Topological condensed matter physics and TGD

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Abstract

There has been a lot of talk about the physics Nobel prize received by Kosterlitz, Thouless, and Haldane. There is an article summarizing the work of KTH in rather detailed manner. The following is an attempt to gain some idea about what are the topics involved.

The notions of topological order, topological physics, and topological materials pop up in the discussions. I have worked for almost 4 decades with Topological Geometrodynamics and it is somehow amusing how little I know about the work done in condensed matter physics.

The pleasant surprise is that topological order seems to have rather precise counterpart in TGD at the level of fundamental physics: in standard model it would emerge. In any case, it is clear that condensed matter physicists have taken the lead during last 32 years when particle physicists have been wandering in stringy landscape.

1 Kosterlitz Thouless phase transition

Kosterlitz Thouless phase transition is one of the basic notions (see http://tinyurl.com/h3xx5ke). KT (or KTB, B for Berezinskii) transition is associated with 2-D systems using so called XY model. For instance, KT transition appears in Josephson junction arrays and thin disordered granular super conducting films at certain critical temperature. Vortices are stabilized by topology so that the phase transitions must change topology and be discontinuous. Vortices are characterized by quantized vorticity. For magnetic vortices magnetic flux is quantized. For velocity vortices quantization occurs for velocity circulation. In KT phase transition for super-conductors the vortices are velocity vortices. Below the critical temperature vortices of opposite vorticity combine to form bound states. Above the critical temperature they decay to free vortices.

KT phase transition is of infinite order. The order of the phase transition tells which is the lowest derivative of free energy, which is dis-continuous as function of temperature. For transitions of infinite order free energy behaves smoothly.

Correlation function is second central statistical notion. In superconducting systems is correlation function for the phase $exp(i\theta)$ of the super-conducting order parameter θ, which is angle like variable, whose gradient is proportional to local velocity. For vortex the circulation of the gradient $\nabla\theta$ around a point is non-vanishing and this point corresponds to singularity. Ordinary hydrodynamical vortex is analogous to quantum vortex. The following list (see http://tinyurl.com/zmdfman) characterizes KT phase transition quantitatively.

1. No spontaneous symmetry breaking (Higgs mechanism is example of this) occurs in KT phase transition. Beta function for coupling constant like parameter is vanishing in the massless phase below criticality so that it does not depend on temperature. Conformal invariance would be natural counterpart for this. Vanishing of beta function means that coupling constant evolution is trivial.

   In TGD framework coupling constant evolution reduces quite generally to a discrete evolution as function of p-adic length scale and effective Planck constant. The evolution takes place as discrete steps as phase transitions at certain critical length scales. This allows to have the extremely beautiful features of $\mathcal{N} = 4$ supersymmetric gauge theories for which coupling constant evolution is trivial in all scales without losing coupling constant evolution, which is an empirical fact.

2. Free energy and all its derivatives are analytic functions of the temperature. This corresponds to the infinite order of the transition. In quantum case temperature would correspond to length scale. Zero energy ontology (ZEO) however suggests that quantum TGD could be regarded as a "square root" of thermodynamics in a well-defined sense. If this is true then thermodynamical and quantum criticalities would fuse to a more general notion.

3. Correlation length diverges exponentially in the vicinity of the critical point, property called essential singularity. For $\beta < \beta_c$, $t = \beta_c/\beta - 1$ (β is inverse temperature) one the dependence of the correlation function $\Gamma(R)$ from distance is in terms of correlation length $\xi(t)$ given by

$$\Gamma(R) \propto exp[\frac{R}{\xi(t)}], \quad \xi \sim exp(t^{-\nu}), \quad \nu = 1/2 \ .$$

   The divergence of correlation length can be interpreted in terms of (quantum) criticality giving rise to long range fluctuations and the analytic form corresponds to essential singularity. Correlation function approaches to constant near criticality.

4. Low temperature phase is a massless phase. Correlation function for the phase $exp(i\theta)$ decreases with power law, which presumably reflects conformal symmetry. Below the critical temperature ($\beta \geq \beta_c$) one has

$$\Gamma \propto \frac{1}{R^{\eta(T)}}, \quad \eta(\beta_c) = 1/4 \ .$$

5. The topological nature of the phase transition means special behaviour of the vortex configurations. The splitting of vortices with opposite spin makes possible the transformation of correlation functions obeying power law to those decaying exponentially although no spontaneous symmetry breaking occur (Higgs mechanism).

Could one relate this to the ideas and applications of TGD? The following is the first attempt.

1. In TGD framework magnetic flux tubes are basic vortex like objects carrying supra currents to which one can assign quantum variant of hydrodynamical vorticity. Thanks to the fact that CP_2 contains non-contractible 2-surfaces through which Kähler magnetic flux is non-vanishing and quantized, they can carry non-vanishing quantized Kähler magnetic flux in which case they are topological objects and apparently associated with magnetic monopoles. There are however no magnetic monopoles since the flux tubes are closed although they can traverse through two space-time sheets connected by pair of wormhole contacts.

   Magnetic flux tubes can also appear as flux tubes with vanishing Kähler magnetic flux and correspond to almost vacuum extremals.

   Magnetic flux tubes can form pairs with opposite or parallel fluxes and these pairs with opposite are central in TGD based model for high temperature superconductivity in anti-ferromagnetic systems [K4, K5]. The tensor network [L1] formed from magnetic flux tubes connecting partonic 2-surfaces could be present in all length scales in TGD Universe.

