models requiring the replacement of points of partonic 2-surfaces with stringy curves connecting the points of two partonic 2-surfaces. One can consider also several strings emanating from a given partonic 2-surface. This leads to an extension of the super-symplectic algebra to a Yangian, whose generators are multi-local (multi-stringy) operators. This picture does not mean loss of effective 2-dimensionality implied by strong form of general coordinate invariance but allows genuine generalization of super-conformal invariance in 4-D context.

1.2 Kähler-Dirac Action

Supersymmetry fixes the interior part of Kähler-Dirac uniquely. The K-D gamma matrices are contractions of the canonical momentum currents of Kähler action with the embedding space gamma matrices and this gives field equations consistent with hermitian conjugation. The modes of K-D equation must be restricted to 2-D string world sheets with vanishing induced W boson fields in order that they have a well-defined em charge. It is not yet clear whether this restriction is part of variational principle or whether it is a property of spinor modes. For the latter option modes one can have 4-D modes if the space-time surface has CP_2 projection carrying vanishing W gauge potentials. Also covariantly constant right-handed neutrino defines this kind of mode.

1.2.1 The boundary terms of Kähler action and Kähler-Dirac action

A long standing question has been whether Kähler action could contain Chern-Simons term cancelling the Chern-Simons contribution of Kähler action at space-time interior at partonic orbit reducing to Chern-Simons terms so that only the contribution at space-like ends of space-time surface at the boundaries of causal diamond (CD) remains. This is however not necessary and super-symmetry would require Chern-Simons-Dirac term as boundary term in Dirac action. This however has unphysical implications since C-S-D Dirac operator acts on CP_2 coordinates only.

The intuitive expectation is that fermionic propagators assignable to string boundaries at light-like partonic orbits are needed in the construction of the scattering amplitudes. These boundaries can be locally space-like or light-like. One could add 1-D massles Dirac action with gamma matrices defined in the induced metric, which is by supersymmetry accompanied by the action defined by geodesic length, which however vanishes for light-like curves. Massless Dirac equation at the boundary of string world sheet fixes the boundary conditions for the spinor modes at the string world sheet. This option seems to be the most plausible at this moment.

1.2.2 Kähler-Dirac equation for induced spinor fields

It has become clear that Kähler-Dirac action with induced spinor fields localized at string world sheets carrying vanishing classical W fields, and the light-like boundaries of the string world sheets at light-like orbits of partonic 2-surfaces carrying massless Dirac operator for induced gamma matrices is the most natural looking option.

The light-like momentum associated with the boundary is a light-like curve of imbedding space and defines light-like 8-momentum, whose M^4 projection is in general time-like. This leads to an 8-D generalization of twistor formalism. The squares of the M^4 and CP_2 parts of the 8-momentum could be identified as mass squared for the embedding space spinor mode assignable to the ground state of super-symplectic representation. This would realize quantum classical correspondence for fermions. The four-momentum assignable to fermion line would have identification as gravitational four-momentum and that associated with the mode of embedding space spinor field as inertial four-momentum.

There are several approaches for solving the Kähler-Dirac (or Kähler-Dirac) equation.

1. The most promising approach assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of both strong form of holography and of number theoretic vision, and also follows from the notion of finite measurement resolution having discretization at partonic 2-surfaces as a geometric correlate. Furthermore, the conditions stating that electric charge is well-defined for preferred extremals forces the localization of the modes to 2-D surfaces in the generic case. This also resolves the interpretational problems related to possibility of strong parity breaking effects since induce W fields and possibly also Z^0 field above weak scale, vanish at these surfaces.

2. One expects that stringy approach based on 4-D generalization of conformal invariance or its 2-D variant at 2-D preferred surfaces should also allow to understand the Kähler-Dirac equation. Conformal invariance indeed allows to write the solutions explicitly using formulas similar to encountered in string models. In accordance with the earlier conjecture, all modes of the Kähler-Dirac operator generate badly broken super-symmetries.

3. Well-definedness of em charge is not enough to localize spinor modes at string world sheets. Covariantly constant right-handed neutrino certainly defines solutions de-localized inside entire space-time sheet. This need not be the case if right-handed neutrino is not covarianty constant since the non-vanishing CP_2 part for the induced gamma matrices mixes it with left-handed neutrino. For massless extremals (at least) the CP_2 part however vanishes and right-handed neutrino allows also massless holomorphic modes de-localized at entire space-time surface and the de-localization inside Euclidian region defining the line of generalized Feynman diagram is a good candidate for the right-handed neutrino generating the least broken super-symmetry.This super-symmetry seems however to differ from the ordinary one in that ν_R is expected to behave like a passive spectator in the scattering. Also for the left-handed neutrino solutions localized inside string world sheet the condition that coupling to right-handed neutrino vanishes is guaranteed if gamma matrices are either purely Minkowskian or CP_2 like inside the world sheet.

1.2.3 Quantum criticality and K-D action

A detailed view about the physical role of quantum criticality results. Quantum criticality fixes the values of Kähler coupling strength as the analog of critical temperature. The recent formulation of quantum criticality states the existence of hierarchy of sub-algebras of super-symplectic algebras isomorphic with the original algebra. The conformal weights of given sub-algebra are n-multiples of those of the full algebra. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter. These sub-algebras correspond to a hierarchy of breakings of super-symplectic gauge symmetry to a sub-algebra. Accordingly the super-symplectic Noether charges of the sub-algebra annihilate physical states and the corresponding classical Noether charges vanish for Kähler action at the ends of space-time surfaces. This defines the notion of preferred extremal. These sub-algebras form an inclusion hierarchy defining a hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

Quantum criticality implies that second variation of Kähler action vanishes for critical deformations defined by the sub-algebra and vanishing of the corresponding Noether charges and super-charges for physical stats. It is not quite clear whether the charges corresponding to broken super-symplectic symmetries are conserved. If this is the case, Kähler action is invariant under brokent symplectic transformations although the second variation is non-vanishing so these deformations contribute to Kähler metric and are thus quantum fluctuating dynamical degrees of freedom.

1.2.4 Quantum classical correspondence

Quantum classical correspondence (QCC) requires a coupling between quantum and classical and this coupling should also give rise to a generalization of quantum measurement theory. The big question mark is how to realize this coupling.

1. As already described, the massless Dirac equation for induced gamma matrices at the boundary of string world sheets gives as solutions for which local 8-momentum is light-like. The M^4 part of this momentum is in general time-like and can be identified as the 8-momentum of incoming fermion assignable to an embedding space spinor mode. The interpretation is as equivalence of gravitational and inertial masses.

2. QCC can be realized at the level of WCW Dirac operator and Kähler-Dirac operator contains only interior term. The vanishing of the normal component of fermion current replaces Chern-Simons Dirac operator at various boundary like surfaces. I have proposed that WCW spinor fields with given quantum charges in Cartan algebra are superpositions of space-time surfaces with same classical charges. A stronger form of QCC at the level of WCW would be that classical correlation functions for various geometric observables are identical with quantal correlation functions.

   QCC could be realized at the level of WCW by putting it in by hand. One can of course consider also the possibility that the equality of quantal and classical Cartan charges is realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system with Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD in zero energy ontology (ZEO) can be regarded as square root of thermodynamics, the procedure looks logically sound.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L1].

2 WCW Spinor Structure: General Definition

The basic problem in constructing WCW spinor structure is clearly the construction of the explicit representation for the gamma matrices of WCW . One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

2.1 Defining Relations For Gamma Matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \quad .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of WCW d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If WCW allows Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma_A^\dagger, \Gamma_B\} = iJ_{AB} \ , \tag{2.1}$$

where J_{AB} denotes the matrix element of the Kähler form of WCW . The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a nontrivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of WCW Dirac operator comes out correctly. Obviously, what one must do is the equivalent of replacing $D^2 = (\Gamma^k D_k)^2$ with $D\hat{D}$ with $\hat{D}$ defined as

$$\hat{D} = iJ^{kl}\Gamma_l^\dagger D_k \ .$$

2.2 General Vielbein Representations

There are two ideas, which make the solution of the problem obvious.

1. Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of WCW it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by TGD counterpart of the massless Dirac action should be coded into the definition of the WCW spinor structure. This leads to the challenge of defining what classical spinor field means.

2. Since classical scalar field in WCW corresponds to second quantized boson fields of the embedding space same correspondence should apply in the case of the fermions, too. The spinor fields of WCW should correspond to second quantized fermion field of the embedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of embedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the embedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of WCW spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

1. The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of WCW spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having WCW as its base space.

2. The gamma matrices of WCW (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned} \Gamma_A^+ &= E_A^n a_n^\dagger \\ \Gamma_A^- &= \bar{E}_A^n a_n \\ iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n \end{aligned} \tag{2.2}$$

where E_A^n are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, WCW gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and WCW metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labeled by the quantum numbers labeling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.

3. A further great idea inspired by the symplectic and Kähler structures of WCW is that configuration gamma matrices are actually generators of super-symplectic symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

2.3 Inner Product For WCW Spinor Fields

The conjugation operation for WCW spinor s corresponds to the standard $ket \rightarrow bra$ operation for the states of the Fock space:

$$\begin{aligned} \Psi &\leftrightarrow |\Psi\rangle \\ \bar{\Psi} &\leftrightarrow \langle\Psi| \end{aligned} \tag{2.3}$$

The inner product for WCW spinor s at a given point of WCW is just the standard Fock space inner product, which is unitary.

$$\bar{\Psi}_1(X^3)\Psi_2(X^3) = \langle\Psi_1|\Psi_2\rangle_{|X^3} \tag{2.4}$$

WCW inner product for two WCW spinor fields is obtained as the integral of the Fock space inner product over the whole WCW using the vacuum functional $exp(K)$ as a weight factor

$$\langle\Psi_1|\Psi_2\rangle = \int \langle\Psi_1|\Psi_2\rangle_{|X^3} exp(K)\sqrt{G}dX^3 \tag{2.5}$$

This inner product is obviously unitary. A modified form of the inner product is obtained by including the factor $exp(K/2)$ in the definition of the spinor field. In fact, the construction of the central extension for the isometry algebra leads automatically to the appearance of this factor in vacuum spinor field.

The inner product differs from the standard inner product for, say, Minkowski space spinors in that integration is over the entire WCW rather than over a time= constant slice of the WCW . Also the presence of the vacuum functional makes it different from the finite dimensional inner product. These are not un-physical features. The point is that (apart from classical non-determinism forcing to generalized the concept of 3-surface) Diff^4 invariance dictates the behavior of WCW spinor field completely: it is determined form its values at the moment of the big bang. Therefore there is no need to postulate any Dirac equation to determine the behavior and therefore no need to use the inner product derived from dynamics.

2.4 Holonomy Group Of The Vielbein Connection

Generalized vielbein allows huge gauge symmetry. An important constraint on physical observables is that they do not depend at all on the gauge chosen to represent the gamma matrices. This is indeed achieved using vielbein connection, which is now quadratic in fermionic oscillator operators. The holonomy group of the vielbein connection is the WCW counterpart of the electro-weak gauge group and its algebra is expected to have same general structure as the algebra of the WCW isometries. In particular, the generators of this algebra should be labeled by conformal weights like the elements of Kac Moody algebras. In present case however conformal weights are complex as the construction of WCW geometry demonstrates.

2.5 Realization Of WCW Gamma Matrices In Terms Of Super Symmetry Generators

In string models super symmetry generators behave effectively as gamma matrices and it is very tempting to assume that WCW gamma matrices can be regarded as generators of the symplectic algebra extended to super-symplectic Kac Moody type algebra. The experience with string models suggests also that radial Virasoro algebra extends to Super Virasoro algebra. There are good reasons to expect that WCW Dirac operator and its square give automatically a realization of this algebra. It this is indeed the case, then WCW spinor structure as well as Dirac equation reduces to mere group theory.

One can actually guess the general form of the super-symplectic algebra. The form is a direct generalization of the ordinary super Kac Moody algebra. The complexified super generators S_A are identifiable as WCW gamma matrices:

$$\Gamma_A = S_A \ . \tag{2.6}$$

The anti-commutators $\{\Gamma_A^\dagger, \Gamma_B\}_+ = i2J_{A,B}$ define a Hermitian matrix, which is proportional to the Kähler form of the configuration space rather than metric as usually. Only in complex coordinates the anti-commutators equal to the metric numerically. This is, apart from the multiplicative constant n, is expressible as the Poisson bracket of the WCW Hamiltonians H_A and H_B. Therefore one should be able to identify super generators $S_A(r_M)$ for each values of r_M as the counterparts of fluxes. The anti-commutators between the super generators S_A and their Hermitian conjugates should read as

$$\{S_A, S_B^\dagger\}_+ = iQ_m(H_{[A,B]}) \ . \tag{2.7}$$

and should be induced directly from the anti-commutation relations of free second quantized spinor fields of the embedding space restricted to the light cone boundary.

The commutation relations between s and super generators follow solely from the transformation properties of the super generators under symplectic transformations, which are same as for the Hamiltonians themselves

$$\{H_{Am}, S_{Bn}\}_- = S_{[Am,Bn]} \ , \tag{2.8}$$

and are of the same form as in the case of Super-Kac-Moody algebra.

The task is to derive an explicit representation for the super generators S_A in both cases. For obvious reason the spinor fields restricted to the 3-surfaces on the light cone boundary $\delta M^4_+ \times CP_2$ can be used. Leptonic/quark like oscillator operators are used to construct Ramond/NS type algebra.

What is then the strategy that one should follow?

1. WCW Hamiltonians correspond to either magnetic or electric flux Hamiltonians and the conjecture is that these representations are equivalent. It turns out that this electric-magnetic duality generalizes to the level of super charges. It also turns out that quark representation is the only possible option whereas leptonic super charges super-symmetrize the ordinary function algebra of the light cone boundary.

2. The simplest option would be that second quantized embedding space spinors could be used in the definition of super charges. This turns out to not work and one must second quantize the induced spinor fields.

3. The task is to identify a super-symmetric variational principle for the induced spinors: ordinary Dirac action does not work. It turns out that in the most plausible scenario the Kähler-Dirac action varied with respect to *both* embedding space coordinates and spinor fields is the fundamental action principle. The c-number parts of the conserved symplectic charges associated with this action give rise to bosonic conserved charges defining WCW Hamiltonians. The second quantization of the spinor fields reduces to the requirement that super charges and Hamiltonians generate super-symplectic algebra determining the anti-commutation relations for the induced spinor fields.

2.6 Central Extension As Symplectic Extension At WCW Level

The earlier attempts to understand the emergence of central extension of super-symplectic algebra were based on the notion of symplectic extension. This general view is not given up although it seems that this abstract approach is not very practical. Symplectic extension emerged originally in the attempts to construct formal expression for the WCW Dirac equation. The rather obvious idea was that the Dirac equation reduces to super Virasoro conditions with Super Virasoro generators involving the Dirac operator of the embedding space. The basic difficulty was the necessity to assign to the gamma matrices of the embedding space fermion number. In the recent formulation the Dirac operator of H does not appear in in the Super Virasoro conditions so that this problem disappears.

The proposal that Super Virasoro conditions should replaced with conditions stating that the commutator of super-symplectic and super Kac-Moody algebras annihilates physical states, looks rather feasible. One could call these conditions as WCW Dirac equation but at this moment I feel that this would be just play with words and mask the group theoretical content of these conditions. In any case, the formulas for the symplectic extension and action of isometry generators on WCW spinor deserve to be summarized.

2.6.1 Symplectic extension

The Abelian extension of the super-symplectic algebra is obtained by an extremely simple trick. Replace the ordinary derivatives appearing in the definition of, say spinorial isometry generator, by the covariant derivatives defined by a coupling to a multiple of the Kähler potential.

$$\begin{aligned} j^{Ak}\partial_k &\rightarrow j^{Ak}D_k \ , \\ D_k &= \partial_k + ikA_k/2 \ . \end{aligned} \tag{2.9}$$

where A_k denotes Kähler potential. The reality of the parameter k is dictated by the Hermiticity requirement and also by the requirement that Abelian extension reduces to the standard form in Cartan algebra. k is expected to be integer also by the requirement that covariant derivative corresponds to connection (quantization of magnetic charge).

The commutation relations for the centrally extended generators J^A read:

$$[J^A, J^B] = J^{[A,B]} + ikj^{Ak}J_{kl}j^{Bl} \equiv J^{[A,B]} + ikJ_{AB} \ . \tag{2.10}$$

Since Kähler form defines symplectic structure in WCW one can express Abelian extension term as a Poisson bracket of two Hamiltonians

$$J_{AB} \equiv j^{Ak}J_{kl}j^{Bl} = \{H^A, H^B\} \ . \tag{2.11}$$

Notice that Poisson bracket is well defined also when Kähler form is degenerate.

The extension indeed has acceptable properties:

1. Jacobi-identities reduce to the form

$$\sum_{cyclic} H^{[A,[B,C]]} = 0 \ , \tag{2.12}$$

and therefore to the Jacobi identities of the original Lie- algebra in Hamiltonian representation.

2. In the Cartan algebra Abelian extension reduces to a constant term since the Poisson bracket for two commuting generators must be a multiple of a unit matrix. This feature is clearly crucial for the non-triviality of the Abelian extension and is encountered already at the level of ordinary (q,p) Poisson algebra: although the differential operators ∂_p and ∂_q commute the

Poisson bracket of the corresponding Hamiltonians p and q is nontrivial: $\{p,q\}=1$. Therefore the extension term commutes with the generators of the Cartan subalgebra. Extension is also local $U(1)$ extension since Poisson algebra differs from the Lie-algebra of the vector fields in that it contains constant Hamiltonian ("1" in the commutator), which commutes with all other Hamiltonians and corresponds to a vanishing vector field.

3. For the generators not belonging to Cartan sub-algebra of CH isometries Abelian extension term is not annihilated by the generators of the original algebra and in this respect the extension differs from the standard central extension for the loop algebras. It must be however emphasized that for the super-symplectic algebra generators correspond to products of δM_+^4 and CP_2 Hamiltonians and this means that generators of say δM_+^4-local $SU(3)$ Cartan algebra are non-commuting and the commutator is completely analogous to central extension term since it is symmetric with respect to $SU(3)$ generators.

4. The proposed method yields a trivial extension in the case of Diff^4. The reason is the (four-dimensional!) Diff degeneracy of the Kähler form. Abelian extension term is given by the contraction of the Diff^4 generators with the Kähler potential

$$j^{Ak}J_{kl}j^{Bl} \quad = \quad 0 \ , \tag{2.13}$$

which vanishes identically by the Diff degeneracy of the Kähler form. Therefore neither 3- or 4-dimensional Diff invariance is not expected to cause any difficulties. Recall that 4-dimensional Diff degeneracy is what is needed to eliminate time like vibrational excitations from the spectrum of the theory. By the way, the fact that the loop space metric is not Diff degenerate makes understandable the emergence of Diff anomalies in string models [B7, B6]
.

5. The extension is trivial also for the other zero norm generators of the tangent space algebra, in particular for the $k_2 = Im(k) = 0$ symplectic generators possible present so that these generators indeed act as genuine $U(1)$ transformations.

6. Concerning the solution of WCW Dirac equation the maximum of Kähler function is expected to be special, much like origin of Minkowski space and symmetric space property suggests that the construction of solutions reduces to this point. At this point the generators and Hamiltonians of the algebra h in the defining Cartan decomposition $g = h + t$ should vanish. h corresponds to integer values of $k_1 = Re(k)$ for Cartan algebra of super-symplectic algebra and integer valued conformal weights n for Super Kac-Moody algebra. The algebra reduces at the maximum to an exceptionally simple form since only central extension contributes to the metric and Kähler form. In the ideal case the elements of the metric and Kähler form could be even diagonal. The degeneracy of the metric might of course pose additional complications.

