Quantum criticality and dark matter: part I

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Abstract

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology. Dark matter as a hierarchy of phases of ordinary matter labelled by the value of effective Planck constant h_{eff} following as prediction of adelic physics suggests a general approach to quantum criticality. In the first part of the chapter about quantum criticality general ideas about quantum criticality and phase transitions are discussed.

1 Introduction

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K10, K24].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could it be that criticality is always accompanied by the generation of dark matter? If this is the case, the recipe would be stupifuingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

5. The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

6. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1 Z_2 e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1 Z_2 e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is

proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

The motivation for this work came from super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity.

1.1 Summary about applications of hierarchy of Planck constants to quantum criticality

During years I have proposed several examples about systems to which I have assigned non-standard value of Planck constant $h_{eff} = n \times h$. If the hypothesis about the connection with criticality is correct they should exhibit criticality and if $h_{eff} = h_{gr}$ hypothesis is true, also phase separation. Also the proposed mechanisms to generate dark matter should involve generation of criticality.

1.1.1 Particle Physics

In particle physics there are some possible applications for the new view about dark matter.

1. The perturbative expansion of scattering amplitudes in terms of gauge coupling strength or gravitational coupling strength ceases to converge at some critical value of the coupling parameter. This can be regarded as a critical phenomenon since a transition to strongly coupled phase with different properties takes place. For instance, in gauge theories according to the electric-magnetic duality the magnetic monopoles replaces charged particles as natural basic entities. The original proposal indeed was that the transition to large h_{eff} phase takes place when the perturbation theory in terms of say electromagnetic coupling strength $Z_1Z_2e^2/\hbar c$ ceases to converge. By replacing h with $h_{em} = Z_1Z_2/e^2h_{eff}$ the convergence is achieved and v_0/c replaces gauge coupling strength as coupling constant. A stronger hypothesis is that $h_{eff} = h \times h = h_{em}$ would connect this hypothesis with generalized conformal invariance and its breaking.

2. One of the earliest applications of TGD notion of color (associated not only with quarks and gluons but also leptons through color partial waves) was to explain anomalous production of electron-positron pairs in heavy ion collisions just above the Coulomb wall [C5, C3, C4, C6]. The TGD inspired hypothesis [K22] was that the electron positron pairs result from the decays of leptopions, which are pion-like color singlet bound states of color octet excitations of electron and positron but one could consider also other options. The identification as positronium is excluded since in this case direct decays would not be kinematically possible. The objection against postulating new elementary light particles is that they should make themselves visible in the decay widths of weak bosons.

   One manner to escape the problem is that spartners are heavy so that the decays of weak bosons to spartner pairs are not possible. Another explanation could be that the exotic particles involved correspond to non-standard value of Planck constant. As a matter fact, these particles could be very massive but due to the large value of h_{eff} would appear as effectively massless particles below the scaled-up Compton length.

   One can consider also other identifications for the new particles possibly involved. TGD predicts that right handed covariantly constant neutrino generates $\mathcal{N} = 2$ supersymmetry. An elegant universal explanation for the absence of spartners would be that they are heavy but can make themselves visible as dark variants in scales below scaled up Compton length. Maybe the lepto-electrons are selectrons possibly moving in color octet partial wave!

   This explanation would apply to all elementary elementary particles and predict that these particles can be produced only in critical systems. This would solve the puzzle created by the non-observation of standard $\mathcal{N} = 1$ SUSY and at LHC. Leptopion production indeed takes place at criticality: just above the Coulomb wall, when the incoming nucleus becomes able to collide directly with the target. It should be noticed that there is experimental evidence also for the leptopions associated with muon and tau [K22].

3. RHIC and later LHC found that the de-confinement phase transition (criticality is obviously involved!) supposed to lead to QCD plasma produced something different. The phase in question has long range correlations and exhibits the presence of string like structures decaying to ordinary hadrons. There is also evidence for strong parity breaking in the system and it is involved with the magnetic fields present [C1]. TGD interpretation could be in terms of a criticality in which long range correlations are generated as dark matter is created. Since strong parity breaking is involved, it seems that the dark particles must be associated with the weak length scale characterized by Mersenne prime M_{89}, which characterizes also the "almost-predicted"! scaled up copy of ordinary hadron physics characterize by Mersenne prime M_{107}. The mass scale is 512 times higher than for ordinary hadrons. Due to darkness the Compton scales of M_{89} hadrons and also weak bosons would be scaled up to about M_{107} p-adic scale if $h_{eff}/h = 2^9$ holds true.

1.1.2 Condensed Matter Physics

By its nature condensed matter physics provides rich repertoire of critical phenomena.

1. Different phases of same substance, say water, can be in phase equilibrium at criticality and dark matter. There are critical regions of parameter space -critical lines and critical points, in which the transitions between different phases are possible. Long range thermodynamical correlations are associated with these systems and the association with dark matter would suggest that dark matter could appear in these critical systems.

2. Different substances can form mixtures (`http://tinyurl.com/286nqx`). For instance, oil can mix to water in some parameter regions. This kind of systems are good candidates for critical systems. There is actually rich spectrum of mixtures. Solutions (`http://tinyurl.com/yz3hvq`), colloids (`http://tinyurl.com/yabljt8l`), dispersions (`http://tinyurl.com/bq3vm2m`) and the substances can be also in different phases (gas, liquid, solid) so that very rich spectrum of possibilities emerges. Is the generation of dark matter involved only with the phase transitions between different types of mixed phases or between mixed and non-mixed phase? Are some phases like gel inherently critical?

3. One example about criticality is phase transition to super-fluidity or super-conductivity. In the transition from super-conducitivity the value of specific hearts diverges having the shape of greek letter λ: hence the name lambda point. This suggests that in transition point the specific heat behaves like N^2 due to the quantum coherence instead of proportionality to N as usually. The strange properties of super-fluid, in particular fountain effect, could be understood in terms of $h_{eff} = h_{gr}$ hypothesis as will be discussed.

1.1.3 Living Matter

Biology is full of critical systems and criticality makes living matter highly sensitive to the external perturbations, gives maximal richness of structure, and makes them quantum coherent in macroscopic scales. Therefore it is not difficult to invent examples. The basic problem is whether the criticality is associated only with the transitions between different systems or with the systems themselves.

1. Sols and gels are very important in biology. Sol is definition a micture solid grains and liquid (say blood of cell liquid). Gel involves fixed solid structure and liquid. Sol-gel phase transition of the cell fluid takes place when nerve pulse travels along axon leading to the expansion of the cell. Is the dark phase generated with the sol-gel transition or does it characterized sol. Perhaps the most logical interpretation is that it is involved with the phase transition.

2. Pollack's fourth phase of water resembles gel [L4]. Charge separation implying that the exclusion zones are negatively charged takes place. Charging takes place because part of protons goes to outside of EZ. TGD proposal is that protons go to magnetic flux tubes outside the region or to flux tubes which are considerably larger than EZ that most of their wave functions is located outside the EZ. Is fourth phase is permanently quantum critical? Or is the quantum criticality associated only with the transition so that magnetic flux tubes would carry protons but they would not be dark after the phase transition. EZs have a strange property that impurities flow out of them. Could the presence of dark flux tubes and $h_{eff} = h_{gr}$ forces the separation of particles with different masses?

3. The chirality selection of bio-molecules is a mystery from the point of view of standard physics. Large h_{eff} phase with so large value of Planck constant that the Compton length of weak bosons defines nanoscale, could explain this: weak bosons would be effectively massless and mediate long range interactions below the scaled up Compton scale. This phase transition could also force phases separation if $h_{gr} = h_{eff}$ holds true. If the masses of biomolecules with different handedness are slightly different also the values of h_{gr} would differ and the molecules would go to flux tubes with different value of h_{eff} - at least in the phase transition. The value of $\hbar_{gr} = GMm/v_0$ is in the range $10^{10} - 10^{11}$ for biomolecules so that the $\Delta n/n \simeq \Delta m/m \simeq 10^{-10} - 10^{-11}$ would be needed: this would correspond to an energy of eV which corresponds to the energy scale of bio-photons and visible light.

4. Neuronal membrane could be permanently a critical system since the membrane potential is slightly above the threshold for nerve pulse generation. Criticality might give rise to the dark magnetic flux tubes connecting lipids to the DNA nucleotides or codons assumed in the model of DNA as topological quantum computer. The braiding of the flux tubes would represent the effect of the nerve pulse patterns and would be generated by the 2-D flow of the lipids of the membrane forming a liquid crystal.

1.1.4 Fringe Physics

If one wants the label of crackpot it is enough to study critical phenomena. Those who try to replicate (or usually, to non-replicate) the claimed findings fail (or rather manage) easily since criticality implies careful tuning of the external parameters to demonstrate the phenomenon. Therefore the tragedy of fringe physicist is to become a victim of the phenomenon that he is studying.

1. Cold fusion involves bombarding of target consisting of Palladium target doped with deuterium using hydrogen atoms as projectiles. Cold fusion is reported to occur in a critical range of doping fraction. This suggests quantum criticality and large h_{eff} phase. One of the TGD based models generalizes the model of Widom and Larsen [C2]. The model assumes that weak interactions involving emission of W boson neutralizing the incoming proton makes possible to overcome the Coulomb wall. What would make the system critical? Does criticality make Palladium a good catalyst? Could the Palladium and with a large surface area define nano-scale variant of partonic 2-surface and large area which quite generally would make it effective as catalyst? Certainly this could hold true for bio-catalysts. Could Pd target be permanently in critical state? Effectiveness of catalyst might mean quantum coherence making chemical reaction rates proportional to N^2 instead of N, which could be the number of reactants of particular kind.

2. Di-electric breakdown in given medium occurs when the electric field strength is just above the critical value. A lot of strange claims have been assigned to these systems by non-professionals: in academic environment these phenomena are kind of taboo. Tesla studied them and was convinced that these phenomena involve new physics [K2]. The basic finding was that charges appeared everywhere: this certainly conforms with long range fluctuations and emergence of flux tubes carrying charged particles as dark matter to the environment. Unfortunately, recent day physicist regards Tesla's demonstrations as a mere entertainment and does not bother to ponder whether Maxwell's theory really explains what happens. It is tragic that the greatest intellectual achievements stop thinking for centuries. $h_{gr} = h_{eff}$ hypothesis allows even to estimate the length scales range in which these phenomena should appear.

Ball lightning (http://tinyurl.com/5jxd7k) is also a good candidate for an analogous phenomenon and has been admitted to be a real phenomenon after sixties even by skeptics.

C. Seward has discovered that di-electric breakdowns generate rather stable torus-like magnetic flux tubes around the breakdown current [H1] (http://tinyurl.com/ybdrpqju), which he calls ESTSs (Electron Spiral Toroid Spheromak) and proposed that ball lightnings might correspond to rotating ESTSs.

In TGD framework the stability might be understood if the toroid corresponds to a magnetic flux tube carrying monopole flux. This would allow to understand stability of the configuration and of ball lightning. Monopole flux tubes could also provide a solution to the plasma confinement problem plaguing hot fusion. Also ordinary lightnings involve poorly understood aspect such as gamma and X-ray bursts and high energy electrons. The common mystery is how the dissipation in atmosphere could allow this phenomena. A possible explanation would be in terms of dark flux tubes generated near criticality to the generation of lightning.

3. So called free energy systems [H3] (for TGD inspired view see the book [K21] include many phenomena claimed to involve a liberation of surplus energy. To my opinion, it is quite possible that over-unity energy production is a transient phenomenon and the dreams about final solution of energy problems will not be fulfilled. What makes these phenomena so interesting to me is that they might involve new physics predicted or at least allowed by TGD.

   The splitting of water represents besides magnetic motors (to be discussed below) a key example of free energy phenomena. In the splitting of water to oxygen and hydrogen the formation of Brown's gas [H3] (Wikipedia article about Brown's gas http://tinyurl.com/5tyl92 provides an amusing example full of "fringe science"s about how skeptic writes about something inducing cognitive dissonance in skeptic's mind) with strange properties was reported long time ago. For instance, Brown gas is reported to melt metals whose melting temperature is thousands of degrees although the Brown's gas itself has temperature of order 100 degrees Celsius.

   I have proposed an interpretation as large h_{eff} phase containing dark proton sequences at magnetic flux tubes and responsible for the liberation of energy as this phase transforms to ordinary one. Brown's gas could be essentially the fourth phase of water containing exclusion zones (EZs) discovered by Pollack [L4]. The TGD inspired model for them [L4] involves magnetic flux tubes at which part of protons in EZ is transferred and forms dark proton sequences- essentially dark protons. There a many ways to generate Brown's gas: for instance, cavitation due to the mechanical agitation and application of electric fields could do it. The expanding and compressing bubble created by acoustic wave in sono-luminescence and reported to have a very high temperature and maybe even allowing nuclear fusion, could be also EZ.

