The Recent View about Twistorialization in TGD Framework

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Contents

Abstract

The recent view about the twistorialization in TGD framework is discussed.

1. A proposal made already earlier is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states.

   This idea does not make sense for incoming/outgoing particles, which light-like momenta unless they are parallel: their total momentum cannot be light-like in the general case. Rather, $P^2 = 0$ applies to the states formed inside CDs from groups of incoming and outgoing particles. BCFW deformation $p_i \rightarrow p_i + zr_i$ describes what happens for the single-particle momenta: they cease to be light-like but the total momenta for subgroups of particles in factorization channels are complex and light-like. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization in terms of “cognitive representations” as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of “world of classical worlds” (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure.

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

1 Introduction

The construction of scattering amplitudes is a dream that I have had since the birth of TGD for four decades ago. Various ideas have gradually emerged, some of them have turned out to be wrong, and some of them have survived. At this age I must admit that the dream about explicit algorithms that any graduate student could apply to construct the scattering amplitudes, would require a collective effort and probably will not be realized during my lifetime.

I have however identified a set of general powerful principles leading to a generalization of the recipes for constructing twistorial amplitudes and already now these principles suggest the possibility of rather concrete realizations. In the sequel several additional insights are developed in more detail. Some of them are discussed already earlier in the formulation of M^8-H duality [L6] in adelic framework [L7, L8] and in the chapters developing the TGD based generalization of twistor Grasmannian approach [K25, K10, K4, K21].

1. A proposal made already earlier [K21] is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states. Single particle momenta cannot be however light-like for this kind of states unless they are parallel. They must be also complex as they indeed are already in classical TGD.

   In fact, BCFW deformation $p_i \to p_i + zr_i$, $r_i \cdot r_j = 0$ creates at z-poles of the resulting amplitude pairs of zero energy states for which complex single particle momenta are not light-like but sum up to massless momentum. One can interpret these zero energy analogs of resonances, states inside CDs formed from massless external particles as they arrive to CD. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization [L7] in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure [K11, K6, K20].

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture [L6] implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

2 General view about the construction of scattering amplitudes in TGD framework

Before twistorial considerations a general vision about the basic principles of TGD and construction of scattering amplitudes in TGD framework is in order.

2.1 General principles behind S-matrix

Although explicit formulas for scattering amplitudes are probably too much to hope, one can try to develop a convincing general view about principles behind the S-matrix.

2.1.1 World of Classical Worlds

The first discovery was what I called the "world of classical worlds" (WCW) [K11, K6, K20] as a generalization of loop space allowing to replace path integral approach failing in TGD work. This led to a generalization of Einstein's geometrization program to an attempt to geometrize entire quantum physics. The geometry of WCW would be essentially unique from its mere existence since the existence of Riemann connection requires already in the case of loop spaces maximal isometries. Super-symplectic and super-conformal symmetries generalizing the 2-D conformal symmetries by replacing 2-D surfaces with light-like 3-surfaces (metrically 2-D!) would define the isometries.

Physical states would be classical spinor fields in the infinite-dimensional WCW and spinors at given point of WCW would be fermionic Fock states. Gamma matrices would be linear combinations of fermionic oscillator operators associated with the analog of massless Dirac equation at space-time surface determined by the variational principle whose preferred extremals the space-time surfaces are. Strong form of holography implied by strong form of general coordinate invariance would imply that it is enough to consider the restrictions of the induced spinor fields at string world sheets and partonic 2-surfaces (actually at discrete points at them defining the ends of boundaries of string world sheets) [K29, K20].

2.1.2 Zero Energy Ontology and generalization of quantum measurement theory to a theory of consciousness

The attempts to understand S-matrix led to the question about what does state function reduction really mean. This eventually led to the discovery of Zero Energy Ontology (ZEO) in which time=constant snapshot as a physical state is replaced with preferred extremal satisfying infinite number of additional gauge conditions [L9]. Temporal pattern becomes the fundamental entity: this conforms nicely with the view neuroscientists and computational scientists for whom behavior and program are basic notions. One can say that non-deterministic state function reduction replaces this kind time evolution with new one. One gets rid of the basic difficulty of ordinary quantum measurement theory.

Causal diamond (CD) is the basic geometric object of ZEO. The members of the state pair defining zero energy state - the analog of physical event characterized by initial and final states - have opposite total conserved quantum numbers and reside at the opposite light-like boundaries of CD being associated with 3-surfaces connected by a space-time surface, the preferred extremal. CDs form a fractal hierarchy ordered by their discrete size scale.

One ends up to a quite radical prediction: the arrow of time changes in "big" state function reduction changing the roles of active and passive boundaries of CD. The state function reductions occurring in elementary reactions represent an example of "big" state function reduction. The sequence of "small" state function reductions - analogs of so called weak measurements - defines self as a conscious entity having CD as embedding space correlate [L9].

In ZEO based view about WCW 3-surfaces X^3 are pairs of 3-surfaces at boundaries of CD connected by preferred extremals of the action principle. WCW spinors are pairs of fermionic Fock states at these 3-surfaces and WCW spinor fields are WCW spinors depending on X^3 . They satisfy the analog of massless Dirac equation which boils down to the analogs of Super Virasoro conditions including also gauge conditions for a sub-algebra of super-symplectic algebra. S-matrix describing time evolution followed by "small" state function reduction relates two WCW spinor fields of this kind.

2.1.3 Generalization of twistor Grassmannian approach to TGD framework

Twistorial approach generalizes from M^4 to $H = M^4 \times CP_2$. One possible motivation could be the fact that ordinary twistor approach describes only scattering of massless particles. In the proposed generalization particles are massless in 8-D sense and in general massive in 4-D sense [K25, K10, K4, K21].

1. The existence of twistor lift of Kähler action as 6-D analog of Kähler action fixes the choice of H uniquely: only M^4 and CP_2 allow twistor space with Kähler structure. The 12-D product of the twistor spaces of M^4 and CP_2 induces twistor structure for 6-D surface X^6 under additional conditions guaranteeing that the X^6 is twistor space of 4-D surface X^4 (S^2 bundle over X^4) - its twistor lift. The conjecture that 6-D Kähler action indeed gives rise to twistor spaces of X^4 as preferred extremals.

2. This conjecture is the analog for Penrose's original twistor representation of Maxwellian fields reducing dynamics of massless fields to homology. There is also an analogy with massless fields. Dimensional reduction of Kähler action occurs for 6-surfaces, which represent twistor spaces and the external particles entering CD would be minimal surfaces defining simultaneous preferred extremals of Kähler action satisfying infinite number of additional gauge conditions. Minimal surfaces indeed satisfy generalization of massless field equations. In the interior of CD defining interaction region there is a coupling to Kähler 4-force and one has analog of massless particle coupling to Maxwellian field.

3. 6-D Kähler action would give the preferred extremals via the analog of dimensional reduction essential for the twistor space property requiring that one has S^2 bundle over space-time surface. I have considered the generalization of the standard twistorial construction of scattering amplitudes of $\mathcal{N} = 4$ SUSY to TGD context. In particular, the crucial Yangian invariance of the amplitudes holds true also now in both M^4 and CP_2 sectors.

4. Skeptic could argue that TGD generalization of twistors does not tell anything about the origin of the Yangian symmetry. During writing of this contribution I however realized that the hierarchy of Grassmannians realizing the Yangian symmetries could be seen as a hierarchy of reduced WCWs associated with the hierarchy of adeles defined by the hierarchy of extensions of rationals. The isometries of Grassmannian would emerge in the reduction of the isometry group of WCW to a finite-D isometry group of Grassmannian and would be caused by finite measurement resolution described number theoretically. Of course, one can consider also more general flag manifolds with Kähler property as candidates for the analogs of Grassmannians. I will represent the argument in more detail later.

   This could also relate to the postulated infinite hierarchy of hyper-finite factors of type II_1 (HFFs) [K28, K8] as a correlate for the finite measurement resolution with included sub-factor inducing transformations which act trivially in the measurement resolution used.

Remark: There is an amusing connection with empiria. Topologist Barbara Shipman observed that honeybee dance allows a description in terms of flag manifold $F = SU(3)/U(1) \times U(1)$, which is the space for the choices of quantization axes of color quantum numbers and also the twistor space in CP_2 degrees of freedom [A2]. This suggest that QCD type physics might make sense in macroscopic length scales. p-Adic length scale hypothesis and the predicted long range classical color gauge fields suggest a hierarchy of QCD type physics. One can indeed construct a TGD based model of honeybee dance with aconcrete interpretation and representation for the points of F at space-time level [L10].

2.1.4 $M^8 - H$ duality

$M^8 - H$ duality provides two equivalent ways to see the dynamics with either M^8 or $H = M^4 \times CP_2$ as embedding space [L6]. One might speak of number theoretic compactification which is a completely non-dynamical analog for spontaneous compactification.

1. In M^8 picture the space-time corresponds to a zero locus for either imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial ($RE(o)$ and $IM(o)$ are defined by the decomposition $o = RE(o) + I_4 IM(o)$, where I_4 is octonion unit orthogonal to quaternionic subalgebra). The dynamics is purely algebraic and ultra-local.

2. At the level of H the dynamics is dictated by variational principle and partial differential equations. Space-time surfaces are preferred extremals of the twistor lift of Kähler action reduced to a sum of 4-D Kähler action and volume term analogous to cosmological term in GRT. The equivalence of these descriptions gives powerful constraints and should follow from the infinite number of gauge conditions at the level of H associated with a sub-algebra of supersymplectic algebra implying the required dramatic reduction of degrees of freedom [K6, K20]. One has a hierarchy of these sub-algebras, which presumably relates to the hierarchy of HFFs and hierarchy of extensions of rationals.

   H picture works very nicely in applications. For instance, the notions of field body and magnetic body are crucial in all applications.

The notion of quaternionicity, which is a central element of $M^8 - H$ duality has a deep connection with causality which I have not noticed earlier. At the level of momentum space quaternionicity means that 8-momenta -, which by $M^8 - H$-duality correspond to 4-momenta at level of M^4 and color quantum numbers at the level of CP_2 - are quaternionic. Quaternionicity means that the time component of 8-momentum, which is parallel to real octonion unit, is non-vanishing. The 8-momentum itself must be time-like, in fact light-like. In this case one can always regard the momentum as momentum in some quaternionic sub-space. Causality requires a fixed sign for the time component of the momentum.

It must be however noticed that 8-momentum can be complex: also the 4-momentum can be complex at the level of $M \times CP_2$ already classically. A possible interpretation is in terms of decay width as part of momentum as it indeed is in phenomenological description of unstable particles.

Could one require that the quaternionic momenta form a linear space with respect to octonionic sum? This is the case if the energy - that is the time-like part parallel to the real octonionic unit - has a fixed sign. The sum of the momenta is quaternionic in this case since the sum of light-like momenta is in general time-like and in special case light-like. If momenta with opposite signs of energy are allowed, the sum can become space-like and the sum of momenta is co-quaternionic.

This result is technically completely trivial as such but has a deep physical meaning. Quaternionicity at the level of 8-momenta implies standard view about causality: only time-like or at most light-like momenta and fixed sign of time-component of momentum.

2.1.5 Adelic physics

The adelization of ordinary physics fusing real number based physics and various p-adic variants of physics in order to describe cognition.

1. Adelic physics [L7, L8] gives powerful number theoretic constraints when combined with $M^8 - H$ duality and leads to the vision about evolutionary hierarchy defined by extensions of rationals. The higher the level in the hierarchy, the higher the dimension n of the extension identified in terms of Planck constant $h_{eff}/h = n$ labelling the levels of dark matter hierarchy.

2. Adelic hypothesis allows to sharpen the strong form of holography to a statement that discrete cognitive representations consisting of a finite number of points identified as points of space-time surface with M^8 coordinates in the extension of rationals fixes the space-time surface itself. This dramatic reduction would be basically due to finite measurement resolution realized as an inherent property of dynamics. Cognitive representation in fact gives the WCW coordinates of the space-time surface in WCW! WCW reduces to a number theoretic discretization of a finite-dimensional space with Kähler structure and presumably maximal isometries.

3. In ZEO space-time surface becomes analogous to a computer program determined in terms of finite net of numbers! Of course, at the QFT limit of TGD giving standard model and GRT space-time is locally much more complex since one approximates the many-sheeted space-time with single slightly curved region of M^4. This is the price paid for getting rid (or losing) the topological richness of the many-sheeted space-time crucial for the understanding living matter and even physics in galactic scales.

4. Skeptic can argue that this discretization of WCW leads to the loss of WCW geometry based on real numbers. One can however consider also continuous values for the points of cognitive representations and assigning metric to the points of cognitive representation. Metric could be defined as kind of induced metric. One slices CD by parallel CDs by shift the CD along the axis connecting its tips. This allows to see the point of cognitive representation as point at one particular CD. One shifts slightly the point along its CD. Embedding space metric allows to deduce the infinitesimal line element ds^2 and to deduce the metric components. This allows a definition of differential geometry so that the analog of WCW metric makes sense as a hierarchy of finite-dimensional metrics for space-time surfaces characterize by the cognitive representations.

   The interpretation in real context would be in terms of finite measurement resolution and the hierarchy would correspond to a hierarchy of hyper-finite factors (HFFs) [K28, K8], whose defining property is that they allow arbitrarily precise finite-dimensional approximations. What would be new is that the hierarchy of extensions of rationals would define a hierarchy of discretizations and hierarchy of HFFs.

Thabove list involves several unproven conjectures, which I can argue to be intuitively obvious with the experience of four decades: I cannot of course expect that a colleague reading for the first time about TGD would share these intuitions.

2.2 Classical TGD

Classical TGD is now rather well understood both in both $H = M^4 \times CP_2$ and M^8 pictures. Applications of classical TGD are in H picture and rather detailed phenomenology has emerged. M^8 picture has led to a rather precise vision about adelic physics and to understanding of finite measurement resolution.

2.2.1 Classical TGD in M^8 picture

Classical TGD in M^8 picture is discussed in [L6].

1. In M^8 picture one ends to an extremely simple number theoretic construction of space-time surfaces fixing only discrete or even finite number of space-time points to obtain space-time surface for a given extension of rationals. The reason is that space-time surfaces are zero loci for $RE(P)$ or $IM(P)$ of octonionic polynomials obtained by continuing real polynomial with coefficients in an extension of rationals to an octonionic polynomial.

   Needless to say, the hierarchy of algebraic extensions of rationals is what makes the dynamics at given level so simple. The coordinates of space-time surface as a point of WCW must be in the extension of rationals. As noticed, the points of space-time surface defining the cognitive representation determining the space-time surface serve as its natural WCW coordinates.

2. The highly non-trivial point is that no variational principle is involved with M^8 construction. Therefore it seems that neither WCW metric nor Kähler function is needed. If this is the case, the exponential of Kähler function definable as action exponential does not appear in scattering amplitudes and must disappear also at H-side from the scattering amplitudes.

3. Skeptic could argue that one loses general coordinate invariance in this approach. This is not true. Linear M^8 coordinates are the only possible option and forced already by symmetries. The choice octonionic and quaternionic structures fixes the linear M^8 coordinates almost uniquely since time direction is associated with real octonion unit and one spatial direction to special imaginary unit defining spin quantization axis. In algebraic approach identifying space-time surface as a zero locus of $RE(P)$ or $IM(P)$ these coordinates define space-time coordinates highly uniquely.

   Skeptic could also argue that number theoretic discretization implies reduction of the basic symmetry groups to their discrete sub-groups. This is true and one can argue that this loss of symmetry is due to the use of cognitive representations with finite resolution. Points with algebraic coordinates could be seen as a choices of representatives from a set of points, which are equivalent as far as measurement resolution is considered.

4. A physically important complication related to M^8 dynamics is the possibility of different octonionic and quaternionic structures. For instance, external particles arriving into CD correspond to different octonionic and quaternionic structures in general since Lorentz boost affects the octonionic structure changing the direction of time axis, which corresponds to the real octonionic unit. In color degrees of freedom one has wave function over different quaternionic structures: essentially color partial waves labelled by color quantum numbers [K12].

   One can apply Poincare transformations and color rotations (or transformation in sub-groups of these groups if one requires that the image points belong to the same extension) to the discrete cognitive representation defining space-time surface. The moduli spaces for these structures are essential for the understanding the standard Poincare and color quantum numbers and standard conservation laws in M^8 picture. Also the size scales of CDs define moduli as also Lorentz boosts leaving either boundary of CD unaffected.

2.2.2 Classical TGD in H picture

At the H side one action principle has partial differential equations and infinite number of gauge conditions associated with a sub-algebra of super-symplectic algebra selecting only extremely few preferred extremals of the action principle in terms of gauge conditions for a sub-algebra of super-symplectic algebra. This dynamics is conjectured to follow from the assumption that 6-D lift of space-time surface X^4 to a CP_1 bundle over X^4 is twistor space of X^4. This condition requires the analog of dimensional reduction since S^2 fiber is dynamically trivial.

For 6-D preferred extremals identifiable as twistor spaces of space-time surfaces the 6-D Kähler action in the product of twistor spaces of M^4 and CP_2 is assumed to dimensionally reduce to 4-D Kähler action plus volume term identifiable as the analog of cosmological constant term. This picture reproduces a description of scattering events highly analogous to that emerging in M^8. External particles correspond to minimal surfaces as analogs of free massless fields and all couplings disappear from the value of the action. The interior of CD corresponds to non-trivial coupling to Kähler 4-force which does not vanish. In M^8 picture one has associative and non-associative regions as counterparts of these regions.

What is remarkable is that the dynamics determined by partial differential equations plus gauge conditions would be equivalent with the number theoretic dynamics determined in terms of zero loci for real or imaginary parts of octonionic polynomials.

2.3 Scattering amplitudes in ZEO

The construction of scattering amplitudes even at the level of principle is far from well-understood. I have discussed rather concrete proposals for the twistorial construction but the feeling is that something is still missing [K25, K10, K4, K21]. This feeling might well reflect my quite too limited mathematical understanding of twistors and experience about practical construction of the scattering amplitudes. Later I will discuss possible identification of the missing piece of puzzle.

Consider first the general picture about the construction of scattering amplitudes suggested by ZEO inspired theory of quantum measurement theory defining also a theory of consciousness.

1. The portions of space-time surfaces outside CD correspond to external particles. They satisfy associativity conditions at M^8 side making possible to map them to minimal surfaces in $H = M^4 \times CP_2$ satisfying various infinite number of gauge conditions for a sub-algebra of super-symplectic algebra isomorphic with it.

   Remark: There is an additional condition requiring that associative tangent space or normal space contains fixed complex subspace of quaternions. It is not quite clear whether this condition can be generalized so that the distribution of these spaces is integrable.

   At both sides the dynamics of external particles is in a well-defined sense critical at both sides and does not depend at all on coupling constants.

2. Inside CDs associativity conditions break down in M^8 and one cannot map this spacetime region - call it X^4 - to H [L6]. It is however possible to construct counterpart of X^4 in H as a preferred extremal for the twistor lift of Kähler action by fixing the 3-surfaces at the boundaries of CD (boundary conditions). The dependence on couplings at the level of H would come from the vanishing conditions for classical Noether charges, which depend on coupling parameters.

3. If the two descriptions of the scattering amplitudes are equivalent, the dependence on coupling parameters in H should have a counterpart in M^8. Coupling constants making sense only at H side are expected to depend on the size scale of CD and on the extension of rationals defining the adele [L7, L8]. Coupling constants should be determined completely by the boundary values of Noether charges at the ends of space-time surface, and therefore by the 3-D ends of associative space-time regions representing external particles at M^8 side. This would suggest that coupling constants are functions of the coefficients of the polynomials and the points of cognitive representation.

2.3.1 Zero energy ontology and the life cycle of self

ZEO meant a decisive step in the understanding of quantum TGD since it solved the basic paradox of quantum measurement problem by forcing to realize that subjective and geometric time are not the same thing [L9].

1. Both the passive boundary of CD and the members of state pairs at it are unaffected during the sequence of state reductions analogous to weak measurements (see `http://tinyurl.com/zt36hpb`) defining self as a generalized Zeno effect. The members of state pairs associated with the active boundary change and the active boundary itself drifts farther away from the passive one in the sequence of "small" state function reductions.

   Also the space-time surfaces connecting passive and active boundaries change during the sequence of weak measurements. Only the 3-surfaces at the passive boundary are unaffected. Hence the geometric past relative to the active boundary changes during the life cycle of self. In positive energy ontology (PEO) this is not possible.

2. In "big" state function reduction the roles of passive and active boundary are changed and the arrow of time identifiable as the direction in which CD grows changes. In consciousness theory "big" state function reduction corresponds to the death of self and subsequent re-incarnations as a self with an opposite arrow of geometric time.

3. In ZEO the life cycle of self corresponds to a sequence of steps. Single step begins with a unitary time evolution in which a superposition of states associated with CDs larger than the original CD emerges. Then follows the analog of weak measurement leading to a localization to a CD in the moduli space of CDs so that it has a fixed and in general larger size. A measurement of geometric time occurs and gives rise to an experience about the flow of time.

This option would allow to identify the total S-matrix as a product of the S-matrices associated with various steps in spirit with the interpretation as a generalized Zeno effect.

Remark: In the usual description one fixes the time interval to which one assigns the S-matrix. There is no division to steps giving rise to the experience of time flow.

4. The measurement of geometric time would be a partial measurement reducing more general unitary time evolution to a unitary time evolution in the standard sense. Can one generalize the notion of partial measurement to other observables so that one would still have unitary time evolution albeit in more restricted sense? Or should one consider giving up the unitary time evolution?

   These observables should commute with the observables having the states at passive boundary as eigenstates: otherwise the state at passive boundary would change. If this picture makes sense, the “big” reduction to the opposite boundary meaning the death of self would necessarily occur when all observables commuting with the eigen observables at the passive boundary have been measured. It could of course occur already earlier.

   Should one allow measurements of all observables commuting with the eigen observables at the passive boundary. This would lead to partial de-coherence of the zero energy state. In TGD inspired quantum biology this could allow to understand ageing as an unavoidable gradual loss of the quantum coherence.

2.3.2 More detailed interpretation of ZEO

There are several questions related to the detailed interpretation of ZEO. The intuitive picture is that inside CD representing self one has collection of sub-CDs representing sub-selves identified as mental images of self. On can loosely say, that sub-CDs represent mind. The sub-CDs are connected by on mass shell lines, which correspond to external particles - matter. Sub-CDs can also have sub-CDs and the hierarchy can have several levels.

The states at the boundaries of CD have opposite total quantum numbers. One can consider two interpretations.

1. In positive energy ontology (PEO) the notion of zero energy state could be seen only as an elegant manner to express conservation laws. This is done in QFT quite generally - also in twistor approach. Also the largest CD would have external particles emanating from its boundaries travelling to the geometric past and future. One would have however have only information about the interior of the CD possessed by conscious entity for which CD plus its sub-CDs (mental images) serve as correlates.

   In this picture the arrow of time is fixed since it must be same for all sub-CDs in order to void inconsistency with the basic idea about self as generalized Zeno effect realized as a sequence of weak measurements.

2. ZEO suggest a more radical interpretation. Zero energy state defines an event. There would be the largest CD defining self and sub-CDs would correspond to mental images. There would be no external particles emanating from the boundaries of the largest CD. In this framework it becomes possible to speak about the death of self as the first state function reduction to the opposite boundary changing the roles of active and passive boundaries of self.

This picture should be consistent with what we know about arrow of time and in TGD framework with the idea that the arrow of time can also change - in particular in living matter.

1. How would the standard arrow of time emerge in ZEO? One could see the emergence of the global arrow of geometric time as a process in which the size of the largest CD increases: the sub-CDs are forced to have the same arrow of time as the largest CD and cannot make state function reductions on opposite boundary (die) independently of it. During evolution the size of the networks with the same arrow of geometric time increases and fixed arrow of geometric time is established in longer scales.

2. This picture cannot be quite correct. The applications of TGD inspired consciousness require that the mental images of self can have arrow of geometric time opposite to that of self. For instance, motor actions could be sensory perceptions in non-standard arrow of time. Memory could be communications with brain of geometric past - seeing in time direction - involving signals to geometric past requiring temporary reversals of the arrow of time at some level of self-hierarchy. Hence space-time regions with different arrows of time but forming a connected space-time surface ought to be possible.

   Many-sheeted space-time means a hierarchy of space-time sheets connected by what I call wormhole contacts having Euclidian signature of the induced metric. Space-time sheets at different levels of the hierarchy are not causally connected in the sense that one cannot speak of signal propagation in the regions of Euclidian signature. This suggests that the space-time sheets connected by wormhole contacts can have different arrows of geometric time and are associated with their own CDs.

   In this manner one would avoid the paradox resulting when sub-self - mental image - dies so that its passive boundary becomes active and the particles emanating from it end up to the passive boundary of CD, where no changes are allowed during the life cycle of self. If the particles emanating from time-reversed sub-self and up to boundaries of parallel CD, the problem is circumvented.

3. Wormhole contacts induce an interaction between Minkowskian space-time sheets that they connect. The interaction is not mediated by classical signals but by boundary conditions at the boundaries between Minkowskian regions and Euclidian wormhole contact. These two boundaries are light-like orbits of opposite wormhole throats (partonic 2-surfaces).

   In number theoretic picture the presence of wormhole contact is reflected in the properties set of points in extension of rationals defining the cognitive representation in turn defining the space-time surface. In particular, the points associated with wormhole contact have space-like distance although they are at opposite boundaries of CD and have time-like distance in the metric of embedding space. This kind of point pairs associated with wormhole contacts serve serve as a tell-tale signature for them.

3 The counterpart of the twistor approach in TGD

The analogs of twistor diagrams could emerge in TGD [K10, K21] in the following manner in ZEO.

1. Portions of space-time surfaces inside CDs would appear as analogs of vertices and the spacetime surfaces connecting them as analogs of propagator lines. The "lines" connecting sub-CDs would carry massless on mass shell states but possibly with complex momenta analogous to those appearing in twistor diagrams. This is true also classically at level of H: the coupling constants appearing in the action defining classical dynamics - at least Kähler coupling strength - are complex so that also conserved quantities have also imaginary parts.

   Remark: At the level of M^8 one does not have action principle and cannot speak of Noether charges. Here the conserved charged are associated with the symmetries of the moduli spaces such as the moduli spaces for octonion and quaternion structures [L6]. The identification of the classical charges in Cartan algebra at H level with the quantum numbers labeling wave functions in moduli space at M^8 level could be seen as a realization of quantum classical correspondence.

2. At space-time level the vertices of twistor diagrams correspond to partonic 2-surfaces in the interior of given CD. In H description fermionic lines along the light-like orbits of partonic 2-surfaces scatter at partonic 2-surfaces. If each partonic 2-surface defining a vertex is surrounded by a sub-CD, these two views about TGD variants of twistor diagrams are unified. Sub-CD can of course contain more complex structures such as pair of wormhole contacts assignable to an elementary particle.

3.1 Could the classical number theoretical dynamics define the hard core of the scattering amplitudes?

The natural hope is that the simple picture about classical dynamics at the level of M^8 should have similar counterpart at the level of scattering amplitudes in M^8. The above arguments suggest that the scattering diagrams correspond to CDs connected by external particle lines representing on mass shell particles. These surfaces are associative at the level of M^8 and minimal surfaces at the level of H. This suggests that scattering amplitude for single CD serves as a building brick for scattering amplitudes: the rest would be "just kinematics" dictated by the enormous symmetries of WCW.

1. Everything in the construction should reduce to a hard core around which one would have integrations (or sums for number theoretic realization of finite measurement resolution) over various moduli characterizing the standard quantum numbers. Twistors for M^4 and CP_2 and the moduli for the choices of CDs should correspond to essentially kinematic contribution involving no genuine dynamics.

2. The scattering amplitudes should make sense in all sectors of adele. This poses powerful constraints on them. The exponential of Kähler function reducing to action exponential can in principle appear in the description at H-side but cannot be present at M^8 side. Therefore it should disappear also at the level of H.

   If the scattering amplitude at the level of H is sum over contributions with the same value of the action exponential, the exponentials indeed cancel and I have proposed that this condition holds true. In perturbative quantum field theory it holds practically always and in integrable theories is exact. This would mean enormous simplification since all information about the action principle in H would appear in the vanishing conditions for the Noether charges of the subalgebra of super-symplectic algebra at the ends of the space-time surface. These Noether changes indeed depend on the action principle and thus on coupling constants.

3. Could the hard core in the construction of the scattering amplitudes be just the choice of the cognitive representation as points if M^8 belonging to the algebraic extension defining the adele and determining space-time surface in terms of octonionic polynomial inside this CD defining the interaction region?

   The set of points of extension of rationals in the cognitive representation defines space-time surface and also its WCW coordinates. The restriction to a cognitive representation with given number of points in given extension of rationals would mean a reduction of WCW to a finite-dimensional sub-space.

   The first wild guess is that this space is Kähler manifold with maximal symmetries - just as WCW is. A further wild guess is that these reduced WCWs are Grassmannians and correspond to those appearing in the twistor Grassmannian approach. A more general conjecture is inspired by the vision that super-symplectic gauge conditions effectively reduce the super-symplectic algebra to a Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to ADE hierarchy. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

4. One must allow the action of Galois group and this gives several options for given set X of points in algebraic extension.

   (a) One can construct $X^4(X)$ in terms of octonionic polynomial and construct a representation of Galois group as superposition of space-time surfaces obtained from space-time surface by the action of Galois group on X giving rise to new sets $X_g = g(X)$.

   (b) One can also consider the action of Galois group on X and get larger set Y of points and construct single multi-sheeted surface $X^4(Y)$. This surface corresponds to Planck constant $h_{eff}/h = n$, where n is the dimension of algebraic extension.

   (c) One can also consider the actions of sub-groups of $H \subset Gal$ to X to get space-time surface with $h_{eff}/h = m$ dividing n. There are several options corresponding to representations for all sub-groups of Galois group. A hierarchy of symmetry breakings seems to be involved with unbroken symmetry associated with the largest value of h_{eff}/h.

5. In this picture the hard core would reduce to the classical number theoretical dynamics of space-time surface in M^8. The additional degrees of freedom would be due to the possibility of different octonionic and quaternionic structures and choices of size scales and Lorentz boosts and translations of CDs. The symmetries would dictate the S-matrix in the moduli degrees of freedom: the dream is that this part of the dynamics reduces to kinematics, so to say.

   The discrete coupling constant evolution would be determined by the hierarchy of extensions of rationals and by the hierarchy of p-adic length scales. The cancellation of radiative corrections in the sense of sub-CDs inside CDs could be achieved by replacing coupling constant evolution with its discrete counterpart.

If this dream has something to do with reality, the construction of scattering amplitudes would reduce to their construction in moduli degrees of freedom and here the generalization of twistorial approach relying on Yangian symmetry allowing to identify scattering amplitudes as Yangian invariants might "trivialize" the situation. It will be found that the Yangian symmetry could corresponds to general coordinate transformations for the reduced WCW forced by the restriction of the spacetime surfaces to those allowed by octonionic polynomials with coefficients in the extensiom of rationals.

3.2 Do loop contributions to the scattering amplitudes vanish in TGD framework?

In TGD scattering amplitudes interpreted as zero energy states would correspond at embedding space level to collections of space-time surfaces inside CDs analogous to vertices and connected by lines defined by the space-time surfaces representing on-mass-shell particles. One would have massless particles in 8-D sense. The quaternionicity of 8-momentum leads to $M^4 \times CP_2$ picture and CP_2 twistors should replace E^4 twistors of M^8 approach.

3.2.1 Why loop corrections should vanish?

There are several arguments suggesting that the loop contributions should vanish in TGD framework. This would give rise to a discrete coupling constant evolution analogous to a sequence of phase transitions between different critical coupling parameters. Amplitudes would be obtained as tree diagrams.

1. In ZEO it is far from clear what the basic operation defining the loop contribution could even mean. One would have zero energy state for which the members of added particle pair have opposite but momenta but the amplitude is superposition of states with varying momenta. Why should one allow zero energy states containing one particle which is not an eigenstate of momentum? This suggests that ZEO does not allow loop contributions at all: the distinction between PEO and ZEO would make itself visible in rather dramatic manner.

2. The restriction of the BCFW to tree diagrams is internally consistent since the loop term is identically vanishing in this case. The first term in the BCFW for diagram with l loops involves a factor with $l > 0$ loops which vanishes. In $l = 1$ case the second term is obtained from $(n + 2, l - 1 = 0)$ diagram by generating loop but this vanishes by assumption.

3. Number theoretic vision does not favor the decomposition of the amplitude to an infinite sum of amplitudes since this is expected to lead to the emergence of transcendental numbers and functions in the amplitude in conflict with the number theoretical universality.

   Loops indeed give logarithms and poly-logarithms of rational functions of external momenta in Grassmannian approach. This violates the number theoretical universality since the p-adic counterpart of logarithm exist only for the argument of form $x = 1 + O(p)$. This condition cannot hold true for all primes simultaneously.

Discrete coupling constant evolution suggests the vanishing of loops. One can imagine two alternative mechanisms for the vanishing of loop contributions. Either the loop contributions do not make sense at all in ZEO, or the sum of loop contributions for the critical values of coupling constants vanishes. The summing up of loop contributions to zero for critical values of couplings should happen for all values of external momenta and other quantum numbers: this does not look plausible.

3.2.2 General number theoretic ideas about coupling constant evolution

The discrete coupling constant evolution would be associated with the scale hierarchy for CDs and the hierarchy of extensions of rationals.

1. Discrete p-adic coupling constant evolution would naturally correspond to the dependence of coupling constants on the size of CD. For instance, I have considered a concrete but rather ad hoc proposal for the evolution of Kähler couplings strength based on the zeros of Riemann zeta [L2]. Number theoretical universality suggests that the size scale of CD identified as the temporal distance between the tips of CD using suitable multiple of CP_2 length scale as a length unit is integer, call it l. The prime factors of the integer could correspond to preferred p-adic primes for given CD.

2. I have also proposed that the so called ramified primes of the extension of rationals correspond to the physically preferred primes. Ramification is algebraically analogous to criticality in the sense that two roots understood in very general sense co-incide at criticality. Could the primes appearing as factors of l be ramified primes of extension? This would give strong correlation between the algebraic extension and the size scale of CD.

In quantum field theories coupling constants depend in good approximation logarithmically on mass scale, which would be in the case of p-adic coupling constant evolution replaced with an integer n characterizing the size scale of CD or perhaps the collection of prime factors of n (note that one cannot exclude rational numbers as size scales). Coupling constant evolution could also depend on the size of extension of rationals characterized by its order and Galois group.

In both cases one expects approximate logarithmic dependence and the challenge is to define "number theoretic logarithm" as a rational number valued function making thus sense also for p-adic number fields as required by the number theoretical universality.

1. Coupling constant evolution with respect to CD size scale

Consider first the coupling constant as a function of the length scale $l_{CD}(n)/l_{CD}(1) = n$.

1. The number $\pi(n)$ of primes $p \leq n$ behaves approximately as $\pi(n) = n/log(n)$. This suggests the definition of what might be called "number theoretic logarithm" as $Log(n) \equiv n/\pi(n)$. Also iterated logarithms such $log(log(x))$ appearing in coupling constant evolution would have number theoretic generalization.

