Beltrami flows, integrable flows and holography = holomorphy hypothesis

November 25, 2025

Matti Pitkänen

orcid:0000-0002-8051-4364.
email: matpitka6@gmail.com,
url: `http://tgdtheory.com/public_html/`,
address: Valtatie 8, as. 2, 03600, Karkkila, Finland.

Contents

Abstract

The field equations of TGD reduce to conservation laws for isometry charges so that TGD is analogous to hydrodynamics. Beltrami flows are indeed a basic aspect of TGD and it is interesting to relate them to the new visions of TGD, in particular the holography = holomorphy principle. The outcome of these considerations is that integrable flows, having interpretation as irrotational and incompressible Beltrami flows, have in TGD an interpretation as generalized complex flows reducing to gradient flows except at singularities. These flows definable for the generalized complex structure of $H = M^4 \times CP_2$ and $X^4 \subset H$ have a natural identification as hydrodynamical flows in the induced Kähler field.

These flows have as singularities are 2-D partonic surfaces and string world sheets. In the view of scattering amplitudes, vertices correspond to partonic 2-surfaces at which the lines of the analogs of Feynman diagrams meet each other whereas string world sheets as intersections of two 4-surfaces with common Hamilton-Jacobi structure characterize interactions as contact interactions.

1 Introduction

Beltrami flows [B1, B5, B3, B4, B2] (see this) appear in several contexts. Google AI informs that Beltrami flow is a force-free flow field at 3-sphere. The simplest Hopf fibration (see this) is from 3-sphere to 2-sphere and fibers correspond to circles and there are numerous generations of Hopf fibration: the fibration $S^5 \to CP_2$ is of special interest in TGD.

1.1 Some background

Some background about Beltrami flows is in order.

1. For the Beltrami flow (see this) velocity field satisfies $curl(v) = \Lambda v$ so that $curl(v)$ is parallel to v. In fluid dynamics Beltrami flow corresponds to a flow for which vorticity $\omega = \nabla \times v$ and velocity v are parallel. $\omega \times v = 0$ gives $\omega = \nabla \times v = \alpha(x,t)v$. Beltrami flows in S^3 satisfy this condition and are exact solutions to Euler equations.

2. In magnetohydrodynamics one can replace velocity field with magnetic field B and of the current j satisfies $j = \nabla \times B = \alpha B$ implying the vanishing of the Lorentz force $j \times B$. The current flows along field lines and in TGD the flow of particles along monopole flux tubes is the counterpart for this flow. These Beltrami flows involve the linking and knotting of magnetic field lines. Similar situation prevails in hydrodynamics.

Jenny Lorraine Nielsen has proposed that the Hopf fibration $S^1 \to S^9 \to CP_4$ could provide a theory of everything (see this) and that Beltrami flows (see this) associated with this kind of fibrations play a key role in physics. The scalar Λ, which depends on position, appearing in the definition of Beltrami flow has dimensions of $1/length$. Mass has dimension of $\hbar/length$ so that $1/\lambda$ should be identified as an analog of Compton length. These flows are topologically very interesting and involve linking and knotting of the flow lines.

The claim of Jenny Nielsen is that it is possible to understand particle massivation in terms of Beltrami flows. Higgs expectation defining the mass spectrum in the standard model is identified as $hbar\Lambda$ for the eigenvalue Λ of the lowest eigenmode of Beltrami flow. It would seem that Λ is assumed to be constant: this is not necessary. It must be possible to relate Λ to the radius of S^3 and one chooses it suitably to get Higgs vacuum expectation. To get masses of fermions one must put them in by hand as couplings of fermions to Higgs so that one does not really predict fermion masses: to my best understanding, the situation remains the same as in the standard model. TGD leads to a predictive model for the masses of elementary fermion [K4, K3] [L2] allowing also to predict hadron masses [L9].

