Combinatorial Hierarchy: two decades later

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Abstract

Combinatorial Hierarchy (CH) is a hierarchy consisting of Mersenne integers $M(n) = M_{M(n-1)} = 2^{M(n-1)} - 1$ and starting from $M_1 = 2$. The first members of the hierarchy are given by $2, 3, 7, 127$, $M_{127} = 2^{127} - 1$ and are primes. The conjecture of Catalan is that the hierarchy continues to some finite prime. It was proposed by Peter Noyes and Ted Bastin that the first levels of hierarchy up to M_{127} are important physically and correspond to various interactions. I have proposed the levels of CH define a hierarchy of codes containing genetic code corresponding to M_7 and also memetic code assignable to M_{127}.

In this article I consider the argument that the hierarchy ends at M_{127} and find that it should end already at M_7 for which the condition used saturates and which corresponds to genetic code in TGD interpretation. The failure of condition at M_{127} level has interesting "Gödelian interpretation". I find also that in TGD Universe genetic code and its memetic counterpart are realized at the level of fundamental particles. Already earlier I have ended up with alternative realizations at the level of dark nucleons and sequences of 3 dark nucleons.

1 Introduction

Combinatorial Hierarchy (CH) [A1, A2] is a hierarchy consisting of Mersenne integers $M(n) = M_{M(n-1)} = 2^{M(n-1)} - 1$ and starting from $M_1 = 2$. The first members of the hierarchy are given by $2, 3, 7, 127, M_{127} = 2^{127} - 1$ and are primes. The conjecture of Catalan is that the hierarchy continues to some finite prime. It was proposed by Peter Noyes and Ted Bastin that the first levels of hierarchy up to M_{127} are important physically and correspond to various interactions (see http://tinyurl.com/hszo9wb). I have proposed the levels of CH define a hierarchy of codes containing genetic code corresponding to M_7 and also memetic code assignable to M_{127} [K1].

Pierre Noyes and Ted Bastin proposed also an argument why CH contains only the levels mentioned above. This has not been part of TGD view about CH: instead of this argument I have considered the possibility that CH does not extend beyond M_{127}. With the inspiration coming from email discussion I tried to understand the argument stating that CH contains M_{127} as the highest level and ended up with a possible interpretation of the condition. Zero energy ontology (ZEO) and the representation of quantum Boolean statements $A \rightarrow B$ as fermionic parts of positive and negative energy parts of zero energy states is essential. This led to several interesting new results.

1. To my best understanding the original argument of Noyes does not allow M_{127} level whereas prime property allows. States at M_{127} level cannot be mapped to zero energy states at M_7 level. Allowing a wild association with Gödel's theorem, one could say that that there is hube number of truths at M_{127} level not realizable as theorems at M_7 level.

   A possible interpretation is that M_{127} level corresponds to next level in the abstraction hierarchy defined by CH and to the transition from imbedding space level to the level of "world of classical worlds" (WCW) in TGD. The possible non-existence of higher levels (perhaps implied if $M_{M_{127}}$ is not prime) could be perhaps interpreted by saying that there is no "world of WCWs"!

2. Rather remarkably, for M_7, which corresponds to genetic code [K1], the inequality serving as consistency condition is saturated. One can say that any set of 64 mutually consistent statements at M_7 level can be represented in terms of 64 Boolean maps at M_3 level representable

in terms of zero energy states. One obtains an explicit identification for the Boolean algebras involved in terms of spin and isospin states of fermions in TGD framework at level M_7 so that genetic code seems to be realized at the fundamental elementary particle level thanks to the dimension $D = 8$ of imbedding space. Even more, the level M_{127} corresponding to memetic code emerges in the second quantization of fermions at M_7 level. Here color triplet property of quarks and color singletness of leptons and the identification of elementary particles as pairs of wormhole contacts are in essential role.

The conclusion would be that in TGD Universe genetic code and its memetic counterpart are realized at the level of fundamental particles. Already earlier I have ended up with alternative realizations at the level of dark nucleons and sequences of 3 dark nucleons [L1].

2 Summary of Combinatorial Hierarchy

I summarize first the basics of CH.

1. One considers the space algebra of Boolean statements of n bits which can be also extended to complex linear space -quantum Boolean algebra. One can give it linear structure as Z_2 algebra for binary coefficients with Z_2 sum having set theoretic interpretation. This linear space has some basis. That the coefficient field for linear structure is Z_2 does not seem to be absolutely essential. In TGD framework one considers the linear space defined by quantum Boolean algebra with qubit interpretation generated by fermionic oscillator operators: one operator for every bit.

2. One assigns to the linear n-D space the n^2-D space of linear maps of it to itself. One can also consider the space of maps of quantum Boolean algebra to itself and also require that this defines a Boolean homomorphism. Dimensions would be the same: only coefficient field would be different.

