Holography and Quantum Error Correcting Codes: TGD View

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Abstract

Preskill et all suggest a highly interesting representation of holography in terms of quantum error correction codes The idea is that time= constant section of AdS, which is hyperbolic space allowing tessellations, can define tensor networks. So called perfect tensors are building bricks of the tensor networks providing representation for holography and at the same time defining error correcting codes by mapping localized interior states (logical qubits) to highly entangled non-local boundary states (physical qubits).

There are three observations that put bells ringing and actually motivated this article.

1. Perfect tensors define entanglement which TGD framework corresponds negentropic entanglement playing key role in TGD inspired theory of consciousness and of living matter.
2. In TGD framework the hyperbolic tesselations are realized at hyperbolic spaces $H_3(a)$ defining light-cone proper time hyperboloids of M^4 light-cone.
3. TGD replaces AdS/CFT correspondence with strong form of holography.

A very attractive idea is that in living matter magnetic flux tube networks defining quantum computational networks provide a realization of tensor networks realizing also holographic error correction mechanism: negentropic entanglement - perfect tensors - would be the key element. As I have proposed, these flux tube networks would define kind of central nervous system make it possible for living matter to experience consciously its biological body using magnetic body.

These networks would also give rise to the counterpart of condensed matter physics of dark matter at the level of magnetic body: the replacement of lattices based on subgroups of translation group with infinite number of tesselations means that this analog of condensed matter physics describes quantum complexity.

1 Introduction

Strong form of holography is one of the basic tenets of TGD, and I have been working with topological quantum computation in TGD framework with the braiding of magnetic flux tubes defining the space-time correlates for topological quantum computer programs [K1]. Flux tubes are accompanied by fermionic strings, which can become braided too and would actually represent the braiding at fundamental level. Also time like braiding of fermionic lines at light-like 3-surfaces and the braiding of light-like 3-surfaces themselvs is involved and one can talk about space-like and time-like braidings. These two are not independent being related by dance metaphor (think dancers at the parquette connected by threads to a wall generating both time like and space-like braidings). I have proposed that DNA and the lipids at cell membrane are connected by braided flux tubes such that the flow of lipids in lipid layer forming liquid crystal would induce braiding storing neural events to memory realized as braiding.

I have a rather limited understanding about error correcting codes. Therefore I was happy to learn that there is a conference in Stanford in which leading gurus of quantum gravity and quantum information sciences are talking about these topics. The first lecture that I listened was about a possible connection between holography and quantum error correcting codes. The lecturer was Preskill and the title of the talk was "Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence" (see `https://www.youtube.com/watch?v=SW2rlQVfnK0` and `http://tinyurl.com/z8fsfh8`). A detailed representation can be found in the article of Preskill *et al* [B4] (see `http://arxiv.org/pdf/1503.06237.pdf`).

The idea is that time= constant section of AdS, which is hyperbolic space allowing tessellations, can define tensor networks. So called perfect tensors are building bricks of the tensor networks providing representation for holography. There are three observations that put bells ringing and actually motivated this article.

1. Perfect tensors define entanglement which TGD framework corresponds negentropic entanglement playing key role in TGD inspired theory of consciousness and of living matter.
2. In TGD framework the hyperbolic tesselations are realized at hyperbolic spaces $H_3(a)$ defining light-cone proper time hyperboloids of M^4 light-cone.
3. TGD replaces AdS/CFT correspondence with strong form of holography.

1.1 Could one replace AdS/CFT correspondence with TGD version of holography?

One can criticize AdS/CFT based holography because it has Minkowski space only as a rather non-unique conformal boundary resulting from conformal compactification. Situation gets worse as one starts to modify AdS by populating it with blackholes. And even this is not enough: one can imagine anything inside blackhole interiors: wormholes connecting them to other blackholes, anything. Entire mythology of mystic creatures filling the white (or actually black) areas of the map. Post-modernistic sloppiness is the problem of recent day theoretical physics - everything goes - and this leads to inflationary story telling. Minimalism would be badly needed.

AdS/CFT is very probably mathematically correct. The question is whether the underlying conformal symmetry - certainly already huge - is large enough and whether its proper extension could allow to get rid of admittedly artificial features of AdS/CFT.

In TGD framework conformal symmetries are generalized thanks due to the metric 2-dimensionality of light-cone boundary and of light-like 3-surfaces in general. The resulting generalization of Kac-Moody group as super-symplectic group replaces finite-dimensional Lie group with infinite-dimensional group of symplectic transformations and leads to what I call strong form of holography in which AdS is replaced with 4-D space-time surface and Minkowski space with 2-D partonic 2-surfaces and their light-like orbits defining the boundary between Euclidian and Minkowskian space-time regions: this is very much like ordinary holography. Also embedding space $M^4 \times CP_2$ fixed uniquely by twistorial considerations plays an important role in the holography.

AdS/CFT realization of holography is therefore not absolutely essential. Even better, its generalization to TGD involves no fictitious boundaries and is free of problems posed by closed time-like geodesics.

1.2 Perfect tensors and tensor networks realized in terms of magnetic body carrying negentropically entangled dark matter

Preskill *et al* suggest a *representation* of holography in terms of tensor networks associated with the tesselations of hyperbolic space and utilizing perfect tensors defining what I call negentropic entanglement. Also Minkowski space light-cone has hyperbolic space as proper time=constant section (light-cone proper time constant section in TGD) so that the model for the tensor network realization of holography cannot be distinguished from TGD variant, which does not need AdS at all.

The interpretational problem is that one obtains also states in which interior local states are non-trivial and are mapped by holography to boundary states are: holography in the standard sense should exclude these states. In TGD this problem disappears since the macroscopic surface is replaced with what I call wormhole throat (something different as GRT wormhole throat for which magnetic flux tube is TGD counterpart) can be also microscopic.

1.3 Physics of living matter as physics condensed dark matter at magnetic bodies?

A very attractive idea is that in living matter magnetic flux tube networks defining quantum computational networks provide realization of tensor networks realizing also holographic error correction mechanism: negentropic entanglement - perfect tensors - would be the key element! As I have proposed, these flux tube networks would define kind of central nervous system make it possible for living matter to experience consciously its biological body using magnetic body.

These networks would also give rise to the counterpart of condensed matter physics of dark matter at the level of magnetic body: the replacement of lattices based on subgroups of translation group with infinite number of tesselations means that this analog of condensed matter physics describes quantum complexity.

I am just a novice in the field of quantum error correction (and probably remain such) but from experience I know that the best way to learn something new is to tell the story with your own words. Of course, I am not at all sure whether this story helps anyone to grasp the new ideas. In any case, if one have a new vision about physical world, the situation becomes considerably easier since creative elements enter to this story re-telling. How these new ideas could be realized in the theory world? This question I try to answer in the following.

The goal in the sequel is therefore an attempt to formulate the connection between quantum holography and error correcting codes in TGD framework bringing in new features relating to the new views about space-time, quantum theory, and living matter and consciousness in relation to quantum physics.

2 Holography

In the following I summarize my understanding about holography.

2.1 Holographies

Holography has become a key notion in attempts to understand gauge theories and gravity. One variant of holography suggests that blackhole horizons containg the information about quantum state assignable to blackhole. The naïve picture is that area unit defined by Planck length corresponds to single bit.

There is also second form of holography. AdS/CFT correspondence states that conformal field theory at the boundary of $AdS_n \times S^{10-n}$ is dual to a string theory or gravitational theory in 10-dimensional space. AdS_n has Minkowski space as $n-1$ dimensional boundary and for conformal field theory in 4-D Minkowski space one would have AdS_5. AdS_n is Minkowski space with 2 time like dimensions realized as a surface $t_1^2+t_2^2-x_1^2-x_2^2-x_3^2-x_4^2 = R^2$, where R is radius of curvature.

I cannot avoid some nitpicking.

1. AdS is somewhat problematic physically since it has closed time-like geodesics as is rather clear from the defining condition (see `http://tinyurl.com/h7xltde`) and also from the $(1,1,-1,-1,-1)$ signature of the metric. The second problem with AdS is that it is rather formal construct.

2. One speaks also about AdS boundary. AdS does not however have an actual boundary. A more precise term is conformal boundary resulting in conformal compactification transforming the metric to Minkowski metric apart from conformal factor. This transformation is fixed apart from a conformal transformation of AdS. The existence of the transformation follows from the conformal flatness of AdS which for dimensions not smaller than four can be formulated in terms of the vanishing of Weyl tensor (see `http://tinyurl.com/y7fsnzk8`). Every manifold with constant sectional curvature is conformally flat. Note that CP_2 is not conformally flat.

   Conformal boundary corresponds to vanishing value of z in the representation of $ds^2 = (ds^2(M^{n-1}) + dz^2)/z^2$ with $z > 0$ giving the half-space and $z = 0$ the $n-1$-D Minkowski space. The metric as making explicit the conformal flatness of AdS, where $\Omega^2 = 1/z^2$ is the conformal scaling factor which becomes infinite in subspace, which is Minkowski space. A second kind of singular behavior would be vanishing of Ω^2 ($z = \infty$) and also this subspace could be identified as conformal boundary.

   Conformal compactification is a map of original space with the property that it performs conformal scaling for the metric tensor. Conformal compactification requires finding of a coordinate transformation taking the metric to the desired form. Note that in the case of Minkowski space with spherical coordinates with metric $dt^2 - dr^2 - r^2 d\Omega^2$ the conformal compactification is induced by the compactifying conformal mapping of $(t,r) \to (T,R)$ satisfying therefore $g_{TT} = -g_{RR} = \Omega^2$ and $r^2 = \Omega^2 R^2$. The map can be explicitly constructed and maps (t,r) plane to a triangle. Ω^2 vanishes or diverges at the conformal boundary. This can be done also for AdS so that the resulting conformal boundary is Minkowski space and has Minkowski metric with vanishing scale factor.

   To my humble opinion, procedures of this kind should be avoided in fundamental theory. On the other hand, the twistor space of Minkowski space used in twistorialization is also conformal compactification. In this case one however has what is known as double fibration [B7] (see `http://tinyurl.com/yb4bt74l`) meaning that one has fibration from $M^4 \times S^2$ -

the trivial S^2 bundle defining the geometric twistor space to the twistors space identified as complex projective space defining conformal compactification of M^4. Double fibration is essential in the twistorialization of TGD [K5].

3. Even worse - to my opinion - is that people are not happy with AdS as such but add to AdS all kinds of stuff such as blackholes making varying assumptions about their interiors. This leads easily to an inflation of sloppily defined notions. Just this happened in super string theory and eventually led to the multiverse mania. Eventually the proposal emerged that the requirement that physical theories should be able to predict something should be given up since the exceptional "beauty" of some of these baroque constructions would be enough to justify them as the only possible theory of everything.

In TGD framework AdS/CFT correspondence is replaced with the strong form of holography.

1. Ordinary holography would state that preferred 3-surfaces - either space-time 3-surfaces at the boundaries of CD or light-like 3-surfaces connecting them - carry same quantum information as space-time surface. Strong form of holography (SH) follows from the conditions that these two identifications of 3-surfaces are equivalent and says that 2-D partonic 2-surfaces and string world sheets carry the quantal information. SHS is very similar to ordinary holography and to e blackhole holography.

   There is no need for conformal compactification and conformal boundary since boundary is something very real being identified as the light-like orbit of partonic 2-surfaces defining the boundary between Euclidian and Minkowskian space-time regions. 4-D space-time surface replaces the 10-D bulk of AdS/CFT correspondence and partonic 2-surfaces and their light-like orbits replace the space-time at boundary of AdS, where one has conformal quantum field theory (CFT).

   In analogy with conformal boundary, 4-D space-time metric becomes singular (determinant vanishes) at the light-like orbits of wormhole throats meaning that the dimension of tangent space degenerates to 4 to 3. These objects are not fictive but completely physical and can be seen as analogs of blackhole horizons making sense as analogs of boundaries for objects of any size. The new element is that the interior has Euclidian signature of induced metric and is interpreted as geometric counterpart for the line of generalized Feynman diagram or twistor diagram representing the orbit of particle [K5].