1.2 Motivations for considering Beltrami flows and integrable flows from the TGD point of view

The field equations of TGD reduce to conservation laws for isometry charges so that TGD is analogous to hydrodynamics. Beltrami flows, generalized to 4-D situation, are indeed a basic aspect of TGD [K2, K1] and it is interesting to try to relate them to the new vision of TGD, in particular the holography = holomorphy principle [L4, L8, L11, L7, L5].

Only irrotational Beltrami flows are possible in the plane. The simplest integrable planar flows, having interpretation as irrotational and incompressible Beltrami flows, reduce to gradient flows except at singularities.

These flows do not correspond to the flows defined by complex analytic maps or symplectic maps of plane, whose generalizations to higher dimensions play a key role in TGD. These flows, definable for the generalized complex structure of $H = M^4 \times CP_2$ and $X^4 \subset H$ accompanied by Kähler structure and symplectic structure, allow global coordinates along their time and have a natural interpretation as hydrodynamical flows in the induced Kähler field.

The complex flows give rise to maps, which have as singularities are 2-D partonic surfaces and string world sheets. In the view of scattering amplitudes, vertices correspond to partonic 2-surfaces at which the lines of the analogs of Feynman diagrams meet each other [L12] whereas string world sheets as intersections of two 4-surfaces with common Hamilton-Jacobi structure [L7] characterize interactions as contact interactions.

2 Beltrami flows and integrable flows in TGD

The generalization of Beltrami flows to 4-D context is one of the key ideas of TGD [K2, K1] but I have not discussed them explicitly in the recent framework based on holography = holomorphy vision (H-H)) [L4, L8].

1. The motivation is that TGD is formally hydrodynamics in the sense that field equations express local conservation of isometry charges of $M^4 \times CP_2$. There is actually infinite-dimensional algebra of conserved charges. The proposal is that in TGD, the Beltrami flows generalize genuinely 4-dimensional flows and correspond to classical field configuration for which the 4-D Lorentz force involving electric components vanishes.

2. The definition of the Beltrami flow is however different since one cannot regard the magnetic field as a vector field in 4 dimensions. For field equations Kähler current typically vanishes but can be also light-like. The counterpart of Beltrami flow states that Kähler current is proportional to the corresponding axial current:

   $$j^{\mu} = D_{\nu} J^{\mu\nu} = \alpha \times \epsilon^{\mu\nu\alpha\beta} A_{\nu} J_{\alpha\beta}$$

   .

   The divergence of j^{μ} vanishes and this must be true also for the instanton current.

   This is the case if the CP_2 projection of the space-time surface is at most 3-dimensional. If it is 4-D the parameter α must vanish since the divergence of the axial current gives instanton density $\epsilon^{\mu\nu\alpha\beta} J_{\mu\nu} J_{\alpha\beta}$, which is non-vanishing for CP_2 and by self-duality proportional to $J^{\mu\nu} J_{\mu\nu}$. Hence the only option is $\alpha = 0$ for $D = 4$.

3. If these Beltrami flows are integrable, they can give a physical realization of some, perhaps all, space-time coordinates as coordinated varying along the flow lines of some isometry current. The time component of 4-force has interpretation as dissipation power and also vanishes. These non-dissipative configurations play a key role in TGD and are natural when space-time surfaces are identified as quantum coherence regions.

4. The key idea sharpening dramatically the notion of Beltrami flow supported by H-H vision is that complex analytic maps $f : z \to f(z)$ allow us to construct integrable flows. What matters physically would be singularities: poles and zeros. Without them these maps would be just general coordinate transformations.

   In TGD, this generalizes to 4 dimensions by the introduction of generalized complex structure in $H = M^4 \times CP_2$. The presence of hypercomplex coordinates in M^4 motivates the term "generalized". In 4-D context, poles and zeros as singularities of a flow correspond to string world sheets and partonic 2-surfaces. The second key idea is that fermions at the flow lines serve as markers and provide information about the flow. In the cognitive sector they realize Boolean logic.