2. If the p-adic variant of $Log(n)$ is mapped to its real counterpart by canonical identification involving the replacement $p \rightarrow 1/p$, the behavior can very different from the ordinary logarithm. $Log(n)$ increases however very slowly so that in the generic case one can expect $Log(n) < p_{max}$, where p_{max} is the largest prime factor of n, so that there would be no dependence on p for p_{max} and the image under canonical identification would be number theoretically universal.

   For $n = p^k$, where p is small prime the situation changes since $Log(n)$ can be larger than small prime p. Primes p near primes powers of 2 and perhaps also primes near powers of 3 and 5 - at least - seem to be physically special. For instance, for Mersenne prime $M_k = 2^k - 1$ there would be dramatic change in the step $M_k \rightarrow M_k + 1 = 2^k$, which might relate to its special physical role.

3. One can consider also the analog of $Log(n)$ as

$$Log(n) = \sum_p k_p Log(p) \quad ,$$

   where p^{k_i} is a factor of n. $Log(n)$ would be sum of number theoretic analogs for primes factors and carry information about them.

One can extend the definition of $Log(x)$ to the rational values $x = m/n$ of the argument. The logarithm $Log_b(n)$ in base $b = r/s$ can be defined as $Log_b(x) = Log(x)/Log(b)$.

4. For $p \in \{2, 3, 5\}$ one has $Log(p) > log(p)$, where for larger primes one has $Log(p) < log(p)$. One has $Log(2) = 2 > log(2) = .693...$, $Log(3) = 3k/2 > log(3) = 1.099$, $Log(5) = 5/3 = 1.666.. > log(5) = 1.609$. For $p = 7$ one has $Log(7) = 7/4 \simeq 1.75 < log(7) \simeq 1.946$. Hence these primes and CD size scales n involving large powers of $p \in \{2, 3, 5\}$ ought to be physically special as indeed conjectured on basis of p-adic calculations and some observations related to music and biological evolution [K16, K17, K19, K13].

   In particular, for Mersenne primes $M_k = 2^k - 1$ one would have $Log(M_k) \simeq klog(2)$ for large enough k. For $Log(2^k)$ one would have $k \times Log(2) = 2k > log(2^k) = klog(2)$: there would be sudden increase in the value of $Log(n)$ at $n = M_k$. This jump in p-adic length scale evolution might relate to the very special physical role of Mersenne primes strongly suggested by p-adic mass calculations [K12].

5. One can wonder whether one could replace the $log(p)$ appearing as a unit in p-adic negentropy [K14] with a rational unit $Log(p) = p/\pi(p)$ to gain number theoretical universality? One could therefore interpret the p-adic negentropy as real or p-adic number for some prime. Interestingly, $|Log(p)|_p = 1/p$ approaches zero for large primes p (eye cannot see itself!) whereas $|Log(p)|_q = 1/|\pi(p)|_q$ has large values for the prime power factors q^r of $\pi(p)$.

2. The dependence of $1/\alpha_K$ on the extension of rationals

Consider next the dependence on the extension of rationals. The natural algebraization of the problem is to consider the Galois group of the extension.

1. Consider first the counterparts of primes and prime factorization for groups. The counterparts of primes are simple groups, which do not have normal subgroups H satisfying $gH = Hg$ implying invariance under automorphisms of G. Simple groups have no decomposition to a product of sub-groups. If the group has normal subgroup H, it can be decomposed to a product $H \times G/H$ and any finite group can be decomposed to a product of simple groups.

   All simple finite groups have been classified (see `http://tinyurl.com/jn44bxe`). There are cyclic groups, alternating groups, 16 families of simple groups of Lie type, 26 sporadic groups. This includes 20 quotients G/H by a normal subgroup of monster group and 6 groups which for some reason are referred to as pariahs.

2. Suppose that finite groups can be ordered so that one can assign number $N(G)$ to group G. The roughest ordering criterion is based on $ord(G)$. For given order $ord(G) = n$ one has all groups, which are products of cyclic groups associated with prime factors of n plus products involving non-Abelian groups for which the order is not prime. $N(G) > ord(G)$ thus holds true. For groups with the same order one should have additional ordering criteria, which could relate to the complexity of the group. The number of simple factors would serve as an additional ordering criterion.

   If its possible to define $N(G)$ in a natural manner then for given G one can define the number $\pi_1(N(G))$ of simple groups (analogs of primes) not larger than G. The first guess is that that the number $\pi_1(N(G))$ varies slowly as a function of G. Since Z_i is simple group, one has $\pi_1(N(G)) \geq \pi(N(G))$.

3. One can consider two definitions of number theoretic logarithm, call it Log_1.

$$\begin{array}{ll} a) & Log_1(N(G)) = \frac{N(G)}{\pi_1(N(G))} \ , \\ b) & Log_1(G) = \sum_i k_i Log_1(N(G_i)) \ , \quad Log_1(N(G_i)) = \frac{N(G_i)}{\pi_1(N(G_i))} \ . \end{array} \tag{3.1}$$

   Option a) does not provide information about the decomposition of G to a product of simple factors. For Option b) one decomposes G to a product of simple groups G_i: $G = \prod_i G_i^{k_i}$ and defines the logarithm as Option b) so that it carries information about the simple factors of G.

4. One could organize the groups with the same order to same equivalence class. In this case the above definitions would give

$$\begin{array}{ll} a) & Log_1(ord(G)) = \frac{ord(G)}{\pi_1(ord(G))} < Log(ord(G)) \ , \\ b) & Log_1(ord(G)) = \sum_i k_i Log(ord(G_i)) \ , \ \ Log_1(ord(G_i)) = \frac{ord(G_i)}{\pi_1(ord(G_i))} \ . \end{array} \tag{3.2}$$

Besides groups with prime orders there are non-Abelian groups with non-prime orders. The occurrence of same order for two non-isomorphic finite simple groups is very rare (see `http://tinyurl.com/ydd6uomb`). This would suggests that one has $\pi_1(ord(G)) < ord(G)$ so that $Log_1(ord(G))/ord(G) < 1$ would be true.

5. For orders $n(G) \in \{2,3,5\}$ one has $Log_1(n(G)) = Log(n(G)) > log(n(G))$ so that the ordes $n(G)$ involving large factors of $p \in \{2,3,5\}$ would be special also for the extensions of rationals. S_3 with order 6 is the first non-abelian simple group. One has $\pi(S_3) = 4$ giving $Log(6) = 6/4 = 1.5 < log(6) = 1.79$ so that S_3 is different from the simple groups below it.

To sum up, number theoretic logarithm could provide answer to the long-standing question what makes Mersenne primes and also other small primes so special.

3.2.3 Considerations related to coupling constant evolution and Riemann zeta

I have made several number theoretic peculations related to the possible role of zeros of Riemann zeta in coupling constant evolution. The basic problem is that it is not even known whether the zeros of zeta are rationals, algebraic numbers or genuine transcendentals or belong to all these categories. Also the question whether number theoretic analogs of ζ defined for p-adic number fields could make sense in some sense is interesting.

1. Is number theoretic analog of ζ possible using $Log(p)$ instead of $log(p)$?

The definition of $Log(n)$ based on factorization $Log(n) \equiv \sum_p k_p Log(p)$ allows to define the number theoretic version of Riemann Zeta $\zeta(s) = \sum n^{-s}$ via the replacement $n^{-s} = exp(-log(n)s) \rightarrow exp(-Log(n)s)$.

1. In suitable region of plane number-theoretic Zeta would have the usual decomposition to factors via the replacement $1/(1-p^{-s}) \rightarrow 1/(1-exp(-Log(p)s)$. p-Adically this makes sense for $s = O(p)$ and thus only for a finite number of primes p for positive integer valued s: one obtains kind of cut-off zeta. Number theoretic zeta would be sensitive only to a finite number of prime factors of integer n.

2. This might relate to the strong physical indications that only a finite number of cognitive representations characterized by p-adic primes are present in given quantum state: the ramified primes for the extension are excellent candidates for these p-adic primes. The size scale n of CD could also have decomposition to a product of powers of ramified primes. The finiteness of cognition conforms with the cutoff: for given CD size n and extension of rationals the p-adic primes labelling cognitive representations would be fixed.

3. One can expand the regions of converge to larger p-adic norms by introducing an extension of p-adics containing e and some of its roots (e^p is automatically a p-adic number). By introducing roots of unity, one can define the phase factor $exp(-iLog(n)Im(s))$ for suitable values of $Im(s)$. Clearly, $exp(-ipIm(s))/\pi(p))$ must be in the extension used for all primes p involved. One must therefore introduce prime roots $exp(i/\pi(p))$ for primes appearing in cutoff. To define the number theoretic zeta for all p-adic integer values of $Re(s)$ and all integer values of $Im(s)$, one should allow all roots of unity $(ep(i2\pi/n))$ and all roots $e^{1/n}$: this requires infinite-dimensional extension.

4. One can thus define a hierarchy of cutoffs of zeta: for this the factorization of Zeta to a finite number of "prime factors" takes place in genuine sense, and the points $Im(s) = ik\pi(p)$

give rise to poles of the cutoff zeta as poles of prime factors. Cutoff zeta converges to zero for $Re(s) \to \infty$ and exists along angles corresponding to allowed roots of unity. Cutoff zeta diverges for $(Re(s) = 0, Im(s) = ik\pi(p))$ for the primes p appearing in it.

Remark: One could modify also the definition of ζ for complex numbers by replacing $exp(log(n)s)$ with $exp(Log(n)s)$ with $Log(n) = \sum_p k_p Log(p)$ to get the prime factorization formula. I will refer to this variant of zeta as modified zeta ($\tilde{\zeta}$) below. $\tilde{\zeta}$ would carry explicit number theoretic information via the dependence of its "prime factors" $1/(1 - exp(-Log(p)s))$.

2. Could the values of $1/\alpha_K$ be given as zeros of ζ or of $\tilde{\zeta}$

In [L2] I have discussed the possibility that the zeros $s = 1/2 + iy$ of Riemann zeta at critical line correspond to the values of complex valued Kähler coupling strength α_K: $s = i/\alpha_K$. The assumption that p^{iy} is root of unity for some combinations of p and y $[log(p)y = (r/s)2\pi]$ was made. This does not allow s to be complex rational. If the exponent of Kähler action disappears from the scattering amplitudes as $M^8 - H$ duality requires, one could assume that s has rational values but also algebraic values are allowed.

1. If one combines the proposed idea about the Log-arithmic dependence of the coupling constants on the size of CD and algebraic extension with $s = i/\alpha_K$ hypothesis, one cannot avoid the conjecture that the zeros of zeta are complex rationals. It is not known whether this is the case or not. The rationality would not have any strong implications for number theory but the existence irrational roots would have (see `http://tinyurl.com/y8bbnhe3`). Interestingly, the rationality of the roots would have very powerful physical implications if TGD inspired number theoretical conjectures are accepted.

   The argument discussed below however shows that complex rational roots of zeta are not favored by the observations [A6] about the Fourier transform for the characteristic function for the zeros of zeta. Rather, the findings suggest that the imaginary parts [L1] should be rational multiples of 2π, which does not conform with the vision that $1/\alpha_K$ is algebraic number. The replacement of $log(p)$ with $Log(p)$ and of 2π with is natural p-adic approximation in an extension allowing roots of unity however allows $1/\alpha_K$ to be an algebraic number. Could the spectrum of $1/\alpha_K$ correspond to the roots of ζ or of $\tilde{\zeta}$?

2. A further conjecture discussed in [L2] was that there is 1-1 correspondence between primes $p \simeq 2^k$, k prime, and zeros of zeta so that there would be an order preserving map $k \to s_k$. The support for the conjecture was the predicted rather reasonable coupling constant evolution for α_K. Primes near powers of 2 could be physically special because $Log(n)$ decomposes to sum of $Log(p)$:s and would increase dramatically at $n = 2^k$ slightly above them.

   In an attempt to understand why just prime values of k are physically special, I have proposed that k-adic length scales correspond to the size scales of wormhole contacts whereas particle space-time sheets would correspond to $p \simeq 2^k$. Could the logarithmic relation between L_p and L_k correspond to logarithmic relation between p and $\pi(p)$ in case that $\pi(p)$ is prime and could this condition select the preferred p-adic primes p?

3. The argument of Dyson for the Fourier transform of the characteristic function for the set of zeros of ζ

Consider now the argument suggesting that the roots of zeta cannot be complex rationals. On basis of numerical evidence Dyson [A6] (`http://tinyurl.com/hjbfsuv`) has conjectured that the Fourier transform for the characteristic function for the critical zeros of zeta consists of multiples of logarithms $log(p)$ of primes so that one could regard zeros as one-dimensional quasi-crystal.

This hypothesis makes sense if the zeros of zeta decompose into disjoint sets such that each set corresponds to its own prime (and its powers) and one has $p^{iy} = U_{m/n} = exp(i2\pi m/n)$ (see the appendix of [L1]). This hypothesis is also motivated by number theoretical universality [K27, L7].

1. One can re-write the discrete Fourier transform over zeros of ζ at critical line as

$$f(x) = \sum_y exp(ixy)) \ , \ y = Im(s) \ .$$

The alternative form reads as

$$f(u) = \sum_s u^{iy} \ , \ u = exp(x) \ .$$

$f(u)$ is located at powers p^n of primes defining ideals in the set of integers.

For $y = p^n$ one would have $p^{iny} = exp(inlog(p)y)$. Note that $k = nlog(p)$ is analogous to a wave vector. If $exp(inlog(p)y)$ is root of unity as proposed earlier for some combinations of p and y, the Fourier transform becomes a sum over roots of unity for these combinations: this could make possible constructive interference for the roots of unity, which are same or at least have the same sign. For given p there should be several values of $y(p)$ with nearly the same value of $exp(inlog(p)y(p))$ whereas other values of y would interfere deconstructively.

For general values $y = x^n$ $x \neq p$ the sum would not be over roots of unity and constructive interference is not expected. Therefore the peaking at powers of p could take place. This picture does not support the hypothesis that zeros of zeta are complex rational numbers so that the values of $1/\alpha_K$ correspond to zeros of zeta and would be therefore complex rationals as the simplest view about coupling constant evolution would suggest.

Remark: Mumford has argued (`http://tinyurl.com/zemw27o`) that the Fourier transform should include also the trivial zeros at $s = -2, -4, -6...$ giving and exponentially small contributions and providing a slowly varying background to the Fourier transform.

2. What if one replaces $log(p)$ with $Log(p) = p/\pi(p)$, which is rational and thus ζ with $\tilde{\zeta}$? For large enough values of p $Log(p) \simeq log(p)$ finite computational accuracy does not allow distinguish $Log(p)$ from $log(p)$. For $Log(p)$ one could thus understand the finding in terms of constructive interference for the roots of unity if the roots of zeta are of form $s = 1/2 + i(m/n)2\pi$. The value of y cannot be rational number and $1/\alpha_K$ would have real part equal to y proportional to 2π which would require infinite-D extension of rationals. In p-adic sectors infinite-D extension does not conform with the finiteness of cognition.

   Remark: It is possible to check by numerical calculations whether the locus of complex zeros of $\tilde{\zeta}$ is at line $Res(2) = 1/2$. If so, then Fourier transform would make sense. One can also check whether the peaks at $nlog(p)$ are shifted to $nLog(p)$: for $p = 2$ one would have $Log(2) = 2 > log(2)$. The positions of peaks should shift to the right for $p = 2, 3, 5$ and to the left for $p > 5$. This should be easy to check by numerical calculations.

3. Numerical calculations have however finite accuracy, and allow also the possibility that y is algebraic number approximating rational multiple of 2π in some natural manner. In p-adic sectors would obtain the spectrum of y and $1/\alpha_K$ as algebraic numbers by replacing 2π in the formula $is = \alpha_K = i/2 + q \times 2\pi$, $q = r/s$, with its approximate value:

$$2\pi \rightarrow sin(2\pi/n)n = i\frac{n}{2}(exp(i2\pi/n) - exp(-i2\pi/n))$$

   for an extension of rationals containing n:th of unity. Maximum value of n would give the best approximation. This approximation performed by fundamental physics should appear in the number theoretic scattering amplitudes in the expressions for $1/\alpha_K$ to make it algebraic number.

   y can be approximated in the same manner in p-adic sectors and a natural guess is that $n = p$ defines the maximal root of unity as $exp(i2\pi/p)$. The phase $exp(ilog(p)y)$ for $y = qsin(2\pi/n(y))$, $q = r/s$, is replaced with the approximation induced by $log(p) \rightarrow Log(p)$ and $2\pi \rightarrow sin(2\pi/n)n$ giving

$$exp(ilog(p)y) \rightarrow exp(iq(y)sin(2\pi/n(y))\frac{p}{\pi(p)}) \ .$$

   If s in $q = r/s$ does not contain higher powers of p, the exponent exists p-adically for this extension and can can be expanded in positive powers of p as

$$\sum_n i^n q^n sin(2\pi/p)^n (p/\pi(p))^n \quad .$$

This makes sense p-adically.

Also the actual complex roots of ζ could be algebraic numbers:

$$s = i/2 + q \times sin(\frac{2\pi}{n(y)})n(y) \quad .$$

If the proposed correlation between p-adic primes $p \simeq 2^k$, k prime and zeros of zeta predicting a reasonable coupling constant evolution for $1/\alpha_K$ is true, one can have naturally, $n(y) = p(y)$, where p is the p-adic prime associated with y: the accuracy in angle measurement would increase with the size scale of CD. For given p there could be several roots y with same $p(y)$ but different $q(y)$ giving same phases or at least phases with same sign of real part.

Whether the roots of $tilde\zeta$ are algebraic numbers and at critical line $Re(s) = 1/2$ is an interesting question.

Remark: This picture allows many variants. For instance, if one assumes standard zeta, one could consider the possibility that the roots y_p associated with p and giving rise to constructive interference are of form $y = q \times (Log(p)/log(p)) \times sin(2\pi/p)p$, $q = r/s$.

4. Could functional equation and Riemann hypothesis generalize?

It is interesting to list the elementary properties of the $\tilde{\zeta}$ before trying to answer to the questions of the title.

1. The replacement $log(n) \rightarrow Log(n) \equiv sum_p k_p Log(p)$ implies that $\tilde{\zeta}$ codes explicitly number theoretic information. Note that $Log(n)$ satisfies the crucial identity $Log(mn) = Log(m) + Log(n)$. $\tilde{\zeta}$ is an analog of partition function with rational number valued $Log(n)$ taking the role of energy and $1/s$ that of a complex temperature. In ZEO this partition function like entity could be associated with zero energy state as a "square root" of thermodynamical partition function: in this case complex temperatures are possible. $|\tilde{\zeta}|^2$ would be the analog of ordinary partition function.

2. Reduction of $\tilde{\zeta}$ to a product of "prime factors" $1/[1-exp(-Log(p)s)]$ holds true by $Log(n) \equiv sum_p k_p Log(p)$, $Log(p) = p/\pi(p)$.

3. $\tilde{\zeta}$ is a combination of exponentials $exp(-Log(n)s)$, which converge for $Re(s) > 0$. For ζ one has exponentials $exp(-log(n)s)$, which also converge for $Re(s) > 0$: the sum $\sum n^{-s}$ does not however converge in the region $Re(s) < 1$. Presumably $\tilde{\zeta}$ fails to converge for $Re(s) \leq 1$. The behavior of terms $exp(-Log(n)s)$ for large values of n is very similar to that in ζ.

4. One can express ζ o in terms of η function defined as

   $$\eta(s) = \sum (-1)^n n^{-s} \quad .$$

   The powers $(-1)^n$ guarantee that η converges (albeit not absolutely) inside the critical strip $0 < s < 1$.

   By using a decomposition of integers to odd and even ones, one can express ζ in terms of η:

   $$\zeta = \frac{\eta(s)}{(-1 + 2^{-s+1})} \quad .$$

   This definition converges inside critical strip. Note the pole at $s = 1$ coming from the factor.

   One can define also $\tilde{\eta}$:

   $$\tilde{\eta}(s) = \sum (-1)^n e^{-Log(n)s} \quad .$$

The formula relating $\tilde{\zeta}$ and $\tilde{\eta}$ generalizes: 2^{-s} is replaced with $exp(-2s)$ $(Log(2) = 2)$:

$$\tilde{\zeta} = \frac{\tilde{\eta}(s)}{-1 + 2exp^{-2s}} \quad .$$

This definition $\tilde{\zeta}$ converges in the critical strip $Re(s) \in (0,1)$ and also for $Re(s) > 1$. $\tilde{\zeta}(1-s)$ converges for $Re(s) < 1$ so that in $\tilde{\eta}$ representation both converge.

Note however that the poles of ζ at $s = 1$ has shifted to that at $s = log(2)/2$ and is below $Re(s) = 1/2$ line. If a symmetrically posioned pole at $s = 1 - log(2)/2$ is not present in $\tilde{\eta}$, functional equation cannot be true.

5. $Log(n)$ approaches $log(n)$ for integers n not containing small prime factors p for which $\pi(n)$ differs strongly from $p/log(p)$. This suggests that allowing only terms $exp(-Log(n)s)$ in the sum defining $\tilde{\zeta}$ not divisible by primes $p < p_{max}$ might give a cutoff $\tilde{\zeta}^{cut,p_{max}}(s)$ behaving very much like ζ from which "prime factors" $1/(1 - exp(-Log(p)s)$, $p < p_{max}$ are dropped of. This is just division of $\tilde{\zeta}$ by these factors and at least formally, this does not affect the zeros of $\tilde{\zeta}$. Arbitrary number of factors can be droped. Could this mean that $\tilde{\zeta}^{cut}$ has same or very nearly same zeros as ζ at critical line? This sounds paradoxical and might reflect my sloppy thinking: maybe the lack of the absolute implies that the conclusion is incorrect.

The key questions are whether $\tilde{\zeta}$ allows a generalization of the functional equation $\xi(s) = \xi(1-s)$ with $\xi(s) = \frac{1}{2}s(s-1)\Gamma(s/2)\pi^{-s/2}\zeta(s)$ and whether Riemann hypothesis generalizes. The derivation of the functional equation is quite a tricky task and involves integral representation of ζ .

1. One can start from the integral representation of ζ true for $s > 0$.

$$\zeta(s) = \frac{1}{(1-2^{1-s})\Gamma(s)} \int_0^{\infty} \frac{t^{s-1}}{e^t + 1} dt \quad , \quad Re(s) > 0 \quad .$$

deducible from the expression in terms of $\eta(s)$. The factor $1/(1 + e^t)$ can be expanded in geometric series $1/(1 + e^t) = \sum (-1)^n exp(nt)$ converning inside the critical strip. One formally performs the integrations by taking nt as an integration variable. The integral gives the result $\sum (-1)^n/n^z)\Gamma(s)$.

The generalization of this would be obtained by a generalization of geometric series:

$$1/(1 + e^t) = \sum (-1)^n exp(nt) \rightarrow \sum (-1)^n e^{exp(Log(n))t}$$

in the integral representation. This would formally give $\tilde{\zeta}$: the only difference is that one takes $u = exp(Log(n))t$ as integration variable.

One could try to prove the functional equation by using this representation. One proof (see `http://tinyurl.com/yak93hyr`) starts from the alternative expression of ζ as

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_1^{\infty} \frac{t^{s-1}}{e^t - 1} dt \quad , \quad Re(s) > 1 \quad .$$

One modifies the integration contour to a contour C coming from $+\infty$ above positive real axis, circling the origin and returning back toc$+\infty$ below the real axes to get a modified representation of ζ:

$$\zeta(s) = \frac{1}{2isin(\pi s)\Gamma(s)} \int_1^{\infty} \frac{(-w)^{s-1}}{e^w - 1} dw \quad , \quad Re(s) > 1 \quad .$$

One modifies the C further so that the origin is circled around a square with vertices at $\pm(2n+1)\pi$ and $\pm i(2n+1)\pi$.

One calculates the integral the integral along C as a residue integral. The poles of the integrand proportional to $1/(1 - e^t)$ are at imaginary axis and correspond to $w = ir2\pi$, $r \in Z$. The residue integral gives the other side of the functional equation.

2. Could one generalize this representation to the recent case? One must generalize the geometric series defined by $1/(e^w-1)$ to $-\sum e^{exp(Log(n))w}$. The problem is that one has only a generalization of the geometric series and not closed form for the counterpart of $1/(exp(w)-1)$ so that one does not know what the poles are. The naïve guess is that one could compute the residue integrals term by term in the sum over n. An equally naïve guess would be that for the poles the factors in the sum are equal to unity as they would be for Riemann zeta. This would give for the poles of n:th term the guess $w_{n,r} = r2\pi/exp(Log(n)$, $r \in Z$. This does not however allow to deduce the residue at poles.Note that the poles of $\tilde{\eta}$ at $s = log(2)/2$ suggests that functional equation is not true.

There is however no need for a functional equation if one is only interested in $F(s) \equiv \tilde{\zeta}(s) + \tilde{\zeta}(1-s)$ at the critical line! Also the analog of Riemann hypothesis follows naturally!

1. In the representation using $\tilde{\eta}$ $F(s)$ converges at critical striple and is *real*(!) at the critical line $Re(s) = 1/2$ as follows from the fact that $1-s = \overline{s}$ for $Re(s) = 1/2$! Hence $F(s)$ is expected to have a large number of zeros at critical line. Presumably their number is infinite, since $F(s)^{cut,p_{max}}$ approaches $2\zeta^{cut,p_{max}}$ for large enough p_{max} at critical line.

2. One can define a different kind of cutoff of $\tilde{\zeta}$ for given n_{max}: $n < n_{max}$ in the sum over $e^{-Log(n)s}$. Call this cutoff $\tilde{\zeta}^{cut,n_{max}}$. This cutoff must be distinguished from the cutoff $\tilde{\zeta}^{cut,p_{max}}$ obtained by dropping the "prime factors" with $p < pmax$. The terms in the cutoff are of the form $u^{\sum k_p p/\pi(p)}$, $u = exp(-s)$. It is analogous to a polymomial but with fractional powers of u. It can be made a polynomial by a change of variable $u \to v = exp(-s/a)$, where a is the product of all $\pi(p)$:s associated with all the primes involved with the integers $n < n_{max}$.

   One could solve numerically the zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ using program modules calculating $\pi(p)$ for a given p and roots of a complex polynomial in given order. One can check whether also all zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ might reside at critical line.

3. One an define also $F(s)^{cut,n_{max}}$ to be distinguished from $F(s)^{cut,p_{max}}$. It reduces to a sum of terms $exp(-Log(n)/2)cos(-Log(n)y)$ at critical line, $n < n_{max}$. Cosines come from roots of unity. $F(s)$ function is not sum of rational powers of $exp(-iy)$ unlike $\zeta(\tilde{s})$. The existence of zero could be shown by showing that the sign of this function varies as function of y. The functions $cos(-Log(n)y)$ have period $\Delta y = 2\pi/Log(n)$. For small values of n the exponential terms $exp(-Log(n)/2)$ are largest so that they dominate. For them the periods Δy are smallest so that one expected that the sign of both $F(s)$ and $F(s)^{cut,n_{max}}$ varies and forces the presence of zeros.

   One could perhaps interpret the system as quantum critical system. The rather large rapidly varying oscillatory terms with $n < n_{max}$ with small $Log(n)$ give a periodic infinite set of approximate roots and the exponentially smaller slowly varying higher terms induce small perturbations of this periodic structure. The slowly varying terms with large $Log(n)$ become however large near the $Im(s) = 0$ so that here the there effect is large and destroys the period structure badly for small root of $\hat{\zeta}$.

3.2.4 Is the vanishing of the loop corrections consistent with unitarity?

Skeptic could argue that the vanishing of loop corrections is not consistent with unitarity. The following argument however shows that the fact that momenta in TGD framework are 8-D light-like momenta could save the situation. If not only single particle states but also *many-particle states* have light-like 8-momenta, the discontinuity of the amplitude at pole $P^2(M^8) = 0$ implies the discontinuity of the amplitude as function of $s \equiv P^2(M^4)$ along s-axis.

Minkowskian contribution to mass squared would essentially the sum of conformal (stringy) contribution from vibrational degrees of freedom and color contribution from CP_2 degrees of freedom. This suggests a weak form of color confinement: many-particle states could have vanishing color hyper charge and isospin but the eigenvalue value of color Casimir operator would be non-vanishing.

To get more concrete view about the situation the reader is encouraged to study the slides of Jaroslav Trnka explaining BCFW recursion formula [B6] (see `http://tinyurl.com/pqjzffj`) or the article [B4] of Elvang and Huang (see `http://tinyurl.com/y9rhbzhk`).

1. Unitarity condition $SS^\dagger = Id$ for S-matrix $S = 1 + iT$ gives $i(T - T^\dagger) = TT^\dagger$. For forward scattering the physical interpretation is that the discontinuity of $-2Im(T) = i(T - T^\dagger)$ in forward scattering as a function of total mass s above kinematical threshold along real axis is essentially the total scattering rate.

2. For a given tree amplitude, which is rational function, one replaces external momenta p_i with $\hat{p}_i = p_i + zr_i$. r_i real, light-like and orthogonal to each other and their sum vanishes. This gives on mass shell scattering amplitude with complex light-like momenta satisfying conservation conditions.

3. One can consider any non-trivial subset I of momenta and for this set one has $\hat{P}_I^2 = P_i^2 + 2zP \cdot R_I$, where one has $P_I = \sum_i p_i$ and $R_I = \sum_i r_i$. This gives

$$\hat{P}_I^2 = -P_I^2 \frac{(z - z_I)}{z_I} \quad , \quad z_I = \frac{P_I^2}{2P_I \cdot R_I} \quad .$$

The poles of the modified amplitude $\hat{A}_n(z)$ come from the propagators at $\hat{P}_I^2 = 0$ and correspond to the points $z = z_I$.

4. From the modified scattering amplitude $\hat{A_n}(z)$ one can obtain the original scattering amplitude by performing a residue integral for $\hat{A_n}(z)/z$ along a curve enclosing the poles z_I. This gives

$$A_n = \hat{A_n}(z = 0) + \sum_{z_I} Res_{z=z_I}(\frac{\hat{A}_n(z)}{z}) + B_n \quad .$$

B_n comes from the possible pole at $z = \infty$ and is often assumed to vanish. If so, the amplitude factorizes into a sum of products

$$Res_{z=z_I} \frac{\hat{A}_n(z)}{z} = \sum_I \hat{A}_L(z_I) \frac{1}{P_I^2} \hat{A}_R(z_I) \quad .$$

The amplitudes appearing in the product are for modified complex momenta.

The vanishing of loop corrections thus implies that the product terms $\hat{A}_L(1/P^2)\hat{A}_R$ in the BCFW formula give rational functions having no cuts just as the number theoretical vision demands. The discontinuities of the imaginary part of the amplitude are at poles and reduce to the products $\hat{A}_L\hat{A}_R$ with complex on-mass- shell light-like momenta as unitarity demands.

For forward scattering the discontinuity would be essentially positive definite total scattering rate. It would be however non-vanishing only at $P^2 = 0$ so that scattering rate could be non-vanishing only for $P^2 = 0$! This does not make sense in 4-D physics. Is it possible to overcome this difficulty in TGD framework?

1. The first thing to notice is that classical TGD predicts complex Noether charges since for instance Kähler coupling strength has imaginary part. This would suggest that the momenta of incoming particles could be complex. Could complex value of $P(M^4) \equiv P$ implying

$$P^2 = Re(P)^2 - Im(P)^2 + i2Re(P) \cdot Im(P) = 0$$

save the situation? The condition requires that $Re(P)$ and $Im(P)$ are light-like and parallel so that one would obtain only light-like four-momenta as total M^4 momenta.

2. However, in TGD light-likeness holds true in 8-D sense for single particle states: this led to the proposed generalization of twistor approach allowing particles to be massive in 4-D sense. $M^8 - H$ duality allows to speak about light-like M^8 momenta satisfying quaternionicity condition. The wave functions in CP_2 degrees of freedom emerge from momentum wave functions in M^8 degrees of freedom respecting quaternionicity. The condition $P^2(M^8) = 0$ implies that $Re[P(M^8)]$ and $Im[P(M^8)]$ are light-like and parallel. $Im[P(M^8)]$ can be arbitrarily small. One has also $Re[P(M^4)]^2 = Re[P(E^4)]^2$ and $Im[P(M^4)]^2 = Im[P(E^4)]^2$.

3. Could one pose the condition $P^2(M^8) = 0$ also on *many-particle states* or only to the many-particle states appearing as complex massless poles in the BCFW conditions? Kind of strong form of conformal invariance would be in question: not only single-particle states but also many-particle states would be massless in 8-D sense. Now $s = Re[P(M^4))]^2 = Re[P(E^4))]^2$ could have a continuum of values. The discontinuity along s-axis required by unitarity would would emerge from the discontinuity due to the pole at $P^2(M^8) = 0$! Hence 8-dimensional light-likeness in strong sense would be absolutely essential for having vanishing loop corrections together with non-vanishing scattering rates!

Here one must be however extremely careful.

1. In BCFW approach the expression of residue integral as sum of poles in the variable z associated with the amplitude obtained by the deformation $p_i \to p_i + zr_i$ of momenta ($\sum r_i = 0$, $r_i \cdot r_j = 0$) leads to a decomposition of the tree scattering amplitude to a sum of products of amplitudes in resonance channels with complex momenta at poles. The products involve $1/P^2$ factor giving pole and the analog of cut in unitary condition. Proof of tree level unitarity is achieved by using complexified momenta as a mere formal trick and complex momenta are an auxiliary notion. The complex massless poles are associated with groups I of particles whereas the momenta of particles inside I are complex and non-light-like.

2. Could BCFW deformation give a description of massless bound states massless particles so that the complexification of the momenta would describe the effect of bound state formation on the single particle states by making them non-light-like? This makes sense if one assumes that all 8-momenta - also external - are complex. The classical charges are indeed complex already classically since Kähler coupling strength is complex [L2]. A possible interpretation for the imaginary part is in terms of decay width characterizing the life-time of the particle and defining a length of four-vector.

3. The basic question in the construction of scattering amplitudes is what happens inside CD for the external particles with light-like momenta. The BCFW deformation leading to factorization suggests an answer to the question. The factorized channel pair corresponds to two CDs inside which analogs of M and $N - M$ particle bound states of external massless particles would be formed by the deformation $p_i \to p_i + zr_i$ making particle momenta non-light-like. The allowed values of z would correspond to the physical poles. The factorization of BCFW scattering amplitude would correspond to a decomposition to products of bound state amplitudes for pairs of CDs. The analogs of bound states for zero energy states would be in question. BCFW factorization could be continued down to the lowest level below which no factorization is possible.

4. One can of course worry about the non-uniqueness of the BCFW deformation. For instance, the light-like momenta r_i must be parallel ($r_i = \lambda_i r$) but the direction of r is free. Also the choice of λ_i is free to a high extent. BCFW expression for the amplitude as a residue integral over z is however unique. What could this non-uniqueness mean?