4. Water memory [I4, I5, I1] is one of the curse words of skeptic and related to scientific attempts to understand the claimed effects of homeopathy, which defines even stronger curse word in the vocabulary of skeptic - of equal strength as "remote mental interaction". The simple idea that the mere presence of original molecules could be replaced by electromagnetic representation of relevant properties of the molecule is utterly impossible for a skeptic to grasp - despite that also skeptic lives in information society. I have developed a model for water memory explaining also claimed homeopathic effects [K9] and this process has been extremely useful for the development of the model of living matter. Same mechanisms that apply to the model of living matter based on the notion of magnetic body, apply also to water memory and remote mental interactions.

   The key idea is that low energy frequency spectrum provides a representation for the bio-active molecules. The spectrum could be identified as cyclotron frequency spectrum associated with the magnetic bodies of EZs and allow them to mimic the bio-active molecule as far as the effects on living matter are considered. The mechanical agitation of the homeopathic remedy could generate EZs just as it generates cavitation. The model for dark proton sequences yields counterparts of DNA, RNA, amino-acids and even tRNA and genetic code based primitive life would be realized at fundamental particle level with biological realization serving as a higher level representation.

The above sections only list examples about systems where dark matter in TGD sense could appear. A lot of details remain to be understood. The basic question whether some of these systems are permanently near critical state or only in phase transitions between different phases.

1.1.5 Proposed Mechanisms For Generating Large h_{eff} Phase

I have proposed several mechanisms, which might generate large h_{eff} phase, and an interesting question is whether these mechanisms generate criticality.

1. Generation of strong electric fields near criticality for the di-electric breakdown is consistent with criticality and living matter would provide a key example in this respect. Teslas's strange findings support the view about presence of dark matter phases.

2. The findings of Cyril Smith [I3] suggesting a pairing between low and high em frequencies such that low frequency irradiation of bio-matter creates regions to which one can assign high frequency and corresponding wavelength as a size scale. TGD explanation would be that the ratio f_h/f_l of high and low frequencies equals to the $h_{eff}/h = n$, and there is a criticality in the sense that for integer values of this frequency ratio a phase transition transforming dark low energy photons to high frequency of same energy or vice versa can take place. The reverse transition might be interpreted as an analog of Bose-Einstein condensation for low frequency photons (recall the n-fold covering property). The criticality would thus be associated with the formation of the analog of Bose-Einstein condensate.

3. I have proposed that rotating systems could in certain circumstances make a transition to a critical state in which large h_{eff} phase is generated.

   (a) First motivation comes from a model for the findings reported by Russian experimentalists Roschin and Godin [H4] who studied a rotating magnetic system probably inspired by the work of british inventor Searl. The experimenters claim several unexpected effects near criticality for mechanical breakdown of the system. For instance, cylindrical magnetic walls of thickness of few centimeters with distance of order.5 meters are formed. The system starts to accelerate spontaneously. Cooling of the nearby environment is reported. Also visible light probably due to di-electric breakdown - another critical phenomenon - are reported.

   One of the proposed TGD inspired explanations [K3] suggests that there is energy and angular momentum transfer from the magnetic walls which could contain dark matter. Dark photons at cyclotron frequencies but possessing energies of visible photons could make the energy transfer very effective. One possibility is the change of direction for spontaneous dark magnetization emitting large amount of energy. Also collective cyclotron transitions reducing the angular momentum of Bose-Einstein condensate like state can be considered.

   (b) Second motivation comes from the magnetic motor of Turkish inventor Yildiz [H2, H5], which run for hours in a public demonstration. I have developed a model of magnetic motor, which might contain the essential elements of the motor of Yildiz.

   The key idea is that radial permanent magnets generate magnetic monopole flux tubes emanating radially through the stator and rotor returning back along z-axis. Monopole character implies that no current to preserve the magnetic field. This I think is essential.

   If the rotor consist of magnets tangential to a circle, a constant torque is generated. Angular momentum and energy conservation of course requires a feed of energy and angular momentum. If dark matter phase is generated, it could come from some magnetic body containing charged particles with spontaneous magnetization and carrying both spin and energy. Also angular momentum of cyclotron Bose-Einstein condensate can be considered. One possibility is that the dark matter associated with Earth estimated later to be a fraction of about $.2 \times 10 \times -4$ of Earth's mass is the provider of angular momentum and energy. The system is certainly critical in the sense that it is near the mechanical breakdown and in some demonstrations the breakdown has also occurred. This of course raises the possibility that the energy feed comes from mechanical tensions.

(c) Third motivation comes from a model of a rotating system to which constant torque is applied. This situation can be described in terms of potential function $V = \tau\phi$ and modelled using Schrödinger equation [K11]. Since V is not periodic function of ϕ, the solution cannot be periodic if τ lasts forever. It is however possible to have a situation in which the duration T of τ is finite. In this case one can consider the possibility that the phase space which is in the simplest situation circle is replaced with its n-fold covering and solutions are periodic with period $n \times 2\pi$ during the period T and before it energy eigenstates for a free system. The average energy for the final state would be differ from that for the initial state and the difference would be the energy fed to the system equal to $\Delta E = \tau\Delta\phi$ classically. During energy feed the systems wave functions have $1/n$-fractional angular momenta unless one assumes $h_{eff} = n \times h$ phase.

What is intriguing that also stationary solutions are obtained: the equation reduces to that for Airy functions in this case. These solutions do not however satisfy periodicity condition for any finite n. Solutions located in a finite covering of circle cannot be energy eigenstates. Could the constancy of energy mean that no dissipation takes place and no energy is feed to the system.

This description brings in mind the general view about large h_{eff} phases as being associated with the breaking of conformal invariance. n could characterize the number of sheets of the covering of S^2. What does criticality correspond to now? Why should angular momentum and energy feed require or imply criticality? There is also a criticality associated with the change of n as the minimum number of periods that τ lasts. If this is the correct identification, the value of n would increase after every turn in positive energy ontology. In ZEO it would be pre-determined and determined by the duration of τ.

The motivation for the model comes from the ATPase molecule (`http://tinyurl.com/y9jxsvr5`), which is a basic tool in energy metabolism. ATPase can be regarded as a molecular motor taking its energy from the change of the energy of protons as they flow through the cell membrane. Three ADPs are transformed to ATP during single turn by giving them phosphate molecule. What could make the system critical? The system in question is not neuronal membrane but there is tendency to consider the possibility that also the mitochondrial membrane potential is near to breakdown value and the flow of protons through it is the counterpart for nerve pulse.

4. TGD inspired model [L3] for the recent findings about microtubules by the group of the group of Bandyonopadhyay. [J1, J3] is based on the assumption that the oscillatory em perturbation of the system induces generation of A type microtubules not present in Nature by a phase transition from B type microtubules. This phenomenon would take for a critical frequency and $f_h/f_l = n$ condition is suggestive. The proposal is that large h_{eff} phase is generated and gives rise to long range correlations at the level of microtubule so that 13-tubulin units combine to form longer units and the broken helical symmetry becomes un-broken symmetry. Quite recently also an observation of short lasting (nanoseconds) super-conductivity at room temperature (`http://tinyurl.com/prvjp6y`) induced by irradiation of high temperature super conductor with infrared light. The mechanism could be similar and involve $f_h/f_l = n$ condition.

In the first part of the chapter the general ideas about quantum criticality and phase transitions inspired by the TGD view about dark matter are discussed with some general applications. A good example is ferromagnetism and the phase transition to ferromagnetic state. In the remaining 3 parts more specific applications, mostly to various anomalies, are discussed.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L1].

2 Criticality In TGD Framework

In the following the proposal that quantum criticality or even criticality (with thermodynamical criticality included) could in TGD framework correspond to phase transition generating dark matter identified as phases of the ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant and residing at dark magnetic flux tubes is discussed.

The precise meaning of quantum criticality has remained frustratingly fuzzy since the long range fluctuations and possible quanta associated with them do not correspond to any of the co-existing phases naturally but rather to transitions between them. Here Zero Energy Ontology (ZEO) in which basic objects are time evolutions suggests an elegant description: the ends of space-time surfaces at opposite boundaries of CD correspond to different values of h_{eff}. This would also give a connection with inclusions of hyper-finite factors: the integer m characterizing the inclusion equals to the radion $m = h_{eff}(f)/h_{eff}(i)$ of Planck constants for final and initial phases.

2.1 Mathematical Approach To Criticality

Concerning the understanding of criticality one can proceed purely mathematically. Consider first 2-dimensional systems and 4-D conformal invariance of Yang-Mills theories.

1. In 2-dimensional case the behavior of the system at criticality is universal and the dependence of various parameters on temperature and possible other critical parameters can be expressed in terms of critical exponents predicted in the case of effectively 2-dimensional systems by conformal field theory discovered by Russian theoreticians Zamolodchikov, Polyakov and Belavin [B1]. To my opinion, besides twistor approach this is one of the few really significant steps in theoretical physics during last forty years.

2. Twistors discovered by Penrose relate closely to 4-D conformal invariance generalized to Yangian symmetry [A1] [B4, B2, B3] in the approach developed by Nima Arkani-Hamed and collaborators recently. 2-dimensional conformal field theories are relatively well-understood and classified. String models apply the notions and formalism of conformal field theories.

3. The notion of conformal symmetry breaking emerges from basic mathematics and is much deeper than its variant based on Higgs mechanism able to only reproduce the mass spectrum but not to predict it: in p-adic thermodynamics based on super-conformal invariance prediction becomes possible [K14].

2.1.1 Basic Building Bricks Of TGD Vision

The big vision is that 2-D conformal invariance generalizes to 4-D context [K5, K10] and the conjecture is that it can be extended to Yangian symmetry assignable - not to finite-D conformal algebra of Minkowski space - but to the infinite-D generalization of 2-D conformal algebra to 4-D context. The details of this generalization are not understood but the building bricks have been identified.

1. One building brick is the infinite-D group of symplectic symmetries of $\delta M^4 - + \times CP_2$ having the structure of conformal algebra but the radial light-like coordinate r_M of δM^4_+ replacing complex coordinate z: r_M presumably allows a continuation to a hyper-complex analog of complex coordinate. One can say that finite-D Lie algebra defining Kac-Moody algebrais replaced with an infinite-D symplectic algebra of $S^2 \times CP_2$ and made local with respect to r_M

2. Second building brick is defined by the conformal symmetries of S^2 depending parametrically on r_M and are due to metric 2-dimensionality of δM^4_+. These symmetries are possible only in 4-D Minkowski space. The isometry algebra of δM^4_+ is isomorphic with that of ordinary conformal transformations (local radial scaling compensates the local conformal scaling).

3. Light-like orbits of the partonic 2-surfaces have also the analog of the extended conformal transformations as conformal symmetries and respect light-likeness.

4. At least in space-time regions with Minkowskian signature of the induced metric spinor modes are localized to string 2-D world sheets from the condition that electric charge is well-defined for the modes. This guarantees that weak gauge potentials are pure gauge at string world sheets and eliminates coupling of fermions to classical weak fields which would be a strong arguments against the notion of induced gauge field. Whether string world sheets and partonic 2-surfaces are actually dual as far as quantum TGD is considered, is still an open question.

The great challenge is to combine these building bricks to single coherent mathematical whole. Yangian algebra, which is multi-local with locus generalized from a point to partonic 2-surface would be the outcome. Twistors would be part of this vision: M^4 and CP_2 are indeed the unique 4-D manifolds allowing twistor space with Kähler structure [A2]. Number theoretic vision involving classical number fields would be part of this vision. 4-dimensionality of space-time surfaces would follow from associativity condition stating that space-time surfaces have associative tangent - or normal space as surfaces in 8-D embedding space endowed with octonionic tangent space structure. 2-dimensionality of the basic dynamical objects would follow from the condition that fundamental objects have commutative tangent - or normal space. String world sheets/partonic 2-surfaces would be commutative/co-commutative or vice versa.

2.1.2 Hierarchy Of Criticalities And Hierarchy Breakings Of Conformal Invariance

The TGD picture about quantum criticality connects it to the failure of classical non-determinism for Kähler action defining the space-time dynamics. A connection with the hierarchy of Planck constants [K8] and therefore dark matter in TGD sense emerges: the number n of conformal equivalence classes for space-time surfaces with fixed ends at the boundaries of causal diamond corresponds to the integer n appearing in the definition of Planck constant $h_{eff} = n \times h$.