2.6.2 Super symplectic action on WCW spinor s

The generators of symplectic transformations are obtained in the spinor representation of the isometry group of WCW by the following formal construction. Take isometry generator in the spinor representation and add to the covariant derivative D_k defined by vielbein connection the coupling to the multiple of the Kähler potential: $D_k \rightarrow D_k + ikAk/2$.

$$\begin{aligned} J^A &= j^{Ak}D_k + D_l j_k \Sigma^{kl}/2 \ , \\ &\rightarrow \hat{J}^A = j^{Ak}(D_k + ikA_k/2) + D_l j^A_k \Sigma^{kl}/2 \ , \end{aligned} \tag{2.14}$$

This induces the required central term to the commutation relations. Introduce complex coordinates and define bosonic creation and annihilation operators as $(1,0)$ and $(0,1)$ parts of the modified isometry generators

$$
\begin{aligned}
B_A^\dagger &= J_+^A = j^{Ak}(D_k + ... \ , \\
B_A &= J_-^A = j^{A\bar{k}}(D_{\bar{k}} + ... \ .
\end{aligned}
\tag{2.15}
$$

where "k" refers now to complex coordinates and "$\bar{k}$" to their conjugates.

Fermionic generators are obtained as the contractions of the complexified gamma matrices with the isometry generators

$$
\begin{aligned}
\Gamma_A^\dagger &= j^{Ak}\Gamma_k \ , \\
\Gamma_A &= j^{A\bar{k}}\Gamma_{\bar{k}} \ .
\end{aligned}
\tag{2.16}
$$

Notice that the bosonic Cartan algebra generators obey standard oscillator algebra commutation relations and annihilate fermionic Cartan algebra generators. Hermiticity condition holds in the sense that creation type generators are hermitian conjugates of the annihilation operator type generators. There are two kinds of representations depending on whether one uses leptonic or quark like oscillator operators to construct the gammas. These will be assumed to correspond to Ramond and NS type generators with the radial plane waves being labeled by integer and half odd integer indices respectively.

The non-vanishing commutators between the Cartan algebra bosonic generators are given by the matrix elements of the Kähler form in the basis of formed by the isometry generators

$$
[B_A^\dagger, B_B] = J(j^{A\dagger}, j^B) \equiv J_{\bar{A}B} \ . \tag{2.17}
$$

and are isometry invariant quantities. The commutators between local $SU(3)$ generators not belonging to Cartan algebra are just those of the local gauge algebra with Abelian extension term added.

The anti-commutators between the fermionic generators are given by the elements of the metric (as opposed to Kähler form in the case of bosonic generators) in the basis formed by the isometry generators

$$
\{\Gamma_A\dagger, \Gamma_B\} = 2g(j^{A\dagger}, j^B) \equiv 2g_{\bar{A}B} \ . \tag{2.18}
$$

and are invariant under isometries. Numerically the commutators and anti-commutators differ only the presence of the imaginary unit and the scale factor R relating the metric and Kähler form to each other (the factor R is same for CP_2 metric and Kähler form).

The commutators between bosonic and fermionic generators are given by

$$
[B_A, \Gamma_B] = \Gamma_{[A,B]} \ . \tag{2.19}
$$

The presence of vielbein and rotation terms in the representation of the isometry generators is essential for obtaining these nice commutations relations. The commutators vanish identically for Cartan algebra generators. From the commutation relations it is clear that Super Kac Moody algebra structure is directly related to the Kähler structure of WCW : the anti-commutator of fermionic generators is proportional to the metric and the commutator of the bosonic generators is proportional to the Kähler form. It is this algebra, which should generate the solutions of the field equations of the theory.

The vielbein and rotational parts of the bosonic isometry generators are quadratic in the fermionic oscillator operators and this suggests the interpretation as the fermionic contribution to the isometry currents. This means that the action of the bosonic generators is essentially non-perturbative since it creates fermion anti-fermion pairs besides exciting bosonic degrees of freedom.

2.7 WCW Clifford Algebra As AHyper-Finite Factor Of Type II_1

The naïve expectation is that the trace of the unit matrix associated with the Clifford algebra spanned by WCW sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained un-noticed until it became clear that there is a connection with von Neumann algebras [A9]. In fact, for a separable Hilbert space defines a standard representation for so called [A6]. This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences.

2.7.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A6].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

2.7.2 von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A10, A4] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A3, A8] relate closely to type II_1 factors. In topological quantum computation [B5] based on braid groups [A11] modular S-matrices they play an especially important role.

2.7.3 Clifford algebra of WCW as von Neumann algebra

The Clifford algebra of WCW provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally. These aspects are discussed in detail in [K15].

3 Under What Conditions Electric Charge Is Conserved For The Kähler-Dirac Equation?

One might think that talking about the conservation of electric charge at 21st century is a waste of time. In TGD framework this is certainly not the case.

1. In quantum field theories there are two ways to define em charge: as electric flux over 2-D surface sufficiently far from the source region or in the case of spinor field quantum mechanically as combination of fermion number and vectorial isospin. The latter definition is quantum mechanically more appropriate.

2. There is however a problem. In standard approach to gauge theory Dirac equation in presence of charged classical gauge fields does not conserve electric charge as quantum number: electron is transformed to neutrino and vice versa. Quantization solves the problem since the non-conservation can be interpreted in terms of emission of gauge bosons. In TGD framework this does not work since one does not have path integral quantization anymore. Preferred extremals carry classical gauge fields and the question whether em charge is conserved arises. Heuristic picture suggests that em charge must be conserved.

It seems that one should pose the well-definedness of spinorial em charge as an additional condition. Well-definedness of em charge is not the only problem. How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? Does this require that classical weak fields vanish in the regions where the modes of induced spinor fields are non-vanishing?

This condition might be one of the conditions defining what it is to be a preferred extremal/solution of Kähler Dirac equation. It is not however trivial whether this kind of additional condition can be posed unless it follows automatically from the recent formulation for Kähler action and Kähler Dirac action. The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string/parton picture part of TGD. The vanishing of classical weak fields has also number theoretic interpretation: space-time surfaces would have quaternionic (hyper-complex) tangent space and the 2-surfaces carrying spinor fields complex (hyper-complex) tangent space.

3.1 Conservation Of EM Charge For Kähler Dirac Equation

What does the conservation of em charge imply in the case of the Kähler-Dirac equation? The obvious guess that the em charged part of the Kähler-Dirac operator must annihilate the solutions, turns out to be correct as the following argument demonstrates.

1. Em charge as coupling matrix can be defined as a linear combination $Q = aI + bI_3$, $I_3 = J_{kl}\Sigma^{kl}$, where I is unit matrix and I_3 vectorial isospin matrix, J_{kl} is the Kähler form of CP_2, Σ^{kl} denotes sigma matrices, and a and b are numerical constants different for quarks and leptons. Q is covariantly constant in $M^4 \times CP_2$ and its covariant derivatives at space-time surface are also well-defined and vanish.

2. The modes of the Kähler-Dirac equation should be eigen modes of Q. This is the case if the Kähler-Dirac operator D commutes with Q. The covariant constancy of Q can be used to derive the condition

$$\begin{aligned} [D,Q]\,\Psi &= D_1\Psi = 0 \ , \\ D &= \hat{\Gamma}^\mu D_\mu \ , \ D_1 = [D,Q] = \hat{\Gamma}^\mu_1 D_\mu \ , \ \hat{\Gamma}^\mu_1 = \left[\hat{\Gamma}^\mu, Q\right] \ . \end{aligned} \tag{3.1}$$

   Covariant constancy of J is absolutely essential: without it the resulting conditions would not be so simple.

   It is easy to find that also $[D_1,Q]\Psi = 0$ and its higher iterates $[D_n,Q]\Psi = 0$, $D_n = [D_{n-1},Q]$ must be true. The solutions of the Kähler-Dirac equation would have an additional symmetry.

3. The commutator $D_1 = [D,Q]$ reduces to a sum of terms involving the commutators of the vectorial isospin $I_3 = J_{kl}\Sigma^{kl}$ with the CP_2 part of the gamma matrices:

$$D_1 = [Q,D] = [I_3,\Gamma_r]\partial_\mu s^r T^{\alpha\mu} D_\alpha \ . \tag{3.2}$$

   In standard complex coordinates in which $U(2)$ acts linearly the complexified gamma matrices can be chosen to be eigenstates of vectorial isospin. Only the charged flat space complexified gamma matrices Γ^A denoted by Γ^+ and Γ^- possessing charges $+1$ and -1 contribute to the right hand side. Therefore the additional Dirac equation $D_1\Psi = 0$ states

$$\begin{aligned} D_1\Psi &= [Q,D]\Psi = I_3(A) e_{Ar}\Gamma^A \partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi \\ &= (e_{+r}\Gamma^+ - e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha \Psi = 0 \ . \end{aligned} \tag{3.3}$$

   The next condition is

$$D_2\Psi = [Q,D]\Psi = (e_{+r}\Gamma^+ + e_{-r}\Gamma^-)\partial_\mu s^r T^{\alpha\mu} D_\alpha\Psi = 0 \ . \tag{3.4}$$

   Only the relative sign of the two terms has changed. The remaining conditions give nothing new.

4. These equations imply two separate equations for the two charged gamma matrices

$$\begin{aligned} D_+\Psi &= T^\alpha_+ \Gamma^+ D_\alpha \Psi = 0 \ , \\ D_-\Psi &= T^\alpha_- \Gamma^- D_\alpha \Psi = 0 \ , \\ T^\alpha_\pm &= e_{\pm r}\partial_\mu s^r T^{\alpha\mu} \ . \end{aligned} \tag{3.5}$$

   These conditions state what one might have expected: the charged part of the Kähler-Dirac operator annihilates separately the solutions. The reason is that the classical W fields are proportional to $e_{r\pm}$.

   The above equations can be generalized to define a decomposition of the energy momentum tensor to charged and neutral components in terms of vierbein projections. The equations state that the analogs of the Kähler-Dirac equation defined by charged components of the energy momentum tensor are satisfied separately.

5. In complex coordinates one expects that the two equations are complex conjugates of each other for Euclidian signature. For the Minkowskian signature an analogous condition should hold true. The dynamics enters the game in an essential manner: whether the equations can be satisfied depends on the coefficients a and b in the expression $T = aG + bg$ implied by Einstein's equations in turn guaranteeing that the solution ansatz generalizing minimal surface solutions holds true [K1].

6. As a result one obtains three separate Dirac equations corresponding to the neutral part $D_0\Psi = 0$ and charged parts $D_\pm\Psi = 0$ of the Kähler-Dirac equation. By acting on the equations with these Dirac operators one obtains also that the commutators $[D_+, D_-]$, $[D_0, D_\pm]$ and also higher commutators obtained from these annihilate the induced spinor field model. Therefore entire -possibly- infinite-dimensional algebra would annihilate the induced spinor fields. In string model the counterpart of Dirac equation when quantized gives rise to Super-Virasoro conditions. This analogy would suggest that Kähler-Dirac equation gives rise to the analog of Super-Virasoro conditions in 4-D case. But what the higher conditions mean? Could they relate to the proposed generalization to Yangian algebra [A2] [B4, B2, B3]? Obviously these conditions resemble structurally Virasoro conditions $L_n|phys\rangle = 0$ and their supersymmetric generalizations, and might indeed correspond to a generalization of these conditions just as the field equations for preferred extremals could correspond to the Virasoro conditions if one takes seriously the analogy with the quantized string.

What could this additional symmetry mean from the point of view of the solutions of the Kähler-Dirac equation? The field equations for the preferred extremals of Kähler action reduce to purely algebraic conditions in the same manner as the field equations for the minimal surfaces in string model. Could this happen also for the Kähler-Dirac equation and could the condition on charged part of the Dirac operator help to achieve this?

This argument was very general and one can ask for simple ways to realize these conditions. Obviously the vanishing of classical W fields in the region where the spinor mode is non-vanishing defines this kind of condition.

3.2 About The Solutions Of Kähler Dirac Equation For Known Extremals

To gain perpective consider first Dirac equation in in H. Quite generally, one can construct the solutions of the ordinary Dirac equation in H from covariantly constant right-handed neutrino spinor playing the role of fermionic vacuum annihilated by the second half of complexified gamma matrices. Dirac equation reduces to Laplace equation for a scalar function and solution can be constructed from this "vacuum" by multiplying with the spherical harmonics of CP_2 and applying Dirac operator [K7]. Similar construction works quite generally thanks to the existence of covariantly constant right handed neutrino spinor. Spinor harmonics of CP_2 are only replaced with those of space-time surface possessing either hermitian structure or Hamilton-Jacobi structure (corresponding to Euclidian and Minkowskian signatures of the induced metric [K1, K16]). What is remarkable is that these solutions possess well-defined em charge although classical W boson fields are present.

This in sense that H d'Alembertian commutes with em charge matrix defined as a linear combination of unit matrix and the covariantly constant matrix $J^{kl}\Sigma_{kl}$ since the commutators of the covariant derivatives give constant Ricci scalar and $J^{kl}\Sigma_{kl}$ term to the d'Alembertian besides scalar d'Alembertian commuting with em charge. Dirac operator itself does not commute with em charge matrix since gamma matrices not commute with em charge matrix.

Consider now Kähler Dirac operator. The square of Kähler Dirac operator contains commutator of covariant derivatives which contains contraction $[\Gamma^\mu, \Gamma^\nu] F^{weak}_{\mu\nu}$ which is quadratic in sigma matrices of $M^4 \times CP_2$ and does not reduce to a constant term commuting which em charge matrix. Therefore additional condition is required even if one is satisfies with the commutativity of d'Alembertian with em charge. Stronger condition would be commutativity with the Kähler Dirac operator and this will be considered in the following.

To see what happens one must consider space-time regions with Minkowskian and Euclidian signature. What will be assumed is the existence of Hamilton-Jacobi structure [K1] meaning complex structure in Euclidian signature and hyper-complex plus complex structure in Minkowskian signature. The goal is to get insights about what the condition that spinor modes have a well-defined em charge eigenvalue requires. Or more concretely: is the localization at string world sheets guaranteeing well-defined value of em charge predicted by Kähler Dirac operator or must one introduce this condition separately? One can also ask whether this condition reduces to commutativity/co-commutativity in number theoretic vision.

1. CP_2 type vacuum extremals serve as a convenient test case for the Euclidian signature. In this case the Kähler-Dirac equation reduces to the massless ordinary Dirac equation in CP_2 allowing only covariantly constant right-handed neutrino as solution. Only part of CP_2 so that one give up the constraint that the solution is defined in the entire CP_2. In this case holomorphic solution ansatz obtained by assuming that solutions depend on the coordinates ξ^i, $i = 1, 2$ but not on their conjugates and that the gamma matrices $\Gamma^{\bar{i}}$, $i = 1, 2$, annihilate the solutions, works. The solutions ansatz and its conjugate are of exactly the same form as in case string models where one considers string world sheets instead of CP_2 region.

   The solutions are not restricted to 2-D string world sheets and it is not clear whether one can assign to them a well-defined em charge in any sense. Note that for massless Dirac equation in H one obtains all CP_2 harmonics as solutions, and it is possible to talk about em charge of the solution although solution itself is not restricted to 2-D surface of CP_2.

2. For massless extremals and a very wide class of solutions produced by Hamilton-Jacobi structure - perhaps all solutions representable locally as graphs for map $M^4 \rightarrow CP_2$ - canonical momentum densities are light-like and solutions are hyper-holomorphic in the coordinates associated with with string world sheet and annihilated by the conjugate gamma and arbitrary functions in transversal coordinates. This allows localization to string world sheets. The localization is now strictly dynamical and implied by the properties of Kähler Dirac operator.

3. For string like objects one obtains massless Dirac equation in $X^2 \times Y^2 \subset M^4 \times Y^2$, Y^2 a complex 2-surface in CP_2. Homologically trivial geodesic sphere corresponds to the simplest choice for Y^2. Modified Dirac operator reduces to a sum of massless Dirac operators associated with X^2 and Y^2. The most general solutions would have Y^2 mass. Holomorphic solutions reduces to product of hyper-holomorphic and holomorphic solutions and massless 2-D Dirac equation is satisfied in both factors.

   For instance, for S^2 a geodesic sphere and $X^2 = M^2$ one obtains M^2 massivation with mass squared spectrum given by Laplace operator for S^2. Conformal and hyper-conformal symmetries are lost, and one might argue that this is quite not what one wants. One must be however resist the temptation to make too hasty conclusions since the massivation of string like objects is expected to take place. The question is whether it takes place already at the level of fundamental spinor fields or only at the level of elementary particles constructed as many-fermion states of them as twistor Grassmann approach assuming massless M^4 propagators for the fundamental fermions strongly suggests [K12].

4. For vacuum extremals the Kähler Dirac operator vanishes identically so that it does not make sense to speak about solutions.

What can one conclude from these observations?

1. The localization of solutions to 2-D string world sheets follows from Kähler Dirac equation only for the Minkowskian regions representable as graphs of map $M^4 \rightarrow CP_2$ locally. For string like objects and deformations of CP_2 type vacuum extremals this is not expected to take place.

2. It is not clear whether one can speak about well-defined em charge for the holomorphic spinors annihilated by the conjugate gamma matrices or their conjugates. As noticed, for embedding space spinor harmonics this is however possible.

3. Strong form of conformal symmetry and the condition that em charge is well-defined for the nodes suggests that the localization at 2-D surfaces at which the charged parts of induced electroweak gauge fields vanish must be assumed as an additional condition. Number theoretic vision would suggest that these surfaces correspond to 2-D commutative or co-commutative surfaces. The string world sheets inside space-time surfaces would not emerge from theory but would be defined as basic geometric objects.

   This kind of condition would also allow duals of string worlds sheets as partonic 2-surfaces identified number theoretically as co-commutative surfaces. Commutativity and co-commutativity would become essential elements of the number theoretical vision.

4. The localization of solutions of the Kähler-Dirac action at string world sheets and partonic 2-surfaces as a constraint would mean induction procedure for Kähler-Dirac matrices from SX^4 to X^2 - that is projection. The resulting em neutral gamma matrices would correspond to tangent vectors of the string world sheet. The vanishing of the projections of charged parts of energy momentum currents would define these surfaces. The conditions would apply both in Minkowskian and Euclidian regions. An alternative interpretation would be number theoretical: these surface would be commutative or co-commutative.

3.3 Concrete Realization Of The Conditions Guaranteeing Well-Defined Em Charge

Well-definedness of the em charge is the fundamental condiiton on spinor modes. Physical intuition suggests that also classical Z^0 field should vanish - at least in scales longer than weak scale. Above the condition guaranteeing vanishing of em charge has been discussed at very general level. It has however turned out that one can understand situation by simply posing the simplest condition that one can imagine: the vanishing of classical W and possibly also Z^0 fields inducing mixing of different charge states.

1. Induced W fields mean that the modes of Kähler-Dirac equation do not in general have well-defined em charge. The problem disappears if the induced W gauge fields vanish. This does not yet guarantee that couplings to classical gauge fields are physical in long scales. Also classical Z^0 field should vanish so that the couplings would be purely vectorial. Vectoriality might be true in long enough scales only. If W and Z^0 fields vanish in all scales then electroweak forces are due to the exchanges of corresponding gauge bosons described as string like objects in TGD and represent non-trivial space-time geometry and topology at microscopic scale.

2. The conditions solve also another long-standing interpretational problem. Color rotations induce rotations in electroweak-holonomy group so that the vanishing of all induced weak fields also guarantees that color rotations do not spoil the property of spinor modes to be eigenstates of em charge.

One can study the conditions quite concretely by using the formulas for the components of spinor curvature [K2] (http://tinyurl.com/z86o5qk).

1. The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{llllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{3.6}$$

   $R_{01} = R_{23}$ and $R_{03} = -R_{31}$ combine to form purely left handed classical W boson fields and Z^0 field corresponds to $Z^0 = 2R_{03}$.