   Complex structure is often accompanied by Kähler structure. Its generalization to the Hamilton-Jacobi structure [L7] of H and M^4 involves hypercomplex structure. Kähler structure involves symplectic structure and the symplectic symmetries of H induce isometries of the "world of classical worlds" (WCW) [L3] as also the generalized holomorphic transformations of H.

   Symplectic *resp.* holomorphic transformations preserve areas *resp.* angles, which in 2-D case are canonically conjugate variables so that these transformations should be very closely related. Symplectic flows are not gradient flows but one can assign to their flow lines a global coordinate the Hamilton canonically conjugate to the Hamilton of the flow. Also complex analytic flows allow this.

2.1 Flows in the complex plane

Flows in the plane are not usually regarded as interesting Beltrami flows since in this case the condition $\nabla \times v = \alpha v$ cannot be satisfied for integrable flows as gradient flows unless the vorticity and a position dependent eigenvalue α vanish. There are however other ways to satisfy the integrability. Complex analytic maps define define integrable flows in more general sense.

One can start from flows in plane, in particular integrable flows.

1. Integrability means that the flow lines of the flow give rise to globally defined coordinate lines which fill the space smoothly. Intuition suggests that without integrability and the existence of a global coordinate along flow lines, the flow would be more like a random motion analogous to the motion of gas particles. Integrability would bring in smoothness and the flow looks like a fluid flow.

2. Integrability in strongest form requires that the velocity v for the flow line is a gradient $v = grad(\phi)$ of the global coordinate in question. This implies $\nabla \times v = 0$ and $\alpha = 0$. This condition is very strong and implies irrotationality so that a rotational flow is only possible in a global sense. There are however milder ways to guarantee the integrability.

3. Note that exotic smooth structures [A2, A3, A1] possible in TGD [L6, L1, L10] would correspond to flows for which smoothness fails at singularities to make possible fermionic interactions, although fermions are free in TGD. But this is possible only for 4-D space-time.

2.1.1 Flows of plane defined by complex analytic maps

The flows defined by complex analytic maps define integrable flows.

1. In the case of complex plane, analyticity conditions for a map $f : z \rightarrow= (u, v)$ give Cauchy-Riemann conditions $\partial_x v^x = \partial_y v^y$ and $\partial_y v^x = -\partial_x v^y$ expressing complex analyticity. Neither $\nabla \times v$ nor $\nabla \cdot v$ vanishes. One has neither gradient flow or incompressible flow.

2. One can also consider velocity fields $j = (j^x, j^y)$ satisfying the Cauchy-Riemann conditions. The exponentiation of v defines a flow as the analytic map $z \rightarrow f(z) = u + iv$ of the complex plane which in the case of the plane is of the same form as the generator of the flow. The flow lines can be identified as coordinate lines of the new coordinates u and v and defined by the conditions $Im(f) = v = constant$ and $Re(f) = u = constant$ so that the flow is integrable.

3. In the case of a complex plane, both the holomorphic vector fields j and maps f can however have poles and zeros as singularities and it is important to make a clear distinction between these two interpretations. Zeros of the map $f = (u, v)$ correspond to point-like vortex cores and poles to point-like sources and sinks at which the analyticity fails. If f is interpreted as an electric or magnetic field, poles correspond to charges as sources of the electric field and vortices to point currents as sources of the magnetic field.

4. One can also allow cuts. They appear if a complex analytic map is many-valued, such as fractional power and it is made discontinuous by taking only a single branch. Second option is to allow a covering in which case the complex plane becomes many-sheeted. In TGD, this picture is generalized to a 4-D situation.

2.1.2 Symplectic flows in plane

One can consider also symplectic flows in plane E^2 endowed with Kähler form $J_{xy} = -J_{yx} = 1$, which is negative of the tensor squared of the metric $g_{ij} = \delta_{ij}$ of E^2. Symplectic flows preserve the signed area defined by the symplectic form which in complex coordinates corresponds to the Kähler form which in complex coordinates defines a geometric representation of the imaginary unit.