A more detailed description for the breaking of conformal invariance is as follows. The statement that sub-algebra V_n of full conformal algebra annihilates physical states means that the generators L_{kn}, $k > 0$, $n > 0$ fixed, annihilate physical states. The generators L_{-kn}, $k > 0$, create zero norm states. Virasoro generators can be of course replaced with generators of Kac-Moody algebra and even those of the symplectic algebra defined above.

Since the action of generators L_m on the algebra spanned by generators $Ln + m$, $m > 0$, does not lead out from this algebra (ideal is in question), one can pose a stronger condition that all generators with conformal weight $k \geq n$ annihilate the physical states and the space of physical states would be generated by generators L_k, $0 < k < n$. Similar picture would hold for also for Kac-Moody algebras and symplectic algebra of $\delta M^4_+ \times CP_2$ with light-like radial coordinate of δM^4_+ taking the role of z. Since conformal charge comes as n-multiples of $\hbar$, one could say that one has $h_{eff} = n \times h$.

The breaking of conformal invariance would transform finite number of gauge degrees to discrete physical degrees of freedom at criticality. The long range fluctuations associated with criticality are potentially present as gauge degrees of freedom, and at criticality the breaking of conformal invariance takes place and these gauge degrees of freedom are transformed to genuine degrees of freedom inducing the long range correlations at criticality.

Changes of symmetry are assigned with criticality since Landau. Could one say that the conformal subalgebra defining the genuine conformal symmetries changes at criticality and this makes the gauge degrees of freedom visible at criticality?

2.1.3 Emergence Of The Covering Spaces Associated With The Hierarchy Of Planck Constants

The original vision was that the hierarchy of Planck constants corresponds to a hierarchy of n-fold singular coverings of the embedding space - or more precisely given causal diamond (CD) forming a book-like structure with pages labelled by the effective value of Planck constant $h_{eff}/h = n$. This view allowed to understand the basic aspects of the hierarchy: in particular, the relative darkness of phases associated with different values of n. The generalization of embedding space is however un-necessary. The non-determinism of Kähler action allows to replace singular coverings of embedding space with the identification of space-time surfaces with their singular coverings.

Space-like 3-surfaces at the opposite boundaries of CD are connected by a multi-sheeted covering with sheets co-inciding at the ends.

How does this picture relate to the breaking of conformal symmetry? The idea is simple. One goes to n-fold covering space by replacing z coordinate by $w = z^{1/n}$. With respect to the new variable w one has just the ordinary conformal algebra with integer conformal weights but in n-fold singular covering of complex plane or sphere. Singularity of the generators explains why $L_k(w)$, $k < n$, do not annihilate physical states anymore. Sub-algebra would consist of non-singular generators and would act as symmetries and also the stronger condition that Lk, $k \geq n$, annihilates the physical states could be satisfied. Classically this would mean that the corresponding classical Noether charges for Kähler action are non-vanishing.

Another manner to look the same situation is to use z coordinate. Now conformal weight is fractionized as integer multiples of $1/n$ and since the generators with fractional conformal weight are singular at origin, one cannot assume that they annihilate the physical states: fractional conformal invariance is broken. Quantally the above conditions on physical states would be satisfied. Sphere - perhaps the sphere assigned with the light-cone boundary or geodesic sphere of CP_2 - would be effectively replaced with its n-fold covering space, and due to conformal invariance one would have n additional discrete degrees of freedom.

These discrete degrees of freedom would define n-dimensional Hilbert space space by the n fractional conformal generators. One can also second quantize by assigning oscillator operators to these discrete degrees of freedom. In this picture the effective quantization of Planck constant would result from the condition that conformal weights for the physical states are integers.

2.1.4 Other Connections

The values of effective Planck constants seems to have profound connections to several key ideas of TGD.

1. As already found, the connection with the hierarchy of broken conformal symmetries is highly suggestive. The integer $h_{eff}/h = n$ would characterize the sub-algebra of gauge conformal symmetries.

2. There seems to be a connection with negentropic entanglement [K13] associated with the density matrix of the state resulting in state function reduction, which is proportional to unit matrix - projector to an eigen space of density matrix. Negentropic entanglement would occur in the new discrete degrees of freedom most naturally. In the special 2-particle case negentropic entanglement corresponds to unitary entanglement encountered in quantum computation: large h_{eff} makes possible long-lived entanglement and its negentropic character implies that Negentropy Maximization Principle [K13] favors its generation. An interesting hypothesis to be killed is that the p-adic prime characterizing the space-time sheet string world sheet or partonic 2-surface divides n.

3. The realization of number theoretic univarsality in terms of strong form holography assumes that string world sheets and partonic 2-surfaces serve as "space-time genes" allowing continuation to preferred extremals. These 2-surfaces are characterized by parameters, which belong to an extension of rationals inducing extensions of p-adic number fields. One has a hierarchy of extensions of increasing complexity. Given extension is characterized by preferred primes known as ramified primes with the property that their decomposition to a products of of primes of extension contains higher powers than one. The product n of ramified rational primes characterizes the extension and is an integer.

   p-Adic continuations identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K15]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. NMP implies a generalization of p-adic length scale hypothesis stating that primes near but below powers of prime are physically favored and thus selected in number theoretic evolution.

$h_{eff}/h = n$ gives the number of sheets of covering and a more plausible identification is as the dimension of covering assignable to number theoretic discretization of space-time surface [L7]. This dimension is the dimension of Galois group for the extension of rationals or its factor is highly suggestive and would lead to a direct connection with the number theoretic view about evolution.

2.1.5 Hierarchy of Planck constants, space-time surfaces as covering spaces, and adelic physics

From the beginning it was clear that $h_{eff}/h = n$ corresponds to the number of sheets for a covering space of some kind. First the covering was assigned with the causal diamonds. Later I assigned it with space-time surfaces but the details of the covering remained unclear. The final identification emerged only in the beginning of 2017.

Number theoretical universality (NTU) leads to the notion of adelic space-time surface (monadic manifold) involving a discretization in an extension of rationals defining particular level in the hierarchy of adeles defining evolutionary hierarchy. The first formulation was proposed in [K25] and more elegant formulation in [L7].

The key constraint is NTU for adelic space-time containing sheets in the real sector and various padic sectors, which are extensions of p-adic number fields induced by an extension of rationals which can contain also powers of a root of e inducing finite-D extension of p-adic numbers (e^p is ordinary p-adic number in Q_p).

One identifies the numbers in the extension of rationals as common for all number fields and demands that embedding space has a discretization in an extension of rationals in the sense that the preferred coordinates of embedding space implied by isometries belong to extension of rationals for the points of number theoretic discretization. This implies that the versions of isometries with group parameters in the extension of rationals act as discrete versions of symmetries. The correspondence between real and p-adic variants of the embedding space is extremely discontinuous for given adelic embedding space (there is hierarchy of them with levels characterized by extensions of rationals). Space-time surfaces typically contain rather small set of points in the extension ($x^n + yn^2 = z^n$ contains no rationals for $n > 2!$). Hence one expects a discretization with a finite cutoff length at space-time level for sufficiently low space-time dimension $D = 4$ could be enough.

After that one assigns in the real sector an open set to each point of discretization and these open sets define a manifold covering. In p-adic sector one can assign 8:th Cartesian power of ordinary p-adic numbers to each point of number theoretic discretization. This gives both discretization and smooth local manifold structure. What is important is that Galois group of the extension acts on these discretizations and one obtains from a given discretization a covering space with the number of sheets equal to a factor of the order of Galois group.

$h_{eff}/h = n$ was identified from the beginning as the dimension of poly-sheeted covering assignable to space-time surface. The number n of sheets would naturally a factor of the order of Galois group implying $h_{eff}/h = n$ bound to increase during number theoretic evolution so that the algebraic complexity increases. Note that WCW decomposes into sectors corresponding to the extensions of rationals and the dimension of the extension is bound to increase in the long run by localizations to various sectors in self measurements [K13]. Dark matter hierarchy represents number theoretical/adelic physics and therefore has now rather rigorous mathematical justification. It is however good to recall that $h_{eff}/h = n$ hypothesis emerged from an experimental anomaly: radiation at ELF frequencies had quantal effects of vertebrate brain impossible in standard quantum theory since the energies $E = hf$ of photons are ridiculously small as compared to thermal energy.

Indeed, since n is positive integer evolution is analogous to a diffusion in half-line and n unavoidably increases in the long run just as the particle diffuses farther away from origin (by looking what gradually happens near paper basket one understands what this means). The increase of n implies the increase of maximal negentropy and thus of negentropy. Negentropy Maximization Principle (NMP) follows from adelic physics alone and there is no need to postulate it separately. Things get better in the long run although we do not live in the best possible world as Leibniz who first proposed the notion of monad proposed!

2.2 Phenomenological Approach To Criticality

These statements do not have any obvious content for an experimentalist. One should have also a more concrete view about criticality. Theoretician would call this phenomenology.

1. Phase transitions and criticality are essential piece of being alive. Criticality means high sensitivity to signals and makes sensory perception possible. Criticality implies also long range correlations making us coherent units. The long range correlations between people who have never seen each other, like most of us, make possibly society, and demonstrate that the criticality appears also at collective levels of life and consciousness: usually biologists dismiss this. For physicist - at least me - the correlation between behaviors of him and his cat looks like a miracle!

2. Self-organization takes place by phase transitions and criticality with long range correlations. In zero energy ontology (ZEO) self-organisation is however self-organisation for entire temporal patterns of space-time dynamics characterised by the 3-surfaces at the ends of causal diamond so that behaviours rather than states emerge. Also the synergy is made possible by criticality.

3. Criticality appears only in a very narrow range of control parameters and is therefore difficult to produce critical systems tend to fall off from criticality: good example is our society which is all the time at the verge of some kind of catastrophe.

One can build refined and highly predictive conformal field theory models but they do not tell what are the microscopic mechanisms behind criticality.

1. What are the space-time correlates for criticality and long range correlations? Something must quite concretely connect the sub-systems, bind them to single coherent unit at criticality. Magnetic flux tubes is of course the TGD based answer! But this is not enough. The long range correlations must be quantal and this requires that Planck constant is large: $h_{eff} = n/times; h$! Dark matter! The emergence of dark matter phase makes system critical! TGD Universe is critical at fundamental level and this implies that this dark matter is present at all length scales.

2. Long range interactions certainly define a basic characteristic of criticality. How do they emerge? Does some universal mechanism exist? $h_{eff} = n \times h$ hypothesis and p-adic length cale hypothesis allow to understand this. Weak bosons are effectively massless below weak boson Compton length - about 10^{-17} meters. When h_{eff} is scaled up by n, this Compton length is scaled up by n too. Weak interactions would become long ranged below much longer length scale, say even cellular scale and among other things explain chiral selection of biomolecules. Similar argument can be carried out for gluons and dark/p-adically scaled down) quarks and gluons would also appear in living matter.

3. Phase separation is key feature of criticality. How does this separation take place? Is there a universal mechanism as suggested by the fact that at criticality everything is universal. The answer relies on the notion of many-sheeted space-time, $h_{eff} = n \times h$ hierarchy, and the notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0$ introduced originally by Nottale [**?**]. The additional hypothesis [K20]

$$h_{eff} = h_{gr}$$

brings in gravitational interaction: the gravitational Planck constant is assigned with gravitation mediated by magnetic flux tubes connecting the two dark systems. The hypothesis predicts that h_{eff} is proportional to particle mass. This means each particle type is at its own dark flux tube/quantum nicely separated from each other. This would explain the phase separation at criticality even if the phase transformed after criticality to ordinary $h_{eff} = h$ phase. Pollack's exclusion zones (EZs) [L4] show the effect too: charge separation occurs and impurities in EZ get put of it. $h_{eff} = h_{gr}$ hypothesis implies that the scaled up Compton length becomes $\lambda_{gr} = GM/v_0$ and does not depend on particle mass at all: and ideal outcome concerning collective quantum coherence. In living matter with dynamics characterized by phase transitions this phase separation of different biologically important molecules would be in crucial role. The cell would not be anymore a random soup of huge number of different biomolecules but nicely arranged archive.

Critical reader - and even me after 9 ears of work! - can of course ask what the mass M appearing in the formula for h_{gr} really is. The logical answer is that it is the portion of matter that is dark: to this dark particles couple. In the Nottale's original model M and in TGD generalization of this model M corresponds to the entire mass of say Sun. This makes sense only if the approximate Bohr orbits in solar system reflect the situation when most of the matter in solar system was dark. Nowadays this is not the case anymore. For Earth the portion of dark matter in TGD sense should be something like 4×10^{-4} as becomes clear by just looking the values of the energies associated with dark cyclotron photons and requiring that they are in the range of bio-photon energies (dark photons would transforming to ordinary photons produce bio-photons). Without this assumption the range of bio-photon energies would be above 40 keV.