   Kähler form is given by

$$J = 2(e^0\wedge e^3 + e^1\wedge e^2) \ . \tag{3.7}$$

2. The vanishing of classical weak fields is guaranteed by the conditions

$$\begin{array}{c} e^0\wedge e^1 - e^2\wedge e^3 = 0 \ , \\ e^0\wedge e^2 - e^3\wedge e^1 \ , \\ 4e^0\wedge e^3 + 2e^1\wedge e^2 \ . \end{array} \tag{3.8}$$

3. There are many ways to satisfy these conditions. For instance, the condition $e^1 = a \times e^0$ and $e^2 = -a \times e^3$ with arbitrary a which can depend on position guarantees the vanishing of classical W fields. The CP_2 projection of the tangent space of the region carrying the spinor mode must be 2-D.

   Also classical Z^0 vanishes if $a^2 = 2$ holds true. This guarantees that the couplings of induced gauge potential are purely vectorial. One can consider other alternaties. For instance, one could require that only classical Z^0 field or induced Kähler form is non-vanishing and deduce similar condition.

4. The vanishing of the weak part of induced gauge field implies that the CP_2 projection of the region carrying spinor mode is 2-D. Therefore the condition that the modes of induced spinor field are restricted to 2-surfaces carrying no weak fields sheets guarantees well-definedness of em charge and vanishing of classical weak couplings. This condition does not imply string world sheets in the general case since the CP_2 projection of the space-time sheet can be 2-D.

How string world sheets could emerge?

1. Additional consistency condition to neutrality of string world sheets is that Kähler-Dirac gamma matrices have no components orthogonal to the 2-surface in question. Hence various fermionic would flow along string world sheet.

2. If the Kähler-Dirac gamma matrices at string world sheet are expressible in terms of two non-vanishing gamma matrices parallel to string world sheet and sheet and thus define an integrable distribution of tangent vectors, this is achieved. What is important that modified gamma matrices can indeed span lower than 4-D space and often do so as already described. Induced gamma matrices defined always 4-D space so that the restriction of the modes to string world sheets is not possible.

3. String models suggest that string world sheets are minimal surfaces of space-time surface or of embedding space but it might not be necessary to pose this condition separately.

In the proposed scenario string world sheets emerge rather than being postulated from beginning.

1. The vanishing conditions for induced weak fields allow also 4-D spinor modes if they are true for entire spatime surface. This is true if the space-time surface has 2-D projection. One can expect that the space-time surface has foliation by string world sheets and the general solution of K-D equation is continuous superposition of the 2-D modes in this case and discrete one in the generic case.

2. If the CP_2 projection of space-time surface is homologically non-trivial geodesic sphere S^2, the field equations reduce to those in $M^4 \times S^2$ since the second fundamental form for S^2 is vanishing. It is possible to have geodesic sphere for which induced gauge field has only em component?

3. If the CP_2 projection is complex manifold as it is for string like objects, the vanishing of weak fields might be also achieved.

4. Does the phase of cosmic strings assumed to dominate primordial cosmology correspond to this phase with no classical weak fields? During radiation dominated phase 4-D string like objects would transform to string world sheets.Kind of dimensional transmutation would occur.

Right-handed neutrino has exceptional role in K-D action.

1. Electroweak gauge potentials do not couple to ν_R at all. Therefore the vanishing of W fields is un-necessary if the induced gamma matrices do not mix right handed neutrino with left-handed one. This is guaranteed if M^4 and CP_2 parts of Kähler-Dirac operator annihilate separately right-handed neutrino spinor mode. Also ν_R modes can be interpreted as continuous superpositions of 2-D modes and this allows to define overlap integrals for them and induced spinor fields needed to define WCW gamma matrices and super-generators.

2. For covariantly constant right-handed neutrino mode defining a generator of super-symmetries is certainly a solution of K-D. Whether more general solutions of K-D exist remains to be checked out.

3.4 Connection With Number Theoretic Vision?

The interesting potential connection of the Hamilton-Jacobi vision to the number theoretic vision about field equations has been already mentioned.

1. The vision that associativity/co-associativity defines the dynamics of space-time surfaces boils down to M^8-H duality stating that space-time surfaces can be regarded as associative/co-associative surfaces either in M^8 or H [K11, K14]. Associativity reduces to hyper-quaternionicity implying that the tangent/normal space of space-time surface at each point contains preferred sub-space $M^2(x) \subset M^8$ and these sub-spaces forma an integrable distribution. An analogous condition is involved with the definition of Hamilton-Jacobi structure.

2. The octonionic representation of the tangent space of M^8 and H effectively replaces $SO(7,1)$ as tangent space group with its octonionic analog obtained by the replacement of sigma matrices with their octonionic counterparts defined by anti-commutators of gamma matrices. By non-associativity the resulting algebra is not ordinary Lie-algebra and exponentiates to a non-associative analog of Lie group. The original wrong belief was that the reduction takes place to the group G_2 of octonionic automorphisms acting as a subgroup of $SO(7)$. One can ask whether the conditions on the charged part of energy momentum tensor could relate to the reduction of $SO(7,1)$

3. What puts bells ringing is that the Kähler-Dirac equation for the octonionic representation of gamma matrices allows the conservation of electromagnetic charge in the proposed sense. The reason is that the left handed sigma matrices (W charges are left-handed) in the octonionic representation of gamma matrices vanish identically! What remains are vectorial=right-handed em and Z^0 charge which becomes proportional to em charge since its left-handed part vanishes. All spinor modes have a well-defined em charge in the octonionic sense defined by replacing embedding space spinor locally by its octonionic variant? Maybe this could explain why H spinor modes can have well-defined em charge contrary to the naïve expectations.

4. The non-associativity of the octonionic spinors is however a problem. Even non-commutativity poses problems - also at space-time level if one assumes quaternion-real analyticity for the spinor modes. Could one assume commutativity or co-commutativity for the induced spinor modes? This would mean restriction to associative or co-associative 2-surfaces and (hyper-)holomorphic depends on its (hyper-)complex coordinate. The outcome would be a localization to a hyper-commutative of commutative 2-surface, string world sheet or partonic 2-surface.

5. These conditions could also be interpreted by saying that for the Kähler Dirac operator the octonionic induced spinors assumed to be commutative/co-commutative are equivalent with ordinary induced spinors. The well-definedness of em charge for ordinary spinors would correspond to commutativity/co-commutativity for octonionic spinors. Even the Dirac equations based on induced and Kähler-Dirac gamma matrices could be equivalent since it is essentially holomorphy which matters.

To sum up, these considerations inspire to ask whether the associativity/co-associativity of the space-time surface is equivalent with the reduction of the field equations to stringy field equations stating that certain components of the induced metric in complex/Hamilton-Jacobi coordinates vanish in turn guaranteeing that field equations reduce to algebraic identifies following from the fact that energy momentum tensor and second fundamental form have no common components? Commutativity/co-commutativity would characterize fermionic dynamics and would have physical representation as possibility to have em charge eigenspinors. This should be the case if one requires that the two solution ansätze are equivalent.

4 Representation Of WCW Metric As Anti-Commutators Of Gamma Matrices Identified As Symplectic Super-Charges

WCW gamma matrices identified as symplectic super Noether charges suggest an elegant representation of WCW metric and Kähler form, which seems to be more practical than the representations in terms of Kähler function or representations guessed by symmetry arguments.

This representation is equivalent with the somewhat dubious representation obtained using symmetry arguments - that is by assuming that the half Poisson brackets of embedding space Hamiltonians defining Kähler form and metric can be lifted to the level of WCW, if the conformal gauge conditions hold true for the spinorial conformal algebra, which is the TGD counterpart of the standard Kac-Moody type algebra of the ordinary strings models. For symplectic algebra the hierarchy of breakings of super-conformal gauge symmetry is possible but not for the standard conformal algebras associated with spinor modes at string world sheets.

4.1 Expression For WCW Kähler Metric As Anticommutators As Symplectic Super Charges

During years I have considered several variants for the representation of symplectic Hamiltonians and WCW gamma matrices and each of these proposals have had some weakness. The key question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians.

The original approach based on flux Hamiltonians did not use Noether currents.

1. Magnetic flux Hamiltonians do not refer to the space-time dynamics and imply genuine rather than only effective 2-dimensionality, which is more than one wants. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed, effective 2-dimensionality might be achieved.

   The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians. It seems that this challenge leads to ad hoc constructions.

2. For the purposes of generalization it is useful to give the expression of flux Hamiltonian. Apart from normalization factors one would have

   $$Q(H_A) = \int_{X^2} H_A J_{\mu\nu} dx^\mu \wedge dx^\nu \quad .$$

   Here A is a label for the Hamiltonian of $\delta M^4_\pm \times CP_2$ decomposing to product of $\delta M^4_\pm$ and CP_2 Hamiltonians with the first one decomposing to a product of function of the radial light-like coordinate r_M and Hamiltonian depending on S^2 coordinates. It is natural to assume that Hamiltonians have well- defined $SO(3)$ and $SU(3)$ quantum numbers. This expressions serves as a natural starting point also in the new approach based on Noether charges.

The approach identifying the Hamiltonians as symplectic Noether charges is extremely natural from physics point of view but the fact that it leads to 3-D expressions involving the induced metric led to the conclusion that it cannot work. In hindsight this conclusion seems wrong: I had not yet realized how profound that basic formulas of physics really are. If the generalization of AdS/CFT duality works, Kähler action can be expressed as a sum of string area actions for string world sheets with string area in the effective metric given as the anti-commutator of the Kähler-Dirac gamma matrices for the string world sheet so that also now a reduction of dimension takes place. This is easy to understand if the classical Noether charges vanish for a sub-algebra of symplectic algebra for preferred extremals.

1. If all end points for strings are possible, the recipe for constructing super-conformal generators would be simple. The embedding space Hamiltonian H_A appearing in the expression of the flux Hamiltonian given above would be replaced by the corresponding symplectic quantum Noether charge $Q(H_A)$ associated with the string defined as 1-D integral along the string. By

replacing Ψ or its conjugate with a mode of the induced spinor field labeled by electroweak quantum numbers and conformal weight nm one would obtain corresponding super-charged identifiable as WCW gamma matrices. The anti-commutators of the super-charges would give rise to the elements of WCW metric labelled by conformal weights n_1, n_2 not present in the naïve guess for the metric. If one assumes that the fermionic super-conformal symmetries act as gauge symmetries only $n_i = 0$ gives a non-vanishing matrix element.

Clearly, one would have weaker form of effective 2-dimensionality in the sense that Hamiltonian would be functional of the string emanating from the partonic 2-surface. The quantum Hamiltonian would also carry information about the presence of other wormhole contacts- at least one- when wormhole throats carry Kähler magnetic monopole flux. If only discrete set for the end points for strings is possible one has discrete sum making possible easy p-adicization. It might happen that integrability conditions for the tangent spaces of string world sheets having vanishing W boson fields do not allow all possible strings.

2. The super charges obtained in this manner are not however entirely satisfactory. The problem is that they involve only single string emanating from the partonic 2-surface. The intuitive expectation is that there can be an arbitrarily large number of strings: as the number of strings is increased the resolution improves. Somehow the super-conformal algebra defined by Hamiltonians and super-Hamiltonians should generalize to allow tensor products of the strings providing more physical information about the 3-surface.

3. Here the idea of Yangian symmetry [K12] suggests itself strongly. The notion of Yangian emerges from twistor Grassmann approach and should have a natural place in TGD. In Yangian algebra one has besides product also co-product, which is in some sense "time-reversal" of the product. What is essential is that Yangian algebra is also multi-local.

   The Yangian extension of the super-conformal algebra would be multi-local with respect to the points of partonic surface (or multi-stringy) defining the end points of string. The basic formulas would be schematically

$$O_1^A = f^A_{BC} T^B \otimes T^B \quad ,$$

   where a summation of B, C occurs and f^A_{BC} are the structure constants of the algebra. The operation can be iterated and gives a hierarchy of n-local operators. In the recent case the operators are n-local symplectic super-charges with unit fermion number and symplectic Noether charges with a vanishing fermion number. It would be natural to assume that also the n-local gamma matrix like entities contribute via their anti-commutators to WCW metric and give multi-local information about the partonic 2-surface and 3-surface.

   The operation generating the algebra well-defined if one an assumes that the second quantization of induced spinor fields is carried out using the standard canonical quantization. One could even assume that the points involved belong to different partonic 2-surfaces belonging even at opposite boundaries of CD. The operation is also well-defined if one assumes that induced spinor fields at different space-time points at boundaries of CD always anti-commute. This could make sense at boundary of CD but lead to problems with embedding space-causality if assumed for the spinor modes at opposite boundaries of CD.

4.2 Handful Of Problems With A Common Resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose Kähler-Dirac action as their solution.

4.2.1 Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of Kähler-Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K4, K11].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

4.2.2 Super-symmetry forces Kähler-Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{4.1}$$

Here T^α_k is canonical momentum current of Kähler action. If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{4.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{4.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{4.4}$$

The requirement that this current vanishes is guaranteed if one assumes that Kähler-Dirac equation

$$\begin{aligned} \hat{\Gamma}^\alpha D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^\alpha &= T_l^\alpha \Gamma^l \ . \end{aligned} \tag{4.5}$$

This equation must be derivable from a Kähler-Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^\alpha D_\alpha \Psi \ . \tag{4.6}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with Kähler-Dirac gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \tag{4.7}$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

As a matter fact, any mode of Kähler-Dirac equation contracted with second quantized induced spinor field or its conjugate defines a conserved super charge. Also super-symplectic Noether charges and their super counterparts can be assigned to symplectic generators as Noether charges but they need not be conserved.

4.2.3 Second quantization of the K-D action

Second quantization of Kähler-Dirac action is crucial for the construction of the Kähler metric of world of classical worlds as anti-commutators of gamma matrices identified as super-symplectic Noether charges. To get a unique result, the anti-commutation relations must be fixed uniquely. This has turned out to be far from trivial.

1. Canonical quantization works after all

The canonical manner to second quantize fermions identifies spinorial canonical momentum densities and their conjugates as $\overline{\Pi} = \partial L_{K_D}/\partial_\Psi = \overline{\Psi}\Gamma^t$ and their conjugates. The vanishing of Γ^t at points, where the induced Kähler form J vanishes can cause problems since anti-commutation relations are not internally consistent anymore. This led me to give up the canonical quantization and to consider various alternatives consistent with the possibility that J vanishes. They were admittedly somewhat ad hoc. Correct (anti-)commutation relations for various fermionic Noether currents seem however to fix the anti-commutation relations to the standard ones. It seems that it is better to be conservative: the canonical method is heavily tested and turned out to work quite nicely.

The canonical manner to second quantize fermions identifies spinorial canonical momentum densities and their conjugates as $\overline{\Pi} = \partial L_{K_D}/\partial_\Psi = \overline{\Psi}\Gamma^t$ and their conjugates. The vanishing of Γ^t at points, where the induced Kähler form J vanishes can cause problems since anti-commutation relations are not internally consistent anymore. This led originally to give up the canonical quantization and to consider various alternatives consistent with the possibility that J vanishes. They were admittedly somewhat ad hoc. Correct commutation relations for various fermionic Noether currents seem however to fix the anti-commutation relations to the standard ones.

Consider first the 4-D situation without the localization to 2-D string world sheets. The canonical anti-commutation relations would state $\{\overline{\Pi}, \Psi\} = \delta^3(x,y)$ at the space-like boundaries of the string world sheet at either boundary of CD. At points where J and thus T^t vanishes, canonical momentum density vanishes identically and the equation seems to be inconsistent.

If fermions are localized at string world sheets assumed to always carry a non-vanishing J at their boundaries at the ends of space-time surfaces, the situation changes since Γ^t is non-vanishing. The localization to string world sheets, which are not vacua saves the situation. The problem is that the limit when string approaches vacuum could be very singular and discontinuous. In the case of elementary particle strings are associated with flux tubes carrying monopole fluxes so that the problem disappears.

It is better to formulate the anti-commutation relations for the modes of the induced spinor field. By starting from

$$\{\overline{\Pi}(x), \Psi(y)\} = \delta^1(x,y) \tag{4.8}$$

and contracting with $\Psi(x)$ and $\Pi(y)$ and integrating, one obtains using orthonormality of the modes of Ψ the result

$$\{b_m^\dagger, b_n\} = \gamma^0 \delta_{m,n} \tag{4.9}$$

holding for the nodes with non-vanishing norm. At the limit $J \rightarrow 0$ there are no modes with non-vanishing norm so that one avoids the conflict between the two sides of the equation.

The proposed anti-commutator would realize the idea that the fermions are massive. The following alternative starts from the assumption of 8-D light-likeness.

2. Does one obtain the analogy of SUSY algebra? In super Poincare algebra anti-commutators of super-generators give translation generator: anti-commutators are proportional to $p^k\sigma_k$. Could it be possible to have an anti-commutator proportional to the contraction of Dirac operator $p^k\sigma_k$ of 4-momentum with quaternionic sigma matrices having or 8-momentum with octonionic 8-matrices?

This would give good hopes that the GRT limit of TGD with many-sheeted space-time replaced with a slightly curved region of M^4 in long length scales has large $\mathcal{N}$ SUSY as an approximate symmetry: $\mathcal{N}$ would correspond to the maximal number of oscillator operators assignable to the partonic 2-surface. If conformal invariance is exact, it is just the number of fermion states for single generation in standard model.

1. The first promising sign is that the action principle indeed assigns a conserved light-like 8-momentum to each fermion line at partonic 2-surface. Therefore octonionic representation of sigma matrices makes sense and the generalization of standard twistorialization of four-momentum also. 8-momentum can be characterized by a pair of octonionic 2-spinors $(\lambda, \overline{\lambda})$ such that one has $\lambda\ \overline{\lambda}) = p^k\sigma_k$.

2. Since fermion line as string boundary is 1-D curve, the corresponding octonionic sub-spaces is just 1-D complex ray in octonion space and imaginary axes is defined by the associated imaginary octonion unit. Non-associativity and non-commutativity play no role and it is as if one had light like momentum in say z-direction.

3. One can select the ininitial values of spinor modes at the ends of fermion lines in such a way that they have well-defined spin and electroweak spin and one can also form linear superpositions of the spin states. One can also assume that the 8-D algebraic variant of Dirac equation correlating M^4 and CP_2 spins is satisfied.

   One can introduce oscillator operators $b_{m,\alpha}^\dagger$ and $b_{n,\alpha}$ with α denoting the spin. The motivation for why electroweak spin is not included as an index is due to the correlation between spin and electroweak spin. Dirac equation at fermion line implies a complete correlation between directions of spin and electroweak spin: if the directions are same for leptons (convention only), they are opposite for antileptons and for quarks since the product of them defines embedding space chirality which distinguishes between quarks and leptons. Instead of introducing electroweak isospin as an additional correlated index one can introduce 4 kinds of oscillator operators: leptonic and quark-like and fermionic and antifermionic.

4. For definiteness one can consider only fermions in leptonic sector. In hope of getting the analog of SUSY algebra one could modify the fermionic anti-commutation relations such that one has

$$\{b^\dagger_{m,\alpha}, b_{n,\beta}\} = \pm i\epsilon_{\alpha\beta}\delta_{m,n} \quad . \tag{4.10}$$

Here α is spin label and ϵ is the standard antisymmetric tensor assigned to twistors. The anti-commutator is clearly symmetric also now. The anti-commmutation relations with different signs $\pm$ at the right-hand side distinguish between quarks and leptons and also between fermions and anti-fermions. $\pm = 1$ could be the convention for fermions in lepton sector.

5. One wants combinations of oscillator operators for which one obtains anti-commutators having interpretation in terms of translation generators representing in terms of 8-momentum. The guess would be that the oscillator operators are given by

$$B^\dagger_n = b^\dagger_{m,\alpha}\lambda^\alpha \quad , \quad B_n = \overline{\lambda}^\alpha b_{m,\alpha} \quad . \tag{4.11}$$

The anti-commutator would in this case be given by

$$\begin{aligned} \{B^\dagger_m, B_n\} &= i\overline{\lambda}^\alpha \epsilon_{\alpha\beta}\lambda^\beta\delta_{m,n} \\ &= Tr(p^k\sigma_k)\delta_{m,n} = 2p^0\delta_{m,n} \quad . \end{aligned} \tag{4.12}$$

The inner product is positive for positive value of energy p^0. This form of anti-commutator obviously breaks Lorentz invariance and this us due the number theoretic selection of preferred time direction as that for real octonion unit. Lorentz invariance is saved by the fact that there is a moduli space for the choices of the quaternion units parameterized by Lorentz boosts for CD.