3. To CH level, which corresponds to Mersenne prime $M(n) = M_{M(n-1)}$ ($n = 2, 3, 7, 127, 2^{127} - 1, ...$) one assigns vector space with dimension

$$D(n-1) = [(M(n-1) + 1)]^2 \ ,$$

and requires that the space formed by

$$D_1(n) = \frac{(M(n) + 1))}{2}$$

bit sequences, which represent a subset of mutually consistent Boolean statements as subset of $M(n) + 1$ bit sequences are representable as a subset of bit sequences with $D(n-1)$ bits. This demands

$$D_1(n) \leq D(n-1)$$

giving

$$\frac{M(n) + 1}{2} \leq [M(n-1) + 1]^2 \ .$$

4. This criterion is satisfied for the primes of CH up to M_7 but not for M_{127}: $2^{127} - 1 > 128^2$ so that M_{127} should not included if I have understood the criterion correctly.

For $M_7 = 2^7 - 1 = 127$ one obtains the condition $2^6 = 64 \leq 8 \times 8 = 64$ so that condition is saturated. Remarkably, 64 is the number of DNA codons!

5. The numbers of CH are also known as Catalan Mersenne numbers. Catalan Mersenne primes are special case of double Mersenne primes M_{M_n} (see http://tinyurl.com/j4tqwch). Catalan conjecture that Catalan Mersennes are primes up to some limit. After the first non-prime the remaining Catalan Mersenne numbers are necessarily composite. The known double Mersennes are given by M_{M_p}: $p = 2, 3, 5, 7$. No other cases are known. These primes are good candidates for labelling scaled up variants of say hadron physics. To my opinion Catalan criterion is more plausible.

6. Classical number fields are in key role in TGD [K7, K8, K9] and have dimensions $D = 1, 2, 4, 8$. Also CH involves these dimensions. $D(n - 1) = M(n - 1) + 1$ giving dimensions $2, 4, 8$ for M_2, M_3, M_7. For M_{127} one would obtain $D = 128$, which does not correspond to any division algebra. This might relate to the above observation.

3 CH as a prediction of quantum TGD

In the following the interpretation of Boolean map in ZEO is proposed. Also it is shown that M_7 level allows a natural realization in terms of spin-isospin states of fermions and that M_{127} level is obtained in second quantization meaning going from the level of imbedding space to the level of WCW.

3.1 Interpretation of the lower level Boolean map in terms of ZEO

One can ask, why one should have this kind of map? One interpretation is that the space of Boolean statements at given level is imbeddable to the space of quantum Boolean maps at previous level. Quantum Boolean maps would represent Boolean rules A→ B, "theorems" or "laws of physics".

1. In TGD framework the interpretation of CH would be as a hierarchy of statements about statements about... The number of statements about N statements is indeed 2^N. One statement corresponding to all bits equal to 0 (in set theoretic realization empty set) is thrown away so that one has $2^N - 1$ statements instead of 2^N.

2. ZEO means that physical states are pairs of states with opposite conserved quantum numbers: they correspond to physical events, which replace states as fundamental entities in ZEO. The fermionic parts of positive and negative energy parts of states would be pairs of many-fermion states allowing interpretation as elements of quantum Boolean algebra. Zero energy states themselves would correspond to pairs of these fermionic states and thus to "theorems" A→ B or maps from Boolean algebra to itself. The allowed statement pairs would satisfy fermion number conservation and conservation of various quantum numbers and would indeed represent laws of physics.

3. A possible interpretation of the map would be that the statements at given level $M(n + 1)$ must be representable as theorems at previous level $M(n)$. For $M(n) > M_7 = 127$ this would not hold true anymore. Could this have some deep mathematical meaning as the wild association with Goedel's theorem suggests?

   In the model of genetic code and its generalizations [K1] I have proposed that each level of CH defines a maximal number of mutually consistent statements identifiable as "axioms": the number is 2^{n-1} for 2^n n-bit statements. For $M_7 = 127$ the number is 64, the number of DNA codons, which would thus have interpretation as axioms or "fundamental truths". In this case the representability would still hold and map would be bijection. At the next level one would have "memetic code" with 2^{126} codons representable as sequences of 21 DNA codons with stop codon included ($126 = 21 \times 6$). By the proposed criterion, at memetic level only vanishingly small subset of truths would be representable as theorems at genetic level.

3.2 Representation of M_7 level in TGD framework

Could the saturation for M_7 have some physical meaning? The maps would be from 8-D space to itself.

1. Bits can be represented in terms of spin and electroweak spin giving $2 \times 2 = 4$ states and imbedding space-spinors ($H = M^4 \times CP_2$) of given H-chirality (quark or lepton like), given fermion number (fermion or antifermion) and physical helicity. If also unphysical helicities with fixed fermion number are allowed one would have $4 + 4 = 8$ states. The condition that helicity is physical would reduce the number of states by one half. This applies to both quarks and leptons since color is not spin like quantum number in TGD (colored states correspond to partial waves in CP_2).

2. What could be the interpretation for $2^7 - 1 = 127$ states containing as subset $n = 2^6$ states. Could $n = 2^6$ correspond to the number of states in the tensor product formed by pairs of 8 leptons and 8 antileptons allowed to have also unphysical polarizations? Same would apply to quarks. Allowing both quark-antiquark and lepton-antilepton type states one would have 128 states. The physicality condition for boson polarizations could drop the number of states to 64. What the dropping of one state would correspond: to the dropping of $\nu_R - \bar{\nu}_R$ pair having no electroweak and color couplings perhaps?