2. Classical holography means that space-time surfaces can be constructed as continuations of string world sheets and partonic 2-surfaces (and possibly also their light-like orbits giving rise to discrete number of non-gauge degrees of freedom) serving as space-time genes. Space-time surfaces are thus preferred extremals of the basic action principle. The preferred extremals would be thus 2-dimensional in information theoretic sense.

   This boils down to the condition that a sub-algebra super-symplectic algebra isomorphic to the algebra itself annihilates the physical states and that the classical Noether charges for the sub-algebra vanish. This huge number of conditions makes space-time surfaces as preferred extremals analogous to Bohr orbits. This picture leads also to the hierarchy of Planck constants $h_{eff} = n \times h$ assignable to the fractal hierarchy of isomorphic subalgebras of the super-symplectic and other conformal algebars. This implies a generalization of quantum theory crucial for biology and quantum computation since large value of h_{eff} means macroscopic quantum coherence.

3. Conformal invariance is extended dramatically and the Lie group defining Kac-Moody group becomes the symplectic group of $\delta M^4_\pm \times CP_2$, $\delta M^4_\pm$ denotes light-cone boundary. The light-like radial coordinate of light-cone boundary takes the role of additional complex coordinate z. Note that the conformal invariance is really huge at the fundamental level. This is what allows to replaced the 10-dimensional space of AdS/CFT with 4-D space-time surface and make the holography free of non-physical auxiliary constructs.

   There is also second difference: embedding space $H = M^4 \times CP_2$ - unique by twistorial considerations - enters into the game too and is expected to take some roles of AdS at the level of embedding space. The hyperbolic character of time=constant sections together with the findings of Preskill *et al* [B4] suggests how this might happen. This will be the main theme of this article.

4. In AdS-CFT correspondence radial direction from boundary is interpreted as renormalization scale. I have considered analogous interpretation for the direction normal to the partonic 2-surfaces in TGD: the idea was that any light-like 3-surface in the slicing by light-like 3-surfaces "parallel" to the orbit of partonic 3-surface is physically equivalent representation for holography. An alternative and perhaps more plausible interpretation is that RG scale corresponds to the proper-time for light-cone boundary labelling the slices $H_3(a)$. It could also naturally correspond to the size scale of CD, perhaps the value of a for which the size of the tessellation contained by CD is largest.

   Quantum criticality is basic aspects of quantum TGD and predicts that various coupling strengths have a spectrum of critical values labelled by the p-adic primes $p \simeq 2^k$, k prime: an attractive conjecture is that their values relate in simple manner to the zeros of Riemann zeta [K3]. These length scales in turn would relate to the size scales of CDs. Coupling constant evolution would become discrete. Perhaps one could interpret the preferred role of the boundaries between Minkowskian and Euclidian regions as counterpart for fixed point property of critical values of coupling constants under RG evolution.

2.2 Blackholes and wormholes

The many-sheeted space-time distinguishes TGD from GRT. GRT is obtained from TGD as an approximation in which the sheets are replaced by single region of Minkowski space with deformed metric obtained by adding the deviations of the metrics of the sheets with their sum representing the deviation of the metric from Minkowski metric. Components of spinor connection are added in the same manner to obtain electroweak gauge fields of the standard model. Classical color gauge potentials are identified as projections of color Killing vector fields and siimlar description applies to them. This picture follows by considering test particle as a small surface having topological contacts with various sheets of the many-sheeted space-time: the classical forces experienced by it are sums over contributions from various sheets.

Blackholes and wormholes are basic notion of GRT and they have also TGD counterparts.

1. Blackholes play akey role in the attempts to relate quantum gravity, holography, and quantum information theory in GRT framework. I am happy to see that theoreticians like Susskind (see `http://tinyurl.com/y927h695`) have the courage to imagine freely. From my own experience there is of course a risk that this leads to endless loose speculations getting us nowhere. To me blackhole interior represents the failure of GRT and the new theory must replace it with something less singular.

   In TGD framework blackholes are not fundamental and the Euclidian space-time regions with 4-D CP_2 projection representing orbits of wormhole contacts between to space-time sheets replace them. Wormhole contacts become basic building bricks of elementary particles and simplest elementary particles consist of a pair of wormhole contacts with magnetic monopole flux flowing from throat to the other one, through contact to second space-time sheet and back.

   These Euclidian regions define the analogs of lines of Feynman diagrams, and one can speak of generalized Feynman (or twistor-) diagrams represented at the level of space-time geometry and topology. There is no need to perform Wick rotation to get well-defined path integral: Euclidian regions provide to the vacuum functional automatically the real exponent guaranteing the mathematical existence of the functional integral.

2. Wormhole is second key notion of GRT. ER-EPR correspondence is a proposal that wormhole throats - kind of flux tubes between distant regions of space-time - serve as correlates of entanglement. An even stronger conjecture is that space-time somehow emerges from entanglement - I cannot get enthusiastic about this idea. Wormhole throats in GRT sense are not however stable but suffer a pinch splitting them.

I have made an analogous proposal much earlier in TGD framework. Magnetic flux tubes would take the role or wormhole throats. Flux tubes would carry Kähler magnetic flux monopole Kähler magnetic flux making them stable.

Remark: To avoid confusion let us make clear that wormhole throat in TGD framework means partonic 2-surface, which can be regarded either as the end of magnetic flux tube replacing wormhole throat understood in GRT sense.

3. Partonic 2-surface is the key notion in TGD, and it turns out that this could define the microscopic representation of 2-surface. The macroscopic continuous 2-surface would be replaced with a union of partonic 2-surfaces and the generalization of Ruy-Taganaki formula [B6] for entanglement entropy would be formulated in terms of partonic 2-surfaces.

2.3 Hyperbolic tessellations are possible for both AdS and Minkowski space

What makes AdS_n so interesting from the point of view of holography and error correcting codes is that the hyperbolic tessellations for time= constant surfaces defining hyperbolic spaces can be used to define quantum holographic codes and states.

1. AdS has a slicing by hyperbolic spaces $t_2^2 - x_1^2 - .. = R^2 - t_1^2$ with slices labelled by $t_1^2 \leq R^2$. Hyperbolic spaces have infinite number of tessellations known also as tilings. The tile is defined as a double coset space $\Gamma \backslash SO(1,n)/SO(n)$, where Γ is an infinite discrete subgroup of $SO(1,n)$ with discontinuous action. Γ is analogous to a discrete subgroup of translations in Euclidian space.

   One can imagine of connecting each face of a given tile to the faces of neighboring tiles and interpret the connection as inputs and outputs and assign to each face an isometry at the level of state space. The tessellation would define a tensor network mapping the inputs assignable to the interior tiles to the boundary states of the network.

2. Remarkably, also Minkowski space appearing as factor of H allows slicing by 3-D hyperbolic spaces $H_3(a)$ defined as hyperboloids $t^2 - r^2 = a^2$ (mass shell in particle physics). This slicing is central in TGD based cosmology with a defining the size scale of the universe. I have already earlier proposed that these hyperbolic tessellations must have fundamental role in TGD.

   For instance, they could naturally define a discretization for $a = constant$ hyperboloids assignable to either tip of the causal diamond CD defined as intersection of future and past directed light-cones. The discretization would be naturally associated with the second boundary of CD and there are indications that astrophysical objects could be at cells of this kind of tessellation analogous to condensed matter lattices. Recession velocity equivalent to equivalently cosmic redshift and by Hubble's law equivalent to distance would be quantized. Evidence for this quantization exists [K7].

3. What is even more significant, a very large fraction of 3-manifolds are known to be hyperbolic manifolds in the sense that they allow hyperbolic metric [K7] [A1]. Thus one can say that they can be mapped to pieces of hyperbolic tessellation. The induce metric of the 3-surface is of course not hyperbolic in general. However, if 3-surface representable as graph from hyperbolic space $H_3(a)$ to CP_2 one can assign a hyperbolic metric to it by considering the metric for the projection to $H_3(a)$ as metric of the 3-surface.

   This seems to work for all 3-surfaces representable as graphs $H_3 \rightarrow CP_2$ but there could be problems due to the presence of boundary. If one allows the boundary of 3-surface to contain broken tiles, there seems to be a very large number of tessellations. The tessellation would be unique only if boundary tiles are required to be unbroken: this condition also implies that the tessellation need not exist.

   Accepting broken boundary tiles, one could say that the hyperbolic tessellation is induced to space-time surface and that space-time surface representable as graph $M^4 \rightarrow CP_2$ allows a slicing by hyperbolic tessellations. One can of course ask, whether the preferred extremal property allows only unbroken tiles. In this case the boundary of tessellation could correspond to light-like 3-surface at which the signature of the induced metric changes to Euclidian. This would fit nicely with the notion of SH.

The 3-volume of hyperbolic manifold defined by the hyperbolic metric is a topological invariant. The Minkowskian 3-volume would be indeed same for all allowed deformations with fixed $H_3(a)$ projection. Hence all deformations preserving the graph property really correspond to the same hyperbolic 3-manifold whose topology would reduce to its boundary topology. Note that preferred extremal property implies effective 2-dimensionality and this might restrict dramatically the number of allowed 3-surfaces. For unbroken tiles the boundary of the 3-surface contributes to the topology. The absence of genuine boundaries apart from the causal boundaries implied by the change of the metric signature is strongly suggested by boundary conditions, and could be achieved by gluing of piece of tessellation and its deformed copy along boundaries and also this brings in non-trivial topology.

4. CP_2 as a compact coset space allows also tessellations by its discrete subgroups with finite number of tiles. These are analogous to Platonic solids. These tessellations might be interesting for 3-surfaces not allowing a representation as a graph of a map from $H_3(a)$ to CP_2. The Euclidian regions obtained as deformations of CP_2 type vacuum extremals are good candidates in this respect. Deformations of string like objects $M^2 \times S^2$, S^2 a geodesic sphere in CP_2 would allow naturally tessellations defined by the tessellations of H_1 and S^2. These objects would be rather simple from the point of view of complexity theory.

These observations suggest that the hyperbolic tessellations realizing the holography can be actualized in quantum TGD and that they could even define quantum holographic codes.

1. The lines connecting faces of the tiles serve as correlates for entanglement. If magnetic flux tubes mediating monopole flux take the role of lines, the only possibility is that tiles and magnetic flux tubes connecting the centers of tiles form a network. The braiding of the flux tubes is also possible and would make possible topological quantum computation.

2. Tiles could contain quantum states and dark matter realized as $h_{eff} = n \times h$ phases for fundamental quarks and leptons serving as building bricks of elementary particles defined a good candidate for these phases. What simplifies the situation is that there are good reasons for the localization of the modes of induced spinor fields at string world sheets and string world sheets can be associated with flux tubes.

3. tessellation should be seen as an idealized model only. Only the topology of the graph defined by the tessellation and perfect tensor character of entanglement seem to be important. Magnetic body could be thus dynamical. Flux tubes could get braided, they could reconnect, they could experience phase transitions changing the value of $h_{eff} = n \times h$ inducing reduction or increase of length also 2-knotting and braiding can be imagined in 4-D space-time and could bring in totally new kind of topological dynamics. These phase transitions are crucial in TGD inspired quantum biology.

Flux tube networks could suffer phase transitions in which the character of the tile as hyperbolic manifold changes just like the condensed matter lattice can change its character and in living matter this kind of phase transitions might be important. This suggests that quantum complexity theory could be seen as counterpart of condensed matter physics at the level of magnetic body.

4. An interesting question is what happens to the tessellations as one approaches boundary of CD, which is part of light-cone boundary. The study of the metric demonstrates that the tessellation degenerates to the tip of CD at this limit and all information is lost. This suggests that the representative capacity of tessellation as a function of light-cone proper time a is measured by the 3-volume of the tessellation in hyperbolic metric. The condition that this volume is restricted inside CD restricts the range of radial coordinate r in $r_M = ar$ and the volume has maximum for some value $0 < a < T$, where T is the distance between the tips of CD. Light-boundary is metrically sphere: could Platonic solids define tessellations at this limit? Could one see this limit as the limit at which ordinary condensed matter physics emerges as the quantum information associated with hyperbolic tessellations disappears.