The anti-commutator vanishes for covariantly constant antineutrino so that it does not generate sparticle states. Only fermions with non-vanishing four-momentum do so and the resulting algebra is very much like that associated with a unitary representation of super Poincare algebra.

6. The recipe gives one helicity state for lepton in given mode m (conformal weight). One has also antilepton with opposite helicity with $\pm = -1$ in the formula defining the anti-commutator. In the similar manner one obtains quarks and antiquarks.

7. Contrary to the hopes, one did not obtain the anti-commutator $p^k\sigma_k$ but $Tr(p^0\sigma_0)$. $2p^0$ is however analogous to the action of Dirac operator $p^k\sigma_k$ to a massless spinor mode with "wrong" helicity giving $2p^0\sigma^0$. Massless modes with wrong helicity are expected to appear in the fermionic propagator lines in TGD variant of twistor approach. Hence one might hope that the resulting algebra is consistent with SUSY limit.

The presence of 8-momentum at each fermion line would allow also to consider the introduction of anti-commutators of form $p^k(8)\sigma_k$ directly making $\mathcal{N} = 8$ SUSY at parton level manifest. This expression restricts for time-like M^4 momenta always to quaternion and one obtains just the standard picture.

8. Only the fermionic states with vanishing conformal weight seem to be realized if the conformal symmetries associated with the spinor modes are realized as gauge symmetries. Super-generators would correspond to the fermions of single generation standard model: 4+4 =8 states altogether. Interestingly, $\mathcal{N} = 8$ correspond to the maximal SUSY for super-gravity. Right-handed neutrino would obviously generate the least broken SUSY. Also now mixing of M^4 helicities induces massivation and symmetry breaking so that even this SUSY is broken. One must however distinguish this SUSY from the super-symplectic conformal symmetry. The space in which SUSY would be realized would be partonic 2-surfaces and this distinguishes it from the usual SUSY. Also the conservation of fermion number and absence of Majorana spinors is an important distinction.

3. What about quantum deformations of the fermionic oscillator algebra?

Quantum deformation introducing braid statistics is of considerable interest. Quantum deformations are essentially 2-D phenomenon, and the experimental fact that it indeed occurs gives a further strong support for the localization of spinors at string world sheets. If the existence of anyonic phases is taken completely seriously, it supports the existence of the hierarchy of Planck constants and TGD view about dark matter. Note that the localization also at partonic 2-surfaces cannot be excluded yet.

I have wondered whether quantum deformation could relate to the hierarchy of Planck constants in the sense that $n = h_{eff}/h$ corresponds to the value of deformation parameter $q = exp(i2\pi/n)$.

A q-deformation of Clifford algebra of WCW gamma matrices is required. Clifford algebra is characterized in terms of anti-commutators replaced now by q-anticommutators. The natural identification of gamma matrices is as complexified gamma matrices. For q-deformation q-anticommutators would define WCW Kähler metric. The commutators of the supergenerators should still give anti-symmetric sigma matrices. The q-anticommutation relations should be same in the entire sector of WCW considered and be induced from the q-anticommutation relations for the oscillator operators of induced spinor fields at string world sheets, and reflect the fact that permutation group has braid group as covering group in 2-D case so that braid statistics becomes possible.

In [A7] (`http://tinyurl.com/y9e6pg4d`) the q-deformations of Clifford algebras are discussed, and this discussion seems to apply in TGD framework.

1. It is assumed that a Lie-algebra g has action in the Clifford algebra. The q-deformations of Clifford algebra is required to be consistent with the q-deformation of the universal enveloping algebra Ug.

2. The simplest situation corresponds to group $su(2)$ so that Clifford algebra elements are labelled by spin $\pm 1/2$. In this case the q-anticommutor for creation operators for spin up states reduces to an anti-commutator giving q-deformation I_q of unit matrix but for the spin down states one has genuine q-anti-commutator containing besides I_q also number operator for spin up states at the right hand side.

3. The undeformed anti-commutation relations can be witten as

$$P^{+kl}_{ij} a_k a_l = 0 \quad , \quad P^{+kl}_{ij} a^\dagger_k a^\dagger_l = 0 \quad , \quad a^i a^\dagger_j + P^{ih}_{jk} a^\dagger_h a^k = \delta^i_j 1 \quad . \tag{4.13}$$

   Here $P^{kl}_{ij} = \delta^i_l \delta^j_k$ is the permutator and $P^{+kl}_{ij} = (1+P)/2$ is projector. The q-deformation reduces to a replacement of the permutator and projector with q-permutator P_q and q-projector and P^+_q, which are both fixed by the quantum group.

4. Also the condition that deformed algebra has same Poincare series as the original one is posed. This says that the representation content is not changed that is the dimensions of summands in a representation as direct sum of graded sub-spaces are same for algebra and its q-deformation. If one has quantum group in a strict sense of the word (quasi-triangularity (genuine braid group) rather that triangularity requiring that the square of the deformed permutator P_q is unit matrix, one can have two situations.

   (a) $g = sl(N)$ (special linear group such as $SL(2, F)$, $F = R, C$) or $g = Sp(N = 2n)$ (symplectic group such as $Sp(2) = SL(2, R)$), which is subgroup of $sl(N)$. Creation (annihilation-) operators must form the N-dimensional defining representation of g.

   (b) $g = sl(N)$ and one has direct sum of M N-dimensional defining representations of g. The M copies of representation are ordered so that they can be identified as strands of braid so that the deformation makes sense at the space-like ends of string world sheet naturally. q-projector is proportional to so called universal R-matrix.

5. It is also shown that q-deformed oscillator operators can be expressed as polynomials of the ordinary ones.

The following argument suggest that the g must correspond to the minimal choices $sl(2, R)$ (or $su(2)$) in TGD framework.

1. The q-Clifford algebra structure of WCW should be induced from that for the fermionic oscillator algebra. g cannot correspond to $su(2)_{spin} \times su(2)_{ew}$ since spin and weak isospin label fermionic oscillator operators beside conformal weights but must relate closely to this group. The physical reason is that the separate conservation of quark and lepton numbers and light-likeness in 8-D sense imply correlations between the components of the spinors and reduce g.

2. For a given H-chirality (quark/ lepton) 8-D light-likeness forced by massless Dirac equation at the light-like boundary of the string world sheet at parton orbit implies correlation between M^4 and CP_2 chiralities. Hence there are 4+4 spinor components corresponding to fermions and antifermions with physical (creation oeprators) and unphysical (annihilation operators) polarizations. This allows two creation operators with given H-chirality (quark or lepton) and fermion number. Same holds true for antifermions. By fermion number conservation one obtains a reduction to $SU(2)$ doublets and the quantum group would be $sl(2) = sp(2)$ for which "special linear" implies "symplectic".

5 Quantum Criticality And Kähler-Dirac Action

The precise mathematical formulation of quantum criticality has remained one of the basic challenges of quantum TGD. The belief has been that the existence of conserved current for Kähler-Dirac equation are possible if Kähler action is critical for the 3-surface in question in the sense that the deformation in question corresponds to vanishing of second variation of Kähler action. The vanishing of the second variation states that the deformation of the Kähler-Dirac gamma matrix is divergence free just like the Kähler-Dirac gamma matrix itself and is therefore very natural.

2-D conformal invariance accompanies 2-D criticality and allows to satisfy these conditions for spinor modes localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This localization is in the generic case forced by the conditions that em charge is well-defined for the spinor modes: this requires that classical W fields vanish and also the vanishing of classical Z^0 field is natural -at least above weak scale. Only 2 Kähler-Dirac gamma matrices can be non-vanishing and this is possible only for Kähler-Dirac action.

5.1 What Quantum Criticality Could Mean?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the embedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to correspond to deformations of a given preferred extremal defining a vanishing second variation of Kähler Kähler function or Kähler action.

   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

(b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A5]. Cusp catastrophe (see `http://tinyurl.com/yddpfdgo`) [A1] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

3. Quantum criticality makes sense also for Kähler action.

   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K5] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

   (b) Also now one expects a hierarchy of criticalitiesandsince criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

   (c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. I have discussed what criticality could mean for Kähler-Dirac action [K16].

   (a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

   (b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since embedding space coordinates appear as parameters in Kähler-Dirac action. Kähler-Dirac equation is satisfied if the first variation of the canonical momentum densities contracted with the embedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. One obtains conserved fermion current associated with deformations only if the deformation of the Kähler-Dirac gamma matrix is divergenceless just like the Kähler-Dirac gamma matrix itself. This conditions requires the vanishing of the second variation of Kähler action.

(c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the Kähler-Dirac gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor.

This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as a matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied without the restriction to 2-surface.

The vacuum degeneracy of Kähler action strongly suggests that the number of critical deformations is always infinite and that these deformations define an infinite inclusion hierarchy of super-conformal algebras. This inclusion hierarchy would correspond to a fractal hierarchy of breakings of super-conformal symmetry generalizing the symmetry breaking hierarchies of gauge theories. These super-conformal inclusion hierarchies would realize the inclusion hierarchies for hyper-finite factors of type II_1.

5.2 Quantum Criticality And Fermionic Representation Of Conserved Charges Associated With Second Variations Of Kähler Action

It is rather obvious that TGD allows a huge generalizations of conformal symmetries. The development of the understanding of conservation laws has been however slow. Kähler-Dirac action provides excellent candidates for quantum counterparts of Noether charges. The problem is that the embedding space coordinates are in the role of classical external fields and induces spinor fields are second quantized so that it is not at all clear whether one obtains conserved charges.

5.2.1 What does the conservation of the fermionic Noether current require?

The obvious anser to the question of the title is that the conservation of the fermionic current requires the vanishing of the first variation of Kähler-Dirac action with respect to embedding space coordinates. This is certainly true but need not mean vanishing of the second variation of Kähler action as thought first. Hence fermionic conserved currents might be obtained for much more general variations than critical ones.

1. The Kähler-Dirac action assigns to a deformation of the space-time surface a conserved charge expressible as bilinears of fermionic oscillator operators only if the first variation of the Kähler-Dirac action under this deformation vanishes.

   The vanishing of the first variation for the Kähler-Dirac action is equivalent with the vanishing of the second variation for the Kähler action. This can be seen by the explicit calculation of the second variation of the Kähler-Dirac action and by performing partial integration for the terms containing derivatives of Ψ and $\overline{\Psi}$ to give a total divergence representing the difference of the charge at upper and lower boundaries of the causal diamond plus a four-dimensional integral of the divergence term defined as the integral of the quantity

$$\begin{aligned} \Delta S_D &= \overline{\Psi}\Gamma^k D_\alpha J^\alpha_k \Psi \ , \\ J^\alpha_k &= \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l_\beta}\delta h^k_\beta + \frac{\partial^2 L_K}{\partial h^k_\alpha \partial h^l}\delta h^l \ . \end{aligned} \tag{5.1}$$

   Here h^k_β denote partial derivative of the embedding space coordinates with respect to space-time coordinates. ΔS_D vanishes if this term vanishes:

$$D_\alpha J_k^\alpha = 0 \ .$$

The condition states the vanishing of the second variation of Kähler action. This can of course occur only for preferred deformations of X^4. One could consider the possibility that these deformations vanish at light-like 3-surfaces or at the boundaries of CD. Note that covariant divergence is in question so that J_k^α does not define conserved classical charge in the general case.

2. This condition is however un-necessarily strong. It is enough that that the deformation of Dirac operator anihilates the spinor mode, which can also change in the deformation. It must be possible to compensate the change of the covariant derivative in the deformation by a gauge transformation which requires that deformations act as gauge transformations on induce gauge potentials. This gives additional constraint and strongly suggests Kac-Moody type algebra for the deformations. Conformal transformations would satisfy this constraint and are suggested by quantum criticality.

3. It is essential that the Kähler-Dirac equation holds true so that the Kähler-Dirac action vanishes: this is needed to cancel the contribution to the second variation coming from the determinant of the induced metric. The condition that the Kähler-Dirac equation is satisfied for the deformed space-time surface requires that also Ψ suffers a transformation determined by the deformation. This gives

$$\delta\Psi = -\frac{1}{D} \times \Gamma^k J_k^\alpha \Psi \ . \tag{5.2}$$

Here $1/D$ is the inverse of the Kähler-Dirac operator defining the counterpart of the fermionic propagator.

4. The fermionic conserved currents associated with the deformations are obtained from the standard conserved fermion current

$$J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi \ . \tag{5.3}$$

Note that this current is conserved only if the space-time surface is extremal of Kähler action: this is also needed to guarantee Hermiticity and same form for the Kähler-Dirac equation for Ψ and its conjugate as well as absence of mass term essential for super-conformal invariance. Note also that ordinary divergence rather only covariant divergence of the current vanishes.

The conserved currents are expressible as sums of three terms. The first term is obtained by replacing Kähler-Dirac gamma matrices with their increments in the deformation keeping Ψ and its conjugate constant. Second term is obtained by replacing Ψ with its increment $\delta\Psi$. The third term is obtained by performing same operation for $\delta\overline{\Psi}$.

$$J^\alpha = \overline{\Psi}\Gamma^k J_k^\alpha \Psi + \overline{\Psi}\hat{\Gamma}^\alpha \delta\Psi + \delta\overline{\Psi}\hat{\Gamma}^\alpha\Psi \ . \tag{5.4}$$

These currents provide a representation for the algebra defined by the conserved charges analogous to a fermionic representation of Kac-Moody algebra.

5. Also conserved super charges corresponding to super-conformal invariance are obtained. The first class of super currents are obtained by replacing Ψ or $\overline{\Psi}$ right handed neutrino spinor or its conjugate in the expression for the conserved fermion current and performing the above procedure giving two terms since nothing happens to the covariantly constant right handed-neutrino spinor. Second class of conserved currents is defined by the solutions of the Kähler-Dirac equation interpreted as c-number fields replacing Ψ or $\overline{\Psi}$ and the same procedure gives three terms appearing in the super current.

6. The existence of vanishing of second variations is analogous to criticality in systems defined by a potential function for which the rank of the matrix defined by second derivatives of the potential function vanishes at criticality. Quantum criticality becomes the prerequisite for the existence of quantum theory since fermionic anti-commutation relations in principle can be fixed from the condition that the algebra in question is equivalent with the algebra formed by the vector fields defining the deformations of the space-time surface defining second variations. Quantum criticality in this sense would also select preferred extremals of Kähler action as analogs of Bohr orbits and the spectrum of preferred extremals would be more or less equivalent with the expected existence of infinite-dimensional symmetry algebras.

It is far from obvious that the criticality conditions or even the weaker conditions guaranteing the existence of (say) isometry charges can be satisfied. It seems that the restriction of spinor modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - implied by the condition that em charge is well-define for them, is the manner to achieve this. The reason is that conformal invariance allows complexification of the Kähler-Dirac gamma matrices and allows to construct spinor modes as holomorphic modes and their conjugates. Holomorphy reduces K-D equation to algebraic condition that Γ^z annihilates the spinor mode. If this is true also the deformation of Γ^z then the existince of conserved current follows. It is essential that only two Kähler-Dirac gamma matrices are non-vanishing and this is possible only for Kähler-Dirac action.

5.2.2 About the general structure of the algebra of conserved charges

Some general comments about the structure of the algebra of conserved charges are in order.

1. Any Cartan algebra of the isometry group $P \times SU(3)$ (there are two types of them for P corresponding to linear and cylindrical Minkowski coordinates) defines critical deformations (one could require that the isometries respect the geometry of CD). The corresponding second order charges for Kähler action are conserved but vanish since the corresponding conjugate coordinates are cyclic for the Kähler metric and Kähler form so that the conserved current is proportional to the gradient of a Killing vector field which is constant in these coordinates.

2. Contrary to the original conclusion, the corresponding fermionic charges expressible as fermionic bilinears are first order in deformation and do not vanish! Four-momentum and color quantum numbers are defined for Kähler action as classical conserved quantities and for Kähler-Dirac action as quantal charges.

5.2.3 Critical manifold is infinite-dimensional for Kähler action

Some examples might help to understand what is involved.

1. The action defined by four-volume gives a first glimpse about what one can expect. In this case Kähler-Dirac gamma matrices reduce to the induced gamma matrices. Second variations satisfy d'Alembert type equation in the induced metric so that the analogs of massless fields are in question. Mass term is present only if some dimensions are compact. The vanishing of excitations at light-like boundaries is a natural boundary condition and might well imply that the solution spectrum could be empty. Hence it is quite possible that four-volume action leads to a trivial theory.

2. For the vacuum extremals of Kähler action the situation is different. There exists an infinite number of second variations and the classical non-determinism suggests that deformations vanishing at the light-like boundaries exist. For the canonical embedding of M^4 the equation for second variations is trivially satisfied. If the CP_2 projection of the vacuum extremal is one-dimensional, the second variation contains a non-vanishing term and an equation analogous to massless d'Alembert equation for the increments of CP_2 coordinates is obtained. Also for the vacuum extremals of Kähler action with 2-D CP_2 projection all terms involving induced Kähler form vanish and the field equations reduce to d'Alembert type equations for CP_2 coordinates. A possible interpretation is as the classical analog of Higgs field. For the deformations of non-vacuum extremals this would suggest the presence of terms analogous to mass terms: these kind of terms indeed appear and are proportional to δs_k. M^4 degrees of freedom decouple completely and one obtains QFT type situation.

3. The physical expectation is that at least for the vacuum extremals the critical manifold is infinite-dimensional. The notion of finite measurement resolution suggests infinite hierarchies of inclusions of hyper-finite factors of type II_1 possibly having interpretation in terms of inclusions of the super conformal algebras defined by the critical deformations.

4. The properties of Kähler action give support for this expectation. The critical manifold is infinite-dimensional in the case of vacuum extremals. Canonical embedding of M^4 would correspond to maximal criticality analogous to that encountered at the tip of the cusp catastrophe. The natural guess would be that as one deforms the vacuum extremal the previously critical degrees of freedom are transformed to non-critical ones. The dimension of the critical manifold could remain infinite for all preferred extremals of the Kähler action. For instance, for cosmic string like objects any complex manifold of CP_2 defines cosmic string like objects so that there is a huge degeneracy is expected also now. For CP_2 type vacuum extremals M^4 projection is arbitrary light-like curve so that also now infinite degeneracy is expected for the deformations.

This leads to the conjecture that the critical deformations correspond to sub-algebras of super-conformal algebras with conformal weights coming as integer multiples of fixed integer m. One would have infinite hierarchy of breakings of conformal symmetry labelled by m. The super-conformal algebras would be effectively m-dimensional. Since all commutators with the critical sub-algebra would create zero energy states. In ordinary conformal field theory one have maximal criticality corresponding to $m = 1$.

5.2.4 Critical super-algebra and zero modes

The relationship of the critical super-algebra to WCW geometry is interesting.

1. The vanishing of the second variation plus the identification of Kähler function as a Kähler action for preferred extremals means that the critical variations are orthogonal to all deformations of the space-time surface with respect to the WCW metric.

   The original expectation was that critical deformations correspond to zero modes but this interpretation need not be correct since critical deformations can leave 3-surface invariant but affect corresponding preferred extremal: this would conform with the non-deterministic character of the dynamics which is indeed the basic signature of criticality. Rather, critical deformations are limiting cases of ordinary deformations acting in quantum fluctuating degrees of freedom.

   This conforms with the fact that WCW metric vanishes identically for canonically imbedded M^4 and that Kähler action has fourth order terms as first non-vanishing terms in perturbative expansion (for Kähler-Dirac the expansion is quadratic in deformation).

   Therefore the super-conformal algebra associated with the critical deformations has genuine physical content.