Besides dark matter also p-adically scaled up variants of weak interaction physics are possible: now weak bosons would be light but not massless above the Compton length which would be scaled up. In the TGD based model of living matter both dark matter and p-adically scaled up variants of particles appear and both are crucial for understanding metabolism. Both kind of phases could appear universally in critical systems. Dark matter would be a critical phenomenon and appear also in thermodynamical phase transitions, not only in quantum phase transitions.

Also so called free energy phenomena, cold fusion, remote mental interactions, etc are critical phenomena and therefore very difficult to replicate unless one knows this so that it is very easy to label researchers of these phenomena crackpots. The researchers in these fields could be seen as victims of the phenomenon they are studying! Life of course is also a critical phenomenon but even the vulgar skeptics are living and conscious beings and usually do not try to deny this!

2.3 Do The Magnetic Flux Quanta Associated With Criticality Carry Monopole Flux?

TGD allows the possibility that the magnetic flux quanta associated with criticality carry monopole flux. In Maxwellian electrodynamics this is not possible. These flux tubes are associated with elementary particles: in this case they have open string like portions at parallel space-time sheets connected at their ends by wormhole contacts to form a closed two-sheeted loop. Since the magnetic monopole flux is conserved along the flux tube, one has full reason to wonder whether these closed magnetic flux tubes can be created from vacuum.

One can imagine two ways to create flux loops: in a continuous energy conserving manner classically or by quantum jump in which quantum sub-Universe associated with given causal diamond (CD) is re-created (recall that causal diamonds define the observable Universes and they have finite size as intersections of future and past directed light-ones)

Consider for simplicity flux tubes which are circular. How the flux tubes can be generated?

1. One possibility is that an existing circular flux tube splits into two. This would take place by self-reconnection: circular flux tubes evolves first a figure eight shape, and after that self-reconnects and splits to two circular flux tubes. Figure eight shape is necessary because the direction of the conserved magnetic flux defines orientation and flux tube portions with opposite orientations cannot join. This mechanism allows replication of flux tubes and could be behind the $1 \rightarrow 2$ decays of elementary particles and the reverse reactions. It could be also behind biological replication at both DNA and cell level, and even higher levels. The reconnection of U-shaped flux tubes for two systems so that they become connected by a pair of flux tubes is the reverse of this process and is proposed to define fundamental mechanism of directed attention.

2. Can one imagine a purely classical mechanism in which flux tubes would be generated from nothing? An idealization as a closed string allows to imagine a closed string which begins from point and expands: in string models this kind of closed strings indeed pop up from vacuum. Energy conservation however forbids the classical occurrence of this process. Therefore this

process is possible only in path integral formalism which allows processes, which are classically impossible.

In TGD framework space-time surfaces appearing in the functional integral are extremals of Kähler action and conserve energy so that this kind of process is impossible. It is difficult to say what happens when the string is replaced with a flux tube having a a finite thickness: could this make it possible an energy conserving process in which initial state would not contain flux tubes but final would contain flux tubes? At elementary particle level this would mean generation of a particle or a pair from vacuum but this does not take place. Note that the development of Higgs expectation can be interpreted as generation of new vacuum state which contains Higgs bosons: TGD counterpart of the ground state would be a superposition of states containing various numbers of flux loops.

3. One can however consider a *quantum jump* generating flux tube from nothing. The sequence of quantum jumps consist of sub-sequences consisting of state function reductions to a fixed boundary of CD ("upper" or "lower"). A sub-sequence defining self corresponds to a sequence of repeated quantum measurements having no effect on the state in ordinary quantum measurement theory. In TGD state function reduction has effect on the second boundary. Or to be precise, on the wave function in the moduli space associated with the second boundary with moduli characterising among other things the temporal distance from the fixed boundary. This effect gives rise to the experienced flow of time as increase of the average temporal distance between the tips of CD and also to its arrow.

   These state function sequences do not last for ever (self has finite lifetime!): Negentropy Maximization Principle (NMP) eventually forces state function reduction at the opposite boundary of CD. The new state can contain flux loops which did not exist in the initial state. These flux loops could exists also outside the CD but this is not relevant for the physics experienced by the conscious observer associated with given CD.

   The generation of this kind of monopole flux loops from nothing could be seen as a direct proof for macroscopic quantum jumps re-creating the Universe. Penrose proposed something similar in Shadows of Mind: quasicrystals are non-periodic lattices which look like lattices but - unlike ordinary crystals - cannot be generated by gradual lattice growth but must pop up in quantal manner to existence.

3 What's New In TGD Inspired View About Phase Transitions?

The comment of Ulla mentioned Kosterlitz-Thouless phases transition and its infinite order. I am not a condensed matter physicist so that my knowledge and understanding are rather rudimentary and I had to go to Wikipedia (see `http://tinyurl.com/ybevezgf`). I realized that I have not payed attention to the classification of types of phase transitions, while speaking of quantum criticality [?]. Also the relationship of ZEO inspired description of phase transitions to that of standard positive energy ontology has remained poorly understood. In the following I try to represent various TGD inspired visions about phase transitions and criticality in organized manner and relate them to the standard description.

3.1 About Thermal And Quantum Phase Transitions

It is good to beging with something concrete. Wikipedia article lists examples about different types of phase transitions. These phase transitions are thermodynamical.

1. In first order phase thermodynamical phase transitions heat is absorbed and phases appear as mixed. Melting of ice and boiling of water represent the basic examples. Breaking of continuous translation symmetry occurs in crystallization and symmetry is smaller at low temperature. One speaks of spontaneous symmetry breaking: thermodynamical fluctuations are not able to destroy the configuration breaking the symmetry.

2. Second order phase transitions are also called continuous and they also break continuous symmetries. Susceptility diverges, correlation range is infinite, and power-law behaviour applies to correlations. Ferromagnetic, super-conducting, and superfluid transitions are examples. Conformal field theory predics power-law behavior and infinite correlation length. Infinite susceptility means that system is very sensitive to external perturbations. First order phase transition becomes second order transition at critical point. Here the reduction by strong form of holography might make sense for high Tc superconductors at least (they are effectively 2-D).

3. Infinite order phase transitions are also possible. Kosterlitz-Thouless phase transition occurring in 2-D systems allowing conformal symmetries represents this kind of transition. These phase transitions are continuous but do not break continuous symmetries as usually.

4. There are also liquid-glass phase transitions. Their existence is hypothetical. The final state depends on the history of transition. Glass state itself is more like an on going phase transition rather than phase.

These phase transitions are thermal and driven by thermal fluctuations. Also quantum phase transitions (see `http://tinyurl.com/yblptwr6`) are possible.

1. According to the standard definition they are possible only at zero temperature and driven by quantum fluctuations. For instance, gauge coupling strength would be analogous to quantum temperature. This is a natural definition in standard ontology, in which thermodynamics and quantum theory are descriptions at different levels.

   Quantum TGD can be seen as a square root of thermodynamics in a well-defined sense and it makes possible to speak about quantum phase transitions also at finite temperature if one can identify the temperature like parameter characterizing single particle states as a kind of holographic representations of the ordinary temperature.

2. The traces of quantum phase transitions are argued to be visible also at finite temperatures if the energy gap is larger than the thermal energy: $\hbar\omega >\geq T$. In TGD framework Planck constant has a spectrum $h_{eff}/h = n$ and allows very large values. This allows quantum phase transitions even at room temperature and TGD inspired quantum biology relies crucially on this. What is of special interest that also ordinary thermal phase transitions might be accompanied by quantum phase transitions occurring at the level of magnetic body and perhaps even inducing the ordinary thermal phase transition.

3. Quantum critical phase transitions occur at critical point and are second order phase transitions so that susceptibility diverges and system is highly sensitive to perturbations and so in wide range around critical temperature (zero in standard theory). Long range fluctuations are generated and this conforms with the TGD vision about the role of large h_{eff} phases and generalized conformal symmetry: which also implies that the region around criticality is wide (exponentially decaying correlations replaced with power law correlations).

3.2 Some Examples Of Quantum Phase Transitions In TGD Framework

TGD suggests some examples of quantum phase transition like phenomena.

1. Bose-Einstein (BE) condensate consisting of bosons in same state would represent a typical quantum phase. I have been talking a lot about cyclotron BE condensates at dark magnetic flux tubes [K12, K17, K18]. The bosonic particles would be in the same cyclotron state. One can consider also the analogs of Cooper pairs with members at flux tubes of a pair of parallel flux tubes with magnetic fields in same or opposite direction. One member at each tube having spin 1 or zero. This would give rise to high T_c superconductivity.

2. One natural mechanism of quantum phase transition would be condensation to a new single particle state. The rate for an additions of new particle to condensate is proportional to $N + 1$ and disappearance of particle from it to N, where N is the number of particles in condensate. The net rate for BE condensation is difference of these and non-vanishing.

Quantum fluctuations induce phase transition between states of this condensate at criticality. For instance, cyclotron condensate could make a spontaneous phase transition to a lower energy state by a change of cyclotron energy state and energy would be emitted as a dark cyclotron radiation. This kind of dark photon radiation could in turn induce cyclotron transition to a higher cyclotron state at some other flux tube. If NMP holds true it could pose restrictions for the occurrence of transitions since one expects that negentropy is reduced. The transitions should involve negentropy transfer from the system.

The irradiation of cyclotron BE condensate with some cyclotron frequency could explain cyclotron phase transition increasing the energy of the cyclotron state. This kind of transition could explain the effecs of ELF em fields on vertebrate brain [J2] in terms of cyclotron phase transition and perhaps serving as a universal communication and control mechanism in the communications of the magnetic body with biological body and other magnetic bodies [K6]. The perturbation of microtubules by an oscillating voltage [J1] (see `http://tinyurl.com/ze366ny`) has been reported by the group of Bandyonophyay [J3] to induce what I have interpreted as quantum phase transition [L6] (see `http://tinyurl.com/yatfreqe`).

External energy feed is essential and dark cyclotron radiation or generalized Josephson radiation from cell membrane acting as generalized Josephson junction and propagating along flux tubes could provide it. Cyclotron energy is scaled up by h_{eff}/h and would be of the order of biophoton energy in TGD inspired model of living matter and considerably above thermal energy at physiological temperature.

3. Also quantum phase transitions affecting the value of h_{eff} are possible [K16] When h_{eff} is reduced and frequency is not changed, energy is liberated and the transition proceeds without external energy feed (NMP might pose restrictions). Another option is increase of h_{eff} and reduce the frequency in such a way that single particle energies are not changed. One can imagine many other possibilities since also p-adic length scale leading to a change of mass scale could change. A possible biological application is to the problem of understanding how biomolecules find each other in the molecular soup inside cell so that catalytic reactions can proceed. Magnetic flux tubes pairs connecting the biomolecules would be generated in the reconnection of U-shaped tentacle like flux tubes associated with the reactants, and the reduction of h_{eff} for the flux tube pair would contract it and force the biomolecules near each other.

4. The model for cold fusion in TGD Universe relies on a process, which is analogous to quantum phase transition [L5] [K4]. Protons from the exclusion zones (EZs) of Pollack [L4] [L4] are transferred to dark protons at magnetic flux tubes outside EZ and part of dark protons sequences transform by dark weak decays and dark weak boson exchanges to neutrons so that beta stable dark nuclei are obtained with binding energy much smaller than nuclear binding energy. This could be seen as dark nuclear fusion and quantum analog of the ordinary thermal nuclear fusion. The transformation of dark nuclei to ordinary nuclei by h_{eff} reducing phase transition would liberate huge energy if allowed by NMP [K13] and explain the reported biofusion.

5. Energetics is clearly an important factor (in ordinary phase transitions for open system thermal energy feed is present). The above considerations assume that ordinary positive energy ontology effectively applies. ZEO [K13] allows to consider a more science fictive possibility. In ZEO energy is conserved when one considers single zero energy state as a time evolution of positive energy state. If single particle realizes square root of thermodynamics, one has superposition of zero energy states for which single particle states appear as pairs of positive and negative energy states with various energies: each state in superposition respects energy conservation. In this kind of situation one can consider the possibility that temperature increases and average single particle energy increases. In positive energy ontology this is impossible without energy feed but in ZEO it is not excluded. I do not understand the situation well enough to decide whether some condition could prevent this. Note however that in TGD inspired cosmology energy conservation holds only in given scale (given CD) and apparent energy non-conservation would result by this kind of mechanism.

3.3 ZEO Inspired View About Phase Transitions

This section begins with questions related to TGD based description of phase transitions, discusses the TGD view about the role of symmetries in phase transitions, and asks what new ZEO can give to the description of phase transitions.

3.3.1 Question related to TGD inspired description of phase transitions

The natural questions are for instance following ones.