There are indications that in living matter and brain this kind of networks are realized and I have proposed that magnetic flux tubes define this kind of quantum computational networks in living matter. One could even say that space-time becomes sensorily conscious about itself by building this kind of networks analogous to coordinate grids.

These observations suggest that quantum holography and error correction codes might be realized at the level of magnetic bodies dynamically in TGD framework. They would not be essential for defining quantum TGD itself but living and intelligent systems could develop the tessellations to develop bodily sensory consciousness.

3 Entanglement and Physics of Quantum Complexity

Both Susskind (see `http://tinyurl.com/y927h695`) and Preskill [B4] emphasize entanglement as the key notion distinguishing between quantum and classical. Entanglement brings in complexity as an exponential decrease of the dimension of the state space and thus the number of states due to the possibility of entanglement. For instance, for classical 3-D spins the configurations space of N spins has dimension $3 \times N$. For quantal spins the complex dimension of state space is 2^N and exponentially larger. It is not perhaps exaggeration to say that the science of complex systems is science of entanglement. Quantum information science has started to seriously study various aspects of entanglement and build simple models.

The notion of entanglement is rather abstract. One can even entangle mutually non-interacting CFTs. This brings in mind Connes tensor product and corresponding entanglement relating to the inclusions of hyper-finite factors [K15]. This entanglement is not forced by interactions but by internal consistency and one might regard it as kinematical. Negentropic entanglement (NE) central for TGD and number theoretic vision is also this kind of kinematic entanglement having very little to do with ordinary dynamics. Surprisingly this is nothing but the entanglement associated with perfect tensors used in the construction of Preskill. Note however that NMP favors its generation.

3.1 Some general results

The understanding of the relationship between holography, geometry, and entanglement has evolved dramatically during the last decade and it was nice to become aware of the work done and find that it complements nicely my own work work done in the framework provided by TGD theory of consciousness as a generalization of quantum measurement theory to ZEO.

1. Ryu-Takayanagi formula [B6] for the entanglement entropy as area of minimal surface is key formula and generalizes the area-entropy relationship for blackholes having also interpretation as entanglement entropy. Another key idea is ER-EPR correspondence stating that entanglement as wormholes connecting the entangled systems as a correlate: magnetic flux tubes as correlate for entanglement is an old idea of TGD.

   I have also considered the possibility that the braiding of magnetic flux tubes could serve as correlate for entanglement. If flux tubes appear as pairs with opposite directions of magnetic flux then entanglement might have braiding as correlate. Braiding could be also - and probably is - something independent.

2. Tensor networks constructed using perfect tensors as a representation for strongly interacting systems is a key idea in the article of Preskill *et al* [B4]. The king idea is the notion of perfect tensor with $2n$ components defining entangled system which in TGD framework would be negentropically entangled in the special sense that all n-dimensional subsystems would be maximally entangled with their complement. These tensors can be also used to define quantum error correcting codes and hyperbolic tessellations allow to construct this kind of codes.

Multiscale Entanglement Renormalization Ansatz (MERA) [B5] (`http://tinyurl.com/y9avp92y`) allows to study numerically hierarchical systems with long range entanglement described by local scale-invariant Hamiltonians. MERA networks allow to estimate the ground state of the system.

1. Hierarchical arrays of tensors represent entanglement in different length scales. MERA can be see as a more general isometry from bulk to boundary than the tensor networks proposed by Preskill *et al.* MERA involves also unitary transformations induced de-entanglement in vertical direction. The isometries $V \rightarrow V \otimes V$ (see Fig. 1 of the article) can be made unitary by extension $|0\rangle \otimes V \rightarrow V \otimes V$. The added tensor factor $|0\rangle$ is analogous to ancilla state in quantum computation. The inverse of this map can be regarded as roughening operation and eventually all entanglement is eliminated as one goes upwards in MERA network.

2. AdS/CFT- MERA connection is proposed first by Swingle [B1] and tensor networks as its realization is mentioned also in the [B4]. AdS/CFT-MERA connection is discussed by Bao *et al* in [B3] (`http://tinyurl.com/y9a9lr29`). AdS/MERA correspondence is suggested to work only in scales longer than AdS scale, and argued to fail to be complete even above AdS scale. The argument starts from the observation that correspondence requires the assignment of length scales to the vertical and horizontal bonds of the MERA network (dimension is 2) and somekind of lattice in AdS is proposed. Authors are not yet aware about the possibility that hyperbolic tessellations might provide an elegant geometrization of MERA network utilizing the discrete symmetries of hyperbolic space and provide the needed scales: scaling invariance allows continuum of scale choices and AdS radius does not seem to matter.

   An open question (to me at least) is whether the unitary transformations are necessary for MERA as claimed by Bao *et al*, and whether the unitary steps are realizable in terms of tessellations: Preskill *et al* do not seem to have them in their construction.

Does MERA have a natural TGD counterpart?

1. The tessellations of H_3 induce tessellations of 3-surfaces in TGD framework and tessellation allows geometrization of the tensor networks as mere graphs.

2. tessellations are scaling invariant but one can ask whether there are natural length scales. Minkowski space does not have any intrinsic scale but the geometric twistor space $M^4 \times S^2$ of M^4 provides such a scale as S^2 radius given by Planck length if the view about twistorialization of TGD is correct [K5]. The physical picture suggests that already CP_2 scale (roughly 10^4 times Planck length) defines a lower bound for scales in which the analog of AdS/MERA correspondence can make sense.

3. p-Adic length scale hierarchy and hierarchy of Planck constants with scaled up versions of p-adic length scales are quantitative formulations for the hierarchies at space-time level and give rise to many-sheeted space-time. These hierarchies are accompanied by several other fractal hierarchies: for instance, in TGD inspired theory of consciousness conscious entities form a fractal hierarchy. A natural guess would be that p-adic length scale and their dark scaled up variants define a hierarchy of length scales below which holography cannot be realized exactly.

From TGD point of view hyperbolic tessellations and MERA have clearly been missing pieces of a puzzle.

3.2 Entanglement in TGD Universe

There are several notions related to entanglement and inspired by the construction of TGD inspired theory of consciousness.

1. Zero Energy Ontology (ZEO) is the basic new element forcing to reconstruct quantum measurement theory. This leads to a theory of consciousness and allows to identify what life and death of conscious entity be from the point of view of quantum physics [K2].

   For dark matter the outcome of state function reduction would not be random below the duration of the sequence of state function reductions to the same boundary of causal diamond (CD) defining the lifetime of conscious entity: life could be seen as generalized Zeno effect. The first reduction to the opposite boundary of CD forced by NMP to eventually occur would mean death of the conscious entity and subsequent re-incarnation at the opposite boundary of CD. This conclusion is not just an idea which happened to pass by: reaching it took almost a quarter of century!

2. The goal of complexity science is to understand and control complex quantum systems and the basic challenge is to overcome quantum decoherence. Here TGD view about dark matter suggests a solution. TGD predicts a hierarchy of quantum phases of ordinary matter labelled by the value of Planck constant $h_{eff} = n \times h$ for which quantum scales are scaled up so that macroscopic quantum coherence becomes possible [K4, **?**, K9]. The time scale for decoherence is expected also to scale up.

   This hierarchy defines a hierarchy of conformal symmetry breakings possible also for ordinary conformal symmetry but for some reason remained un-recognized. This makes possible fractal symmetry breaking: the symmetry broken sub-algebra is isomorphic to the original one: symmetry breaking without symmetry breaking!

   Second mystery of theoretical physics of last decades is the failure to realize that conformal invariance has a generalization to 4-D context due to the fact that light-cone boundary in 4-D Minkowski space is metrically 2-D. This symmetry is crucial for the construction of TGD as Kähler geometry and spinor structure of "World of Classical Worlds" (WCW). Unfortunately, my attempts to communicate this simple fact have been fruitless. Maybe the reasons are basically sociological: "people from Harward" simply refuse to take seriously what lower level organisms try to explain to them.

3. A further new element is Negentropy Maximization Principle (NMP) [K8], which serves as the basic variational principle of TGD inspired quantum measurement theory and also that of TGD inspired theory of consciousness. NMP states roughly that the negentropy gain in state function reduction is maximal. Mathematically NMP is very much like second law but applies to number-theoretic entanglement negentropy characterizing information content of the entire entangled system rather than ensemble entropy characterizing either member of of entangled pair so that no conflict with second law need to be implied. It indeed seems, that second law becomes scale dependent notion holding true only above the life-time of conscious entities involved.

4. NE is a further new element is brought by number theoretical vision. NE is possible if entanglement probabilities belong to an algebraic extension of rationals to which the parameters characterizing string world sheets and partonic 2-surfaces are assumed to belong.

   Remark: The restriction of parameters to be in algebraic extension means realization of finite measurement resolution at the level of WCW allowing to get rid of the problems related to breaking of basic symmetries in the discretization realized at space-time level.

   NE is defined as entangelement negentropy is defined as p-adic variant of entanglement entropy for a prime for which the entanglement entropy is minimal. This makes sense if the probabilities belong to algebraic extension.

   The p-adic entanglement entropy defined by the same formula as Shannon entropy but replacing ordinary logarithm of probability with the logarithm of the p-adic norm of probability can be negative so that it is better to talk about negentropy. The entanglement negentropy has interpretation as a measure of coscious information about the state of entangled system rather as entropy characterizing the lack of information about the state of either sub-system defining thermodynamical entropy.

   An important special case corresponds to NE for which density matrix is proportional to unit matrix: in this case the entanglement negentropy is maximal. The interpretation is that entanglement negentropy accompanies positively colored conscious experiences like experience of understanding and love. In the case of unit matrix it would correspond to meditative experience in which all distinctions are reported to disappear: this could relate to the fact that in this case any state basis is eigenstate of the density matrix.

5. Which states are characterized by NE and thus by algebraic entanglement probabilities? Could all states associated with the values $h_{eff}/h > 1$ be such states? This would make sense if the entanglement is in the discrete degrees of freedom implied by the n-dimensional covering of space-time surface.

6. There is a nice connection to quantum error correcting codes. Preskill *et al* [B4] introduce the notion of perfect tensor as a building brick of the tensor networks which they propose

to give rise to a representation of holography using hyperbolic tessellations. Tensors define entangled states and perfect tensors have $2n$ k-valued indices such that any decomposition of the system to $2n$ states possesses maximal entanglement. This system has also the property that there is isometric embedding of any $n-k$ states to the state space spanned by $n+k$ states in the complement. This kind of system is ideal for error correcting quantum code and would represent the stable subspace of states for which error correction is possible. The definition of the negentropically entangled system stable under is essentially the same!

7. NMP favors generation of NE. NE can be transferred between systems and can thus reduce for a given subsystem (just like thermodynamical entropy), it tends to increase for the entire Universe. NE is assigned with dark matter with non-standard value of Planck constant. This forces to reconsider the possibility that second law, which could be seen as a consequence of the randomness of the outcome of state function reduction, holds only for the visible matter with standard value of Planck constant and characterized by ordinary entanglement. A more precise formulation is that second law holds true in time scales longer than the time scale of the living systems involved: the death of conscious entity means ordinary state function at the opposite boundary of CD and therefore generates ensemble entropy. For NE the outcome is not random during the reduction sequence defining self as Zeno effect.

If this vision is on the correct track, the science of complexity would be also physics of dark matter, living matter, consciousness, and cognition.

4 Quantum Error Correcting Codes, Holography, and Tensor Networks

Quantum computer must be isolated from the environment that is keep the state of the computing system pure. In the world described by standard quantum theory alone it is however extremely difficult to prevent the generation of entanglement with environment in turn implying that the state obtained by tracing over the environmental degrees of freedom is non-pure: entanglement entropy is generated and decoherence occurs.

To avoid this error correction mechanisms have been developed.

1. The basic idea is that one has system, environment, and auxiliary system called ancilla. In the initial state the systems are unentangled but perturbations generate entanglement between system and environment (why not between ancilla and environment?). One can however perform a unitary transformation for the whole system transferring the entanglement from system-environment to environment-ancilla pair by performing a unitary transformation and by replacing ancilla essentially with a new one. Ancilla would function as trash bin.