The flows defining infinitesimal generators of the symplectic transformations are in the general case of the form $j^k = J^{kl} H_l$, where index raising is by the metric. In the case of plane E^2 the explicit expression is $(j^x, j^y) = (\partial_y H, -\partial_x H)$, where H is the Hamiltonian of the flow, which defines conserved "energy" constant along flow lines. The vanishing of the divergence $D_k j^k$ means the preservation of the area.

Symplectic flow is not a gradient flow but it allows a global coordinate varying along the flow lines. This follows from the existence of the canonical conjugate H^c of H, whose Poisson bracket with H equals to on: $\{H^c, H\} = \partial_k H^c J^{kl} \partial_l H = 1$. The equation for H^c along the flow lines of H is $dH^c/dt = \{H^c, H\} = 1$ and is solve by $H^c = t$ so that H_c defines the gradient flow giving rise to a global coordinate. The plane decomposes to a union of flow lines as $H = E$ surfaces.

2.2 The 2-dimensional flows related to the simplest Hopf fibration $S^3 \to S^2$

Consider first the Hopf fibration $S^3 \to S^2$. The simplest visualization of the fibration is in terms of inverse images of the circles S^1 of S^2 in S^3 under bundle projection. The fibers associated with the points S^2 correspond to linked, non-intersecting circles in S^3. The twist or linkage is characterized by an integer known as Chern number. That the inverse images are smooth 2-surfaces, is highly non-trivial and is due to the fact that the flow in S^2 is integrable. Any integrable flow allows similar smooth lift.

For visualization purposes, one can represent S^2 as E^2 and S^3 as E^3. For instance, the inverse images of the circles $S^1 \subset S^2$ with a constant latitude θ, identified as flow lines, define a slicing of $E^3 \setminus Z$, where Z is z-axis, by tori $S^1 \subset S^1$ the origin of E^3 and projecting to a circle with center point at the origin of E^2. Poles of S^2 correspond to tori which degenerate to a single point, the origin E^3. The inverse images of closed flow lines in S^3 are tori for any integrable flow.

The flows of S^3 consistent with the Hopf fibration are unions of toric flows at the tori $S^1 \times S^1$ characterized by 2 winding numbers (n_1, n_2) project to circles $S^1 \subset S^2$. Note that the flow in S^2 is not geodesic flow. The flows of charged particles along closed cosmic strings with homologically trivial $S^2 \subset CP_2$ as cross section and define analogs of these flows.

Besides Betrami flows $\nabla \times v = \alpha v$ in S^3 also other flows S^2 loosely related to Hopf fibrations and its generalization are interesting in the TGD framework. Since S^2 has complex and Kähler structures, the integrable flows of S^2 should be reducible to analytic maps $f : z \to f(z)$ of S^2 to itself. From the TGD point of view, especially interesting flows flow are magnetohydrodynamics geodesic flows of CP_1 (and CP_2) coupled to its Kähler form as $U(1)$ field for which S^3 (S^5) define the fiber of U^1 bundle.

1. At the fermionic the presence of the S^1 as fiber of S^3 brings in a coupling of S^2 spinors to a covariantly constant Kähler form of S^2, which corresponds to a $U(1)$ symmetry assignable to S^1. In the case of S^2, the coupling is not necessary but in the case of CP_2 the Hopf fibration $S^5 \to CP_2$ allows Spin$_c$ structure and leads to the standard model couplings and symmetries in TGD.

2. S^2 with Kähler structure can be visualized for the standard embedding $S^2 \to E^3$ as a covariantly constant magnetic field B orthogonal to S^2. Another way to describe B is as a covariantly constant antisymmetric 2-tensor in S^2.