2. Since the action of X^4 local Hamiltonians of $\delta M^4_\times CP_2$ corresponds to the action in quantum fluctuating degrees of freedom, critical deformations cannot correspond to this kind of Hamiltonians.

3. The notion of finite measurement resolution suggests that the degrees of freedom which are below measurement resolution correspond to vanishing gauge charges. The sub-algebras of critical super-conformal algebra for which charges annihilate physical states could correspond to this kind of gauge algebras.

4. The conserved super charges associated with the vanishing second variations cannot give WCW metric as their anti-commutator. This would also lead to a conflict with the effective 2-dimensionality stating that WCW line-element is expressible as sum of contribution coming from partonic 2-surfaces as also with fermionic anti-commutation relations.

5.2.5 Connection with quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question how to make TGD unique if Kähler function for WCW is defined by the Kähler action for a preferred extremal assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obviou idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength is analogous to critical temperature so that TGD would be more or less uniquely defined.

To understand the delicacies it is convenient to consider various variations of Kähler action first.

1. The variation can leave 3-surface invariant but modify space-time surface such that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface and perhaps also at light-like 3-surfaces. In this case the correspondence between X^3 and $X^4(X^3)$ would not be unique and one would have non-deterministic dynamics characteristic for critical systems. This criticality would correspond to criticality of Kähler action at X^3. Note that the original working hypothesis was that $X^4(X^3)$ is unique. The failure of the strict classical determinism implying spin glass type vacuum degeneracy indeed suggets that this is the case.

2. The variation could act on zero modes which do not affect Kähler metric which corresponds to (1, 1) part of Hessian in complex coordinates for WCW . Only the zero modes characterizing 3-surface appearing as parameters in the metric WCW would be affected and the result would be a generalization of conformal transformation. Kähler function would change but only due to the change in zero modes. These transformations do not seem to correspond to critical transformations since Kähler function changes.

3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would of course affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that (1, 1) part of Hessian is degenerate. This could mean that in the vicinity of X^3 the Kähler form has non-definite signature: physically this is unacceptable since inner product in Hilbert space would not be positive definite.

Critical transformations might relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

1. The critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical transformations then critical transformation would extend WCW to a bundle for which 3-surfaces would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.

2. Critical deformations could correspond to H or sub-group of H (which dependes on X^3). For other 3-surfaces than X^3 the action of H is non-trivial as the case of $CP_2 = SU(3)/U(2)$ makes easy to understand.

3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies of a given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1. For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical. Basic example of criticality is bifurcation diagram for cusp catastrophe. Quantum criticality realized as the vanishing of the second variation gives hopes about a more or less unique identification of preferred extremals and considered alternative identifications such as absolute minimization of Kähler action which is just the opposite of criticality.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to deformations of four-surface and second for the Kähler function itself with respect to deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

1. The criticality for preferred extremals would make 4-D criticality a property of all physical systems.

2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1, 1) part of Hessian in complex coordinates must be positive definite. Thus criticality might imply problems.

   This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no zero modes causing troubles in Gaussian approximation to functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function.

3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW . These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interaction seems to be that these degrees of freedom correspond to deformations in zero modes.

Both the super-symmetry of D_K and conservation Dirac Noether currents for Kähler-Dirac action have thus a connection with quantum criticality.

1. Finite-dimensional critical systems defined by a potential function $V(x^1, x^2, ..)$ are characterized by the matrix defined by the second derivatives of the potential function and the rank of system classifies the levels in the hierarchy of criticalities. Maximal criticality corresponds to the complete vanishing of this matrix. Thom's catastrophe theory classifies these hierarchies, when the numbers of behavior and control variables are small (smaller than 5). In the recent case the situation is infinite-dimensional and the criticality conditions give additional field equations as existence of vanishing second variations of Kähler action.

2. The vacuum degeneracy of Kähler action allows to expect that this kind infinite hierarchy of criticalities is realized. For a general vacuum extremal with at most 2-D CP_2 projection the matrix defined by the second variation vanishes because $J_{\alpha\beta} = 0$ vanishes and also the matrix $(J^{\alpha}_{\ k} + J_k^{\ \alpha})(J^{\beta}_{\ l} + J_l^{\ \beta})$ vanishes by the antisymmetry $J^{\alpha}_{\ k} = -J_k^{\ \alpha}$.

   The formulation of quantal version of Equivalence Principle (EP) in string picture demonstrates that the conservation of of fermionic Noether currents defining gravitational four-momentum and other Poincare quantum numbers requires that the deformation of the Kähler-Dirac equation obtained by replacing Kähler-Dirac gamma matrices with their deformations is also satisfied. Holomorphy can guarantee this. The original wrong conclusion was that this condition is equivalent with much stronger condition stating the vanishing of the second variation of Kähler action, which it is not. There is analogy for this: massless Dirac equation does not imply the vanishing of four-momentum.

3. Conserved bosonic and fermionic Noether charges would characterize quantum criticality. In particular, the isometries of the embedding space define conserved currents represented in

terms of the fermionic oscillator operators if the second variations defined by the infinitesimal isometries vanish for the Kähler-Dirac action. For vacuum extremals the dimension of the critical manifold is infinite: maybe there is hierarchy of quantum criticalities for which this dimension decreases step by step but remains always infinite. This hierarchy could closely relate to the hierarchy of inclusions of hyper-finite factors of type II_1. Also the conserved charges associated with super-symplectic and Super Kac-Moody algebras would require infinite-dimensional critical manifold defined by the spectrum of second variations.

4. Phase transitions are characterized by the symmetries of the phases involved with the transitions, and it is natural to expect that dynamical symmetries characterize the hierarchy of quantum criticalities. The notion of finite quantum measurement resolution based on the hierarchy of Jones inclusions indeed suggests the existence of a hierarchy of dynamical gauge symmetries characterized by gauge groups in ADE hierarchy [K5] with degrees of freedom below the measurement resolution identified as gauge degrees of freedom.

5. Does this criticality have anything to do with the criticality against the phase transitions changing the value of Planck constant? If the geodesic sphere S^2_I for which induced Kähler form vanishes corresponds to the back of the CP_2 book (as one expects), this could be the case. The homologically non-trivial geodesic sphere S^12_{II} is as far as possible from vacuum extremals. If it corresponds to the back of CP_2 book, cosmic strings would be quantum critical with respect to phase transition changing Planck constant. They cannot however correspond to preferred extremals.

5.3 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

The vanishing of the second variation in interior of $X^4(X^3_l)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago! The question whether these extremals correspond to absolute minima remains however open.

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).

2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.

3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.

4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

5. There is a possible connection with the notion of self-organized criticality [B1] introduced to explain the behavior of systems like sand piles. Self-organization in these systems tends to lead "to the edge". The challenge is to understand how system ends up to a critical state, which by definition is unstable. Mechanisms for this have been discovered and based on phase transitions occurring in a wide range of parameters so that critical point extends to a critical manifold. In TGD Universe quantum criticality suggests a universal mechanism of this kind. The criticality for the preferred extremals of Kähler action would mean that classically all systems are critical in well-defined sense and the question is only about the degree of criticality. Evolution could be seen as a process leading gradually to increasingly critical systems. One must however distinguish between the criticality associated with the preferred extremals of Kähler action and the criticality caused by the spin glass like energy landscape like structure for the space of the maxima of Kähler function.

5.4 Quantum Criticality And Electroweak Symmetries

In the following quantum criticali and electroweak symmetries are discussed for Kähler-Dirac action.

5.4.1 What does one mean with quantum criticality?

Quantum criticality is one of the basic guiding principles of Quantum TGD. What it means mathematically is however far from clear and one can imagine several meanings for it.

1. What is obvious is that quantum criticality implies quantization of Kähler coupling strength as a mathematical analog of critical temperature so that the theory becomes mathematically unique if only single critical temperature is possible. Physically this means the presence of long range fluctuations characteristic for criticality and perhaps assignable to the effective hierarchy of Planck constants having explanation in terms of effective covering spaces of the embedding space. This hierarchy follows from the vacuum degeneracy of Kähler action, which in turn implies 4-D spin-glass degeneracy. It is easy to interpret the degeneracy in terms of criticality.

2. At more technical level one would expect criticality to corresponds to deformations of a given preferred extremal defining a vanishing second variation of Kähler Kähler function or Kähler action.

   (a) For Kähler function this criticality is analogous to thermodynamical criticality. The Hessian matrix defined by the second derivatives of free energy or potential function becomes degenerate at criticality as function of control variables which now would be naturally zero modes not contribution to Kähler metric of WCW but appearing as parameters in it. The bevavior variables correspond to quantum fluctuating degrees of freedom and according to catastrophe theory a big change can in quantum fluctuating degrees of freedom at criticality for zero modes. This would be control of quantum state by varying classical variables. Cusp catastrophe is standard example of this. One can imagined also a situation in which the roles of zero modes and behavior variables change and big jump in the values of zero modes is induced by small variation in behavior variables. This would mean quantum control of classical variables.

(b) Zero modes controlling quantum fluctuating variables in Kähler function would correspond to vanishing of also second derivatives of potential function at extremum in certain directions so that the matrix defined by second derivatives does not have maximum rank. Entire hierarchy of criticalities is expected and a good finite-dimensional model is provided by the catastrophe theory of Thom [A5]. Cusp catastrophe (see `http://tinyurl.com/yddpfdgo`) [A1] is the simplest catastrophe one can think of, and here the folds of cusp where discontinuous jump occurs correspond to criticality with respect to one control variable and the tip to criticality with respect to both control variables.

3. Quantum criticality makes sense also for Kähler action.

   (a) Now one considers space-time surface connecting which 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K5] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

   (b) Also now one expects a hierarchy of criticalitiesandsince criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

   (c) The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. I have discussed what criticality could mean for Kähler-Dirac action [K16].

   (a) I have conjectured that it leads to the existence of additional conserved currents defined by the variations which do not affect the value of Kähler action. These arguments are far from being mathematically rigorous and the recent view about the solutions of the Kähler-Dirac equation predicting that the spinor modes are restricted to 2-D string world sheets requires a modification of these arguments.

   (b) The basic challenge is to understand the mechanism making this kind of currents conserved: the same challenge is met already in the case of isometries since embedding space coordinates appear as parameters in Kähler-Dirac action. The existence of conserved currents does not actually require the vanishing of the second variation of Kähler action as claimed earlier. It is enough that the first variation of the canonical momentum densities contracted with the embedding space gamma matrices annihilates the spinor mode. Situation is analogous to massless Dirac equation: it does not imply the vanishing of four-momentum, only the vanishing of mass. Hence conserved currents are obtained also outside the quantum criticality.

   (c) It is far from obvious that these conditions can be satisfied. The localization of the spinor modes to string world sheets or partonic 2-surfaces guaranteeing in the generaic case that em charge is well-defined for spinor modes implies holomorphy allowing to formulate current conservation for currents associated with the deformations of the space-time surface for second quantized induced spinor field. The crux is that the deformation respects the holomorphy properties of the modified gamma matrices at string world sheet and thus does not mix Γ^z with $\Gamma^{\overline{z}}$. The deformation of Γ^z has only z-component and also annihilates the holomorphic spinor. This mechanism is possible only for Kähler-Dirac action since the Kähler-Dirac gamma matrices in directions orthogonal to the 2-surface must vanish and this is not possible for other actions. This also means that energy momentum tensor has rank 2 as matrix. Cosmic string solutions are an exception since in this case CP_2 projection of space-time surface is 2-D and conditions guaranteing vanishing of classical W fields can be satisfied.

In the following these arguments are formulated more precisely. The unexpected result is that critical deformations induce conformal scalings of the modified metric and electro-weak gauge transformations of the induced spinor connection at X^2. Therefore holomorphy brings in the Kac-Moody symmetries associated with isometries of H (gravitation and color gauge group) and quantum criticality those associated with the holonomies of H (electro-weak-gauge group) as additional symmetries.

5.4.2 The variation of modes of the induced spinor field in a variation of space-time surface respecting the preferred extremal property

Consider first the variation of the induced spinor field in a variation of space-time surface respecting the preferred extremal property. The deformation must be such that the deformed Kähler-Dirac operator D annihilates the modified mode. By writing explicitly the variation of the Kähler-Dirac action (the action vanishes by Kähler-Dirac equation) one obtains deformations and requiring its vanishing one obtains

$$\delta\Psi = D^{-1}(\delta D)\Psi \ . \tag{5.5}$$

D^{-1} is the inverse of the Kähler-Dirac operator defining the analog of Dirac propagator and δD defines vertex completely analogous to $\gamma^k\delta A_k$ in gauge theory context. The functional integral over preferred extremals can be carried out perturbatively by expressing δD in terms of δh^k and one obtains stringy perturbation theory around X^2 associated with the preferred extremal defining maximum of Kähler function in Euclidian region and extremum of Kähler action in Minkowskian region (stationary phase approximation).

What one obtains is stringy perturbation theory for calculating n-points functions for fermions at the ends of braid strands located at partonic 2-surfaces and representing intersections of string world sheets and partonic 2-surfaces at the light-like boundaries of CDs. δD- or more precisely, its partial derivatives with respect to functional integration variables - appear atthe vertices located anywhere in the interior of X^2 with outgoing fermions at braid ends. Bosonic propagators are replaced with correlation functions for δh^k. Fermionic propagator is defined by D^{-1}.

After 35 years or hard work this provides for the first time a reasonably explicit formula for the N-point functions of fermions. This is enough since by bosonic emergence these N-point functions define the basic building blocks of the scattering amplitudes. Note that bosonic emergence states that bosons corresponds to wormhole contacts with fermion and anti-fermion at the opposite wormhole throats.

5.4.3 What critical modes could mean for the induced spinor fields?

What critical modes could mean for the induced spinor fields at string world sheets and partonic 2-surfaces. The problematic part seems to be the variation of the Kähler-Dirac operator since it involves gradient. One cannot require that covariant derivative remains invariant since this would require that the components of the induced spinor connection remain invariant and this is quite too restrictive condition. Right handed neutrino solutions de-localized into entire X^2 are however an exception since they have no electro-weak gauge couplings and in this case the condition is obvious: Kähler-Dirac gamma matrices suffer a local scaling for critical deformations:

$$\delta\Gamma^{\mu} = \Lambda(x)\Gamma^{\mu} . \quad (5.6)$$

This guarantees that the Kähler-Dirac operator D is mapped to ΛD and still annihilates the modes of ν_R labelled by conformal weight, which thus remain unchanged.

What is the situation for the 2-D modes located at string world sheets? The condition is obvious. Ψ suffers an electro-weak gauge transformation as does also the induced spinor connection so that D_{μ} is not affected at all. Criticality condition states that the deformation of the space-time surfaces induces a conformal scaling of Γ^{μ} at X^2. It might be possible to continue this conformal scaling of the entire space-time sheet but this might be not necessary and this would mean that all critical deformations induced conformal transformations of the effective metric of the space-time surface defined by $\{\Gamma^{\mu}, \Gamma^{\nu}\} = 2G^{\mu\nu}$. Thus it seems that effective metric is indeed central concept (recall that if the conjectured quaternionic structure is associated with the effective metric, it might be possible to avoid problem related to the Minkowskian signature in an elegant manner).

In fact, one can consider even more general action of critical deformation: the modes of the induced spinor field would be mixed together in the infinitesimal deformation besides infinitesimal electroweak gauge transformation, which is same for all modes. This would extend electroweak gauge symmetry. Kähler-Dirac equation holds true also for these deformations. One might wonder whether the conjectured dynamically generated gauge symmetries assignable to finite measurement resolution could be generated in this manner.

The infinitesimal generator of a critical deformation J_M can be expressed as tensor product of matrix A_M acting in the space of zero modes and of a generator of infinitesimal electro-weak gauge transformation $T_M(x)$ acting in the same manner on all modes: $J_M = A_M \otimes T_M(x)$. A_M is a spatially constant matrix and $T_M(x)$ decomposes to a direct sum of left- and right-handed $SU(2)\times U(1)$ Lie-algebra generators. Left-handed Lie-algebra generator can be regarded as a quaternion and right handed as a complex number. One can speak of a direct sum of left-handed local quaternion $q_{M,L}$ and right-handed local complex number $c_{M,R}$. The commutator $[J_M, J_N]$ is given by $[J_M, J_N] = [A_M, A_N] \otimes \{T_M(x), T_N(x)\} + \{A_M, A_N\} \otimes [T_M(x), T_N(x)]$. One has $\{T_M(x), T_N(x)\} = \{q_{M,L}(x), q_{N,L}(x)\} \oplus \{c_{M,R}(x), c_{N,R}(x)\}$ and $[T_M(x), T_N(x)] = [q_{M,L}(x), q_{N,L}(x)]$. The commutators make sense also for more general gauge group but quaternion/complex number property might have some deeper role.

Thus the critical deformations would induce conformal scalings of the effective metric and dynamical electro-weak gauge transformations. Electro-weak gauge symmetry would be a dynamical symmetry restricted to string world sheets and partonic 2-surfaces rather than acting at the entire space-time surface. For 4-D de-localized right-handed neutrino modes the conformal scalings of the effective metric are analogous to the conformal transformations of M^4 for $\mathcal{N} = 4$ SYMs. Also ordinary conformal symmetries of M^4 could be present for string world sheets and could act as symmetries of generalized Feynman graphs since even virtual wormhole throats are massless. An interesting question is whether the conformal invariance associated with the effective metric is the analog of dual conformal invariance in $\mathcal{N} = 4$ theories.

Critical deformations of space-time surface are accompanied by conserved fermionic currents. By using standard Noetherian formulas one can write

$$J_i^{\mu} = \overline{\Psi}\Gamma^{\mu}\delta_i\Psi + \delta_i\overline{\Psi}\Gamma^{\mu}\Psi . \quad (5.7)$$

Here $\delta\Psi_i$ denotes derivative of the variation with respect to a group parameter labeled by i. Since $\delta\Psi_i$ reduces to an infinitesimal gauge transformation of Ψ induced by deformation, these currents are the analogs of gauge currents. The integrals of these currents along the braid strands at the ends of string world sheets define the analogs of gauge charges. The interpretation as Kac-Moody charges is also very attractive and I have proposed that the 2-D Hodge duals of gauge potentials could be identified as Kac-Moody currents. If so, the 2-D Hodge duals of J would define the quantum analogs of dynamical electro-weak gauge fields and Kac-Moody charge could be also seen as non-integral phase factor associated with the braid strand in Abelian approximation (the interpretation in terms of finite measurement resolution is discussed earlier).

One can also define super currents by replacing $\overline{\Psi}$ or Ψ by a particular mode of the induced spinor field as well as c-number valued currents by performing the replacement for both $\overline{\Psi}$ or Ψ.

As expected, one obtains a super-conformal algebra with all modes of induced spinor fields acting as generators of super-symmetries restricted to 2-D surfaces. The number of the charges which do not annihilate physical states as also the effective number of fermionic modes could be finite and this would suggest that the integer $\mathcal{N}$ for the supersymmetry in question is finite. This would conform with the earlier proposal inspired by the notion of finite measurement resolution implying the replacement of the partonic 2-surfaces with collections of braid ends.

Note that Kac-Moody charges might be associated with "long" braid strands connecting different wormhole throats as well as short braid strands connecting opposite throats of wormhole contacts. Both kinds of charges would appear in the theory.

5.4.4 What is the interpretation of the critical deformations?

Critical deformations bring in an additional gauge symmetry. Certainly not all possible gauge transformations are induced by the deformations of preferred extremals and a good guess is that they correspond to holomorphic gauge group elements as in theories with Kac-Moody symmetry. What is the physical character of this dynamical gauge symmetry?

1. Do the gauge charges vanish? Do they annihilate the physical states? Do only their positive energy parts annihilate the states so that one has a situation characteristic for the representation of Kac-Moody algebras. Or could some of these charges be analogous to the gauge charges associated with the constant gauge transformations in gauge theories and be therefore non-vanishing in the absence of confinement. Now one has electro-weak gauge charges and these should be non-vanishing. Can one assign them to deformations with a vanishing conformal weight and the remaining deformations to those with non-vanishing conformal weight and acting like Kac-Moody generators on the physical states?

