Gödel's Undecidability Theorem and TGD

December 25, 2022

Matti Pitkänen

Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

$M^8 - H$ duality relates number theoretic and geometric views of physics. Gödel's incompleteness theorem relates to number theory. In zero energy ontology, space-time surfaces obey almost exact holography and are analogous to proofs of theorems. Could one consider a geometric and physical interpretation of Gödel's incompleteness theorem in the TGD framework based on the idea that the conscious experience accompanying a proof of a theorem corresponds to a localization of a zero energy state in the discretization of the "world of classical worlds" (WCW) to a 4-surface representing the theorem? Could the unprovability of Gödel's incompleteness theorem correspond to an impossibility to localize the zero energy state to the corresponding space-time surface? Can one identify the explicit form of Gödel sentences involved? These are the questions considered below.

Contents

1 Introduction

$M^8 - H$ duality [L2, L3] relates number theoretic and geometric views of physics [L7, ?]. Gödel's incompleteness theorem relates to number theory. Could one consider a geometric and physical interpretation of Gödel's incompleteness theorem in the TGD framework?

The following response to Lawrence Crowell in the discussion group "The Road to Unifying Relativistic and Quantum Theories" indeed suggests such an interpretation. The topic of discussion related to Gödel's theorem and its possible connection with consciousness proposed by Penrose [J1].

My own view is that quantum jump as state function reduction (SFR) cannot reduce to a deterministic computation and can be seen as a moment of re-creation or discovery of a new truth not following from an existing axiomatic system summarizing the truths already discovered. Zero energy ontology allows to solve the basic paradox of quantum measurement theory [L1, L4].

My emphasis in the sequel is on how the number theoretic vision of the TGD [L2, L3, L7, L6] proposed to provide a mathematical description of (also mathematical) cognition could allow us to interpret the unprovable Gödel sentence and its negation. There is no need to emphasize that these considerations are highly speculative.

2 What Gödel's theorem could mean in the TGD Universe?

The basic question concerns the physical and consciousness theoretic interpretation of the Göedel's undecidability theorem in the TGD Universe.

2.1 Some TGD background

In the following some necessary conceptual background will be introduced.

1. The polynomials P define space-time surfaces and one possible interpretation is that the ramified primes of P define external particles for a space-time region representing particle scattering. The polynomials P which reduce to single ramified prime would represent forward scattering of a single "elementary" particle.

2. In zero energy ontology (ZEO) [L5], ordinary quantum states are replaced by superpositions of almost deterministic time evolutions so that also "elementary" particle would correspond to a scattering event.

   What exists would be events, and what we call states would reduce to particular events. One could call ZEO as an "eastern" ontology. ZEO would predict not only scattering events but densities of particles as single particle scattering events inside a given causal diamond causal diamond (CD) representing quantization volume [L6].

3. Single space-time surface in $H = M^4 \times CP_2$ is obtained by $M^8 - H$ duality from a 4-surface in M^8 and satisfies in H almost exact holography forced by the general coordinate invariance. At the level of M^8 its preimage obeys number theoretic dynamics forcing the associativity of its normal space [L2, L3]. This 4-surface connects mass shells $H^3_a \subset M^4 \subset M^8$, which correspond to the roots of a polynomial P with integer coefficients.

   Almost holographic space-time surfaces represent a profound deviation from the standard physics view. They can be regarded as analogs of computations or proofs of theorems, counterparts of behaviors in neuroscience, and counterparts of biological functions. Quantum states are their superpositions. Number theoretically realized finite measurement resolution means that the superposition of space-time surfaces having the same theoretic discretization effectively represents a single space-time surface.

   Therefore the idea that the SFRs localizing the state to this kind of surfaces, could represent a physical realization of a mathematical theorem, looks natural. Gödel's theorem could correspond to a space-time surface to which localization by SFR is not possible.

4. The additional hypothesis [L6] motivated by $M^8 - H$ duality is that the values of WCW Kähler function H for its maxima defined by preferred extremals in H and analogous to Bohr orbits have values of vacuun functional $exp(K)$, which is equal to $1/D^k$, where the integer k defines analog of temperature and is inversely proportional the discrete running Kähler coupling strength $1/\alpha_k$. Zero energy states correspond to scattering amplitudes so that this would predict the scattering probabilities in WCW geometric degrees of freedom.