2. The mirror image of this idea is actually familiar from TGD inspired theory of consciousness. NMP predicts that negentropy tends to increase in quantum jumps. The negentropy of individual systems can be however reduced. In particular, NE from the system can be stolen: this would define the quantum correlate for stealing as crime! Religious myths catch this idea. Eve could not avoid the temptation to eat the fruit from the tree of Good and Bad Knowledge! The error correcting code would perform just the opposite by cleaning away the parasitic entanglement.

3. In TGD based biological quantum computation this mechanism could be used to transfer conscious information from quantum computing system to other system. Cleaning would become picking of fruits! I have proposed that the fundamental function of metabolism is basically transfer of NE from nutrients to organism [K9].

In he ordinary computation error correcting codes replace bits with bit sequences for which additional bits serve as check bits allowing to detect whether one or more bits have changed. If the number of bits is below some number defining the code, the errors can be corrected. For an unpractical person like me, it was quite a surprise to learn that without this kind of error correction mechanisms IT technology would not be possible. Errors are not rare events but do occur all the time.

In quantum situation bits are replaced with qubits and error correcting code is defined by subspace of the full code space. Quantum error correcting code maps logical qubits by Hilbert space isometry to an entangled state formed by a larger number of physical qubits. The image under the isometry defines code subpace.

A kind of quantum hologram in which the information is distributed between larger number of qubits is created. If the number of erratic qubits is below some threshold characterizing the code, the errors can be corrected by applying a unitary transformation determined by the erased bits. This is highly analogous to the properties of the ordinary hologram. These codes allow also also to protect some maximal number of logical bits and also to detect whether quantal eves-dropping has taken place.

This suggests a beautiful correspondence between the classical and quantal holographies in TGD framework.

1. Classical holography would be realized in terms of string world sheets and partonic 2-surfaces serving as holograms coding space-time surfaces as preferred extremals whereas quantum logical hologram would be realized in terms of fermions whose many-particle states at string world sheets and partonic 2-surfaces define quantum Boolean logic.

   WCW gamma matrices are constructed as combinations of the fermionic oscillator operators so that the classical and quantum holographies would correspond to WCW orbital degrees of freedom and spin degrees of freedom. Note also that the modes of WCW spinor fields are formally classical. In fact super-conformal symmetries relate the orbital and spin degrees of freedom of WCW so that also the two holographies might be closely related at deeper level.

2. This observation raises a question related to the interpretation and fundamental formulation of TGD [K16, K12]. The recent formulation assumes that Kähler-Dirac action is defined at entire space-time but that the solutions are localized to string world sheets: this guarantees the well-definedness of em charge if induced W gauge fields vanish at string world sheets. There are several other excellent reasons for assuming the localization.

   Could it be that also the induced spinor fields in the complement of partonic 2-surfaces and string world sheets are present just like interior degrees of freedom for space-time surfaces? Should one one follow supersymmetric instincts and holograpically continue also the spinor modes at string world sheets to the space-time interior just as one continues the string world sheets themselves? The holographic localization of the modes at string world sheets making possible conformal invariance at them would mean that all information about many-fermion states are carried by string world sheets.

4.1 Tensor networks

The concept of tensor network relies on the notion of isometry between Hilbert spaces, to the notion of perfect tensor defining a basic building brick for the networks defining isometry of logical qubits in the interior of 3-surface with the entangled states formed by physical qubits at the boundary. Hyperbolic tessellation is a further key concept. Two key results are that the isometry defining quantum holographic mapping of interior quantum states to boundary states can be realized in terms of tensor network assignable to the tessellation and entanglement entropy can be expressed as area for the surface separating subset of the boundary from its complement.

Hyperbolicity means negative curvature and for hyperbolic spaces means the existence of infinite number of tessellations with infinite number of cells allowing to build tensor networks. In 2-D case this is basically due to the negative sign of the angle deficit for a geodesic polygon: this favors regular tiles with large number of faces.

Group-theoretically this means that the discrete group defining the tessellation is infinite and there is infinite number of them. For constant curvature spaces with positive curvature this group is finite as also tessellation itself. Also the number of tessellations is finite - at least in the case of sphere (the 5 Platonic solids). By the way, Platonic solids might be interesting at the light-like boundary of CD resulting as a limit of hyperbolic space since it reduces to sphere metrically.

4.2 Isometries and perfect tensors

Unitary transformations of Hilbert space satisfying $UU^\dagger = Id$ are isometries preserving the inner product. The isometries from n-dimensional Hilbert space H_n to $n+k$-dimensional Hilbert space H_{n+k} are possible and define embeddings.The inverse transformation is defined only in the subspace defined by the n-dimensional image and has k-dimensional kernel. If there are unitary transformations U and U' of the two Hilbert spaces satisfying $TU = U'T$ then $T^\dagger T \propto Id_n$ is satisfied.

For an isometry $T : H_1 \otimes H_2 \to H$ one can construct isometry $\tilde{T} : H_1 \to H \otimes H_2$ such that $\tilde{T}^\dagger \tilde{T} = dim(H_2)Id$ by simply moving a_2 from $|a_2a_1\rangle$ to the right-hand side in the formula

$$T : |a_2a_1\rangle \to \sum_b |b\rangle T_{ba_2a_1} \quad ,$$

and by tracing to get

$$\tilde{T} : |a_1\rangle \to \sum_{ba_2} |ba_2\rangle T_{ba_2a_1} \quad .$$

Tensor $T_{a_1....a_{2n}}$ defines entangled states by a contraction with quantum states in labelled by a_i defining the set of $2n$ indices with v values.

Perfect tensors have the defining property that for any decomposition of the index set to two parts A and its complement A^c, $|A| \leq |A^c|$ the map defined by the tensor from A to A^c is isometry. Furthermore, perfect tensor maps $m \leq n$ spins isometrically to $2n - m$ spins. This obviously defines code as mapping of $m \leq n$- spin states to $2n - m$-spin states. In particular, single spin is map to $2n - 1$ spins isometrically. For any (n, n) decomposition perfect tensor defines absolutely maximally entangled states.

In the case considered spin is coded by a linear map to entangled $2n-1$-spins state of physical spins and there is 1 protected logical spin. There is stability against the erasure of $n - 1$ physical spins. A party holding n spins has complete information about logical spin because the erasure of $n - 1$ spins is correctable. Party holding $n - 1$ spins has no information about the full state. One could say that local physics in interior is coded to non-local physics at boundary, which is just what happens also in the ordinary hologram.

This picture suggest a deep connection between quantum information theory, quantum measurement theory according to ZEO, and consciousness: NE is optimal from the point of view of error correction and therefore it is natural to assume NMP and adelic physics bringing in various p-adic physics as correlates for cognition.

4.3 Hyperbolic tessellations and holographic quantum states and codes

The key results of the work of Preskill *et al* [B4] is what they call toy model for the realization of holography at the level of quantum states using tensor networks associated with tessellations of 2-D hyperbolic space forming time= constant section of AdS_3. The results are believed to hold true also in higher dimensions.

There is infinite number of tessellations labelled by the discrete infinite groups of Lorentz group so that the number of tensor networks is infinite reflecting faithfully the fact that entanglement can be equated with quantum complexity. The tensor network mapping the localized interior states isometrically to entangled boundary states is highly non-unique. For instance, one can choose the part of boundary involved in many ways and there is infinite number of tessellations. This suggests also a physical definition of complexity. Complexity should be defined in terms of the minimal tensor network allowing to realize this isometry.

Preskill *et al* define holographic quantum states and quantum codes. For holographic quantum states each tile of the tessellation carries quantum numbers meaning that not all of the spins are contracted. One could interpret these states as an explicit realization of the isometric mapping of localized interior states to boundary states with their image defining the stable code space. These states should not be however possible if one interprets holography in strict sense. This forces to consider whether the notion of holography used is general enough - this will be considered later from TGD point of view. For quantum codes all interior spins are contracted and the state is pure boundary state. Two examples about tessellations involving hexagons and pentagons are discussed explicitly.

4.4 Entanglement structure of holographic states

In the case considered 3-surface is replaced with 2-surface and in this case Ryu-Takayanagi formula [B6] represents the entanglement entropy between sets A and A^c at the boundary of 2-surface as a length of a minimal geodesic of hyperbolic plane connecting the boundary points of A. In 3-D case one would have minimal 2-surface.

The unit of "area" for d-dimensional boundary used is $4G_{d+2}$, where G_D is Newtons's constant of D-dimensional gravitational theory. The formula follows by assuming Einstein's equations at low energies. The argument of Ryu and Takayanagi involves AdS/CFT duality and introduction of AdS blackhole. One has string theory description of gravitation in AdS, one takes long length scale limit of this theory by making AdS dynamical and then introduces blackholes: reader can decide whether to take these steps leading to the desired outcome seriously. The source of my own skepticism is that the long length scale limit of string theory at long length scales involves too much hand waving and has led to the landscape problem in super string models.

Remark: In TGD framework strong form of holography implies that for $d = 2$ AdS_4 is replaced by 4-D space-time surface. One avoids the S^6 factor of 10-D fictive embedding space altogether. 8-D embedding space is present but is completely non-dynamical. Possible blackhole like entities (the light-like orbits of partonic 2-surfaces) are associated in TGD based holography with space-time surface. Strings in AdS correspond to string worlds sheets in space-time. Clearly there are strong similarities but also crucial differences.

Preskill *et al* [B4] represent an argument for how Ryu-Takayanagi formula could be understood in terms of the isometry relating local states in interior and non-local states at boundary. The minimal geodesic defines what is called causal wedge. The local operators within the causal wedge can be mapped isometrically to those at the boundary. Or stated differently: logical qubits in interior can be isometrically mapped to physical many-qubit states at the boundary.

As far as I understand, only the graph property of the tessellation and perfect tensor serving as building brick of the isometry are essential so that deformations of the tessellations are possible. In the argument the minimal curve reduces to the number of tiles along the boundary of the causal wedge and the formula expresses only upper bound for the entanglement entropy. The formula is purely combinatorial and based on the number of spin states at each node and on the number of nodes along the minimal geodesic.

Also an algorithm for finding the mininal geodesic is considered. So called greedy geodesic constructible numerically by starting from a given perfect tensor at boundary and pushing it to interior by isometries defined by perfect tensor as far as this is possible is discussed. For tessellations the greedy geodesics associated with A and A^c co-incide and define the minimal geodesic.

The notion of entanglement wedge is also considered. Within it the local operators in interior are mappable by isometry to the boundary operators. The authors consider multipartite entanglement and entanglement distillation but I am not specialist enough to attempt to relate this to TGD. Quantum error correction in holographic codes is also discussed in more detail. Some comments about information theoretic interpretation of blackholes are also represented and the problematic interpretation of holographic states is considered. Creation of blackhole would be analogous to an elimination of tiles in the interior and generating in this manner free induces and disjoint component to boundary.

5 TGD View about the Holographic States and Codes

6 TGD View about the Holographic States and Codes

The considerations of the article generalize to TGD rather easily. In TGD one has 3-D hyperbolic space H^3 as a sub-manifold of future light-cone (actually part of H^3 as sub-manifold of CD) so that possible problems due to the existence of closed time-like geodesics are avoided as also the objections due to the artificial nature of the conformal boundary. I have already described the geometric ideas.

6.1 Realization of the holographic states in terms of flux tube networks

For TGD point of view the most interesting question is whether also holographic states de

ning the code explicitly are possible as genuine physical states: it would be disappointing if the beautiful mathematics of hyperbolic 3-manifolds would not be realized physically. One could indeed consider of putting many-fermion states to the nodes of the tessellation in which the faces of tiles are connected by flux tubes and this would give kind of physical realization of holography.

Dark matter at magnetic flux tubes would represent higher level able to concretely represent physics at the lower level by realizing strong form of holography. Dark matter would have kind of sensory representations about the matter at lower levels of dark matter hierarchy. Biological systems would be this kind of systems since they build representations about environment - send themselves - some of them even about laws of physics (!)-, and there is evidence that in living matter kind of coordinate grids are realized [K11]: the proposed TGD interpretation is in terms of flux tube networks defining tensor networks mapping boundary states to interior states.