   Suppose one accepts the number theoretic vision that scattering amplitudes are representations for sequences of algebraic manipulations. These representations are bound to be highly non-unique since very many sequences can connect the same initial and final expressions. The space-time surface associated with given representation of the scattering amplitude is not unique since each computation corresponds to different space-time surface. There however exists a representation with maximal simplicity.

   Could these two kinds of non-uniqueness relate?

It is indeed easy to see that many-particle states with light-like single particle momenta cannot have light-like momenta unless the single-particle momenta are parallel so that in non-parallel case one must give up light-likeness condition also in complex sense.

1. The condition of light-likeness in complex sense allows the vanishing of real and imaginary mass squared for individual particles

$$Im(p_i) = \lambda_i Re(p_i) \ , \quad (Re(p_i))^2 = (Im(p_i))^2 = 0 \ . \tag{3.3}$$

Real and imaginary parts are parallel and light-like in 8-D sense. All λ_i have same sign and p_i has positive or negative time component depending on whether positive or negative energy part of zero energy state is in question.

2. The remaining two conditions come from the vanishing of the real and imaginary parts of the total mass squared:

$$\sum_{i\neq j} Re(p_i)\cdot Re(p_j) - Im(p_i)\cdot Im(p_j) = 0 \ , \quad \sum_{i\neq j} Re(p_i)\cdot Im(p_j) = 0 \ . \tag{3.4}$$

By using proportionality of $Im(p_i)$ and $Re(p_i)$ one can express the conditions in terms of the real momenta

$$\sum_{i\neq j}(1-\lambda_i\lambda_j) Re(p_i)\cdot Re(p_j) = 0 \ , \quad \sum_{i\neq j} \lambda_j Re(p_i)\cdot Re(p_j) = 0 \ . \tag{3.5}$$

For positive/negative energy part of zero energy state the sign of time component of momentum is fixed and therefore λ_i have fixed sign. Suppose that λ_i have fixed sign. Since the inner products $p_i \cdot p_j$ of time-like vectors with fixed sign of time compomemet are all positive or negative the second term can vanish only if one has $p_i \cdot p_j = 0$. If the sign of λ_i can vary, one can satisfy the condition linear in λ_i but not the first condition as is easy to see in 2-particle case.

3. States with light-like parallel 8-momenta are allowed and one can ask whether this kind of states might be realized inside magnetic flux tubes identified as carriers of dark matter in TGD sense. The parallel light-like momenta in 8-D sense would give rise to a state analogous to super-conductivity. Could this be true also for quarks inside hadrons assumed to move in parallel in QCD based model. This also brings in mind the earlier intuitive proposal that the momenta of fermions and antifermions associated with partonic 2-surfaces must be parallel so that the propagators for the states containing altogether n fermions and antifermions would behave like $1/(p^2)^{n/2}$ and would not correspond to ordinary particles.

These arguments are formulated in M^8 picture. What could this mean in $M^4\times CP_2$ picture?

1. The intuitive expectation is that $Re[P(E^4)]^2$ corresponds to the eigenvalue Λ of CP_2 d'Alembertian so that the higher the momentum, the larger the value of Λ. CP_2 d'Alembertian would be essentially the M^4 mass squared of the state. This would allow vanishing color quantum numbers Y and I_3 but force symmetry breaking $SU(3)\rightarrow SU(2)\times U(1)$. This picture is not quite accurate: also the vibrational degrees of freedom contribute to the mass squared what might be called stringy contribution.

2. Could the geometry of CP_2 induce this symmetry breaking? For instance, Kähler gauge potential depends on the $U(2)$ invariant "radial" coordinate of CP_2 and is invariant only under $U(2)$ rotations and changes by gauge transformation in other color rotations. Could one assign the symmetry breaking to the choice of color quantization axes boiling down at the classical level to the fixing of CP_2 Kähler function would?

   One would have color confinement in weak sense: in QCD picture physical states correspond to color singlet representations. This is certainly very strong statement in a sharp conflict with the standard view about color confinement. It would make sense in TGD framework, where color as a spin like quantum number is replaced with angular momentum like quantum number. One could say that macroscopic systems perform macroscopic color rotation. The model for the honeybee dance [L10] conforms with this view and actually led to the proposal for a modification of cosmic string type extremals $X^4 = X^2\times Y^2 \subset M^4\times CP_2$ by putting Y^2 in 2-D rigid body color rotation along both time axis and spatial axis of the string world sheet X^2.

3. This picture raises again the old question about the relationship of color and electroweak quantum numbers in TGD framework. Could one regard electroweak quantum numbers as a spin related to color group $SU(3)$ just as one can relate ordinary spin with Lorentz transformations? Color quantum numbers of say quarks would be analogous to orbital angular momentum. The realization of the action of the electroweak $U(2)_{ew}$ on CP_2 spinors indeed involves also geometric color rotation affecting the gauge potentials in the general case and $U(2)_{ew}$ can be identified as holonomy group of CP_2 spinor connection and sugroup of $SU(3)$. One could also see electroweak symmetry breaking as a further symmetry breaking $U(2) \rightarrow U(1) \times U(1)$ assignable with the flag manifold $SU(3)/U(1) \times U(1)$ parameterizing different choices of color quantization axes and having interpretation as CP_2 twistor space.

Remark: Number theoretic vision means that the quaternionic M^8-momenta are discrete with components having values in the extension of rationals. $P^2(M^4)$ becomes discrete if one poses $P^2(M^8) = 0$ condition for all states. The values of discontinuity of $Im(T)$ correspond now to a discrete sequence of poles along s-axis approximating cut. At the continuum limit this discrete sequence of poles becomes cut. Continuum limit would correspond to a finite measurement resolution in which one cannot distinguish the poles from each other.

3.3 Grassmannian approach and TGD

Grassmannian approach has provided besides technical progress deeper views about twistorialization and also led to the understanding of the Yangian symmetry.

3.3.1 Grassmannian twistorialization - or what I understand about it

The twistorialization of the scattering amplitudes works for planar amplitudes in massless theories and involves the following ingredients.

1. All scattering amplitudes are expressible in terms of on-mass-shell scattering amplitudes with massless on-mass-shell particles in complex sense.

2. The scattering amplitude is sum over contributions with varying number of loops. BCFW recursion relation allows to construct scattering amplitudes from their singularities using 3-particle amplitudes as building brick amplitudes. There are two types of singularities.

   For the first type of singularity one has on-shell internal line and one obtains a sum over all possible decompositions of the scattering amplitude to a product of on-mass-shell scattering amplitudes multiplied by delta function for momentum squared of the internal line. Second type of singularity corresponds to the so called forward limit and is obtained from $(n+2, k)$ amplitude by contracting two added adjacent particles to form a loop so that their momenta are opposite and integrating over the momentum.

3. The singular term is algebraically analogous to an exterior derivative of the scattering amplitude and can be integrated explicitly: the integration adds BCFW bridge to the both terms such that the forward limit loop in the second term is under the bridge. The outcome is BCFW formula for l-loop amplitude with n external particles with k negative helicities consisting of these two terms.

Twistor Grassmannian approach expresses the on mass shell scattering amplitudes appearing as building bricks as residue integrals over Grassmannian $Gr(n, k)$, where n is the number of particles and k is the number of negative helicities. The Grassmannian approach is described in a concise form in the slides by Jaroslav Trnka [B6] (see `http://tinyurl.com/pqjzffj`).

1. The construction of the on-mass-shell scattering amplitudes appearing in BCFW formula as residue integrals in Grassmannians follows by expressing the momentum conserving delta functions in twistor description in terms of auxiliary variables serving as coordinates of Grassmannian $G(n, k, C)$ for the on mass shell tree amplitude with n external particles having k negative helicities. Grassmannian has dimension $d = (n-k)k$ and can be identified as the space of k-planes - or equivalently $n-k$-planes in C^N. Grasmannian has a representation

as homogenous space $G(n,k,C) = U(n)/U(n-k) \times U(k)$ having $SU(n)$ as the group of isometries. For $k = 1$ one obtains projective space which is also symmetric space (allowing reflection along geodesic lines as isometries).

2. Grassmannians emerge as an auxiliary construct, and the multiple residue integral over Grassmannian gives sum of residues so that the introduction of Grassmannians might look like un-necessary complication. The selection of points of Grassmannian for given external quantum numbers by residue integral given at the same time the value of the amplitude might however have some deeper meaning.

   The construction involves standard mathematics, which is however new for physicists. For instance, notions such as Plücker coordinates, Schubert cells and cell decomposition appear. One can relate to each other various widely different looking expressions for the amplitudes as being associated with different cell decompositions of Grassmannian. The singularities of the integrand of the scattering amplitude defined as a multiple residue integral over $G(k,n)$ define a hierarchy of Schubert cells.

3. The so called positive Grassmannian [B5] defines a subset of singularities appearing in the scattering amplitudes of $\mathcal{N} = 4$ SUSY. The points of positive Grassmannian $Gr_+(k,n)$ are representable as $k \times n$ matrices with positive $k \times k$ determinants. The singularities correspond to the boundaries of $Gr_+(k,n)$ with some $k \times k$ determinants vanishing. For tree diagrams the singularities correspond to poles appearing in the factorized term of the BCFW decomposition of the scattering amplitude. The positivity conditions hold true also for the twistors representing external particles.

4. Positivity conditions guarantee the convexity of the integration region determined by the C-matrix as point of $Gr_+(k,n)$ appearing in the conditions dictating the integration region.

   To better understand the meaning of positivity one can first consider triangle call it T - as a representation of positive Grassmannian $Gr_+(1,3) = P^2_+$. Any interior points of T can be regarded as center of mass for suitable positive masses at the vertices of the triangle. These conditions generalizes to the case of general polygons, which must be convex. If the number of vertices of the polygon is larger than 3, convexity is not automatically satisfied, and requires additional conditions.

   This description generalizes to Grassmannians $Gr_+(k,n)$. Masses define the analog of C-matrix as element of $Gr_+(k,n)$ appearing in the twistor approach and the vertices of the triangle are analogous to the twistors associated with external particles combining to form a point of $Gr(4,n)$. Positivity condition is generalized to the condition that $k \times k$ minors of the $k \times n$ matrix are positive.

5. Also the twistors associated with the external particles must satisfy analogs of the positivity conditions. This involves the replacement of $Gr(4,n)$ associated with twistors of the external particles with $Gr_+(k+4,n)$. The additional k components of the twistors are Grassman numbers and determined by the superparts of the twistors (see the slides of Trnka at `http://tinyurl.com/pqjzffj`. I must admit that I did not understand this.

6. Residue integral can be defined in terms of what is called canonical form Ω - analog of volume form - having logarithmic singularities at the boundaries of the $Gr_+(k,n)$. Hence one can perform a reduction of the residue integral to a sum of integrals over $G(k,k+4)$ instead of $G(k,n)$ (actually not so surprising since the residue integrals give as outcome the residues at discrete points!).

   This leads to a reduction of the residue integral over $Gr_+(k,n)$ to a sum of lower dimensional residue integrals over triangulation defined by $Gr_+(k,k+4)$ represented as surfaces of $Gr_+(k,n)$ glued together along sides. The geometric analog would be decomposition of polygon to a union of triangles.

   This simplifies the situation dramatically [B7, B6, B5] and leads to the notion of amplituhedron [B2, B1]. What is so remarkable, is the simplicity of the expressions for all-loop amplitudes and the fact that positivity implies locality and unitarity for $\mathcal{N} = 4$ SUSY.

7. It should be possible to construct Ω explicitly having the desired singularities which would be in TGD framework poles with $P^2(M^8) = P^2(M^4 \times CP_2) = 0$ if the proposed realization of unitary makes sense? Could one just assumes that Ω vanishes for that part of the boundary of $Gr_+(k,n)$, which gives loop singularities? Could these points $Gr_+(k,n)$ be transcendental and excluded for this reason?

If loop corrections are vanishing as ZEO strongly suggests, only tree amplitudes are needed. Therefore it is appropriate to summarize what I have managed to understand about the construction of the tree amplitudes with general value of k in the amplituhedron approach.

1. The notion of amplituhedron relies on the mapping of $G(k,n)$ to $G_+(k,k+m)$ $n \geq k+m$. Actually a map from $G(k,n) \times G(k+4,n) \rightarrow G_+(k,k+m)$ is in question. $m = 4$ identifiable as the apparent dimension of twistor space without projective identification giving the actual dimension $d = 3$. n is the number of external particles and k the number of negative helicities.

   The value of m is $m = 4$ and follows from the conditions that amplitudes come out correctly. The constraint $Y = C \cdot Z$, where Y corresponds to point of $G_+(k,k+4)$ and Z to the point of $G(k+4,n)$ performs this mapping, which is clearly many-to one. One can decompose integral over $G_+(k,n)$ to integrals over positive regions $G_+(k,k+4)$ intersecting only along their common boundary portions. The decomposition of a convex polygon in plane to triangles represent the basic example of this kind of decomposition. Obviously there are several decompositions of this kind.

2. Each decomposition defines a sum of contributions to the scattering amplitude involving integration of a projectively invariant volume form over the positive region in question. The form has a logarithmic singularity at the boundaries of the integration region but spurious singularities cancel so that only the contribution of the genuine boundary of $G_+(k,k+4)$ remains. There are additional delta function constraints fixing the integral completely in real case.

3. In complex case one has residue integral. The proposed generalization to the complex case is by analytic continuation. TGD inspired proposal is that the positivity condition in the real case is generalized to the condition that the positive coordinates are replaced by complex coordinates of hyperbolic space representable as upper half plane or equivalently as the unit disk obtained from upper half plane by exponential mapping $w = exp(iz)$. The measure $d\alpha/\alpha$ would correspond to $dz = dw/w$. If taken over boundary circle labelled by discrete phase factors $exp(i\phi)$ given by roots of unity the integral would be numerically a discrete Riemann sum making no sense p-adically but residue theorem could allow to avoid the discretizaton and to define the p-adic variant of the integral by analytic continuation. These conditions would be completely general conditions on various projectively invariant moduli involved.

4. One must extend the bosonic twistors Z_a of external particles by adding k coordinates. This extension looks very difficult to understand intuitively. Somewhat surprisingly, these coordinates are anti-commutative super-coordinates expressible as linear combinations of fermionic parts of super-twistor using coefficients, which are also Grassmann numbers. Integrating over these one ends up with the standard expression of the amplitude using canonical integration measure for the regions in the decomposition of amplituhedron. An interesting question is whether the addition of k-dimensional anti-commutative parts to Z_a expressible in terms of super-coordinates is only a trick or whether it could have some physical interpretation.

3.3.2 Grassmannians as reduced WCWs?

Grassmannians appear as auxiliary spaces in twistor approach. Could Grassmannians and the procedure assigning to external momenta and helicities discrete set of points of Grassmannian and scattering amplitude have some concrete interpretation in TGD framework?

1. The points of cognitive representation define WCW coordinates for space-time surface. For a fixed number of points in cognitive representation WCW is effectively replaced with a finite-dimensional reduced WCW. These points would naturally correspond to the points defining ends of fermionic lines at partonic 2-surfaces. WCW has Kähler metric with Euclidian signature. This could be true also for its reduction.

2. The experience with twistorialization suggests that these spaces could be simply Grassmannians $Gr(n,r,C)$ consisting or r-dimensional complex planes of n-dimensional complex space representable as coset spaces $U(n)/U(n-r)\times U(r)$ appearing as auxiliary spaces in the construction of twistor amplitudes.

   Note that the correlation between quantum states and geometry would be present since n corresponds to the number of external particles and r to those with negative helicity in ordinary twistor Grassmann approach. In TGD framework discretized variants of these spaces corresponding to the extension of rationals used would appear. Yangian symmetries could correspond to general coordinate transformations for the reduced WCW acting as gauge symmetry. These transformations act as diffeomorphisms for so called positive Grassmannians also in the standard twistorialization. If the reduced WCWs indeed correspond to twistor Grassmannians, one would have a completely unexpected connection with supersymmetric QFTs.

3. The reduction of WCW to a finite dimensional Kähler manifold suggests that also WCW spinors become ordinary spinors for Kähler manifold so that gamma matrices form a finite-D fermionic oscillator operator algebra. WCW has maximal symmetries and it would not be surprising if also the finite-D Kähler manifold would possess maximal symmetries. Note that WCW gamma matrices together with isometry generators of WCW give rise to a super-symplectic algebra involving a generalization of 2-D conformal invariance replacing 2-D surfaces with light-like 3-surfacs.

4. The interpretation of supersymmetry would be different from the standard one. Kähler structure implies that $\mathcal{N}$ is even and Majorana spinors are absent and both baryon and lepton number can be conserved separately. The ordinary fermionic oscillator algebra is a Clifford algebra and could be interpreted in terms of a broken supersymmetry.

Also more general flag manifolds than Grassmannians can be considered. If these spaces are homogenous spaces they have maximal isometries. They should have also Kähler structure. Compactness looks also a highly desirable property. The gauge conditions for the subalgebra of super-symplectic algebra state that the sub-algebra and its commutator with the entire algebra annihilate physical states and give rise to vanishing classical Noether charges. This would effectively reduce the super-symplectic algebra to a finite-D Lie group or Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to the ADE hierarchy as the hierarchy of inclusions of HFFs as an alternative correlate for the realization of finite measurement resolution suggests. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

3.3.3 Interpretation for Grassmannian residue integrations

The identification of Grassmannians (or possibly more general spaces) as reduced WCWs would give a genuine physical interpretation for the Grassmannian integrations as residue integrations over reduced WCW. What looks mysterious and maybe even frustrating is that the outcome of the entire process is sum over discrete residues: what does this mean?

1. The residue integration is only over a surface of reduced WCW with dimension equal to one half of that of WCW. One has integrand, which depends on the external quantum numbers coded in terms of twistors and on coordinates of reduced WCW. The residue integration is analogous to summation over amplitude associated with space-time surfaces coded by different cognitive representations.

2. One can argue that a continuous residue integral over Grassmannian is not consistent with the number theoretic discretization. The outcome is however discrete set of space-time surfaces labelled by cognitive representations as points of Grassmannian. Of the points in question are in the extension and if this is equivalent with the corresponding property for the coordinates of Grassmannian, there should be no problems. The restriction of external momenta to the extension of rationals might guarantee this.

3. The full multiple residue integral leaves only pole contributions, which correspond to a discrete collection of space-time surfaces (at least the set of space-time surfaces obtained by the action of Galois group), that is discrete set of points of reduced WCW. It seems that the entire residue integration is just a way to realize quantum classical correspondence by associating to the external quantum numbers space-time surfaces and corresponding cognitive representations - and of course, also the scattering amplitude.

4. One can also ask whether the positivity of Grassmannian might relate to the fact that p-adic numbers as ordinary integers are always non-negative (most of them infinite). The positivity might be necessary in order to have number theoretic universality. If the minors associated with the C-matrix serve as coordinates for $Gr_+(k,n)$ they could be interpreted also as p-adic numbers. If they are allowed to be negative, one encounters problems since p-adic numbers are not well-ordered and one cannot say whether p-adic number is negative or positive.

3.3.4 Posible description of SUSY and its breaking in TGD framework

Although twistor description make sense also in the absence of supersymmetry, super-symmetry is an essential part of the elegance of the Grassmannian approach. For the ordinary SUSY one has gluons and their superpartners characterized in terms of super-twistors. In TGD one has two pictures [L6, K21].

1. At the level H fermions as fundamental particles are described in terms of second quantized induced spinor fields, whose oscillator operators can be used to build gamma matrices for WCW [K29, K20]. In TGD universe all known elementary particles would be composites of fundamental fermions represented as lines at the light-like orbits of partonic 2-surfaces (wormhole throats) and ordinary elementary particles involve a pair of wormhole contacts with throats containing these fermion lines. It is assumed that the fermions are at different points: this allows to avoid problems due to infinities.

   In the proposed generalization of twistor approach $2 \rightarrow 2$ fermion scattering in the classical fields at partonic 2-surface would define the basic $2 \rightarrow 2$-vertex replacing 3-vertices of twistorial SUSY. Essentially one has only two-vertices describing the redistribution of fermions at partonic 2-surface between orbits of the partonic 2-surfaces meeting at it. This is different from $\mathcal{N} = 4$ SUSY [K10]. If one allows completely local multi-fermion states at the level of H one cannot avoid fermionic contact interactions.

   The many-fermion states associated with partonic 2-surfaces would define the analogs of super-multiplets. One can wonder whether a SUSY type description could exist as a limit when the partonic 2-surface is approximated with single point so that also positions of fermions are approximated as single point. SUSY would be only approximate.

2. At the level of M^8 I have proposed the use of polynomials P of super-octonion serving as analogs of super-gluon fields to construct scattering amplitudes [L6]. This allows geometric description of all particles using super-multiplets. Each monomial of theta parameters would give rise to its own space-time surface by the condition that either $IM(P)$ or $RE(P)$ vanishes for the corresponding polynomial P. This condition would reduce the components of super-field to algebraic surfaces.

   There is however an important difference from H picture. The members of super-multiplet defined by P correspond to the coefficients of monomials of theta parameters having interpretation as analogs of oscillator operators. Super-partners would be in this sense point-like objects unlike in H approach, where this can hold true only approximately.

Could H- and M^8 pictures be equivalent and could one understand the breaking of SUSY in this framework?

1. $M^8 - H$ correspondence as a map of associative space-time regions from M^8 to minimal surfaces in H makes sense for the external particles and thus at boundaries of CDs. It assigns to a point of the partonic 2-surface $X^2 \subset X^4 \subset M^8$ the quaternionic tangent space of X^4 at it characterized by a point of CP_2. M^4 point is mapped to itself. There is additional condition requiring that quaternionic tangent space contains fixed complex sub-space but this is not relevant now.

2. Could this map be one-to-many so that super-field component describing purely many-fermion state would be mapped to several points at the image of X^2 in H describing multi-local many-fermion state? This is possible if the points in M^8 are singular in the sense that the action of a normal subgroup H of Galois group Gal leaves the point invariant so that Gal reduces to Gal/H: symmetry breaking takes place.

   The tangent spaces of the degenerate points are however different and are mapped to different points of CP_2 in M^8-H correspondence making sense at boundaries of CDs but not in their interiors. One would have several fermions with same M^4 coordinates but different CP_2 coordinates and the outcome would be many-fermion state. In the case of 2-fermion state the different values of CP_2 coordinates would be associated with the opposite throats of a wormhole contact whose orbit defines light-like 3-surface. Could light-likeness inducing the reduction of the metric dimension of the tangent space from 4 to 3 somehow induce also this degeneration?

3. Could symmetry breaking as a degeneration of Gal action to that for Gal/H take place for the conditions defining the 4-surfaces associated with the higher components of super-octonion and induce the breaking of SUSY at the level of M^8 manifesting as the non-locality of the fermion state at the level of H? This degeneration would be a typical manifestation of quantum criticality: criticality in general means co-incidence of two roots.

3.4 Summary

Since the contribution means in well-defined sense a breakthrough in the understanding of TGD counterparts of scattering amplitudes, it is useful to summarize the basic results deduced above as a polished answer to a Facebook question.

There are two diagrammatics: Feynman diagrammatics and twistor diagrammatics.

1. Virtual state is an auxiliary mathematical notion related to Feynman diagrammatics coding for the perturbation theory. Virtual particles in Feynman diagrammatics are off-mass-shell.

2. In standard twistor diagrammatics one obtains counterparts of loop diagrams. Loops are replaced with diagrams in which particles in general have complex four-momenta, which however light-like: on-mass-shell in this sense. BCFW recursion formula provides a powerful tool to calculate the loop corrections recursively.

3. Grassmannian approach in which Grassmannians $Gr(k,n)$ consisting of k-planes in n-D space are in a central role, gives additional insights to the calculation and hints about the possible interpretation.

4. There are two problems. The twistor counterparts of non-planar diagrams are not yet understood and physical particles are not massless in 4-D sense.

In TGD framework twistor approach generalizes.

1. Massless particles in 8-D sense can be massive in 4-D sense so that one can describe also massive particles. If loop diagrams are not present, also the problems produced by non-planarity disappear.

2. There are no loop diagrams- radiative corrections vanish. ZEO does not allow to define them and they would spoil the number theoretical vision, which allows only scattering amplitudes, which are rational functions of data about external particles. Coupling constant evolution - something very real - is now discrete and dictated to a high degree by number theoretical constraints.

3. This is nice but in conflict with unitarity if momenta are 4-D. But momenta are 8-D in M^8 picture (and satisfy quaternionicity as an additional constraint) and the problem disappears! There is single pole at zero mass but in 8-D sense and *also many-particle states* have vanishing mass in 8-D sense: this gives all the cuts in 4-D mass squared for all many-particle state. For many-particle states not satisfying this condition scattering rates vanish:

these states do not exist in any operational sense! This is certainly the most significant new discovery in the recent contribution.

BCFW recursion formula for the calculation of amplitudes trivializes and one obtains only tree diagrams. No recursion is needed. A finite number of steps are needed for the calculation and these steps are well-understood at least in 4-D case - even I might be able to calculate them in Grassmannian approach!

4. To calculate the amplitudes one must be able to explicitly formulate the twistorialization in 8-D case for amplitudes. I have made explicit proposals but have no clear understanding yet. In fact, BCFW makes sense also in higher dimensions unlike Grassmannian approach and it might be that the one can calculate the tree diagrams in TGD framework using 8-D BCFW at M^8 level and then transform the results to $M^4 \times CP_2$.

What I said above does yet contain anything about Grassmannians.

1. The mysterious Grassmannians $Gr(k,n)$ might have a beautiful interpretation in TGD: they could correspond at M^8 level to reduced WCWs which is a highly natural notion at $M^4 \times CP_2$ level obtained by fixing the numbers of external particles in diagrams and performing number theoretical discretization for the space-time surface in terms of cognitive representation consisting of a finite number of space-time points.

   Besides Grassmannians also other flag manifolds - having Kähler structure and maximal symmetries and thus having structure of homogenous space G/H - can be considered and might be associated with the dynamical symmetries as remnants of super-symplectic isometries of WCW.

2. Grassmannian residue integration is somewhat frustrating procedure: it gives the amplitude as a sum of contributions from a finite number of residues. Why this work when outcome is given by something at finite number of points of Grassmannian?!

   In M^8 picture in TGD cognitive representations at space-time level as finite sets of points of space-time determining it completely as zero locus of real or imaginary part of octonionic polynomial would actually give WCW coordinates of the space-time surface in finite resolution.

   The residue integrals in twistor diagrams would be the manner to realize quantum classical correspondence by associating a space-time surface to a given scattering amplitude by fixing the cognitive representation determining it. This would also give the scattering amplitude.

   Cognitive representation would be highly unique: perhaps modulo the action of Galois group of extension of rationals. Symmetry breaking for Galois representation would give rise to supersymmetry breaking. The interpretation of supersymmetry would be however different: many-fermion states created by fermionic oscillator operators at partonic 2-surface give rise to a representation of supersymmetry in TGD sense.

4 New insights about quantum criticality for twistor lift inspired by analogy with ordinary criticality

Quantum criticality (QC) is one of the basic ideas of TGD. Zero energy ontology (ZEO) is second key notion and leads to a theory of consciousness as a formulation of quantum measurement theory making observer part of the quantum system in terms of notion of self identified as a generalized Zeno effect or analog for a sequence of weak measurements, and solving the basic paradox of standard quantum measurement theory, which one usually tries to avoid by introducing some "interpretation".

ZEO allows to see quantum theory could be seen as "square root" of thermodynamics. It occurred to me that it would be interesting to apply this vision in the case of quantum criticality to perhaps gain additional insights about its meaning. We have a picture about criticality in the framework of thermodynamics: what would be the analogy in ZEO based interpretation of Quantum TGD? Could it help to understand more clearly the somewhat poorly understood views about the notion of self, which as a quantum physical counterpart of observer becomes in ZEO a key concept of fundamental physics?

The basic ingredients involved are discrete coupling constant evolution, zero energy ontology (ZEO) implying that quantum theory is analogous to "square root" of thermodynamics, self as generalized Zeno effect as counterpart of observer made part of the quantum physical system, $M^8 \leftrightarrow M^4 \times CP_2$ duality, and quantum criticality. A further idea is that vacuum functional is analogous to a thermodynamical partition function as exponent of energy $E = TS - PV$.

The correspondence rules are simple. The mixture of phases with different 3-volumes per particle in a critical region of thermodynamical system is replaced with a superposition of space-time surfaces of different 4-volumes assignable to causal diamonds (CDs) with different sizes. Energy E is replaced with action S for preferred extremals defining Kähler function in the "world of classical worlds" (WCW). S is sum of Kähler action and 4-volume term, and these terms correspond to entropy and volume in the generalization $E = TS - PV \rightarrow S$. P resp. T corresponds to the inverse of Kähler coupling strength α_K resp. cosmological constant Λ. Both have discrete spectrum of values determined by number theoretically determined discrete coupling constant evolution. Number theoretical constraints force the analog of micro-canonical ensemble so that S as the analog of E is constant for all 4-surfaces appearing in the quantum superposition. This implies quantization rules for Kähler action and volume, which are very strong since α_K is complex.

This kind of quantum critical zero energy state is created in unitary evolution created in single step in the process defining self as a generalized Zeno effect. This unitary process implying time de-localization is followed by a weak measurement reducing the state to a fixed CD so that the clock time identified as the distance between its tips is well-defined. The condition that the action is same for all space-time surfaces in the superposition poses strong quantization conditions between the value of Kähler action (Kähler coupling strength is complex) and volume term proportional to cosmological constant. The outcome is that after sufficiently large number of steps no space-time surfaces satisfying the conditions can be found, and the first reduction to the opposite boundary of CD must occur - self dies. This is the classical counterpart for the fact that eventually all state function reduction leaving the members of state pairs at the passive boundary of CD invariant are made and the first reduction to the opposite boundary remains the only option.

The generation of magnetic flux tubes provides a way to satisfy the constancy conditions for the action so that the existing phenomenology as well as TGD counterpart of cyclic cosmology as re-incarnations of cosmic self follows as a prediction. This picture allows to add details to the understanding of the twistor lift of TGD at classical level and allows an improved understanding of the p-adic length scale evolution of cosmological constant solving the standard problem caused by the huge value of Λ. The sign of Λ is predicted correctly.

This picture generalizes to the twistor lift of TGD and cosmology provides an interesting application. One ends up with a precise model for the p-adic coupling constant evolution of the cosmological constant Λ explaining the positive sign and smallness of Λ in long length scales as a cancellation effect for M^4 and CP_2 parts of the Kähler action for the sphere of twistor bundle in dimensional reduction, a prediction for the radius of the sphere of M^4 twistor bundle as Compton length associated with Planck mass (2π times Planck length), and a prediction for the p-adic coupling constant evolution for Λ and coupling strength of M^4 part of Kähler action giving also insights to the CP breaking and matter antimatter asymmetry. The observed two values of Λ could correspond to two different p-adic length scales differing by a factor of $\sqrt{2}$.

4.1 Some background

Some TGD background is needed to understand the ideas proposed in the sequel.

4.1.1 Discrete coupling constant evolution

The most obvious implication is discrete coupling constant evolution in which the set of values for coupling constants is discrete and analogous to the set of the critical values of temperature [L17] (see `http://tinyurl.com/y9hlt3rp`). Zeros of Riemann Zeta or its slight modification suggest themselves as the spectrum for the Kähler coupling strength. This discrete coupling constant evolution requires that loop corrections vanish. This vision is realized concretely in the generalization of the twistorial approach to the construction of scattering amplitudes [L17].

Non-manifest unitarity is the basic problem of the twistor Grassmann approach. A generalization of the BCFW formula without the loop corrections gives scattering amplitudes satisfying unitary constraints. The needed cuts are be replaced by sequences of massless poles in 8-D sense and cuts approximate these sequences (consider electrostatic analogy in which line charge approximates a discrete sequences of poles). The replacement cuts with sequences of poles is forced by the number theoretic discretization of momenta so that they belong to an extension of rationals defining the adele [L7] (see `http://tinyurl.com/ycbhse5c`).

Non-planar loop diagrams are a chronic problem of twistor approach since there is no general rule loop integrations allowing to combine them neatly. Also this problem disappears now.

$M^8 - H$ duality plays key role in the twistorial approach [L6] (see `http://tinyurl.com/yd43o2n2`). In the ordinary twistor approach all momenta are light-like so that it does not apply to massive particles. TGD solves this problem: at M^8 level one has quaternionic light-like 8-D momenta, which correspond to massive 4-D momenta in M^8 picture. In $H = M^4 \times CP_2$ picture ground states of super-conformal representations are constructed in terms of spinor harmonics of in $M^4 \times CP_2$, which are products plane-waves characterized by massive 4-momenta and color wave functions associated with massless Dirac equation in H. Also the analog of Dirac equation for the induced spinor fields at space-time surface is massless [K29] (see `http://tinyurl.com/yc2po5gf`).

4.1.2 ZEO and self as generalized Zeno effect

ZEO allows to see self as generalized Zeno effect [L9](see `http://tinyurl.com/ycxm2tpd`).

1. Generalized Zeno effect can be regarded as a sequence of "small" state function reductions analogous to weak measurements performed at active boundary of causal diamond (CD). In usual Zeno effect the state is unaffected under repeated measurements: now the same is true at passive boundary of CD whereas the members of state pairs at the active boundary change. The unitary evolutions followed by these evolutions leave thus passive boundary and states at it invariant whereas active boundary shifts farther away from the passive boundary and the members of state pairs at it are affected. This gives rise to the experienced flow of time.

   The change of states is characterized unitary S-matrix. Each unitary evolution involves delocalization in the space of CDs so that one has quantum superposition of CDs with sizes not smaller than the CD to which the state was localized at previous reduction. This gives rise to a steady increase of clock time defined as the distance between the tips of CD. Self dies and reincarnates as a self with opposite direction of clock time when the first unitary evolution at the passive boundary followed by a weak measurement at it takes place. Self dies when all observables leaving the states at passive boundary invariant are measured. There are no choices to be made anymore.

2. Quantum TGD as "square root " of thermodynamics means that the partition function of thermodynamics is replaced by its "square root" defined by the vacuum functional identified as exponent of Kähler function of "world of classical worlds" (WCW). Kähler function is analogous to energy $E = TS - PV$ in thermodynamics with T replaced with the inverse of complex Kähler coupling strength and P with cosmological constant, which have discrete spectrum of values.

   One has the analog of micro-canonical ensemble for which only states with given energy are possible. Now the action (Kähler function) is same for the space-time surfaces assignable to the zero energy states involved. This condition allows to get rid of the exponentials defining the vacuum functional otherwise appearing in the scattering amplitudes. This condition is strongly suggested by number theoretic universality for which these exponentials are extremely troublesome since both the exponent and exponential should belong to the extension of rationals used.

   This implies a huge simplification in the construction of the amplitudes [L6] (see `http://tinyurl.com/yd43o2n2`) because finite measurement resolution effectively replaces space-time surfaces with their cognitive representation defined by a discrete set of space-time points with embedding space coordinates in the extension of rationals defining the adele.This representation codes for the space-time surface if it corresponds to zero locus of real or imaginary part (in quaterionic sense) of an octonionic polynomial with real coefficients. WCW coordinates are given by the cognitive representation and are discrete. One is led to enumerative algebraic geometry.

