About Twistor Lift of TGD

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Abstract

The twistor lift of classical TGD is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative. Cosmological constant and thus twistor lift make sense only in zero energy ontology (ZEO) involving causal diamonds (CDs) in an essential manner.

One motivation for introducing the hierarchy of Planck constants was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter twistor lift is studied in detail.

1. The first working hypothesis is that the values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. The first interesting finding is that allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.
2. The other working hypothesis is $\alpha_K(M^4) = \alpha_K(CP_2)$. The small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs. This option looks more natural.

Both options lead to a generalization of Chladni mechanism to a "dynamics of avoidance" meaning that at least asymptotically the two dynamics decouple. This leads to an interpretation with profound implications for the views about what happens in particle physics experiment and in quantum measurement, for consciousness theory and for quantum biology.

A related observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density assignable to cosmological constant and be therefore a fundamental physics length scale.

1 Introduction

The twistor lift of classical TGD [K8] is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Volume term in action removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative.

What is remarkable that twistor lift is possible only in zero energy ontology (ZEO) since the volume term would be infinite by infinite volume of space-time surface in ordinary ontology: by the finite size of causal diamond (CD) the space-time volume is however finite in ZEO. Furthermore, the condition that the destructive interference does not cancel vacuum functional implies Bohr quantization for the action in ZEO. The scale of CD corresponds naturally to the length scale $L_\Lambda = \sqrt{8\pi/\Lambda}$ defined by the cosmological constant.

One motivation for introducing the hierarchy of Planck constants [K7, **?**] was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter two options for the twistor lift are studied in detail.

1. Option I (the original option): The values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. Allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.

   It has turned out that this option has several shortcomings. First of all, $\alpha_K(M^4) \neq \alpha_K(CP_2)$ looks like ad hoc assumption tailored to make cosmological constant small. Secondly, the decoupling between Kähler action and volume term implies separately conserved Noether charges which looks strange. Thirdly, for $\sqrt{g_4}$ instead of $\sqrt{|g_4|}$ in the volume element assumed hitherto, there is no charge transfer between Minkowski and Euclidian regions.

2. Option II: $\alpha_K(M^4) = \alpha_K(CP_2)$ is satisfied. Now entire action is identified as the cosmological term. A small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. Minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs, where the analog of free geodesic motion as minimal surfaces is expected. For $\sqrt{|g_4|}$ option there is charge transfer between Minkowski and Euclidian regions.

The two options provide different generalizations of Chladni mechanism [K13] [L5, L6] (see "An Amazing Resonance Experiment" at `http://tinyurl.com/kcbmrzz`)to a "dynamics of avoidance". Both options have profound implications for the views about what happens in particle physics experiment and in quantum measurement, and for consciousness theory and for quantum biology. It is however clear that Option II is the favored one.

The need to understand the twistor lift leads to a critics of the formulation of the basic action principle and the outcome is a more elegant formulation with non-trivial physical consequences.

1. Dimensionless gauge field is obtained from dimension 2 induced Kähler form by division with constant R_1^2 with dimension two. This parameter defines a hidden coupling parameter in the action and the identification in terms of CP_2 radius made hitherto rather implicitly is probably reasonable but ad hoc. The simple idea is to use the induced Kähler form as basic object and formulate the action principle accordingly. This brings in the dimensional parameter $1/R_1^4$ compensating for the dimension of $\sqrt{g_4}$ in the action.

2. One ends up to a general formulation of both bosonic and fermionic action principles showing that the overall scaling factor of fermionic and bosonic actions - call it X, disappears from classical dynamics so that extremals have no explicit independence on X. This is crucial for number theoretical universality.

   Quantum Classical Correspondence (QCC) realized as the condition that classical Noether charges in Cartan algebra correspond to eigenvalues of quantal fermionic charges however breaks the invariance with respect to scalings of action via fermionic anticommutation relations which depend on the scaling factor. The new formulation leads to a unique guesses for the 6-D actions, their 4-D dimensionally reduced variants, and 2-D effective actions.

3. The formulation helps to realize that Number Theoretical Universality (NTU) requires that $\sqrt{|g_4|}$ option is the only possible one. Physically the need to have charge transfer between Euclidian and Minkowskian space-time regions implies the same result.

This leads to two different views about cosmological constant.

1. For Option I the explanation for dark energy is in terms of volume term of the action and small value of cosmological constant obeying p-adic coupling constant evolution as function of p-adic length scale. For Option II the cancellation of Kähler action and volume term would give rise to a small value of cosmological constant and its p-adic evolution.

2. Either $L_\lambda = \sqrt{8\pi/\Lambda}$ or the length L characterizing vacuum energy density as $\rho_{vac} = \hbar/L^4$ or both can obey p-adic length scale hypothesis as analogs of coupling constant parameters.

The third option makes sense if the ratio R/l_P of CP_2 radius and Planck length is power of two: it can be indeed chosen to be $R/l_P = 2^{12}$ within measurement uncertainties. $L(now)$ corresponds to the p-adic length scale $L(k) \propto 2^{k/2}$ for $k = 175$, size scale of neuron and axon.

3. A microscopic explanation for the vacuum energy realizing strong form of holography (SH) is in terms of vacuum energy for radial flux tubes emanating from the source of gravitational field. The independence of energy from the value of $h_{eff}/h = n$ implies analog of Uncertainty Principle: the product Nn for the number N of flux tubes and the value of n defining the number of sheets of the covering associated with $h_{eff} = n \times h$ is constant. This picture suggests that holography is realized in biology in terms of pixels whose size scale is characterized by L rather than Planck length.

4. A interesting observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density characterized by cosmological constant and be therefore a fundamental physics length scale. An especially interesting result is that in the recent cosmology the size scale of a large neuron would be fundamental physical length scale determined by cosmological constant. This gives additional boost to the idea that biology and fundamental physics could relate closely to each other: the size scale of neuron would not be an accident but "determined in stars" and even beyond them!

2 More about twistor lift of Kähler action

The following piece of text was motivated by some observations relating to the twistor lift of Kähler action forcing a criticism of the earlier view about twistor lift.

The first observation was that the correct formulation of 6-D Kähler action in the framework of adelic physics implies that the *classical* physics of TGD does not depend on the overall scaling of Kähler action. This implies that the preferred extremals need not be minimal surface extremals of Kähler action. It is enough that they are so asymptotically - near the boundaries of CDs where they behave like free particles. This also nicely conforms with the physical idea that they are 4-D generalizations for orbits of particles in induced Kähler field.

The independence of the classical physics on the scale of the action inspires a detailed discussion of the number theoretic vision. Quantum Classical Correspondence (QCC) breaks the invariance with respect to the scalings via fermionic anti-commutation relations and Number Theoretical Universality (NTU) can fix the spectrum of values of the over-all scaling parameter of the action. One ends up to a condition guaranteeing NTU of the action exponential and finds an answer to the nagging question whether one should use $\sqrt{g_4}$ (imaginary in Minkowskian regions) or $\sqrt{|g_4|}$ in the action. Complex α_K allows $\sqrt{|g_4|}$ and NTU assuming that $1/\alpha_K = s$, $s = 1/2 + iy$ zero of Riemann zeta, implies $y = q\pi$, q rational as proposed also in [L3].

Second observation relates to cosmological constant. The proposed vision for the p-adic evolution of cosmological constant assumes that $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are different for the twistor lift. One however finds that single value of α_K is the natural choice. This destroys the original proposal for the p-adic length scale evolution of cosmological constant explaining why it is so small in cosmological scale.

The solution to the problem of the cosmological constant would be that the *entire* 6-D action decomposing to 4-D Kähler action and volume term is identified in terms of cosmological constant. The cancellation of Kähler electric contribution and remaining contributions would explain why the cosmological constant is so small in cosmological scales and also allows to understand p-adic coupling constant evolution of cosmological constant. One must however remain cautious: also the original proposal can be defended.

2.1 Kähler action contains overall scale as a hidden coupling parameter

The first observation leads to a more precise understanding of 6-D Kähler action relates to the induction procedure.

1. Kähler form has dimension two since its square gives metric: $J^2 = -g$. Gauge fields are however 2-forms, which are usually taken to be dimensionless (this requires that coupling constant g is included as multiplicative factor to gauge potential). Accordingly, I have assumed that induced Kähler form is obtained by diving Kähler form by $1/R^2$, R the radius of CP_2 identified as the radius of its geodesic sphere. One can however argue that the identification of the scaling factor is ad hoc since its value does not affect classical field equations.

2. What would happen if one induces the dimensional Kähler form as such? Kähler action density $L_K\sqrt{g_4}$ would have dimension of volume so that $1/\alpha_K$ must be replaced with $1/8\pi\alpha_K R_1^4$, where R_1 a fundamental coupling constant with dimension of length. This coupling however disappears from the classical field equations and in the recent adelic formulation also from quantum theory [L8].

3. For the 6-D twistor lift of Kähler action one must introduce an additional dimensional factor to get a dimensionless action. One has $R_1^4 \to R_1^4 R_0^2$, where R_0^2 has dimensions of area. The 4-D action density obtained from dimensional reduction for twistor sphere $S^2(X^4)$ assuming that the induced Kähler form for the sphere satisfies $J^4 = -g$ for $S^2(X^4)$ is proportional to

$$L = X \times (J \cdot J - 2)\sqrt{g_4} \ , \quad X = \frac{1}{2\alpha_K}\frac{Area(S^2(X^4))}{S_0}\frac{1}{R_1^4} \ , \quad S_0 = 4\pi R_0^2 \ . \tag{2.1}$$

   The shift of Kähler action density by -2 comes from $S^2(X^4)$ part of 6-D Kähler action.

4. From this form one can immediately see that the factor X in Eq. 2.1 disappears from field equations, and the functional form of preferred extremals has no dependence on coupling parameters! The quantum classical correspondence (QCC) stating that fermionic Noether charges in Cartan algebra have eigenvalues equals to their classical counterparts however implies this dependence.

2.1.1 Modified Dirac action and string world sheet action in the new formalism

What about the modified Dirac action related super-symmetrically to Kähler action in the new formalism? The 6-D formalism for the induced spinors doubles the number of spinor components and dimensional reduction must eliminate half of them to give something equivalent with the ordinary induced spinor structure. Chirality condition is the most plausible manner to achieve this. This answers the old question whether one could assume only leptonic spinors as fundamental spinors and construct quarks as some of anyonic leptons. This would require two chirality conditions and this is very probably not possible. The 6-D modified Dirac action can be written using the same rules as applied in 4-D case. The possible delicacies of the fermionic dimensional reduction require a separate discussion.

The 4-D dimensionally reduced part of 6-D modified Dirac action must reduce to the 4-D modified Dirac action associated with the full bosonic action. The modified gamma matrices Γ^α are expressible as contractions of the canonical momentum currents with embedding space gamma matrices (this applies also in $D = 6$). Therefore they are proportional to the dimensionless quantity $X\sqrt{g_4}$. Γ^α has dimension $1/L$ so that induced spinors must have dimension $L^{1/2}$. In the usual approach the dimension would be $1/L^{3/2}$.

With these conventions X apparently drop from the equations stating QCC as identity of eigenvalues of fermionic Noether charges and corresponding classical Noether charges in Cartan algebra. This not true. The anti-commutations for Ψ and time component J^0 of the canonical momentum density $J^\alpha = \partial L/\partial(\partial_\alpha\Psi) = \overline{\Psi}\Gamma^\alpha$ involve X and affect the scale of anti-commutation relations and therefore QCC. That the anti-commutations can be indeed realized under these dimensional constraints, requires a proof.

What about the spinors restricted to 2-D string world sheets and corresponding space-time action? Perhaps the most plausible option is that they do not appear at the fundamental level and appear only as the effective action suggested by SH. If this is the case, it is rather easy to guess the form of the bosonic and fermion 2-D effective actions. Their forms could be exactly the same as the form of 4-D actions. The only modification would be in the bosonic case the replacement of

$1/R_1^4$ with $1/R_1^2$ to get the dimensions correctly! The bosonic action would dictate the fermionic action by above rules.

The bosonic string world sheet action would differ from the area action. The action density would be $XR_1^2(J \cdot J - 2)\sqrt{g}$ in complete analogy with the 4-D case. Two special cases deserve to be mentioned.

1. This action vanishes for string world sheets with $J \cdot J = 2$. This is the case if one has $J = M(M^4)$ and J is self-dual. This is true if string world sheet is the preferred plane M^2 defining the symplectic structure of M^4 (there is moduli space form them in order to gain Lorentz invariance and giving rise to sectors of WCW).

   Small deformations of this plane would give rise to strings with small string tension and be naturally relating to the small value of the cosmological constant. These strings should accompany long strings mediating gravitational interaction in long length scales. The small action would require large value of $h_{eff}/h = n = h_{gr}$ for the perturbation theory to work.

2. Second special case corresponds to Lagrangian surfaces for which $J(M^4) + J(CP_2)$ induced to string world sheet vanishes. One would have ordinary strings with area action. String tension would be determined by CP_2 size scale. The appearance of also light strings would distinguish between TGD and super string models.

Kähler action can contain also a topological instanton term affecting the field equations only via boundary condition. This term could induce to the string world sheet action a magnetic flux term reducing to a boundary term at the boundaries of string world sheets adding an interaction term to the usual action defined by word-line length. The outcome would be equation of motion for a point-like particle experiencing Kähler force. These topological terms give additional terms to corresponding modified Dirac equations.

It would seem that the new approach to action principle allows a more unified approach to the details of the variational principle in dimension $D = 4$ and allows also to deduce the general form of 6-D and 2-D effective action. It must be however made clear that one could have brane like hierarchy of structures already at fundamental level. Also in this case the new approach applies.