   For elementary particles sfor which D reduces to a single prime $D = P$, $1/\alpha_k$ would roughly behave like logarith of P. This would unify the logarithmic dependence of p-adic coupling constant evolution with the p-adic length scale hypothesis [L6].

2.2 Gödel numbering in TGD framework and the first for guess for the undecidable statement

Polynomials with integer coefficients (no common factor coefficients) to which all rational polynomials can be scaled without changing the roots define the space-time surfaces. One can pose additional physically well-motivated conditions to these polynomials. These conditions will be discussed later.

What the assignment of a Gödel number to this kind of polynomial could mean? Most of the classical physical content, if not all of it, can be coded by the coefficients $[a_0, , ...a_N]$ of the polynomial.

The Gödel number G associated with polynomial P would be rather naturally

$$G(P) = p_0^{a_0} p_2^{a_1} ... p_N^{a_N} \quad ,$$

where p_i is i:th prime and is an injection. Note that one has $p_0 = 2, p_1 = 3, p_2 = 5, ...$

The discriminant D (`https://en.wikipedia.org/wiki/Discriminant`) is the determinant of an $(2N-1) \times (2N-1)$-matrix defined by P and its derivative dP/dx ($[a_1, 2a_2, ..., Na_N]$) and is an integer decomposing to a product of ramified primes of P.

The first guess for Gödels' undecidable statement would that there exist a polynomial P for which one has $G = D$. The number D coding a sentence, whatever it is, would be its own Gödel number. Why this guess? At least this statement is short. Can this statement be undecidable? What undecidability could mean physically?

1. The equation involves both D as a polynomial of a_i and G involving transcendental functions $p_i^{a_i}$ (essentially exponential functions) so that one goes outside the realm of rationals and algebraic numbers.

2. $D = G$ is an analogue of Diophantine equation for $a_1, ..., a_N$ and both powers and exponential $p_i^{a_i}$ appear. If the coefficients a_i are allowed to be a complex numbers, one can ask whether the complex solutions of $G = D$ could form an N-1-D manifold. One can however assume this since $p_i^{a_i}$ leads outside the realm of algebraic numbers and one does not have a polynomial equation.

3. The existence of an integer solution to $D = G$ would mean that the primes p_i for which a_i are non-vanishing, correspond to ramified primes of P with multiplicity a_i so that the polynomials would be very special if solutions exist.

4. It might be possible to solve the equation for any finite field G_p, that is in modulo P approximation. Here one can use Fermat's little theorem $p_i^p = p_i \ mod \ p$. If integer solutions exist, they exist for every G_p.

2.3 About the number theoretical content of $G = D$ sentence

It is interesting to look at the number theoretical content of $G = D$ sentence.

1. Integer D would express the sentence/statement. D codes for the ramified primes. Their number is finite and we know them once we know P. Does the unprovable Gödel sentence say that there exists a polynomial P of some degree N, whose ramified primes are the primes p_i associated with a_i? Or does it say that there exists a polynomial satisfying $G = D$ in the set of polynomials of fixed degree N. Note that a priori one does not pose constraints on the values of coefficients a_i.

2. Is it that we cannot prove the existence of integer solution a_i to $P = G$ using a finite computation. Is this due to the appearance of the functions $p_i^{a_i}$ or allowance of arbitrarily large coefficients a_i? The p-adic solutions associated with finite field solutions have an infinite number of coefficients and can be p-adic transcendentals rather than rationals having periodic pinary extensions.

3. Polynomials of degree N satisfying $D = G$ are very special. The ramified primes are contained in a set of $N + 1$ first primes p_i so that D is rather small unless the coefficients a_i are large. D is a determinant of $2N - 1 \times 2N - 1$ matrix so that its maximum value increases rapidly with N even when one poses the constraint $a_i < N$. Rough estimates and explicit numerical calculations demonstrate that determinants involving very large primes are possible, in particular those involving single ramified prime identified as analogues of elementary particles, D can reduce to single large prime: $D = P$.

What about the polynomials P in the vicinity of points of the space of polynomials of degree N satisfying $D = 0$: they correspond to $N+1$ ramified primes, which are minimal (note that the number of roots is N). D is a product of the root differences and 2 or more roots coincide for $D = 0$. D is a smooth function of real arguments restricted to the integer coefficients. The value of D in the neighborhood of $D = 0$ can be however rather large. Note that the proposed Gödel numbering fails for $D = 0$, and therefore makes sense only for polynomials without multiple roots.