In TGD framework magnetic flux tubes carrying monopole Kahler magnetic flux are correlates for entanglement and could also define the edges of the tensor networks based on H^3 tessellations. Partonic 2-surfaces are however not homologically trivial and therefore not boundaries of anything: this is what makes them behave as Kähler monopoles and stabilizes both wormhole contacts and magnetic flux tubes. As a consequence, they appear as throats of wormhole contacts behaving like opposite sign magnetic charges. Even this is not enough: by the fact that magnetic flux lines must be closed, one must have pairs of wormhole contacts as a minimal structure and elementary particles are indeed modelled in this manner.

6.2 Generalization of the area formula for entanglement entropy

How the area law for entanglement entropy could generalize?

1. The replacement of entropy with its number theoretical variant, which becomes negative for NE, looks very natural in living matter. The growth of the network during time evolution discussed by Susskind in this lecture (see `http://tinyurl.com/y8zhhca5`) would not correspond to approach to chaos of thermal equilibrium but generation of NE and evolution at dark matter level! Second law would be manifested only at the level of visible matter since entanglement is always entropic for it.

   The phase transitions increasing Planck constant would generate NE and would scale up quantum length by $h_{eff}/h = n$ so that the networks would increase.

   Remark: It has become clear that instead of h one must have minimal value h_0 of h_{eff} which could be smaller than h [?, ?].

   Susskind assigns the growth of the network to blackhole interiors containing kind of invisible growing part of the network. In TGD Universe the emergence of wormholes connected by flux tubes would correspond to the growth of the tensor network. The hyperbolic tessellations of H^3 defining hyperbolic structure at 3-surfaces representable as graphs $H^3 \rightarrow CP_2$ have scaling invariance as a symmetry.

   The growth of the tensor network would occur in quantum critical phase transitions increasing the value of Planck constant. It is known that there is no cosmic expansion in local scales: this seems to be conflict with expansion in cosmological scales. The explanation could be that the expansion in given scale occurs in jerkwise manner - perhaps via phase transitiond increasing h_{eff} permanently or possiblty temporarily followed by a phase transition increasing p-adic length scale and reducing h_{eff} back to the standard value so that the scale is not changed. I have actually proposed a model for Expanding Earth (motivation comes from the observations that continents fit nicely together if the radius of Earth is by a factor 1/2 smaller than it is now) in terms of a phase transition increasing Earth radius by a factor of two. Sudden increase of the information content of flux tube network would be in question - kind of eureka experience of Mother Gaia [K5]! The model explains also the mysterious emergence of highly evolved multicellular lifeforms in Cambrian explosion as life forms evolved in the underground oceans and burst to the surface of Earth in the phase transition creating also the oceans. Before Cambrian Explosion Earth would have been like Mars, which by the way has radius equal to 1/2 of the Earth's radius.

2. Tensor network discretizes the continuum view: in particular, the continuous minimal surface is replaced with a set consisting of tiles of the tessellation. Could this description emerge from TGD as a genuine microscopic description in which macroscopic area consists of sum over microscopic areas? Wormhole throats from which flux tubes defining the links of the tensor network emerge could indeed specify the boundary of 3-D surface at microscopic level. Consider a subset A consisting of wormholes throats and the complement A^c of this set.

3. Ryu-Takayanagi formula involves a 2-surface, whose boundary is same as the boundary between A and A^c. What is essential that this surface is separating. The entanglement entropy for $A - A^c$ pair is given by the area of the separating surface and the discretized version of this can be understood quite concretely by studying the tensor network. One should identify a separating 2-surface for this set and its complement in TGD framework.

   To separate the throat from environment one must cut both the throat and flux tubes emerging from it. One could do this for all flux tubes in the set leading to complement of the set - or to environment. The entanglement entropy (or maybe negentropy for dark particles) would be proportional to the sum of the cross sectional areas divided by some unit for area. This could be the area the area of CP_2 geodesic sphere. One can argue that the cutting of monopole flux tube is not possible physically since it would create two opposite monopole charges. One can certainly imagine open strings like objects with CP_2 projection, which is homologically trivial but preferred extremal property and boundary conditions probably do not allow this. A more plausible realization would be as a pair of parallel flux tubes with opposite directions of magnetic fluxes with are identical. Reconnection for this pair would cut the flux tube pair. This kind of U-shaped flux tube pairs are central in TGD model for living matter. They act as kind of tentacles sniffing the environment, and when two flux tubes pairs of this kind meet, they can reconnect if the fluxes are identical and magnetic field strengths are suciently near to each other - the fluxes are quantized by the monopole character of flux.

This microscopic picture seems to be considerably more flexible than the picture based on the consideration of the 3-surfaces with macroscopic boundary. For instance, the entanglement negentropy of blackhole horizon (or actually the surface defining the causal horizon of any astrophysical object as surface at which the signature of the induced metric changes) with environment could be expressed as sum of cross-sectional areas of flux tubes connecting the horizon to the environment. These flux tubes would mediate gravitational interaction. It would be essential that the networks emerge at the level of magnetic body and dark matter, not the ordinary matter.

What is somewhat troubling in the construction of Preskill *et al* involving interior spins is that these should not be present in macroscopic holography. Authors suggest the interpretation of the holes of the tensor network as blackholes giving rise to horizon as new part of boundary. In TGD framework spins as physical states would always have as building bricks wormhole throats so that one would have counterparts of boundary states also now so that this problem disappears.

TGD interpretation generalizes the interpretation of Preskill *et al* by allowing single wormhole throat as a minimal blackhole like entity. In fact, the counterparts of blackholes would in TGD framework correspond to macroscopic wormhole throats. Also anyonic systems in condensed matter physics could correspond to this kind of systems elementary particles would be glued to this large boundary surface by 3-D topological condensation somewhat like various objects like plants to the Earth's surface [K10].

6.3 Summary

AdS/CFT correspondence is not essential for the realization of tensor networks. TGD based holography works equally well and the hyperbolic space H^3 emerges naturally in this framework. TGD approach allows also to get rid of various problematic aspects of AdS/CFT correspondence (artificial character of AdS, time-like closed geodesics, AdS boundary is not real) and a microscopic generalization of the Ruy-Taganaki formula is possible. Only the combinatorial structure of the tessellation and the notion of perfect tensor matter from information theoretic point of view (Ryu-Takayanagi formula however requires Einstein's equations) and this allows considerable flexibility in the realization of tensor works. There are many ways to induce the tessellation from M^4 since boundary tiles need not be complete. Holography might allow much more general representations than ideal tessellations since all that matters is the topological structure of the graph involved and that the tensors used as building bricks are perfect. The maps expressing holography can be also composed from a large variety of different perfect tensors.

The tensor networks might be realized in TGD inspired quantum biology and rely on NE assignable to discrete dark matter degrees of freedom. I have already earlier considered the hypothesis that the coordinate grid like structure formed from flux tubes could define kind of template for the self-organization of biosystem in 4-D sense implied by ZEO. Quantum complexity could force a generalization of condensed matter physics to that associated with tessellations of H^3, the number of which is infinite!

There are many interesting questions not discussed. Quantum TGD can be regarded as "complex square root" of thermodynamics. What can one say about the first law of this quantum thermodynamics? What about the analog of the first law for the area law?

7 Tensor Networks and S-matrices

The concrete construction of scattering amplitudes has been the toughest challenge of TGD and the slow progress has occurred by identification of general principles with many side tracks. One of the key problems has been unitarity. The intuitive expectation is that unitarity should reduce to a local notion somewhat like classical field equations reduce the time evolution to a local variational principle. The presence of propagators have been however the obstacle for locally realized unitarity in which each vertex would correspond to unitary map in some sense.

TGD suggests two approaches to the construction of S-matrix.

1. The first approach is generalization of twistor program [K13]. What is new is that one does not sum over diagrams but there is a large number of equivalent diagrams giving the same outcome. The complexity of the scattering amplitude is characterized by the minimal diagram. Diagrams correspond to space-time surfaces so that several space-time surfaces give rise to the same scattering amplitude. This would correspond to the fact that the dynamics breaks classical determinism. Also quantum criticality is expected to be accompanied by quantum critical fluctuations breaking classical determinism. The strong form of holography would not be unique: there would be several space-time surfaces assignable as preferred extremals to given string world sheets and partonic 2-surfaces defining "space-time genes".

2. Second approach relies on the number theoretic vision and interprets scattering amplitudes as representations for computations with each 3-vertex identifiable as a basic algebraic operation [K13]. There is an infinite number of equivalent computations connecting the set of initial algebraic objects to the set of final algebraic objects. There is a huge symmetry involved: one can eliminate all loops moving the end of line so that it transforms to a vacuum tadpole and can be snipped away. A braided tree diagram is left with braiding meaning that the fermion lines inside the line defined by light-like orbit are braided. This kind of braiding can occur also for space-like fermion lines inside magnetic flux tubes and defining correlate for entanglement. Braiding is the TGD counterpart for the problematic non-planarity in twistor approach.

Third approach involving local unitary as an additional key element is suggested by tensor networks relying on the notion of perfect entanglement discussed by Preskill *et al* [B4].

1. Tensor networks provide an elegant representation of holography mapping interior states isometrically (in Hilbert space sense) to boundary states or vice versa for selected subsets of states defining the code subspace for holographic quantum error correcting code. Again the tensor net is highly non-unique but there is some minimal tensor net characterizing the complexity of the entangled boundary state.

2. Tensor networks have two key properties, which might be abstracted and applied to the construction of S-matrix in zero energy ontology (ZEO): perfect tensors define isometry for any subspace defined by the index subset of perfect tensor to its complement and the non-unique graph representing the network. As far as the construction of Hilbert space isometry

between local interior states and highly non-local entangled boundary states is considered, these properties are enough.

One cannot avoid the question whether these three constructions could be different aspects of one and same construction and that tensor net construction with perfect tensors representing vertices could provide and additional strong constraint to the long sought for explicit recipe for the construction of scattering amplitudes.

7.1 Objections

It is certainly clear from the beginning that the possibly existing description of S-matrix in terms of tensor networks cannot correspond to the perturbative QFT description in terms of Feynman diagrams.

1. Tensor network description relates interior and boundary degrees in holography by a isometry. Now however unitary matrix has quite different role. It could correspond to U-matrix relating zero energy states to each other or to the S-matrix relating to each other the states at boundary of CD and at the shifted boundary obtained by scaling. These scalings shifting the second boundary of CD and increasing the distance between the tips of CD define the analog of unitary time evolution in ZEO. The U-matrix for transitions associated with the state function reductions at fixed boundary of CD effectively reduces to S-matrix since the other boundary of CD is not affected.

   The only manner one could see this as holography type description would be in terms of ZEO in which zero energy states are at boundaries of CD and U-matrix is a representation for them in terms of holography involving the interior states representing scattering diagram in generalized sense.

2. The appearance of small gauge coupling constant tells that the entanglement between "states" in state spaces whose coordinates formally correspond to quantum fields is weak and just opposite to that defined by a perfect tensor. Quite generally, coupling constant might be the fatal aspect of the vertices preventing the formulation in terms of perfect entanglement.

   One should understand how coupling constant emerges from this kind of description - or disappears from standard QFT description. One can think of including the coupling constant to the definition of gauge potentails: in TGD framework this is indeed true for induced gauge fields. There is no sensical manner to bring in the classical coupling constants in the classical framework and the inverse of Kähler coupling strength appears only as multiplier of the Kähler action analogous to critical temperature.

   More concretely, there are WCW spin degrees of freedom (fermionic degrees of freedom) and WCW orbital degrees of freedom involving functional integral over WCW. Fermionic contribution would not involve coupling constants whereas the functional integral over WCW involving exponential of vacuum functional could give rise to the coupling constants assignable to the vertices in the minimal tree diagram.

3. The decomposition $S = 1 + iT$ of unitary S-matrix giving unitarity as the condition $-i(T - T^\dagger) + T^\dagger T = 0$ reflects the perturbative thinking. If one has only isometry instead of unitary transformation, this decomposition becomes problematic since T and $T^\dagger$ whose some appears in the formula act in different spaces. One should have the generalization of Id as a "trivial" isometry. Alternatively, one should be able to extend the state space H_{in} by adding a tensor factor mapped trivially in isometry.