2.1.2 Action principle, quantum classical correspondence, and number theoretical universality

The above observations force to reconsider the interpretation of the action principle. Here the adelic physics based vision can be used as a guideline.

1. It is good to list the geometric parameters and coupling constant like parameters of TGD. CP_2 scale $R(CP_2)$ certainly appears in the theory. The radius of $S^2(M^4)$ makes l_P^2 a natural scale factor of M^4 metric. One can re-scale $J(M^4)$ and the M^4 part of the metric of $T(M^4)$ but not the entire metric.

2. $r = R_1/R(CP_2)$ can be seen as a dimensionless coupling constant like parameter and in principle quantum criticality allows it to have a spectrum values determined by the extension of rationals defining adeles. The QCC condition stating the quantized values of the fermionic Noether charges are equal to their classical counterparts having non-local expressions forces to consider the possibility that the value of R_1 can indeed vary and has value guaranteeing that QCC holds true. Also α_K has spectrum of values: one possible spectrum corresponds to the zeros of Riemann zeta [L3]. Even the number theoretically problematic exponent of action could belong to the extension with a suitable choice of R_1.

   This would allow to speak about the exponent of action and of Kähler function making sense also p-adically in the intersection of real and p-adic WCWs. Both action and its exponent should exist in the extension. This is true if the action is of form $q_1 + q_2\pi$, q_i rational numbers. One might hope that a suitable choice of R_1 could make possible to realize QCC and this condition.

2.1.3 QCC and the value spectrum of R_1

Classical field equations do not depend at all on the value on the overall coefficient X of the action in Eq. 2.1. Also boundary conditions are independent of the scaling of X. Does this mean that one has projective invariance in the sense that the value of R_1 does not matter at all? No!

1. QCC for the Cartan algebra of fermionic and classical Noether charges gives meaning for the scale R_1. QCC states that the eigenvalues of the Cartan algebra charges are equal to the corresponding values of classical Noether charges. Since the normalization of quantal charges is fixed by the value of $\hbar$, this fixes the normalization of classical charges and thus the parameter R_1. If Ψ is taken dimensionless, the modified Dirac action can be taken to be proportional to factor $1/R_1^3$. Therefore R_1 has physical meaning. The above argument suggests that R_1 is fixed by quantum criticality and characterizes the extension of rationals.

2. Could one require that the values of classical charges belong to the extension of rationals defining the adeles in question? This condition involves in real context integral over 3-surface and is thus a non-local operation. How can one know, which 3-surfaces satisfy the condition? Is the choice of R_1 dictated by this condition so that it depends on the extension of rationals involved and obeys number theoretic coupling constant evolution?

   Note that classical Noether charges serve as WCW coordinates, and the interpretation would be the same as at space-time level: these special 3-surfaces would form a kind of cognitive representation analogous to that formed by the points of space-time surface with coordinates in extension. The quantization of these WCW coordinates would give a cognitive representation!

3. The action would be same for the symmetry related 3-surfaces and one could have WCW wave functions at the orbits of symmetries with coordinates which are conjugate variables for the quantized Noether charges. For the orbits of symmetry groups the allowed points in WCW would correspond to values of group parameters in the extension. Besides isometries and corresponding Kac-Moody algebras supersymplectic symmetry gives rise to this kind of wave functions. In case of four-momentum, the basic number theoretic conditions would be for rest masses.

   Strong form of holography (SH) could be realized by the reduction of both bosonic and fermionic action to an effective action restricted to string world sheets and partonic 2-surfaces. This option looks more attractive from the point of view of SH than fundamental action containing terms located at lower-dimensional surfaces.

2.1.4 Number theoretical universality and action exponential

In adelic physics number theoretical universality plays a key role.

1. Adelic physics leads to the proposal that the action exponentials appearing in the scattering amplitudes disappear. The normalization factor defined by functional integral of action exponential to which also the scattering amplitude is proportional would cancel them as in QFTs [L8].

   This would require that each maximum of Kähler function with respect to variations of 3-surface and having fixed topological scattering diagram defined by light-like partonic orbits and same action defines its own zero energy state as functional integral and these states can be freely superposed. One would not functionally integrate over different topological scattering diagrams: this would allow to interpret topological scattering diagram as a representation of computation.

2. At the level of scattering amplitudes - but not at the level of WCW geometry - the absence of exponents would allow to get rid of the grave difficulty posed by the fact that the exponent of Kähler action belongs to an extension of rationals only when powerful additional conditions are satisfied. The cancellation of exponents of action from scattering amplitudes looks compelling if one requires number theoretical universality since there are no practical means for checking that the exponent of action is in the extension of rationals for an arbitrary preferred

extremal. Also the definition of the action as integral is problematic in p-adic context and the only possible means to define it seems to be in terms of algebraic continuations from the real sector.

One can however argue against number theoretical extremism. Action exponentials are needed for the interpretation of the theory. Maxima of Kähler function, which also correspond to stationary phase correspond to the most probable 3-surfaces. Hence one can argue that the exponents should appear in the scattering amplitudes. Number theoretical cognition theorist could however argue that the points of WCW, which correspond to maxima have WCW coordinates in an extension of rationals and thus define cognitive representation at the level of WCW. Furthermore, one can argue that scattering amplitudes are not the entire physics. Kähler action and its exponent have real meaning independent of scattering amplitudes.

3. On the other hand, if the value of R_1 adjusts to guarantee that the action is of form

$$S = q_1 + iq_2\pi \quad . \tag{2.2}$$

exponents can appear in the amplitudes and the standard approach allowing functional integral giving sum of several exponents makes sense. In this case the scattering amplitudes are proportional to X_i/X, $X = \sum_i X_i$, where X_i denotes action exponent for a particular maximum of action as function of WCW coordinates. Note however that the action itself is not number theoretically universal: only its exponent. This is completely analogous with the fact that angles do not make sense p-adically and one can speak about corresponding phases identified as roots of unity.

Number theoretical universality (NTU) allows two options to consider depending on whether the action exponentials can appear in the scattering amplitudes or not. In WCW geometry action and also its exponent certainly appear.

1. The elimination of exponents of 6-D action from the scattering amplitudes would be a huge simplification and make practical calculations possible. This kind of assumption is in practice made also in standard path integral approach as approximation. ZEO allows this and the interpretation is in terms of the notion of quantum phase of matter: different topologies for partonic 2-surfaces correspond to different phases and the localization to single phase for zero energy states is possible: space-time would be much more classical object than without localization. One must however remain critical: the value of R_1 depending on extension of rationals could allow to achieve QCC conditions.

2. If something is gained, something is also lost. The earlier arguments involving exponent of Kähler function are lost if the exponentials do not appear in scattering amplitudes. In particular, the estimate for the value of gravitational coupling strength in terms of exponent of Kähler function and α_K (see the last section of [K26]) is lost if exponents do not appear anywhere. One can argue that this argument was actually lost already when the twistor lift was introduced and Planck length was transformed to a fundamental parameter appearing as scaling factor of M^4 Kähler form and metric.

There is a further challenge for the adelic physics. What could fix the value of the fundamental parameter $l_P^2/R^2(CP_2)$ (of order 10−7)? It seems that quantum criticality cannot help here. Both l_P^2 and R^2 appear in the induced metric of space-time surface and number theoretical universality for field equations demands that $l_P^2/R^2(CP_2)$ is a rational number. The p-adic evolution scenario of cosmological constant and empirical input for the cosmological constant gives $l_P^2/R^2(CP_2) = 2^{-12}$ [K2]. Why power of 2 which having unit p-adic norm for all odd primes and why just this power?

To sum up, a more precise adelic formulation of the classical action has allowed to detect a hitherto hidden scaling parameter in the action appearing as an additional coupling parameter depending on the extension of rationals, to understand better the number theoretical role of QCC, and allowed to answer a nagging question about whether to use metric determinant or its absolute value in the action assuming NTU for the exponential of action, and deduce the earlier conjecture for the zeros of zeta.

2.1.5 Answer to an old nagging question

Eq. 2.2 can be applied to the situation in which the extremal is known. For CP_2 type extremals volume and Kähler action (-4 times volume) are indeed known. Quite surprisingly, this suggest a solution to an old problem whether one should use $\sqrt{g_4}$ giving imaginary volume element in Minkowskian space-time regions or $\sqrt{|g_4|}$ used usually.

1. The action exponent

   $$e^{\frac{x}{2\alpha_K}} \quad , \quad x = \frac{6Vol(CP_2)}{R_1^4}$$

   is a number in an extension of rationals guaranteed if one has

   $$(1/2)Re(\frac{1}{\alpha_K}) \times x = q_1 \quad , \quad (1/2)Im(\frac{1}{\alpha_K}) \times x = q_2\pi \quad .$$

   .

2. Suppose that the volume integral uses volume element $\sqrt{g_4}$, which is imaginary in Minkowskian space-time regions and real in Euclidian regions. The motivation is that for real α_K the action exponential from Minkowskian space-time regions is phase as QFT picture demands.

   For $1/\alpha_K = is = i/2 + y$, s a complex zero of zeta, the phase of the action exponential coming from Minkowskian regions is proportional to iy and in a good approximation equal to $1/Re(\alpha_K)$. The conditions give $Vol(CP_2)/R_1^4 \propto \pi$ and $y = q$. Note that $Vol(CP_2)$ is proportional to π^2 so that the normalization volume R_1^4 would be proportional to π. Since $R_1^4 = q \times Vol(CP_2)$ is natural normalization factor one would have expected x to be rational. This does not look promising.

   That the zeros of zeta should be complex rationals is totally unexpected but would conform with the number theoretical universality. This would be of course very nice from TGD point of view strongly suggesting that zeros belong to some extension of rationals. I have proposed that the zeros of zeta appear as conformal weights in TGD framework [L3].

3. Suppose that the volume element is given by $\sqrt{|g_4|}$ as was done originally. If α_K is complex, the phase factor is obtained in any case. This option favours $1/\alpha_K = s$, s a complex zero of zeta. Eq. 2.2 would predict $Vol(CP_2)/R_1^4 = q$ and $y = q\pi$. These predictions conform with the physical intuition. I have proposed earlier [L3] that the exponents of imaginary parts for the zeros of zeta could correspond to roots of unity. Only the exponents of zeros of zeta would be number theoretically universal and continuable to the p-adic sectors.

To sum up, a more precise adelic formulation of the classical action has allowed to detect a hitherto hidden scaling parameter in the action appearing as an additional coupling parameter depending on the extension of rationals, to understand better the number theoretical role of QCC, and allowed to answer a nagging question about whether to use metric determinant or its absolute value in the action assuming NTU for the exponential of action, and deduce the earlier conjecture for the zeros of zeta.

There is however a further challenge for the adelic physics. What could fix the value of the fundamental parameter $l_P^2/R^2(CP_2)$ (of order 10−7)? It seems that quantum criticality cannot help here. Both l_P^2 and R^2 appear in the induced metric of space-time surface and number theoretical universality for field equations demands that $l_P^2/R^2(CP_2)$ is a rational number. The p-adic evolution scenario of cosmological constant and empirical input for the cosmological constant gives $l_P^2/R^2(CP_2) = 2^{-12}$ [K2]. Why power of 2 which having unit p-adic norm for all odd primes and why just this power?

2.2 The problem with cosmological constant

Second (unpleasant) observation was that the previous proposal for the twistor lift of Kähler action has an ad hoc feature.

2.2.1 Can the original proposal for the twistor lift of Kähler action be correct?

Consider first the unpleasant observation about cosmological constant.

1. α_K is also assumed to be complex and the conjecture [L3] has been that its values correspond to zeros of Riemann zeta. In the earlier proposal for twistor lift cosmological constant and α_K are assumed to obey independent p-adic evolutions, and cosmological constant was assumed to be real and to behave like $1/p$ as function of p-adic prime in p-adic length scale evolution so that its extreme smallness in cosmological scales could be understood [K8, K2].

   The motivation for the proposal was the decomposition $T(H) = T(M^4) \times T(CP_2)$ of the twistor space of H. It was argued that this allows to decompose the Kähler action of $T(H)$ to a sum of two parts with *different* values of α_K. For M^4 part the value of α_K, call it $\alpha_K(M^4)$, would be enormous and the resulting volume term in the dimensionally reduced 6-D Kähler action would have cosmological constant $\hbar/l_D^4$ as its coefficient: l_D would be of the order of the size about 10^{-4} meters of a large neuron in cosmological length scales.

2. If the value of $\alpha_K(M^4)$ is real or has different phase than $1/\alpha_K$, whose spectrum is proposed to correspond to zeros of zeta [L3], the action is complex, and one has separate field equations for real and imaginary part of action. The extremals would be minimal surface extremals of Kähler action. That all known extremals of Kähler action have this property was seen as a support for the hypothesis.

   The physically problematic aspect is that Kähler action and volume term effectively decouple. This would make sense asymptotically but looks strange as a general property [**?**] On the other hand, the independence of the extremals on coupling constants is a highly desirable outcome from the point of view of number theoretical universality.

3. The assumption about different Kähler coupling strengths admittedly looks somewhat ad hoc. If one assumes that also M^4 possesses Kähler form $J(M^4)$ [L9], and induced Kähler form corresponds to the sum $J(M^4) + J(M^2)$, universal value of α_K is the natural option. This assumption however allowed to understand the smallness of cosmological term in 4-D action and also the p-adic coupling constant evolution for the cosmological constant.