4. For $D(P) = 0$ one has a problem with the equation $G = D$. $G(P)$ is well-defined also now. The condition $D(P) = 0 = G(P)$ does not however make sense. The first guess is that for 2 identical roots, P is replaced with dP/dx in the definition of D: $D(P) - -> D(dP/dx)$. D is nonvanishing and the ramified primes p_i do exist for dP/dx. Therefore the condition $D(dP/dx) = G(P)$ makes sense. For N identical roots one must use have $D(d^{n-1}P/dx^{n-1}) = G(P)$.

2.4 About the physical interpretation of the undecidability

What about the physical interpretation of the undecidabililty in the TGD Universe? What kind of scattering events would these analogues of Gödel sentences correspond? Representations of new mathematical axioms as scattering events, not provable from existing axioms, perhaps?

Exactly what we cannot prove to be true or not true for the possibly existing very special polynomials satisfying $G = D$? What could the $G = D$ sentece state? What "proving" could mean from the point of physics and TGD view of consciousness? Could it mean a conscious experience of proof as a localization to the corresponding space-time surface in WCW? The almost deterministic space-time surface would represent the almost deterministic sequence of logical steps for the proof?

Could $G = D$ sentence be a space-time surface to which a localization in WCW is not possible for the simple reason that the additional natural physical conditions on the physical states do not allow its existence in superpositions definition zero energy states?

1. In TGD, the hypothesis [L6] that the coefficients of polynomials of degree N are smaller than N, is physically very natural and would make the number of polynomials to be considered finite so that in this case one can check the existence of a $G = D$ sentence in a finite time. It looks rather plausible that for given N, no $G = D$ sentence, which satisfies the conditions $a_i \leq N$, does exist.

2. One can of course criticize the hypothesis $a_i \leq N$ implying a strong correlation between the degree N of P and the maximal size of ramified primes of P identified as p-adic primes characterizing elementary particles. One can argue that in absence of this correlation predictivity is lost. This hypothesis also makes also finite fields basic building bricks of number theoretic vision of TGD [L6].

3. Could this give rise to a realization of undecidability at the level of conscious experience and cognition relying on number theoretic notions? How?

   Quantum states are superpositions of space-time surfaces determined by polynomials P and if the holography of consciousness is true, conscious experience reflects the number theoretic properties of these polynomials if associated to a localization to a given polynomial P in a "small" SFR (SSFR). This would be position measurement in the "world of classical worlds" (WCW)? The proof of the statement $G = D$ would mean that a cognizing system becomes conscious of the $G = D$ space-time surface by a localization to it.

   Suppose that for a given finite N and condition $a_i \leq N$, $G = D$ sentences do not exist. Hence one can say that $G = D$ sentences go outside the axiomatic system realized in terms of the polynomials considered. Even the space of all allowed polynomials identified as a union of spaces with varying value for degree N would not allow this. $G = D$ sentences would be undecidable by the condition $a_i \leq N$.

REFERENCES

Neuroscience and Consciousness

[J1] Penrose R. *Shadows of the Mind.* Vintage, 1994.

Articles about TGD

[L1] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: https:/tgdtheory.fi/public_html/articles/zeoquestions.pdf., 2019.

[L2] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part I. Available at: https:/tgdtheory.fi/public_html/articles/M8H1.pdf., 2020.

[L3] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part II. Available at: https:/tgdtheory.fi/public_html/articles/M8H2.pdf., 2020.

[L4] Pitkänen M. Some questions concerning zero energy ontology. https://tgdtheory.fi/public_html/articles/zeonew.pdf., 2021.

[L5] Pitkänen M. About the number theoretic aspects of zero energy ontology. https://tgdtheory.fi/public_html/articles/ZEOnumber.pdf., 2022.

[L6] Pitkänen M. Finite Fields and TGD. https://tgdtheory.fi/public_html/articles/finitefieldsTGD.pdf., 2022.

[L7] Pitkänen M. Some New Ideas Related to Langlands Program *viz.* TGD. https://tgdtheory.fi/public_html/articles/Langlands2022.pdf., 2022.