3. At the hydrodynamical level, one can consider hydrodynamics in which geodesic free motion couples to the magnetic field defined by the Kähler form via Lorentz force. The magnetic force causes a twisting so that the motion is not anymore along a big circle. The flow lines tend to turn towards the North Pole or South Pole and approach/or leave the poles from South or North. Chiral symmetry is clearly violated.

   For the lift of this flow to S^3 flow lines define a union of non-intersecting linked circles S^1 as fibers of $S^3 \to S^2$ giving rise to tori in the case of closed flow lines. If the S^2 flow is integrable, it is possible to label the fiber circles by a time coordinate, so that they are expected to combine to form a smooth 2-D manifold. Vortex singularities must correspond to single fiber S^1, possibly contracted to a point.

4. The basic question is whether a given flow is integrable rather than like a random motion of gas molecules for which flow lines can intersect and do not form a smooth filling of the space. Complex and Kähler structures make sense also for S^2. The conclusion is that analytic maps $z \to f(z)$ of a complex coordinate of S^2 define an integrable flow. The real and imaginary parts of $f(z)$ define the velocity field v. Also symplectic flows define flows global coordinate along the flow lines so that the flow lines allow a lift to tori in S^3.

5. There are two kinds of singularities at which the analyticity fails: zeros correspond to vortices and poles to sources and sinks. Everywhere else the flow is locally incompressible and irrotational so that both the divergence and rotor of the velocity field vanish. If the flow has no singularities it can be regarded as a mere coordinate change. Singularities contain the physics. It would seem that only integrable flows allow a lift to flows in S^3.

2.3 Hopf fibration $S^5 \to CP_2$

In TGD, the projection $S^5 \to CP_2$ is the crucial Hopf fibration since it makes it possible to provide CP_2 with a respectable spinor structure. The Kähler coupling gives rise to the standard model couplings and symmetries and $H = M^4 \times CP_2$ is physically unique: weak interactions are color interactions in CP_2 spin degrees of freedom (charge and weak isospin). What is essential is the coupling of the Kähler gauge potential to spinors. This in turn leads to a Dirac equation in $H = M^4 \times CP_2$ and the induced Dirac equation at the space-time surface X^4.

1. At the hydrodynamical level one has a geodesic flow coupled to the self-dual Kähler form of CP_2 . One has Euclidian analogs of constant electric and magnetic fields, which are of the same magnitude. They would be orthogonal in E^4 but in CP_2 their inner product gives constant instanton density. In this case the inverse images of the flow lines not linked.

2. Also now complex analytic maps $f : CP_2 \to CP_2$ define integrable flows with singularities guaranteeing that the inverse images of flow lines in S^5 are 2-D smooth manifolds. There are two complex coordinates and one can have poles with respect to both of them. Both poles and zeros are replaced with 2-D surfaces and also the analogs of cuts appearing if many-valued maps f are allowed.

3. Also symplectic maps define flows global coordinate along the flow lines so that the flow lines allow a lift to tori in S^5.

2.4 CP_2 type extremals

At the next level one can consider CP_2 type extremals, which are deformations of the canonical embedding of CP_2 as an Euclidean 4-surface of $H = M^4 \times CP_2$ for which M^4 coordinates are constant. They can be said to define basic building bricks of particles in TGD. The CP_2 type extremal has locally the same induced metric and Kähler structure as CP_2 but its M^4 projection is a light-like curve, light-like geodesic in the simplest situation. It also ends, that is holes realized as 3-surfaces.

1. The above situation for which time is time parameter as 5:th coordinate is replaced with M^4 time coordinate u varying along the light-like curve. Also now the complex analytic functions $f : CP_2 \to CP_2$ define integrable flows. Time coordinate labels 3-D sections of the flow.

2. Now these flows would carry real physics. Induced Dirac equation effectively reduces to 1-D Dirac equation for fermion lines identified and holomorphy solves it, very much like in string models.