4.1.3 $M^8 - H$ duality

$M^8 - H$ duality [L6] (see http://tinyurl.com/yd43o2n2) states that the purely algebraic dynamics determined by the vanishing of real or imaginary part for octonionic polynomial is dual to the dynamics dictated by partial differential equations for an action principle.

1. There are two options for how to identify M^8 counterparts of space-time surfaces in terms roots of four polynomials defining real or imaginary part of an octonionic polynomial obtained as a continuation of real polynomial.

   (a) One can allow all roots $x + iy$ and project them to M^4 or M^8 from M^8_c. One can decompose these surfaces to regions with associative (quaternionic) tangent space or normal space and they are analogous to external particles of a twistor diagram entering CD and to interaction regions in which associativity does not hold true and which correspond to interiors of CD. One can criticizes the projection as somewhat adhoc process.

   (b) It became later clear that one can also consider space-time surface as Minkowskian real regions so that the projection to a sub-space $M^4 \subset M^8_c$ of complexified octonions is invariant under the conjugation $i \to -i, I_k \to -I_k$, where I_k are quaternionic units. M^4_c parts of space-time coordinates would be form $m = m^0 + iI_k m^k$, m^0, m^k real. This conditions need not or even cannot be posed on E^4_c coordinates since $M^8 - H$ duality assigns to the tangent space of space-time surface a CP_2 point irrespective of whether the point is in M^8_c or M^8.

2. At the level of H external particles correspond to minimal surfaces, which are also extremals of Kähler action and in accordance with the number theoretical universality and quantum criticality do not depend on the coupling parameters at all. They are obtained by a map taking the 4-surfaces in M^8 to those in H. These conditions should be equivalent with the condition that the 6-D surfaces X^6 in 12-D twistor space of H define twistor bundles of space-time surfaces X^4.

3. The space-time regions in the interiors of CDs are not minimal surfaces so that Kähler action and volume term couple dynamically and coupling parameters characterize the extremals. The analog is motion of point like particle in the Maxwell field defined by induced Kähler form: this is generalize to the motion of 3-D object with purely internal Kähler field and that associated with wormhole contacts and mediating interaction with larger and smaller space-time sheets.

   In these regions the map mediating $M^8 - H$ duality does not exist since one cannot label the tangent spaces of space-time surface by points of CP_2. The non-existence of this map is due to the failure of either associativity of tangent space or normal space at M^8 level. The initial values at boundaries of CD for the incoming preferred extremals however allows to fix the time evolution in the interior of CD. This is essentially due to the infinite number of gauge conditions for the super-symplectic algebra.

   It has later turned out [L17] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.

4.1.4 Quantum criticality

Quantum criticality is a further key notion of TGD and was originally motivated by the idea that Kähler coupling strength must be unique in order that the theory is unique.

1. The first implication of quantum criticality is quantization of various coupling strengths as analogs of critical temperature and of other critical parameters such as pressure. This quantization is required also by number theoretical universality in the adelic approach: coupling constant parameters must belong to the extension of rationals used.

2. Second implication of quantum criticality is a huge generalization of conformal symmetries to their 4-D analogs. The key observation is that 3-D light-like surfaces allow a generalization of conformal invariance to get the Kac-Moody algebra associated with the isometries of H (at least) as symmetries. In the case of boundary of CD this leads to what I call supersymplectic invariance: the symplectic transformations of the two components of $\delta CD \times CP_2$ act as isometries of WCW. This algebra allows a fractal hierarchy of sub-algebras isomorphic to the algebra itself and gauge conditions state that this kind of sub-algebra and its commutator with the entire algebra annihilate physical states and classical Noether charges for them vanish [L17] (see `http://tinyurl.com/y9hlt3rp`). By quantum classical correspondence (QCC) the eigenvalues of quantum charges are equal to the classical Noether charges in Cartan algebra of supersymplectic algebra.

3. The third implication is the understanding of preferred extremals in $H = M^4 \times CP_2$ and their counterparts at the level of M^8. Associativity condition at the level of M^8 satisfied by the spacetime surfaces representing external particles arriving into CD corresponds to quantum criticality posing conditions on the coefficients of octonionic polynomials. The space-time regions inside CD the space-time surfaces do not satisfy associativity conditions and are not critical.

4. TGD as "square root" of thermodynamics idea suggests a fourth application of quantum criticality. This analogy might allow a better understanding of self as Zeno effect. This application will be studied in the sequel.

4.2 Analogy of the vacuum functional with thermodynamical partition function

Consider first the thermodynamical view about criticality. I have discussed criticality from slightly different perspective in [L13] (see `http://tinyurl.com/ydhknc2c`).

1. Thermodynamical states in critical region, where phases with different densities - say liquid and gas - are present serves as a basic example. This situation is actually a problem of the approach relying on partition function as van der Waals equation predicting 3 different densities for the density of molecules as function of pressure and temperature. Cusp catastrophe gives a view about situation: number density n is behavior variable and P and T are the control variables.

2. The experimental fact is that the density is constant as function of volume V for fixed temperature T whereas van der Waals predicts dependence on V. The phase corresponding to the middle sheet of the cusp is not at all present and the portions of liquid and gas phases vary. Maxwell's rules (area rule and lever rule) allow to solve the problem plaguing actually all approaches based on partition function. Lever rule assumes that there are actually two kinds of "elements" present. Molecules are the first element but what the second element could be? TGD identification is as magnetic tubes [L13].

3. In the more general case in which the catastrophe is more general than cusp and has more sheets, two or more phases with different volumes are present and their volumes and possibly other behavior variables analogous to volume vary at criticality.

4. If one applies criticality in stronger sense by requiring that the function which has extremum as function of n at the surface represented by cusp catastrophe has same value at different

sheets of the cusp, only the boundary line of the cusp having V-shaped projection in (p,T)-plane remains.

4.2.1 Generalization of thermodynamical criticality to TGD context

The generalization of this picture to TGD framework replaces the mixture of thermodynamical phases with different volumes with quantum superposition of space-time surfaces with different 4-volumes assignable to CDs with different quantized sizes (by number theoretical constraints).

1. Vacuum functional, which is exponent of Kähler function of WCW expressible as Kähler action for its preferred extremal, can be regarded as a complex "square root" of thermodynamical partition function Z meaning that its real valued modulus squared is analogous to partition function [K25, K10, K4, K21].

   Action S, whose value for preferred extremal defines Kähler function of WCW serves as the analog of energy assumed to have expression $E = PV - TS$, which is not generally true but implied by the condition that E is homogenous as function of conjugate variable pairs P,V and T,S. The analogs of P and T correspond to coupling constant parameters. Pressure p is replaced with the coefficient of volume term in action - essentially cosmological constant. T is replaced with the coefficient $1/\alpha_K$ of Kähler action representing entropy (or negentropy depending on situation).

   Remark: Note that T corresponds now to $1/\alpha_K$ rather than α_K analogous to temperature when Kähler action S_K is regarded as analog of energy E rather than entropy S.

2. Quantum criticality in the sense of ZEO is the counterpart for the criticality in thermodynamics. The mixture of thermodynamical phases with different 3-volumes is replaced with quantum superposition of zero energy states with 4-surface having same action S but different 4-volumes assignable to different CDs. Critical system consists of several phases with same values of coupling parameters α_K and Λ but different 4-volume.

   There is also a number theoretic constraint identifiable as the counterpart of the constant energy condition defining micro-canonical ensemble. The exponent of action S must cancel from the scattering amplitudes to avoid serious existence problems in the p-adic sectors of adele associated with given extension of rationals. Criticality means thus that $exp(S)$ has same value for all preferred extremals involved. Real parts are same for all of them and imaginary parts of the action exponential are fixed modulo multiple of 2π. The analog in the case of van der Waals equation of state that the allowed states are associated with the boundary of the projection of the cusp catastrophe to (p,T) plane.

   Critical quantum states are superpositions of space-time surfaces with different 4-volumes associated with CDs with quantized size scales (distance between tips) and are generated by unitary evolution. The value of time as size of CD (distance between its tips) is not well-defined in these states.

   Remark: Quantum critical states are "timeless" as meditative practices would express it.

   This kind of superposition is created by unitary evolution operator at each step in the sequence of unitary evolutions followed by a state function reduction measuring clock time as the distance between the tips of CD. Localization to single CD is the outcome and only superposition with same time-scale and same S but possibly different 4-volumes.

3. The condition that action is same is very strong and applies to both real and imaginary parts of action (α_K is complex). The proposal [L2, L17] (see `http://tinyurl.com/yas6ofhv` and `http://tinyurl.com/y9hlt3rp`) is that the coupling constant evolution as p-adic length scale $p \simeq 2^k$, k prime corresponds to zero of Riemann ζ for $1/\alpha_K$ or is proportional to it by rational multiplier q. For $q = 1$ $Re(1/\alpha_K)$ analogous to the ordinary temperature would be equal to $Re(s) = 1/2$ for the zeros at the critical line and imaginary parts would correspond to the imaginary parts $Im(s)$ of the zeros. Constancy of the action S would boil down to the conditions

$$Re(S_K) + Re(S_{vol}) = constant \ \ , \ \ Im(S_K) + Im(S_{vol}) = constant \ \ mod \ \ 2\pi \ \ . \qquad (4.1)$$

Note that the condition for imaginary part is a typical quantization condition.

4-volume can can have arbitrary large values but for S_K this is probably not the case - this already by the quantization conditions. Hence one expects that there is some maximal possible volume for preferred extremals and thus maximal distance between the tips of CDs involved.

When the zero energy state is a superposition of only space-time surfaces with this maximal volume, further unitary evolutions are not possible and the first state function reduction to the opposite boundary of CD happens (death of self and reincarnation with opposite direction of clock time). Self has finite lifetime! This would be the classical correlate for the situation in which no quantum measurements leaving invariant the members of state pairs at the passive boundary of CD are possible.

4.2.2 The constancy of $Re(S)$

How the cancellation of real part of $\Delta(Re(S_K)) + \Delta(Re(S_{vol}))$ could take place?

1. The physical picture is that the time evolution giving rise to self starts from flux tube dominated phase obtained in the first state function reduction to the opposite boundary of CD and that also asymptotically one obtains flux tube dominated phase again but the flux tubes are scaled up. This is the TGD view about quantum cosmology as a sequences of selves and of their time reversals [K23] [L3] (see `http://tinyurl.com/y7fmaapa`). This picture suggests that the generation of magnetic flux tubes allows to satisfy the $\Delta Re(S_K) + \Delta Re(S_{vol}) = 0$ condition: in Minkowskian regions the change magnetic part of $\Delta Re(S_K)$ tends to cancel $\Delta Re(S_{vol})$ whereas the electric part is of the same sign. Therefore magnetic flux tubes are favored.

   If the sign of the volume term is negative the exponential defining the vacuum functional decreases with volume. If the relative sign of S_K and S_{vol} is negative, the magnetic part of the action is positive. The generation of flux tubes generates positive magnetic action ΔS_K helping to cancel the change ΔS_{vol}.

   The additional conditions coming from the imaginary parts are analogous to semiclassical quantization conditions.

2. The proposed picture can be realized by a proper choice of the relative signs of volume term and Kähler action term. The relative sign comes automatically correct for a positive value of cosmological constant Λ. For this choice the total action density is

$$L_{tot} = (L_K + \frac{\Lambda}{8\pi G})\sqrt{g_4} \quad . \tag{4.2}$$

   This choice gives positive vacuum energy density associated with the volume term.

3. The density of Kähler action associated with CP_2 degrees of freedom is

$$L_{K,CP_2} = -\frac{1}{4g^2} J^{\mu\nu} J_{\mu\nu} \quad . \tag{4.3}$$

   The action is proportional to $E^2 - B^2$ in Minkowskian regions and magnetic term has sign opposite to that of volume term so that these terms can compensate with the condition guaranteeing constant action. The overall sign of action in the exponent can be chosen so that the exponential vanishes for large volumes. This suggests that the volume term is negative in the vacuum functional (Kähler function as negative of the action for preferred extremal). Euclidian regions, where CP_2 part of Kähler action is of form $B^2 + E^2$ and tends to cancel the volume term.

4. There is also Kähler action in M^4 degrees of freedom. In twistor lift dimensional reduction occurs for 6-D Kähler action and M^4 part and CP_2 part contribute to Kähler action. The S^2 parts of these actions must give rise to a cosmological constant decreasing like the inverse of p-adic length scale squared. This is achieved if the Kähler contributions have opposite signs so that M^4 contribution has a non-standard sign. This is possible if M^4 Kahler form is proportional to imaginary unit and M^4 Kähler coupling strength contains additional scaling factor.

   The induced Kähler form must be sum of the M^4 parts and CP_2 parts and also the action must be sum of M^4 and CP_2 parts. This is achieved if the charge matrices of these two Kähler forms are orthogonal (the trace of their product vanishes). Since CP_2 part couples to both 1 and Γ_9 giving rise to Kähler charges proportional to 1 for quarks and 3 for leptons having opposite chiralities, the corresponding charges would be proportional to 3 for quarks and -1 for leptons.

   The imaginary unit multiplying M^4 Kähler form disappears in action and field equations and one obtains

$$L_K = -\frac{1}{4g_K^2}(\epsilon^2 J^2(M^4) + J^2(CP_2) \ , \tag{4.4}$$

   where ϵ is purely imaginary so that one has $\epsilon^2 < 0$. Since the fields are induced, negative sign for M^4 Kähler action is not expected to lead to difficulties if M^4 term is small.

Some examples are in order.

1. For cosmic string extremals Kähler action is multiple of volume action. The condition that the two actions cancel would give a constraint between Λ and α_K. Net string tension would be reduced from the value determined by CP_2 scale to a rather small value. This need not occur generally but might be true for very short p-adic length scales, where Λ is large as required by the large value of string tension associated with Kähler action. For thickened cosmic strings (magnetic flux tubes) the value of string tension assignable to Kähler action is reduced and the condition can be satisfied for smaller values of Λ.

2. For CP_2 type extremals assignable to wormhole contacts serving as basic building bricks of elementary particles the action would be finite for all size scales of CD. Both magnetic and electric contribution to the action are of same sign. For Euclidian regions with 4-D space-time projection with so strong electric field that it changes the signature of the induced metric the same is true.

3. One can ask whether blackhole interiors as Euclidian regions correspond to these Euclidian space-time sheets or to highly tangled magnetic flux tubes with length considerably longer than Schwartschild radius for which cancellation also can occur (see `http://tinyurl.com/ydhknc2c`). Both pictures are consistent in many-sheeted space-time: magnetic flux tube tangle could topologically condense to a space-time sheet with Euclidian signature. Cancellation cannot last for ever so that also blackholes are unstable against big state function reduction changing the arrow of time. Blackhole evaporation might relate to this instability.

4.2.3 The constancy of $Im(S)$ modulo 2π

If cosmological constant is real, the condition for the constancy of imaginary part of ΔS modulo 2π applies only to the case of S_K and implies that ΔS_K is fixed modulo 2π in the superposition of space-time surfaces. If zeros of ζ [L2] (see `http://tinyurl.com/yas6ofhv`) or its modification *Zeta*) [L17]) (see `http://tinyurl.com/y9hlt3rp`) give the spectrum of $1/\alpha_K$ the value of $\Delta S_{K,red} = \int Tr(J^2)dV$ is given as multiples of $2\pi n/y$, where y is imaginary part for a zero of zeta. The constancy of $Re(S)$ implies that the 4-volume ΔV is quantized as multiples of $2\pi n/\Lambda$. These conditions bring in mind semiclassical quantization of the action in multiples of $\hbar$.

It however turns out that twistor lift forces same phase for M^4 and CP_2 parts of the Kähler action so that the quantization condition for volume is lost. The reason is that $1/\alpha_K(M^4)$ and $1/\alpha_K(CP_2)$ are proportional to

$$\frac{1}{\alpha_{K,6}} = \frac{1}{\alpha_{K,4}R^2} \ , \tag{4.5}$$

where R^2 has dimensions of length squared.

4.3 Is the proposed picture consistent with twistor lift of Kähler action?

Is it possible to realize the cancellation of real parts of ΔS_{vol} and ΔS_K (modulo 2π for imaginary part) for the twistor lift of Kähler action? Does the sign of the cosmological constant Λ come out correctly (wrong sign of Λ is the probably fatal problem of M-theory)? Can one understand the p-adic evolution of the cosmological constant Λ implying that Λ becomes small in long p-adic length scales and thus solving the key problem related to Λ?

4.3.1 Dimensional reduction of the twistor lift

The condition that the induction of the product of twistor bundles of M^4 and CP_2 to the space-time surface gives the twistor bundle of the space-time surface is conjectured to determine the dynamics of the space-time surfaces. A generalization of 4-D Kähler action to 6-D Kähler action is proposed to give this dynamics, and to dimensionally reduce to a sum of Kähler actions associated with M^4 and CP_2 Kähler forms plus cosmological term.

1. Twistor bundles are sphere bundles. For the extremals of 6-D Kähler action dimensional reduction takes place since 6-D extremals must be twistor bundle of corresponding space-time surface. Therefore S^2 degrees of freedom are frozen and become non-dynamical.

   One could say that the spheres appearing as fibers of twistor bundles of M^4 and CP_2 are identified in the embedding map. The simplest correspondence between $S^2(M^4)$ and $S^2(CP_2)$ identifies (θ_1, ϕ_1) for $S^2(M^4)$ with (θ_2, ϕ_2) for $S^2(CP_2)$. This means that $S^2(X^6)$ is mapped in the same manner to $S^2(M^4)$ and $S^2(CP_2)$.

   One can imagine also correspondence with n-fold winding based on the identification $(\theta_1, \phi_1) = (\theta_2, n\phi_2)$. The area of $S^2(M^4)$ are becomes n-fold and the S^2 part of the Kähler action using θ_2 as coordinate transforms as $S_K(S^2(M^4)n = 1) \rightarrow S_K(S^2(M^4)n) = n^2 S_K(S^2(M^4))$. $n = 1$ is the most plausible option physically.

2. What the proposed general vision implies for cosmological constant as a sum of $S^2(M^4)$ and $S^2(CP_2)$ parts of 6-D Kähler action giving in dimensional reduction 4-D volume term responsible for the cosmological constant and 4-D Kähler action. If the charge matrices of M^4 and CP_2 parts of Kähler form are orthogonal one can induce Kähler form. If the coupling to M^4 Kähler form is imaginary, M^4 and CP_2 contributions to the total Kähler action have opposite signs. M^4 and CP_2 parts have opposite signs of magnetic terms and the sign of CP_2 magnetic part is opposite to the volume term.

3. The dimensionally reduced action is obtained by integrating the 6-D Kähler action over S^2 fiber. The integration gives the area $A(S^2)$ of the S^2 fiber, which in the metric induced from the spheres of twistor space of X^4 is given by

   $$A(S^2) = (1 + r^2)4\pi R^2(S^2(CP_2)) \ , \quad r = \frac{R(S^2(CP_2))}{R(S^2(M^4))} \ . \tag{4.6}$$

   The very natural but un-checked assumption is that the radius of $S^2(CP_2)$ equals to the radius $R(CP_2)$ of the geodesic sphere of CP_2:

   $$R(S^2(CP_2)) = R(CP_2) \ . \tag{4.7}$$

One obtains

$$L = -\frac{1}{16\pi\alpha_{K,6}}\left[J^2(CP_2) + \epsilon^2 J^2(M^4) + J^2(S^2(CP_2)) + \epsilon^2 J^2(S^2(M^4))\right] A(S^2) \quad . \tag{4.8}$$

The immediate conclusion is that the phases of Kähler action and volume term are same so that the quantization condition for imaginary part of the action is not obtained.

4. The Kähler coupling strengths $\alpha_K(CP_2)$ and $\alpha_K(M^4)$ can be read from the first term

$$\begin{aligned} \frac{1}{\alpha_K(CP_2)} &= \frac{1}{\alpha_{K,4}4\pi(1+r^2)}\frac{R^2(CP_2)}{R^2} \quad , \\ \frac{1}{\alpha_K(M^4)} &= \frac{\epsilon^2}{\alpha_K(CP_2)} \quad . \end{aligned} \tag{4.9}$$

One can choose the factor R^2 to be the area of S^2 by suitably renormalizing $1/\alpha_K$. This would give simpler expression

$$\begin{aligned} \frac{1}{\alpha_K(CP_2)} &= \frac{1}{\alpha_{K,4}} \quad , \\ \frac{1}{\alpha_K(M^4)} &= \frac{\epsilon^2}{\alpha_K(CP_2)} \quad . \end{aligned} \tag{4.10}$$

5. One can deduce constraints on the value of the ϵ^2 from the smallness of the contributions of the corresponding $U(1)$ gauge potential to the ordinary Coulomb potential affecting the energies of atoms by a coupling proportional to mass number A rather than Z as for Coulomb potential. This allows to distinguish between isotopes. This gives very stringent bounds on ϵ^2. I have earlier derived an upper bound treating this term as a perturbation and by considering the contribution to the Coulomb energy of hydrogen atom [L5] (see `http://tinyurl.com/y8xcem2d`). One obtains $\epsilon^2 \leq 10^{-10}$. The upper bound is also the size scale of CP breaking induced by M^4 part and characterizes also matter-antimatter asymmetry.

4.3.2 Cosmological constant

Consider next the prediction for the cosmological constant term.

1. The S^2 parts of the actions have constant values. The natural normalization of Kähler form of $J(S^2(X))$, $X = M^4, CP_2$ is as $J^2 = -2$. This a convention is the overall scale of normalization can be chosen freely by rescaling $1/\alpha_{K,4}$. Taking into account the fact that index raising is carried out by induced metric one finds that the cosmological term given the sum of M^4 and CP_2 contributions to S^2 part of Kähler action multiplied by $A(S^2)$

$$\Lambda = \frac{1}{16\pi\alpha_K}\frac{2}{(1+r^2)R^2(CP_2)}(1+\frac{\epsilon^2}{r^4}) \quad . \tag{4.11}$$

If ϵ is imaginary one can achieve the cancellation giving rise to small cosmological constant.

2. The empirical condition on cosmological constant (see `https://en.wikipedia.org/wiki/Cosmological_constant`) can be expressed in terms of critical mass density corresponding to flat 3-space as

$$\begin{aligned} \Lambda &= 3\Omega_\Lambda H^2 \quad , \qquad \Omega \simeq .691 \quad , \\ H &= \frac{da}{dt}a \qquad \frac{da}{dt} = \frac{1}{\sqrt{g_{aa}}} \quad . \end{aligned} \tag{4.12}$$

Here a corresponds to the proper time for the light-cone M^4_+ and t for the proper time for the space-time surface, which is Lorentz invariant under the Lorentz group leaving the boundary δM^4_+.

From this one obtains a condition for allowing to get idea about the discrete evolution of Λ with p-adic length scale occurring in jumps:

$$1+\frac{\epsilon^2}{r^4} = 24\pi\alpha_K(1+r^2)R^2(CP_2)\times\Omega_\Lambda H^2 \ . \tag{4.13}$$

In an excellent approximation one must have $\epsilon \simeq r^2$, $r = R(M^4)/(CP_2)$. One can consider two obvious guesses. One has either $R(M^4) = L_{Pl} = \sqrt{G}$ - that is Planck length - or one has the Compton length associated with Planck mass given by $R(M^4) = 2\pi l_{Pl}$. The first option gives in reasonable approximation $r = 2^{-11}$ and $\epsilon^2 = r^4 = 2^{-44} \sim .6\times 10^{-13}$. The second option gives $\epsilon^2 \simeq .9\times 10^{-10}$. This values corresponds roughly to the CP_2 breaking parameter and matter-antimatter asymmetry and M^4 part of the Kähler action indeed gives rise to CP_2 breaking. I have earlier derived an upper bound for ϵ by demanding that the Kähler U(1) forces does not give rise to observable effects in the energy levels of hydrogen atom. The upper bound is of the same magnitude as the estimate for ϵ^2 for the Compton scale option.

3. If one accepts p-adic length scale hypothesis $L_p \propto \sqrt{p}$, $p \simeq 2^k$ [K15], one expects $\Lambda(k) \propto 1/L(k)^2$ [K4] (see `http://tinyurl.com/ybrhguux`). How to achieve this? The only possibility is that the parameter ϵ^2 is subject to coupling constant evolution. One would have for the cosmological constant

$$\Lambda(k) \propto \frac{\epsilon^2}{r^4} - 1 \propto \frac{1}{L^2(k)} \propto 2^{-k} \ . \tag{4.14}$$

This would suggest for the 2-adic coupling constant evolution of ϵ the expression

$$\epsilon^2 = -r^4(1-X) \ , \quad X = 24\pi\alpha_K(1+r^2)R^2(CP_2)\times\Omega_\Lambda H^2 = q\times 2^{-k} \ . \tag{4.15}$$

where q is rational number. Note that from p-adic length scale hypothesis one has $2^{-k} \propto 1/L^2(k)$. One can consider also p-adic primes near powers of small prime in which case one obtains different evolution.

4. For Ω_Λ constant this would predict quantization of Hubble constant as $\Omega_\Lambda H^2 \propto 1/L(k)^2$ determined by naïve scaling dimension. The ratio of Hubble constants for two subsequent scales would be $H(k)/H(k+1) = \sqrt{2}$ if Ω is constant. The observed - and poorly understood - variation of Hubble constant from cosmological studies and distance ladder studies is in the range $50 - 73.2$ km/s/Mpc. Cosmological studies correspond to longer scales so that the smaller value of H is consistent with the decrease of H. The ratio of these upper and lower bounds is $1.46 < \sqrt{2} \simeq 1.141$ (see `http://tinyurl.com/yd6m8sca` and `http://tinyurl.com/ycr4ffm4`).

Remark: The uncertainty in the value of Hubble constant is reflected as uncertainty in the distances D deduced from cosmic redshift $z \simeq HD/c$. This is taken into account in the definition of cosmological distant unit $h^{-1}Mpc$, where h is in the range $.5-.75$ corresponding to a scale factor 1.5 rather near to $\sqrt{2}$.

5. Piecewise constant evolution means that acceleration parameter is positive since constant value of H gives

$$\frac{d^2a}{dt^2} = \frac{(da/dt)^2}{a} = aH^2 > 0 \ . \tag{4.16}$$

If the phase transitions reducing H by factor 1/2 occur at $a(k) = 2^{k/2}a_0$, one has

$$\frac{d^2a}{dt^2} \propto 2^{-k/2} \ . \tag{4.17}$$

Acceleration would be reduced gradually with rate determined by its naïve scaling dimension.

4.3.3 Solution of Hubble constant discrepancy from the length scale dependence of cosmological constant

One can critize this proposal. The recent best values of the Hubble constant are 67.0 km/s/Mpc and and 73.5 km/s/Mpc and their ratio is about 1.1 rather than $\sqrt{2}$. Therefore the hypothesis that H satisfies p-adic length scale hypothesis might be too strong. In the following a proposal in which the variation of H could be due to the variation of cosmological constant Λ satisfying p-adic length scale hypothesis is discussed.

The discrepancy of the two determinations of Hubble constant has led to a suggestion that new physics might be involved (see `http://tinyurl.com/yabszzeg`).

1. Planck observatory deduces Hubble constant H giving the expansion rate of the Universe from CMB data something like 360,000 y after Big Bang, that is from the properties of the cosmos in long length scales. Riess's team deduces H from data in short length scales by starting from galactic length scale and identifies standard candles (Cepheid variables), and uses these to deduce a distance ladder, and deduces the recent value of $H(t)$ from the redshifts.

2. The result from short length scales is 73.5 km/s/Mpc and from long scales 67.0 km/s/Mpc deduced from CMB data. In short length scales the Universe appears to expand faster. These results differ too much from each other. Note that the ratio of the values is about 1.1. There is only 10 percent discrepancy but this leads to conjecture about new physics: cosmology has become rather precise science!

TGD could provide this new physics. I have already earlier considered this problem but have not found really satisfactory understanding. The following represents a new attempt in this respect.

1. The notions of length scale are fractality are central in TGD inspired cosmology. Many-sheeted space-time forces to consider space-time always in some length scale and p-adic length scale defined the length scale hierarchy closely related to the hierarchy of Planck constants $h_{eff}/h_0 = n$ related to dark matter in TGD sense. The parameters such as Hubble constant depend on length scale and its value differ because the measurements are carried out in different length scales.

2. The new physics should relate to some deep problem of the recent day cosmology. Cosmological constant Λ certainly fits the bill. By theoretical arguments Λ should be huge making even impossible to speak about recent day cosmology. In the recent day cosmology Λ is incredibly small.

3. TGD predicts a hierarchy of space-time sheets characterized by p-adic length scales (Lk) so that cosmological constant Λ depends on p-adic length scale $L(k)$ as $\Lambda \propto 1/GL(k)^2$, where $p \simeq 2^k$ is p-adic prime characterizing the size scale of the space-time sheet defining the sub-cosmology. p-Adic length scale evolution of Universe involve as sequence of phase transitions increasing the value of $L(k)$. Long scales $L(k)$ correspond to much smaller value of Λ.

4. The vacuum energy contribution to mass density proportional to Λ goes like $1/L^2(k)$ being roughly $1/a^2$, where a is the light-cone proper time defining the "radius" $a = R(t)$ of the Universe in the Robertson-Walker metric $ds^2 = dt^2 - R^2(t)d\Omega^2$. As a consequence, at long length scales the contribution of Λ to the mass density decreases rather rapidly.

   Must however compare this contribution to the density ρ of ordinary matter. During radiation dominated phase it goes like $1/a^4$ from $T \propto 1/a$ and form small values of a radiation dominates over vacuum energy. During matter dominated phase one has $\rho \propto 1/a^3$ and also now matter dominates. During predicted cosmic string dominated asymptotic phase one has $\rho \propto 1/a^2$ and vacuum energy density gives a contribution which is due to Kähler magnetic energy and could be comparable and even larger than the dark energy due to the volume term in action.

5. The mass density is sum $\rho_m + \rho_d$ of the densities of matter and dark energy. One has $\rho_m \propto H^2$. $\Lambda \propto 1/L^2(k)$ implies that the contribution of dark energy in long length scales is considerably smaller than in the recent cosmology. In the Planck determination of H it is however assumed that cosmological constant is indeed constant. The value of H in long length scales is under-estimated so that also the standard model extrapolation from long to short length scales gives too low value of H. This is what the discrepancy of determinations of H performed in two different length scales indeed demonstrate.

A couple of remarks are in order.

1. The twistor lift of TGD [K25, K4] [L14] suggests an alternative parameterization of vacuum energy density as $\rho_{vac} = 1/L^4(k_1)$. k_1 is roughly square root of k. This gives rise to a pair of short and long p-adic length scales. The order of magnitude for $1/L(k_1)$ is roughly the same as that of CMB temperature T: $1/L(k_1) \sim T$. Clearly, the parameters $1/T$ and R correspond to a pair of p-adic length scales. The fraction of dark energy density becomes smaller during the cosmic evolution identified as length scale evolution with largest scales corresponding to earliest times. During matter dominated era the mass density going like $1/a^3$ would to dominate over dark energy for small enough values of a. The asymptotic cosmology should be cosmic string dominated predicting $1/GT^2(k)$. This does not lead to contradiction since Kähler magnetic contribution rather than that due to cosmological constant dominates.

2. There are two kinds of cosmic strings: for the other type only volume action is non-vanishing and for the second type both Kähler and volume action are non-vanishing but the contribution of the volume action decreases as function of the length scale.

5 Further comments about classical field equations in TGD framework

In the sequel some remarks about field equations defining space-time surfaces in TGD framework are made.

First three dualities at the level of field equations are discussed. These dualities are rather obvious but extremely important concerning the physical interpretation of TGD.

The earlier proposal that external particles correspond to minimal surfaces is strengthened. Also the interaction regions would correspond to minimal surfaces. The strongest condition would be that the minimal surface property break down at reaction vertices only associated with partonic 2-surfaces defining the 2-D counterparts of vertices: this would mean physical exchange of classical conserved charges between volume part of the action and Kähler action just at these points. This condition might be too strong.

The strongest condition could mean strengthening of the strong form of holography to $M^4 \times CP_2$ counterpart of the proposed number theoretic holography based on the notion of cognitive representation at the level of M^8 [L6] and also justification for the proposed construction of twistor Grassmannian variants of scattering amplitudes involving also data at a discrete set of points [L17].

5.1 Three dualities at the level of field equations

The basic field equations of TGD allow several dualities. There are 3 of them at the level of basic field equations (and several other dualities such as $M^8 - M^4 \times CP_2$ duality).

1. The first duality is the analog of particle-field duality. The spacetime surface describing the particle (3-surface of $H = M^4 \times CP_2$ instead of point-like particle) corresponds to the particle aspect whereas the fields inside it geometrized in terms of sub-manifold geometry correspond to the field aspect. Particle orbit serves as wave guide for field, one might say.

2. Second duality is particle-spacetime duality. Particle identified as 3-D surface means that particle orbit is space-time surface glued to a larger space-time surface by topological sum contacts. It depends on the scale used, whether it is more appropriate to talk about particle or of space-time.

3. The third duality is hydrodynamics- massless field theory duality. Hydrodynamical equations state local conservation of Noether currents. Field equations indeed reduce to local conservation conditions of Noether currents associated with the isometries of H. One the other hand, these equations have interpretation as non-linear geometrization of massless wave equation with coupling to Maxwell fields. This realizes the ultimate dream of theoretician: symmetries dictate the dynamics completely. This is expected to be realized also at the level of scattering amplitudes and the generalization of twistor Grassmannian amplitudes could realize this in terms of Yangian symmetry.

Hydrodynamics-wave equations duality generalizes to the fermionic sector and involves superconformal symmetry.

1. What I call modified gamma matrices Γ^α are obtained as contractions of the partial derivatives of the action defining space-time surface with respect to the gradients of embedding space coordinate with embedding space gamma matrices [K29]. The divergence $D_\alpha\Gamma^\alpha$ vanishes by field equations for the space-time surface and this is necessary for the internal consistency the Dirac equation ($\overline{\Psi}$ satisfies essentially the same equation as Ψ). Γ^α reduce to ordinary ones if the space-time surface is M^4 and one obtains ordinary massless Dirac equation.

2. Modified Dirac equation [K29] expressess conservation of super current and actually infinite number of super currents obtained by contracting second quantized induced spinor field with the solutions of modified Dirac. This corresponds to the super-hydrodynamic aspect. On the other hand, modified Dirac equation corresponds to fermionic analog of massless wave equation.

5.2 Are space-time surfaces minimal surfaces everywhere except at 2-D interaction vertices?

If one starts from the analogy with complex analysis, the natural hypothesis would be that singular surfaces are co-dimension 2 surfaces - string world sheets and partonic 2-surfaces, which are at the ends of space-time surfaces and define topological reaction vertices. Light-like 3-surfaces as partonic orbits would be formally analogous to cuts of analytic function.