1. The general classification of thermodynamical phase transitions is in terms of order: the order of the lowest discontinuous derivative of the free energy with respect to some of its arguments. In catastrophe theoretic description one has a hierarchy of criticalities of free energy as function of control variables (also other behavior variables than free energy are possible) and phase transitions with phase transitions corresponding to catastrophe containing catastrophe.... such that the order increases. For intance, for cusp catatrophe one has lambda-shaped critical line and critical point at its tip. Thom's catastrophe theory description is mathematically very attractive but I think that it has problems at experimental side. It indeed applies to flow dynamics defined by a gradient of potential and thermodynamics is something different.

   In TGD framework the sum of Kähler function defined by real Kähler action in Euclidian space-time regions and imaginy Kähler action from Minkowskian space-time regions defining a complex quanty replaced free energy. This is in accordance with the vision that quantum TGD can be seen as a complex square root of thermodynamics. Situation is now infinite-dimensional and catastrophe set would be also infinite-D. The hierarchy of isomorphic super-conformal algebras defines an infinite hierarchy of criticalities with levels labelled by Planck constants and catastrophe theoretic description seems to generalize.

   Does this general description of phase transitions at the level of dark magnetic body (field body is more general notion but I will talk about magnetic body (MB) in the sequel) allow to understand also thermodynamical phase transitions as being induced from those for dark matter at MB?

2. Quantum TGD can be formally regarded as a square root of thermodynamics. Does this imply "thermal holography" meaning that single particle states can represent ensemble state as square root of the thermal state of ensemble. Could one unify the notions of thermal and quantum phase transition and include also the phase transitions changing h_{eff}? Could MB make this possible?

3. How does the TGD description relate to the standard description? TGD predicts that conformal gauge symmetries correspond to a fractal hierarchy of isomorphic conformal sub-algebras. Only the lowest level with maximal conformal symmetry matters in standard theory. Are the higher "dark" levels something totally new or do they appear in the description of also ordinary phase transitions? What is the precise role of symmetries and symmetry changes in TGD description and is this consistent with standard description. Here the notion of field body is highly suggestive: the dynamics of field body could induce the dynamics of ordinary matter also in phase transitions.

There is a long list of questions related to various aspects of TGD based description of phase transitions.

1. In TGD framework NMP applying to single system replaces second law applying to ensemble as fundamental description. Second law follows from the randomness of the state function reduction for ordinary matter and in long length and time scales from the ultimate occurrence of state function reductions to opposite boundary of CD in ensemble. How does this affect the description of phase transitions? NMP has non-trivial implications only for dark matter at MB since it NMP does favor preservation and even generation of negentropic entanlement (NE). Does NMP imply that MB plays a key role in all phase transitions?

2. Does strong form of holography of TGD reduce all transitions in some sense to this kind of 2-D quantum critical phase transitions at fundamental level? Note that partonic 2-surfaces can be seen as carriers of effective magnetic charges and string world sheets carrying spinor modes accompany magnetic flux tubes. Could underlying conformal gauge symmetry and its change have practical implications for the description of all phase transitions, even 3-D and thermodynamical phase transitions?

3. Could many-sheetedness of space-time - in particular the associated p-adic length scale hierarchy - be important and could one identify the space-time sheets whose dynamics controls the transition? Could the fundamental description in terms of quantum phase transitions relying on strong form of holography apply to all phase transitions? Could dark phases at MB be the key to the description of also ordinary thermodynamical phase transitions? Could one see dark MB as master and ordinary matter as slave and redue the description of all phase transitions to dark matter level.

   Could the change of h_{eff} for dark matter at field body accompany any phase transition - even thermodynamical - or only quantum critical phase transition at some level in the hierarchy of space-time sheets? Or are also phase transitions involving no change of h_{eff} possible? Do ordinary phase transitions correspond to these. What is the role of h_{eff} changing "transitons" and their dynamical symmetries?

4. The huge vacuum degeneracy of Kähler action implies that any space-time surface with CP_2 projection that is Lagrangian manifold and has therefore dimension not larger than two, is vacuum extremal. The small deformations of these vacuum extremals define preferred extremals. One expects that this vacuum degeneracy implies infinite number of ground states as in the case of spin glass (magnetized system consisting of regions with different direction of magnetization). One can speak of 4-D spin glass. It would seem that the hierarchy of Planck constants labelling different quantum phases and the phase transitions between these phases can be interpreted in terms of 4-D spin glass property? Besides phases one would have also phase transitions having "transitons" as building bricks.

   It seems that one cannot assign 4-D spin glass dynamics to MB. If magnetic flux tubes are carriers of monopole flux, they cannot be small local deformations of vacuum extremals for which Kähler form vanishes. Hence 4-D spin glass property can be assigned to flux tubes carrying vanishing magnetic flux. Early cosmology suggests that cosmic strings as infinitely flux tubes having 2-DCP_2 projection and carrying monopole flux are deformed to magnetic flux tubes and suffer topological condensation around vacuum extremals and deform them during the TGD counterpart of inflationary period.

   Remark: Glass state looks like a transition rather than state and ZEO and 4-D spin glass description would seem to fit naturally to his situation: glass would be a 4-D variant of spin glass. The time scale of transition is long and one might think that h_{eff} at the space-time sheet "controlling" transition is rather large and also the change of h_{eff} is large.

3.3.2 Symmetries and phase transitions

The notion of symmetry is considerably more complex in TGD framework than in standard picture based on positive energy ontology. There are dynamical symmetries of dark matter states located at the boundaries of CD. For space-time sheets describing phase transitions there are also dynamical symmetries but they are different. In standard physics one has just states and their symmetries. Conformal gauge symmetries forming a hierarchy: conformal field theories this symmetry is maximal and the hierarchy is absent.

1. There is importance and very delicate difference between thermal and thermodynamical symmetries. Thermal symmetries are due to thermal equilibrium implying symmetries in *statistical sense*. Quantal symmetries correspond to representations of symmetry group and are possible if thermal fluctuations do not transform the states of the representations the states of other representation.

   Dark dynamical symmetries are quantum symmetries. The breaking of thermal translational symmetry of liquid leads to discrete translational symmetry of crystal having interpretation as quantum symmetry. The generation of continuous thermal translational symmetry from discrete quantum symmetry means loss of quantum symmetry. To my opinion, standard thinking is sloppy here.

2. For thermodynamical phase transitions temperature reduction induces spontaneous breaking of symmetry: consider only liquid-to-crystal transition. Analogously, in gauge theories the reduction gauge coupling strength leads to spontaneous symmetry breaking: quantum fluctuations combine representation of sub-group to a representation of larger group. It would seem that spontaneous symmetry breaking actually brings in a symmetry and the ubroken symmetry is "thermal" or pure gauge symmetry. QCD serves as an example: as strong coupling strength (analogous to temperature) becomes large confinement occurs and color symmetry becomes pure gauge symmetry.

3. In TGD the new feature is that there are two kinds of symmetries for dark conformal hierarchies. Symmetries are either pure gauge symmetries or genuine dynamical symmetries affecting the dark state at field body physically. As h_{eff} increases, the conformal pure gauge symmetry is reduced (the conformal gauge algebra annilating the states becomes smaller) but dynamical symmetry associated with the degrees of freedom above measurement resolution increases. In ordinary conformal theories pure gauge conformal symmetry is always maximal so that this phenomenon does not occur.

   The intuitive picture is that the increase of dynamical symmetry induced by the reduction of pure gauge conformal symmetry occurs as temperature is lowered and quantum coherence in longer scales becomes possible. This conforms with the thermodynamical and gauge theory views if pure gauge symmetry is identified as counterpart of symmetry as it is understood in thermodynamics and gauge theories.

   The dynamical symmetry of dark matter however increases. This symmetry is something new and would be genuine quantum symmetry in the sense that quantum fluctuations respect the representations of this group. The increase of h_{eff} indeed implies reduction of Kähler coupling strength analogous to reduction of temperature so that these quantum symmetries can emerge.

4. There is also a dynamical symmetry associated with phase transitions $h_{eff}(f) = m \times h_{eff}(i)$ such that m would define the rank of ADE Lie group G classifying states of "transitons". Lie groups with ranks $n_{eff}(i)$ and $n_{eff}(f)$ would be ranks for the Lie group G in the initial and final states. G would correspond to either gauge (not pure gauge) or Kac-Moody symmetry as also for corresponding dynamical symmetry groups associated with phases.

5. An interesting question relates to Kosterlitz-Thouless Thouless phase transition (see `http://tinyurl.com/yce24jr9`), which is 2-D and for which symmetry is not changed. Could one interpret it as a phase transition changing h_{eff} for MB: symmetry group as abstract group would not change although the scale in which acts would change: this is like taking zoom. The dynamical symmetry group assignable to dark matter at flux tubes would however change but remain hidden.

To sum up, the notion of magnetic (field) body might apply even to the ordinary phase transitions. Dark symmetries - also discrete translational and rotational symmetries - would be assigned with dark MB possibly present also in ordinary phases. The dynamical symmetries of MB would bring a new element to the description. Ordinary phase transitions would be induced by those of MB. This would generalize the vision that MB controls biological body central for TGD view about living matter. In the spirit of slaving hierarchy and TGD inspired vision about quantum biology, ordinary matter would be slave and MB the master and the description of the phase transitions in terms of dynamics of master could be much more simpler than the standard description. This would be a little bit like understanding technical instrument from the knowledge of its function and from control level rather than from the mere physical structure.

3.3.3 Quantum phase transitions and 4-D spin glass energy landscape

TGD has led to two descriptions for quantum criticality. The first one relies on the notion of 4-D spin glass degeneracy and emerged already around 1990 when I discovered the unique properties of Kähler action. Second description relies on quantum phases and quantum phase transitions and I have tried to explain my understanding about it above. The attempt to understand how these two approaches relate to each other might provide additional insights.

1. Vacuum degeneracy of Kähler action is certainly a key feature of TGD and distinguishes it from all classical field theories. Small deformations of the vacua probably induced by gluing of magnetic flux tubes (primordially cosmic strings) to these vacuum space-time sheets deforms them slightly and would give rise to TGD Universe analogous to 4-D spin glass. The challenge is to relate this description to the vision provided by quantum phases and quantum phase transitions.

2. In condensed matter physics one speaks of fractal spin glass energy landscape with free energy minima inside free energy minima. This landscape obeys ultrametric topology: p-adic topologies are ultra metric and this was one of the original motivations for the idea that p-adic physics might be relevant for TGD. Free energy is replaced with the sum of Kähler function - Kähler action of Euclidian space-time regions and imaginary Kähler action from Minkowskian space-time regions.

3. In the fractal spin glass energy landscape there is an infinite number of minima of free energy. The presence of several degenerate minima leads to what is known as frustration. In TGD framework all the vacuum extremals have the same vanishing action so that there is infinite degeneracy and infinite frustration (also created by the attempt to understand what this might imply physically!). The diffeomorphisms of M^4 and symplectic transformations of CP_2 map vacuum extremals to each other and acts therefore as gauge symmetries. Symplectic transformations indeed act as U(1) gauge transformations. Besides this each Lagrangian sub-manifold of CP_2 defines its own space of vacuum-extremals as orbit of this symplectic group.

   As one deforms vacuum extremals slightly to non-vacuum extremals, classical gravitational energy becomes non-vanishing and Kähler action does not vanish anymore and the above gauge symmetries become dynamical symmetries. This picture serves as a useful guideline in the attempts to physically interpret. In TGD inspired quantum biology gravitation plays indeed fundamental role (gravitational Planck constant h_{gr}).

4. Can one identify a quantum counterpart of the degeneracy of extremals? The notion of negentropic entanglement (NE) is cornerstone of TGD. In particular, for maximal negentropic entanglement density matrix is proportional to unit matrix so that states are degenerate in the same sense as the states with same energy in thermodynamics. Energy has Kähler function as analogy now: hence the degeneracy of density matrix could correspond to that for Kähler function. More general NE corresponds to algebraic entanglement probabilities and allows to identify unique basis of eigenstates of density matrix. NE is favored by NMP and serves key element of TGD inspired theory of consciousness.

   In standard physics degeneracy of density matrix is extremely rare phenomenon as is also entanglement with algebraic entanglement probabilities. These properties are also extremely unstable. TGD must be somehow special. The vacuum degeneracy of Kähler action indeed distinguishes TGD from quantum field theories, and an attractive idea is that the degeneracy associated with NE relates to that for extremals of Kähler action. This is not enough however: NMP is needed to stabilize NE and this occurs only for dark matter ($h_{eff}/h > 1$ equals to the dimension of density matrix defining NE).