4. Also boundary conditions are problematic for this option. It would be highly desirable to have flow of classical Noether charges between Euclidian and Minkowskian space-time regions as a correlate for classical interactions between physical objects having Euclidian regions as space-time correlates (analogous to lines of scattering diagrams). The conditions stating the conservation of sums of complex Kähler and volume charges from Minkowskian and Euclidian regions however give 2+2 conditions if the phases of Kähler action and volume term are different and the metric determinant $\sqrt{g_4}$ is imaginary for Minkowskian regions. It is easy to see that Kähler and volume charges are conserved separately and that there is no charge transfer between Euclidian and Minkowskian regions. The alternative $\sqrt{|g_4|}$ allows the flow of real and imaginary charges between the two regions. One can however insist that the existence of two separate conserved energies should have been discovered long time ago.

What if one gives up the assumption $\alpha_K(M^4) \neq \alpha_K(CP_2)$?

1. The volume term would be also proportional to $1/\alpha_K$ so that the phases of both Kähler action and volume term would be identical. The pleasant surprise is that coupling constants disappear from the field equations altogether! It is not necessary to postulate minimal surface property of the preferred extremals anymore to guarantee number theoretical universality.

   Minimal surface property could be however asymptotic so that there would be no exchange of conserved quantities between these degrees of freedom. This would conform with the idea that incoming and outgoing particles are free and thus minimal surfaces as 4-D generalization of a geodesic line resulting when 4-D generalization of Abelian Maxwell force vanishes. Causal diamond (CD) would represent a region with the property that the extremals approach minimal surfaces at its boundary. One can loosely say that interactions are coupled on and off near the opposite boundaries of CD: CD corresponds to scattering volume.

The vertices of topological diagrams defined by as 2-D intersections of the ends of orbits of partonic 2-surfaces - analogous to vertices of Feynman diagrams - would be also accompanied by transient regions, where there the motion of 3-surface is not geodesic. The results are extremely nice from the point of view of number theoretical universality.

2. Also in this case the charge transfer between Euclidian and Minkowskian regions is impossible if $\sqrt{g_4}$ defines volume element (imaginary in Minkowskian regions). $\sqrt{|g_4|}$ this is not the case. As found, also NTU favors this option.

3. The above result is extremely nice. What makes the shower cold is that one ends up with problems with cosmological constant since Kähler and volume terms in the action are of same order of magnitude. Also the proposed p-adic evolution scenario for the cosmological constant is lost. The only cure that I can imagine is that the entire 4-D action has interpretation as a cosmological term, and that a cancellation between Kähler action and volume term take place giving rise to a very small effective value of cosmological constant.

2.2.2 Can one understand the p-adic evolution of cosmological constant?

The above findings lead to a problem with cosmological constant.

1. If the cosmological constant corresponds to the volume term in the dimensionally reduced 6-D Kähler action with scaling factor $X = 1/2\alpha_K R_1^2 S_0$, one has from Eq. 2.1

$$\rho_{vac} = \frac{1}{l_D^4} = \frac{2}{\alpha_K R_1^4}\frac{Area(S^2(X^4))}{S_0} = \frac{\Lambda}{8\pi l_P^2} \quad . \tag{2.3}$$

   Here l_D corresponds to a length scale which is roughly the size 10^{-4} meters of large neuron for cosmological constant in cosmic scales. Also Kähler action would be extremely small. It would however seem that the ratio of these actions should be extremely small. The simplest solution corresponds to $\frac{Area(S^2(X^4))}{S_0} = 1$.

2. The Kähler action for CP_2 type extremal with light-like geodesic as M^4 projection the action would be

$$S = -3\frac{Vol(CP_2)}{l_D^4} \quad .$$

   The action has totally different order of magnitude than assumed earlier if R_1 corresponds to the value of cosmological constant. If one assumes $R_1 = R(CP_2)$, cosmological constant is enormous. Something seems to go wrong.

How could one overcome this problem?

1. Could l_D be small and imply large cosmological constant? Could the parameter $X = \frac{Area(S^2(X^4)}{S_0}$ be small and increase the effective size of l_D? Could the time-like signature for $S^2(M^4)$ allow this by reducing the value of $Area(S^2(X^4)$?

   One can study the embedding of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers n_1 and n_2. One can choose S_0 to be the area for the embedding with $n_1 = n_2 = 1$. This gives $\frac{Area(S^2(X^4)}{S_0} = (n_1X^2 - n_2)(X^2 - 1)$, $X^2 = (R^2(CP_2)/l_P^2)$ for time-like signature for $S^2(M^4)$. The condition $\frac{Area(S^2(X^4)}{S_0} = 1/p$ would give p-adic length scales but could be satisfied for finite number of primes p only. Second problem is that this would *not* affect the ratio of Kähler and volume contributions to the action.

2. Could effective cosmological constant correspond to the entire action so that Kähler would cancel the real cosmological term in cosmological scales?

   Could $J \cdot J - 2$ should become small in Minkowskian regions and be necessarily large in Euclidian regions? The positive Kähler electric contribution to the action should sum up to

almost zero with the negative magnetic contribution and cosmological term. This cancellation should take place in cosmic scales at least and require long range induced Kähler electric fields. They are assumed to be present in the model for large voids. If M^4 Kähler form is present as CP breaking and some other arguments suggest [L9] [K2], it could give a large Kähler electric contribution in long scales if CP_2 contribution becomes small as one might expect.

The values of 6-D Kähler action should have tendency to concentrate around values inversely proportional to prime p near power of 2 (also other small primes can be considered). The values of Kähler action for the maxima of Kähler function could have this property. This conjecture was made earlier in an attempt to understand gravitational constant in terms of p-adic length scale hypothesis and the exponent of Kähler action for CP_2 type extremals (see the last section of [K26]).

3. This interpretation would mean that for strings like objects having both vanishing induced M^4 and CP_2 parts of induced Kähler fields the action would be large and coming from cosmological constant in CP_2 scale, and one could at least formally say that the situation is perturbative. Strings could however carry non-vanishing and large M^4 parts of Kähler electric fields and the action could be small in this case.

4. I must be added that the interpretation of cosmological constant has varied during years. For the 4-D Kähler action the proposal was that cosmological constant corresponds to the magnetic part of Kähler action with magnetic tension responsible for the negative pressure. The twistor lift in turn led to ask whether Kähler action and volume term could provide alternative, dual ways to understand cosmological constant. For the recent option the small effective cosmological constant results from the cancellation of Kähler action and volume term.

The cautious conclusion would be following. If the 6-D Kähler action contains only single α_K, the cosmological constant is very large at short scales and for Euclidian space-time regions. The cancellation of Minkowskian Kähler electric contribution and Kähler magnetic action in 6-D sense however makes the effective value of cosmological very small. The solution of the problem of cosmological constant would be dynamical. The previous option for which Kähler action decomposes to M^4 and CP_2 parts with different values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ $\leq\leq$ $\alpha_K(M^4)$ cannot be however excluded.

3 Twistor lift of TGD, hierarchy of Planck constant, quantum criticality, and p-adic length scale hypothesis

Kähler action is characterized by enormous vacuum degeneracy: any four-surface, whose CP_2 projection is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähker form, defines a vacuum extremal. The perturbation theory around canonically imbedded M^4 in $M^4 \times CP_2$ defined in terms of path integral fails completely as also canonical quantization. This led to the construction of quantum theory in "world of classical worlds" (WCW) and to identification of quantum theory as classical physics for the spinor fields of WCW: WCW spinors correspond to fermionic Fock states. The outcome is 4-D spin glass degeneracy realizing non-determinism at classical space-time level [K9, K5, K30, K20].

The twistor lift of TGD is based on unique properties of the twistor spaces of M^4 and CP_2. Note that M^4 allows two notions of twistor space. The first one involves conformal compactification allowing only conformal equivalence class of metrics. Second one is equal to Cartesian product $M^4 \times S^2$ [**?**] (see `http://tinyurl.com/yb4bt74l`). CP_2 has flag manifold $SU(3)/U(1) \times U(1)$ as twistor space having interpretation as the space for the choices for quantization axis of color hypercharge and isospin. Both these spaces Kähler structure (strictly speaking E^4 and S^4 allow it but the notion generalizes to M^4) and there are no others. Therefore TGD is unique both from standard model symmetries and twistorial considerations.

The existence of Kähler structure is a unique hint for how to proceed in the twistorial formulation of classical TGD. One must lift Kähler action to that in the twistor space of space-time surface having also S^2 as a fiber and identify the preferred extremals of this 6-D Kähler action as those of

dimensionally reduced Kähler action, which is 4-D Kähler action plus volume term identifiable in terms of cosmological constant. As found, there are two options to consider.

1. Option I: The values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. Allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges. In this case the cosmological constant would correspond to running $\alpha_K(M^4)$ and would behave like $1/p$, p p-adic prime. This was the original proposal.

2. Option II: $\alpha_K(M^4) = \alpha_K(CP_2)$ is satisfied. A small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs, where the analog of free geodesic motion as minimal surfaces is expected. In this case effective cosmological constant would correspond to the *entire action*: volume term and Kähler action receiving also M^4 contribution would cancel almost completely in cosmic scales.

   One can in fact argue that one cannot distinguish between Kähler and volume contributions to the action so that Option II remains the only possible one. Option I also breaks the symmetry between Kähler forms of M^4 and CP_2. It is natural that the induced Kähler form is the sum of both and appears in the Kähler action: hence $\alpha_K(M^4) = \alpha_K(CP_2)$.

Option I might be argued to be adhoc but at this moment it is not yet wise to select between these two options. The most conservative assumption is that the twistorial approach is only an alternative for the space-time formulation: in this formulation preferred extremal property might reduce to twistor space property.

Kähler action gives as fundamental constants the radius $R \simeq 2^{12} l_P$ of CP_2 serving as the TGD counterpart of the unification scale of GUTs and Kähler coupling strength α_K in terms of which gauge coupling strengths can be expressed. Twistor lift gives 2 additional dimensional constants. The radius of S^2 fiber of M^4 twistor space $M^4 \times S^2$ is essentially Planck length $l_P = \sqrt{G/\hbar}$, and the cosmological constant $\Lambda = 8\pi G \rho_{vac}$ defining vacuum energy density is dynamical in the sense that it allows p-adic coupling constant evolution as does also α_K.

For both Option I and II one can imagine two options for the p-adic coupling constant evolution of cosmological constant.

1. $\rho_{vac} = k_1 \times \hbar/L_p^4$, where $p \simeq 2^k$ characterizes a given level in the p-adic length scale hierarchy for space-time sheets. Here one can in principle allow $k_1 \neq 1$.

2. $\Lambda/8\pi = k_2/L_\Lambda^2 \propto \frac{1}{p_\Lambda^2}$. Also k_2 could differ from unity. Number theoretical universality suggests $k_1 = k_2 = 1$. The that here secondary p-adic length scale is assumed.

The first option seems more natural physically. During very early cosmology $\Lambda R^2/8\pi$ approaches l_P^2/R^2 for the first option, where $R \simeq 2^{12} l_P$ is the size scale of CP_2 so that one has $\Lambda R^2/8\pi \simeq 2^{-24} \simeq 6 \times 10^{-8}$ at this limit. Therefore perturbation theory would fail for Option I also in early cosmology near vacuum extremals. In the recent cosmology Λ is extremely small. Note that vacuum energy density would be always smaller than $\hbar/R^4$ and thus by a factor $(l_P/R)^4 \simeq 2^{-48} \simeq 3.6 \times 10^{-15}$ lower than in GRT based cosmology.

It it is good go recall that the earlier identification of the cosmological constant was in terms of the effective description for the magnetic energy density of the magnetic flux tubes. Magnetic tension would give rise to effective negative pressure. For Option II the cosmological constant would correspond to the entire action with magnetic and volume contributions slightly larger than Kähler electric contribution. For Option I it wold correspond to the volume term.

3.1 Twistor lift brings volume term back

Concerning volume term the situation changed as I introduced twistor lift of TGD. One could say that twistor lift forces cosmological constant. As already described, there are two options: Option I and Option II. The following arguments developed for Option I apply with small modifications also to Option II. The only difference is that the volume term has complex phase for complex α_K [L3] and effective cosmological constant follows from the compensation of Kähler and volume contributions.

1. The twistor lift of Kähler action is 6-D Kähler action for the twistor space $T(X^4)$ of space-time surface X^4. The analog of twistor structure would be induced from the product $T(M^4)\times T(CP_2)$, of twistor spaces $T(M^4) = M^4 \times S^2$ of M^4 [**?**] and $T(CP_2) = SU(3)/U(1) \times U(1)$ of CP_2 having Kähler structure so that the induction of Kähler structure to $T(X^4)$ makes sense. Besides M^4 and CP_2 only the spaces E^4 and the S^4, which are variants of M^4 have twistor space with Kähler structure or analog of it. The induction conditions would imply dimensional reduction so that the 6-D Kähler action for the twistor lift would reduce to 4-D Kähler action plus volume term identifiable in terms of cosmological constant Λ.

2. 4-D Kähler action has Kähler coupling strength α_K as coupling parameter and volume term has coefficient $1/L^4$ identifiable in terms of cosmological constant

$$\frac{1}{L^4} \equiv \frac{\Lambda}{8\pi l_P^2} \quad .$$

   $l_P = \sqrt{G/\hbar}$ would correspond to the radius of twistor sphere for M^4 and thus becomes fundamental length scale of twistorially lifted TGD besides radius of CP_2. Note that the radius of twistor sphere of CP_2 is naturally CP_2 radius.

   L is in the role of coupling constant and expected to obey discrete p-adic coupling constant evolution $L \propto \sqrt{p}$, prime or prime near power of two if p-adic length scale hypothesis is accepted. In the recent cosmology L could correspond to the p-adic length scale $L(175) \simeq 40$ μm, the size of large neuron.