2. The simplest option is that the critical Kac-Moody charges/gauge charges with non-vanishing positive conformal weight annihilate the physical states. Critical degrees of freedom would not disappear but make their presence known via the states labelled by different gauge charges assignable to critical deformations with vanishing conformal weight. Note that constant gauge transformations can be said to break the gauge symmetry also in the ordinary gauge theories unless one has confinement.

3. The hierarchy of quantum criticalities suggests however entire hierarchy of electro-weak Kac-Moody algebras. Does this mean a hierarchy of electro-weak symmetries breakings in which the number of Kac-Moody generators not annihilating the physical states gradually increases as also modes with a higher value of positive conformal weight fail to annihilate the physical state?

   The only manner to have a hierarchy of algebras is by assuming that only the generators satisfying $n \ mod \ N = 0$ define the sub-Kac-Moody algebra annihilating the physical states so that the generators with $n \ mod \ N \neq 0$ would define the analogs of gauge charges. I have suggested for long time ago the relevance of kind of fractal hierarchy of Kac-Moody and Super-Virasoro algebras for TGD but failed to imagine any concrete realization.

   A stronger condition would be that the algebra reduces to a finite dimensional algebra in the sense that the actions of generators Q_n and Q_{n+kN} are identical. This would correspond to periodic boundary conditions in the space of conformal weights. The notion of finite measurement resolution suggests that the number of independent fermionic oscillator operators is proportional to the number of braid ends so that an effective reduction to a finite algebra is expected.

   Whatever the correct interpretation is, this would obviously refine the usual view about electro-weak symmetry breaking.

These arguments suggests the following overall view. The holomorphy of spinor modes gives rise to Kac-Moody algebra defined by isometries and includes besides Minkowskian generators associated with gravitation also SU(3) generators associated with color symmetries. Vanishing second variations in turn define electro-weak Kac-Moody type algebra.

Note that criticality suggests that one must perform functional integral over WCW by decomposing it to an integral over zero modes for which deformations of X^4 induce only an electro-weak gauge transformation of the induced spinor field and to an integral over moduli corresponding to the remaining degrees of freedom.

5.5 The Emergence Of Yangian Symmetry And Gauge Potentials As Duals Of Kac-Moody Currents

Yangian symmetry plays a key role in $\mathcal{N} = 4$ super-symmetric gauge theories. What is special in Yangian symmetry is that the algebra contains also multi-local generators. In TGD framework multi-locality would naturally correspond to that with respect to partonic 2-surfaces and string world sheets and the proposal has been that the Super-Kac-Moody algebras assignable to string worlds sheets could generalize to Yangian.

Witten has written a beautiful exposition of Yangian algebras [B2]. Yangian is generated by two kinds of generators J^A and Q^A by a repeated formation of commutators. The number of commutations tells the integer characterizing the multi-locality and provides the Yangian algebra with grading by natural numbers. Witten describes a 2-dimensional QFT like situation in which one has 2-D situation and Kac-Moody currents assignable to real axis define the Kac-Moody charges as integrals in the usual manner. It is also assumed that the gauge potentials defined by the 1-form associated with the Kac-Moody current define a flat connection:

$$\partial_\mu j^A_\nu - \partial_\nu j^A_\nu + [j^A_\mu, j^A_\nu] = 0 \ . \tag{5.8}$$

This condition guarantees that the generators of Yangian are conserved charges. One can however consider alternative ways to obtain the conservation.

1. The generators of first kind - call them J^A - are just the conserved Kac-Moody charges. The formula is given by

$$J_A = \int_{-\infty}^{\infty} dx j^{A0}(x,t) \ . \tag{5.9}$$

2. The generators of second kind contain bi-local part. They are convolutions of generators of first kind associated with different points of string described as real axis. In the basic formula one has integration over the point of real axis.

$$Q^A = f^A_{BC} \int_{-\infty}^{\infty} dx \int_x^{\infty} dy j^{B0}(x,t) j^{C0}(y,t) - 2\int_{-\infty}^{\infty} j^A_x dx \ . \tag{5.10}$$

These charges are indeed conserved if the curvature form is vanishing as a little calculation shows.

How to generalize this to the recent context?

1. The Kac-Moody charges would be associated with the braid strands connecting two partonic 2-surfaces - Strands would be located either at the space-like 3-surfaces at the ends of the space-time surface or at light-like 3-surfaces connecting the ends. Kähler-Dirac equation would define Super-Kac-Moody charges as standard Noether charges. Super charges would be obtained by replacing the second quantized spinor field or its conjugate in the fermionic bilinear by particular mode of the spinor field. By replacing both spinor field and its conjugate by its mode one would obtain a conserved c-number charge corresponding to an anti-commutator of two fermionic super-charges. The convolution involving double integral is however not number theoretically attractive whereas single 1-D integrals might make sense.

2. An encouraging observation is that the Hodge dual of the Kac-Moody current defines the analog of gauge potential and exponents of the conserved Kac-Moody charges could be identified as analogs for the non-integrable phase factors for the components of this gauge potential.

This identification is precise only in the approximation that generators commute since only in this case the ordered integral $P(exp(i\int Adx))$ reduces to $P(exp(i\int Adx))$.Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization implying that Abelian approximation works. This conforms with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

This would make possible a direct identification of Kac-Moody symmetries in terms of gauge symmetries. For isometries one would obtain color gauge potentials and the analogs of gauge potentials for graviton field (in TGD framework the contraction with M^4 vierbein would transform tensor field to 4 vector fields). For Kac-Moody generators corresponding to holonomies one would obtain electroweak gauge potentials. Note that super-charges would give rise to a collection of spartners of gauge potentials automatically. One would obtain a badly broken SUSY with very large value of $\mathcal{N}$ defined by the number of spinor modes as indeed speculated earlier [**?**].

3. The condition that the gauge field defined by 1-forms associated with the Kac-Moody currents are trivial looks unphysical since it would give rise to the analog of topological QFT with gauge potentials defined by the Kac-Moody charges. For the duals of Kac-Moody currents defining gauge potentials only covariant divergence vanishes implying that curvature form is

$$F_{\alpha\beta} = \epsilon_{\alpha\beta}[j_\mu, j^\mu] \ , \tag{5.11}$$

so that the situation does not reduce to topological QFT unless the induced metric is diagonal. This is not the case in general for string world sheets.

4. It seems however that there is no need to assume that j_μ defines a flat connection. Witten mentions that although the discretization in the definition of J^A does not seem to be possible, it makes sense for Q^A in the case of $G = SU(N)$ for any representation of G. For general G and its general representation there exists no satisfactory definition of Q. For certain representations, such as the fundamental representation of $SU(N)$, the definition of Q^A is especially simple. One just takes the bi-local part of the previous formula:

$$Q^A = f^A_{BC}\sum_{i<j} J_i^B J_j^C \ . \tag{5.12}$$

What is remarkable that in this formula the summation need not refer to a discretized point of braid but to braid strands ordered by the label i by requiring that they form a connected polygon. Therefore the definition of J^A could be just as above.

5. This brings strongly in mind the interpretation in terms of twistor diagrams. Yangian would be identified as the algebra generated by the logarithms of non-integrable phase factors in Abelian approximation assigned with pairs of partonic 2-surfaces defined in terms of Kac-Moody currents assigned with the Kähler-Dirac action. Partonic 2-surfaces connected by braid strand would be analogous to nearby points of space-time in its discretization. This would fit nicely with the vision about finite measurement resolution as discretization in terms partonic 2-surfaces and braids.

The resulting algebra satisfies the basic commutation relations

$$\left[J^A, J^B\right] = f^{AB}_C J^C \ , \ \left[J^A, Q^B\right] = f^{AB}_C Q^C \ . \tag{5.13}$$

plus the rather complex Serre relations described in [B2].

6 Kähler-Dirac Equation And Super-Symmetries

The previous considerations concerning super-conformal symmetries and space-time SUSY have been based on general arguments. The new vision about preferred extremals and Kähler-Dirac equation however leads to a rather detailed understanding of super-conformal symmetries at the level of field equations and is bound to modify the existing vision about super-conformal symmetries.

Whether TGD predicts some variant of space-time SUSY or not has been a long-standing issue: the reason is that TGD does not allow Majorana spinors since fermion number conservation is exact. The more precise formulation of field equations made possible by the realization that spinor modes are localized at string world sheets allows to conclude that the analog of broken $\mathcal{N}=8$ SUSY is predicted at parton level and that right-handed neutrino generates the minimally broken $\mathcal{N}=2$ sub-SUSY.

One important outcome of criticality is the identification of gauge potentials as duals of Kac-Moody currents at the boundaries of string world sheets: quantum gauge potentials are defined only where they are needed that is string curves defining the non-integrable phase factors. This gives also rise to the realization of the conjectured Yangian in terms of the Kac-Moody charges and commutators in accordance with the earlier conjecture.

6.1 Super-Conformal Symmetries

It is good to summarize first the basic ideas about Super-Virasoro representations. TGD allows two kinds of super-conformal symmetries.

1. The first super-conformal symmetry is associated with $\delta M^4_\pm \times CP_2$ and corresponds to symplectic symmetries of $\delta M^4_\pm \times CP_2$. The reason for extension of conformal symmetries is metric 2-dimensionality of the light-like boundary $\delta M^4_\pm$ defining upper/lower boundary of causal diamond (CD). This super-conformal symmetry is something new and corresponds to replacing finite-dimensional Lie-group G for Kac-Moody symmetry with infinite-dimensional symplectic group. The light-like radial coordinate of $\delta M^4_\pm$ takes the role of the real part of complex coordinate z for ordinary conformal symmetry. Together with complex coordinate of S^2 it defines 3-D restriction of Hamilton-Jacobi variant of 4-D super-conformal symmetries. One can continue the conformal symmetries from light-cone boundary to CD by forming a slicing by parallel copies of $\delta M^4_\pm$. There are two possible slicings corresponding to the choices δM^4_+ and δM^4_- assignable to the upper and lower boundaries of CD. These two choices correspond to two arrows of geometric time for the basis of zero energy states in ZEO.

2. Super-symplectic degrees of freedom determine the electroweak and color quantum numbers of elementary particles. Bosonic emergence implies that ground states assignable to partonic 2-surfaces correspond to partial waves in $\delta M^4_\pm$ and one obtains color partial waves in particular. These partial waves correspond to the solutions for the Dirac equation in embedding space and the correlation between color and electroweak quantum numbers is not quite correct. Super-Kac-Moody generators give the compensating color for massless states obtained from tachyonic ground states guaranteeing that standard correlation is obtained. Super-symplectic degrees are therefore directly visible in particle spectrum. One can say that at the point-like limit the WCW spinors reduce to tensor products of embedding space spinors assignable to the center of mass degrees of freedom for the partonic 2-surfaces defining wormhole throats.

   I have proposed a physical interpretation of super-symplectic vibrational degrees of freedom in terms of degrees of freedom assignable to non-perturbative QCD. These degrees of freedom would be responsible for most of the baryon masses but their theoretical understanding is lacking in QCD framework.

3. The second super-conformal symmetry is assigned light-like 3-surfaces and to the isometries and holonomies of the embedding space and is analogous to the super-Kac-Moody symmetry of string models. Kac-Moody symmetries could be assigned to the light-like deformations of light-like 3-surfaces. Isometries give tensor factor $E^2 \times SU(3)$ and holonomies factor

$SU(2)_L \times U(1)$. Altogether one has 5 tensor factors to super-conformal algebra. That the number is just five is essential for the success p-adic mass calculations [K8, K7].

The construction of solutions of the Kähler-Dirac equation suggests strongly that the fermionic representation of the Super-Kac-Moody algebra can be assigned as conserved charges associated with the space-like braid strands at both the 3-D space-like ends of space-time surfaces and with the light-like (or space-like with a small deformation) associated with the light-like 3-surfaces. The extension to Yangian algebra involving higher multi-linears of super-Kac Moody generators is also highly suggestive. These charges would be non-local and assignable to several wormhole contacts simultaneously. The ends of braids would correspond points of partonic 2-surfaces defining a discretization of the partonic 2-surface having interpretation in terms of finite measurement resolution.

These symmetries would correspond to electroweak and strong gauge fields and to gravitation. The duals of the currents giving rise to Kac-Moody charges would define the counterparts of gauge potentials and the conserved Kac-Moody charges would define the counterparts of non-integrable phase factors in gauge theories. The higher Yangian charges would define generalization of non-integrable phase factors. This would suggest a rather direct connection with the twistorial program for calculating the scattering amplitudes implies also by zero energy ontology.

Quantization recipes have worked in the case of super-string models and one can ask whether the application of quantization to the coefficients of powers of complex coordinates or Hamilton-Jacobi coordinates could lead to the understanding of the 4-D variants of the conformal symmetries and give detailed information about the representations of the Kac-Moody algebra too.

6.2 WCW Geometry And Super-Conformal Symmetries

The vision about the geometry of WCW has been roughly the following and the recent steps of progress induce to it only small modifications if any.

1. Kähler geometry is forced by the condition that hermitian conjugation allows geometrization. Kähler function is given by the Kähler action coming from space-time regions with Euclidian signature of the induced metric identifiable as lines of generalized Feynman diagrams. Minkowskian regions give imaginary contribution identifiable as the analog of Morse function and implying interference effects and stationary phase approximation. The vision about quantum TGD as almost topological QFT inspires the proposal that Kähler action reduces to 3-D terms reducing to Chern-Simons terms by the weak form of electric-magnetic duality. The recent proposal for preferred extremals is consistent with this property realizing also holography implied by general coordinate invariance. Strong form of general coordinate invariance implying effective 2-dimensionality in turn suggests that Kähler action is expressible string world sheets and possibly also areas of partonic 2-surfaces.

2. The complexified gamma matrices of WCW come as hermitian conjugate pairs and anti-commute to the Kähler metric of WCW . Also bosonic generators of symplectic transformations of $\delta M^4_\pm \times CP_2$ a assumed to act as isometries of WCW geometry can be complexified and appear as similar pairs. The action of isometry generators co-incides with that of symplectic generators at partonic 2-surfaces and string world sheets but elsewhere inside the space-time surface it is expected to be deformed from the symplectic action. The super-conformal transformations of $\delta M^4_\pm \times CP_2$ acting on the light-like radial coordinate of $\delta M^4_\pm$ act as gauge symmetries of the geometry meaning that the corresponding WCW vector fields have zero norm.

3. WCW geometry has also zero modes which by definition do not contribute to WCW metric expect possibly by the dependence of the elements of WCW metric on zero modes through a conformal factor. In particular, induced CP_2 Kähler form and its analog for sphere $r_M = constant$ of light cone boundary are symplectic invariants, and one can define an infinite number of zero modes as invariants defined by Kähler fluxes over partonic 2-surfaces and string world sheets. This requires however the slicing of CD parallel copies of δM^4_+ or δM^4_-.

The physical interpretation of these non-quantum fluctuating degrees of freedom is as classical variables necessary for the interpretation of quantum measurement theory. Classical variable would metaphorically correspond the position of the pointer of the measurement instrument.

4. The construction receives a strong philosophical inspiration from the geometry of loop spaces. Loop spaces allow a unique Kähler geometry with maximal isometry group identifiable as Kac-Moody group. The reason is that otherwise Riemann connection does not exist. The only problem is that curvature scalar diverges since the Riemann tensor is by constant curvature property proportional to the metric. In 3-D case one would have union of constant curvature spaces labelled by zero modes and the situation is expected to be even more restrictive. The conjecture indeed is that WCW geometry exists only for $H = M^4 \times CP_2$: infinite-D Kähler geometric existence and therefore physics would be unique. One can also hope that Ricci scalar is finite and therefore zero by the constant curvature property so that Einstein's equations are satisfied.

5. The matrix elements of WCW Kähler metric are given in terms of the anti-commutators of the fermionic Noether super-charges associated with symplectic isometry currents. A given mode of induced spinor field characterized by embedding space chirality (quark or lepton), by spin and weak spin plus conformal weight n. If the super-conformal transformations for string modes act gauge transformations only the spinor modes with vanishing conformal weight correspond to non-zero modes of the WCW metric and the situation reduces essentially to the analog of $\mathcal{N} = 8$ SUSY.

   The WCW Hamiltonians generating symplectic isometries correspond to the Hamiltonians spanning the symplectic group of $\delta M^4_\pm \times CP_2$. One can say that the space of quantum fluctuating degrees of freedom is this symplectic group of $\delta M^4_\pm \times CP_2$ or its subgroup or coset space: this must have very deep implications for the structure of the quantum TGD.

   An interesting possibility is that the radial conformal weights of the symplectic algebra are linear combinations of the zeros of Riemann Zeta with integer coefficients. Also this option allows to realize the hierarchy of super-symplectic conformal symmetry breakings in terms of sub-algebras isomorphic to the entire super-symplectic algebra. WCW would have fractal structure corresponding to a hierarchy of quantum criticalities.

6. The localization of the induced spinors to string world sheets means that the super-symplectic Noether charges are associated with strings connecting partonic 2-surfaces. The physically obvious fact that given partonic surface can be accompanied by an arbitrary number of strings, forces a generalization of the super-symplectic algebra to a Yangian containing infinite number of n-local variants of various super-symplectic Noether charges. For instance, four -momentum is accompanied by multi-stringy variants involving four-momentum P_0^A and angular momentum generators. At the first level of the hierarchy one has $P_1^A = f^A_{BC} P_0^B \otimes J^C$. This hierarchy might play crucial role in understanding of the four-momenta of bound states.

7. Zero energy ontology brings in additional delicacies. Basic objects are now unions of partonic 2-surfaces at the ends of CD. One can generalize the expressions for the isometry generators in a straightforward manner by requiring that given isometry restricts to a symplectic transformation at partonic 2-surfaces and string world sheets.

8. One could criticize the effective metric 2-dimensionality forced by the general consistency arguments as something non-physical. The WCW Hamiltonians are expressed using only the data at partonic 2-surfaces and string string world sheets: this includes also 4-D tangent space data via the weak form of electric-magnetic duality so that one has only effective 2-dimensionality. Obviously WCW geometry must huge large gauge symmetries besides zero modes. The hierarchy of super-symplectic symmetries indeed represent gauge symmetries of this kind.

   Effective 2-dimensionality realizing strong form of holography in turn is induced by the strong form of general coordinate invariance. Light-like 3-surfaces at which the signature of the induced metric changes must be equivalent with the 3-D space-like ends of space-time surfaces at the light-boundaries of space-time surfaces as far as WCW geometry is considered. This requires that the data from their 2-D intersections defining partonic 2-surfaces should dictate the WCW geometry. Note however that Super-Kac-Moody charges giving information about the interiors of 3-surfaces appear in the construction of the physical states.

6.3 The Relationship Between Inertial Gravitational Masses

The relationship between inertial and gravitational masses and Equivalence Principle have been on of the longstanding problems in TGD. Not surprisingly, the realization how GRT space-time relates to the many-sheeted space-time of TGD finally allowed to solve the problem.

6.3.1 ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

6.3.2 Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

1. At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

2. The most recent view (2014) about understanding how EP emerges in TGD is described in [K13] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

3. The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the Kähler-Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

6.3.3 Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD space-time and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case also now but one can consider also more general solutions in which one has two cosmological constants which are not genuine constants anymore: this approach is not promising.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

6.3.4 Constraints from p-adic mass calculations and ZEO

A further important physical input comes from p-adic thermodynamics forming a core element of p-adic mass calculations.

1. The first thing that one can get worried about relates to the extension of conformal symmetries. If the conformal symmetries generalize to $D = 4$, how can one take seriously the results of p-adic mass calculations based on 2-D conformal invariance? There is no reason to worry. The reduction of the conformal invariance to 2-D one for the preferred extremals takes care of this problem. This however requires that the fermionic contributions assignable

to string world sheets and/or partonic 2-surfaces - Super- Kac-Moody contributions - should dictate the elementary particle masses. For hadrons also symplectic contributions should be present. This is a valuable hint in attempts to identify the mathematical structure in more detail.