   The physical interpretation is very concrete. The addition of fermions to fermion lines serves as an addition of a marker making the flow visible. Fermions as markers allow to get information about the underlying geometric flow making itself visible via the time evolution of the many-fermion state.

   In TGD, fermions also realize Boolean logic at quantum level and the time evolutions between fermionic states can be seen as logical implication $A \to B$. Spinor structure as square root of metric structure fuses logic and geometry to a larger structure.

2.5 Flows at space-time surfaces $X^4 \subset H$

In holography = holomorphy vision space-time surfaces are roots for a pair $f = (f_1, f_2) : H \to C^2$ of two generalized analytic functions f_i of one real hypercomplex coordinate u of M^4, and the remaining 3 complex coordinates of H. Let us denote one of the complex coordinates by w, which can be either an M^4 or CP_2 coordinate.

1. The roots give space-time surfaces as minimal surfaces solving the field equations for any classical action as long as it is general coordinate invariant and constructible in terms of induced geometry. The extremely nonlinear field equations reduce to local algebraic conditions and Riemannian geometry to algebraic geometry.

2. X^4 shares one hypercomplex coordinate and one complex coordinate with H and both X^4 and H have generalized complex structure. X^4 has hypercomplex coordinate u ($u = t - z$ of M^2 in the simplest situation) and complex coordinate w (coordinate of complex plane E^2 in the simplest situation). This defines the Hamilton-Jacobi structure of X^4.

3. Complex analytic maps of X^4 are of the form by $(u \to f(u), w \to g(u,w))$. Integrable flows are induced by these maps. If there are no singularities they correspond to general coordinate transformations. The map by f having singularities generates a new Hamilton-Jacobi structure.

4. Poles and zeros in the w-plane correspond to 2-D string world sheets. The counterparts of zeros and poles for hypercomplex plane, parameterized by a discrete set of values of the real hypercomplex coordinate u correspond to singular partonic 2-surfaces with complex coordinate w at the light-like orbit of a partonic 2-surface.

   These singular partonic 2-surfaces can be identified as TGD counterparts analogs of vertices at which fermionic lines can change their direction. At these surfaces the trace H of the second fundamental form vanishing everywhere else by minimal surface property has a delta function like singular. Its CP_2 part has an interpretation as analog of Higgs vacuum expectation value. The claim of Jenny Nielsen is analogous to this result. In TGD also the M^4 part of H is non-vanishing and corresponds to a local acceleration concentrated at the singularity. An analog of Brownian motion is in question.

   One could very loosely say that the parameter α for Beltrami flow vanishes everywhere except at singularities where it has interpretation as value of the analog of Higgs expectation as the trace of the second fundamental form.

   String world sheets in turn mediate interactions since they connect to each other the light-like orbits of partonic 2-surfaces. This view conforms with the basic physical picture of TGD.

A summary of the situation would look like follows.

1. It would seem that in TGD the flows in CP_1 and CP_2 are more important than flows in S^3 and S^5 but that the integrable flows allow a lift of the flow lines to smooth manifolds of the total space. The spheres provide the needed Kähler form guaranteeing the twisting of the flow and making in the case of S^2 possible arbitrarily complex flow topologies as knotting, braiding, and linking. Also 2-knots are possible in 4-D context.

2. The flows with a coupling to the induced Kähler form have a clear physical interpretation and the fermion lines central in the TGD based view of scattering amplitudes could correspond to the flow lines. The flows without singularities define general coordinate transformations. What about the Kähler flows expected to have singularities? Could they have some physical interpretation?

   String world sheets are identifiable as intersections of two space-time surfaces with the same H-J structure, this applies also to self-intersections. Partonic 2-surfaces in turn are counterparts of vertices at which the TGD counterparts of Feynman lines meet [L12]. These singularities play a key role in the construction of scattering amplitudes in the TGD framework. Also the singularities of the complex flows in the presence of Kähler force have this kind of singularities as counterparts vortices and sinks and sources. Could the flow singularities correspond to self intersections and partonic 2-surfaces?