   Ł $\simeq 40\mu$m corresponds to the energy scale $E = 1/L \simeq .031$ eV, which is thermal energy at temperature of 310 K (40 C) - the physiological temperature. A deep connection with quantum biology is suggestive. Also the energy scale defined by cell membrane potential is in this energy scale. This energy scale about 10 times smaller than the mass scale of neutrinos.

   Also $L_\Lambda = \sqrt{8\pi/\Lambda}$ would satisfy p-adic coupling constant evolution as already discussed. Now the p-adic length scale would be secondary p-adic length scale $L_\Lambda = L(2,p) = \sqrt{p} \times (R/l_P)$, l_P Planck length. p-Adic length scale hypothesis demands that R/l_P - the ratio for the radii of CP_2 and twistor sphere is power of 2. p-Adic mass calculations indeed allow this ratio can be indeed chosen to be equal to $R/l_P = 2^{12}$.

3.2 ZEO and twistor lift

The volume term, which I gave up 38 years ago, has creeped back to the theory! The infinite value of volume for space-time surfaces of infinite duration? This would not make the notion of vacuum functional poorly defined. Should one forget twistor lift because of this? No! ZEO saves the situation.

In ZEO given CD defines a sub-WCW consisting of space-time surfaces inside CD. This implies that the volumes for the M^4 projections of allowed space-time surfaces are smaller than CD volume having the order of magnitude $L^4(CD)$, $L(CD)$ is the temporal distance between the tips of CD (one has $c = 1$). I have also proposed that $L(CD)$ is quantized in multiples of integers, primes or primes near power of two so that the identification might make sense. $L(CD) = L$ is not possible due to the small value 40 μm of L but $L(CD) = L_\Lambda$ could make sense.

3.2.1 Stationary phase condition and ZEO

The preferred extremal property realizing SH poses extremely strong constraints on the value of total action and it should force the phase defined by action to be stationary so that interference effects would be practically absent. This argument assumes that the action exponentials indeed appear in the scattering amplitudes defined by the WCW spinor fields in ZEO. NTU however forces to challenge this assumption unless one assumes that action is quantized as $q_1 + iq_2\pi$: this might be achieved by the quantization of the overall scale factor X of the action. The construction of twistor scattering amplitude suggests that the cancellation of action exponentials might be indeed achieved. If the exponents are present, the question is how the stationarity of phase could be achieved.

1. The most general possibility is that the phase of the vacuum functional can be large but is localized around very narrow range of values. The imaginary part of the action S_{Im} for preferred extremals should be around values $S_{Im} = A_0 + n2\pi$. Standard Bohr orbitology indeed assumes the quantization of action in this manner. One could also argue that just the absence of destructive interference demands Bohr quantization of the action in the vacuum functional. Whether preferred extremal property indeed gives rise to this kind of Bohr quantization, is an open problem. The real exponent of the vacuum functional should in turn be large enough and positive values are favored. They are however bounded in ZEO because of the finite size of CDs.

2. To proceed further one must say something about the value spectrum of α_K. In the most general situation α_K is complex number: the proposal of [L3] is that the discrete p-adic coupling constant evolution for $1/\alpha_K$ corresponds to a complex zero $s = 1/2 + iy$ of Riemann zeta: also the trivial real zeros can be considered. For large values of y the imaginary part of y would determine $1/\alpha_K$ and $Re(s) = 1/2$ would be responsible for complex value of α_K. This makes sense since quantum TGD can be regarded formally as a complex square root of thermodynamics.

3. Denote by $S = S_{Re} + iS_{Im}$ the exponent of vacuum functional. For complex values of $1/\alpha_K$ S_{Im} and S_{Re} receive a contribution from both Euclidian and Minkowskian regions and a contribution also from the Minkowskian regions. For S_{Im} the contributions should obey the condition

$$S_{Im} \quad = \quad S_{Im}(M) + S_{Im}(E) \simeq A_0 + n2\pi \tag{3.1}$$

   to achieve constructive interference.

   For real parts the condition $S_{Re} = S_{Re}(M) + S_{Re}(E)$ must be small if negative. Large positive values of S_{Re} are favored. S_{Re} automatically selects the configurations, which contribute most and among these configurations the phase $exp(iS_{Im})$ must be stationary. The conditions for S_{Im} relate the values of action in the Euclidian and Minkowskian regions. If α_K is real, one has $S_{Im}(M) \simeq A_0 + n2\pi$ and $S_{Re}(E)$ small if negative and Euclidian and Minkowskian regions effectively decouple in the conditions. It seems that complex values of α_K are indeed needed.

4. $S_{Re}(E) = S_{Re}(M) + S_{Re}(E)$ receives a positive contribution from Euclidian regions. Minkowskian regions a contributions for complex value α_K. Both positive and negative contributions are present and the character of these contributions depends on sign of the imaginary part of α_K. Depending on the sign factor ± 1 of $Im(1/\alpha_K)$ Minkowskian regions give negative (positive) contribution from the space-time regions dominated by Kähler electric fields and positive (negative) contribution from the volume term and the regions dominated by Kähler magnetic field.

   The option "+" for which Kähler magnetic action and volume term give positive contribution to $S_{Re}(M)$ looks physically attractive. "+" option would have no problems in ZEO since the contribution to S_{Re} would be automatically positive but bounded by the finite size of CD: this would give a deep reason for the notion of CD (also the realization of super-symplectic

symmetries gives it). For "-" option Minkowskian regions containing Kähler electric fields would be essential in order to obtain $S_{Re} > 0$: Kähler magnetic fields would not be favored and the unavoidable volume term would give wrong sign contribution to $S_{Re} > 0$.

3.2.2 The condition $S_{Im} \leq \pi/2$ is not realistic

One can look what the mere volume term contributes to S_{Im} assuming $S_{Im} \leq \pi/2$. Volume term dominates for near to vacuum extremals with a small Kähler action: in particular, for string like objects $X^2 \times S^2$, S^2 a homologically trivial geodesic sphere with vanishing induced Kähler form. It turns out that these conditions are not physically plausible and that $S_{Im} \simeq A_0 + n2\pi$ is the only realistic option.

1. Cosmological constant (parametrizable using the scale L) together with the finite size of CD gives a very stringent upper bound for the volume term of the action: $A = vol(X^4)/L^4$. The rough estimate is that for the largest CDs involved the volume action is not much larger than $L^4\pi/2$ in the recent cosmology. In the recent cosmology L would be only about 40 μm so that the bound is extremely strong! and suggests that $S_{Im} < \pi/2$ is not a realistic condition.

2. $L(CD) = L$ is certainly excluded. Can one have $L(CD) = L_\Lambda$? How can one achieve space-time volume not much larger that L^4 for space-time surfaces with duration $L(CD)$? Could magnetic flux tubes help! For the simplest string like objects $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and Y^2 a 2-D surface (complex sub-manifold of CP_2) the volume action is essentially

$$Action = \frac{V}{l_P^2 L_\Lambda^2} = \frac{Area(X^2)}{L_\Lambda^2} \times \frac{Area(Y^2)}{l_P^2} \quad . \tag{3.2}$$

   The conservative condition for the absence of destructive interference is roughly $Action < \pi/2$.

3. To get a more concrete idea about the situation one can use the parameterization

$$Area(string) = L(CD) \times L(string) \quad , \quad Area(Y^2) = x \times 4\pi R^2 \quad . \tag{3.3}$$

   x is a numerical parameter, which can be quite large for deformations of cosmic strings with thick transversal M^4 projection. The condition for the absence of destructive interference is roughly

$$\frac{L(CD) \times L(string))}{L_\Lambda^2} \times x \times \frac{4\pi R^2}{l_P^2} < \frac{\pi}{2} \quad . \tag{3.4}$$

   For $L(string) \ll L(CD)$ one can have space-time surfaces of temporal duration $L(CD) = L_\Lambda$. For these the condition reduces to

$$\begin{aligned} y \times x &< \pi \frac{l_P^2}{4\pi R^2} = 2^{-13}\pi \quad , \\ y &\equiv \frac{L(string)}{L_\Lambda} \quad . \end{aligned} \tag{3.5}$$

   For deformations the transversal area of string like object can be also chosen to be considerably larger than the area of geodesic sphere. For flux tubes of length of order 1 AU the one have $y \sim 10^{-16}$. This would require $x \leq 10^{13}$. This would correspond to a radius $L(Y^2)$ about $10^6 R$ much smaller than required.

   For $L(string) \sim L$ this would give $y \sim 10^{-31}$ giving $x \leq 10^{28}$ $L(Y^2) \leq 10^{14} R$, which corresponds to elementary particle scale. Still this fails to fit with intuitive expectations, which are of course inspired by the standard positive energy ontology.

4. One could try to invent mechanisms making volume term small. The required reduction would be enormous. This does look sensible. One can have vacuum extremals of Kähler action for which CP_2 projection is a geodesic line: $\Phi = \omega t$. The time component $g_{tt} = 1 - R^2\omega^2$ of the flat metric can be arbitrarily small so that the volume proportional to $\sqrt{gtt}$ can be arbitrarily small. One expects that this happens in early cosmology but as a general mechanism this is not plausible. Also very rapidly rotating string like objects with small area of string world sheet are in principle possible but do not represent a realistic option.

The cautious conclusion is that Bohr quantization $S_{Im} \simeq A_0 + n2\pi$ is the only sensible option. The hypothesis that the coupling constant evolution for $1/\alpha_K$ is given in terms of zeros of Riemann zeta seems to be consistent with this picture and correlates the values of actions in Minkowskian and Euclidian regions.

3.3 Hierarchy of Planck constants

One motivation (besides motivations from bio-electromagnetism and Nottale's work [E1]) for the hierarchy of Planck constants $h_{eff} = n \times h$ identified as gravitational Planck constants $\hbar_{gr} = GMm/v_0$ at the magnetic flux tubes mediating the gravitational interaction was that it effectively replaces the large coupling parameter GMm with dimensionless coupling $v_0/c < 1$. This assumes quantum coherence in even astrophysical length and time scales. For gauge interaction corresponding to gauge coupling g one $\hbar_g = Q_1Q_2\alpha/v_0$. Also Kähler coupling strength α_K to α_K/n and makes perturbation theory converging for large enough value of n.

The geometric interpretation for $h_{eff} = n \times h$ emerges if one asks how to make the action large for very large value of coupling parameter to guarantee convergence of functional integral.

1. The answer is simple: space-time surfaces are replaced with n-fold coverings of a space-space giving n-fold action and effectively scaling h to $h_{eff} = n \times h$ so that coupling strength scale down by $1/n$. The coverings would be singular in the sense that at the 3-D ends of space-time surface at the boundaries of causal diamond (CD) the sheets co-incide.

2. The branches of the space-time surface would be related by discrete symmetries. The symmetry group could be Galois group in number theoretic vision about finite measurement resolution realized in terms of what I call monadic or adelic geometries [L4] [K10].

   On the other hand, the twistor lift suggests that covering could be induced by the covering of the fiber $S^2(X^6)$ by the spheres $S^2(M^4 \times S^2)$ and the twistor space $S^2(SU(3)/U(1) \times U(1))$ defining fibers of twistor spaces of M^4 and CP_2. There would be gauge transformations transforming the light-like parton orbits to each other and the discrete set would consists of gauge equivalence classes. These two identifications for the symmetries could be equivalent.

$h_{eff} = h_{gr} = n \times h$ would make perturbation theory possible for the space-time surfaces near vacuum extremals. For far from vacuum extremals Kähler action dominates and one would have $h_{eff} = h_{g_K} = n \times h$. This picture would conform with the idea that gravitational interactions are mediated by massless extremals (MEs) topologically condensed at magnetic flux tubes obtained as deformations of string like objects $X^2 \times S^2_I$, S^2_I a homologically trivial geodesic sphere of CP_2. The other interactions could be mediated in the similar manner. The flux tubes would be deformations of $X^2 \times S^2_{II}$, S^2_{II} a homologically non-trivial sphere so that the flux tubes would carry monopole flux.

The enormously small value of cosmological constant would require large value of $h_{eff}/h = n$ explaining the huge value of h_{gr} whereas for other interactions the value of n would be much smaller. Since only the size of the action matters, this is true for both Option I and Option II. One can consider also variants of this working hypothesis. For instance, all long range interactions mediated by massless quanta could correspond to extremals for which cosmological constant is small.

What smallness requires depends on option. For Option I the reason is that very long homologically non-trivial magnetic flux tubes tend to have large energy (the energy goes as $1/S$) so that homologically trivial flux tubes having only vacuum energy are favored. For Option II the cancellation of Kähler action and volume term is necessary. The compensating Kähler electric

action could come from the M^4 Kähler from $J(M^4)$. These flux tubes could be also homologically non-trivial

Quantum criticality would suggest that both homologically trivial and non-trivial phases are important. In TGD inspired quantum biology [K11] I have considered the possibility that structures with size scaled by $h_{eff}/h = n$ can transform to structures with $n = 1$ but p-adic length scale scaled up by n. Here n would be power of two by p-adic length scale hypothesis.

This would have interpretation in terms of quantum criticality. Homologically non-trivial string like objects with given string tension determined by Kähler action would be transformed to homologically trivial string like objects with the same string tension but determined by the cosmological constant term. This would give a condition on the value of the cosmological constant and thickness of flux tubes to be discussed later.