One can argue [L19] that the singular surface defines a sub-manifold giving a deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to the energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. There must be one time-like or light-like direction and singular points do not satisfy this condition. There can be however an exchange of conserved charged between Kähler and volume degrees of freedom for the singular surfaces [L19]. One can also consider the possibility that the exchange is non-vanishing at singular points only. This option, which is perhaps non-realistic would be the strongest and will be discussed below.

String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations (more general option allows also string world sheets as carriers of currents).

The action S determining space-time surfaces as preferred extremals follows from twistor lift [K25, K21, K4, L17] and equals to the sum of volume term Vol multiplied by the TGD counterpart of cosmological constant and Kähler action S_K. The field equation is a geometric generalization of d'Alembert (Laplace) equation in Minkowskian (Eucidian) regions of space-time surface coupled with induced Kähler form analogous to Maxwell field. Generalization of equations of motion for particle by replacing it with 3-D surface is in question and the orbit of particle defines a region of space-time surface.

1. Zero energy ontology (ZEO) suggests that the external particles arriving to the boundaries of given causal diamond (CD) are like free massless particles and correspond to minimal surfaces as a generalization of light-like geodesic. This dynamic reduces to mere algebraic conditions and there is no dependence on the coupling parameters appearing in S. In contrast to this, in the interaction regions inside CDs there could be a coupling between Vol and S_K due to the non-vanishing divergences of energy momentum currents associated with the two terms in action cancelling each other.

2. Similar algebraic picture emerges from $M^8 - H$ duality [L6] at the level of M^8 and from what is known about preferred extremals of S assumed to satisfy infinite number of super-symplectic gauge conditions at the 3-surfaces defining the ends of space-time surface at the opposite boundaries of CD.

   At M^8 side of $M^8 - H$ duality associativity is realized as quaternionicity of either tangent or normal space of the space-time surface. The condition that there is 2-D integral distribution of sub-spaces of tangent spaces defining a distribution of complex planes as subspaces of octonionic tangent space implies the map of the space-time surface in M^8 to that of H. Given point m_8 of M^8 is mapped to a point of $M^4 \times CP_2$ as a pair of points (m_4, s) formed by $M^4 \subset M^8$ projection m_4 of m_8 point and by CP_2 point s parameterizing the tangent space or the normal space of $X^4 \subset M^8$.

   Remark: The assumption about integrable distribution of $M^2(x)$ defining string world sheet in M^4 might be too general: $M^2x)$ could not depend on x.

   If associativity or even the condition about the existence of the integrable distribution of 2-planes fails, the map to $M^4 \times CP_2$ is lost. One could cope with the situation since the gauge conditions at the boundaries of CD would allow to construct preferred extremal connecting the 3-surfaces at the boundaries of CD if this kind of surface exists at all. One can however wonder whether giving up the map $M^8 \to H$ is necessary.

3. Number theoretic dynamics in M^8 involves no action principle and no coupling constants, just the associativity and the integrable distribution of complex planes $M^2(x)$ of complexified octonions. This suggests that also the dynamics at the level of H involves coupling constants only via boundary conditions. This is the case for the minimal surface solutions suggesting that $M^8 - H$ duality maps the surfaces satisfying the above mentioned conditions to minimal surfaces. The universal dynamics conforms also with quantum criticality.

4. One can argue that the dependence of field equations on coupling parameters of S leading to a perturbative series in coupling parameters in the interior of the space-time surface inside CD spoils the extremely beautiful purely algebraic picture about the construction of solutions of field equations using conformal invariance assignable to quantum criticality. Classical perturbation series is also in conflict with the vision that the TGD counterparts twistorial Grassmannian amplitudes do not involve any loop contributions coming as powers of coupling constant parameters [L17].

To sum up, both $M^8 - H$ duality, number theoretic vision, quantum criticality, twistor lift of TGD reducing dynamics to the condition about the existence of induced twistor structure, and the proposal for the construction of twistor scattering amplitudes suggest an extremely simple picture about the situation. The divergences of the energy momentum currents of Vol and S_K would be non-vanishing delta function type singularities only at discrete points at partonic 2-surfaces defining generalized vertices so that minimal surface equations would hold almost everywhere as the original proposal indeed stated.

1. The fact that all the known extremals of field equations for S are minimal surfaces conforms with the idea. This might be due to the fact that these extremals are especially easy to

construct but could be also true quite generally apart from singular points. The divergences of the energy momentum currents associated with S_K and Vol vanish separately: this follows from the analog of holomorphy reducing the field equations to purely algebraic conditions.

It is essential that Kähler current j_K vanishes or is light-like so that its contraction with the gradients of the embedding space coordinates vanishes. Second condition is that in transversal degrees of freedom energy momentum tensor is tensor of form (1,1) in the complex sense and second fundamental form consists of parts of type (1,1) and (-1-1). In longitudinal degrees of freedom the trace H^k of the second fundamental form $H^k_{\alpha\beta} = D_\beta \partial_\alpha h^k$ vanishes.

2. Minimal surface equations are a non-linear analog of massless field equation but one would like to have also the analog of massless particle. The 3-D light-like boundaries between Minkowskian and Euclidian space-time regions are indeed analogs of massless particles as are also the string like word sheets, whose exact identification is not yet fully understood. In any case, they are crucial for the construction of scattering amplitudes in TGD based generalization of twistor Grassmannian approach. At M^8 side these points could correspond to singularities at which Galois group of the extension of rationals has a subgroup leaving the point invariant. The points at which roots of polynomial as function of parameters co-incide would serve as an analog.

   The intersections of string world sheets with the orbits of partonic 2-surface are 1-D light-like curves X^1_L defining fermion lines. The twistor Grassmannian proposal [L17] is that the ends of the fermion lines at partonic 2-surfaces defining vertices provide the information needed to construct scattering amplitudes so that information theoretically the construction of scattering amplitudes would reduce to an analog of quantum field theory for point-like particles.

3. Number theoretic vision discretizes coupling constant evolution: the values of coupling constants are labelled by parameters of extension of rationals and p-adic primes. This implies that twistor scattering amplitudes for given discrete values of coupling constants involve no radiative corrections [L17]: the construction of twistor Grassmannian amplitudes would be extremely simple. Note that infinite perturbation series would break the expression of scattering amplitudes as rational functions with coefficients int he extension of rationals defining the adele [L7, L8]. The cuts for the scattering amplitudes would be replaced by sequences of poles. This is unavoidable also because there is number theoretical discretization of momenta from the condition that their components belong to an extension of rationals defining the adele.

What could the reduction of cuts to poles for twistorial scattering amplitudes at the level of momentum space [L17] mean at space-time level?

1. Poles of an analytic function are co-dimension 2 objects. d'Alembert/Laplace equations holding true in Minkowskian/Euclidian signatures express the analogs of analyticity in 4-D case. Co-dimension 2 rule forces to ask whether partonic 2-surfaces defining the vertices and string world sheets could serve analogs of poles at space-time level? In fact, the light-like orbits X^3_L of partonic 2-surfaces allow a generalization of 2-D conformal invariance since they are metrically 2-D so that X^3_L and string world sheets could serve in the role of poles.

   X^3_L could be seen as analogs of orbits of bubbles in hydrodynamical flow in accordance with the hydrodynamical interpretations. Particle reactions would correspond to fusions and decays of these bubbles. Strings would connect these bubbles and give rise to tensor networks and serve as space-time correlates for entanglement. Reaction vertices would correspond to common ends for the incoming and outgoing bubbles. They would be analogous to the lines of Feynman diagram meeting at vertex: now vertex would be however 2-D partonic 2-surface.

2. What can one say about the singularities associated with the light-like orbits of partonic 2-surfaces? The divergence of the Kähler part T_K of energy momentum current T is proportional to a sum of contractions of Kähler current j_K with gradients ∇h^k of H coordinates. j_K need not be vanishing: it is enough that its contraction with ∇h^k vanishes and this is true if j_K is light-like. This is the case for so called massless extremals (MEs). For the other known extremals j_K vanishes.

Could the Kähler current j_K be light-like and non-vanishing and singular at X^3_L and at string world sheets? This condition would provide the long sought-for precise physical identification of string world sheets. This would also induce to the modified Dirac action a 2-D contribution. Minimal surface equations would hold true also at these two kinds of surfaces apart from possible singular points. Even more: j_K could be non-vanishing and thus also singular only at the 1-D intersections X^1_L of string world sheets with X^3_L - I have called these curves fermionic lines.

What it means that j_K is singular - that is has 2-D delta function singularity at string world sheets? j_K is defined as divergence of the induced Kähler form J so that one can use the standard definition of derivative to define j_K at string world sheet as the limiting value $j^\alpha_K = (Div_{+-}J)^\alpha = lim_{\Delta x^n \to 0}(J^{\alpha n}_+ - J^{\alpha n}_-)/\Delta x^n$, where x^n is a coordinate normal to the string world sheet. If J is discontinuous, this gives rise to a singular current located at string world sheet. This current should be light like to guarantee that energy momentum currents are divergenceless. If J is not light-like, it gives rise to isometry currents with non-vanishing divergence at string world sheet. This is guaranteed if the isometry currents $T^{\alpha A}$ are continuous through the string world sheet.

3. If the light-like j_K at partonic orbits is localized at fermionic lines X^1_L, the divergences of isometry currents could be non-vanishing and singular only at the vertices defined at partonic 2-surfaces at which fermionic lines X^1_L meet. The divergences $DivT_K$ and $DivT_{Vol}$ would be non-vanishing only at these vertices. They should of course cancel each other: $DivT_K = -DivT_{Vol}$.

4. $DivT_K$ should be non-vanishing and singular only at the intersections of string world sheets and partonic 2-surfaces defining the vertices as the ends of fermion lines. How to translate this statement to a more precise mathematical form? How to precisely define the notions of divergence at the singularity?

   The physical picture is that there is a sharing of conserved isometry charges of the incoming partonic orbit $i = 1$ determined T_K between 2 outgoing partonic orbits labelled by $j = 2, 3$. This implies charge transfer from $i = 1$ to the partonic orbits $j = 2, 3$ such that the sum of transfers sum up to the total incoming charge. This must correspond to a non-vanishing divergence proportional to delta function. The transfer of the isometry charge for given pair i, j of partonic orbits that is $Div_{i\to j}T_K$ must be determined as the limiting value of the quantity $\Delta_{i\to j}T^{\alpha,A}_K/\Delta x^\alpha$ as Δx^α approaches zero. Here $\Delta_{i\to j}T^{\alpha,A}_K$ is the difference of the components of the isometry currents between partonic orbits i and j at the vertex. The outcome is proportional delta function.

5. Similar description applies also to the volume term. Now the trace of the second fundamental form would have delta function singularity coming from $Div_{i\to j}T_K$. The condition $Div_{i\to j}T_K = -Div_{i\to j}T_{Vol}$ would bring in the dependence of the boundary conditions on coupling parameters so that space-time surface would depend on the coupling constants in accordance with quantum-classical correspondence. The manner how the coupling constants make themselves visible in the properties of space-time surface would be extremely delicate.

This picture conforms with the vision about scattering amplitudes at both M^8 and H sides of $M^8 - H$ duality.

1. M^8 dynamics based on algebraic equations for space-time surfaces [L6] leads to the proposal that scattering amplitudes can be constructed using the data only at the points of space-time surface with M^8 coordinates in the extension of the rationals defining the adele [L8, L7]. I call this discrete set of points cognitive representation with motivations coming from TGD inspired theory of consciousness [K16].

2. At H side the information theoretic interpretation would be that all information needed to construct scattering amplitudes would come from points at which the divergences of the energy momentum tensors of S_K and Vol are non-vanishing and singular.

Both pictures would realize extremely strong form of holography, much stronger than the strong form of holography that stated that only partonic 2-surfaces and string world sheets are needed.

6 Still about twistor lift of TGD

Twistor lift of TGD led to a dramatic progress in the understanding of TGD but also created problems with previous interpretation. The new element was that Kähler action as analog of Maxwell action was replaced with dimensionally reduced 6-D Kähler action decomposing to 4-D Kähler action and volume term having interpretation in terms of cosmological constant.

One can of course ask whether the resulting induced twistor structure is acceptable. Certainly it is not equivalent with the standard twistor structure. In particular, the condition $J^2 = -g$ is lost. In the case of induced Kähler form at X^4 this condition is also lost. For spinor structure the induction guarantees the existence and uniqueness of the spinor structure, and the same applies also to the induced twistor structure being together with the unique properties of twistor spaces of M^4 and CP_2 the key motivation for the notion.

There are some potential problems related to the definition of Kähler function. The most natural identification is as 6-D dimensionally reduced Kähler action.

1. WCW metric must be Euclidian - that positive definite. Since it is defined in terms of second partial derivatives of the Kähler function with respect to complex WCW coordinates and their conjugates, the preferred extremals must be completely stable to guarantee that this quadratic form is positive definite. This condition excludes extremals for which this is not the case. There are also other identifications for the preferred extremal property and stability condition would is a obvious additional condition. Note that at quantum criticality the quadratic form would have some vanishing eigenvalues representing zero modes of the WCW metric.

2. Vacuum functional of WCW is exponent of Kähler function identified as negative of Kähler action for a preferred extremal. The potential problem is that Kähler action contains both electric and magnetic parts and electric part can be negative. For the negative sign of Kähler action the action must remain bounded, otherwise vacuum functional would have arbitrarily large values. This favours the presence of magnetic fields for the preferred extremals and magnetic flux tubes are indeed the basic entities of TGD based physics.

3. One can ask whether the sign of Kähler action for preferred extremals is same as the overall sign of the diagonalized Kähler metric: this would exclude extremals dominated by Kähler electric part of action or at least force the electric part be so small that WCW metric has the same overall signature everywhere.

If one accepts the proposal that the preferred extremals are minimal surfaces (the known extremals are), extremal property is satisfied for both 4-D Kähler action and volume term separately except at finite set of singular points at which there is transfer of conserved charges between the two degrees of freedom. In this principle this would allow the identification of Kähler function as either 4-D Kähler function or 4-D volume term (actually magnetic S^2 part of 6-D Kähler action). This option looks however rather ad hoc.

6.1 Is the cosmological constant really understood?

The interpretation of the coefficient of the volume term as cosmological constant has been a long-standing interpretational issue and caused many moments of despair during years. The intuitive picture has been that cosmological constant obeys p-adic length scale scale evolution meaning that Λ would behave like $1/L_p^2 = 1/p \simeq 1/2^k$ [K4].

This would solve the problems due to the huge value of Λ predicted in GRT approach: the smoothed out behavior of Λ would be $\Lambda \propto 1/a^2$, a light-cone proper time defining cosmic time, and the recent value of Λ - or rather, its value in length scale corresponding to the size scale of the observed Universe - would be extremely small. In the very early Universe - in very short length scales - Λ would be large.

A simple solution of the problem would be the p-adic length scale evolution of Λ as $\Lambda \propto 1/p$, $p \simeq 2^k$. The flux tubes would thicken until the string tension as energy density would reach stable minimum. After this a phase transition reducing the cosmological constant would allow further thickening of the flux tubes. Cosmological expansion would take place as this kind of phase transitions (for a mundane application of this picture see [K9]).

This would solve the basic problem of cosmology, which is understanding why cosmological constant manages to be so small at early times. Time evolution would be replaced with length scale evolution and cosmological constant would be indeed huge in very short scales but its recent value would be extremely small.

I have however not really understood how this evolution could be realized! Twistor lift seems to allow only a very slow (logarithmic) p-adic length scale evolution of Λ [L16]. Is there any cure to this problem?

1. The magnetic energy decreases with the area S of flux tube as $1/S \propto 1/p \simeq 1/2^k$, where $\sqrt{p}$ defines the transversal length scale of the flux tube. Volume energy (magnetic energy associated with the twistor sphere) is positive and increases like S. The sum of these has minimum for certain radius of flux tube determined by the value of Λ. Flux tubes with quantized flux would have thickness determined by the length scale defined by the density of dark energy: $L \sim \rho_{vac}^{-1/4}$, $\rho_{dark} = \Lambda/8\pi G$. $\rho_{vac} \sim 10^{-47}$ GeV^4 (see `http://tinyurl.com/k4bwlzu`) would give $L \sim 1$ mm, which would could be interpreted as a biological length scale (maybe even neuronal length scale).

2. But can Λ be very small? In the simplest picture based on dimensionally reduced 6-D Kähler action this term is not small in comparison with the Kähler action! If the twistor spheres of M^4 and CP_2 give the same contribution to the induced Kähler form at twistor sphere of X^4, this term has maximal possible value!

   The original discussions in [K25, K4] treated the volume term and Kähler term in the dimensionally reduced action as independent terms and Λ was chosen freely. This is however not the case since the coefficients of both terms are proportional to $(1/\alpha_K^2)S(S^2)$, where $S(S^2)$ is the area of the twistor sphere of 6-D induced twistor bundle having space-time surface as base space. This are is same for the twistor spaces of M^4 and CP_2 if CP_2 size defines the only fundamental length scale. I did not even recognize this mistake.

The proposed fast p-adic length scale evolution of the cosmological constant would have extremely beautiful consequences. Could the original intuitive picture be wrong, or could the desired p-adic length scale evolution for Λ be possible after all? Could non-trivial dynamics for dimensional reduction somehow give it? To see what can happen one must look in more detail the induction of twistor structure.

1. The induction of the twistor structure by dimensional reduction involves the identification of the twistor spheres S^2 of the geometric twistor spaces $T(M^4) = M^4 \times S^2(M^4)$ and of T_{CP_2} having $S^2(CP_2)$ as fiber space. What this means that one can take the coordinates of say $S^2(M^4)$ as coordinates and embedding map maps $S^2(M^4)$ to $S^2(CP_2)$. The twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ have in the minimal scenario same radius $R(CP_2)$ (radius of the geodesic sphere of CP_2. The identification map is unique apart from $SO(3)$ rotation R of either twistor sphere possibly combined with reflection P. Could one consider the possibility that R is not trivial and that the induced Kähler forms could almost cancel each other?

2. The induced Kähler form is sum of the Kähler forms induced from $S^2(M^4)$ and $S^2(CP_2)$ and since Kähler forms are same apart from a rotation in the common S^2 coordinates, one has $J_{ind} = J + RP(J)$, where R denotes a rotation and P denotes reflection. Without reflection one cannot get arbitrary small induced Kähler form as sum of the two contributions. For mere reflection one has $J_{ind} = 0$.

   Remark: It seems that one can do with reflection if the Kähler forms of the twistor spheres are of opposite sign in standard spherical coordinates. This would mean that they have have opposite orientation.

   One can choose the rotation to act on (y,z)-plane as $(y,z) \rightarrow (cy + sz, -sz + cy)$, where s and c denote the cosines of the rotation angle. A small value of cosmological constant is obtained for small value of s. Reflection P can be chosen to correspond to $z \rightarrow -z$. Using coordinates $(u = cos(\Theta), \Phi)$ and their primed counterparts and by writing the reflection followed by rotation explicitly in coordinates (x,y,z) one finds $u' = -cu - s\sqrt{1-u^2}sin(\Phi)$, $\Phi' = arctan[(su/\sqrt{1-u^2}cos(\Phi) + ctan(\Phi)]$. In the lowest order in s one has $u' = -u - s\sqrt{1-u^2}sin(\Phi)$, $\Phi' = \Phi + scos(\Phi)(u/\sqrt{1-u^2})$.

3. Kähler form J^{tot} is sum of unrotated part $J = du \wedge d\Phi$ and $J' = du' \wedge d\Phi'$. J' equals to the determinant $\partial(u',\Phi')/\partial(u,\Phi)$. A suitable spectrum for s could reproduce the proposal $\Lambda \propto 2^{-k}$ for Λ. The S^2 part of 6-D Kähler action equals to $(J^{tot}_{\theta\phi})^2/\sqrt{g_2}$ and in the lowest order proportional to s^2. For small values of s the integral of Kähler action for S^2 over S^2 is proportional to s^2.

   One can write the S^2 part of the dimensionally reduced action as $S(S^2) = s^2F^2(s)$. Very near to the poles the integrand has $1/[sin(\Theta) + O(s)]$ singularity and this gives rise to a logarithmic dependence of F on s and one can write: $F = F(s, log(s))$. In the lowest order one has $s \simeq 2^{-k/2}$, and in improved approximation one obtains a recursion formula $s_n(S^2, k) = 2^{-k/2}/F(s_{n-1}, \log(s_{n-1})$ giving renormalization group evolution with k replaced by anomalous dimension $k_{n,a} = k + 2log[F(s_{n-1}, log(s_{n-1})]$ differing logarithmically from k.

4. The sum $J+RP(J)$ defining the induced Kähler form in $S^2(X^4)$ is covariantly constant since both terms are covariantly constant by the rotational covariance of J.

5. The embeddings of $S^2(X^4)$ as twistor sphere of space-time surface to both spheres are holomorphic since rotations are represented as holomorphic transformations. Also reflection as $z \rightarrow 1/z$ is holomorphic. This in turn implies that the second fundamental form in complex coordinates is a tensor having only components of type $(1,1)$ and $(-1,-1)$ whereas metric and energy momentum tensor have only components of type $(1,-1)$ and $(-1,1)$. Therefore all contractions appearing in field equations vanish identically and $S^2(X^4)$ is minimal surface and Kähler current in $S^2(X^4)$ vanishes since it involves components of the trace of second fundamental form. Field equations are indeed satisfied.

6. The solution of field equations becomes a family of space-time surfaces parameterized by the values of the cosmological constant Λ as function of S^2 coordinates satisfying $\Lambda/8\pi G = \rho_{vac} = J \wedge (*J)(S^2)$. In long length scales the variation range of Λ would become arbitrary small.

7. If the minimal surface equations solve separately field equations for the volume term and Kähler action everywhere apart from a discrete set of singular points, the cosmological constant affects the space-time dynamics only at these points. The physical interpretation of these points is as seats of fundamental fermions at partonic 2-surface at the ends of light-like 3-surfaces defining their orbits (induced metric changes signature at these 3-surfaces). Fermion orbits would be boundaries of fermionic string world sheets.

   One would have family of solutions of field equations but particular value of Λ would make itself visible only at the level of elementary fermions by affecting the values of coupling constants. p-Adic coupling constant evolution would be induced by the p-adic coupling constant evolution for the relative rotations R combined with reflection for the two twistor spheres. Therefore twistor lift would not be mere manner to reproduce cosmological term but determine the dynamics at the level of coupling constant evolution.

8. What is nice that also $\Lambda = 0$ option is possible. This would correspond to the variant of TGD involving only Kähler action regarded as TGD before the emergence of twistor lift. Therefore the nice results about cosmology [K23] obtained at this limit would not be lost.

6.2 Does p-adic coupling constant evolution reduce to that for cosmological constant?

One of the chronic problems if TGD has been the understanding of what coupling constant evolution could be defined in TGD.

1. The notion of quantum criticality is certainly central. The continuous coupling constant evolution having no counterpart in the p-adic sectors of adele would contain as a sub-evolution discrete p-adic coupling constant evolution such that the discrete values of coupling constants allowing interpretation also in p-adic number fields are fixed points of coupling constant evolution.

Quantum criticality is realized also in terms of zero modes, which by definition do not contribute to WCW metric. Zero modes are like control parameters of a potential function in catastrophe theory. Potential function is extremum with respect to behavior variables replaced now by WCW degrees of freedom. The graph for preferred extremals as surface in the space of zero modes is like the surface describing the catastrophe. For given zero modes there are several preferred extremals and the catastrophe corresponds to the regions of zero mode space, where some branches of co-incide. The degeneration of roots of polynomials is a concrete realization for this.

Quantum criticality would also mean that coupling parameters effectively disappear from field equations. For minimal surfaces (generalization of massless field equation allowing conformal invariance characterizing criticality) this happens since they are separately extremals of Kähler action and of volume term.

Quantum criticality is accompanied by conformal invariance in the case of 2-D systems and in TGD this symmetry extends to its 4-D analogas isometries of WCW.

2. In the case of 4-D Kähler action the natural hypothesis was that coupling constant evolution should reduce to that of Kähler coupling strength $1/\alpha_K$ inducing the evolution of other coupling parameters. Also in the case of the twistor lift $1/\alpha_K$ could have similar role. One can however ask whether the value of the 6-D Kähler action for the twistor sphere $S^2(X^4)$ defining cosmological constant could define additional parameter replacing cutoff length scale as the evolution parameter of renormalization group.

3. The hierarchy of adeles should define a hierarchy of values of coupling strengths so that the discrete coupling constant evolution could reduce to the hierarchy of extensions of rationals and be expressible in terms of parameters characterizing them.

4. I have also considered number theoretical existence conditions as a possible manner to fix the values of coupling parameters. The condition that the exponent of Kähler function should exist also for the p-adic sectors of the adele is what comes in mind as a constraint but it seems that this condition is quite too strong.

   If the functional integral is given by perturbations around single maximum of Kähler function, the exponent vanishes from the expression for the scattering amplitudes due to the presence of normalization factor. There indeed should exist only single maximum by the Euclidian signature of the WCW Kähler metric for given values of zero modes (several extrema would mean extrema with non-trivial signature) and the parameters fixing the topology of 3-surfaces at the ends of preferred extremal inside CD. This formulation as counterpart also in terms of the analog of micro-canonical ensemble (allowing only states with the same energy) allowing only discrete sum over extremals with the same Kähler action [L15].

5. I have also considered more or less ad hoc guesses for the evolution of Kähler coupling strength such as reduction of the discrete values of $1/\alpha_K$ to the spectrum of zeros of Riemann zeta or actually of its fermionic counterpart [L2]. These proposals are however highly ad hoc.

As I started once again to consider coupling constant evolution I realized that the basic problem has been the lack of explicit formula defining what coupling constant evolution really is.

1. In quantum field theories (QFTs) the presence of infinities forces the introduction of momentum cutoff. The hypothesis that scattering amplitudes do not depend on momentum cutoff forces the evolution of coupling constants. TGD is not plagued by the divergence problems of QFTs. This is fine but implies that there has been no obvious manner to define what coupling constant evolution as a continuous process making sense in the real sector of adelic physics could mean!

2. Cosmological constant is usually experienced as a terrible head ache but it could provide the helping hand now. Could the cutoff length scale be replaced with the value of the length scale defined by the cosmological constant defined by the S^2 part of 6-D Kähler action? This parameter would depend on the details of the induced twistor structure. It was shown above that if the moduli space for induced twistor structures corresponds to rotations of S^2 possibly

combined with the reflection, the parameter for coupling constant restricted to that to $SO(2)$ subgroup of $SO(3)$ could be taken to be taken $s = sin(\epsilon)$.

3. RG invariance would state that the 6-D Kähler action is stationary with respect to variations with respect to s. The variation with respect to s would involve several contributions. Besides the variation of $1/\alpha_K(s)$ and the variation of the $S(^2)$ part of 6-D Kähler action defining the cosmological constant, there would be variation coming from the variations of 4-D Kähler action plus 4-D volume term . This variation vanishes by field equations. As matter of fact, the variations of 4-D Kähler action and volume term vanish separately except at discrete set of singular points at which there is energy transfer between these terms. This condition is one manner to state quantum criticality stating that field equations involved no coupling parameters.

   One obtains explicit RG equation for α_K and Λ having the standard form involving logarithmic derivatives. The form of the equation would be

$$\frac{dlog(\alpha_K)}{ds} = -\frac{S(S^2)}{S_K(X^4) + S(S^2)} \frac{dlog(S(S^2))}{ds} \quad . \tag{6.1}$$

   The equation contains the ratio $S(S^2)/(S_K(X^4) + S(S^2))$ of actions as a parameter. This does not conform with idea of micro-locality. One can however argue that this conforms with the generalization of point like particle to 3-D surface. For preferred extremal the action is indeed determined by the 3 surfaces at its ends at the boundaries of CD. This implies that the construction of quantum theory requires the solution of classical theory.

   In particular, the 4-D classical theory is necessary for the construction of scattering amplitudes. and one cannot reduce TGD to string theory although strong form of holography states that the data about quantum states can be assigned with 2-D surfaces. Even more: $M^8 - H$ correspondence implies that the data determining quantum states can be assigned with discrete set of points defining cognitive representations for given adel This set of points depends on the preferred extremal!

4. How to identify quantum critical values of α_K? At these points one should have $dlog(\alpha_K)/ds = 0$. This implies $dlog(S(S^2)/ds = 0$, which in turn implies $dlog(\alpha_K)/ds = 0$ unless one has $S_K(X^4) + S(S^2) = 0$. This condition would make exponent of 6-D Kähler action trivial and the continuation to the p-adic sectors of adele would be trivial. I have considered also this possibility [L16].

   The critical values of coupling constant evolution would correspond to the critical values of S and therefore of cosmological constant. The basic nuisance of theoretical physics would determine the coupling constant evolution completely! Critical values are in principle possible. Both the numerator $J_{u\Phi}^2$ and the numerator $1/\sqrt{det(g)}$ increase with ϵ. If the rate for the variation of these quantities with s vary it is possible to have a situation in which the one has

$$\frac{dlog(J_{u\Phi}^2)}{ds} = -\frac{dlog(\sqrt{det(g)})}{ds} \quad . \tag{6.2}$$

5. One should demonstrate that the critical values of s are such that the continuation to p-adic sectors of the adele makes sense. For preferred extremals cosmological constant appears as a parameter in field equations but does not affect the field equations expect at the singular points. Singular points play the same role as the poles of analytic function or point charges in electrodynamics inducing long range correlations. Therefore the extremals depend on parameter s and the dependence should be such that the continuation to the p-adic sectors is possible.

   A naïve guess is that the values of s are rational numbers. Above the proposal $s = 2^{-k/2}$ motivated by p-adic length scale hypothesis was considered but also $s = p^{-k/2}$ can be considered. These guesses might be however wrong, the most important point is that there is that one can indeed calculate $\alpha_K(s)$ and identify its critical values.

6. What about scattering amplitudes and evolution of various coupling parameters? If the exponent of action disappears from scattering amplitudes, the continuation of scattering amplitudes is simple. This seems to be the only reasonable option. In the adelic approach [L7] amplitudes are determined by data at a discrete set of points of space-time surface (defining what I call cognitive representation) for which the points have M^8 coordinates belong to the extension of rationals defining the adele.

   Each point of $S^2(X^4)$ corresponds to a slightly different X^4 so that the singular points depend on the parameter s, which induces dependence of scattering amplitudes on s. Since coupling constants are identified in terms of scattering amplitudes, this induces coupling constant evolution having discrete coupling constant evolution as sub-evolution.

The following argument suggests a connection between p-adic length scale hypothesis and evolution of cosmological constant but must be taken as an ad hoc guess: the above formula is enough to predict the evolution.

1. p-Adicization is possible only under very special conditions [L7], and suggests that anomalous dimension involving logarithms should vanish for $s = 2^{-k/2}$ corresponding to preferred p-adic length scales associated with $p \simeq 2^k$. Quantum criticality in turn requires that discrete p-adic coupling constant evolution allows the values of coupling parameters, which are fixed points of RG group so that radiative corrections should vanish for them. Also anomalous dimensions Δk should vanish.

2. Could one have $\Delta k_{n,a} = 0$ for $s = 2^{-k/2}$, perhaps for even values $k = 2k_1$? If so, the ratio c/s would satisfy $c/s = 2^{k_1} - 1$ at these points and Mersenne primes as values of c/s would be obtained as a special case. Could the preferred p-adic primes correspond to a prime near to but not larger than $c/s = 2^{k_1} - 1$ as p-adic length scale hypothesis states? This suggest that we are on correct track but the hypothesis could be too strong.

3. The condition $\Delta d = 0$ should correspond to the vanishing of dS/ds. Geometrically this would mean that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

6.3 Appendix: Explicit formulas for the evolution of cosmological constants

What is needed is induced Kähler form $J(S^2(X^4)) \equiv J$ at the twistor sphere $S^2(X^4) \equiv S^2$ associated with space-time surface. $J(S^2(X^4))$ is sum of Kähler forms induced from the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$.

$$J(S^2(X^4) \equiv J = P[J(S^2(M^4)) + J(S^2(CP_2))] \quad , \tag{6.3}$$

where P is projection taking tensor quantity T_{kl} in $S^2(M^4) \times S^2(CP_2)$ to its projection in $S^2(X^4)$. Using coordinates y^k for $S^2(M^4)$ or $S^(CP_2)$ and x^μ forS^2, P is defined as

$$P : T_{kl} \to T_{\mu\nu} = T_{kl} \frac{\partial y^k}{\partial x^\mu} \frac{\partial y^l}{\partial x^\nu} \quad . \tag{6.4}$$

For the induced metric $g(S^2(X^4)) \equiv g$ one has completely analogous formula

$$g = P[g(J(S^2(M^4)) + g(S^2(CP_2))] \quad . \tag{6.5}$$

The expression for the coefficient K of the volume part of the dimensionally reduced 6-D Kähler action density is proportional to

$$L(S^2) = J^{\mu\nu} J_{\mu\nu} \sqrt{det(g)} \quad . \tag{6.6}$$

(Note that $J_{\mu\nu}$ refers to S^2 part 6-D Kähler action). This quantity reduces to

$$L(S^2) = (\epsilon^{\mu\nu} J_{\mu\nu})^2 \frac{1}{\sqrt{det(g)}} \quad . \tag{6.7}$$

where $\epsilon^{\mu\nu}$ is antisymmetric tensor density with numerical values +,-1. The volume part of the action is obtained as an integral of K over S^2:

$$S(S^2) = \int_{S^2} L(S^2) = \int_{-1}^{1} du \int_0^{2\pi} d\Phi \frac{J_{u\Phi}^2}{\sqrt{det(g)}} \quad . \tag{6.8}$$

$(u, \Phi) \equiv (cos(\Theta, \Phi)$ are standard spherical coordinates of S^2) varying in the ranges $[-1, 1]$ and $[0, 2\pi]$.

This the quantity that one must estimate.

6.3.1 General form for the embedding of twistor sphere

The embedding of $S^2(X^4) \equiv S^2$ to $S^2(M^4) \times S^2(CP_2)$ must be known. Dimensional reduction requires that the embeddings to $S^2(M^4)$ and $S^2(CP_2)$ are isometries. They can differ by a rotation possibly accompanied by reflection

One has

$$(u(S^2(M^4)), \Phi(S^2(M^4)) = (u(S^2(X^4), \Phi(S^2(X^4)) \equiv (u, \Phi) \ ,$$

$$\left[u(S^2(CP_2)), \Phi(S^2(CP_2))\right] \equiv (v, \Psi) = RP(u, \Phi)$$

where RP denotes reflection P following by rotation R acting linearly on linear coordinates (x,y,z) of unit sphere S^2). Note that one uses same coordinates for $S^2(M^4)$ and $S^2(X^4)$. From this action one can calculate the action on coordinates u and Φ by using the definite of spherical coordinates.

The Kähler forms of $S^2(M^4)$ resp. $S^2(CP_2)$ in the coordinates $(u = cos(\Theta), \Phi)$ resp.(v, Ψ) are given by $J_{u\Phi} = \epsilon = \pm 1$ resp. $J_{v\Psi} = \epsilon = \pm 1$. The signs for $S^2(M^4)$ and $S^2(CP_2)$ are same or opposite. In order to obtain small cosmological constant one must assume either

1. $\epsilon = -1$ in which case the reflection P is absent from the above formula (RP $\rightarrow$ R).
2. $\epsilon = 1$ in which case P is present. P can be represented as reflection $(x, y, z) \rightarrow (x, y, -z)$ or equivalently $(u, \Phi) \rightarrow (-u, \Phi)$.