3. Could the analytic maps with singularities defined by Kähler flow allow to define Hamilton-Jacobi structure in geometric terms using the information about its singularities as self-intersections.

4. The realization that fermion lines very concretely serve as markers of a hydrodynamic flow.

REFERENCES

Mathematics

[A1] Etesi G. Exotica or the failure of the strong cosmic censorship in four dimensions, 2015. Available at: http://arxiv.org/abs/1503.04945v4.

[A2] Gompf RE. Three exotic R^4's and other anomalies. *J. Differential Geometry*, 18:317–328, 1983.Available at: http://projecteuclid.org/download/pdf_1/euclid.jdg/1214437666.

[A3] Asselman-Maluga T and Brans CH. World Scientific. https://doi.org/10.1142/4323., 2007.

Theoretical Physics

[B1] Lakthakia A. *Beltrami Fields in Chiral Media*, volume 2. World Scientific, Singapore, 1994.

[B2] Ghrist G Etnyre J. Generic hydrodynamic instability of curl eigen fields, 2003. Available at: https://arxiv.org/abs/math/0306310.

[B3] Ghrist R Etnyre J. An index for closed orbits in Beltrami field, 2001. Available at: https://arxiv.org/abs/math/010109.

[B4] Marsh GE. *Helicity and Electromagnetic Field Topology.* World Scientific, 1995.

[B5] Bogoyavlenskij OI. Exact unsteady solutions to the Navier-Stokes equations and viscous MHD equations. *Phys Lett A*, pages 281–286, 2003.

Books related to TGD

[K1] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/prext.pdf, 2023.

[K2] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/class.pdf, 2023.

[K3] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/elvafu.pdf, 2023.

[K4] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mless.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD. https://tgdtheory.fi/public_html/articles/finitefieldsTGD.pdf., 2022.

[L2] Pitkänen M. Two objections against p-adic thermodynamics and their resolution. https://tgdtheory.fi/public_html/articles/padmass2022.pdf., 2022.

[L3] Pitkänen M. Symmetries and Geometry of the "World of Classical Worlds" . https://tgdtheory.fi/public_html/articles/wcwsymm.pdf., 2023.

[L4] Pitkänen M. About Langlands correspondence in the TGD framework. https://tgdtheory.fi/public_html/articles/Frenkel.pdf., 2024.

[L5] Pitkänen M. Holography=holomorphy vision in relation to quantum criticality, hierarchy of Planck constants, and $M^8 - H$ duality. https://tgdtheory.fi/public_html/articles/holoholonumber.pdf., 2024.

[L6] Pitkänen M. TGD as it is towards the end of 2024: part II. https://tgdtheory.fi/public_html/articles/TGD2024II.pdf., 2024.

[L7] Pitkänen M. Holography and Hamilton-Jacobi Structure as 4-D generalization of 2-D complex structure, 2024. Available at: https://tgdtheory.fi/public_html/articles/HJ.pdf.

[L8] Pitkänen M. A more detailed view about the TGD counterpart of Langlands correspondence. https://tgdtheory.fi/public_html/articles/Langlands2025.pdf., 2025.

[L9] Pitkänen M. A refined view of the phenomenology of hadron physics and p-adic mass calculations. https://tgdtheory.fi/public_html/articles/padmass2025.pdf., 2025.

[L10] Pitkänen M. Comparing the S-matrix descriptions of fundamental interactions provided by standard model and TGD. https://tgdtheory.fi/public_html/articles/hadroQCDTGD.pdf., 2025.

[L11] Pitkänen M. Holography= holomorphy vision and a more precise view of partonic orbits . https://tgdtheory.fi/public_html/articles/HHpartons.pdf., 2025.

[L12] Pitkänen M. M^8H duality reduces to local G_2 invariance. https://tgdtheory.fi/public_html/articles/M8Hstill.pdf., 2025.