4. There are 3- and 4-vertices rather than only -say, 3-vertices as in tensor networks. For non-Abelian Chern-Simons term for simple Lie group one would have besides kinetic term only 3-vertex $Tr(A \wedge A \wedge A)$ defining the analog of perfect tensor entanglement when interpreted as co-product involving 3-D permutation symbol and structure constants of Lie algebra. Note also that for twistor Grassmannian approach the fundamental vertices are 3-vertices. It must be however emphasized that QFT description emerges from TGD only at the limit when one identifies gauge potentials as sums of induced gauge potentials assignable to the space-time sheets, which are replaced with single piece of Minkowski space.

5. Tensor network description does not contain propagators since the contractions are between perfect tensors. It is to make sense propagators must be eliminated. The twistorial factorization of massless fermion propagator suggest that this might be possible by absorbing the twistors to the vertices.

These reasons make it clear that the proposed idea is just a speculative question. Perhaps the best strategy is to look this crazy idea from different view points: the overly optimistic view developing big picture and the approach trying to debunk the idea.

7.2 The overly optimistic vision

With these prerequisites on one can follow the optimistic strategy and ask how tensor networks could the allow to generalize the notion of unitary S-matrix in TGD framework.

1. Tensor networks suggests the replacement of unitary correspondence with the more general notion of Hilbert space isometry. This generalization is very natural in TGD since one must allow phase transitions increasing the state space and it is quite possible that S-matrix represents only isometry: this would mean that $S^\dagger S = Id_{in}$ holds true but $SS^\dagger = Id_{out}$ does not even make sense. This conforms with the idea that state function reduction sequences at fixed boundary of causal diamonds defining conscious entities give rise evolution implying that the size of the state space increases gradually as the system becomes more complex. Note that this gives rise to irreversibility understandandable in terms of NMP [K8]. It might be even impossible to formally restore unitary by introducing formal additional tensor factor to the space of incoming states if the isometric map of the incoming state space to outgoing state space is inclusion of hyperfinite factors.

2. If the huge generalization of the duality of old fashioned string models makes sense, the minimal diagram representing scattering is expected to be a tree diagram with braiding and should allow a representation as a tensor network. The generalization of the tensor network concept to include braiding is trivial in principle: assign to the legs connecting the nodes defined by perfect tensors unitary matrices representing the braiding - here topological QFT allows realization of the unitary matrix. Besides fermionic degrees of freedom having interpretation as spin degrees of freedom at the level of "World of Classical Worlds" (WCW) there are also WCW orbital degrees of freedom. These two degrees of freedom factorize in the generalized unitarity conditions and the description seems much simpler in WCW orbital degrees of freedom than in WCW spin degrees of freedom.

3. Concerning the concrete construction there are two levels involved, which are analogous to descriptions in terms of boundary and interior degrees of freedom in holography. The level of fundamental fermions assignable to string world sheets and their boundaries and the level of physical particles with particles assigned to sets of partonic 2-surface connected by magnetic flux tubes and associated fermionic strings. One could also see the ends of causal diamonds as analogous to boundary degrees of freedom and the space-time surface as interior degrees of freedom.

The description at the level of fundamental fermions corresponds to conformal field theory at string world sheets.

1. The construction of the analogs of boundary states reduces to the construction of N-point functions for fundamental fermions assignable to the boundaries of string world sheets. These boundaries reside at 3-surfaces at the space-like space-time ends at CDs and at light-like 3-surfaces at which the signature of the induced space-time metric changes.

2. In accordance with holography, the fermionic N-point functions with points at partonic 2-surfaces at the ends of CD are those assignable to a conformal field theory associated with the union of string world sheets involved. The perfect tensor is assignable to the fundamental 4-fermion scattering which defines the microscopy for the geometric 3-particle vertices having twistorial interpretation and also interpretation as algebraic operation.

What is important is that fundamental fermion modes at string world sheets are labelled by conformal weights and standard model quantum numbers. No four-momenta nor color quantum numbers are involved at this level. Instead of propagator one has just unitary matrix describing the braiding.

3. Note that four-momenta emerging in somewhat mysterious manner to stringy scattering amplitudes and mean the possibility to interpret the amplitudes at the particle level.

Twistorial and number theoretic constructions should correspond to particle level construction and also now tensor network description might work.

1. The 3-surfaces are labelled by four-momenta besides other standard model quantum numbers but the possibility of reducing diagram to that involving only 3-vertices means that momentum degrees of freedom effectively disappear. In ordinary twistor approach this would mean allowance of only forward scattering unless one allows massless but complex virtual momenta in twistor diagrams. Also vertices with larger number of legs are possible by organizing large blocks of vertices to single effective vertex and would allow descriptions analogous to effective QFTs.

2. It is highly non-trivial that the crucial factorization to perfect tensors at 3-vertices with unitary braiding matrices associated with legs connecting them occurs also now. It allows to split the inverses of fermion propagators into sum of products of two parts and absorb the halves to the perfect tensors at the ends of the line. The reason is that the inverse of massless fermion propagator (also when masslessness is understood in 8-D sense allowing M^4 mass to be non-vanishing) to be express as bilinear of the bi-spinors defining the twistor representing the four-momentum. It seems that this is absolutely crucial property and fails for massive (in 8-D sense) fermions.

7.3 Twistorial and number theoretic visions

Both twistorial and number theoretical ideas have given a strong boost to the development of ideas.

1. With experience coming from twistor Grassmannian approach, twistor approach is conjectured to allow an extension of super-symplectic and other superconformal symmetry algebras to Yangian algebras by adding a hierarchy of multilocal generators [K13]. The twistorial diagrams for $\mathcal{N} = 4$ SUSY can be reduced to a finite number and there is large number of equivalent diagrams. One expects that this is true also in TGD framework.

   Twistorial approach is extremely general and quite too demanding to my technical skills but its is a useful guideline. An important outcome of twistor approach is that the intermediate states are massless on-mass-shell states but with complex momenta. Does this generalize and could each vertex define unitary scattering event with complex four-momenta in possibly complexified Minkowski space? Or could even real momenta be possible for massive particles, which would be massless in 8-D sense thanks to the existence of octonionic tangent space structure of 8-D embedding space? And what is the role of the unique twistorial properties of M^4 and CP_2?

2. Number theoretical vision suggests that the scattering amplitudes correspond to sequences of algebraic operations taking inputs and producing outputs, which in turn serve as inputs for a neighboring node [K13]. The vertices form a diagram defining a network like structure defining kind of distributed computations leading from given inputs to given outputs. A computation leading from given inputs to given outputs is suggestive. There exists an infinite number of this kind of computations and there must be the minimal one which defines the complexity of the scattering. The maximally simplifying guess is that this diagram would correspond to a braided tree diagram. At space-time level these diagrams would correspond to different space-time surfaces defining same physics: this is because of holography meaning that only the ends of space-time surfaces at boundaries of CD matter.

   This vision generalizes of the old-fashioned stringy duality. It states that all diagrams can be reduced to minimal diagrams. This is achieved by by moving the ends of internal lines so that loops becomes vacuum tadpoles and can be snipped off. Tree diagrams must be however allowed to braid and outside the vertices the diagrams look like braids. Braids for which threads can split and glue together is the proper description for what the diagrams could be. Braiding would provide the counterpart for the non-planar twistor diagrams.

The fermion lines inside the light-like 3-surfaces can get braided. Smaller partonic 2-surfaces can topologically condense at given bigger partonic 2-surface (electronic parton surface can topologically condense to nano-scopic parton surface) and the orbits of the condensed partonic 2-surfaces at the light-like orbit of the parton surface can get braided. This gives rise to a hierarchy of braids with braids.

7.4 Generalization of the notion of unitarity

The understanding of unitarity has been the most difficult issue in my attempts to understand S-matrix in TGD framework. When something turns out to be very difficult to understand, it might make sense to ask whether the definition of this something involves un-necessary assumptions. Could unitarity be this kind of notion?

The notion of tensor network suggests that unitarity can generalized and that this generalization allows the realization of unitarity in extremely simple manner using perfect tensors as building bricks of diagrams.

1. Both twistorial and number theoretical approaches define M-matrix and associated S-matrix as a map between the state spaces H_{in} and H_{out} assignable to the opposite boundaries of CD - say positive and negative energy parts of zero energy state. In QFT one has $H_{in} = H_{out}$ and the map would be Hilbert space unitary transformation satisfying $SS^\dagger = S^\dagger S = Id$.

2. The basic structure of TGD (NMP favoring generation of negentropic entanglement, the hierarchy of Planck constants, length scale hierarchies, and hierarchy of space-time sheets) suggests that the time evolution leads to an increasingly complex systems with higher-dimensional Hilbert space so that $H_{in} = H_{out}$ need not hold true but is replaced with $H_{in} \subset H_{out}$. This view is very natural since one must allow quantum phase transitions increasing the value of h_{eff} and the value of p-adic prime defining p-adic length scale.

   S-matrix would thus define isometric map $H_{in} \subset H_{out}$. Isometry property requires $U^\dagger U = Id_{in}$. If the inclusion of H_{in} to H_{out} is a genuine subspace of H_{out}, the condition $UU^\dagger = Id_{out}$ does not make sense anymore. This means breaking of reversibility and is indeed implied by the quantum measurement theory based on ZEO.

3. It would be at least formally possible to fuse all state spaces to single very large state space by replacing isometry $H_{in} \subset H_{out}$ with unitary map $H_{out} \to H_{out}$ by adding a tensor factor in which the map acts as identity transformation. This is not practical since huge amounts of redundant information would be introduced. Also the information about hierarchical structure essential for the idea of evolution would be lost. This hierarchical of inclusions should also be crucial for understanding the construction of S-matrix or rather, the hierarchy of S-matrices of isometric inclusions including as a special case unitary S-matrices.

4. There is also a further intricacy, which might prevent the formal unitarization by the addition of an inert tensor factor. I have talked a lot about HFFs referring to hyper-finite factors of type II_1 (possibly also of type III_1) and their inclusions [K15]. The reason is that WCW spinors form a canonical representation for these von Neumann algebras.

   Could the isometries replacing unitary S-matrix correspond to inclusions of HFFs? In the recent interpretation the included factor (now H_{in}) corresponds to the degrees of freedom below measurement resolution. Certainly this does not make sense now. The interpretation in terms of finite measurement resolution need not however be the only possible interpretation and the interpretation in terms of measurement resolution might of course be wrong. Therefore one can ask whether the relation between H_{in} and H_{out} could be more complex than just $H_{out} = H_{in} \otimes H_1$ so that formal unitarization would fail.

7.5 Scattering diagrams as tensor networks constructed from perfect tensors

Preskill's tensor network construction [B4] realizes isometric maps as representations of holography and as models for quantum error correcting codes. These tensor networks have remarkable similarities with twistorial and number theoretical visions, which suggests that it could be used to construct scattering amplitudes. A further idea inspired by holography is that the description of scattering amplitudes in terms of fundamental fermions and physical particles are dual to each other.

1. In the construction of quantum error codes tensor network defines an isometric embedding of local states in the interior to strongly entangled non-local states at boundary. Their vertices correspond to tensors, which in the proposal of Preskill *et al* [B4] are perfect tensors such that one can take any m legs of the vertex and the tensor defines isometry from the state space of m legs to that of $n-m$ legs. When the number of indices is $2n$, the entanglement defined by perfect tensor between any n-dimensional subspace and its complement is maximal

   TGD framework maximal entanglement corresponds to negentropic entanglement with density matrix proportional to identity matrix. What is important that the isometry is constructed by composing local isometries associated with a network. Given isometry can be constructed in very many ways but there is some minimal realization.

2. The tensor networks considered in [B4] are very special since they are determined by tessellations of hyperbolic space H_2. This kind of tessellations of H_3 could be crucial for understanding the analog of condensed matter physics for dark matter and could appear in biology [K6]. What is crucial is that only the graph property and perfect tensor property matter as far as isometricity is considered so that it is possible to construct very general isometries by using tensor networks.