   The strong form of holography takes this idea further: 2-D string world sheets and partonic 2-surfaces are labelled by parameters, which belong to algebraic extension of rationals. This replaces effectively infinite-D WCW with discrete spaces characterized by these extensions and allows to unify real and p-adic physics to adelic physics. This hierarchy of algebraic extensions would be behind various hierarchies of quantum TGD, also the hierarchy of deformations of vacuum extremals.

5. In 3-D spin glass different phases assignable to the bottoms of potential wells in the fractal spin energy landscape compete. In 4-D spin glass energy of TGD also time evolutions

compete, and degeneracy and frustration chacterize also time evolutions. In biology the notions of function and behavior corresponds to temporal patterns: functions and behaviors are fighting for survival rather than only organisms.

At quantum level the temporal patterns would correspond to phase transitions perhaps induced by quantum phase transitions for dark matter at the level of magnetic bodies. Phase transitions changing the value of h_{eff} would define correlates for "behaviors" and the above proposed description could apply to them.

6. Conformal symmetries (the shorthand "conformal" is understood in very general sense here) allow to understand not only quantum phases but also quantum phase transitions at fundamental level and "transitons" transforming according to representations of Kac-Moody group or gauge group assignable to the inclusion of hyperfinite factors characterized by the integer m in $h_{eff}(f) = m \times h_{eff}(i)$ could allow precise quantitative description. Fractal symmetry breaking leads to conformal sub-algebra isomorphic with the original one

   What could this symmetry breaking correspond in spin energy landscape? The phase transition increasing the dynamical symmetry leads to a bottom of a smaller well in spin energy landscape. The conformal gauge symmetry is reduced and dynamical symmetry increased, and the system becomes more critical. Indeed, the smaller the potential well, the more prone the system is for being kicked outside the well by quantum fluctuations. The smaller the well, the large the value of h_{eff}. At space-time level this corresponds to a longer scale. At the level of WCW (4-D spin energy landscape) this corresponds to a shorter scale.

3.3.4 What ZEO can give to the description of criticality?

One should clarify what quantum criticality exactly means in TGD framework. In positive energy ontology the notion of state becomes fuzzy at criticality. It is difficult to assign long range fluctuations and associated quanta with any of the phases co-existent at criticality since they are most naturally associated with the phase change. Hence Zero Energy Ontology (ZEO) might show its power in the description of (quantum) critical phase transitions.

1. Quantum criticality could correspond to zero energy states for which the value of h_{eff} differs at the opposite boundaries of causal diamond (CD). The space-time surface between boundaries of CD would describe the transition classically. If so, then quanta for long range fluctuations would be genuinely 4-D objects - "transitons" - allowing proper description only in ZEO. This could apply quite generally to the excitations associated with quantum criticality. Living matter is key example of quantum criticality and here "transitons" could be seen as building bricks of behavioral patterns. Maybe it makes sense to speak even about Bose-Einstein condensates of "transitons".

2. Quantum criticality would be associated with the transition increasing $n_{eff} = h_{eff}/h$ by integer factor m or its reversal. Large h_{eff} phases as such would not be quantum critical as I have sloppily stated in several contexts earlier. $n_{eff}(f) = m \times n_{eff}(i)$ would correspond to a phase having longer long range correlations as the initial phase. Maybe one could say that at the side of criticality (say the "lower" end of CD) the $n_{eff}(f) = m \times n_{eff}(i)$ excitations are pure gauge excitations and thus "below measurement resolution" but become real at the other side of criticality (the "upper" end of CD)? The integer m would have clear geometric interpretation: each sheet of n_i-fold coverings defining space-time surface with sheets co-inciding at the other end of CD would be replaced with its m-fold covering. Several replications of this kind or their reversals would be possible.

3. The formation of m-fold covering could be also interpreted in terms of an inclusion of hyperfinite factors labelled by integer m. This suggests a deep connection with symmetries of dark matter. Generalizing the McKay correspondence between finite subgroups of SU(2) characterizing the inclusions and ADE type Lie groups, the Lie group G characterizing the dynamical gauge group or Kac-Moody group for the inclusion of HFFs characterized by m would have rank given by m (the dimension of Cartan algebra of G).

These groups are expected to be closely related to the inclusions for the fractal hierarchy of isomorphic sub-algebras of super-symplectic subalgebra. $h_{eff}/h = n$ could label the sub-algebras: the conformal weights of sub-algebra are n-multiples of those of the entire algebra. If the sub-algebra with larger value of n_{eff} annihilates the states, it effectively acts as normal subgroup and one can say that the coset space of the two super-conformal groups acts either as gauge group or (perhaps more naturally) Kac-Moody group. The inclusion hierarchy would allow to realize all ADE groups as dynamical gauge groups or more plausibly, as Kac-Moody type symmetry groups associated with dark matter and characterizing the degrees of freedom allowed by finite measurement resolution.

4. If would be natural to assign "transitions" with light-like 3-surfaces representing parton orbits between boundaries of CD. I have indeed proposed that Kac-Moody algebras are associated with parton orbits where super-symplectic algebra and conformal algebra of light-one boundary is associated with the space-like 3-surfaces at the boundaries of CD. This picture would provide a rather detailed view about symmetries of quantum TGD.

The number-theoretic structure of h_{eff} reducing transitions is of special interest.

1. A phase characterized by $h_{eff}/h = n_{eff}(i)$ can make a phase transition only to a phase for which $n_{eff}(f)$ divides $n_{eff}(i)$. This in principle allows purely physics based method of finding the divisors of very large integers (gravitational Planck constant $\hbar_{gr} = GMm/v_0 = \hbar_{eff} = n \times \hbar$ defines huge integer).
2. In TGD inspired theory of consciousness a possible application is to a model for how people known as idiot savants unable to understand what the notion of prime means are able to decompose large integers to prime factors [K19]. I have proposed that the division to prime factors is a spontaneous process analogous to the splitting of a periodic wave characterized by wave length $\lambda/\lambda_0 = n_i$ to a wave with wavelength $\lambda/\lambda_0 = n_{eff}(f)$ with $n_{eff}(f)$ a divisor of $n_{eff}(i)$. This process might be completely spontaneous sequence of phase transitions reducing the value of n_{eff} realized geometrically as the number of sheets of the singular covering defining the space-time sheet and somehow giving rise to a direct sensory percept.

3.4 Maxwell's lever rule and expansion of water in freezing: two poorly understood phenomena

The view about condensed matter as a network with nodes identifiable as molecules and bonds as flux tubes is one of the basic predictions of TGD and obviously means a radical modification of the existing picture. In the sequel two old anomalies of standard physics are explained in this conceptual framework.

3.4.1 Maxwell's lever rule as an indication for the presence of magnetic flux tubes

Van der Waals equation of state (`http://tinyurl.com/yayjgehm`) is a simple model for two phase system used for mostly pedagogical purposes. The model is not realistic. In particular, it has difficulties in the critical region, where two phases are present. The latter difficulties are actually encountered also in the partition function approach of statistical mechanics.

1. Van der Waals equation of state

Consider first the van der Waals equation of state.

1. Van der Waals equation of state has variables (n, T) so that the natural thermodynamical function is free energy F. The equation is of form

$$P = (\frac{\partial F}{\partial V})_T = \frac{n}{1-nb_1}T - a_1 n^2 \quad . \tag{3.1}$$

Here one has $n = N/V$, where N is particle number and constant parameter. $(b_1 = 0, a_1 = 0)$ gives the equation of state for ideal gas. The interpretation of the parameters is discussed in `http://tinyurl.com/yayjgehm`.

2. To deduce free energy F and internal energy E one would need also the partial derivative of free energy

$$S = (\frac{\partial F}{\partial T})_V \quad , \tag{3.2}$$

so that $dF = SdT - pdV$ could be integrated. The information about entropy is not included in van der Waals.

3. The expressions of both E and F can be fixed by assuming that E is a homogenous funktion of (S, T, P, V):

$$E = TS - PV \quad . \tag{3.3}$$

This is additional assumption, which of course need not be true.

(a) The assumption would give for the free energy per particle the expression

$$f = \frac{F}{N} = \frac{E - TS}{N} = \frac{PV}{N} = \frac{P}{n} \tag{3.4}$$

In the case of van der Waals one obtains by using the expression for the pressure already given

$$f = \frac{P}{n} = \frac{T}{1-nb_1} - a_1 n \quad . \tag{3.5}$$

(b) The entropy per particle is given by

$$s = \frac{S}{N} = \frac{(\frac{\partial p}{\partial T})_V}{n} = \frac{1}{1-nb_1} \quad . \tag{3.6}$$

$s = S/N$ does not depend on temperature at all.

(c) For single particle energy $e = E/N$ one has

$$e = \frac{TS-PV}{N} = a_1 n \quad . \tag{3.7}$$

Also e depends on n only.

4. Van der Waals indeed allows 2 phases and they appear simultaneously in the critical region. The equation of state can be written as a condition for the vanishing of 3rd degree polynomial $P_3(n, T)$ as a function of n

$$\begin{array}{ll} P_3(n,t) = \sum_{k=0}^{3} p_k^{n^k} = 0 \quad p_3 = 1 \ , & p_2 = -\frac{1}{b_1} \ , \\ p_1 = \frac{P}{a_1 b_1} + \frac{T}{a_1 b_1} \ , & p_0 = -\frac{P}{a_1 b_1} \ . \end{array} \tag{3.8}$$

The number of the real roots for n is odd: either 3 or 1. In the critical region, which corresponds to a cusp catastrophe (see `http://tinyurl.com/ngfa9t3`) having n as behaviour variable, the number of real roots is 3, call them $n_{max} \geq n_0 \geq n_{min}$. The largest root n_{max} and smallest root n_{min} correspond to liquid and gas phases. The middle root n_0 is unstable if the polynomial equation is interpreted as a vanishing of the derivative of a fourth-order polynomial of n having p and T as control parameters. It has no physical identification.

The projection of the cusp (see http://tinyurl.com/ngfa9t3) to (p,T) has shape of V with curved edges. The tip of V corresponds to critical point and at the edges of V a phase transition takes place between vapour phase and critical phase or liquid phase and critical phase. Above the tip one cannot say whether the phase is gas or liquid and the continuous transformation of gas to liquid can be also regarded as poorly understood.

At the right (left) hand side of V there is single real root n_G (n_L). $n_G < n_L$ allows the interpretation in terms of gas and liquid phases.

2. The problems of van der Waals

Consider now the problems of van der Waals in the critical region.

1. Van der Waals allows besides gas phase also liquid phase but the model does now work well in liquid phase. In the critical region where both gas and liquid phases are possible, the model works badly. Equation of state forces the system to a 2-dimensional surface in (n,p,T) space ($n = N/V$, also V can be used as variable since N is constant parameter).

   The standard interpretation is that both phases are present as pure phases and only their fractions vary. The intermediate phase allowed by van der Walls is not present. The empirical finding that the pressure for given temperature does not depend on V. $p(V,T) = p(T)$ condition states that the pressures of the two phases are same: this can be interpreted as equilibrium condition. It follows from van der Waals naturally for different roots n for the equation of state.

2. Already Maxwell proposed a modification of van der Waals. Area rule (for a visualization see http://tinyurl.com/ycabjdhh) tells how the oscillatory behaviour of $p(T,V)$ as function of V as one moves in transversal direction (in which p varies) to V along cusp from lowest sheet of cusp (n_-) to the highest sheet (n_+) by increasing V is replaced with constant behavior. In other words, the curve along cusp connecting constant T curves connecting the points at upper and lower sheet of cusp with the same value of p is replaced with a straight vertical line. The condition is that the area below the line is same as the area below the oscillatory curve of constant p.

   Lever rule (http://tinyurl.com/ybuq7aye) is needed to understand the proportions of the two phases. Usually the rule is applied to metal alloys. Consider two pure phases α ja β and their mixture γ. Let the fractions of phases α and β be X_α and X_β. Assume that the phases contain two "elements" A and B. Let the proportions of B be a in α, b in β and c in γ. The lever rule

$$X_\alpha = \frac{c-b}{a-b} \tag{3.9}$$

   follows trivially from the fact that in mixed phase γ one has $c = X_\alpha a + (1 - X_\alpha b = X_\alpha(a-b) + b$.

   In critical regions a and b should vary. To me this picture however represents a problem. What the two "elements" are in the case of say water? If water molecule corresponds to A, what does B correspond to? A different state of water molecule? Or does the system contain also some other "element" than water molecule?

As a consequence of this problem the working models are numerical since analytical models cannot explain the lever rule. This problem is not only the problem of van der Waals but quite general problem of statistical models relying on partition function giving free energy F.

3. TGD based explanation of the lever rule

The TGD interpretation for the situation could be following.