2. ZEO suggests that all particles, even virtual ones correspond to massless wormhole throats carrying fermions. As a consequence, twistor approach would work and the kinematical constraints to vertices would allow the cancellation of divergences. This would suggest that the p-adic thermal expectation value is for the longitudinal M^2 momentum squared (the definition of CD selects $M^1 \subset M^2 \subset M^4$ as also does number theoretic vision). Also propagator would be determined by M^2 momentum. Lorentz invariance would be obtained by integration of the moduli for CD including also Lorentz boosts of CD.

3. In the original approach one allows states with arbitrary large values of L_0 as physical states. Usually one would require that L_0 annihilates the states. In the calculations however mass squared was assumed to be proportional L_0 apart from vacuum contribution. This is a questionable assumption. ZEO suggests that total mass squared vanishes and that one can decompose mass squared to a sum of longitudinal and transversal parts. If one can do the same decomposition to longitudinal and transverse parts also for the Super Virasoro algebra then one can calculate longitudinal mass squared as a p-adic thermal expectation in the transversal super-Virasoro algebra and only states with $L_0 = 0$ would contribute and one would have conformal invariance in the standard sense.

4. In the original approach the assumption motivated by Lorentz invariance has been that mass squared is replaced with conformal weight in thermodynamics, and that one first calculates the thermal average of the conformal weight and then equates it with mass squared. This assumption is somewhat ad hoc. ZEO however suggests an alternative interpretation in which one has zero energy states for which longitudinal mass squared of positive energy state derive from p-adic thermodynamics. Thermodynamics - or rather, its square root - would become part of quantum theory in ZEO. M-matrix is indeed product of hermitian square root of density matrix multiplied by unitary S-matrix and defines the entanglement coefficients between positive and negative energy parts of zero energy state.

5. The crucial constraint is that the number of super-conformal tensor factors is $N = 5$: this suggests that thermodynamics applied in Super-Kac-Moody degrees of freedom assignable to string world sheets is enough, when one is interested in the masses of fermions and gauge bosons. Super-symplectic degrees of freedom can also contribute and determine the dominant contribution to baryon masses. Should also this contribution obey p-adic thermodynamics in the case when it is present? Or does the very fact that this contribution need not be present mean that it is not thermal? The symplectic contribution should correspond to hadronic p-adic length prime rather the one assignable to (say) u quark. Hadronic p-adic mass squared and partonic p-adic mass squared cannot be summed since primes are different. If one accepts the basic rules [K9], longitudinal energy and momentum are additive as indeed assumed in perturbative QCD.

6. Calculations work if the vacuum expectation value of the mass squared must be assumed to be tachyonic. There are two options depending on whether one whether p-adic thermodynamics gives total mass squared or longitudinal mass squared.

   (a) One could argue that the total mass squared has naturally tachyonic ground state expectation since for massless extremals longitudinal momentum is light-like and transversal momentum squared is necessary present and non-vanishing by the localization to topological light ray of finite thickness of order p-adic length scale. Transversal degrees of freedom would be modeled with a particle in a box.

   (b) If longitudinal mass squared is what is calculated, the condition would require that transversal momentum squared is negative so that instead of plane wave like behavior exponential damping would be required. This would conform with the localization in transversal degrees of freedom.

6.4 Realization Of Space-Time SUSY In TGD

The generators of super-conformal algebras are obtained by taking fermionic currents for second quantized fermions and replacing either fermion field or its conjugate with its particular mode. The resulting super currents are conserved and define super charges. By replacing both fermion and its conjugate with modes one obtains c-number valued currents. In this manner one also obtains the analogs of super-Poincare generators labelled by the conformal weight and other spin quantum numbers as Noether charges so that space-time SUSY is suggestive.

The super-conformal invariance in spinor modes is expected to be gauge symmetry so that only the generators with vanishing string world sheet conformal weight create physical states. This would leave only the conformal quantum numbers characterizing super-symplectic generators (radial conformal weight included) under consideration and the hierarchy of its sub-algebras acting as gauge symmetries giving rise to a hierarchy of criticalities having interpretation in terms of dark matter.

As found in the earlier section, the proposed anti-commutation relations for fermionic oscillator operators at the ends of string world sheets can be formulated so that they are analogous to those for Super Poincare algebra. The reason is that field equations assign a conserved 8-momentum to the light-like geodesic line defining the boundary of string at the orbit of partonic 2-surface. Octonionic representation of sigma matrices making possible generalization of twistor formalism to 8-D context is also essential. As a matter, the final justification for the analog of space-time came from the generalization of twistor approach to 8-D context.

By counting the number of spin and weak isospin components of embedding space spinors satisfying massless algebraic Dirac equation one finds that broken $\mathcal{N}=8$ SUSY is the expected space-time SUSY. $\mathcal{N}=2$ SUSY assignable to right-handed neutrino is the least broken sub-SUSY and one is forced to consider the possibility that spartners correspond to dark matter with $h_{eff}=n\times h$ and therefore remaining undetected in recent particle physics experiments.

6.4.1 Super-space viz. Grassmann algebra valued fields

Standard SUSY induces super-space extending space-time by adding anti-commuting coordinates as a formal tool. Many mathematicians are not enthusiastic about this approach because of the purely formal nature of anti-commuting coordinates. Also I regard them as a non-sense geometrically and there is actually no need to introduce them as the following little argument shows.

Grassmann parameters (anti-commuting theta parameters) are generators of Grassmann algebra and the natural object replacing super-space is this Grassmann algebra with coefficients of Grassmann algebra basis appearing as ordinary real or complex coordinates. This is just an ordinary space with additional algebraic structure: the mysterious anti-commuting coordinates are not needed. To me this notion is one of the conceptual monsters created by the over-pragmatic thinking of theoreticians.

This allows allows to replace field space with super field space, which is completely well-defined object mathematically, and leave space-time untouched. Linear field space is simply replaced with its Grassmann algebra. For non-linear field space this replacement does not work. This allows to formulate the notion of linear super-field just in the same manner as it is done usually.

The generators of super-symmetries in super-space formulation reduce to super translations, which anti-commute to translations. The super generators Q_α and $\overline{Q}_{\dot{\beta}}$ of super Poincare algebra are Weyl spinors commuting with momenta and anti-commuting to momenta:

$$\{Q_\alpha,\overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\ \alpha\ \dot{beta}}P_\mu \quad . \tag{6.1}$$

One particular representation of super generators acting on super fields is given by

$$\begin{aligned} D_\alpha &= i\frac{\partial}{\partial_{\theta_\alpha}} \quad , \\ D_{\dot{\alpha}} &= i\frac{\partial}{\partial_{\overline{\theta}_{\ \dot{alpha}}}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \end{aligned} \tag{6.2}$$

Here the index raising for 2-spinors is carried out using antisymmetric 2-tensor $\epsilon^{\alpha\beta}$. Super-space interpretation is not necessary since one can interpret this action as an action on Grassmann algebra valued field mixing components with different fermion numbers.

Chiral superfields are defined as fields annihilated by $D_{\dot{\alpha}}$. Chiral fields are of form $\Psi(x^\mu + i\overline{\theta}\sigma^\mu\theta, \theta)$. The dependence on $\overline{\theta}_{\dot{\alpha}}$ comes only from its presence in the translated Minkowski coordinate annihilated by $D_{\dot{\alpha}}$. Super-space enthusiast would say that by a translation of M^4 coordinates chiral fields reduce to fields, which depend on θ only.

6.4.2 The space of fermionic Fock states at partonic 2-surface as TGD counterpart of chiral super field

As already noticed, another manner to realize SUSY in terms of representations the super algebra of conserved super-charges. In TGD framework these super charges are naturally associated with the modified Dirac equation, and anti-commuting coordinates and super-fields do not appear anywhere. One can however ask whether one could identify a mathematical structure replacing the notion of chiral super field.

In [?] it was proposed that generalized chiral super-fields could effectively replace induced spinor fields and that second quantized fermionic oscillator operators define the analog of SUSY algebra. One would have $\mathcal{N} = \infty$ if all the conformal excitations of the induced spinor field restricted on 2-surface are present. For right-handed neutrino the modes are labeled by two integers and de-localized to the interior of Euclidian or Minkowskian regions of space-time sheet.

The obvious guess is that chiral super-field generalizes to the field having as its components many-fermions states at partonic 2-surfaces with theta parameters and their conjugates in one-one correspondence with fermionic creation operators and their hermitian conjugates.

1. Fermionic creation operators - in classical theory corresponding anti-commuting Grassmann parameters - replace theta parameters. Theta parameters and their conjugates are not in one-one correspondence with spinor components but with the fermionic creation operators and their hermitian conjugates. One can say that the super-field in question is defined in the "world of classical worlds" (WCW) rather than in space-time. Fermionic Fock state at the partonic 2-surface is the value of the chiral super field at particular point of WCW .

2. The matrix defined by the $\sigma^\mu\partial_\mu$ is replaced with a matrix defined by the Kähler-Dirac operator D between spinor modes acting in the solution space of the Kähler-Dirac equation. Since Kähler-Dirac operator annihilates the modes of the induced spinor field, super covariant derivatives reduce to ordinary derivatives with respect the theta parameters labeling the modes. Hence the chiral super field is a field that depends on θ_m or conjugates $\overline{\theta}_m$ only. In second quantization the modes of the chiral super-field are many-fermion states assigned to partonic 2-surfaces and string world sheets. Note that this is the only possibility since the notion of super-coordinate does not make sense now.

3. It would seem that the notion of super-field does not bring anything new. This is not the case. First of all, the spinor fields are restricted to 2-surfaces. Second point is that one cannot assign to the fermions of the many-fermion states separate non-parallel or even parallel four-momenta. The many-fermion state behaves like elementary particle. This has non-trivial implications for propagators and a simple argument [?] leads to the proposal that propagator for N-fermion partonic state is proportional to $1/p^N$. This would mean that only the states with fermion number equal to 1 or 2 behave like ordinary elementary particles.

6.5 Comparison Of TGD And Stringy Views About Super-Conformal Symmetries

The best manner to represent TGD based view about conformal symmetries is by comparison with the conformal symmetries of super string models.

6.5.1 Basic differences between the realization of super conformal symmetries in TGD and in super-string models

The realization super conformal symmetries in TGD framework differs from that in string models in several fundamental aspects.

1. In TGD framework super-symmetry generators acting as configuration space gamma matrices carry either lepton or quark number. Majorana condition required by the hermiticity of super generators which is crucial for super string models would be in conflict with the conservation of baryon and lepton numbers and is avoided. This is made possible by the realization of bosonic generators represented as Hamiltonians of X^2-local symplectic transformations rather than vector fields generating them [K4]. This kind of representation applies also in Kac-Moody sector since the local transversal isometries localized in X_l^3 and respecting light-likeness condition can be regarded as X^2 local symplectic transformations, whose Hamiltonians generate also isometries. Localization is not complete: the functions of X^2 coordinates multiplying symplectic and Kac-Moody generators are functions of the symplectic invariant $J = \epsilon^{\mu\nu} J_{\mu\nu}$ so that effective one-dimensionality results but in different sense than in conformal field theories. This realization of super symmetries is what distinguishes between TGD and super string models and leads to a totally different physical interpretation of super-conformal symmetries. The fermionic representations of super-symplectic and super Kac-Moody generators can be identified as Noether charges in standard manner.

2. A long-standing problem of quantum TGD was that stringy propagator $1/G$ does not make sense if G carries fermion number. The progress in the understanding of second quantization of the modified Dirac operator made it however possible to identify the counterpart of G as a c-number valued operator and interpret it as different representation of G [K3].

3. The notion of super-space is not needed at all since Hamiltonians rather than vector fields represent bosonic generators, no super-variant of geometry is needed. The distinction between Ramond and N-S representations important for $N = 1$ super-conformal symmetry and allowing only ground state weight 0 an $1/2$ disappears. Indeed, for $N = 2$ super-conformal symmetry it is already possible to generate spectral flow transforming these Ramond and N-S representations to each other (G_n is not Hermitian anymore).

4. If Kähler action defines the Kähler-Dirac operator, the number of spinor modes could be finite. One must be here somewhat cautious since bound state in the Coulomb potential associated with electric part of induced electro-weak gauge field might give rise to an infinite number of bound states which eigenvalues converging to a fixed eigenvalue (as in the case of hydrogen atom). Finite number of generalized eigenmodes means that the representations of super-conformal algebras reduces to finite-dimensional ones in TGD framework. Also the notion of number theoretic braid indeed implies this. The physical interpretation would be in terms of finite measurement resolution. If Kähler action is complexified to include imaginary part defined by CP breaking instanton term, the number of stringy mass square eigenvalues assignable to the spinor modes becomes infinite since conformal excitations are possible. This means breakdown of exact holography and effective 2-dimensionality of 3-surfaces. It seems that the inclusion of instanton term is necessary for several reasons. The notion of finite measurement resolution forces conformal cutoff also now. There are arguments suggesting that only the modes with vanishing conformal weight contribute to the Dirac determinant defining vacuum functional identified as exponent of Kähler function in turn identified as Kähler action for its preferred extremal.

5. What makes spinor field mode a generator of gauge super-symmetry is that is c-number and not an eigenmode of $D_K(X^2)$ and thus represents non-dynamical degrees of freedom. If the number of eigen modes of $D_K(X^2)$ is indeed finite means that most of spinor field modes represent super gauge degrees of freedom.

6.5.2 The super generators G are not Hermitian in TGD!

The already noticed important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that

WCW gamma matrices possess a well defined fermion number. The hermiticity of the WCW gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D\ mod\ 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

WCW gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates anti-fermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and anti-fermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and anti-fermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates anti-fermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{aligned} \{G_m, G_n^\dagger\} &= 2L_{m+n} + \tfrac{c}{3}(m^2 - \tfrac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} &= 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} &= 0 \ . \end{aligned} \tag{6.3}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

6.5.3 What could be the counterparts of stringy conformal fields in TGD framework?

The experience with string models would suggest the conformal symmetries associated with the complex coordinates of X^2 as a candidate for conformal super-symmetries. One can imagine two counterparts of the stringy coordinate z in TGD framework.

1. Super-symplectic and super Kac-Moody symmetries are local with respect to X^2 in the sense that the coefficients of generators depend on the invariant $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ rather than being completely free [K4]. Thus the real variable J replaces complex (or hyper-complex) stringy coordinate and effective 1-dimensionality holds true also now but in different sense than for conformal field theories.

2. The slicing of X^4 by string world sheets Y^2 and partonic 2-surfaces X^2 implied by number theoretical compactification implies string-parton duality and involves the super conformal fermionic gauge symmetries associated with the coordinates u and w in the dual dimensional reductions to stringy and partonic dynamics. These coordinates define the natural analogs of stringy coordinate. The effective reduction of X_l^3 to braid by finite measurement resolution implies the effective reduction of $X^4(X^3)$ to string world sheet. This implies quite strong resemblance with string model. The realization that spinor modes with well- define em

charge must be localized at string world sheets makes the connection with strings even more explicit [K16].

One can understand how Equivalence Principle emerges in TGD framework at space-time level when many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) is replaced with effective space-time lumping together the space-time sheets to M^4 endowed with effective metric. The quantum counterpart EP has most feasible interpretation in terms of Quantum Classical Correspondence (QCC): the conserved Kähler four-momentum equals to an eigenvalue of conserved Kähler-Dirac four-momentum acting as operator.

3. The conformal fields of string model would reside at X^2 or Y^2 depending on which description one uses and complex (hyper-complex) string coordinate would be identified accordingly. Y^2 could be fixed as a union of stringy world sheets having the strands of number theoretic braids as its ends. The proposed definition of braids is unique and characterizes finite measurement resolution at space-time level. X^2 could be fixed uniquely as the intersection of X_l^3 (the light-like 3-surface at which induced metric of space-time surface changes its signature) with $\delta M^4_\pm \times CP_2$. Clearly, wormhole throats X_l^3 would take the role of branes and would be connected by string world sheets defined by number theoretic braids.

4. An alternative identification for TGD parts of conformal fields is inspired by M^8-H duality. Conformal fields would be fields in WCW . The counterpart of z coordinate could be the hyper-octonionic M^8 coordinate m appearing as argument in the Laurent series of WCW Clifford algebra elements. m would characterize the position of the tip of CD and the fractal hierarchy of CDs within CDs would give a hierarchy of Clifford algebras and thus inclusions of hyper-finite factors of type II_1. Reduction to hyper-quaternionic field -that is field in M^4 center of mass degrees of freedom- would be needed to obtained associativity. The arguments m at various level might correspond to arguments of N-point function in quantum field theory.

7 Still about induced spinor fields and TGD counterpart for Higgs

The understanding of the modified Dirac equation and of the possible classical counterpart of Higgs field in TGD framework is not completely satisfactory. The emergence of twistor lift of Kähler action [K6] [L2] inspired a fresh approach to the problem and it turned out that a very nice understanding of the situation emerges.

More precise formulation of the Dirac equation for the induced spinor fields is the first challenge. The well-definedness of em charge has turned out to be very powerful guideline in the understanding of the details of fermionic dynamics. Although induced spinor fields have also a part assignable space-time interior, the spinor modes at string world sheets determine the fermionic dynamics in accordance with strong form of holography (SH).

The well-definedness of em charged is guaranteed if induced spinors are associated with 2-D string world sheets with vanishing classical W boson fields. It turned out that an alternative manner to satisfy the condition is to assume that induced spinors at the boundaries of string world sheets are neutrino-like and that these string world sheets carry only classical W fields. Dirac action contains 4-D interior term and 2-D term assignable to string world sheets. Strong form of holography (SH) allows to interpret 4-D spinor modes as continuations of those assignable to string world sheets so that spinors at 2-D string world sheets determine quantum dynamics.

Twistor lift combined with this picture allows to formulate the Dirac action in more detail. Well-definedness of em charge implies that charged particles are associated with string world sheets assignable to the magnetic flux tubes assignable to homologically non-trivial geodesic sphere and neutrinos with those associated with homologically trivial geodesic sphere. This explains why neutrinos are so light and why dark energy density corresponds to neutrino mass scale, and provides also a new insight about color confinement.

A further important result is that the formalism works only for embedding space dimension $D=8$. This is due the fact that the number of vector components is the same as the number of spinor components of fixed chirality for $D=8$ and corresponds directly to the octonionic triality.

p-Adic thermodynamics predicts elementary particle masses in excellent accuracy without Higgs vacuum expectation: the problem is to understand fermionic Higgs couplings. The observation that CP_2 part of the modified gamma matrices gives rise to a term mixing M^4 chiralities contain derivative allows to understand the mass-proportionality of the Higgs-fermion couplings at QFT limit.

7.1 More precise view about modified Dirac equation

Consistency conditions demand that modified Dirac equation with modified gamma matrices Γ^α defined as contractions $\Gamma^\alpha = T^{\alpha k}\gamma_k$ of canonical momentum currents $T^{\alpha k}$ associated with the bosonic action with embedding space gamma matrices γ_k [K16, K10]. The Dirac operator is not hermitian in the sense that the conjugation for the Dirac equation for Ψ does not give Dirac equation for $\overline{\Psi}$ unless the modified gamma matrices have vanishing covariant divergence as vector at space-time surface. This says that classical field equations are satisfied. This consistency condition holds true also for spinor modes possibly localized at string world sheets to which one can perhaps assign area action plus topological action defined by Kähler magnetic flux. The interpretation is in terms of super-conformal invariance.

The challenge is to formulate this picture more precisely and here I have not achieved a satisfactory formulation. The question has been whether interior spinor field Ψ are present at all, whether only Ψ is present and somehow becomes singular at string world sheets, or whether both stringy spinors Ψ_s and interior spinors Ψ are present. Both Ψ and Ψ_s could be present and Ψ_s could serve as source for interior spinors with the same H-chirality.