7.6 Eigenstates of Yangian co-algebra generators as a way to generate maximal entanglement?

Negentropically entangled objects are key entities in TGD inspired theory of consciousness and also of tensor networks, and the challenge is to understand how these could be constructed and what their properties could be. These states are diametrically opposite to unentangled eigenstates of single particle operators, usually elements of Cartan algebra of symmetry group. The entangled states should result as eigenstates of poly-local operators. Yangian algebras involve a hierarchy of poly-local operators, and twistorial considerations inspire the conjecture that Yangian counterparts of super-symplectic and other algebras made poly-local with respect to partonic 2-surfaces or end-points of boundaries of string world sheet at them are symmetries of quantum TGD [K5]. Could Yangians allow to understand maximal entanglement in terms of symmetries?

1. In this respect the construction of maximally entangled states using bi-local operator $Q^z = J_x \otimes J_y - J_x \otimes J_y$ is highly interesting since entangled states would result by state function. Single particle operator like J_z would generate un-entangled states. The states obtained as eigenstates of this operator have permutation symmetries. The operator can be expressed as $Q^z = f^z_{ij} J^i \otimes J^j$, where f^A_{BC} are structure constants of SU(2) and could be interpreted as co-product associated with the Lie algebra generator J^z. Thus it would seem that unentangled states correspond to eigenstates of J^z and the maximally entangled state to eigenstates of co-generator Q^z. Kind of duality would be in question.

2. Could one generalize this construction to n-fold tensor products? What about other representations of SU(2)? Could one generalize from SU(2) to arbitrary Lie algebra by replacing Cartan generators with suitably defined co-generators and spin 1/2 representation with fundamental representation? The optimistic guess would be that the resulting states are maximally entangled and excellent candidates for states for which negentropic entanglement is maximized by NMP [K8].

3. Co-product is needed and there exists a rich spectrum of algebras with co-product (quantum groups, bialgebras, Hopf algebras, Yangian algebras). In particular, Yangians of Lie

algebras are generated by ordinary Lie algebra generators and their co-generators subject to constraints. The outcome is an infinite-dimensional algebra analogous to one half of Kac-Moody algebra with the analog of conformal weight N counting the number of tensor factors. Witten gives a nice concrete explanation of Yangian [B2] for which co-generators of T^A are given as $Q^A = \sum_{i<j} f^A_{BC} T^B_i \otimes T^C_j$, where the summation is over discrete ordered points, which could now label partonic 2-surfaces or points of them or points of string like object (see `http://tinyurl.com/y727n8ua`). For a practically totally incomprehensible description of Yangian one can look at the Wikipedia article (see `http://tinyurl.com/y7heufjh`).

4. This would suggest that the eigenstates of Cartan algebra co-generators of Yangian could define an eigen basis of Yangian algebra dual to the basis defined by the totally unentangled eigenstates of generators and that the quantum measurement of poly-local observables defined by co-generators creates entangled and perhaps even maximally entangled states. A duality between totally unentangled and completely entangled situations is suggestive and analogous to that encountered in twistor Grassmann approach where conformal symmetry and its dual are involved. A beautiful connection between generalization of Lie algebras, quantum measurement theory and quantum information theory would emerge.

7.7 Two different tensor network descriptions

The obvious question is whether also unitary S-matrix of TGD could be constructed using tensor network built from perfect tensors. In ZEO the role of boundary would be taken by the ends of the space-time at upper and lower light-like boundaries of CD carrying the particles characterized by standard model quantum numbers. Strong form of holography would suggest that partonic surfaces and strings at the ends of CD provide information for the description of zero energy states and therefore of scattering amplitudes. The role of interior would be taken by the space-time surface - in particular the light-like orbits of partonic surfaces carrying the fermion lines identified as boundaries of string world sheets. Conformal field theory description would apply to fermions residing at string world sheets with boundaries at light-like orbits of partonic 2-surfaces.

In QFT Feynman diagrammatics one obtains a sum over diagrams with arbitrary numbers of loops. In both twistorial and number theoretic approach however only a finite number of diagrams with possibly complex on mass shell massless momenta are needed. If the vertices are however such that particles remain on-mass-shell but are allowed to have complex four-momenta then the integration over internal momenta (loops) is not present and tensor network description could make sense. This encourages the conjecture that tensor networks could be used to construct the scattering amplitudes in TGD framework.

What could perfect tensor property mean for the vertices identified as nodes of a tensor network? There are two levels to be considered: the geometric level identifying particles as 3-surfaces with net quantum numbers and the fermion level identifying particles as fundamental fermions at the boundaries of string world sheets.

1. At the geometric level vertices corresponds to light-like orbits of partonic 2-surfaces meeting at common end which is partonic 2-surface. This is 3-D generalization of Feynman diagram as a geometric entity. At the level of fermion lines associated with the light-like 3-surfaces one the basic interaction corresponds to the scattering of 2-fermions leading to re-sharing of fermion lines between outgoing light-like 3-surfaces, which include also representations for virtual particles. One has 4-fermion vertex but not in the sense that it appears in the interaction of weak interactions at low energies.

   Geometrically the basic vertex could be 3-vertex: $n > 3$-vertices are unstable against deformation to lower vertices. For 3-vertex perfect tensor property means that the tensor defining the vertex maps any 1-particle subspaces to 2-particle subspace isometrically. The geometric vertices define a network consisting of 3-D "lines" and 2-D vertices but one cannot tell what is within the 3-D lines and what happens in the 2-D nodes. The lines would consist of braided fundamental fermion lines and in nodes the basic process would be 2+2 scattering for fermions. In the case of 3-vertex momentum conservation would effectively eliminate the four-momentum and the state spaces associated with vertex would be effectively discrete. This is p-adically of utmost importance.

2. At the level of fundamental fermion lines in the interior of particle lines one would have 4-vertices and if a perfect tensor describes it, it gives rise to a unitary map of any 2-fermion subspace to its complement plus isometric maps of 1-fermion subspaces to 3-fermion subspaces. In this case momenta cannot act as labels of fermion lines for rather obvious reasons: the solution of the problem is that conformal weights label fundamental fermion lines

   The conservation of discrete quark and lepton numbers allows only vertices of type $qL \rightarrow qL$ and its variants obtained by crossing. In this case the isometries might allow realization. The isometries must be defined to take into account quark and lepton number conservation by crossing replacing fermion with antifermion. By allowing the states of Hilbert space in node to be both quarks and leptons, difficulties can be avoided.

7.7.1 Tensor network description in terms of fundamental fermions and CFT

Consider first fundamental fermions. What are the labels characterizing the states of fundamental fermions fermions propagating along the lines? There are two options: the labels are either conformal weights or four-momenta.

1. Since fermions corresponds to strings defining the boundaries of string world sheets and since strong form of holography implies effective 2-dimensionality also in fermion sector, the natural guess is that the conformal weights plus some discrete quantum numbers - standard model quantum numbers at least - are in question. The situation would be well-defined also p-adically for this option. In this case one can hope that conformal field theory at partonic 2-surface could define the fermionic 4-vertex more or less completely. There would be no need to assign propagators between different four-fermion vertices. The scattering diagram would define a composite formed from light-like 3-surfaces and one would have single isometry build from 4-fermion perfect tensors. There would be no integrations over internal momenta.

2. Second option is that fundamental fermions are labelled by four-momenta. The outgoing four-momenta in 4-vertices would not be completely fixed by the values of the incoming momenta and this extends the state space. Concerning p-adicization this integral is not desirable and this forces to consider seriously discrete labelling. The unitarity condition for 2+2 scattering would involve integral over 2-sphere. Four-fermion scattering must be unitary process in QFT so that this condition might be possible to satisfy. The problem would be how to fix this fundamental scattering matrix uniquely. This option does not look attractive number theoretically.

The most plausible option is that holography means that conformal field theory describes the scattering of fundamental fermions and QFT type description analogous to twistorial approach describes the scattering of physical fermions. If only 3-vertices are allowed, and if masslessness corresponds to masslessness in 8-D sense, one obtains non-trivial scattering vertices (for ordinary twistor approach all massless momenta would be collinear if real).

7.7.2 Tensor network description for physical particles

Could the twistorial description expected to correspond to the description in terms of particles allow tensor network description?

1. Certainly one must assign four-momenta to incoming *physical* particles - also fermions - but they correspond to pairs of wormhole contacts rather than fundamental fermions at the boundaries of string world sheets. It would be natural to assign four-momenta also to the virtual *physical* fermions appearing in the diagram and the geometric view about scattering would allow only 3-vertices so that momentum conservation would eliminate momentum degrees of freedom effectively. This would be a p-adically good news.

2. At the level of fundamental fermions entanglement is described as a tensor contraction of the CFT vertices. This locality is natural since the vertices are at null distance from each other. At QFT limit the entanglement between the ends of the line is characterized the propagator.

   One must get rid of propagators in order to have tensor network description. The inclusion of propagators to the fundamental tensor diagrams would break the symmetry between the legs of vertex since the propagator cannot be included to its both ends. Situation changes if one can represent the propagator as a bilinear of something more primitive and include the halves to the opposite ends of the line. Twistor representation of four-momentum indeed defines this kind of representation as a bilinear $p^{a\tilde{b}} = \lambda\tilde{\mu}^{\tilde{b}}$ of twistors λ and $\tilde{\mu}$. There is problem due to the diverging $1/p^2$ factor but residue integral eliminates this factor and one can write directly the fermionic propagator factors as $p^{a\tilde{b}}$.

3. In QFT description the perturbative expansion is in powers of coupling constant. If the reduction to braided tree diagrams analogous to twistor diagrams occurs, power g^{N-2} of coupling constant is expected to factorize as a multiplier of a tree diagram with N external legs. One should understand this aspect in the tensor net-work picture.

   For $\mathcal{N} = 4$ SUSY there is coupling constant renormalization. Similar prediction is expected from TGD. Coupling constant evolution is expected to be discrete and induced by the discrete evolution of Kähler coupling strength defined by the spectrum of its critical values. The conjecture is that critical values are naturally labelled by p-adic primes $p \simeq 2^k$, k prime, labelling p-adic length scales. Therefore one might hope that problems could be avoided.

These observations encourage the expectation that twistorial approach involving only 3-vertices allows to realize tensor network idea also at the level of physical particles. It might be essential that twistors can be generalized to 8-D twistors. Octonionic representation of gamma matrices might make this possible. Also the fact twistorial uniqueness of M^4 and CP_2 might be crucial.

Gauge theory follows as QFT limit of TGD so that one cannot in principle require that gauge theory vertices satisfy the isometricity conditions. Nothing however prevents from checking whether gauge theory limit might inherit this property.

1. For instance, could 3-vertices of Yang-Mills theory define isometric embedding of 1-particle states to 2 particle states? For a given gauge boson there should exist always a pair of gauge bosons, which can fuse to it. Consider a basis for Lie-algebra generators of the gauge group. If the generator T is such that there exists no pair $[A, B]$ with the property $[A, B] = T$, Jacobi identity implies that T must commute with all generators and one has direct sum of Lie algebras generated by T and remaining generators.

2. In the case of weak algebra $SU(2) \times U(1)$ the weak mixing of Y and I_3 might allow the isometric embeddings of type $1 \rightarrow 2$. Does this mean that Weinberg angle must be non-vanishing in order to have consistent theory? A realistic manner to get rid of the problem is to allow at QFT limit the lines to be also fermions so that also U(1) gauge boson can be constructed as fermion pair.

7.7.3 How the two tensor network descriptions would be related?

There are two descriptions for the zero energy states providing representation of scattering amplitudes: the CFT description in terms of fundamental fermions at the boundaries of string world sheets, and the description in terms of physical particles to which one can assign light-like 3-surfaces as virtual lines and total quantum numbers.

1. CFT description in terms of fundamental fermions in some aspects very simple because of its 2-dimensionality and conformal invariance. The description is in terms of physical particles having light-like 3-surfaces carrying some total quantum numbers as correlates and is simpler in different sense. These descriptions should be related by an Hilbert space isometry.

2. The perfect tensor property for 4-fermion vertices makes fundamental fermion states analogous to physical states realizing logical qubits as highly entangled structures. Geometric description in terms of 3-surfaces is in turn analogous to the description in terms of logical qubits.