1. In the liquid phase molecules can be connected by flux tubes. They are also possible in gas phase but their number is smaller. In particular, in vapour phase intermediate between liquid

and gas also gas molecules can be connected by flux tubes to form connected networks.Only single connected network could be present in liquid phase.

The number of flux tubes per particle can depend on the thermodynamical parameters (V, T) and is expected to be considerably smaller in gas phase in regions where one can distinguish liquid phase from vapor phase (not below the tip of V).

In liquid state the flux tubes could be shorter than in gas phase. In liquid phase there are large connected structures - maybe only single one - whereas in gas phase these structures are smaller. At criticality they might correspond to vapour droplets. Gas phase would be different from gas phase far from criticality.

2. In critical region there are regions, which form connected networks differing with respect to the number of bonds per particle characterizing the networking. The volume of the mixed phase depends on the relative volumes of these two phases since they have widely differing densities. Large number of networked molecules gives a smaller volume. The pressures in these two kinds of regions are same in mechanical equilibrium.

3. What could be the counterparts for the two "elements" A and B? Could A correspond water molecule and B to flux tube? The portion of flux tubes would distinguish between the two phases at criticality. They are present also in gas phase unless one has $b = 0$ identically. In this case a must however vary inside critical region. For $b = 0$, perhaps realized far from the left edge of V, gas phase would have no flux tubes. In liquid phase to the right from V but not below the tip of V a would be large. At tip and below it one would have $a = b$ along some line and one can say that gas phase transforms to liquid phase. As one goes around the tip the fraction a in liquid phase becomes b for gas phase.

4. What distinguishes liquid and gas phases? What suggests itself is that when the number N_b of flux tube bonds per molecule is above critical value $N_{b,crit}$, a transition to liquid phase takes place and the density is reduced to that of liquid. Below the tip of V and left to V this phase transition does not take place. To the right of the edge of we it would take place. Inside V there are both kinds of regions. What this means that the parameters a and b are new parameters characterizing the state of liquid and gas phases. This could allow better understanding of vapour phase.

5. The appearance of flux tubes could be understood in two ways. .

   (a) New flux tube pairs could emerge by reconnection of flux tube loops associated with molecules.

   (b) Already existing long flux tubes (or flux tube pairs) between molecules shorten in a phase transition reducing $h_{eff} = n \times h$ to its standard value and forces the molecules connected by them to become close together. Since the phases with non-standard value of Planck constant quite generally have higher energy (for instance, bond energies are higher and atomic binding energies lower) this implies that energy is liberated in this connection process.

It seems that flux tube picture could explain the lever rule, which works but cannot be understood in thermodynamics and statistical physics. This would be seen as a direct indication for the reality of flux tubes.

3.4.2 Strangeness in the freezing of water

Water has hundreds of anomalies as one learns from the excellent web pages of Martin Chaplin (see `http://www1.lsbu.ac.uk/water/`). I have discussed these anomalies in [K7]. One of them relates to the freezing of water. Usually the volume per particle is reduced in freezing but now it increases. Second biologically enormously important anomaly is the decrease of the molecular volume instead of increase as the temperature grows form 0 °C to 4 °C.

In TGD framework the anomalies of water can be seen as a support for the existence of two phases in water: dark phase identified as a phase with non-standard value $h_{eff}/h = n$ of Planck constant [?] and ordinary phase. On basis of the model explaining Maxwell's rule at criticality, one can ask whether the dark and ordinary phases correspond to those for the flux tubes rather than molecules. In the case of water the flux tubes could be assigned to hydrogen bonds, which could have quite long lengths for large values of $h_{eff}/h = n$. They would be present also for other liquids. Maybe the flux tubes carrying $h_{eff}/n = n$ dark protons associated with hydrogen bonds distinguish water from other liquids.

Dark states have higher energy than ordinary ones so that the formation of dark phase requires energy. The natural assumption is that the dark phase transforms to ordinary one in the freezing of water. Long dark flux tubes would get shorter. Alternatively, dark flux loops reconnect and form short flux tube pairs between molecules assignable to hydrogen bonds. Why this should lead to an expansion of the molecular volume?

To answer this question it is useful consider first the second anomaly. Why the volume increases as one reduces temperature from 4 °C to 0 °C? As $h_{eff}/h = n$ for flux tube or reconnecting flux tubes decreases, the length of flux tube as quantal length shortens and the result could be a rather rigid short stick. There exists a proposal that these rigid flux tubes reduce the motility of water molecules belonging to the water clusters, which correspond to connected flux tube networks. Since molecules cannot move freely anymore, empty volume is generated. The outcome is an increase of the average molecular volume.

What about freezing?

1. Above boiling point water has 3.4 hydrogen bonds to its neighbors, which is nearly the maximal number 4 realized for ice (see `http://tinyurl.com/ydcedet4`). Either all existing long flux tubes would have shortened or all loops would have shortened and reconnected to flux tube pairs.

2. In freezing the dark energy is liberated so that the latent heat should be higher when a phase with a non-standard value of $h_{eff}/h = n$ is present in the liquid phase. This could explain the especially high latent heat 334 kJ/kg for water.

3. Only ammonia NH^3 (see `http://tinyurl.com/yc6zcl6o`) has comparable latent heat 332.17 kJ/kg (see `http://tinyurl.com/h3lvm43`) . Interestingly, also NH_3 molecules form hydrogen bonds and for this reason ammonium is easily miscible to water. This property might relate also to the biological importance of NH_3 and nitrogen and hydrogen containing molecules.

4. Also O and F form hydrogen bonds. More generally, any atom containing lone electron pairs, that is pairs of valence electrons, which do not belong to valence bonds, can form hydrogen bonds. A possible explanation is that the lone pair goes to a flux tube pair associated with the hydrogen bond and gives rise to a Cooper pair making possible high T_c superconductivity by the mechanism discussed in [K17, K18]. Flux tube pair would contain also the dark proton delocalized to both flux tubes.

3.5 TGD based view about ferromagnetism

I received a link to a highly interesting popular article (`https://tinyurl.com/y8b86df3`) about ferromagnetism. According to the article, Yi Li, a physicist working at John Hopkins University and his two graduate students, Eric Bobrow and Keaton Stubis, seem to have made a considerable progress in understanding how the system of electron spins in lattice ends up to a ferromagnetic sate [D1] (`https://tinyurl.com/y9ycj3nt`). This ferromagnetism is known as itinerant ferromagnetism and involves vacancies, sites without electron, which can be moved freely without affecting the energy of the state. This article inspired train of thought allowing to develop TGD view about ferromagnetism.

3.5.1 The ideas related to the work of Li *et al*

The problem considered by Li *et al* is how the ferromagnetic state could emerge from an arbitrary state with some numbers of spin up and spin down states at lattice sites connected by edges.

1. Permutation of electrons with same spin leave the ferromagnetic state invariant and does not cost energy while permutations in arbitrary configuration can do so.

2. Li *et al* [D1] considered a simple 4×4 lattice with single vacancy and noticed a connection with so called 15-puzzle involving 15 tiles and single vacancy with neighboring tiles of vacancy able to move to its position. The observation is following. If one has spin lattice containing single vacancy, one can number the sites by a number running from 1 to N (now 15) in arbitrary manner. If so called connectedness condition holds true one can realize any permutation of these numbers. This means that 15-puzzle has always a solution. In particular, one can arrange the situation that the numbers form an ordered sequence from 1 to N so that numbers n and $n+1$ are nearest neighbors.

   The result found by Li *et al* first for 2-D 4×4 lattice with single vacancy generalizes to lattices, which are non-separable in the sense the removal of a lattice site does not separate any pair of spins - they are still connected by an edge-loop.

3. The curve solving the 15-puzzle goes through all points of the 4×4 lattice and is generally known as Hamiltonian curve. It becomes Hamiltonian cycle if the numbers 1 and N are nearest neighbors.

4. The basic problem of this approach is that the theorem is true only for single vacancy and does not allow generalization to a larger number of vacancies. It is however known that ferromagnetism is possible up to fraction 1/3 for vacancies. The challenge is to generalize the result of Li *et al*.

3.5.2 Some reasons to get interested

In TGD framework there are good reasons of getting interested on these results.

1. The result of Li *et al* states that ferromagnetic phase transition might be understood in terms of shifting of lattice vacancy if the lattice with single defect allows deformation of any configuration of spin labelled by numbers N running from 1 to N to a closed curve connecting nearest neighbors along which N increases. Could there be a connection with Hamiltonian curves making sense for lattice like structures (actually all graphs)? Could Hamiltonian curve have some deeper physical meaning or is it only an auxialiary notion useful for representing the possibility to realize all permutations for the points of a lattice with vacancy by shifting it suitably?

   Hamiltonian curve connects neighboring points of a lattice and goes through all points without self-intersections. Icosahedral geometry appears in biology and one can ask whether this kind of cycles could be actually realized physically - say as flux tubes at icosahedron and tetrahedron, which play key role in TGD inspired biology. Flux tube are actually fundamental objects in TGD Universe in all scales. For instance, final states of stars could correspond to flux tube spaghettis consisting of single volume filling flux tube [L8] (`https://tinyurl.com/tkkyyd2`).

2. If the Hamiltonian cycle is something physical it could correspond to flux tube. The notion of magnetic flux tube central in TGD might allow application to ferromagnetism. TGD predicts two kinds of flux tubes: Maxwellian ones and monopole flux tubes with magnetic fields requiring no currents to generate them: they are not not allowed by Maxwell's theory.

   The preservation of the Earth's magnetic field predicted to decay rather rapidly as currents generating it dissipate supports the view that it contains monopole flux part which from biological input would correspond to endogenous magnetic field B_{end}, which is a fraction 2/5 about the nominal value of $B_E = .5$ Gauss. The presence of magnetic fields in cosmological scales is also a mystery finding a solution in terms of monopole flux tubes.

3. Monopole flux tubes must be closed. Closed non-intersecting flux tubes connecting nearest neighbors in lattice would correspond to Hamiltonian cycles. In TGD inspired biology Hamiltonian cycles associated with icosahedron and tetrahedron provide a realization of the vertebrate genetic code [L2] (`https://tinyurl.com/yad4tqwl`) but it is still somewhat of mystery why the points of icosahedron and tetrahedron, which are lattices (tessellations) at sphere, would be connected by a curve. Quantum classical correspondence suggests that magnetization corresponds to flux tubes connecting magnetic dipoles as formal analogs of

monopole-antimonopole pairs. Could magnetic flux tubes provide a concrete realizaon for these Hamiltonan cycles?

4. Closed monopole flux tubes seem to be unrealistic for the description of ferromagnetism, which suggests the presence of N parallel flux tubes carrying magnetization M and defining a braid connecting opposite ends of ferromagnet. The monopole fluxes could arrive as single flux along parallel space-time sheet carrying field H defined by single thick flux tube. Test particle would experience $B = M + H$.

The following considerations are not much more than first impressions and probably require updating.

3.5.3 TGD based view

Flux tubes are the new element of condensed matter physics predicted by TGD. Could they provide insights into ferromagnetism?

1. Starting from text book picture about ferromagnetism

To develop TG view about ferromagnetism it is best to start from the text book picture.

1. In the standard model of ferromagnetism one assumes the presence of field B identified as sum $B = M + H$ of magnetization and field H equal to B outside the magnet. M is due to magnetic dipoles besides magnetic field B and the interaction of spins with H is important. B is usually regarded as the fundamental field M and and H appear as auxiliary notions and their relation to B requires a model for the system: typically H, M, and B are assumed to be linearly related.

2. The field M could be naturally assigned with a flux tube connecting the spins - perhaps at nearest neighbor lattice points. What about H? In standard model H and B are parallel for the ferromagnetic configuration. If B is assigned with the flux tube connecting the magnetic moments and B is parallel to H, this would suggest a flux tube consisting of long straight portions parallel to each other.

   In the many-sheeted space-time of TGD M and H can reside at different space-time sheets, which are parallel so that they are on top of each other in $H = M^4 \times CP_2$. H is at large space-time sheet including also the environment. The decomposition to sum would have representation as a set theoretic union.

   The test particle would experience the sum of the magnetic fields associated with the two sheets. Could M and H as the return flux associated with M and superposing with the external contribution to H correspond to these two space-time sheets so that particle would experience their sum as $B = M + H$? If so, ferromagnetism could be seen as a direct signature of many-sheeted space-time.

2. Could also monopole flux tubes be important?

There is still one important aspect related to the TGD view about magnetic field which might play important role. TGD predicts two kind of flux tubes. The first kind of flux tubes could be called Maxwellian and the corresponding magnetic fields require current to generate them. There are also flux tubes having closed cross section and carrying monopole fluxes. No currents are required to generate corresponding magnetic fields. Could also these flux tubes having no current as sources be present? This would mean new physics.