3.4 Magnetic flux tubes as mediators of interactions

The gravitational Planck constant $\hbar_{gr} = GMm/v_0$ [K23, K15, K16, **?**] introduced originally by Nottale [E1] depends on the large central mass M and small mass m. This makes sense only if h_{gr} characterizes a magnetic flux tube connecting the two masses. Similar conclusion holds true for h_g. This leads to a picture in which mass M has involves a collection of radial flux tubes emanating radially from it. This assumption makes sense in many-sheeted space-time since the fluxes can go to the another space-time sheets through wormhole contacts associated also with elementary particles. For single-sheeted space-time one should have genuine magnetic charges.

This picture encourages a strongly simplified vision about how holography is realized. From center mass flux tubes emanate and in given size scale of the space-time sheet from by the flux tubes having say spherical boundary, the boundary is decomposed of pixels representing finite number of qubits. Each pixel receives one flux tube.

3.4.1 Vacuum energy for Options I and II

For Option I and magnetic flux tubes with vanishing Kähler form carry mere vacuum energy and are candidates for the mediators of long range interactions including gravitation. The homologically trivial flux tubes carry vacuum energy, which by flux conservation is proportional to $1/S$, where S is surface area. Long flux tubes are necessarily thick.

For Option II the thin magnetic flux tubes with vanishing induced Kähler form have very large tension and could be perturbative so that there would be no need for large values of $h_{eff}/h = n$. These flux tubes are expected to be short. The string world sheets mediating gravitational interaction should be long and have small string tension. They would naturally carry non-vanishing Kähler electric field in the direction of string (and flux tube).

1. Gravitational action (interaction energy from $J(M^4)$) and volume action (energy) would compensate to give a small cosmological constant forcing $h_{eff}/h = n$ hierarchy describing dark matter. Thus $J(M^4)$ crucial for understanding CP breaking and matter antimatter asymetry would be also crucial for the smallness of cosmological constant. This option looks physically rather attractive.

2. For flux tubes with vanishing induced $J(CP_2)$ the condition for cancellation would be $J \cdot J - 2 \simeq 0$. The compensating Kähler field would be electric and would naturally due to $J(M^4)$ and also responsible for the gravitational field along flux tube at QFT limit. Compensation of actions giving a small and scale dependent cosmological constant requiring large $h_{eff}/h = n = h_{gr}/h$ is possible.

3. For flux tubes with Kähler magnetic tube carrying magnetic monopole flux the cancellation condition would $J(M^4) \cdot J(M^4) - 2 - J(CP_2) \cdot J(CP_2) \simeq 0$. The thickening of flux tubes weakening the value of $J(CP_2)$ behaving from flux conservation like $J(CP_2) \propto 1/S$, S the cross sectional area of the flux tube, should make approximate cancellation possible. Elementary particles would represent an example of structures formed by closed monopole flux tubes assignable with a pair of space-time sheets. Homologically non-trivial magnetic flux tubes with small string tension could explain the mysterious cosmic magnetic fields: homological non-triviality implies that no current is needed to create the fields.

3.4.2 Magnetic flux tubes as carriers of magnetic energy

The holographic picture leads to a picture about vacuum energy. The following arguments developed originally for Option I should apply to both options since it is enough that magnetic flux tubes have only low vacuum energy density. Possible delicacies relate to the fact that small Kähler action ($E^2 - B^2$) does not necessarily mean small Kähler energy. For Option II this situation is however not encountered.

1. Vacuum energy can be expressed as a sum of energies assignable to the flux tubes. Same applies to Kähler interaction energy. The contribution of individual flux tube is proportional to its length given by radius r of the large sphere considered. The total vacuum energy must be proportional to r^3 so that the number of flux tubes must be proportional to r^2. This implies that single flux tube correponds to constant area ΔS of the boundary sphere for given value of cosmological constant. The natural guess is that ΔS is of the same order of magnitude as the area defined by the length scale defined L by the vacuum energy density $\rho_{vac} = \Lambda/8\pi G$ allowing parameterization $\rho_{vac} = k_1\hbar/L^4$.

2. In the recent cosmology one has $\hbar/L(now) \simeq .029$ eV, which equals roughly to $M/10$, where $M = \sum m(\nu_i) \simeq .032 \pm 0.081$ eV is the sum of the three neutrino masses. L is given as a geometric mean

$$L = \sqrt{L_\Lambda l_P} \simeq .42 \times 10^{-4}$$

   meters of length scales $l_P = \sqrt{G/\hbar}$ and $L_\Lambda = (8\pi/\Lambda)^{1/2}$. $L(now)$ corresponds to the size scale of large neuron. This is perhaps not an accident.

   The area of pixel must be of order $L^2(now)$ suggesting strongly a p-adic length scale assignable with neuron: maybe neuronal system wold realize holography. $L(151) = 10$ nm (cell length scale thickness) and $L(k) \propto \sqrt{p} \simeq 2^{k/2}$ gives the estimate $p \simeq 2^k$, $k = 175$: the p-adic length scale is 4 per cent smaller than $L(now)$.

3. The pixel area would be by a factor $L^2(now)/l_P^2$ larger than Planck length squared usually assumed to define the pixel size but would conform with the p-adic variant of Hawking-Bekenstein law in which p-adic length scale replaces Planck length [K14].

The value of the vacuum energy density for a given flux tube is proportional to the value of $h_{eff}/h = n$ by the multi-sheeted covering property. Vacuum energy cannot however depend on n. There are two ways to achieve this: local and global.

1. For the local option the energy of each flux tube would remain invariant under $h \to n \times h$ as would also the number N of flux tubes. This requires that the cross section S of the radial gravitational flux tube to which energy is proportional, scales down as S/n. This looks strange.

2. For the global option flux tubes are not changed but the number N of the radial flux tubes scales down as $N \propto 1/n$: one has $Nn = constant$. In the situation in which Kähler magnetic energy dominant local option demands $S \propto n$ and global option $N \propto 1/n$. Nn constant conditions brings in mind something analogous to Uncertainty Principle. The resolutions characterized by N and n are associated with complementary variables.

The global option applies to both homologically trivial and non-trivial options and is more promising.

3.4.3 Could the value of endogenous dark magnetic field relate to cosmological constant?

TGD development of inspired model for quantum biology was initiated by the observation [J1] that ELF em fields have non-trivial effects on the brain physiology and behavior of vertebrates [K18, K19]. Since the energies of ELF photons (with frequencies in EEG range) are many orders of magnitude below thermal energy, the proposal was that one has dark photons having $h_{eff}/h = n$

increasing the value of the energy $E = h_{eff}f$ of ELF photons above thermal energy, possibly even to the energies of bio-photons in visible and UV range identified as resulting in a phase transition reducing h_{eff} to its value for visible matter.

The effects appear at multiples of cyclotron frequencies of biologically important ions in endogenous ("dark") magnetic field of $B_{end} \simeq .2$ Gauss. This corresponds to magnetic length $1/\sqrt{eB}$ not far from the size of large neuron. Could this field strength correspond to the Kähler magnetic field assignable to the flux tubes carrying monopole magnetic field, whose strength is determined by the value of cosmological constant? This would give a direct connection between cosmology and biology!

1. In recent cosmology the value of B_K (more precisely, $g_K B_K$ using ordinary conventions) at criticality would be

$$B_K = \frac{\Phi_0}{4\pi}\frac{1}{L^2(175)} \quad .$$

B_K corresponds to the U(1) magnetic field in standard model and is therefore as such not the ordinary magnetic field. For S^2_{II} Kähler magnetic field is non-vanishing. If Z^0 field vanishes, classical em field (with e included as normalization factor) equals to $\gamma = 3J$, where J is K"ahler induced Kähler form (see [L1]. One has

$$B_K = \frac{eB_{em}}{3} \quad . \tag{3.6}$$

2. An interesting question is whether one could identify physically the ordinary magnetic field assignable to the critical Kähler magnetic field.

Earth's magnetic field $B_E = .5$ Gauss corresponds to magnetic length $L_B = \sqrt{\hbar}eB = 5\mu$m. Endogenous magnetic field $B_{end} \simeq 2B_E/5$ explaining the findings of Blackman [J1] about the effects of ELF em fields on vertebrate brain in terms of cyclotron transitions corresponds to $L_B = 12.5$ μm to be compared with the p-adic length scale $L(175) = 40$ μm. Also these findings served as inspiration of $h_{eff} = n \times h$ hypothesis [K18, K17].

I have assigned large Planck constant phases with the flux tubes of B_{end}, which have however remained somewhat mysterious entity. Could B_{end} correspond to quantum critical value of B_K and therefore relate directly to cosmology?

One can check whether $B_K = eB_{end}/3$ holds true. The hypothesis would give

$$eB_{end} = \frac{1}{L_B^2} = 3 \times \frac{\Phi_0}{4\pi\hbar}\frac{1}{L^2(175)} \quad .$$

implying

$$r = \frac{L^2(175)}{L_B^2} = \frac{3\Phi_0}{4\pi\hbar} \quad .$$

The left hand side gives $r = 10.24$. For $\Phi_0 = 8\pi\hbar$ the right hand side gives $r = 6$. $B_E = .34$ Gauss left and right hand sides of the formula are identical.

3. One can wonder the proposed formulas might be exact for preferred extremals satisfying extremely powerful conditions to guarantee strong form of holography. This would require in both cases bundle structure with transversal cross section action as fiber. In the case of extremals of Kähler this would require that induce Kähler magnetic field is covariantly constant.

3.5 Two variants for p-adic length scale hypothesis for cosmological constant

There are two options for the dependence string tension Tandarea S of the cross section of the flux tube on p-adic length scale: either $L_\Lambda = \sqrt{8\pi/\Lambda}$ or $L = (\hbar/\rho_{vac})^{1/4}$ satisfies p-adic length scale hypothesis. The "boundary condition" is that the radius of flux tubes would be of the order of neutron size scale in recent cosmology.

1. $L(now) = L_p$ scaling gives

$$S = S(now)\frac{p(now)}{p} \tag{3.7}$$

with $p_{now} \simeq 2^{175}$ by p-adic length scale hypothesis. $L(175)$ is by about 4 per cent smaller than the Compton length assignable to $\hbar/L(now) = .029$ eV.

If one wants $L(now) = L(175)$ exactly, one must increase R by 4 per cent, which is allowed by p-adic mass calculations fixing the value of R only with 10 per cent accuracy. Indeed, the second order contribution in p-adic mass calculations is uncertain and the ratio of maximal and minimal values of R is $R_{max}/R_{min} = \sqrt{6/5} \simeq 1.1$.

As already noticed, $L(now)$ corresponds to neutron size scale, which conforms with p-adic mass calculations since the radius of flux tubes would correspond to p-adic length scale. This option looks more natural and suggest a profound connection with biology and fundamental physics.

2. $L_\lambda \equiv \sqrt{8\pi/\Lambda}$ could be proportional to secondary p-adic length scale $L(2, p_\Lambda) \equiv \sqrt{p_\Lambda}L_{p_\Lambda}$. The scaling law

$$L_\Lambda \propto \frac{p_\Lambda(now)}{p_\Lambda} \tag{3.8}$$

gives

$$L^2_\Lambda(now) = \frac{8\pi}{\Lambda(now)} = (\frac{p}{p(now)})^2 \times \frac{L^4(now)}{l_P^2} \quad . \tag{3.9}$$

$L_\Lambda(now) \sim 50$ Gly (roughly the age of the Universe) holds true. Note that one has $S \propto \sqrt{p_{now}/p}S(now)$ and $T = T_{now}\sqrt{p/p_{now}}$.

$1/p$-dependence for the string tension T looks more natural in light of p-adic mass calculations. One must however notice that the $L = L(175)$ is 4 per cent small than $L(now)$.

The density of dark energy is uncertain by few per cent at least and one can ask whether $L(now) = L(175)$ could fix it. The change induced to ρ_{vac} by that of $L(now)$ is

$$\frac{\Delta\rho_{vac}}{\rho_{vac}} = -4\frac{\Delta L(now)}{L(now)}$$

and the reduction L by 4 per cent would reduce vacuum density by 16 per cent, which looks rather large change. The value of R can be determined by 10 per cent accuracy and the increase of R by four per cent is another manner to achieve $L(now) = L(175)$.

One can of course ask, whether both variants of p-adic length scale hypothesis could be correct. The reader night protest that this leads to the murky waters of p-adic numerology.

1. Could L_Λ be proportional to the secondary p-adic length scale $L(p,2) = \sqrt{p}L_p = 2^{k/2} \times L(k)$ associated with p characterizing L such that the proportionality constant is power of $\sqrt{2}$. The application of the condition defining L in terms of $L_\Lambda^2 = 8\pi/\Lambda$ gives

$$L_\Lambda^2 = \frac{L^4}{l_P^2} \quad .$$

Using $L_\Lambda = \sqrt{p_\Lambda}R$ and taking square roots, this gives

$$\sqrt{p_\Lambda} = pk^2 \quad , \quad k = \frac{R_{CP_2}}{l_P} \quad . \tag{3.10}$$

This conforms with the p-adic length scales hypothesis in its simplest form if k is power of $\sqrt{2}$.

2. The estimate from p-adic mass calculations for $r \equiv R(CP_2)/l_P$ is $r = 4.167 \times 10^3$ and is 2 per cent larger than 2^{12}. Could the $R(CP_2)/l_P = 2^{12}$ for the radii of CP_2 and M^4 twistorial sphere be an exact formula between fundamental length scales? As noticed, the second order contribution in p-adic mass calculations is uncertain by 10 per cent. This would allow the reduction of $R(CP_2)$ by 2 percent.