3. Holography-like correspondence between these descriptions of zero energy states (scattering diagrams) should exist. Physical particles should correspond to the level, at which resolution is smaller and which should be isometrically mapped to the strongly entangled level defined by

fundamental fermions and analogous to boundary degrees of freedom (fundamental fermions *are* at the boundaries of string world sheets!).

The map relating the two descriptions seems to exist. One can assign four-momenta to the legs of conformal four-point function as parameters so that one obtains a mapping from the states labelled by conformal weights to the states labelled by four-momenta! The appearance of 4-momenta from conformal theory is somewhat mysterious looking phenomenon but this duality makes it rather natural.

7.8 Taking into account braiding and WCW degrees of freedom

One must also take intro account braiding and orbital degrees of freedom of WCW. The generalization of tensor network to braided tensor network is trivial. Thanks to the properties of tensor network orbital and spinor degrees of freedom factorize so that also the treatment of WCW degrees of freedom seems to be possible.

7.8.1 What about braiding?

The scattering diagrams would be tree diagrams with braiding of fermionic lines along light-like 3-surfaces - dance of fundamental quarks and leptons at parquette defined by the partonic 2-surface one might say. Also space-like braiding at magnetic flux tubes at the ends of CD is possible and its time evolution between the ends of space-time surfaces defines 2-braiding which is generalization of the ordinary braiding but will not be discussed here. This gives rise to a hierarchy of braidings. One can talk about flux tubes within flux tubes and about light-like 3-surface within light-like 3-surfaces. The smaller light-like 3-surface would be glued by a wormhole contact to the larger one and contact could have Euclidian signature of induced metric.

How can one treat the braiding in the tensor network picture? The answer is simple. Braiding corresponds to an element of braid group and one can represent it by a unitary matrix as one does in topological QFT as one constructs knot invariants. In particular, the trace of this unitary matrix defines a knot invariant. The generalization of the tensor network is simple. One attaches to the links connecting two nodes unitary transformation defining a representation of the braid involved. Local variant of unitarity would mean isometricity at nodes and unitarity at links.

7.8.2 What about WCW degrees of freedom?

The above considerations are about fermions that its WCW spinor degrees of freedom and the space-time surface itself has been regarded as a fixed background. How can one take into account WCW degrees of freedom?

The scattering amplitude involves a functional integral over the 3-surfaces at the ends of CD. The functional integration over WCW degrees of freedom gives an expression depending on Kähler coupling strength α_K and determines the dependence on various gauge coupling strengths expressible in terms of α_K. This makes it possible to have the tensor network description in fermionic degrees of freedom without losing completely the dependence of the scattering amplitudes on gauge couplings. By strong form of holography the functional integral should reduce to that over partonic 2-surfaces and strings connecting them. Number theoretic discretization with a cutoff determined by measurement resolution forces the parameters characterizing the 2-surfaces to belong to an algebraic extension of rationals and is expected to reduce functional integral to a sum over discretized WCW so that it makes sense also in p-adic sectors [K12, K14].

A brief summary of quantum measurement theory in ZEO is necessary. The repeated state function reduction shifts active boundary A of CD and affects the states at it. The passive boundary of CD- call it P - and the states at it - remain unaffected. The repeated state function reductions leaving P unaffected and giving usually rise to Zeno effect, correspond now to the TGD counterpart of unitary time evolution by shifts between subsequent state function reductions. Call A and its shifted version A_{in} and A_{out} and the corresponding state spaces H_{in} and H_{out}. The unitary (or more generally isometric) S matrix represents this shift. This is the TGD counterpart of a unitary evolution of QFTs. S forms a building brick of a more general unitary matrix U acting in the space of zero energy states but U is not considered now.

Consider now the isometricity conditions.

1. Unitarity conditions generalized to isometricity conditions apply to S. Isometricity conditions $S^\dagger S = Id_{in}$ can be applied at A_{in}. The states appearing in the isometry conditions as initial and final states correspond to A_{in} and A_{out}. There is a trace over WCW spin indices (labels for many-fermion states) of H_{out} in the conditions $S^\dagger S = Id_{in}$. Isometricity conditions involve also an integral over WCW orbital degrees of freedom at both ends: these degrees of freedom are strongly correlated and for a strict classical determinism the correlation between the ends is complete. If the tensor network idea works, the summation over spinor degrees of freedom at A_{out} gives just a unit matrix in the spinor indices at A_{in} and leaves only the WCW orbital degrees of freedom in consideration. This factorization of spinor and orbital WCW degrees of freedom simplifies the situation dramatically.

2. One can express isometricity conditions for modes with $\Psi_{in,M}$ and $\Psi_{out,N}$ at A_{in} and A_{out}: this requires functional integration over 3-surfaces WCW at A_{in} and A_{out}. The conditions are formulated in terms of the labels - call them M_{in}, N_{in} - of WCW spinor modes at A_{in} including standard model quantum numbers and labels characterizing the states of supersymplectic and super-conformal representations. The trace is over the corresponding indices R_{out} at A_{out}. The WCW functional integrals in the generalized unitarity conditions are therefore over A_{in} and A_{out} and should give Kronecker delta $\sum_{R_{out}} S^\dagger_{M_{in}R_{out}} S_{R_{out}N_{in}} = \delta_{M_{in},N_{in}}$.

3. The simplest view would be that Kähler action with boundary conditions implies completely deterministic dynamics. The conditions expressing strong form of holography state that sub-algebras of super-symplectic algebra and related conformal algebras isomorphic to the entire algebra give rise to vanishing Noether charges. Suppose that these conditions posed at the ends of CD are so strong that they fix the time evolution of the space-time surface as preferred extremal completely when posed at either boundary. In this case the isometricity conditions would be so strong that the double functional integration appearing in the matrix product reduces to that at A_{in} and the isometricity conditions would state just the orthonormality of the basis of WCW spinor modes at A_{in}.

4. Quantum criticality and in particular, the hierarchy of Planck constants providing a geometric description for non-deterministic long range fluctuations, does not support this view. Also the fact that string world sheets connect the boundaries of CD suggests that determinism must be broken. The inner product defining the completeness of the WCW state basis in orbital degrees of freedom can be however generalized to a bi-local inner product involving functional integration over 3-surfaces at both A_{in} and A_{out}. There is however a very strong correlation so that integration volume at A_{out} is expected to be small. This also suggests that one can have only isometricity conditions.

7.9 How do the gauge couplings appear in the vertices?

Reader is probably still confused and wondering how the gauge couplings appear in the vertices from the functional integral over WCW degrees of freedom. In twistorial approach, the vanishing of loops in $\mathcal{N} = 4$ SYM theory gives just g^N, N the number of 3-vertices. Each vertex should give gauge coupling. Or equivalently, each propagator line connecting vertices should give α_K. The functional integral should give this factor for each propagator line. Generalization of conformal invariance is expected to give this picture.

To proceed some basic facts about N-point functions of CFTs are needed.

1. In conformal field theory the functional form of two-point function is completely fixed by conformal symmetry:

$$\begin{aligned} G^{(2)}(z_i, \overline{z}_i) &= \frac{C_{12}}{z_{12}^{2h}\overline{z}_{12}^{2\overline{h}}} \ , \\ z_{ij} &= z_i - z_j \ , \ \overline{z}_{ij} = \overline{z}_i - \overline{z}_j \ , \\ h_1 = h_2 = h &= h_a + ih_b \ , \ \overline{h} = \overline{h}_a + i\overline{h}_b \ . \end{aligned} \tag{7.1}$$

$h_1 = h_2 \equiv h$ and its conjugate $\overline{h}$ are conformal weights of conformal field and its conjugate. Note that the conformal weights of conformal fields Φ_1 and Φ_2 must be same. In TGD context C_{12} is expected to be proportional to α_K and this would give to each vertex g_K when couplings are absorbed into vertices.

2. The 3-point function for 3 conformal fields Φ_i, $i = 1, 2, 3$ is dictated by conformal symmetries apart from constant C_{123}:

$$G^{(3)}(z_i, \overline{z}_i) = C_{123} \times \frac{1}{z_{12}^{h_1+h_2-h_3} z_{23}^{h_2+h_3-h_1} z_{31}^{h_3+h_1-h_2}} \times \frac{1}{\overline{z}_{12}^{\overline{h}_1+\overline{h}_2-\overline{h}_3} \overline{z}_{23}^{\overline{h}_2+\overline{h}_3-\overline{h}_1} \overline{z}_{31}^{\overline{h}_3+\overline{h}_1-\overline{h}_2}} . \quad (7.2)$$

Here C_{123} should bef fixed by super-symplectic and related symmetries and determined the numerical coefficients various couplings when expressed in terms of g_K.

3. 4-point functions have analogous form

$$\begin{aligned} G^{(4)}(z_i, \overline{z}_i) &= f_{1234}(x, \overline{x}) \prod_{i<j} z_{ij}^{-(h_i+h_j)+h/3} \prod_{i<j} \overline{z}_{ij}^{-(\overline{h}_i+\overline{h}_j)+\overline{h}/3} , \\ h &= \sum_i h_i , \end{aligned} \quad (7.3)$$

but are proportional to an arbitrary function f_{1234} of conformal invariant $x = z_{12}z_{34}/z_{13}z_{24}$ and its conjugate.

If only 3-vertices appear/are needed for physical particles - as both twistorial and number theoretic approaches strongly suggest - the conformal propagators and vertices are fixed apart from constants C_{ijk}, which in turn should be fixed by the huge generalization of conformal symmetries. α_K emerges in the expected manner.

This picture seems to follow from first principles.

1. One can fix the partonic 2-surfaces at the boundaries of CD but there is a functional integral over partonic 2-surfaces defining the vertices: their deformations induce deformations of the legs. One can expand the exponent of Kähler action and in the lowest order the perturbation term is trilinear and non-local in the perturbations. This gives rise to 3-point function of CFT nonlocal in z_i. The functional integral over perturbations gives the propagators in legs proportional to α_K in terms of two point function of CFT. Note that the external propagator legs can be eliminated in S-matrix.

2. The cancellation of higher order perturbative corrections in WCW functional integral is required by the quantum criticality and means trivial coupling constant evolution for α_K and other coupling constants. Coupling constant evolution is discretized with values of α_K analogous to critical temperatures and should correspond to p-adic coupling constant evolution [K3].

3. This picture leaves a lot of details open. An integration over the values of z_i is needed and means a kind of Fourier analysis leading from complex domain. The analog of Fourier analysis would be for deformations of partonic 2-surface labelled by some natural labels. Conformal weights could be natural labels of this kind.

It is easy to get confused since there are several diagrammatics involved: the topological diagrammatics of 3-surfacse assignable to the physical particles with partonic 2-surfaces as vertices, the diagrammatics associated with the perturbative functional integral for the Kähler action, and the fermionic diagrammatics suggested to reduce to tensor network. The conjectures are as follows.

1. The "primary" vertices $G^{(n)}$, $n > 3$ assignable to single partonic 2-surface and coming from a functional integral for Kähler action vanishes. This corresponds to quantum criticality and trivial RG evolution.

2. $G^{(n)}$, $n > 3$ in the sense of topological diagrammatics without loops and involving n partonic 2-surfaces do not vanish. One can construct the analog of $G^{(4)}$ from two $G^{(3)}$:s at different partonic 2-surfaces and propagator defined by 2-point function connecting them as string diagram.

   Also topological variant of $G^{(4)}$ assignable to single partonic 2-surface can be constructed by allowing the 3-D propagator "line" to return back to the partonic 2-surface. This would correspond to an analog of loop. Similar construction applies to "primary" $G^{(n)}$,$n > 4$. In number theoretic vision these loops are eliminated as redundant representations so that one has only braided tree diagrams. Also twistor Grassmann approach supports this view.

To sum up, the tensor network description would apply to fermionic degrees of freedom. In bosonic degrees of freedom functional integral would give CFT picture with 3-vertex as the only "primary" vertex and from this twistorial and number theoretic visions follow via the super-symplectic symmetries of the vertex coefficients C_{ijk} extended to Yangian symmetries.

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