Rotation R can represented as a rotation in (y,z)-plane by angle ϕ which must be small to get small value of cosmological constant. When the rotation R is trivial, the sum of induced Kähler forms vanishes and cosmological constant is vanishing.

6.4 Induced Kähler form

One must calculate the component $J_{u\Phi}(S^2(X^4)) \equiv J_{u\Phi}$ of the induced Kähler form and the metric determinant $det(g)$) using the induction formula expressing them as sums of projections of M^4 and CP_2 contributions and the expressions of the components of $S^2(CP_2)$ contributions in the coordinates for $S^2(M^4)$. This amounts to the calculation of partial derivatives of the transformation R (or RP) relating the coordinates (u, Φ) of $S^2(M^4)$ and to the coordinates (v, Ψ) of $S^2(CP_2)$.

In coordinates (u, Φ) one has $J_{u\Phi}(M^4) = \pm 1$ and similar expression holds for $J(v\Psi)S^2(CP_2)$. One has

$$J_{u\Phi} = 1 + \frac{\partial(v, \Psi)}{\partial(u, \Phi)} \quad . \tag{6.9}$$

where right-hand side contains the Jacobian determinant defined by the partial derivatives given by

$$\frac{\partial(v,\Psi)}{\partial(u,\Phi)} = \frac{\partial v}{\partial u}\frac{\partial \Psi}{\partial \Phi} - \frac{\partial v}{\partial \Phi}\frac{\partial \Psi}{\partial u} \quad . \tag{6.10}$$

6.4.1 Induced metric

The components of the induced metric can be deduced from the line element

$$ds^2(S^2(X^4) \equiv ds^2 = P[ds^2(S^2(M^4)) + ds^2(S^2(CP_2))] \ .$$

where P denotes projection. One has

$$P(ds^2(S^2(M^4))) = ds^2(S^2(M^4)) = \frac{du^2}{1-u^2} + (1-u^2)d\Phi^2 \ .$$

and

$$P[ds^2(S^2(CP_2))] = P[\frac{(dv)^2}{1-v^2} + (1-v^2)d\Psi^2] \ ,$$

One can express the differentials $(dv, d\Psi)$ in terms of $(du, d\Phi)$ once the relative rotation is known and one obtains

$$P[ds^2(S^2(CP_2))] = \frac{1}{1-v^2}[\frac{\partial v}{\partial u}du + \frac{\partial v}{\partial \Phi}d\Phi]^2 + (1-v^2)[\frac{\partial \Psi}{\partial u}du + \frac{\partial \Psi}{\partial \Phi}d\Phi]^2 \ .$$

This gives

$$\begin{aligned}
&P[ds^2(S^2(CP_2))] \\
&= [(\tfrac{\partial v}{\partial u})^2\tfrac{1}{1-v^2} + (1-v^2)(\tfrac{\partial \Psi}{\partial u})^2]du^2 \\
&+[(\tfrac{\partial v}{\partial \Phi})^2\tfrac{1}{1-v^2} + (\tfrac{\partial \Psi}{\partial \Phi})^2 1-v^2]d\Phi^2 \\
&+2[\tfrac{\partial v}{\partial u}\tfrac{\partial v}{\partial \Phi}\tfrac{1}{(1-v^2)} + \tfrac{\partial \Psi}{\partial u}\tfrac{\partial \Psi}{\partial \Phi}(1-v^2)]dud\Phi \ .
\end{aligned}$$

From these formulas one can pick up the components of the induced metric $g(S^2(X^4)) \equiv g$ as

$$\begin{aligned}
g_{uu} &= \tfrac{1}{1-u^2} + (\tfrac{\partial v}{\partial u})^2\tfrac{1}{1-v^2} + (1-v^2)(\tfrac{\partial \Psi}{\partial u})^2] \ , \\
g_{\Phi\Phi} &= 1-u^2 + (\tfrac{\partial v}{\partial \Phi})^2\tfrac{1}{1-v^2} + (\tfrac{\partial \Psi}{\partial \Phi})^2(1-v^2) \\
g_{u\Phi} = g_{\Phi u} &= \tfrac{\partial v}{\partial u}\tfrac{\partial v}{\partial \Phi}\tfrac{1}{(1-v^2)} + \tfrac{\partial \Psi}{\partial u}\tfrac{\partial \Psi}{\partial \Phi}(1-v^2) \ .
\end{aligned} \tag{6.11}$$

The metric determinant $det(g)$ appearing in the integral defining cosmological constant is given by

$$det(g) = g_{uu}g_{\Phi\Phi} - g_{u\Phi}^2 \ . \tag{6.12}$$

6.4.2 Coordinates (v, Ψ) in terms of (u, Φ)

To obtain the expression determining the value of cosmological constant one must calculate explicit formulas for (v, Ψ) as functions of (u, Φ) and for partial derivations of (v, Ψ) with respect to (u, Φ).

Let us restrict the consideration to the RP option.

1. P corresponds to $z \rightarrow -z$ and to

$$u \rightarrow -u \ . \tag{6.13}$$

2. The rotation R $(x, y, z) \rightarrow (x', y', z')$ corresponds to

$$x' = x, \quad y' = sz + cy = su + c\sqrt{1-u^2}sin(\Phi) \ , \quad z' = v = cu - s\sqrt{1-u^2}sin(\Phi) \ . \tag{6.14}$$

Here one has $(s, c) \equiv (sin(\epsilon), cos(\epsilon)$, where ϵ is rotation angle, which is extremely small for the value of cosmological constant in cosmological scales.

From these formulas one can pick v and $\Psi = arctan(y'/x)$ as

$$v = cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \tag{6.15}$$

3. RP corresponds to

$$v = -cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[-\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \tag{6.16}$$

6.4.3 Various partial derivatives

Various partial derivates are given by

$$\begin{aligned}
\frac{\partial v}{\partial u} &= -1 + s\frac{u}{\sqrt{1-u^2}}sin(\Phi) \ , \\
\frac{\partial v}{\partial \Phi} &= -s\frac{u}{\sqrt{1-u^2}}cos(\Phi) \ , \\
\frac{\partial \Psi}{\partial \Phi} &= (-s\frac{u}{\sqrt{1-u^2}}sin(\Phi) + c)\frac{1}{X} \ , \\
\frac{\partial \Psi}{\partial u} &= \frac{scos(\Phi)(1+u-u^2)}{(1-u^2)^{3/2}}\frac{1}{X} \ , \\
X &= cos^2(\Phi) + [-s\frac{u}{\sqrt{1-u^2}} + csin(\Phi)]^2 \ .
\end{aligned} \tag{6.17}$$

Using these expressions one can calculate the Kähler and metric and the expression for the integral giving average value of cosmological constant. Note that the field equations contain S^2 coordinates as external parameters so that each point of S^2 corresponds to a slightly different space-time surface.

6.4.4 Calculation of the evolution of cosmological constant

One must calculate numerically the dependence of the action integral S over S^2 as function of the parameter $s = sin(\epsilon)$). One should also find the extrema of S as function of s.

Especially interesting values are very small values of s since for the cosmological constant becomes small. For small values of s the integrand (see Eq. 6.8) becomes very large near poles having the behaviour $1/\sqrt{g} = 1/(sin(\Theta)+O(s))$ coming from $\sqrt{g}$ approaching that for the standard metric of S^2. The integrand remains finite for $s \neq 0$ but this behavior spoils the analytic dependence of integral on s so that one cannot do perturbation theory around $s = 0$. The expected outcome is a logarithmic dependence on s.

In the numerical calculation one must decompose the integral over S^2 to three parts.

1. There are parts coming from the small disks D^2 surrounding the poles: these give identical contributions by symmetry. One must have criterion for the radius of the disk and the natural assumption is that the disk radius is of order s.

2. Besides this one has a contribution from S^2 with disks removed and this is the regular part to which standard numerical procedures apply.

One must be careful with the expressions involving trigonometric functions which give rise to infinite if one applies the formulas in straightforward manner. These infinities are not real and cancel, when one casts the formulas in appropriate form inside the disks.

1. The limit $u \to \pm 1$ at poles involves this kind of dangerous quantities. The expression for the determinant appearing in $J_{u\Phi}$ remains however finite and $J^2_{u\phi}$ vanishes like s^2 at this limit. Also the metric determinant $1/\sqrt{g}$ remains finite expect at $s = 0$.

2. Also the expression for the quantity X in $\Psi = arctan(X)$ contains a term proportional to $1/cos(\Phi)$ approaching infinity for $\Phi \rightarrow \pi/2, 3\pi/2$. The value of $\Psi = arc(tan(X)$ remains however finite and equal to $\pm\Phi$ at this limit depending on on the sign of us.

Concerning practical calculation, the relevant formulas are given in Eqs. 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, and 6.17.

The calculation would allow to test the conjectures already discussed.

1. There indeed exist extrema satisfying thus $dS/ds = 0$.

2. These extrema correspond to $s = 2{-}k$ or more generally $s = p^{-k}$. This conjecture is inspired by p-adic length scale hypothesis.

3. A further conjecture is that for certain integer values of integer k the integral $S(S^2)$ of Eq. 6.8 is of form $S(S^2) = xs^2$ for $s = 2^{-k}$, where x is a universal numerical constant.

   This would realize the idea that p-adic length scales realized as scales associated with cosmological constant correspond to fixed points of renormalization group evolution implying that radiative corrections otherwise present cancel. In particular, the deviation from $s = 2^{-d/2}$ would mean anomalous dimension replacing $s = 2^{-d/2}$ with $s^{-(d+\Delta d)/2}$ for $d = k$ the anomalies dimension Δd would vanish.

4. The condition $\Delta d = 0$ should be equivalent with the vanishing of the dS/ds. Geometrically this means that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

7 More about the construction of scattering amplitudes in TGD framework

The construction of scattering amplitudes in TGD framework has been a longstanding problem, and I have considered several proposals - perhaps the most realistic proposal relies on the generalization of twistor Grassmann approach to TGD context [L17]. These approaches have however suffered from their ad hoc character.

One reason for the slow progress might be the fact that I have not conditioned Feynman diagrams into my spine: I have intentionally avoided this in the fear that it would prevent genuine thinking. Second reason is that TGD is really different and my mathematical skills are rather limited. For instance, in TGD classical theory is an exact part of quantum theory and particles are replaced with 3-surfaces: there is no hope of starting from Lagrangian with simple non-linearities and writing Feynman rules and deducing beta functions.

There are several questions waiting for an answer. How to achieve unitarity? What it is to be a particle in classical sense? Can one identify TGD analogs of quantum fields? Could scattering amplitudes have interpretation as Fourier transforms of n-point functions for the analogs of quantum fields?

Unitarity is certainly the issue #1 and in the sequel almost trivial solution to unitarity problem is proposed. Also quantum classical correspondence is discussed.

7.1 Some background

7.1.1 Supersymplectic algebra

Let us collect what I think is known in TGD framework.

1. The "world of classical worlds" (WCW) [K20] geometry does not exist without maximal group of isometries and WCW is assumed to possess super-symplectic algebra (SSA) assignable to light-cone boundary (boundaries of causal diamonds (CDs)) as isometries. Also Kac-Moody algebras for isometries of embedding space realized at the light-like partonic orbits serving as boundaries between Euclidian and Minkowskian regions of space-time surface are expected to be of key importance (for p-adic mass calculations applying these symmetries see [K12].

SSA has a fractal hierarchy of isomorphic sub-algebras and the proposal is that one has hierarchy of criticalities such that sub-SSA and its commutator with SSA annihilate the physical states so that SSA effectively reduces to a finite-D Lie-algebra generating the physical states. Sub-SSA takes the role of gauge algebra and one could say that it represents finite measurement resolution. This hierarchy would correspond to a hierarchies of inclusions of von Neumann algebras known as hyper-finite factors of type II_1 [K28, K8].

It seems obvious to me that the scattering amplitudes should allow a formulation in terms of SSA effectively reducing to finite-D Lie-algebra of corresponding Kac-Moody algebra plus Kac-Moody algebras associated with embedding space isometries.

Remark: Conformal weights of SSA associated with the radial light-like coordinate are non-negative so that one has analogy with Yangian algebra. The TGD variant of twistor Grassmann approach [K21] [L17] strongly suggests that SSA extends to Yangian having multi-local generators with locus corresponding to partonic 2-surface.

2. There are both classical and fermionic Noether charges associated with SSA and the Kac-Moody algebras [K6, K29, K20]. Quantum-classical correspondence (QCC) suggests that the eigenvalues for Cartan algebra Noether charges in the fermionic representation correspond to bosonic charges assignable to the dimensionally reduced Kähler action. One obtains also fermionic super-charges in 1-1 correspondence with the modes of the induced spinor field. Super-charges are very much like oscillator operators creating or annihilating fermions and there is a temptation to think that these fermionic SSA and Kac-Moody charges take the role of operators creating fermionic and bosonic states.

   One could think of constructing many-particle states at both boundaries of causal diamond (CD) by decomposing SSA to Cartan algebra and to parts acting like creation and annihilation operators. States would be created by the generators acting like oscillator operators.

   The time evolution dictated by preferred extremals and corresponding modified Dirac equation would transform initial states at boundary A of CD to final states at boundary B. This time evolution is determined by preferred extremal property and by modified Dirac equation [K29]. Time evolution is not obtained by exponentiating quantum Hamiltonian as in QFT approach. The existence of infinite-D SSA of Noether changes should make it possible to prove unitarity.

7.1.2 General argument for unitarity

The argument for unitarity is very general and based on zero energy ontology (ZEO). Causal diamond (CD) containing space-time surfaces having ends at its opposite boundaries is central for ZEO. Zero energy states are quantum superpositions of space-time surfaces, which are preferred extremals of dimensionally reduced 6-D Kähler action decomposing to 4-D Kähler action and volume term. CD has two boundaries: the active boundary (B) and passive boundary (A) and space-time surfaces as preferred extremals have ends at these boundaries [L9].

In ZEO one has two kinds of state function reductions.

1. At the active boundary (B) one has "small" state function reductions as counterparts of weak measurements following unitary time evolutions shifting the active boundary B farther from passive boundary A in statistical sense. During each unitary time evolution there is a de-localization with respect to the distance between the tips of CD followed by localization serving also as time measurement. This would yield the correlation between experienced time as sequence of these weak measurements and geometric time identified as distance between the tips of CD.

   Also measurements of observables commuting with the observables, whose eigenstates the states at boundary A are, are possible. Passive boundary (A) and the members of zero energy states associated with it do not change, and this gives rise to what one might call generalized Zeno effect.

   S-matrix would correspond to the evolution between two weak measurements for the states at the active boundary of CD and expected to be unitary. At passive boundary of CD and states at it would not be affected. The time evolution in the fermionic sector would be induced by the modified Dirac equation. Now one can express the states at new active boundary in terms of those at old active boundary and one would obtain unitary S-matrix by expressing the final states in terms of the state basic for the original boundary.

2. In "big" state function reduction the roles of passive boundary A and active boundary B are changed. The states at B are superpositions of states in the state basis for SSA. Unitary S-matrix would be obtained by expressing these states in terms of SSA basis.

Unitarity does not seem to be a problem since the conservation of Cartan charges for SSA in the fermionic representation would not allow breaking of unitarity. The time evolution would be induced by the preferred extremal property and modified Dirac equation.

Scattering amplitudes would involve an integration over positions of particles meaning that instead of single 4-surface one would have large number of them contributing to single scattering amplitude. Different position would correspond to different values of zero modes not contributing to WCW metric. Number theoretical vision [L7, L8] demands that the exponent of action is same for all of these surfaces: with inspiration coming from the idea about quantum TGD as square root of thermodynamics, I have indeed proposed [L15] this quantum analog of micro-canonical ensemble (for which energy is constant) as a way to get rid of difficulties in the realization of number theoretical universality. The number theoretically cumbersome action exponents would cancel out from the scattering amplitudes.

7.2 Does 4-D action generate lower-dimensional terms dynamically?

The original proposal was that the action defining the preferred extremals is 4-D Kähler action. Later it became obvious that there must be also 2-D string world sheet term present and probably also 1-D term associated with string boundaries at partonic 2-surfaces. The question has been whether these lower-D terms in the action are primary of generated dynamically. By superconformal symmetry the same question applies to the fermionic part of the action. The recent formulation based on the twistor lift of TGD contains also volume term but the question remains the same.

Quantum criticality would be realized as a minimal surface property realized by holomorphy in suitably generalized sense [L18, L16]. The reason is that the holomorphic solutions of minimal surface equations involve no coupling parameters as the universality of the dynamics at quantum criticality demands.

Minimal surface equation would be true apart from possible singular surfaces having dimension $D = 2, 1, 0$. $D = 2$ corresponds to string world sheets and partonic 2-surfaces. If there are 0-D singularities they would be associated with the ends of orbits of partonic 2-surfaces at boundaries of causal diamond (CD). Minimal surfaces are solutions of non-linear variant of massless d'Alembertian having as effective sources the singular surfaces at which d'Alembertian equation fails. The analogy with gauge theories is highly suggestive: singular surfaces would act as sources of massless field.

Strings world sheets seem to be necessary. The basic question is whether the singular surfaces are postulated from the beginning and there is action associated with them or whether they emerge dynamical from 4-D action. One can consider two extreme options.

Option I: There is an explicit assignment of action to the singular surfaces from the beginning. A transfer of Noether charges between space-time interior and string world sheets is possible. This kind of transfer process can take place also between string world sheets and their light-like boundaries and happens if the normal derivatives of embedding space coordinates are discontinuous at the singular surface.

Option II: No separate action is assigned with the singular surfaces. There could be a transfer of Noether charges between 4-D Kähler and volume degrees of freedom at the singular surfaces causing the failure of minimal surface property in 4-D sense. But could singular surfaces carry Noether currents as 2-D delta function like densities?

This is possible if the discontinuity of the normal derivatives generates a 2-D singular term to the action. Conservation laws require that at string world sheets energy momentum tensor should degenerate to a 2-D tensor parallel to and concentrated at string world sheet. Only 4-D action would be needed - this was actually the original proposal. Strings and particles would be essentially edges of space-time - this is not possible in GRT. Same could happen also at its boundaries giving rise to point like particles. Super-conformal symmetry would make this possible also in the fermionic sector.

For both options the singular surfaces would provide a concrete topological picture about the scattering process at the level of single space-time surface and telling what happens to the initial state. The question is whether Option I actually reduces to Option II. If the 2-D term is generated to 4-D action dynamically, there is no need to postulate primary 2-D action.

7.2.1 Can Option II generate separate 2-D action dynamically?

The following argument shows that Option II with 4-D primary action can generate dynamically 2-D term into the action so that no primary action need to be assigned with string world sheets.

1. Dimensional hierarchy of surfaces and strong form of holography

String world sheets having light-like boundaries at the light-like orbits of partonic 2-surfaces are certainly needed to realize strong form of holography [K29]. Partonic 2-surfaces emerge automatically as the ends of the orbits of wormhole contacts.

1. There could (but neet not) be a separate terms in the primary action corresponding to string world sheets and their boundaries. This hierarchy bringing in mind branes would correspond to the hierarchy of classical number fields formed by reals, complex numbers, quaternions (space-time surface), and octonions (embedding space in M^8-side of M^8 duality). The tangent - or normal spaces of these surfaces would inherit real, complex, and quaternionic structures as induced structure. The number theoretic interpretation would allow to see these surfaces as images of those surfaces in M^8 mapped to H by $M^8 - H$ duality. Therefore it would be natural to assign action to these surfaces.

2. This makes in principle possible the transfer of classical and quantum charges between space-time interior and string world sheets and between from string world sheets to their light-like boundaries. TGD variant of twistor Grassmannian approach [K21, L17] relies on the assumption that the boundaries of string world sheets at partonic orbits carry quantum numbers. Quantum criticality realized in terms of minimal surface property realized holomorphically is central for TGD and one can ask whether it could play a role in the definition of S-matrix and identification of particles as geometric objects.

3. For preferred extremals string world sheets (partonic 2-surfaces) would be complex (co-complex) manifolds in octonionic sense. Minimal surface equations would hold true outside string world sheets. Conservation of various charges would require that the divergences of canonical momentum currents at string world sheet would be equal to the discontinuities of the normal components of the canonical momentum currents in interior. These discontinuities would correspond to discontinuities of normal derivatives of embedding space coordinates and are acceptable. Similar conditions would hold true at the light-like boundaries of string world sheets at light-like boundaries of parton orbits. String world sheets would not be minimal surfaces and minimal surface property for space-time surface would fail at these surfaces.

   Quantum criticality for string world sheets would also correspond to minimal surface property. If this is realized in terms of holomorphy, the field equations for Kähler and volume parts at string world sheets would be satisfied separately and the discontinuities of normal components for the canonical momentum currents in the interior would vanish at string world sheets.

4. The idea about asymptotic states as free particles would suggest that normal components of canonical momentum currents are continuous near the boundaries of CD as boundary conditions at least. The same must be true at the light-like boundaries of string world sheets. Minimal surface property would reduce to the property of being light-like geodesics at light-like partonic 2-surface. If this is not assumed, the orbit is space-like. Even if these conditions are realized, one can imagine the possibility that at string world sheets 4-D minimal surface equation fails and there is transfer of charges between Kähler and volume degrees of freedom (Option II) and therefore breaking of quantum criticality.

If the exchange of Noether charges vanishes everywhere at string world sheets and boundaries, one could argue that they represent independent degrees of freedom and that TGD reduces to string model. The proposed equation for coupling constant evolution however contains a coefficients depending on the total action so that this would not be the case.

5. Assigning action to the lower-D objects requires additional coupling parameters. One should be able to express these parameters in terms of the parameters appearing in 4-D action (α_K and cosmological constant). For string sheets the action containing cosmological term is 4-D and Kähler action for $X^2 \times S^2$, where S^2 is non-dynamical twistor sphere is a good guess. Kähler action gets contributions from X^2 and S^2. If the 2-D action is generated dynamically as a singular term of 4-D action its coupling parameters are those of 4-D action.

6. There is a temptation to interpret this picture as a realization of strong form of holography (SH) in the sense that one can deduce the space-time surfaces by using data at string world sheets and partonic 2-surfaces and their light-like orbits. The vanishing of normal components of canonical momentum currents would fix the boundary conditions.

   If double holography $D = 4 \rightarrow D = 2 \rightarrow D = 1$ were satisfied it might be even possible to reduce the construction of S-matrix to the proposed variant of twistor Grassmann approach. This need not be the case: p-adic mass calculations rely on p-adic thermodynamics for the excitions of massless particles having CP_2 mass scale and it would seem that the double holography can makes sense for massless states only.

   In M^8-picture [L6] the information about space-time surface is coded by a polynomial defined at real line having coefficients in an extension of rationals. This real line for octonions corresponds to the time axis in the rest system rather than light-like orbit as light-like boundary of string world sheet.

2. Stringy quantum criticality?

The original intuition [L18] was that there are canonical momentum currents between Kähler and volume degrees of freedom at singular surfaces but no transfer of canonical momenta between interior and string world sheets nor string world sheets and their boundaries. Also string world sheets would be minimal surfaces as also the intuition from string models suggests. Could also the stringy quantum criticality be realized?

1. Some embedding space coordinates h^k must have discontinuous partial derivatives in directions normal to the string world sheet so that 3-surface has 1-D edge along fermionic string connecting light-like curves at partonic 2-surfaces in both Minkowskian and Euclidian regions. A closed highly flattened rectangle with long and short edges would be associated with closed monopole flux tube in the case of wormhole contact pairs assigned with elementary particles. 3-surfaces would be "edgy" entities and space-time surfaces would have 2-D and 1-D edges. In condensed matter physics these edges would be regarded as defects.

2. Quantum criticality demands that the dynamics of string world sheets and of interior effectively decouple. Same must take place for the dynamics of string world sheets and their boundaries. Decoupling allows also string world sheets to be minimal surfaces as analogs of complex surfaces whereas string world sheet boundaries would be light-like (their deformations are always space-like) so that one obtains both particles and string like objects.

3. By field equations the sums for the divergences of stringy canonical momentum currents and the corresponding singular parts of these currents in the interior must vanish. By quantum criticality in interior the divergencespf Kähler and volume terms vanish separately. Same must happen for the sums in case of string world sheets and their boundaries. The discontinuity of normal derivatives implies that the contribution from the normal directions to the divergence reduces to the sum of discontinuities in two normal directions multiplied by 2-D delta function. Thid contribution is in the general case equal to the divergence of corresponding stringy canonical momentum current but must vanish if one has quantum criticality also at string world sheets and their boundaries.

The separate continuity of Kähler and volume parts of canonical momentum currents would guarantee this but very probably implies the continuity of the induced metric and Kähler form and therefore of normal derivatives so that there would be no singularity. However, the condition that total canonical momentum currents are continuous makes sense, and indeed implies a transfer of various conserved charges between Kähler action and volume degrees of freedom at string world sheets and their boundaries in normal directions as was conjectured in [L18].

4. What about the situation in fermionic degrees of freedom? The action for string world sheet X^2 would be essentially of Kähler action for $X^2 \times S^2$, where S^2 is twistor sphere. Since the modified gamma matrices appearing in the modified Dirac equation are determined in terms of canonical momentum densities assignable to the modified Dirac action, there could be similar transfer of charges involved with the fermionic sector and the divergences of Noether charges and super-charges assignable to the volume action are non-vanishing at the singular surfaces. The above mechanism would force decoupling between interior spinors and string world sheets spinors also for the modified Dirac equation since modified gamma matrices are determined by the bosonic action.

   Remark: There is a delicacy involved with the definition of modified gamma matrices, which for volume term are proportional to the induced gamma matrices (projections of the embedding space gamma matrices to space-time surface). Modified gamma matrices are proportional to the contractions $T^{\alpha}_k\Gamma^k$ of canonical momentum densities $T^{\alpha_k} = \partial L/\partial(\partial_\alpha h^k)$ with embedding space gamma matrices Γ^k. To get dimension correctly in the case of volume action one must divide away the factor $\Lambda/8\pi G$. Therefore fermionic super-symplectic currents do not involve this factor as required.

It remains an open question whether the string quantum criticality is realized everywhere or only near the ends of string world sheets near boundaries of CD.

3. String world sheet singularities as infinitely sharp edges and dynamical generation of string world sheet action

The condition that the singularities are 2-D string world sheets forces 1-D edges of 3-surfaces to be infinitely sharp.

Consider an edge at 3-surface. The divergence from the discontinuity contains contributions from two normal coordinates proportional to a delta function for the normal coordinate and coming from the discontinuity. The discontinuity must be however localized to the string rather than 2-surface. There must be present also a delta function for the second normal coordinate. Hence the value of also discontinuity must be infinite. One would have infinitely sharp edge. A concrete example is provided by function $y = |x|^\alpha$ $\alpha < 1$. This kind of situation is encountered in Thom's catastrope theory for the projection of the catastrophe: in this case one has $\alpha = 1/2$. This argument generalizes to 3-D case but visualization is possible only as a motion of infinitely sharp edge of 3-surface.

Kähler form and metric are second degree monomials of partial derivatives so that an attractive assumption is that $g_{\alpha\beta}$, $J_{\alpha\beta}$ and therefore also the components of volume and Kähler energy momentum tensor are continuous. This would allow $\partial_{n_i} h^k$ to become infinite and change sign at the discontinuity as the idea about infinitely sharp edge suggests. This would reduce the continuity conditions for canonical momentum currents to rather simple form

$$T^{n_i n_j}\Delta\partial_{n_j} h^k = 0 \ . \tag{7.1}$$

which in turn would give

$$T^{n_i n_j} = 0 \tag{7.2}$$

stating that canonical momentum is conserved but transferred between Kähler and volume degrees of freedom. One would have a condition for a continuous quantity conforming with the intuitive view about boundary conditions due to conservation laws. The condition would state that energy momentum tensor reduces to that for string world sheet at the singularity so that the system becomes effectively 2-D. I have already earlier proposed this condition.

The reduction of 4-D locally to effectively 2-D system raises the question whether any separate action is needed for string world sheets (and their boundaries)? The generated 2-D action would be similar to the proposed 2-D action. By super-conformal symmetry similar generation of 2-D action would take place also in the fermionic degrees of freedom. I have proposed also this option already earlier. This would mean that Option II is enough.

The following gives a more explicit analysis of the singularities. The vanishing on the discontinuity for the sum of normal derivative gives terms with varying degree of divergence. Denote by n_i resp. t_i the coordinate indices in the normal resp. tangent space. Suppose that some derivative $\partial_{n_i} h^k$ become infinite at string. One can introduce degree n_D of divergence for a quantity appearing as part of canonical momentum current as the degree of the highest monomial consisting of the diverging derivatives $\partial_{n_i} h^k$ appearing in quantity in question. For the leading term in continuity conditions for canonical momentum currents of total action one should have $n_D = 2$ to give the required 2-D delta function singularity.

- $\partial_{n_i} h^k$ has $n_D \leq 1$. If it is also discontinuous - say changes sign - one has $n_D = 2$ for $\Delta \partial_{n_i} h^k$ in direction n_i.
- One has $n_D(g_{t_i t_j}) = 0$, $n_D(g_{t_i n_j}) = 1$, $n_D(g_{n_i n_i}) = 2$ and $n_D(g_{n_i n_j}) = 1$ or 2 for $i \neq j$. One has $n_D(g) = 4$ ($g = det(g_{\alpha\beta})$). For contravariant metric one gas $n_D(g^{t_i t_j}) = 0$ and $n_D(g^{n_i j}) = n_D(g^{n_i n_j}) = -2$ as is easy to see from the formula for $g^{\alpha\beta}$ in terms of cofactors.
- Both Kähler and volume terms in canonical momentum current are proportional to $\sqrt{g}$ with $n_D(\sqrt{g}) = 2$ having leading term proportional to 2-determinant $\sqrt{det(g_{n_i n_j})}$. In Kähler action the leading term comes from tangent space part J_{ij} and has $n_D = -1$ coming from the partial derivative. The remaining parts involving $J_{t_i n_j}$ or $J_{n_i n_j}$ have $n_D < 0$.
- Consider the behavior of the contribution of volume term to the canonical momentum currents. For $g^{n_i t_j} \partial_{t_j} h^k \sqrt{g}$ one has $n_D = 0$ so that this term is finite. For $g^{n_i n_j} \partial_{n_j} h^k \sqrt{g}$ one has $n_D \leq 1$ and this term can be infinite as also its discontinuity coming solely from the change of sign for $\partial_{n_j} h^k$. If $\partial_{n_j} h^k$ is infinite and changes sign, one can have $n_D = 2$ as required by 2-D delta function singularity.

  The continuity condition for the canonical momentum current would state the vanishing of $n_D = 2$ discontinuity but would not imply separate vanishing of discontinuity for Kähler and volume parts of canonical momentum currents - this in accordance with the idea about canonical momentum transfer. If the sign of partial derivative only changes the coefficient of the partial derivative must vanish so that the condition reduces to the condition $T^{n_i n_j} = 0$ already given for the components of the total energy momentum tensor, which would be continuous by the above assumption.

4. A connection with Higgs vacuum expectation?

What about the physical interpretation of the singular divergences of the isometry currents J_A of the volume action located at string world sheet?

1. The divergences of J_A are proportional to the trace of the second fundamental form H formed by the covariant derivatives of gradients $\partial_\alpha h^k$ of H-coordinates in the interior and vanish. The singular contribution at string world sheets is determined by the discontinuity of the isometry current J_A and involves only the first derivatives $\partial_\alpha h^k$.
2. One of the first questions after ending up with TGD for 41 years ago was whether the trace of H in the case of CP_2 coordinates could serve as something analogous to Higgs vacuum expectation value. The length squared for the trace has dimensions of mass squared. The discontinuity of the isometry currents for $SU(3)$ parts in $h = u(2)$ and its complement t, whose complex coordinates define $u(2)$ doublet. $u(2)$ is in correspondence with electroweak algebra and t with complex Higgs doublet. Could an interpretation as Higgs or even its vacuum expectation make sense?

3. p-Adic thermodynamics explains fermion masses elegantly (understanding of boson masses is not in so good shape) in terms of thermal mixing with excitations having CP_2 mass scale and assignable to short string associated with wormhole contacts. There is also a contribution from long strings connecting wormhole contacts and this could be important for the understanding of weak gauge boson masses. Could the discontinuity of isometry currents in t determine this contribution to mass. Edges/folds would carry mass.

4. The non-singular part of the divergence multiplying 2-D delta function has dimension 1/length squared and the square of this vector in CP_2 metric has dimension of mass squared. Could the interpretation of the discontinuity as Higgs expectation make sense? If so, Higgs expectation would vanish in the space-time interior.

   Could the interior modes of the induced spinor field - or at least the interior mode of right-handed neutrino ν_R having no couplings to weak or color fields - be massless in 8-D or even 4-D sense? Could ν_R and $\overline{\nu}_R$ generate an unbroken $\mathcal{N}=2$ SUSY in interior whereas inside string world sheets right-handed neutrino and antineutrino would be eaten in neutrino massivation and the generators of $\mathcal{N}=2$ SUSY would be lost somewhat like charged components of Higgs!

   If so, particle physicists would be trying to find SUSY from wrong place. Space-time interior would be the correct place. Would the search of SUSY be condensed matter physics rather than particle physics?

7.2.2 Summarizing the recent view about elementary particles

It is interesting to see how elementary particles and their basic interaction vertices could be realized in this framework.

1. In TGD framework particle would correspond to pair of wormhole contact associated with closed magnetic flux tube carrying monopole flux. Strongly flattened rectangle with Minkowskian flux tubes as long edges with length given by weak scale and Euclidian wormhole contacts as short edges with CP_2 radius as lengths scale is a good visualization. 3-particle vertex corresponding to the replication of this kind of flux tube rectangle to two rectangles would replace 3-vertex of Feynman graph. There is analogy with DNA replication. Similar replication is expected to be possible also for the associated closed fermionic strings.

2. Denote the wormhole contacts by A and B and their opposite throats by A_i and B_i, $i=1,2$. For fermions A_1 can be assumed to carry the electroweak quantum numbers of fermion. For electroweak bosons A_1 and A_2 (for instance) could carry fermion and anti-fermion, whose quantum numbers sum up to those of ew gauge boson. These "corner fermions" can be called *active*.

   Also other distributions of quantum numbers must be considered. Fermion and anti-fermion could in principle reside at the same throat - say A_1. One can however assume that second wormhole contact, say A has quantum numbers of fermion or weak boson (or gluon) and second contact carries quantum numbers screening weak isospin.

3. The model assumes that the weak isospin is neutralized in length scales longer than the size of the flux tube structure given by electro-weak scale. The screening fermions can be called *passive*. If the weak isospin of $W^{\pm}$ boson is neutralized in the scale of flux tube, 2 $\nu_L\overline{\nu}_R$ pairs are needed (lepton number for these pairs must vanish) for W^-. For Z $\nu_L\overline{\nu}_R$ and $\overline{\nu}_L nu_R$ are needed. The pairs of passive fermions could reside in the interior of flux tube, at string world sheet or at its corners just like active fermions. The first extreme is that the neutralizing neutrino-antineutrino pairs reside in interior at the opposite long edges of the rectangular *flux tube*. Second extreme is that they are at the corners of rectangular *closed string*.