This looks an attractive option. The bad news is that the *increase* of $R(CP_2)$ by about 4 per cent to achieve $L(now) = L(175)$ is in conflict with its *reduction* by 2 per cent to achieve $R(CP_2)/l_P = 2^{12}$: this would reduce $L(175)$ by 2 per cent and increase ρ_{vac} by about 8 per cent. ρ_{vac} is however an experimental parameter depending on theoretical assumption and it value could allow this tuning. Therefore

$$\begin{aligned} \frac{R_{CP_2}}{l_P} &= 2^{12} \quad , \\ p_\Lambda &= 2^{48} \times p^2 \quad . \end{aligned} \tag{3.10}$$

is an attractive option fixing completely the value of $R(CP_2)/l_P$ and predicting relation between cosmological scale L_Λ and a fundamental scale in recent biology, which could be assigned to magnetic flux tubes assignable to axons. Note that for $k_{now} = 175$ the value of $k_\Lambda = k_{now} + 48$ is $k_\Lambda = 175 + 48 = 223$ which corresponds to p-adic length scale of 64 m.

3. Needless to say that one must be take these estimates with a big grain of salt. Number theoretical universality suggests that one might apply number theoretical constraints to fundamental constants like R, l_P, and Λ but one should be very critical concerning the values of empirical parameters such as ρ_{vac} depending on theoretical assumptions. Furthermore, p-adic length scale hypothesis is applied at the level of embedding space metric and one can ask whether it actually applies for the induced metric (Robertson-Walker metric now).

4 What happens for the extremals of Kähler action in twistor lift

As I started to work with TGD around 1977, I adopted path integral and canonical quantization as the first approaches. One of the first guesses for the action principle was 4-volume in induced metric giving minimal surfaces as preferred extremals. The field equations are a generalization of massless field equation and at least in the case of string models Hamiltonian formalism and second quantization is possible. The reason why for giving up this option was that for space-time surfaces of infinite duration the volume is infinite. This is not pleasant news concerning quantization since subtraction of exponent of infinite volume factor looked really ugly thing to do. At that time I did of course have no idea about ZEO and CDs.

For Kähler action there is however infinite vacuum degeneracy. All space-time surfaces with CP_2 projection, which is Lagrangian manifold (at most 2-dimensional) are vacuum extremals and

canonical quantization fails completely. This implies classical non-determinism also for non-vacuum extremals obtained as small deformations of vacuum extremals. This feature seems to have nice implications such as 4-D spin glass degeneracy. It would however make WCW metric singular for nearly vacuum extremals.

The twistor lift brings volume term to the action. For option II there is also coupling between Kähler action and volume term but asymptotically one expects minimal surface extremals as analogs for free geodesic motion. The question is what happens to the known extremals of Kähler action, most of which are minimal surfaces.

4.1 The coupling between Kähler action and volume term

The addition of the volume term to Kähler action has very nice interpretation as a generalization of equations of motion for a world-line extended to a 4-D space-time surface. The field equations generalize in the same manner for 3-D light-like surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian, for 2-D string world sheets, and for their 1-D boundaries defining world lines at the light-like 3-surfaces. For 3-D light-like surfaces the volume term is absent. Either light-like 3-surface is freely choosable in which case one would have Kac-Moody symmetry as gauge symmetry or that the extremal property for Chern-Simons term fixes the gauge.

The condition that the dynamics based on Kähler action and volume term is number theoretically universal demands that coupling constants do not appear in it. This leaves only Option I ($\alpha_K(M^4) \neq \alpha_K(CP_2)$ with different phases) and option II ($\alpha_K(M^4) = \alpha_K(CP_2)$ with the same phase). This condition is taken as granted in the following.

4.1.1 The dynamics of twistor lift as a generalization of the dynamics of point like particle coupling to Maxwell field

Almost all the known non-vacuum extremals are minimal surface extremals of Kähler action [K3, K1] and it might well be that the preferred extremal property realizing SH quite generally demands this. CP_2 type vacuum extremals are also minimal surfaces if one assumes that the M^4 projection is light-like geodesic rather than only geodesic line.

The addition of the volume term could however make Kähler coupling strength a manifest coupling parameter also classically when the phases of Λ and α_K are same. Therefore quantum criticality for Λ and α_K would have a precise local meaning also classically in the interior of space-time surface. The equations of motion for a world line of U(1) charged particle would generalize to field equations for a "world line" of 3-D extended particle.

This is an attractive idea consistent with standard wisdom but for Option I one can invent strong objections against it.

1. The conjecture is that α_K has zeros of zeta as its spectrum of critical values [L3]. If so then all preferred extremals are minimal surface extremals of Kähler action for a real value of cosmological constant Λ possible for Option I ($\alpha_K(M^2)$ would be real). Hence the two actions decouple: this does not look nice. For Option II the phase is same and there is interaction between these degrees of freedom. One could of course force also the phase for Option I to be same.

2. All known non-vacuum extremals of Kähler action are minimal surfaces and the minimal surface vacuum extremals of Kähler action become non-vacuum extremals. This allows to consider the possibility that preferred extremals are minimal surface extremals of Kähler action so that the two dynamics apparently decouple. For Option II this makes sense since the solutions do not depend at all on the common over-all scaling factor of Kähler action and volume term. Minimal surface extremals are analogs for geodesics in the case of point-like particles: one might say that one has only gravitational interaction. This conforms with SH stating that gauge interactions at boundaries (orbits of partonic 2-surfaces and 2-surfaces at the ends of CD) correspond classically to the gravitational dynamics in the space-time interior.

   Note that at the boundaries of the string world sheets at light-like 3-surfaces the situation is different: one has equations of motion for geodesic line coupled to induce Kähler gauge

potential and gauge coupling indeed appears classically as one might expect! For string world sheets one has only the topological magnetic flux term and minimal surface equation in string world sheet. Magnetic flux term gives the Kähler coupling at the boundary.

3. For Option I decoupling implied by extremal property of both real and imaginary parts of action would allow to realize number theoretical universality [K29] since the field equations would not depend on coupling parameters at all. For Option II same is achieved even without decoupling.

4. One can argue that the decoupling for Option I makes it impossible to understand coupling constant evolution. This need not be the case. The point is that the classical charges assignable to super-symplectic algebra are sums over contributions from Kähler action and volume term and therefore depend on the coupling parameters. Their vanishing conditions for sub-algebra and its commutator with entire algebra give boundary conditions on preferred extremals so that coupling constant evolution creeps in classically!

   Quantum classical correspondence realized as the condition that the eigenvalues of fermionic charge operators are equal to the classical charges brings in the dependence of quantum charges on coupling parameters. Since the elements of scattering matrix are expected to involve as building bricks the matrix elements of super-symplectic algebra and Kac-Moody algebra of isometry charges, one expects that discrete coupling constant evolution creeps in also quantally via the boundary conditions for preferred extremals.

4.1.2 Options I and II and Chladni mechanism

One can compare Options I and II.

1. For Option I the coupling between the two dynamics could be induced just by the condition that the space-time surface becomes an analog of geodesic line by arranging its interior so that the U(1) force vanishes! This would generalize Chladni mechanism (see `http://tinyurl.com/j9rsyqd`)!

   The interaction would be present but be based on going to the nodal surfaces! Also the dynamics of string world sheets is similar: if the string sheets carry vanishing W boson classical fields, em charge is well-defined and conserved. One would also avoid the problems produced by large coupling constant between the two-dynamics present already at the classical level. At quantum level the fixed point property of quantum critical couplings would be the counterparts for decoupling. This option however seems to be missing the transient phase preceding the Chladni configuration.

2. For Option II the coupling would be present during transient periods leading to decoupling. The alternative view is that the deviation from minimal surface and can act as a controller of the dynamics defined by the volume term providing a small push or pull now and then. Could this sensitivity relate to quantum criticality and to the view about morphogenesis relying on Chladni mechanism in which field patterns control the dynamics with charged flux tubes ending up to the nodal surfaces of (Kähler) electric field [L5]? Magnetic flux tubes containing dark matter would in turn control and serve as template for the dynamics of ordinary matter.

   Chladni mechanism would not be instantaneous but lead via transient phase to minimal surface extremals near either or both boundaries of CDs analogous to external particles in particle reaction. The space-time regions assignable to particle interaction vertices identified as 2-surfaces at which the ends of three 3-D light-like partonic orbits meet, would correspond to transient regions, where the coupling is present. This option looks clearly more realistic.

Admittedly Option II looks more attractive.

As an example one can consider a typical particle physics experiment. There are incoming and outgoing free particles moving along geodesics, these particles interact, and emanate as free particles from the interaction volume. This phenomenological picture does not follow from QFT but is put in by hand, in particular the idea about interaction couplings becoming non-zero is involved. Also the role of the observer remains poorly understood.

The motion of incoming and outgoing particles is analogous to free motion along geodesic lines with particles generalized to 3-D extended objects. For both options these would correspond to the preferred extremals in the complement of CD within larger CD representing observer or measurement instrument. Decoupling would take place. In interaction volume interactions are "coupled on" and particles interact inside the volume characterized by causal diamond (CD). What could be the TGD view translation of this picture?

1. For Option I one would still have decoupling and the interpretation would be in terms of twistor picture in which one always has also in the internal lines on mass shell particles but with complex four-momenta. In TGD framework the momenta would be always complex due to the contribution of Euclidian regions defining the lines of generalized scattering diagrams. Note however that the real and imaginary parts of the conserved charges are predicted to be proportional to each other. This result is obtained also in twistor approach from 8-D light-likeness and is crucial for twistorialization in TGD sense [L8]. As explained, coupling constant evolution can be understood also in this case and also classical dynamics depends on coupling parameters via the boundary conditions. There would be no counterpart for transitory period (interaction on) leading to the decoupled situation so that Option I is not attractive.

2. For Option II the transitory period would correspond to the coupling between the two classical dynamics in regions assignable to the vertices of topological scattering diagrams at which the ends of the parton orbits meet. Near the ends the dynamics would decouple and one would have the analog of free geodesic motion.

Second example comes from biology. The free geodesic line dynamics with vanishing $U(1)$ Kähler force indeed brings in mind the proposed generalization of Chladni mechanism generating nodal surfaces at which charged magnetic flux tubes are driven [K13] [L5, L6] . Chlandi mechanism could be seen as a basic mechanism behind morphogenesis.

1. For Option I the interiors of all space-time surfaces would be analogous to nodal surfaces and "big" state function reductions would correspond to transition periods between different nodal surfaces. The decoupling would be dynamics of avoidance and could highly analogous to Chladni mechanism.

2. For Option II transition period would correspond to a period during which nodal surfaces are formed.

It seems that Option II is favored by both SH, number theoretical universality, and generalization of Chladni mechanism to a dynamics of avoidance.

4.2 Twistor lift and the extremals of Kähler action

The addition of the volume term makes Kähler coupling strength a genuine coupling parameter also classically when the variation of Kähler action is non-vanishing. Therefore quantum criticality for Λ and α_K gets precise meaning also classically. The equations of motion for a worldline of U(1) charged particle generalize to field equations for a "world line" of 3-D extended particle.

The field equations generalize in the same manner for 3-D light-like surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian, for 2-D string world sheets, and for their 1-D boundaries defining world lines at the light-like 3-surfaces. For 3-D light-like surfaces the volume term is absent. Either light-like 3-surface is freely choosable in which case one would have Kac-Moody symmetry as gauge symmetry or that the extremal property for Chern-Simons term fixes the gauge.

4.2.1 What happens to the extremals of Kähler action?

What happens to the extremals of Kähler action when volume term is introduced?

1. The known non-vacuum extremals [K3, K1] such as massless extremals (topological light rays) and cosmic strings are minimal surfaces.

2. For $J(M^4) = 0$ these extremals remain extremals for both Option I and II and only the classical Noether charges receive an additional volume term. In particular, string tension is modified by the volume term. Homologically non-trivial cosmic strings are of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and $Y^2 \subset CP_2$ is complex 2-surface and therefore also minimal surface.

3. For $J(M^4) \neq 0$ essential for obtaining small cosmological constant for Option II, the situation changes and minimal surface property is possible only under additional conditions. For instance, one can have minimal surfaces of form $X^2 \times Y^2 \subset M^4 \times Y^2$, where Y^2 is minimal surface in CP_2. X^2 cane be $M^2 \subset N^2 \times E^2$ defining the $J(M^4)$ giving $J(M^4) \cdot J(M^4) - 2 = 0$. X^2 can be also minimal surface, which is an analog of Lagrangian manifold for $J(M^4)$.

4. Vacuum degeneracy is lifted for both options. For $J(M^4) = 0$ vacuum extremals, which are minimal surfaces survive as extremals for both options. For $J(M^4) \neq 0$ the situation is more complex.

4.2.2 Vacuum extremals

For CP_2 type vacuum extremals [K3, K1] the roles of M^4 and CP_2 are changed. M^4 projection is light-like curve, and can be expressed as $m^k = f^k(s)$ with light-likeness conditions reducing to Virasoro conditions. These surfaces are isometric to CP_2 and have same Kähler and symplectic structures as CP_2 itself. What is new as compared to GRT is that the induced metric has Euclidian signature. The interpretation is as lines of generalized scattering diagrams. The addition of the volume term forces the random light-like curve to be light-like geodesic and the action becomes the volume of CP_2 in the normalization provided by cosmological constant. What looks strange is that the volume of any CP_2 type vacuum extremals equals to CP_2 volume but only the extremal with light-like geodesic as M^4 projection is extremal of volume term. A little calculation shows that for CP_2 type extremals the contribution of the volume term to the action would be completely negligible as compared to the Kähler action.

Consider next vacuum extremals, which have vanishing induced Kähler form and are thus have CP_2 projection belonging to at most 2-D Lagrangian manifold of CP_2 [K3, K1].