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   Formation of a flux loop defining Hamiltonian cycle would be a new kind of phenomenon analogous to spontaneous magnetization requiring no external field H. Spontaneous magnetization would be however something different. A trivial braid consisting of N parallel strands representing M and parallel to it locally with return flux arriving along single large flux tube carrying H would be formed in ferromagnetic transition and also in spontaneous magnetization.

3. Bringing in thermodynamics

One can try to make this more concrete by bringing in thermodynamics.

1. Assume that there exists single flux tube - connecting all the lattice points (magnetic moments) or possibly N flux tubes parallel to local magnetization M and giving rise to a braid like structure representing the topology of flux lines of M connecting opposite boundaries of magnet.

2. In the general case, the points of the lattice could be connected by a flux tubes connecting points, which need not be nearest neighbors. The first guess is that the magnetic interaction energy of spins at the ends of the flux tube portion connecting them decreases with the distance between spins. There should be also magnetic energy associated with the field H at the space-time sheet carrying the return flux. Thermodynamics would bring in entropy and free energy $F = E - TS$ would be mimimized. Entropy maximization would favor long random flux tubes and energy minimization short flux tubes.

   One expects that flux tube has free energy F increasing with flux tube length. If one does not allow self-intersections - as suggested by repulsive Coulomb interaction and Fermi statistics - the flux tube could be either Hamiltonian cycle or consist of analogs of braid strands: in the case of ferromagnetism the strands would be parallel to each other. The interaction energy would be same for all Hamiltonian cycles if determined by nearest neighbour interactions.

3. In the general case with lattice replaced by graph one expects that a large number of Hamiltonian cycles not related by rotation to each other exists so that one would have large number of states with same minimum energy. Could this somehow correspond to spin glass state allowing large number of degenerate states? The flux tube need not be closed. In ferromagnetic configuration this would be the case.

4. How would the assignment of spin direction to the lattice points affect the situation? Could the numbers N_+ and N_- of spin up and spin down electrons determine the flux tube configuration (say braid) by (Gibbs) energy minimization?

4. Could 2-braid describe the transition to ferromagnetism?

In the work of Li *et al* discussed in the article, the permutations of lattice points are induced by moving the vacancy around. This picture inspired the considerations above but is too limited. In fact the work of Li *et al* only directed attention to Hamiltonian cycles and braids formed by the non-closed analogs.

1. TGD picture brings in mind braid-knot connection. One can replace the braid associated with M with a knot by connecting the magnetic moments at the opposite ends of the braid by strands of a trivial braid at parallel space-time sheet. This trivial braid would carry the return flux having interpretation in terms of field H.

   The flux tubes of trivial braid could also fuse to single thicker flux tube or flow to a larger space-time sheet carrying the total return flux associated with M. This would conform with the idea that H provides a description of the system in longer length scale being analogous to a smoothed out total magnetic field acting as self-consistent background.

   Remark: Could one assume that only H assignable to big flux tube has constant direction and magnitude and that M is represented as flux tubes connecting dipoles can in principle correspond to any permutation of atoms? For this option the spontaneous magnetization would correspond to a superposition of different configurations with same weights and would be invariant under permutations as in the argument of Li *et al* involving no flux tubes. This option does not look attractive.

2. What braid picture allows to say about the transition to ferromagnetism? Could the transition be realized by deforming the flux tubes associated with M and forming a non-trivial braid be induced by permutation of the lattice points taking the non-trivial braid to trivial one? This would be like opening the braid. The lattice points in the initial and final state would correspond to the ends of a dynamical evolution. The permutation would be realized as a time-like braiding with braid strands in time direction.

   Mathematically braid group corresponds to the covering group of permutation group and quantum group representations correspond to the representations of braid groups. The description of the transition could provide a new application of quantum groups.

The description as time-like braiding is not however complete since there is an additional structure involved: the flux tubes connecting the magnetic dipoles in lattice and defining a braid or even more complex configuration having flux tube connections between non-neighboring magnetic moments.

1. If there is no return flux assignable to H, M corresponds to a closed flux tube carrying monopole flux the dynamical time-like dynamical braiding would lead to a Hamiltonian cycle in this case and the number of final state configurations would be finite, there is degeneracy. Could spin glass phase correspond to this situation?

2. In ferromagnetism final state would contain N parallel strands carrying the monopole flux assignable to M and the return flux H would arrive along parallel thick flux tube. In general configuration these strands can be braided. The transition to ferromagnetism would represent time-like braiding of an ordinary 3-D braiding of flux tube strands connecting the opposite boundaries of ferromagnetic. In the initial state braid would be non-trivial and the flux tubes of braid would not have minimal length and minimum energy. In the final ferromagnetic state braid would be trivial with parallel flux tubes.

   Mathematically this process would correspond to what is called 2-braiding: I have proposed that 2-braidings are important in TGD inspired biology as a topological description of dynamical processes. An interesting interpretation is as a topological analog for problem solving. I have also proposed that in bio-systems topological quantum computation programs are represented as this kind 2-braidings for flux tubes [K1, K23] (`https://tinyurl.com/ycvgjccq` and `https://tinyurl.com/ydylud6c`.)

   Ferromagnetism would correspond to an opening of a non-trivial braid. If the return flux arrives along flux tubes this is possible smoothly only if the knot defined in this manner is trivial. To achieve opening, the 2-braiding must involve reconnections, which correspond to cutting the knot strand and reconnecting the pieces in new manner: this is how Alexander opened his knot. Fermi statistics and repulsive Coulomb interaction do not favour this mechanism. If the return flux arrives along single flux tube, the opening could correspond to a smooth deformation without reconnections transferring the braiding to the parallel space-time sheet, where it is "neutralized" by fusing the flux tubes to single flux tube.

I received a link to a highly interesting popular article (https://tinyurl.com/y8b86df3) about ferromagnetism. According to the article, Yi Li, a physicist working at John Hopkins University and his two graduate students, Eric Bobrow and Keaton Stubis, seem to have made a considerable progress in understanding how the system of electron spins in lattice ends up to a ferromagnetic sate [D1] (https://journals.aps.org/prb/abstract/10.1103/PhysRevB.98.180101). This ferromagnetism is known as itinerant ferromagnetism and involves vacancies, sites without electron, which can be moved freely without affecting the energy of the state. This article inspired train of thought allowing to develop TGD view about ferromagnetism.

3.5.4 The ideas related to the work of Li *et al*

The problem considered by Li *et al* is how the ferromagnetic state could emerge from an arbitrary state with some numbers of spin up and spin down states at lattice sites connected by edges.

1. Permutation of electrons with same spin leave the ferromagnetic state invariant and does not cost energy while permutations in arbitrary configuration can do so.

2. Li *et al* [**?**]onsidered a simple 4×4 lattice with single vacancy and noticed a connection with so called 15-puzzle involving 15 tiles and single vacancy with neighboring tiles of vacancy able to move to its position. The observation is following. If one has spin lattice containing single vacancy, one can number the sites by a number running from 1 to N (now 15) in arbitrary manner. If so called connectedness condition holds true one can realize any permutation of these numbers. This means that 15-puzzle has always a solution. In particular, one can arrange the situation that the numbers form an ordered sequence from 1 to N so that numbers n and $n + 1$ are nearest neighbors.

   The result found by Li *et al* first for 2-D 4×4 lattice with single vacancy generalizes to lattices, which are non-separable in the sense the removal of a lattice site does not separate any pair of spins - they are still connected by an edge-loop.

3. The curve solving the 15-puzzle goes through all points of the 4×4 lattice and is generally known as Hamiltonian curve. It becomes Hamiltonian cycle if the numbers 1 and N are nearest neighbors.

4. The basic problem of this approach is that the theorem is true only for single vacancy and does not allow generalization to a larger number of vacancies. It is however known that ferromagnetism is possible up to fraction 1/3 for vacancies. The challenge is to generalize the result of Li *et al*.

3.5.5 Some reasons to get interested

In TGD framework there are good reasons of getting interested on these results.

1. The result of Li *et al* states that ferromagnetic phase transition might be understood in terms of shifting of lattice vacancy if the lattice with single defect allows deformation of any configuration of spin labelled by numbers N running from 1 to N to a closed curve connecting nearest neighbors along which N increases. Could there be a connection with Hamiltonian curves making sense for lattice like structures (actually all graphs)? Could Hamiltonian curve have some deeper physical meaning or is it only an auxialiary notion useful for representing the possibility to realize all permutations for the points of a lattice with vacancy by shifting it suitably?

   Hamiltonian curve connects neighboring points of a lattice and goes through all points without self-intersections. Icosahedral geometry appears in biology and one can ask whether this kind of cycles could be actually realized physically - say as flux tubes at icosahedron and tetrahedron, which play key role in TGD inspired biology. Flux tube are actually fundamental objects in TGD Universe in all scales. For instance, final states of stars could correspond to flux tube spaghettis consisting of single volume filling flux tube [L8] (https://tinyurl.com/tkkyyd2).

2. If the Hamiltonian cycle is something physical it could correspond to flux tube. The notion of magnetic flux tube central in TGD might allow application to ferromagnetism. TGD predicts two kinds of flux tubes: Maxwellian ones and monopole flux tubes with magnetic fields requiring no currents to generate them: they are not not allowed by Maxwell's theory.

   The preservation of the Earth's magnetic field predicted to decay rather rapidly as currents generating it dissipate supports the view that it contains monopole flux part which from biological input would correspond to endogenous magnetic field B_{end}, whichisafraction2/5aboutthenominalvalu
   .5Gauss.Thepresenceofmagneticfieldsincosmologicalscalesisalsoamysteryfindingasolutionintermsofmc

3. Monopole flux tubes must be closed. Closed non-intersecting flux tubes connecting nearest neighbors in lattice would correspond to Hamiltonian cycles. In TGD inspired biology Hamiltonian cycles associated with icosahedron and tetrahedron provide a realization of the vertebrate genetic code [L2] (`https://tinyurl.com/yad4tqwl`) but it is still somewhat of mystery why the points of icosahedron and tetrahedron, which are lattices (tessellations) at sphere, would be connected by a curve. Quantum classical correspondence suggests that magnetization corresponds to flux tubes connecting magnetic dipoles as formal analogs of monopole-antimonopole pairs. Could magnetic flux tubes provide a concrete realizaon for these Hamiltonan cycles?

4. Closed monopole flux tubes seem to be unrealistic for the description of ferromagnetism, which suggests the presence of N parallel flux tubes carrying magnetization M and defining a braid connecting opposite ends of ferromagnet. The monopole fluxes could arrive as single flux along parallel space-time sheet carrying field H defined by single thick flux tube. Test particle would experience $B = M + H$.

The following considerations are not much more than first impressions and probably require updating.

3.5.6 TGD based view

Flux tubes are the new element of condensed matter physics predicted by TGD. Could they provide insights into ferromagnetism?

1. Starting from text book picture about ferromagnetism

To develop TG view about ferromagnetism it is best to start from the text book picture.

1. In the standard model of ferromagnetism one assumes the presence of field B identified as sum $B = M + H$ of magnetization and field H equal to B outside the magnet. M is due to magnetic dipoles besides magnetic field B and the interaction of spins with H is important. B is usually regarded as the fundamental field M and and H appear as auxiliary notions and their relation to B requires a model for the system: typically H, M, and B are assumed to be linearly related.

2. The field M could be naturally assigned with a flux tube connecting the spins - perhaps at nearest neighbor lattice points. What about H? In standard model H and B are parallel for the ferromagnetic configuration. If B is assigned with the flux tube connecting the magnetic moments and B is parallel to H, this would suggest a flux tube consisting of long straight portions parallel to each other.

   In the many-sheeted space-time of TGD M and H can reside at different space-time sheets, which are parallel so that they are on top of each other in $H = M^4 \times CP_2$. H is at large space-time sheet including also the environment. The decomposition to sum would have representation as a set theoretic union.

   The test particle would experience the sum of the magnetic fields associated with the two sheets. Could M and H as the return flux associated with M and superposing with the external contribution to H correspond to these two space-time sheets so that particle would experience their sum as $B = M + H$? If so, ferromagnetism could be seen as a direct signature of many-sheeted space-time.

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   Formation of a flux loop defining Hamiltonian cycle would be a new kind of phenomenon analogous to spontaneous magnetization requiring no external field H. Spontaneous magnetization would be however something different. A trivial braid consisting of N parallel strands representing M and parallel to it locally with return flux arriving along single large flux tube carrying H would be formed in ferromagnetic transition and also in spontaneous magnetization.

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of states with same minimum energy. Could this somehow correspond to spin glass state allowing large number of degenerate states? The flux tube need not be closed. In ferromagnetic configuration this would be the case.

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