4. Rectangular closed string containing active fermion at wormhole A (say) and with members of isospin neutralizing neutrino-antineutrino pair at the throats of B serves as basic units. In scales shorter than string length the end A would behave like fermion with weak isospin. At longer scales physical fermion would be hadron like entity with vanishing isospin and one could speak of confinement of weak isospin.

From these physical fermions one can build gauge bosons as bound states. Weak bosons and also gluons would be pairs of this kind of fermionic closed strings connecting wormhole contacts A and B. Gauge bosons (and also gravitons) could be seen as composites of string like physical fermions with vanishing net isospin rather than those of point like fundamental fermions.

5. The decay of weak boson to fermion-antifermion pair would be flux tube replication in which closed strings representing physical fermion and anti-fermion continue along different copies of flux tube structure. The decay of boson to two bosons - say $W \rightarrow WZ$ - by replication of flux tube would require creation of a pair of physical fermionic closed strings representing Z. This would correspond to a V-shaped vertex with the edge of V representing closed fermionic closed string turning backwards in time. In decays like $Z \rightarrow W^+W^-$ two closed fermion strings would be created in the replication of flux tube. Rectangular fermionic string would turns backwards in time in the replication vertex and the rectangular strings of Z would be shared between W^+ and W^-.

This mesonlike picture about weak bosons as bound states of fermions sounds complex as compared with standard model picture. On the other hand only the spinor fields assignable to single fermion family are present.

A couple of comments concerning this picture are in order.

1. M^8 duality provides a different perspective. In M^8 picture these vertices could correspond to analogs of local 3 particle vertices for octonionic superfield, which become nonlocal in the map taking $M^8 = M^4 \times CP_2$ surfaces to surfaces in $H = M^4 \times CP_2$. The reason is that M^4 point is mapped to M^4 point but the tangent space at E^4 point is mapped to a point of CP_2. If the point in M^8 corresponds to a self-intersection point the tangent space at the point is not unique and point is mapped to two distinct points. There local vertex in M^8 would correspond to non-local vertex in H and fermion lines could just begin. This would mean that at H-level fermion line at moment of replication and V-shaped fermion line pair beginning at different point of throat could correspond to 3-vertex at M^8 level.

2. The 3-vertex representing replication could have interpretation in terms of quantum criticality: in reversed direction of time two branches of solution of classical field equations would co-incide.

7.2.3 Gravitation as a square of gauge interaction

I encountered in FB a link to an interesting popular article (see `http://tinyurl.com/y5r4glgg`) about theoretical physicist Henrik Johansson who has worked with supergravity in Wallenberg Academy. He has found strong mathematical evidence for a new duality. Various variants of super quantum gravity support the view that supersymmetric quantum theories of gravitation can be seen as a double copy of a gauge theory. One could say that spin 2 gravitons are gluons with color charge replaced with spin. Since the information about charges disappears, gluons can be understood very generally as gauge bosons for given gauge theory, not necessarily QCD.

The article of C. D. White [B3] (see `https://arxiv.org/pdf/1708.07056.pdf`) entitled "The double copy: gravity from gluons" explains in more detail the double copy duality and also shows that it relates in many cases also exact classical solutions of Einsteins equations and YM theories. One starts from L-loop scattering amplitude involving products of kinematical factors n_i and color factors c_i and replaces color factors with extra kinematical factors $\tilde{n}_i$. The outcome is an L-loop amplitude for gravitons.

As if gravitation could be regarded as a gauge theory with polarization and/or momenta identified giving rise to effective color charges. This is like taking gauge potential and giving it additional index to get metric tensor. This naïve analogy seems to hold true at the level of scattering amplitudes and also for many classical solutions of field equations. Could one think that gravitons as states correspond to gauge singlets formed from two gluons and having spin 2? Also spin 1 and spin 0 states would be obtained and double copies involve also them.

TGD view about elementary particles indeed predicts that gravitons be regarded in certain sense pairs of gauge bosons. Consider now gravitons and assume for simplicity that spartners of fundamental fermions - identifiable as local multi-fermion states allowed by statistics - are not involved: this does not change the situation much [L21]. Graviton's spin 2 requires 2 fermions and 2 anti-fermions: fermion or anti-fermion at each throat. For gauge bosons fermion and anti-fermion at two throats is enough. One could therefore formally see gravitons as pairs of two gauge bosons in accordance with the idea about graviton is a square of gauge boson.

The fermion contents of the monopole flux tube associated with elementary particle determines quantum numbers of the flux tube as particle and characterizes corresponding interaction. The interaction depends also on the charges at the ends of the flux tube. This leads to a possible interpretation for the formation of bound states in terms of flux tubes carrying quantum numbers of particles.

1. These long flux tubes can be arbitrarily long for large values of $\hbar_{eff} = n \times \hbar_0$ assigned to the flux tube. A plausible guess for for the expression of $\hbar$ in terms of $_0$ is as $\hbar = 6 \times \hbar_0$ [L4, L11]. The length of the flux tube scales like $\hbar_{eff}$.

2. Nottale [?] proposed that it makes sense to speak about gravitational Planck constant h_{gr}. In TGD this idea is generalized and interpreted in framework of generalized quantum theory [K22, K18, K3]. For flux tubes assignable to gravitational bound states along which gravitons propagate, one would have $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is parameter with dimensions of velocity. One could write interaction strength as

$$GMm = v_0 \times \hbar_{gr} \quad .$$

3. $\hbar_{gr}$ obtained from this formula must satisfy $\hbar_{gr} > \hbar$. This generalizes to other interactions. For instance, one has one would have

$$Z_1 Z_2 e^2 = \frac{v_0 \hbar_{em}}{}$$

for electromagnetic flux tubes in the case that ones $h_{em} > \hbar$. The interpretation of the velocity parameter v_0 is discussed in [K3].

One could even turn the situation around and say that the value of $\hbar_{eff}$ fixes the interaction strength. $\hbar_{eff}$ would depend on fermion content and thus of virtual particle and also on the masses or other charges at the ends of the flux tube. The longer the range of the interaction, the larger the typical value of $\hbar_{eff}$.

4. The interpretation could be in terms long length scale quantum fluctuations at quantum criticality. Particles generate U-shaped monopole flux tubes with varying length proportional to $\hbar_{gr}$. If these U-shaped flux tubes from two different particles find each other, they reconnect to flux tube pairs connecting particles and give rise to interaction. What comes in mind is tiny curious and social animals studying their environment.

5. I have indeed proposed this picture in biology: the U-shaped flux tubes would be tentacles with which bio-molecules (in particular) would be scanning their environment. This scanning would be the basic mechanism behind immune system. It would also make possible for bio-molecules to find each in molecular crowd and provide a mechanism of catalysis. Could this picture apply completely generally? Would even elementary particles be scanning their environment with these tentacles?

6. Could one interpret the flux tubes as analogs of virtual particles or could they replace virtual particles of quantum field theories? The objection is that flux tubes would have time-like momenta whereas virtual particle analogs would have space-like momenta. The interpretation makes sense only if the associated momenta are between space-like and time-like that is light-like so that flux tube would correspond to mass shell particle. But this is the case in twistor approach to gauge theories also in TGD [L21] (see `http://tinyurl.com/y62no62a`).

Perhaps the following interpretation is more appropriate. Flux tubes are accompanied by strings and string world sheets can be interpreted as stringy description of gravitation and other interactions.

7.2.4 Kähler calibrations: an idea before its time?

While updating book introductions I was surprised to find that I had talked about so called calibrations of sub-manifolds as something potentially important for TGD and later forgotten the whole idea! A closer examination however demonstrated that I had ended up with the analog of this notion completely independently later as the idea that preferred extremals are minimal surfaces apart form 2-D singular surfaces, where there would be exchange of Noether charges between Kähler and volume degrees of freedom.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

   Calibrated k-manifolds are minimal surfaces in their homology class, in other words their volume is minimal. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces and representing fundamental fermions play a key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality outside singular surfaces would be realized as decoupling of the two parts of the action. May be all preferred extremals be regarded as calibrated in generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

   It is interesting to recall that the original proposal for the preferred extremals as absolute minima of Kähler action has transformed during years to a proposal that they are absolute minima of volume action within given homology class and having fixed ends at the boundaries of CD.

4. The experience with CP_2 would suggest that the Kähler structure of M^4 defining the counterpart of form ϕ is unique. There is however infinite number of different closed self-dual Kähler forms of M^4 defining what I have called Hamilton-Jacobi structures. These forms can have subgroups of Poincare group as symmetries. For instance, magnetic flux tubes correspond to given cylindrically symmetry Kähler form. The problem disappears as one realizes that Kähler structures characterize families of preferred extremals rather than M^4 itself.

If the notion of calibration indeed generalizes, one ends up with the same outcome - preferred extremals as minimal surfaces with 2-D string world sheets and partonic 2-surfaces as singularities - from many different directions.

1. Quantum criticality requires that dynamics does not depend on coupling parameters so that extremals must be separately extremals of both volume term and Kähler action and therefore minimal surfaces for which these degrees of freedom decouple except at singular 2-surfaces, where the necessary transfer of Noether charges between two degrees of freedom takes place at these. One ends up with string picture but strings alone are of course not enough. For instance, the dynamical string tension is determined by the dynamics for the twistor lift.

2. Almost topological QFT picture implies the same outcome: topological QFT property fails only at the string world sheets.

3. Discrete coupling constant evolution, vanishing of loop corrections, and number theoretical condition that scattering amplitudes make sense also in p-adic number fields, requires a representation of scattering amplitudes as sum over resonances realized in terms of string world sheets.

4. In the standard QFT picture about scattering incoming states are solutions of free massless field equations and interaction regions the fields have currents as sources. This picture is realized by the twistor lift of TGD in which the volume action corresponds to geodesic length and Kähler action to Maxwell action and coupling corresponds to a transfer of Noether charges between volume and Kähler degrees of freedom. Massless modes are represented by minimal surfaces arriving inside causal diamond (CD) and minimal surface property fails in the scattering region consisting of string world sheets.

5. Twistor lift forces M^4 to have generalize Kähler form and this in turn strongly suggests a generalization of the notion of calibration. At physics side the implication is the understanding of CP breaking and matter anti-matter asymmetry.

6. M^8-H duality requires that the dynamics of space-time surfaces in H is equivalent with the algebraic dynamics in M^8. The effective reduction to almost topological dynamics implied by the minimal surface property implies this. String world sheets (partonic 2-surfaces) in H would be images of complex (co-complex sub-manifolds) of $X^4 \subset M^8$ in H. This should allows to understand why the partial derivatives of embedding space coordinates can be discontinuous at these edges/folds but there is no flow between interior and singular surface implying that string world sheets are minimal surfaces (so that one has conformal invariance).

The analogy with foams in 3-D space deserves to be noticed.

1. Foams can be modelled as 2-D minimal surfaces with edges meeting at vertices. TGD space-time could be seen as a dynamically generated foam in 4-D many-sheeted space-time consisting of 2-D minimal surfaces such that also the 4-D complement is a minimal surface. The counterparts for vertices would be light-like curves at light like orbits of partonic 2-surfaces from which several string world sheets can emanate.

2. Can one imagine something more analogous to the usual 3-D foam? Could the light-like orbits of partonic 2-surfaces define an analog of ordinary foam? Could also partonic 2-surfaces have edges consisting of 2-D minimal surfaces joined along edges representing strings connecting fermions inside partonic 2-surface?

   For years ago I proposed what I called as symplectic QFT (SQFT) as an analog of conformal QFT and as part of quantum TGD [K5]. SQFT would have symplectic transformations as symmetries, and provide a description for the symplectic dynamics of partonic 2-surfaces. SQFT involves an analog of triangulation at partonic 2-surfaces and Kähler magnetic fluxes associated with them serve as observables. The problem was how to fix this kind of network. Partonic foam could serve as a concrete physical realization for the symplectic network and have fundamental fermions at vertices. The edges at partonic 2-surfaces would be space-like geodesics. The outcome would be a calibration involving objects of all dimensions $0 \leq D \leq 4$ - a physical analog of homology theory.

7.3 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

7.3.1 Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K25]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A4]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

M^8-H duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L16].

7.3.2 Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in

twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L7]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

7.3.3 Number-theoretic approach to unitarity

Twistorialization leads to the proposal that cuts in the scattering amplitudes are replaced with sums over poles, and that also many-particle states have discrete momentum and mass squared spectrum having interpretation in terms of bound states. Gravitation would be the natural physical reason for the discreteness of the mass spectrum and in string models it indeed emerges as "stringy" mass spectrum. The situation is very similar to that in dual resonance models, which were precedessors of super string theories.

Number theoretical discretization based on the hierarchy of extensions of rationals defining extensions of p-adic number fields gives rise to cognitive representatations as discrete sets of space-time surface and discretization of 4-momenta and S-matrix with discrete momentum labels. In number theoretic discretization cuts reduce automatically to sequences of poles. Whether this discretization is an approximation reflecting finite cognitive resolution or whether finite cognitive representation is a property of physical states reflecting itself as a condition that various parameters characterizing them belong to the extension considered, remains an open question.

One can approach the unitarity conditions also number theoretically. In the discretization forced by the extension of rationals the amplitudes are defined between states having a discrete spectrum of 4-momenta. Unitarity condition reduces to a purely algebraic condition involving only sums. In these conditions the Dirac delta functions associated with the mass squared of the resonances are replaced with Kronecker deltas.

1. For given extension of rationals the unitary conditions are purely algebraic equations

$$i(T_{mn}+\overline{T}_{nm})=\sum_r T_{mr}\overline{T}_{nr}=T_{mn}\overline{T}_{nn}+T_{mm}\overline{T}_{mn}+\sum_{r\neq m,n}T_{mr}\overline{T}_{nr} \quad .$$

   where T_{mn} belongs the extension. Complex imaginary unit i corresponds to that appearing in the extension of octonions in M^8-H duality [L6].

2. In the forward direction $m=n$ one obtains

$$2Im(T_{mm})=Re(T_{mm})^2+Im(T_{mm})^2+P_m \quad , \quad P_m=\sum_{r\neq m}T_{mr}\overline{T}_{mr} \quad .$$

   P_m represents total probability for non-forward scattering.

3. One can think of solving $Im(T_{mm})$ algebraically from this second order polynomial in the lowest order approximation in which $T_{mn}=0$ for $m\neq n$. This gives

$$2Im(T_{mm})=1+\sqrt{1-P_m-Re(T_{mm})^2} \quad .$$

   Reality requires $1-Re(T_{mm})^2-P_m\geq 0$ giving

$$Re(T_{mm})^2+P_m\leq 1 \quad .$$

   This condition is identically true by unitarity since probability for scattering cannot be larger than 1.

   Besides this the real root must belong to the original extension of rationals. For instance, if the extension of rationals is trivial, the quantity $1-P_m-Re(T_{mm})^2$ must be a square of rational y giving $1-P_m=y^2+Re(T_{mm})^2$. In the case of extension y is replaced with a number in the extension. I am not enough of number theorist to guess how powerful this kind of number theoretical conditions might be. In any case, the general ansatz for S is a unitary matrix in extension of rationals and this kind of matrices form a group so that there is no hope about unique solution.

4. One could think of iterative solution of the conditions by assuming in the zeroth order approximation $T_{mn} = 0$ for $m \neq n$ giving $Re(T_{mm})^2 + Im(T_{mm})^2 = 1$ reducing to $cos^2(\theta) + sin^2(\theta) = 1$. For trivial extension of rationals θ would correspond to Pythagorean triangle.

   For non-diagonal elements of T_{mn} one would obtain at the next step the conditions

$$i(T_{mn} + \overline{T}_{nm}) = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{nm} \quad .$$

   This gives a 2 linear equations for T_{mn}.

5. These conditions are not enough to give unique solution. Time reversal invariance gives additional conditions and might help in this respect. T invariance is slightly broken but CPT symmetry could replace T symmetry in the general situation.

   Time reversal operator T (to be not confused with T_{mn} above) is anti-unitary operator and one has $S^\dagger = T(S)$. In wave mechanics one can show that T-invariant S-matrix and thus also T-matrix is symmetric: $S = S^T$. The matrices of this kind do not form a group so that the conditions can be very powerful.

   Combined with the above equations symmetry gives

$$2Im(T_{mn}) = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn} \quad .$$

   The two conditions for T_{mn} in principle fix it completely in this order.

   One obtains from the real part of the equation

$$2Im(T_{mn}) = Re[T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn}] \quad .$$

   The vanishing of the imaginary part gives

$$Im[T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn}] = 0 \quad .$$

   giving a linear relation between the real and imaginary parts of T_{mn}. No new number theoretical conditions emerge. This relation requires that real and imaginary parts belong to the extension.

6. At higher orders one must feed the resulting ansatz to the unitarity conditions for the diagonal elements T_{nn}. One can hope that the lowest order ansatz leads to rather unique solution by iteration of the unitarity conditions. In higher order conditions the higher order corrections appear linearly so that no new number theoretic conditions emerge at higher orders.

   Physical picture suggests that the S-matrices could be obtained by an iterative procedure. Since infinitely long procedure very probably leads out of the extension, one can ask whether the procedure should stop after finite steps. This property would pose an additional conditions to the S-matrix.

7. Diagonal matrices are solutions to the conditions and for then the diagonal elements are roots of unity in the extension of rationals considered. The automorphisms $S_d \rightarrow US_dU^{-1}$ produce new S-matrices and if the unitary matrix U is orthogonal real matrix in algebraic extension satisfying therefore $UU^T = 1$, the condition $S = S^T$ is satisfied. There are therefore a large number of solutions.

   S-matrices diagonalizable in the extension are not the only solutions. The diagonalization of a unitary matrix $S = S^T$ in general gives a diagonal S-matrix, for which the roots of unity in general do not belong to the extension. Also the diagonalizating matrix fails to be in the extension. This non-diagonalizability might have deep physics content and explain why the physically natural state basis is not the one in which S-matrix is diagonal. In the case of density matrix it would guarantee stability of entanglement.

To sum up, number theoretic conditions could give rise to highly unique discrete S-matrices, when CPT symmetry can be formulated purely algebraically and be combined with unitarity. CPT symmetry might not however allow formulation in terms of automorphisms of diagonal unitary matrices analogous to orthogonal transformations.

7.4 Summary

It seems that unitarity of S-matrix reduces to the existence of maximal group of WCW isometries. The conservation of charges implies conservation of probability and unitarity.

Disjoint 3-surfaces and also those topologically condensed at larger space-time sheets would have interpretation as topological representations of particles in this approach. The special role of the partonic orbits suggests holography in the sense that these orbits have particle interpretation. Similar holography would make sense true for string world sheets and their boundaries. Action could therefore contain parts associated with $D = 2$ and $D = 1$ surfaces so that oscillator operators associated with these would be involved in the construction of states.

The transfer of quantum numbers from space-time interior to string world sheets could take place in interaction regions for Option I for which one assigns action to singular surfaces identified as surfaces having complex or real tangent space at M^8 level. The transfer would naturally vanish near the boundaries of CD. Same applies to the transfer from string world sheets to their boundaries. For Option II two the string world sheets would not carry Noether currents and only minimal surface property could fail at these surfaces: therefore this option is not realistic. Also for Option I there could be breaking of minimal surface property in this sense and the discontinuity of normal component for Noether currents would imply it automatically.

When this picture is combined with the twistor Grassmannian inspired view about scattering amplitudes using the constraints coming from quantum criticality, discreteness of the coupling constant evolution, and the existence of amplitudes as rational functions with coefficients in a extension of rationals allowing p-adic variants, one ends up to a picture in which amplitudes reduces to sums over resonances - this was just what was assumed in Veneziano model besides s-t duality.

This picture does not conform with QFT picture in superstring framework, where one has single large string tension so that poles cannot be approximated by cuts for low energies. In TGD framework this can be the case since string tension has spectrum reducing to that for cosmological constant. Since momenta are already classically predicted to be complex, resonance poles have finite width and one can in principle understand also unitarity. Therefore twistorialization in TGD framework leads to string models, and strings are indeed an essential part of twistorialization in TGD framework.

8 Scattering amplitudes and orbits of cognitive representations under subgroup of symplectic group respecting the extension of rationals

Number theorist Minhyong Kim has speculated about very interesting general connection between number theory and physics [A7, A9] (see `http://tinyurl.com/y86bckmo`). The reading of a popular article about Kim's work revealed that number theoretic vision about physics provided by TGD has led to a very similar ideas and suggests a concrete realization of Kim's ideas [L20]. The identification of points of algebraic surface with coordinates, which are rational or in extension of rationals, gives rise to what one can call identification problem. In TGD framework the embedding space coordinates for points of space-time surface belonging to the extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by the extension. These points define what I call cognitive representation, whose construction means solving of the identification problem.

Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) taking the role of the space of spaces in Kim's approach. The symmetries of this space are proposed by Kim to help to solve the identification problem. The maximal isometries of WCW necessary for the existence of its Kähler geometry provide symmetries identifiable as symplectic symmetries. The discrete subgroup respecting extension of rationals acts as symmetries of cognitive representations of space-time surfaces in WCW, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

This picture could be applied to the construction of scattering amplitudes with finite cognitive precision in terms of cognitive representations and their orbits under subgroup S_D of symplectic group respecting the extension of rationals defining the adele. One could pose to S_D the additional condition that it leaves the value of action invariant: call this group $S_{D,S}$: this would define what I have called micro-canonical ensemble (MCE).

The obvious question is whether the simplest zero energy states could correspond to single orbit of S_D or whether several orbits are required. For the more complex option zero energy states would be superposition of states corresponding to several orbits of S_D with coefficients constructed of symplectic invariants. The following arguments lead to the conclusion that MCE and single orbit orbit option are non-realistic, and raise the question whether the orbits of S_D could combine to an orbit of its Yangian analog. A generalization of the formula for scattering amplitudes in terms of n-point functions emerges and somewhat surprisingly one finds that the unitarity is an automatic consequence of state orthonormalization in zero energy ontology (ZEO).

8.1 Zero energy states

The degrees of freedom at WCW level can be divided to zero modes, which do not contribute to WCW metric and correspond to symplectic invariants and to dynamical degrees of freedom which correspond to the orbits of symplectic group of $\delta M^4_\pm \times CP_2$. The assumption is that symplectic group indeed acts as isometries. The general proposal for the state construction in continuum case should have a discrete analog. There are good reasons to hope that the zero energy states in the degrees of freedom corresponding to the orbits of the discrete variant S_D of the symplectic grop are analogous to spherical harmonics and are dictated completely by symmetry considerations.

Quantum superposition of space-time surfaces - preferred extremals - defines zero energy state. The natural question is whether zero energy state could correspond to single orbit of S_D or whether several of them are needed.

1. Preferred extremal is fixed more or less uniquely by its ends, which are 3-surfaces at the opposite light-like boundaries of CD. The interpretation is in terms of holography forced also by general coordinate invariance requiring that one must be able to assign to a given 3-surface a unique space-time surface at which general coordinate transformations act. In ZEO 3-surface means union of 3-surface at opposite ends of CD.

   The idea about preferred extremals as analogs of Bohr orbits suggests that the 3-surface at the either end determines the 3-surface at the opposite end highly uniquely. The proposal that preferred extremals are minimal surfaces apart from singular 2-surfaces identifiable as string world sheet, means that they are separately extremals of both Kähler action and volume term supports this expectation as also the condition that sub-algebra of symplectic group Lie algebra isomorphic to it gives rise to vanishing Noether charges and also the Noether charges associated with its commutator with the full algebra vanish.

   The condition that the zero energy state at the active boundary of CD is superposition of many-particle states with different particle number in topological sense suggests that this is not the case.

   Even stronger form of holography would be that the data at string world sheets and partonic 2-surfaces determines the preferred extremal completely. In number theoretic vision one can consider even stronger number theoretic holography: if octonionic polynomials code for the space-time surfaces as $M^8 - H$ holography suggests [L6], cognitive representation consisting of discrete set of points with M^8 coordinates in extension of rationals would determine the preferred extremals.

2. Also fermionic degrees of freedom at the ends are involved. Quantum classical correspondence (QCC) states that the classical charges in Cartan sub-algebra of symmetries are equal to the eigenvalues of quantal charges constructible in terms of fermionic oscillator operator algebra. Many-fermion states would correspond to preferred extremals and the fermionic statistics requires that one has superposition over corresponding 4-surfaces. The state at second end of CD is quantum entangled, and fermionic statistics suggests entanglement at both ends.

Symplectic isometries have subgroup with parameters in the extension of rationals defining the adele: call this subgroup S_D. Denote the subgroup of S_D leaving action invariant by $S_{D,S}$. The representations of S_D (or possibly $S_{D,S}$) are expected to be important concerning the construction of scattering amplitudes and on basic of zero energy state property one expects that the action of S_D ($S_{D,S}$) on the opposite ends of space-time surface compensate each other for zero energy states.

A reasonable looking question is whether simplest zero energy states could corresponds to single orbit of S_D. One expects that the number of points defining the cognitive representation is same for all preferred extremals at its orbit. There are several questions to be answered.

1. The existence of preferred extremals connecting given 3-surface with fixed topological particular number to 3-surface at the second end of CD having varying topological particle number looks rather plausible. Topological particle number can be identified either as number of disjoint 3-surfaces and number of disjoint partonic 2-surfaces carrying fermions.

   Can single orbit of S_D contain space-time surfaces with varying topological particle number at the other end of CD? If not , one must allow some minimal number of orbits of S_D in the definition of minimal zero energy state. This option looks the most realistic one.

2. What is the precise definition of cognitive representation?

3. Micro-canonical ensemble (MCE) hypothesis states that action is same for all space-time surfaces appearing in zero energy state. Can this hypothesis be consistent with the presence of many-particle states with different topological particle number? CP_2 type extremals represent particles and have non-vanishing actions. Also the action of symplectic group in general changes the Kähler action although the action is constant at co-dimension 1 surface of WCW so that the subgroup $S_{D,S}$ should act at this surface. It would seem that one must allow the variation of action and this is a challenge for number theoretic universality since the number theoretically non-universal part of action exponentials must be common to all space-time surfaces involves and must cancel in S-matrix.

What does one mean with cognitive representation? Is single orbit of S_D enough? Can one assume MCE? These are the key questions to be considered.

8.2 The action of symplectic isometries on cognitive representations

The action of S_D on cognitive representation defining the adele is straightforward. It is not however quite clear how to identify the cognitive representation.

1. Cognitive representation in question corresponds to a set of points of space-time surface with M^8 coordinates in extension of rationals defining the adele (a stronger condition is that also $M^4 \times CP_2$ coordinates satisfy the same condition).

2. Does cognitive representation contain only the points at the ends of CD, either end, or also interior points? Or does cognitive representation consists of singular points at which non-trivial subgroup of Galois group leaves the point invariant? The singular points could correspond to fundamental fermions at partonic 2-surfaces.

   Remark: If the fermionic lines are light-like geodesic they would correspond as cognitive representations exceptionally informative and easy ones containing infinite number of points of extensions essentially the number line defined by the extension. This raises the question whether the simplest string world sheets identifiable as planes M^2 could be the most interesting singularities of preferred extremals identified as singular minimal surfaces. Canonical embedding of M^4 is also cognitively easy.

   The condition that the actions of symplectic group at opposite boundaries of CD compensate each other makes sense only if one restricts the cognitive representations at either boundary of CD. This would exclude interior points.

   Could one allow also points in the interior of space-time surface by generalizing the view about symplectic invariance of zero energy state? For instance, could the partonic 2-surface defining vertices in the interior contain points of the cognitive representation. Does the allowance of the points of cognitive representation in interior mean giving up strict determinism and does the variational principle with volume term allow it (mere 4-D Kähler action allows huge vacuum degeneracy).

3. When does the point of cognitive representation correspond to a fundamental fermion? I have proposed [L6] that this is the case if the point is critical in number theoretical sense meaning that there is subgroup of Galois group leaving it invariant: the sheets corresponding to different elements of Galois sub-group would co-incide at critical point. The number of singular points and thus number of fundamental fermions might vary.

4. Could the number of singular points vary for the 4-surfaces at the orbit so that the number of fundamental fermions would vary too? Could this allow to have superposition of many-particle states as active part of the zero energy state? This does not seem plausible since the number of points of cognitive representations must be S_D invariant. Several orbits of S_D seem to be required.

The role of Galois group of extension of rationals must be important.

1. Galois group act do not affect space-time surface but only inside the cognitive representation. Galois group can also have subgroup leaving invariant given point. A possible interpretation is as number theoretic correlate for fundamental fermion.

2. A natural hypothesis is that the sub-group of symplectic group leaving the cognitive representation invariant acts as Galois group. A goo analogous for Galois group is provide by the rotation group $SO(3)$ serving as isotropy group of time-like 4-momentum having vanishing 3-momentum in the rest system. For induced representations $SO(3)$ acts in spin degrees of freedom. In the recent case Galois group could act in number theoretic spin degrees of freedom. Could the action of Galois group be physically non-trivial. For instance, could the ordinary symmetries be represented as Galois transformations in fermionic degrees of freedom?

Symplectic invariants characterize the representation and Kähler fluxes for M^4 and CP_2 Kähler forms define this kind of invariants. Also higher fluxes are possible. The general state as superposition of states associated with the over orbits of S_D would have functions of these invariants as coefficients.

8.3 Zero energy states and generalization of micro-canonical ensemble

The space-time surfaces in micro-canonical ensemble (MCE) [L15] would have same action so that Kähler function would be constant. It is interesting to discuss this hypothesis in light of the idea that simplest zero energy state corresponds to a finite set of orbits of $S_{D,S}$.

8.3.1 Is micro-canonical ensemble consistent with zero energy state- S_D orbit correspondence?

The assumption that action is constant at the orbit is not problematic. Kähler function must vary in order to give rise to non-trivial Kähler metric. Kähler function is however constant at co-dimension 1 surfaces of WCW. For instance, the Kähler function of CP_2 is function of the radial coordinate invariant under subgroups invariant under $U(2)$ but not under $SU(3)$.

1. The simplest variant of MCE is that single space-time surface is involved. The action of $S_{D,S}$ would be essentially trivial - zero momentum would be more familiar Minkowski analogy. One would get rid of the action exponentials: this would solve the problems related to number theoretical universality caused by the fact that the exponential need not exist in various p-adic number fields.

2. A more realistic hypothesis is that $S_{D,S}$ has several 4-surfaces at its orbit. If the number of surfaces is N the sum of action exponentials is N-fold and the exponential disappears from the S-matrix elements in analogy with what happens in the full theory without discretization by cancellation of the exponential strong suggested by what happens in QFTs.

MCE has however problems.

1. It is not at all clear whether one can make restriction to a subgroup preserving the action. To gain some perspective, not that in the case of CP_2 this would mean restriction to $r = constant$ surface of CP_2 and this is not possible. In the case of rotation group this would mean restriction to sphere.

   Physically it is also obvious that one should allow in the zero energy state all 4-surfaces which are allowed by the conditions posed by preferred extremal property and there seems no good reason to prevent final states with varying particle topological particle number.

2. Also the standard view about S-matrix suggests at active boundary of CD a superposition of final states with different topological particle numbers having different number disjoint 3-surfaces or same number of disjoint 3-surfaces but varying number of partonic 2-surfaces. That the action of S_D changes the number of the disjoint 3-surfaces is in conflict with naïve intuitions but one must remember that number theoretic discretization loses information about connectedness.

3. If the zero energy state has at the active boundary 3-surfaces with a varying topological particle number identified as a number of CP_2 type extremals with unique maximal action, one expects that action exponential is not constant along the orbit of S_D. If the subgroup of S_D, call it $S_{D,S}$, preserves the value of the action, one must allow orbits of S_D with varying value of action. This would give superposition MCEs. Action preserving subgroup would be analogous to the little group of Poincare group preserving the momentum of particle. As notice, also several orbits of S_D must be allowed.

The conclusions seems to be that MCE is physically non-realistic.

8.3.2 Can one generalize micro-canonical ensemble to single orbit of S_D?

Suppose that the orbit of S_D contains many-particle states having in final state varying particle numbers measured as number disjoint 3-surfaces or partonic 2-surfaces. Is there any hope of understanding these many-particle states in terms of single representation of S_D?

1. The orbit of S_D must have 4-surfaces with varying value of action. This is possible if the action exponentials differ by a multiplicative rational number so that the number theoretically problematic part cancels out from the S-matrix since it appears in both denominator and numerator of the expression defining S-matrix element.

2. That cognitive representations at the orbit would have same number of points at all points of orbits is intuitively in conflict with varying topological particle number. If Galois group has a subgroup of order $m > 1$ acting trivially on points representing fundamental fermions, the number of points in the representation is effectively reduced since m points are replaced by 1 point. This could allows to have a varying particle numbers identified as the number of points of cognitive representation.

   If CP_2 type extremals in the final state serve as correlates for particles, one should understand how their addition is possible. Their addition to the state would require that some non-degenerate points of representation become degenerate. If the number N points is large, it is quite possible to have rather large number of fundamental fermions in the final state. The degeneration of these points would give rise to fermions. There is however an upper bound which also comes from infrared cutoff for energy.

3. It is not clear whether S_D can transform to each other points with different value of m. The problem is that idea that S_D maps some points to single point is in conflict with the idea that S_D action is bijective. It seems that this idea simply fails.

The conclusion seems to be that one must allow several orbits on basis of purely classical picture and QCC suggesting the possibility of finals states with varying topological particles number.

8.3.3 Could ZEO allow to understand the possibility of particle creation and annihilation?

The idea about quantum superposition of states with varying particle number in topological sense is natural if one believes in QFT based intuition. Just for fun one can ask whether ZEO could provide a loophole.

In ZEO "self" corresponds to a sequence of unitary time evolutions changing the state at active boundary. The active boundary itself becomes de-localized. "Small" state function reduction induces localization of the active boundary. This means measurement of clock time as temporal distance between CDs. The time increment ΔT between subsequent values of clock time varies, and one expects that particle number changes in each unitary evolution. The big state function reduction occurs at some time T, the lifetime of self, and one can assume that the value of T varies statistically.

Could one think that the particle number in topological is actually well-defined after each small reduction? The ensemble of detected particle reactions providing the data allowing to deduce the cross sections. Could the variation of intervals ΔT and the variation for the duration T gives rise to a variation of detected particle numbers in the final state. If this is the case the unitary time evolutions and "small" state function reductions would be very "classical". If so ZEO would simplify dramatically the structure of S-matrix.

To make this mechanism more detailed, one can add the existing wisdom about CP_2 type extremals as building bricks of particles.

1. The action is expected to depend on particle number and different numbers of CP_2 type extremals assignable to which fundamental fermions are assigned correspond to different values of actions. This is not a problem now since would not have have superposition over states with different number of CP_2 type extremals and even micro-canonical ensemble could make sense.