1. Vacuum extremals with 2-D projections to CP_2 and M^4 are possible and are of form $X^2 \times Y^2$, X^2 arbitrary 2-surface and Y^2 a Lagrangian manifold. Volume term forces X^2 to be a minimal surface and Y^2 is Lagrangian minimal surface unless the minimal surface property destroys the Lagrangian character.

   If the Lagrangian sub-manifold is homologically trivial geodesic sphere, one obtains string like objects with string tension determined by the cosmological constant alone.

   Do more general 2-D Lagrangian minimal surfaces than geodesic sphere exist? For general Kähler manifold there are obstructions but for Kähler-Einstein manifolds such as CP_2, these obstructions vanish (see `http://tinyurl.com/gtkpya6`). The case of CP_2 is also discussed in the slides "On Lagrangian minimal surfaces on the complex projective plane" (see `http://tinyurl.com/jrhl6gy`). The discussion is very technical and demonstrates that Lagrangian minimal surfaces with all genera exist. In some cases these surfaces can be also lifted to twistor space of CP_2.

2. More general vacuum extremals have 4-D M^4 projection. Could the minimal surface condition for 4-D M^4 projection force a deformation spoiling the Lagrangian property? The physically motivated expectation is that string like objects give as deformations magnetic flux tubes for which string is thicknened so that it has a 2-D cross section. This would suggest that the deformations of string like objects $X^2 \times Y^2$, where Y^2 is Lagrangian minimal surface, give rise to homologically trivial magnetic flux tubes. In this case Kähler magnetic field would vanish but the spinor connection of CP_2 would give rise to induced magnetic field reducing to some U(1) subgroup of U(2). In particular, electromagnetic magnetic field could be present.

3. p-Adically Λ behaves like $1/p$ as also string tension. Could hadronic string tension be understood also in terms of cosmological constant in hadronic p-adic length scale for strings if one

assumes that cosmological constant for given space-time sheet is determined by its p-adic length scale?

4.2.3 Maxwell phase

What might be called Maxwell phase which would correspond to small perturbations of M^4 is also possible for 4-D Kähler action. For the twistor lift the volume term makes this phase possible. Maxwell phase is highly interesting since it corresponds to the intuitive view about what QFT limit of TGD could be. The following arguments apply only for $J(M^4) = 0$.

1. The field equations are a generalization of massless field equations for fields identifiable as CP_2 coordinates and with a coupling to the deviation of the induced metric from M^4 metric. It representes very weak perturbation. Hence the linearized field equations are expected to be an excellent approximation. The general challenge would be however the construction of exact solutions. One should also understand the conditions defining preferred extremals and stating that most of symplectic Noether charges vanish at the ends of space-time surface about boundaries of CD.

2. Maxwell phase is the TGD analog for the perturbative phase of gauge theories. The smallness of the cosmological constant in cosmic length scales would make the perturbative approach useless in the path integral formulation. In TGD approach the path integral is replaced by functional integral involving also a phase but also now the small value of cosmological constant is a problem in long length scales. As proposed, the hierarchy of Planck constants would provide the solution to the problem.

3. The value of cosmological constant behaving like $\Lambda \propto 1/p$ as the function of p-adic prime could be in short p-adic length scales large enough to allow a converging perturbative expansion in Maxwellian phase. This would conform with the idea that Planck constant has its ordinary value in short p-adic length scales.

4. Does Maxwell phase allow extremals for which the CP_2 projection is 2-D Lagrangian manifold - say a perturbation of a minimal Lagrangian manifold? This perturbation could be seen also as an alternative view about thickened minimal Lagrangian string allowing also M^4 coordinates as local coordinates. If the projection is homologically trivial geodesic sphere this is the case. Note that solutions representable as maps $M^4 \to CP_2$ are also possible for homologically non-trivial geodesic sphere and involve now also the induced Kähler form.

5. The simplest deformations of canonically imbedded M^4 are of form $\Phi = k \cdot m$, where Φ is an angle coordinate of geodesic sphere. The induced metric in M^4 coordinates reads as $g_{kl} = m_{kl} - R^2 k_k k_l$ and is flat and in suitably scaled space-time coordinates reduces to Minkowski metric or its Euclidian counterpart. k_k is proportional to classical four-momentum assignable to the dark energy. The four-momentum is given by

$$P^k = A \times \hbar k^k \quad , \quad A = \frac{Vol(X^3)}{L_\Lambda^4} \times \frac{1+2x}{1+x} \quad , \quad x = R^2 k^2 \quad .$$

   Here k^k is dimensionless since the the coordinates m^k are regarded as dimensionless.

6. There are interesting questions related to the singularities forced by the compactness of CP_2. Eguchi-Hanson coordinates (r, θ, Φ, Ψ) [L1] (see `http://tinyurl.com/z86o5qk`) allow to get grasp about what could happen.

   For the cyclic coordinates Ψ and Φ periodicity conditions allow to get rid of singularities. One can however have n-fold coverings of M^4 also now.

   (r, θ) correspond to canonical momentum type canonical coordinates. Both of them correspond to angle variables ($r/\sqrt{1+r^2}$ is essentially sine function). It is convenient to express the solution in terms of trigonometric functions of these angle variables. The value of the trigonometric function can go out of its range $[-1, 1]$ at certain 3-surface so that the solution ceases to be well-defined. The intersections of these surfaces for r and θ are 2-D surfaces. Many-sheeted space-time suggests a possible manner to circumvent the problem by gluing

two solutions along the 3-D surfaces at which the singularities for either variable appear. These surfaces could also correspond to the ends of the space-time surface at the boundaries of CD or to the light-like orbits of the partonic 2-surfaces.

Could string world sheets and partonic 2-surfaces correspond to the singular 2-surfaces at which both angle variables go out of their allowed range. If so, 2-D singularities would code for data as assumed in strong form of holography (SH). SH brings strongly in mind analytic functions for which also singularities code for the data. Quaternionic analyticity which makes sense would indeed suggest that co-dimension 2 singularities code for the functions in absence of 3-D counterpart of cuts (light-like 3-surfaces?) [K8].

7. A more general picture might look like follows. Basic objects come in two classes. Surfaces $X^2 \times Y^2$, for which Y^2 is either homologically non-trivial complex minimal 2-surface of CP_2 of Lagrangian minimal surface. The perturbations of these two surfaces would also produce preferred extremals, which look locally like perturbations of M^4. Quaternionic analyticity might be shared by both solution types. Singularities force many-sheetedness and strong form of holography.

4.2.4 Astrophysical and cosmological solutions

Cosmological constant is expected to obey p-adic evolution and in very early cosmology the volume term becomes large. What are the implications for the vacuum extremals representing Robertson-Walker metrics having arbitrary 1-D CP_2 projection? [K3, K1, K24]. One can also ask what is the fate of spherically symmetric solutions of GRT providing a model of star.

Already the existing physical picture explaining $h_{gr}/hh_{eff}/h = n$ in terms of flux tubes mediating gravitational interactions suggests that Robertson-Walker metrics and spherically symmetric metrics are possible only at QFT limit. The presence of covariantly constant $J(M^4)$ breaking Lorentz symmetry and rotational symmetry makes this obvious. One could consider variants of $J(M^4)$ invariant under Lorentz group or some subgroup of Lorentz group but $J(M^4)$ would not be covariantly constant anymore. It is not clear when it makes sense to extend the moduli space for $J(M^4)$.

1. The TGD inspired cosmology involves primordial phase during a gas of cosmic strings in M^4 with 2-D M^4 projection dominates. The value of cosmological constant at that period could be fixed from the condition that homologically trivial and non-trivial cosmic strings have the same value of string tension. After this period follows the analog of inflationary period when cosmic strings condense are the emerging 4-D space-time surfaces with 4-D M^4 projection and the M^4 projections of cosmic strings are thickened. A fractal structure with cosmic strings topologically condensed at thicker cosmic strings suggests itself.

2. GRT cosmology is obtained as an approximation of the many-sheeted cosmology as the sheets of the many-sheeted space-time are replaced with region of M^4, whose metric is replaced with Minkowski metric plus the sum of deformations from Minkowski metric for the sheet. The vacuum extremals with 4-D M^4 projection and arbitrary 1-D projection could serve as an approximation for this GRT cosmology. Note however that this representability is not required by basic principles.

3. For cosmological solutions with 1-D CP_2 projection minimal surface property forces the CP_2 projection to belong to a geodesic circle S^1. Denote the angle coordinate of S^1 by Φ and its radius by R. For the future directed light-cone M^4_+ use the Robertson-Walker coordinates $(a = \sqrt{m_0^2 - r_M^2}, r = ar_M, \theta, \phi)$, where (m^0, r_M, θ, ϕ) are spherical Minkowski coordinates. The metric of M^4_+ is that of empty cosmology and given by $ds^2 = da^2 - a^2 d\Omega^2$, where Ω^2 denotes the line element of hyperbolic 3-space identifiable as the surface $a = constant$.

   One can can write the ansatz as a map from M^4_+ to S^1 given by $\Phi = f(a)$. One has $g_{aa} = 1 \to g_{aa} = 1 - R^2(df/da)^2$. The field equations are minimal surface equations and the only non-trivial equation is associated with Φ and reads $d^2f/da^2 = 0$ giving $\Phi = \omega a$, where ω is analogous to angular velocity. The metric corresponds to a cosmology for which mass density goes as $1/a^2$ and the gravitational mass of comoving volume (in GRT sense) behaves is proportional to a and vanishes at the limit of Big Bang smoothed to "Silent whisper

amplified to rather big bang" for the critical cosmology for which the 3-curvature vanishes. This cosmology is proposed to results at the limit when the cosmic temperature approaches Hagedorn temperature [K24].

4. The TGD counterpart for inflationary cosmology corresponds to a cosmology for which CP_2 projection is homologically trivial geodesic sphere S^2 (presumably also more general Lagrangian (minimal) manifolds are allowed). This cosmology is vacuum extremal of Kähler action. The metric is unique apart from a parameter defining the duration of this period serving as the TGD counterpart for inflationary period during which the gas of string like objects condensed at space-time surfaces with 4-D M^4 projection. This cosmology could serve as an approximate representation for the corresponding GRT cosmology.

   The form of this solution is completely fixed from the condition that the induced metric of $a = constant$ section is transformed from hyperbolic metric to Euclidian metric. It should be easy to check whether this condition is consistent with the minimal surface property. It seems that one cannot satisfy minimal surface equations.

5. For $J(M^4) \neq 0$ the spherical and Lorentz symmetries are lost and the only cosmological solution are light-cones $M^4_{\pm}$. Also the existence of stationary spherically symmetric minimal surface extremals is impossible for $J(M^4) \neq 0$. Spherically symmetric metrics and Robertson-Walker metric would serve only as long length scale approximations providing a statistical description of the gravitational interaction described microscopically in terms of a flux tube network.

4.3 Are minimal surface extremals of Kähler action holomorphic surfaces in some sense?

If the spectrum for the critical value of Kähler coupling strength is complex - say given by the complex zeros of zeta [L3] - the preferred extremals of Kähler action are minimal surfaces for Option I. For Option II they correspond to asymptotic solutions.

I have considered several ansätze for the general solutions of the field equations for the preferred extremals. One proposal is that preferred extremals as 4-surfaces of embedding space with octonionic tangent space structure have quaternionic tangent space or normal space (so called $M^8 - H$ duality [K25]). Second proposal is that preferred extremals can be seen as quaternion analytic [**?**] surfaces [K20, K27] [L2]. Third proposal relies on a fusion of complex and hypercomplex structures to what I call Hamilton-Jacobi structure [K28, K1]. In Euclidian regions this would correspond to complex structure. Twistor approach [K8] suggests that the condition that the twistor lift of the space-time surface to a 6-D surface in the product of twistor spaces of M^4 and CP_2 equals to the twistor space of CP_2. This proposal is highly interesting since twistor lift works only fr $M^4 \times CP_2$. The intuitive picture is that the field equations are integrable and all these views might be consistent.

Preferred extremals of Kähler action as minimal surfaces would be a further proposal. Can one make conclusions about general form of solutions assuming that one has minimal surface extremals of Kähler action?

In $D = 2$ case minimal surfaces are holomorphic surfaces or they hyper-complex variants and the embedding space coordinates can be expressed as complex-analytic functions of complex coordinate or a hypercomplex analog of this. Field equations stating the vanishing of the trace $g\alpha\beta H^k_{\alpha\beta}$ if the second fundamental form $H^k_{\alpha\beta} \equiv D_\alpha \partial_\beta h^k$ are satisfied because the metric is tensor of type $(1,1)$ and second fundamental form of type $(2,0) \oplus (2,0)$. Field equations reduce to an algebraic identity and functions involved are otherwise arbitrary functions. The constraint comes from the condition that metric is of form $(1,1)$ as holomorphic tensor.

This raises the question whether this finding generalizes to the level of 4-D space-time surfaces and perhaps allows to solve the field equations exactly in coordinates generalizing the hypercomplex coordinates for string world sheet and complex coordinates for the partonic 2-surface.

Almost all the known non-vacuum extremals are minimal surface extremals of Kähler action [K3, K1] and it might well be that the preferred extremal property realizing SH quite generally demands this. CP_2 type vacuum extremals are also minimal surfaces if one assumes that the M^4 projection is light-like geodesic rather than only geodesic line. The common feature suggested

already earlier to be common for all preferred extremals is the existence of generalization of complex structure.