The strong form of holography (SH) suggests that interior spinor modes Ψ_n are obtained as continuations of the stringy spinor modes $\Psi_{s,n}$ and one has $\Psi = \Psi_s$ at string world sheets. Dirac action would thus have a term localized at strong world sheets and bosonic action would contain similar term by the requirement of super-conformal symmetry. Can one realize this intuition?

1. Suppose that Dirac action has interior and stringy parts. For the twistor lift of TGD [L2] the interior part with gamma matrices given by the modified gamma matrices associated with the sum of Kähler action and volume action proportional to cosmological constant Λ. The variation with respect to the interior spinor field Ψ gives modified Dirac equation in the interior with source term from the string world sheet. The H-chiralites of Ψ and Psi_s would be same. Quark like and leptonic H-chiralities have different couplings to Kähler gauge potential and mathematical consistency strongly encourages this.

   What is important is that the string world sheet part, which is bilinear in interior and string world sheet spinor fields Ψ and Ψ_s and otherwise has the same form as Dirac action. The natural assumption is that the stringy Dirac action corresponds to the modified gamma matrices assignable to area action.

2. String world sheet must be minimal surface: otherwise hermiticity is lost. This can be achieved either by adding to the Kähler action string world sheet area term. Whatever the correct option is, quantum criticality should determine the value of string tension. The first string model inspired guess is that the string tension is proportional to gravitational constant $1/G = 1/l_P^2$ defining the radius fo M^4 twistor sphere or to $1/R^2$, R CP_2 radius. This would however allow only strings not much longer than l_P or R. A more natural estimate is that string tension is proportional to the cosmological constant Λ and depends on p-adic length scale as $1/p$ so that the tension becomes small in long length scales. Since Λ coupling contant type parameter, this estimate looks rather reasonable.

3. The variation of stringy Dirac action with action density

$$L = [\overline{\Psi}_s D_s^{\rightarrow}\Psi - \overline{\Psi}_s D_s^{\leftarrow}\Psi]\sqrt{g_2} + h.c. \qquad (7.1)$$

   with respect to stringy spinor field Ψ_s gives for Ψ Dirac equation $D_s\Psi = 0$ if there are no Lagrange multiplier terms (see below). The variation in interior gives $D\Psi = S = D_s\Psi_s$, where the source term S is located at string world sheets. Ψ satisfies at string world sheet the analog of 2-D massless Dirac equation associated with the induced metric. This is just what stringy picture suggests.

The stringy source term for D equals to $D_s\Psi_s$ localized at string world sheets: the construction of solutions would require the construction of propagator for D, and this does not look an attractive idea. For $D_s\Psi_s = 0$ the source term vanishes. Holomorphy for Ψ_s indeed implies $D_s\Psi = 0$.

4. $\Psi_s = \Psi$ would realize SH as a continuation of Ψ_s from string world sheet to Ψ in the interior. Could one introduce Lagrange multiplier term

$$L_1 = \overline{\Lambda}(\Psi - \Psi_s) + h.c.$$

to realize $\Psi_s = \Psi$? Lagrange multiplier spinor field Λ would serve a source in the Dirac equation for $\Psi = \Psi_s$ and Ψ should be constructed at string world sheet in terms of stringy fermionic propagator with Λ as source. The solution for Ψ_s would require the construction of 2-D stringy propagator for Ψ_s but in principle this is not a problem since the modes can be solved by holomorphy in hypercomplex stringy coordinate. The problem of this option is that the H-chiralities of Λ and Ψ would be opposite and the coupling of opposite H-chiralities is not in spirit with H-chirality conservation.

A possible cure is to replace the Lagrange multiplier term with

$$L_1 = \overline{\Lambda}^k\gamma_k(\Psi - \Psi_s) + h.c. \quad . \tag{7.2}$$

The variation with respect to the spin 3/2 field Λ^k would give 8 conditions - just the number of spinor components for given H-chirality - forcing $\Psi = \Psi_s$! $D = 8$ would be in crucial role! In other embedding space dimensions the number of conditions would be too high or too low.

One would however obtain

$$D_s\Psi = D_s\Psi_s = \Lambda^k\gamma_k \quad . \tag{7.3}$$

One could of course solve Ψ at string world sheet from $\Lambda^k\gamma_k$ by constructing the 2-D propagator associated with D_s. Conformal symmetry for the modes however implies $D_s\Psi = 0$ so that one has actually $\Lambda^k = 0$ and Λ^k remains mere formal tool to realize the constraint $\Psi = \Psi_s$ in mathematically rigorous manner for embedding space dimension $D = 8$. This is a new very powerful argument in favor of TGD.

5. At the string world sheets Ψ would be annihilated both by D and D_s. The simplest possibility is that the actions of D and D_s are proportional to each other at string world sheets. This poses conditions on string world sheets, which might force the CP_2 projection of string world sheet to belong to a geodesic sphere or circle of CP_2. The idea that string world sheets and also 3-D surfaces with special role in TGD could correspond to singular manifolds at which trigonometric functions representing CP_2 coordinates tend to go outside their allowed value range supports this picture. This will be discussed below.

   (a) For the geodesic sphere of type II induced Kähler form vanishes so that the action of 4-D Dirac massless operator would be determined by the volume term (cosmological constant). Could the action of D reduce to that of D_s at string world sheets? Does this require a reduction of the metric to an orthogonal direct sum from string world sheet tangent space and normal space and that also normal part of D annihilates the spinors at the string world sheet? The modes of Ψ at string world sheets are locally constant with respect to normal coordinates.

   (b) For the geodesic sphere of type I induced Kähler form is non-vanishing and brings an additional term to D coming from CP_2 degrees of freedom. This might lead to trouble since the gamma matrix structures of D and D_s would be different. One could however add to string world sheet bosonic action a topological term as Kähler magnetic flux. Although its contribution to the field equations is trivial, the contribution to the modified gamma matrices is non-vanishing and equal to the contraction $J^{\alpha k}\gamma_k$ of half projection of the Kähler form with CP_2 gamma matrices. The presence of this term could allow the reduction of $D\Psi_s = 0$ and $D_s\Psi_s = 0$ to each other also in this case.

7.2 A more detailed view about string world sheets

In TGD framework gauge fields are induced and what typically occurs for the space-time surfaces is that they tend to "go out" from CP_2. Could various lower-D surfaces of space-time surface correspond to sub-manifolds of space-time surface?

1. To get a concrete idea about the situation it is best to look what happens in the case of sphere $S^2 = CP_1$. In the case of sphere S^2 the Kähler form vanishes at South and North poles. Here the dimension is reduced by 2 since all values of ϕ correspond to the same point. $sin(\Theta)$ equals to 1 at equator - geodesic circle - and here Kähler form is non-vanishing. Here dimension is reduced by 1 unit. This picture conforms with the expectations in the case of CP_2 These two situations correspond to 1-D and 2-D geodesic sub-manifolds.

2. CP_2 coordinates can be represented as cosines or sines of angles and the modules of cosine or sine tends to become larger than 1 (see `http://tinyurl.com/z3coqau`). In Eguchi-Hanson coordinates (r, Θ, Φ, Ψ) the coordinates r and Θ give rise to this kind of trigonometric coordinates. For the two cyclic angle coordinates (Φ, Ψ) one does not encounter this problem.

3. In the case of CP_2 only geodesic sub-manifolds with dimensions $D = 0, 1, 2$ are possible. 1-D geodesic submanifolds carry vanishing induce spinor curvature. The impossibility of 3-D geodesic sub-manifolds would suggest that 3-D surfaces are not important. CP_2 has two geodesic spheres: S^2_I is homologically non-trivial and S^2_{II} homologically trivial (see `http://tinyurl.com/z3coqau`).

   (a) Let us consider S^2_I first. CP_2 has 3 poles, which obviously relates to SU(3), and in Eguchi Hanson coordinates (r, θ, Φ, Ψ) the surface $r = \infty$ is one of them and corresponds - not to a 3-sphere - but homologically non-trivial geodesic 2- sphere, which is complex sub-manifold and orbits of $SU(2) \times U(1)$ subgroup. Various values of the coordinate Ψ correspond to same point as those of Φ at the poles of S^2. The Kähler form J and classical Z^0 and γ fields are non-vanishing whereas W gauge fields vanish leaving only induced γ and Z^0 field as one learns by studying the detailed expressions for the curvature of spinor curvature and vierbein of CP_2.

   String world sheet could have thus projection to S^2_I but both γ and Z^0 would be vanishing except perhaps at the boundaries of string world sheet, where Z^0 would naturally vanish in the picture provided by standard model. One can criticize the presence of Z^0 field since it would give a parity breaking term to the modified Dirac operator. SH would suggest that the reduction to electromagnetism at string boundaries might make sense as counterpart for standard model picture. Note that the original vision was that besides induced Kähler form and em field also Z^0 field could vanish at string world sheets.

   (b) The homologically trivial geodesic sphere S^2_{II} is the orbit of $SO(3)$ subgroup and not a complex manifold. By looking the standard example about S^2_I, one finds that the both J, Z_0, and γ vanish and only the W components of spinor connection are non-vanishing. In this case the notion of em charge would not be well-defined for S^2_{II} without additional conditions. Partonic 2-surfaces, their light-like orbits, and boundaries of string world sheets could do so since string world sheets have 1-D intersection with with the orbits. This picture would make sense for the minimal surfaces replacing vacuum extremals in the case of twistor lift of TGD.

   Since em fields are not present, the presence of classical W fields need not cause problems. The absence of classical em fields however suggests that the modes of induced spinor fields at boundaries of string worlds sheets must be em neutral and represent therefore neutrinos. The safest but probably too strong option would be right-handed neutrino having no coupling spinor connection but coupling to the CP_2 gamma matrices transforming it to left handed neutrino. Recall that ν_R represents a candidate for super-symmetry.

Neither charged leptons nor quarks would be allowed at string boundaries and classical W gauge potentials should vanish at the boundaries if also left-handed neutrinos are allowed: this can be achieved in suitable gauge. Quarks and charged leptons could reside only at string world sheets assignable to monopole flux tubes. This could relate to color confinement and also to the widely different mass scales of neutrinos and other fermions as will be found.

To sum up, the new result is that the distinction between neutrinos and other fermions could be understood in terms of the condition that em charge is well-defined. What looked originally a problem of TGD turns out to be a powerful predictive tool.

7.3 Classical Higgs field again

A motivation for returning back to Higgs field comes from the twistor lift of Kähler action.

1. The twistor lift of TGD [K6] [L2] brings in cosmological constant as the coefficient of volume term resulting in dimensional reduction of 6-D Kähler action for twistor space of space-time surface realized as surface in the product of twistor space of M^4 and CP_2. The radius of the sphere of M^4 twistor bundle corresponds to Planck length. Volume term is extremely small but removes the huge vacuum degeneracy of Kähler action. Vacuum extremals are replaced by 4-D minimal surfaces and modified Dirac equation is just the analog of massless Dirac equation in complete analogy with string models.

2. The well-definedness and conservation of fermionic em charges and SH demand the localization of fermions to string world sheets. The earlier picture assumed only em fields at string world sheets. More precise picture allows also W fields.

3. The first guess is that string world sheets are minimal surfaces and this is supported by the previous considerations demanding also string area term and Kähler magnetic flux tube. Here gravitational constant assignable to M^4 twistor space would be the first guess for the string tension.

What one can say about the possible existence of classical Higgs field?

1. TGD predicts both Higgs type particles and gauge bosons as bound states of fermions and antifermions and they differ only in that their polarization are in M^4 resp. CP_2 tangent space. p-adic thermodynamics [K7] gives excellent predictions for elementary particle masses in TGD framework. Higgs vacuum expectation is not needed to predict fermion or boson masses. Standard model gives only a parametrization of these masses by assuming that Higgs couplings to fermions are proportional to their masses, it does not predict them.

   The experimental fact is however that the couplings of Higgs are proportional to fermion masses and TGD should be able to predict this and there is a general argument for the proportonality, which however should be deduced from basic TGD. Can one achieve this?

2. Can one imagine any candidate for the classical Higgs field? There is no covariantly constant vector field in CP_2, whose space-time projection could define a candidate for classical Higgs field. This led years ago before the model for how bosons emerge from fermions to the wrong conclusion that TGD does not predict Higgs.

   The first guess for the possibly existing classical counterpart of Higgs field would be as CP_2 part for the divergence of the space-time vector defined modified gamma matrices expressible in terms of canonical momentum currents having natural interpretation as a generalization of force for point like objects to that for extended objects. Higgs field in this sense would however vanish by above consistency conditions and would not couple to spinors at all.

Classical Higgs field should have only CP_2 part being CP_2 vector. What would be also troublesome that this proposale for classical Higgs field would involve second derivatives of embedding space coordinates. Hence it seems that there is no hope about geometrization of classical Higgs fields.

3. The contribution of the induced Kähler form gives to the modified gamma matrices a term expressible solely in terms of CP_2 gamma matrices. This term appears in modified Dirac equation and mixes M^4 chiralities - a signal for the massivation. This term is analogous to Higgs term expect that it contains covariant derivative.

   The question that I have not posed hitherto is whether this term could at QFT limit of TGD give rise to vacuum expectation of Higgs. The crucial observation is that the presence of derivative, which in quantum theory corresponds roughly to mass proportionality of chirality mixing coupling at QFT limit. This could explain why the coupling of Higgs field to fermions is proportional to the mass of the fermion at QFT limit!

4. For S^2_{II} type string world sheets assignable to neutrinos the contribution to the chirality mixing coupling should be of order of neutrino mass. The coefficient $1/L^4$ of the volume term defining cosmological constant [L2] separates out as over all factor in massless Dirac equation and the parameter characterizing the mass scale causing the mixing is of order $m = \omega_1\omega_2 R$. Here ω_1 characterizes the scale of gradient for CP_2 coordinates. The simplest minimal surface is that for which CP_2 projection is geodesic line with $\Phi = \omega_1 t$. ω_2 characterizes the scale of the gradient of spinor mode.

   Assuming $\omega_1 = \omega_2 \equiv \omega$ the scale m is of order neutrino mass $m_\nu \simeq .1$ eV from the condition $m \sim \omega^2 R \sim m_\nu$. This gives the estimate $\omega \sim \sqrt{m_{CP_2} m_\nu} \sim 10^2 m_p$ from $m_{CP_2} \sim 10^{-4} m_P$, which is weak mass scale and therefore perfectly sensible. The reduction $\Delta c/c$ of the light velocity from maximal signal velocity due the replacement $g_{tt} = 1 - R^2\omega^2$ is $\Delta c/c \sim 10^{-34}$ and thus completely negligible. This estimate does not make sense for charged fermions, which correspond to S^2_I type string world sheets.

   A possible problem is that if the value of the cosmological constant Λ evolves as $1/p$ as function of the length mass scale the mass scale of neutrinos should increase in short scales. This looks strange unless the mass scale remains below the cosmic temperature so that neutrinos would be always effectively massless.

5. For S^2_I type string world sheets assignable to charged fermions Kähler action dominates and the mass scales are expected to be higher than for neutrinos. For S^2_I type strings the modified gamma matrices contain also Kähler term and a rough estimate is that the ratio of two contributions is the ratio of the energy density of Kähler action to vacuum energy density. As Kähler energy density exceeds the value corresponding to vacuum energy density $1/L^4$, $L \sim 40$ μm, Kähler action density begins to dominate over dark energy density.

To sum up, this picture suggest that the large difference between the mass scales of neutrinos and em charged fermions is due to the fact that neutrinos are associated with string world sheet of type II and em charged fermions with string world sheets of type I. Both strings world sheets would be accompanied by flux tubes but for charged particles the flux tubes would carry Kähler magnetic flux. Cosmological constant forced by twistor lift would make neutrinos massive and allow to understand neutrino mass scale.

REFERENCES

Mathematics

[A1] Catastrophe Theory. Available at: `https://en.wikipedia.org/wiki/Catastrophe_theory#Cusp_catastrophe`.

[A2] Yangian symmetry. Available at: `https://en.wikipedia.org/wiki/Yangian`.

[A3] Kassel C. *Quantum Groups.* Springer Verlag, 1995.

[A4] Witten E. Quantum field theory and the Jones polynomial. *Comm Math Phys*, 121:351–399, 1989.

[A5] Zeeman EC. *Catastrophe Theory.* Addison-Wessley Publishing Company, 1977.

[A6] Jones FR. *Braid groups, Hecke algebras and type II_1 factors.* 1983.

[A7] Fiore G. On q-Deformations of Clifford Algebras, 2000. Available at: https://arxiv.org/pdf/math/0002194v2.pdf.

[A8] Sierra G Gomez C, Ruiz-Altaba M. *Quantum Groups and Two-Dimensional Physics.* Cambridge University Press, Cambridge, 1996.

[A9] Dixmier J. *Von Neumann Algebras.* North-Holland, Amsterdam, 1981. First published in French in 1957: Les Algebres d'Operateurs dans l'Espace Hilbertien, Paris: Gauthier-Villars.

[A10] Sawin S. Links, Quantum Groups, and TQFT's, 1995.Available at: https://arxiv.org/abs/q-alg/9506002.

[A11] Ge ML Yang CN. *Braid Group, Knot Theory, and Statistical Mechanics.* World Scientific, 1989.

Theoretical Physics

[B1] Self-Organized Criticality. Available at: https://en.wikipedia.org/wiki/Self-organized_criticality.

[B2] Witten E Dolan L, Nappi CR. Yangian Symmetry in $D = 4$ superconformal Yang-Mills theory, 2004. Available at: https://arxiv.org/abs/hep-th/0401243.

[B3] Plefka J Drummond J, Henn J. Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory, 2009. Available at: https://cdsweb.cern.ch/record/1162372/files/jhep052009046.pdf.

[B4] Arkani-Hamed N et al. The All-Loop Integrand For Scattering Amplitudes in Planar N=4 SYM, 2010. Available at: https://arxiv.org/abs/1008.2958.

[B5] Wang Z Freedman M, Larsen H. A modular functor which is universal for quantum computation. *Comm Math Phys*, 1(2):605–622, 2002. Available at: https://arxiv.org/abs/quant-ph/0001108.

[B6] Witten E Green MB, Schwartz JH. *Superstring Theory.* Cambridge University Press, Cambridge, 1987.

[B7] Schwartz JH. *Super strings. The first 15 years of Superstring Theory.* World Scientific, 1985.

Cosmology and Astro-Physics

Books related to TGD

[K1] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/class.pdf, 2023.

[K2] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://www.tgdtheory.fi/public_html/pdfpool/append.pdf, 2023.

[K3] Pitkänen M. Construction of Quantum Theory: M-matrix. In *Quantum TGD: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/towards.pdf, 2023.

[K4] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/compl1.pdf, 2023.

[K5] Pitkänen M. Does TGD Predict a Spectrum of Planck Constants? In *Dark Matter and TGD:* *https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/Planck, 2023.

[K6] Pitkänen M. From Principles to Diagrams. In *Quantum TGD: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/diagrams.pdf, 2023.

[K7] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mless.pdf, 2023.

[K8] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: https://www.tgdtheory.fi/tgdhtml/padphys.html., 2023.

[K9] Pitkänen M. p-Adic Particle Massivation: Hadron Masses. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mass3.pdf, 2023.

[K10] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/wcwnew.pdf, 2023.

[K11] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionb.pdf, 2023.

[K12] Pitkänen M. The classical part of the twistor story. In *Quantum TGD: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/twistorstory.pdf, 2023.

[K13] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/tgdgrt.pdf., 2023.

[K14] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/numbervision.pdf, 2023.

[K15] Pitkänen M. Was von Neumann Right After All? In *TGD and Hyper-finite Factors.* *https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/vNeumann.pdf, 2023.

[K16] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/cspin.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. Available at: https://www.tgdtheory.fi/tgdglossary.pdf., 2014.

[L2] Pitkänen M. How the hierarchy of Planck constants might relate to the almost vacuum degeneracy for twistor lift of TGD? Available at: `https://tgdtheory.fi/public_html/articles/hgrtwistor.pdf`., 2016.