   One can imagine two alternative identifications for the two tensor factors.

   (a) In TGD framework fundamental bosons correspond to fermion antifermion pairs with members at opposite throats of wormhole contact connecting two space-time sheets. Could the genetic code correspond to 64 elementary bosons with physical polarizations and the maps to those assigning to 8 fermions 8 antifermions?

   (b) An alternative identification is suggested by ZEO. The tensor product of fermionic Boolean algebras at opposite boundaries of causal diamond (CD) would replace that at opposite wormhole throats. This would in accordance with the interpretation of zero energy states as statements $A \to B$ represented as Boolean maps.

3.3   Representation of M_{127} level in TGD framework

What about the physical interpretation of M_{127} level in TGD framework?

1. The first thing to observe is that physically $p = M_{127}$ corresponds in TGD to the p-adic prime p characterizing electron in p-adic mass calculations: Compton length is proportional to the p-adic length scale and thus proportional to $\sqrt{p}$. The remaining Mersenne primes correspond to completely super-astrophysical Compton lengths. Hence M_{127} has a very special role. The Mersenne primes $3, 7, 31, 127$ giving rise to double Mersenne primes correspond to extremely short p-adic length scales.

   Recall that the ratio of m_{CP_2}/m_e is approximately $m_{CP_2}/m_e = 2^{127/2}/\sqrt{5+x}$, where $x \in [0, 1]$ characterizes the second order contribution to electron mass from p-adic mass calculations [K3]. The ratio of Planck mass to proton mass equals to $m_{Pl}/m_p = 1.307 \times 10^{19}$. For $x = 0$ this gives $m_{Pl}/m_{CP_2} = (m_p/m_e) \times 3.96 = 7.271 \times 10^3$, which is not far from $2^{13} \simeq 8.912 \times 10^3$. The value of 2^{13} is very attractive number theoretically and would be obtained for $x = .5$, again power of 2.

2. The states at this level should correspond to statements about statements at the lower level represented in terms of quark lepton state space as many-fermion states assignable to wormhole throat or several wormhole throats (elementary corresponds to two wormhole contants and 4 wormhole throats). The construction of infinite primes can be interpreted as a process of forming repeatedly statements about statements and the physical analog is repeated second quantization [K6].

   In the recent situation second quantization would correspond to the formation of many-fermion states at partonic 2-surfaces defined by the throats of wormhole contacts. This would automatically give rise to M_{127} states if one has 127 single fermion states to begin with.

   Physically this step would correspond to a step from the spinor modes of imbedding space to the spinor modes of WCW identifiable as fermionic Fock states assignable to partonic 2-surfaces so that indeed a huge abstraction is in question. I have proposed that anyonic

states could be this kind of states for large value of $h_{eff} = n \times h$ implying that the size of wormhole throat becomes nano-scopic [K5].

3. One has 127 boson states but how to obtain 127 (or $128 = 2^7$) single fermion states? Counting only spin and weak isospin gives $n = 8 + 8 = 2^4$ ($n = 4 + 4 = 2^3$) single fermion states if one allows (does not allow) also unphysical polarizations. The simplest option is that each single fermion state has 2^3 (2^4) additional states. The location of fermion at one of the 4 wormhole throat could give 4 additional degrees of freedom. This would leave 2 (4) additional states per fermion state still missing.

4. A good guess is that quark color realized as color partial waves comes in rescue and gives the needed states. Light quarks must move in color triplet states and leptons in singlet states. Thefore quarks have $3 \times 8 = 24$ modes and leptons 8 modes giving altogether 32 modes altogether. There are 4 wormhole throats so that $4 \times 32 = 128$ modes are obtained and if right-handed neutrino is thrown out one has 127 states as required if no constraints on polarizations are posed. It therefore seems that TGD physics codes CH naturally at elementary particle level!

There is indeed a rich set of "vibrational" degrees of freedom giving also rise to color degrees of freedom. The symplectic group of $\Delta M_{\pm}^4$ assignable to either boundary of causal diamond (CD) defined as the intersection of future and past directed light-cones of M^4 with points replaced with CP_2 gives rise to products of S^2 and CP_2 partial waves. Besides this there is a conformal weight labelling the states correlating with $S^2 \times CP_2$ partial wave Light quarks massless before massivation by p-adic thermodynamics move in color partial waves and color triplets are obtained as the color excitations for them corresponding to higher conformal weights and having CP_2 mass as mass scale.

I have already earlier ended up with the proposal that genetic code is realized at the level of dark nuclear physics. Either the states of dark proton or sequence of 3 protons could be organized naturally states corresponding to 64 DNAs, 64 RNAs, 20 aminoacids, and 40 tRNAs and vertebrate genetic code follows from very simple assumption that opposite spins are paired [K4, K2] [L1] (see `http://tinyurl.com/jgfjlbe`). These findings suggest that genetic code and memetic code are also realized at the elementary particle level.

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