1. For Minkowskian regions this structure would correspond to what I have called Hamilton-Jacobi structure [K28, K1]. The tangent space of the space-time surface X^4 decomposes to local direct sum $T(X^4) = T(X^2) \oplus T(Y^2)$, where the 2-D tangent places $T(X^2)$ and $T(Y^2)$ define an integrable distribution integrating to a decomposition $X^4 = X^2 \times Y^2$. The complex structure is generalized to a direct some of hyper-complex structure in X^2 meaning that there is a local light-like direction defining light-like coordinate u and its dual v. $Y2$ has complex complex coordinate $(w, \overline{w})$. Minkowski space M^4 has similar structure. It is still an open question whether metric decomposes to a direct sum of orthogonal metrics assignable to X^2 and Y^2 or is the most general analog of complex metric in question. g_{uv} and $g_{w\overline{w}}$ are certainly non-vanishing components of the induced metric. Metric could allow as non-vanishing components also g_{uw} and $g_{v\overline{w}}$. This slicing by pairs of surfaces would correspond to decomposition to a product of string world sheet and partonic 2-surface everywhere.

   In Euclidian regions ne would have 4-D complex structure with two complex coordinates (z, w) and their conjugates and completely analogous decompositions. In CP_2 one has similar complex structure and actually Kähler structure extending to quaternionic structure. I have actually proposed that quaternion analyticity could provide the general solution of field equations.

2. Assuming minimal surface property the field equations for Kähler action reduce to the vanishing of a sum of two terms. The first term comes from the variation with respect to the induced metric and is proportional to the contraction

$$A = J^{\alpha}_{\ \gamma} J^{\gamma\beta} H^k_{\alpha\beta} \quad . \tag{4.1}$$

   Second term comes from the variation with respect to induced Kähler form and is proportional to

$$B = j^{\alpha} P^k_{\ s} J^s_{\ l} \partial_{\alpha} h^l \quad . \tag{4.2}$$

   Here P^k_l is projector to the normal space of space-time surface and $j^{\alpha} = D_{\beta} J^{\alpha\beta}$ is the conserved Kähler current.

   For the known extremals j vanishes or is light-like (for massless extremals) in which case A and B vanish separately.

3. An attractive manner to satisfy field equations would be by assuming that the situation for 2-D minimal surface generalizes so that minimal surface equations are identically satisfied. Extremal property for Kähler action could be achieved by requiring that energy momentum tensor also for Kähler action is of type $(1, 1)$ so that one would have $A = 0$. This implies $j^{\alpha} \partial_{\alpha} s^k = 0$. This is true if j vanishes or is light-like as it is for the known extremals. In Euclidian regions one would have $j = 0$.

4. The proposed generalization is especially interesting in the case of cosmic string extremals of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface (string world sheet) and Y^2 is complex homologically non-trivial sub-manifold of CP_2 carrying Kähler magnetic charge. The generalization would be that the two transversal coordinates $(w, \overline{w})$ in the plane orthogonal to the string world sheet defining polarization plane depend holomorphically on the complex coordinates of complex surface of CP_2. This would transform cosmic string to flux tube.

5. There are also solutions of form $X^2 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 with vanishing Kähler magnetic charge and their deformations with $(w, \overline{w})$ depending on the complex coordinates of Y^2 (see the slides "On Lagrangian minimal surfaces on the complex projective plane" at `http://tinyurl.com/jrhl6gy`). In this case Y^2 is not complex sub-manifold of CP_2 with arbitrary genus and induced Kähler form vanishes. The simplest choice

for Y^2 would be as homologically trivial geodesic sphere. Because of its 2-dimensionality Y^2 has a complex structure defined by its induced metric so that solution ansatz makes sense also now.

5 About string like objects

String like objects and partonic 2-surfaces carry the information about quantum states and about space-time surfaces as preferred extremals if strong form of holography (SH) holds true. SH has of course some variants. The weakest variant states that fundamental information carrying objects are metrically 2-D. The light-like 3-surfaces separating space-time regions with Minkowskian and Euclidian signature of the induced metric are indeed metrically 2-D, and could thus carry information about quantum state.

The original observation was that string world sheets should carry vanishing W boson fields in order that the em charge for the modes of the induced spinor field is well-defined. This condition can be satisfied in certain situations also for the entire space-time surface. This raises several questions. What is the fundamental condition forcing the restriction of the spinor modes to string world sheets - or more generally, to a surface of given dimension?

Can one have an analog of brane hierarchy in which also higher-D objects can carry modes of induced spinor field [K22]. Or should one identify 2-surfaces in terms of effective action, which by SH allows to describe the dynamics in terms of 2-D data? Both options have their nice features.

5.1 Two options for fundamental variational principle

String world sheets and partonic 2-surfaces seems to be fundamental for TGD - especially so in the fermionic sector - but also the 4-D action seems to necessary and supersymmetry forces 4-D modified Dirac action too. The interpretation of the situation is far from obvious. One ends up to two options for the fundamental variational principle.

Option A: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K22].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option B: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If the induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes!

If string world sheets are generalized Lagrangian sub-manifolds, only the induced em field would be non-vanishing and electroweak symmetry breaking would be a fundamental prediction. This however requires that M^4 has the analog of symplectic structure suggested also by twistorialization. This in turn provides a possible explanation of CP breaking and matter-antimatter asymmetry. In this case 4-D spinor modes do not define space-time super-symmetries.

The latter option conforms with number theoretically broken SH and would mean that the theory is amazingly simple. String world sheets together with number theoretical space-time discretization meaning small breaking of SH would provide the basic data determining classical and quantum dynamics. The Galois group of the extension of rationals defining the number-theoretic space-time discretization would act as a covering group of the covering defined by the discretization of the space-time surface, and the value of $h_{eff}/h = n$ would correspond to the

dimension of the extension dividing the order of its Galois group. The phase transitions reducing $ord(G) \geq n$ would correspond to spontaneous symmetry breaking leading from Galois group to a subgroup H so that $ord(H)$ would divide $ord(G)$ and the new value of n would divide n.

The ramified primes of the extension would be preferred primes of given extension. The extensions for which the number of p-adic space-time surfaces representable also as a real algebraic continuation of string world sheets to preferred extrenal is especially large would be physically favored as also corresponding ramified primes. In other words, maximal number of p-adic imaginations would be realizable so that these extensions and corresponding ramified primes would be winners in the number-theoretic fight for survival. Whether this conforms with p-adic length scale hypothesis, remains an open question.

An attractive possibility is that this information is basically topological. For instance, the value of Planck constant $h_{eff} = n \times h$ would tell the number sheets of the singular covering defining this surface such that the sheets co-incide at partonic 2-surfaces at the ends of space-time surface at boundaries of CD. In the following some questions related to string world sheets are considered. The information could be also number theoretical. Galois group for the algebraic extension of rationals defining particular adelic physics would transform to each other the number theoretic discretizations of light-like 3-surfaces and give rise to covering space structure. The action is trivial at partonic 2-surfaces should be trivial if one wants singular covering: this would mean that discretizations of partonic 2-surfaces consist of rational points. $h_{eff}/h = n$ could in this case be a factor of the order of Galois group.

5.2 How to achieve low value of string tension?

String tension should be low for string world sheets in long scales. If string actions are effective actions (Option B), the same should be true for the string tensions of the magnetic flux tubes accompanying strings. Minimal surface property for string world sheets is natural. Let us consider only Option B in the following.

1. Could the analogs of Lagrangian sub-manifolds of $X^4 \subset M^4 \times CP_2$ satisfying $J(M^4) + J(CP_2) = 0$ define string world sheets and their variants with varying dimension? For Option I ($\alpha_K(M^4) \neq \alpha_K(CP_2)$) this could make sense if the flux tubes are homologically trivial. Homologically non-trivial (monopole) flux tubes should be thick enough to have small enough string tension, which is inversely proportional to the cross sectional area of the flux tube.

2. For Option II ($\alpha_K(M^4) = \alpha_K(CP_2)$) the action density is proportional to $J \cdot J - 2$ also for stringy action and this does not seem to make sense. Could the additional condition be $J(M^4) \cdot J(M^4) - 2 \sim 0$ holding true in 4-D sense for space-time regions with a small value of cosmological constant behaving like $1/p$, p preferred p-adic prime near power of 2. That low string tension and small cosmological constant would have the same origin, would be nice.

   The cancellation mechanism involving in an essential manner $J(M^4)$ would give rise to low mass strings and light hadron like particles and small cosmological constant instead of only high mass strings as in super string models. p-Adic thermodynamic for CP_2-mass excitations assignable to wormhole throats would determine elementary particle masses and long monopole flux tubes with small string tension connecting pairs of wormhole contacts would give stringy contribution to particle masses. In the case of hadrons this contribution from color magnetic flux tubes would dominate over quark masses. Clearly, Option II seems to conform with the existing picture about masses of elementary particles and hadrons.

5.3 How does the gravitational coupling emerge?

The appearance of $G = l_P^2$ has coupling constant remained for a long time actually somewhat of a mystery in TGD. l_P defines the radius of the twistor sphere of M^4 replaced with its geometric twistor space $M^4 \times S^2$ in twistor lift. G makes itself visible via the coefficients $\rho_{vac} = 8\pi\Lambda/G$ volume term but not directly and if preferred extremals are minimal surface extremals of Kähler action ρ_{vac} makes itself visible only via boundary conditions. How G appears as coupling constant?

Somehow the M^4 Kähler form should appear in field equations. $1/G$ could naturally appear in the string tension for string world sheets as string models suggest. p-Adic mass calculations

identify the analog of string tension as something of order of magnitude of $1/R^2$ [K12]. This identification comes from the fact that the ground states of super-conformal representations correspond to embedding space spinor modes, which are solutions of Dirac equation in $M^4 \times CP_2$. This argument is rather convincing and allows to expect that the p-adic mass scale is not determined by string tension.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One expects long gravitational flux tubes mediating gravitational interactions. The hypothesis $\hbar_{eff} = n\hbar = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is a parameter with dimensions of velocity, suggests that the string tension assignable to the flux tubes mediating gravitational interaction between masses M and m is apart from a numerical factor equal to Λ_{gr}^{-2}, where gravitational Compton length is $\Lambda_{gr} = h_{gr}/m = GM/v_0$ so that the length of the flux tubes is of order Λ_{gr}.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One would like to have long gravitational flux tubes mediating gravitational interactions. Strong form of holography (SH) indeed suggests that stringy action appears as effective action expressing 4-D space-time action and modified Dirac action as 2-D actions assignable to string world sheets [L7] (see `http://tinyurl.com/zylrd7w`). This view would allow to understand the localization of spinor modes to string world sheets carrying vanishing W fields in terms as an effective description implying well-defineness of classical em charge and conservation of em charge at the level of scattering amplitudes. In fact that the introduction of the Kähler form $J(M^4)$ would allow to understand string world sheets as analogs of Lagrangian sub-manifolds.

5.4 Non-commutative embedding space and strong form of holography

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. The non-commutativity is introduced by postulating the Minkowskian analog of symplectic form and $J(M^4)$ forced by Option II indeed is symplectic form. The loss of Lorentz invariance induced by $J(M^4)$ is the basic stumbling block. In TGD framework the moduli space for $J(M^4)$ emerges already when one introduces the moduli space for CDs. $J(M^4)$ would define quantization axis of energy (rest system) and quantization axis of spin. The nice features of $J(M^4)$ is that it could allow to understand CP breaking and matter antimatter asymmetry at fundamental level.

5.4.1 The analog of non-commutative space-time in TGD framework

In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

General Coordinate Invariance (GCI) poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in embedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 embedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure

J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Käction also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing embedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

5.4.2 The analog of brane hierarchy at fundamental level or from SH?

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive (note however that SH does not favour it). Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4)+J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions. As already found, for the physically favoured Option II the more plausible option is $J(M^4) \cdot J(M^4) - 2 \sim 0$ for space-time regions with small cosmological constant. Despite this one can discuss this idea.

This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4)+J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K22].

Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would couple only to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [L1] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of electro-weak group to electromagnetic gauge group.

It seems relatively easy to construct am infinite family of Lagrangian string world sheets satisfying $J(M^4) + J(CP_2) = 0$ using generalized symplectic transformations of M^4 and CP_2 as Hamiltonian flows to generate new ones from a given Lagrangian string world sheets. One must pose minimal surface property as a separate condition. Consider a piece of M^2 with coordinates (t, z) and homologically non-trivial geodesic sphere S^2 of CP_2 with coordinates $(u = cos(\Theta), \Phi)$. One has $J(M^4)_{tz} = 1$ and $J_{u\Phi} = 1$. Identify string world sheet via map $(u, \Phi) = (kz, \omega t)$ from M^2 to S^2. The induced CP_2 Kahler form is $J(CP_2)_{tz} = k\omega$. $k\omega = -1$ guarantees $J(M^4)+J(CP_2) = 0$. The strings have necessarily finite length from $L = 1/k \leq z \leq L$. One can perform symplectic

transformations of CP_2 and symplectic transformations of M^4 to obtain new string world sheets. In general these are not minimal surfaces and this condition would select some preferred string world sheets.

5.4.3 Number theoretic vision about the analog of brane hierarchy

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in embedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of embedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces plus number theoretic discretization of space-time surface would be enough to construct quantum states. Therefore SH would be thus slightly broken. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD.

To sum up, one cannot exclude the possibility that $J(M^4)$ is present implying a universal transversal localization of embedding space spinor harmonics and the modes of spinor fields in the interior of X^4: this could perhaps relate to somewhat mysterious de-coherence interaction producing locality and to CP breaking and matter-antimatter asymmetry. The moduli space for M^4 Kähler structures proposed by number theoretic considerations would save from the loss of Poincare invariance and the number theoretic vision based on quaternionic and octonionic structure would have rather concrete realization. This moduli space would only extend the notion of WCW.

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