2. The addition of particle to the final state during the unitary evolution taking the active boundary farther away from the passive boundary would correspond to a creation of CP_2 type extremal. Simplest mechanism is 3-vertex defined by partonic 2-surface at which CP_2 type extremal replicates. The outgoing lines in the analogs of twistor diagrams would be unstable against replication. Replication is suggested to be universal process in TGD and the replication of magnetic body (MB) would induce DNA replication in TGD inspired quantum biology.

3. A possible interpretation would be in terms of quantum criticality. CP_2 type extremals would be unstable against decay. One could also interpret the analog of twistor diagram as a sequence of algebraic operations.

In this framework the scattering rates would be determined by a hierarchy of S-matrices labelled by different values of total durations $T_n \sum_{k=1}^{n} \Delta T_k$ for a sequence of unitary evolution followed by time localization. In standard picture they would correspond to single infinitely long time evolution. It would not be surprising if this difference could exclude the proposal as unrealistic.

8.3.4 Could one regard zero energy state involving several orbits of S_D as an orbit of Yangian analog of S_D?

QCC suggest strongly that one must allow zero energy states, which correspond to several orbits of S_D. An interesting possibility is that these orbits could be integrated to a representation of a larger group. What suggests itself is the possibly existing Yangian variant of S_D in which the group action is not local anymore even at the level of WCW. The Yangian of projective transformations of M^4 indeed appears in twistor Grassmannian approach and gives rise to huge symmetries behind the success of twistor Grassmannian approach. I have proposed that super-symplectic variant of Grassmannian indeed exists [K25, K21, K4, L17].

8.4 How to construct scattering amplitudes?

Lubos Motl (see `http://tinyurl.com/y5lndpn3`) told about two new hep-th papers, by Pate, Raclariu, and Strominger (see `http://tinyurl.com/yxqx237b`) and by Nandan, Schreiber, Volovich, Zlotnikov (see `http://tinyurl.com/y642yspf`) related to a new approach to scattering amplitudes based on the replacement of the quantum numbers associated with Poincare group labelling particles appearing in the scattering amplitudes with quantum numbers associated with the representations of Lorentz group.

Why I got interested was that in zero energy ontology (ZEO) the key object is causal diamond (CD) defined as intersection of future and past directed M^4 light-cones with points replaced with CP_2. Space-time surfaces are inside CD and have ends at its light-like boundaries. The Lorentz symmetries associated with ithe boundaries of CD could be more natural than Poincare symmetry, which would emerge in the integration over the positions of CDs of external particles arriving to the opposite light-like boundaries of the big CD defining the scattering region where preferred extremal describing the scattering event resides.

I did my best to understand the articles and - of course relate these ideas to TGD, where the construction of scattering amplitudes is the basic challenge. My technical skills are too limited for to meet this challenge at the level of explicit formulas but I can try to understand the physics and mathematics brought in by TGD.

While playing with more or less crazy and short-lived ideas inspired by the reading of the articles I finally realized that there is perhaps no point in starting from quantum field theories. TGD is not quantum field theory and I must start from TGD itself.

In TGD framework the picture inspired by adelic physics [L8, L7] is roughly following.

1. Cognitive representations realizing number theoretic universality of adelic physics consist of points of embedding space with coordinates in the extension of rationals. The number of points is typically finite. Cognitive representation should contain as subset the points associated with n-point functions, which are essentially correlation functions.

   Fundamental fermions are building bricks of elementary particles, and a good guess is that fundamental fermions correspond to singular points for which the action of subgroup of Galois group of extension is trivial so that several points collapse together.

2. One must sum over the orbits of a subgroup S_D of symplectic group of light-cone boundary acting as isometries of both boundaries of CD. S_D consists of isometries with parameters in the extension of rationals defining the adele. All orbits needed to represent the pairs of initial and final 3-surfaces at the boundaries of CD allowed by the action principle must be realized so that single orbit very probably is not enough.

3. Correlations code for the quantum dynamics. In quantum field theories quantum fluctuations of fields at distinct points of space-time correlate and give rise to n-point functions expressible in terms of propagators and vertices: massless fields and conformal fields define the basic example. Operator algebra or path integral describes them mathematically.

   In TGD correlations between embedding space points belonging to the space-time surface result from classical deterministic dynamics: the points of 3-surface at opposite boundaries of CD are not independent.

   This dynamics is non-linear geometric analog for the dynamics of massless fields: space-time sheets as preferred extremals are indeed minimal surfaces with string world sheets appearing as singularities. Minimal surface property is forced by the volume action implied by the twistor lift and having interpretation in terms of cosmological constant. The correlation between points at the same boundary of CD are expected to be independent since these 3-surfaces chosen rather freely as analogs of boundary values for fields.

   Fermionic dynamics governed by modified Dirac action is dictated completely by super-symplectic and super-conformal symmetries. Second quantization of fermions at space-time level is necessary to realized WCW spinor structure: WCW gamma matrices are linear combinations of fermionic oscillator operators.

4. This suggests that the attempts to guess the conformal field theory producing the correlation functions makes things much more complex than they actually are. It should be possible to understand how these correlations emerge from the classical dynamics of space-time surfaces.

As the first brave guess one could try to calculate directly the correlations of spinor harmonics of embedding space assigned with these points.

1. Sum over the symplectic orbits of cognitive representations must be involved as also vacuum expectation values in the fermionic sector for fermionic fields which must appear in vertices for external particles. At the level of cognitive representations anti-commutators for oscillator operators involve Kronecker deltas so that one has discretized variant of second quantization.

2. This could be achieved by expanding the restriction $\Psi^A_{|X^3}$ of the embedding space harmonic Ψ^A restricted to 3-surface at end of space-time surface as sum of modes Ψ_n of the induced spinor field. This would be counterpart for the induction procedure. One can assign to singular points bilinear of type $\overline{\Psi}^A_{|X^3} D^{\leftrightarrow} \Psi$, where Ψ is second quantized induced spinor field expressible as sum over its modes multiplied by oscillator operators. D is modified Dirac operator. This gives as vacuum expectations propagators connecting fermions vertices at the opposite ends of space-time surface.

3. A more concrete picture must rely on a concrete model for elementary particles. Elementary particles have as building bricks pair of wormhole contacts with fermion lines at the light-like orbits of the throats at which the signature of the metric changes from Minkowskian to Euclidian. Particle is necessarily a pair of two wormhole contacts and flux tube connects them at both space-time sheets and forms a closed flux tube carrying monopole flux.

   All particles consist of fundamental fermions and anti-fermions: for instance gauge bosons involve fermion and anti-fermion responsible for the quantum numbers at the opposite throats of second wormhole contact. Second wormhole contact involves neutrino pair neutralizing electroweak isospin in scales longer than the size of the flux tube structure.

4. The topological counterpart of 3-vertex appearing in Feynman diagram corresponds to a replication of this kind of 3-surface highly analogous to bio-replication. In replication vertex, there is no singularity of 3-surface analogous to that appearing in the vertices of stringy diagrams but space-time surface is singular just like 1-D manifold is singular for at vertex of Feynman diagram.

   These singular replicating 3-surfaces and the partonic 2-surfaces give rise to the counterparts of interaction vertices. Fermionic 4-vertex is impossible and fermion lines can only be re-shared between outgoing partonic orbits. This is however not enough as will be found. It will be found that also the creation of fermion pair as effective turning of fermion lines entering along "upper" wormhole throat and turning back at Euclidian wormhole throat and continuing along the orbit of "lower" wormhole throat must be possible.

To see how this conclusion emerges consider the following problem. One should obtain also emission of bosons identified as fermion pairs from fermion line. One has incoming fermion and outgoing fermion and fermion pair describing boson which represents gauge boson or graviton with vanishing B and L. Fermionic 4-vertex is not allowed since this would bring in divergences.

1. The appearance of a sub-CD assignable to the partonic 2-surface is possible but does not solve the problem considered. There would be incoming fermion line at lower boundary and 1 fermion line and fermion and anti-fermion line associated with the boson at the "upper" boundary. There would be non-locality in the scale of the partonic 2-surface and sub-CD meaning that the lines can end to vacuum. Now one would encounter the same difficulty but only in shorter scale.

2. Could one say that fermion line turns backwards in time? A line turning back could be described as an annihilation of fermion pair to vacuum carrying classical gauge field, which is standard process. In QFT picture this would be achieved if a bilinear $\overline{\Psi} D \Psi$ is allowed in the vertex where annihilation takes place. Not in TGD: fermionic action vanishes identically by field equations expressing essentially the conservation of fermion current and various super currents obtained as contractions fermion field with modes.

   Could fermion-anti-fermion pair creation occur at singular points associated with partonic surfaces representing the turning of fermion line backwards in time. This looks still too singular.

   Rather, the turning backwards in time should mean that a fermion line arriving from future along the orbit of "upper" throat (say) goes through Euclidian wormhole throat and continues along the orbit of "lower" throat back to future than making discontinuous turn-around. Euclidian regions of space-time surface representing one key distinction between GRT and TGD would thus be absolutely essential for the generalized scattering diagrams. An exchange of momentum with classical field would be Feynman diagrammatic manner to say this.

New oscillator operator pairs emerge at the partonic vertices and would correspond to the above described turn-around for fermion line at wormhole contact. Fermion pairs present at the "lower" boundary of CD could also disappear.

3. The anti-commutation relations fermions are modified due to the presence of vacuum gauge fields so that the anti-commutator of fermionic creation operators $a_m^\dagger$ and anti-fermionic creation operators $b_n^\dagger$ is non-vanishing. A proper formulation of the fermionic anti-commutation relations at the ends of space-time surface is needed and in discretization provided by cognitive representation this should be relatively straightforward.

   One can imagine that although standard anti-commutation relations at the lower end of space-time surface hold true, the time evolution of Ψ in the presence of vacuum gauge potentials implies that the vacuum expectations $\langle vac|a_m^\dagger b_n^\dagger|vac\rangle$ are non-vanishing. This would require that for instance $b_n^\dagger$ and a_n are mixed.

There are still questions to be answered.

1. Is the first guess enough? It is not as becomes clear after a thought about the continuum limit. The WCW degrees of freedom are described at continuum limit in terms of super-symplectic algebra (SSA) acting on ground state are neglected. Embedding space spinor modes characterizee only the ground staes of these representations. These degrees of freedom are essential already in elementary particle physics [K12].

   Sub-algebra SSA$_m$ of SSA with conformal weights coming as m-multiples of those of SSA and its commutator with SSA annihilate the physical states, and one obtains a hierarchy. How to describe these states in the discretization? The natural possibility are the representations of S_D such that $(S_D)_m$ and the subgroup generated by the commutator algebra are represented trivially. One has non-trivial $(S_D)_m$ representations at both ends of WCW such that the action of S_D on the tensor product acts trivially.

   There are also fermionic degrees of freedom. The challenge is to identify among other things WCW gamma matrices as fermionic super charges and it would be nice if all charges were Noether charges. The simplest guess is that the algebra generated by fermionic Noether charges Q^A for symplectic transformations $h^k \rightarrow h^k + j^{Ak}$ assumed to induce isometries of WCW and Noether supercharges Q_n and their conjugates for the shifts $\Psi \rightarrow \Psi + \epsilon u_n$, where u_n is a solution of the modified Dirac equation, is enough.

   The commutators $\Gamma_n^A = [Q^A, Q_n]$ are super-charges labelled by (A, n). One would like to identify them as gamma matrices of WCW. The problem is that they are labelled by (A, n) whereas isometry generators are labelled by A only. There should be one-one correspondence. Do all supercharges Γ_n^A except Γ_0^A corresponding to $u_0 = constant$ annihilate the physical states so that one would have 1-1 correspondence. This would be analogous to what happens quite generally in super-conformal algebras.

   The generators of this fermionic algebra could be used to generate more general states. One should also construct the discretized versions of the generators as sums over points of the cognitive representation at the ends of space-time surface. Note that this requires tangent space data.

2. What about the conservation of four-momentum and other conservation laws? This can be handled by quantum classical correspondence (QCC). The momentum and color labels defined by fermionic quantum numbers in Cartan algebra can be assumed to be equal to the corresponding classical Noether charges for particle-like space-time surfaces entering to CD. The technical problem is that if one knows only the discretization - even with tangent space data - one does not know the values of these charges! It might be that M^8-H correspondence in which M^8 side fixes space-time surfaces as roots for real or imaginary parts of octonionic polynomials from the data at discrete set of points is needed.

3. ZEO means deviations from ordinary description. S_D invariance of zero energy state forces sum over the 4-surfaces of the orbit with identical coefficients. Symplectic invariance implies time-like entanglement. One can describe this in terms of hermitian square root Ψ of density matrix satisfying $\Psi^\dagger\Psi = \rho$. The coefficients of different orbits need not be same and allows description in terms of dynamical density matrix. If there is Yangian symmetry also this entanglement is analogous to the entanglement due to statistics.

   Surprisingly - and somewhat disappointingly after decades of attempts to understand unitarity in TGD - unitarity is trivial in ZEO since state basis is defined essentially by the rows of matrices and orthogonality conditions their orthogonality and therefore unitarity. More concretely, for single state at the passive end state function normalization to unity defined by inner product as sum over 3-surfaces at active end would give conservation of probability. Orthogonality of the state basis with inner product as sum over surfaces passive boundary gives orthogonality for the coefficients defining rows of a matrix and therefore unitarity. In the case that single orbit or even several of them defines the states one obtains the same result.

   What then guarantees the orthogonality of zero energy states? In ordinary quantum mechanics the property of being eigenstates of some hermitian operator guarantees orthogonality. In TGD zero energy states would be solutions of the analog of massless Dirac equation in WCW consisting of pairs of 3-surfaces with members at the ends of preferred extremals inside CD. This generalizes Super Virosoro conditions of superconformal theories and would provide the orthonormal state basis.

The outcome would be amazingly simple. There would be no propagators, no vertices, just spinor harmonics of embedding assigned with these $n = n_1 + n_2$ points at the boundaries of CD, and summation over the orbits of the symplectic group. All these mathematical objects would emerge from classical dynamics. The sum over the orbits for chosen spinor harmonics would produce n-point functions, vertices and propagators. It is difficult to imagine anything simpler and quantum classical correspondence would be complete.

9 Minimal surfaces: comparison of the perspectives of mathematician and physicist

The popular article "*Math Duo Maps the Infinite Terrain of Minimal Surfaces*" (see `http://tinyurl.com/yyetb7c7`) was an exceptional representative of its species. It did not irritate the reader with non-sense hype but gave very elegant and thought provoking representation of very abstract ideas in mathematics.

9.1 Progress in the understanding of closed minimal surfaces

The article tells about the work of mathematicians Fernando Coda Marques and Andre Neves based on a profound and - as they tell - extremely hard-to-understand work of Jon Pitts forgotten by mathematics community. It is comforting that at least in mathematics good work is eventually recognized.

The results of Marques and Neves are about minimal hyper-surfaces imbedded in various spaces with dimension varying between 3 and 7 and clearly extremely general. These spaces have varying topologies and are called "shapes" in the popular article.

9.1.1 Some examples of minimal surfaces

To begin it is good to have some examples about minimal surfaces.

1. For mathematician any lower-dimensional manifold in some embedding space is surface, even 1-D curve! Simplest minimal surfaces are indeed 1-D geodesic lines. In flat 3-space they are straight lines of infinite length but at the surface of sphere they are big circles.

2. Soap films are 2-D minimal surfaces spanned by frames and familiar for every-one. Frame is necessary for having minimal surface, which does not collapse to a point or extend to infinity and possibly self-intersect.

   Why minimal surfaces are not nice closed surfaces of finite size not intersecting themselves is due to the fact that the equations for minimal surface state the vanishing of the sum of external curvatures defined by the trace of so called second fundamental form defined by the covariant derivatives of tangent vectors of the minimal surface.

   One can say that for 2-D minimal surface the external curvatures in 2 orthogonal directions at given point of surface are of opposite sign. Surface looks locally like saddle rather than sphere. In n-dimensional case the sum of n principal curvatures - eigenvalues of second fundamental form as matrix- sum up to zero for each normal direction: more general saddle.

   In flat embedding space this implies the saddle property always but in curved space it might happen that the covariant derivatives replacing the ordinary derivatives in the definition of second fundamental form - having interpretation as generalized acceleration - can change the situation and the question is whether non-flat closed embedding space could contain closed minimal surfaces.

   Indeed, in compact spaces with non-flat metric minimal surface can be closed and there is a century old theorem by Birkhoff stating that sphere has always at least one closed geodesic independent of metric. In the case of ordinary sphere this geodesic is big circle, the equator. In complex projective space CP_2 there is infinite number of 2-D minimal surfaces which are closed: geodesic spheres are the simplest examples.

3. A good example about a non-closed 1-D surface is generic geodesic in torus with points labelled by two angles (ϕ_1, ϕ_2) in flat metric. The geodesic lines are of form $\phi_1 = \alpha\phi_2$). For non-rational value of α the curve winds the torus infinitely many times and has infinite length. For $\alpha = m/n$ the curve winds m times around second non-contractible circles and n times around the second one. Note that now the geodesic line is absolute minimum: this is caused by the non-contractibility. It can be only shifted in both directions so that the minimum has 2-D degeneracy.

4. In spaces allowing Kähler structure - means that imaginary unit i satisfying $i^2 = -1$ has a representation as antisymmetric tensor - any complex algebraic surfaces representable as root for a set of polynomials, whose number is smaller than complex dimension of the space, is a minimal surface. This huge variety of minimal surfaces is due to the presence of complex structure.

9.1.2 What does minimal surface property mean?

Consider now what minimal surface property really means.

1. Strictly speaking, minimal surfaces are stationary with respect to the *local* variations of volume only. This is practically always true for physical variational principles defined by an action. For a great circle at sphere the minimality of length with respect to small variations is easy to understand by drawing to see what this variation means. With respect to non-local variations meaning shift toward North or East the area decreases so that one has maximum! This leads to the term Minimax principle used by Jon Pitts and his followers as a powerful guideline.

   In fact, minimal surfaces can be both minima and maxima for volume simultaneously. The general extremum as solution of equations defined by a general action principle is saddle. Minimum with respect to some variables and maximum with respect to others and minimal surfaces are this kind of objects in the general case.

2. There is a deep connection with Morse theory in topology (see `http://tinyurl.com/ych4chg9`). Morse function gives information about the topology of space. Morse function is a continuous monotonously increasing function from the space to real line and its extrema provide information about the topology of the space. Morse function can be seen as a kind of height function, a particular coordinate for the space.

The height as z-coordinate for torus imbedded in 3-space gives a classical example of height function. As z varies on obtains 1-D intersections of torus. The minimum of z corresponds to a single point, above it one has circle, then circle decomposes to 2 circles at lower saddle, and circles fuse back to circle at upper saddle, which becomes a point at maximum. Therefore the extrema of heigh function tell about how the topology of the cross section of the torus varies with height: point-circle-2 circles-circle-point. The area of surface serves as a Morse function and minimal points are analogous to the points of the torus at which cross section changes its topology.

A good guess is that the volume of the surface serves as a Morse function and thus gives information about the topology of rather abstract infinite-dimensional space: the space of surfaces. Minimal surfaces would be analogous to the critical points of height function at torus: points at which the cross section changes its topology.

3. Minimax property states the fact that minimal surfaces are in in generic situation saddle points in the space of surfaces. There would be a strange correspondence. The points of minimal surfaces are locally saddles in the finite-dimensional embedding space H and minimal surfaces represent saddle points in the finite-dimensional space of surfaces in H. This strange local-global correspondence bringing in mind holography might be behind a general principle: saddle property could have representations at two levels: points of the surface and points of the space of surfaces.

   Are minimal surfaces a rare exception or could it be that for a general action principle the extremals are saddles locally and that the space of all field configurations (not only extremals) contains the extremals as saddle points?

   Remark: Minimal surfaces might be very special and related to what corresponds in physics to criticality implying that the dynamics in certain sense universal. The space of surfaces corresponds in TGD as the space of 3-surfaces and is analogous to Wheeler's superspace consisting of 3-metrics. By holography forced by 4-D general coordinate invariance 3-surfaces in question must be in one-one correspondence with 4-D surfaces identified as space-time surfaces. I have christened this space world of classical worlds (WCW). Space-time surfaces are 4-D minimal surfaces in 8-D $H = M^4 \times CP_2$ but possessing lower dimensional singularities having interpretation as orbits of string like objects and point like particles. Minimal surface property would be a correlate for quantum criticality so that minimal surface would be very special.

9.1.3 The question and the answer

The question that Marquez and Neves posed to themselves was under which conditions compact space allows a closed minimal surface not intersecting itself or whether all candidates intersect themselves or have infinite volume. In fact, Marquez and Neves restricted the consideration to hyper-surfaces. A possible good reason for this is that there is only one field like dynamical degree of freedom for co-dimension 1 - the coordinate in the normal direction- and this is expected to simplify the situation considerably. From the tone of the article - "-hyper" has been dropped away - one has a temptation to guess that the results are much more general.

The basic result of Marques and Neves was rather astonishing. In almost all closed spaces with dimension between 3 and 7 there exists an infinite series of imbedded *closed* minimal hyper-surfaces (embedding means that there are no self-intersections). No frames needed! The irony was that they could not prove their result for spaces with roundest metrics (no bumps making metric positively curved, which in turn helps to have minimal surface property without local saddle property). Song however generalized this result to apply for arbitrary closed embedding spaces [A1] (see `http://tinyurl.com/yycbw4lx`).

What helped in the proof was a surprising result by Marques, Neves, and Liokumovich that the volume for these minimal hyper-surfaces depends on the volume of the compact embedding space only [A8] (see `http://tinyurl.com/y59pdawj`)!

This dependence suggests that these closed minimal hyper-surfaces manage to visit a dense set of points of the embedding space without intersecting themselves: in this manner they could "measure" the volume. Marques, Neves and Irie show that there is infinite set of imbedded minimal hyper-surfaces in spaces of dimension $3 \leq n \leq 7$ intersecting any given ball of the embedding space [A5] (see `http://tinyurl.com/y3u3bvnc`). Even more, these minimal surfaces tend to fill space in some sense evenly.

A natural guess inspired by Minimax Principle is that minimals surfaces correspond to saddle points for the volume as functional of surface defining Morse function. The volume is analogous to action in TGD framework.

Two remarks are in order.

1. As noticed, the popular article says that these results hold for minimal surfaces. The articles however restrict the consideration to minimal hyper-surfaces.

2. The theorem about the dependence of volume of hyper-surface on the volume of embedding space was inspired by a result proven by Weyl for the high frequencies of drum defined as a boundary of some space: these frequencies depend on the volume of the space, not on the shape of drum! One can understand this intuitively by the fact that high frequency vibrations correspond to short wave lengths and therefore depend only on the *local* properties of the space and not on the global topology. The dependence on volume comes from boundary conditions at the boundaries of the volume.

   In the case of minimal hyper-surfaces the analogy would suggests that the addition of details to the minimal hyper-surface corresponds to the increase of the frequency for drum. Boundary conditions for drum would be replaced by the compactness of the embedding space leading to the quantization of the volume analogous to that for frequency.

3. The infinite geodesic on flat torus described above is a rough analog for omni-presence although it is not closed. Also complex surfaces in CP_2 defined as zero loci of polynomials of complex coordinates (ξ^1, ξ^2) modified to contain irrational powers of ξ^i could define this kind of omni-present surfaces having however infinite area. There is however infinite number of minimal surfaces defined by complex polynomials, which are closed but not omni-present.

9.2 Minimal surfaces and TGD

In TGD framework surfaces satisfying minimal surface equations almost everywhere - play a central role.

9.2.1 Space-time surfaces as singular minimal surfaces

From the physics point this is not surprising since minimal surface equations are the geometric analog for massless field equations.

1. The boundary value problem in TGD is analogous to that defining soap films spanned by frames: space-time surface is thus like a 4-D soap film. Space-time surface has 3-D ends at the opposite boundaries of causal diamond of M^4 with points replaced with CP_2: I call this 8-D object just causal diamond (CD). Geometrically CD brings in mind big-bang followed by big crunch.

   These 3-D ends are like the frame of a soap film. This and the Minkowskian signature guarantees the existence of minimal surface extremals. Otherwise one would expect that the non-compactness does not allow minimal surfaces as non-self-intersecting surfaces.

2. Space-time is a 4-surface in 8-D $H = M^4 \times CP_2$ and is a minimal surface, which can have 2-D or 1-D singularities identifiable as string world sheets having 1-D singularities as light-like orbits - they could be geodesics of space-time surface.

   Remark: I considered in [L12] the possibility that the minimal surface property could fail only at the reaction vertices associated with partonic 2-surfaces defining the ends of string world sheet boundaries. This condition however seems to be too strong. It is essential that the singular surface defines a sub-manifold giving deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. Singular points do not satisfy this condition.

String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations.

3. Geometrically string world sheets are analogous to folds of paper sheet. Space-time surfaces are extremals of an action which is sum of volume term having interpretation in terms of cosmological constant and what I call Kähler action - analogous to Maxwell action. Outside singularities one has minimal surfaces stationary with respect to variations of both volume term and Kähler action - note the analogy with free massless field. At singularities there is an exchange of conserved quantities between volume and Kähler degrees of freedom analogous to the interaction of charged particle with electromagnetic field. One can see TGD as a generalization of a dynamics of point-like particle coupled to Maxwell field by making particle 3-D surface.

4. The condition that the exchange of conserved charges such as four-momentum is restricted to lower-D surfaces realizes preferred extremal property as a consequence of quantum criticality demanding a universal dynamics independent of coupling parameters [L18]. Indeed, outside the singularities the minimal surfaces dynamics has no explicit dependence on coupling constants provided local minimal surface property guarantees also the local stationarity of Kähler action.

   Preferred extremal property has also other formulations. What is essential is the generalization of super-conformal symmetry playing key role in super string models and in the theory of 2-D critical systems so that field equations reduce to purely algebraic conditions just like for analytic functions in 2-D space providing solutions of Laplace equations.

5. TGD provides a large number of specific examples about closed minimal surfaces [K2]. Cosmic strings are objects, which are Cartesian products of minimal surfaces (string world sheets) in M^4 and of complex algebraic curves (2-D surfaces). Both are minimal surfaces and extremize also Kähler action. These algebraic surfaces are non-contractible and characterized by homology charge having interpretation as Kähler magnetic charge. These surfaces are genuine minima just like the geodesics at torus.

   CP_2 contains two kinds of geodesic spheres, which are trivially minimal surfaces. The reason is that the second fundamental form defining as its trace the analogs of external curvatures in the normal space of the surfaces vanishes identically. The geodesic sphere of the first kind is non-contractible minimal surface and absolute minimum. Geodesic spheres of second kind is contractible and one has Minimax type situation.

   These geodesic spheres are analogous to 2-planes in flat 3-space with vanishing external curvatures. For a generic minimal surface in 3-space the principal curvatures are non-vanishing and sum up to zero. This implies that minimal surfaces look locally like saddles. For 2-plane the curvatures vanish identically so that saddle is not formed.

9.2.2 Kähler action as Morse function in the space of minimal surfaces

It was found that surface volume could define a Morse function in the space of surfaces. What about the situation in TGD, where volume is replaced with action which is sum of volume term and Kähler action [L17, L16, L18]?

Morse function interpretation could appear in two ways. The first possibility is that the action defines an analog of Morse function in the space of 4-surfaces connecting given 3-surfaces at the boundaries of CD. Could it be that there is large number of preferred extremals connecting given 3-surfaces at the boundaries of CD? This would serve as analogy for the existence of infinite number of closed surfaces in the case of compact embedding space. The fact that preferred extremals extremize almost everywhere two different actions suggests that this is not the case but one must consider also this option.

1. The simplest realization of general coordinate invariance would allow only single preferred extremal but I have considered also the option for which one has several preferred extremals. In this case one encounters problem with the definition of Kähler function which would become many-valued unless one is ready to replace 3-surfaces with its covering so that each preferred extremal associated with the given 3-surface gives rise to its own 3-surface in the covering space. Note that analogy with the definition of covering space of say circle by replacing points with the set of homologically equivalence classes of closed paths at given point (rotating arbitrary number of times around circle).

2. Number theoretic vision [K27, K10] suggests that these possibly existing different preferred extremals are analogous to same algebraic computation but performed in different ways or theorem proved in different ways. There is always the shortest manner to do the computation and an attractive idea is that the physical predictions of TGD do not depend on what preferred extremal is chosen.

3. An interesting question is whether the "drum theorem" could generalize to TGD framework. If there exists infinite series of preferred extremals which are singular minimal surfaces, the volume of space-time surface for surfaces in the series would depend only on the volume of the CD containing it. The analogy with the high frequencies and drum suggests that the surfaces in the series have more and more local details. In number theoretic vision this would correspond to emergence of more and more un-necessary pieces to the computation. One cannot exclude the possibility that these details are analogs for what is called loop corrections in quantum field theory.

4. If the action defines Morse action, the preferred extremals give information about its topology. Note however that the requirement that one has extremum of both volume term and Kähler action almost everywhere is an extremely strong additional condition and corresponds physically to quantum criticality.

   Remark: The original assumption was that the space-time surface decomposes to critical regions which are minimal surfaces locally and to non-critical regions inside which there is flow of canonical momentum currents between volume and Kähler degrees of freedom. The stronger hypothesis is that this flow occurs at 2-D and 1-D surfaces only.

9.2.3 Kähler function as Morse function the space of 3-surfaces

The notion of Morse function can make sense also in the space of 3-surfaces - the world of classical worlds which in zero energy ontology consists of pairs of 3-surfaces at opposite boundaries of CD connected by preferred extremal of Kähler action [K6, K20, L17, L16]. Kähler action for the preferred extremal is assumed to define Kähler function defining Kähler metric of WCW via its second derivates $\partial_K\partial_{\overline{L}}K$. Could Kähler function define a Morse function?

1. First of all, Morse function must be a genuine function. For general Kähler metric this is not the case. Rather, Kähler function K is a section in a $U(1)$ bundle consisting of patches transforming by real part of a complex gradient as one moves between the patches of the bundle. A good example is CP_2, which has non-trivial topology, and which decomposes to 3 coordinate patches such that Kähler functions in overlapping patches are related bythe analog of U(1) gauge transformation.

   Kähler action for preferred extremal associated with given 3-surface is however uniquely defined unless one includes Chern-Simons term which changes in U(1) gauge transformation for Kähler gauge potential of CP_2.

2. What could one conclude about the topology of WCW if the action for preferred extremal defines a Morse function as a functional of 3-surface? This function cannot have saddle points: in a region of WCW around saddle point the WCW metric depending on the second derivatives of Morse function would not be positive definite, and this is excluded by the positivity of Hilbert space inner product defined by the Kähler metric essential for the unitarity of the theory. This would suggest that the space of 3-surfaces has very simple topology if Kähler function.

   This is too hasty conclusion! WCW metric is expected to depend also on zero modes, which do not contribute to the WCW line element. What suggests itself is bundle structure. Zero modes define the base space and dynamical degrees of freedom contributing to WCW line element as fiber. The space of zero modes can be topologically complex.

There is a fascinating open problem related to the metric of WCW.

1. The conjecture is that WCW metric possess the symplectic symmetries of $\Delta M^4_+ \times CP_2$ as isometries. In infinite dimensional case the existence of Riemann/Kähler geometry is not at all obvious as the work of Dan Freed demonstrated in the case of loops spaces [A3], and the maximal group of isometries would guarantee the existence of WCW Kähler geometry. Geometry would be determined by symmetries alone and all points of the space would be metrically equivalent. WCW would be an infinite-dimensional analog of symmetric space.

2. Isometry group property does not require that symplectic symmetries leave Kähler action, and even less volume term for preferred extremal, invariant. Just the opposite: if the action would remain invariant, Kähler function and Kähler metric would be trivial!

3. The condition for the existence of symplectic isometries must fix the ratio of the coefficients of Kähler action and volume term highly uniquely. The physical interpretation is in terms of quantum criticality realized mathematically in terms of the symplectic symmetry serving as analog of ordinary conformal symmetry characterizing 2-D critical systems. Note that at classical level quantum criticality realized as minimal surface property says nothing outside singular surfaces since the field equations in this regions are algebraic. At singularities the situation changes. Note also that the minimal surface property is a geometric analog of masslessness which in turn is a correlate of criticality.

4. Twistor lift of TGD [**?**]eads to a proposal for the spectra of Kähler coupling strength and cosmological constant allowed by quantum criticality [L16]. What is surprising that cosmological constant identified as the coefficient of the volume term takes the role of cutoff mass in coupling constant evolution in TGD framework. Coupling constant evolution discretizes in accordance with quantum criticality which must give rise to infinite-D group of WCW isometries. There is also a connection with number theoretic vision in which coupling constant evolution has interpretation in terms of extensions of rationals [K27, L7, L6].

9.2.4 Can one apply the mathematical results about closed minimal surfaces to TGD?

The general mathematical thinking involved with the new results is applied also in TGD as should be clear from the above. But can one apply the new mathematical results described above to TGD? Unfortunately not as such. There are several reasons for this.

1. The dimension of $H = M^4 \times CP_2$ is $D = 8 > 7$. M^4 is non-compact and also the signature of M^4 metric is Minkowskian rather than Euclidian. Could one apply these results to special kinds of 4-surface such as stationary surfaces $M^1 \times X^3$, $X^3 \subset E^3 \times CP_2$. No: the problem is that E^3 is non-compact.

2. In TGD one does not consider closed space-time surfaces but analogs of soap films spanned by a frame defined by the 3-surfaces at the opposite ends of CD. Note that the singular surfaces of dimension $D = 2, 1$ are analogous to frames with boundaries at the ends of space-time surface.

3. In TGD framework preferred extremal property requires that space-time surface is both minimal surface and extremal of Kähler action outside singularities. This is known to be the case for all known extremals. This poses very strong conditions on extremals and seems to mean the existence of a generalization of Kähler structure and conformal invariance to 4-D situation. This drops a large number of minimal surface extremals from consideration

4. Minimal surfaces filling space evenly do not have any reasonable physical interpretation. Maybe this could be used to argue that one must have $D = 8$ and that signature must be Minkowskian in order to have soap films rather than closed minimal surfaces.

   What about E^4 with Euclidian signature instead of M^4 and closed space-time surfaces in analogy with Euclidian field theories? Would the projections of closed minimal 4-surfaces in $E^4 \times CP_2$ which are also extremals of Kähler action reduce to a point in E^4 and complex 2-surfaces in CP_2: Euclidianized TGD would degenerate to an Euclidian version of string model. Also in $H = S^4 \times CP_2$ the situation might be same since the property of being extremal of Kähler action is very powerful. It is however essential that also M^4 has analog of Kähler structure: S^4 does not have it although it allows twistor structure so that this options drops out.

5. Can one apply the results of Marques, Neves and others about hyper-surfaces to TGD? What comes in mind is a minimal 4-surface, which is a Cartesian product of geodesic line $M^1 \subset M^4$ and 3-D hyper-surface $X^3 \subset CP_2$ visiting all points of CP_2 and having a finite volume. If the action would contain only the volume term, this extremal would be possible. The action however contains Kähler action and this very probably excludes this extremal.

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