2. In TGD framework one can imagine a counterpart for the phase transitions, which do not break symmetry. Super-symplectic algebra is fractal and allows infinite hierarchy of sub-algebras isomorphic to it. The phase transitions between different phases with symmetries characterized by different but isomorphic sub-algebras are possible. This would mean kind of zoom-up or zoom-down of the phase without changing it otherwise. It is not clear whether this situation corresponds to what happens in KT phase transitions. If this is the case, the massless phase should appear also above critical temperature but in smaller scale.

3. KT transitions have also quantum counterpart and in TGD framework these would involve at criticality phase transitions changing the value of $h_{eff} = n \times h$ transforming different dark phases of matter to each other or to visible matter or vice versa. Cell membrane is in TGD framework a Josephson junction array with membrane proteins serving as Josephson junctions [K6, K1]. The associated magnetic flux tubes through the membrane would serve as ionic channels and pumps allowing to transfer dark ions through the membrane. It would not be surprising if KT transitions would take place in these systems. Vortices could be

associated with the 2-D flow of lipids or various ions at membrane inducing the braiding of the magnetic flux tubes connecting lipid layer to DNA codons or tubulins inside microtubules. This would make possible topological quantum computation [K2].

2 Topological order

Topological order (see `http://tinyurl.com/juu34ng`) is a further key notion and refers to phases whose proper description requires topological quantum field theory which describes those dynamical degrees of freedom which are not metric. TGD can be seen as almost topological QFT and it is interesting to see whether and how TGD based vision about quantum physics correspond to the vision of condensed matter theorists about topological order (see `http://tinyurl.com/juu34ng`).

It is indeed amazing to see how precisely the general concepts correspond to each other. There are also differences. In TGD one considers quantum phases and quantum phase transitions whereas topological order is thermodynamical notion. In ZEO the conformal symmetries of 2-D systems are also replaced with much higher symmetries. Also new space-time concept and more general view about quantum theory (ZEO and the hierarchy of Planck constants $h_{eff} = n \times h$) is necessary. ZEO suggests a unified description of thermodynamical and quantum phase transitions.

Consider first a brief summary about the notion of topological order (my understanding is not that of condensed matter specialist).

1. The notion of emergence is central in condensed matter physics and topological phases would mean emergence by a generation of long range entanglement. Whether long range entanglement is really possible is the basic problem. Dynamical gauge symmetries are also proposed to emerge.

2. Ordinary phase transitions are characterized by symmetry change and Landau's symmetry breaking theory theory provides a standard description. This description however fails for high temperature superconductors.

   This was learned by an attempt to model high temperature super conductors as spin liquids. Chiral spin liquid breaks time reversal and parity symmetry but not spin rotation symmetry was a good candidate. It was however found that it has large number of chiral spin states with the same symmetry so that symmetry was not enough to describe these states and the realization than Landau's paradigm is not enough. The resolution of the problem was the realization that the systems in question are not characterized by mere symmetry but also by topological order. One problem of standard quantum theory based description is parity breaking in long length scales encountered also in biology: here phases with large values of Planck constant identifiable as dark matter could come in rescue.

   Topological order is macroscopically characterized by the topological degeneracy of the ground state and non-Abelian geometric phases (Berry phases) provides a macroscopic description of the situation. Microscopic descriptions rely on long range entanglement.

   One can assign to Berry phases assignable to adiabatic evolutions of quantum states Abelian and even non-Abelian effective gauge fields and these can give rise to the analogs of magnetic monopoles. These monopoles do not reside in space-time but in parameter space in which the adiabatic time evolution takes place. For instance, the space of wave vectors can be this space in the case of quantum Hall effect.

3. Ground states of topological orders have description in terms of topological quantum field theories - 2+1-D Chern-Simons action serves as the basic example. Charge fractionization and the emergence of gap states as boundary states carrying fractional charges is a basic implication. Edge excitations are 1-D analogs of 2-D boundary states.

4. Local unitary gauge transformations realized as adiabatic evolutions change entanglement pattern locally and produced ground states equivalent with the original so that topological orders correspond to this kind of equivalence classes. Hence topological orders are characterized by long range entanglement (see `http://tinyurl.com/z7qjdv9`). This leads to tensor nets and classification of topological orders in terms of entanglement patterns.

Consider now the TGD view about topological order.

1. The notion of emergence is in TGD framework somewhat different. In TGD both elementary fermions and bosons emerge from fundamental fermions but space-time does not emerge as is fashionable to postulate. Neither would the various condensed matter phases emerge but be implied by the new physics predicted by TGD. Space-time does not emerge in TGD Universe: what emerges is conscious experience about space-time and flux tube networks make this possible.

   For instance, the generation of phases with entanglement in long length scale is somewhat a matter of belief in standard physics and is obtained only in large N limit of gauge theory description. In TGD Universe topological orders would be present already in the fundamental theory.

   In standard approach dynamical gauge symmetries as opposed to fundamental ones are assumed to emerge. In TGD one has besides standard model gauge symmetries also dynamically generated local gauge symmetries or Kac-Moody symmetries related to the hierarchy of inclusions of so called hyper-finite factors of type II_1.

   More precisely, given sub-algebra of super-symplectic algebra isomorphic to it and its commutator with the entire algebra would give rise to vanishing classical Noether charges realizing the strong form of holography (SH) and making 3-surface almost 2-D. This would leave gauge group or - more probably - Kac-Moody group assignable to an ADE Lie-group acting as dynamical symmetries. The hierarchy of Planck constants is closely related to this. These symmetries would be the counterpart for emergent gauge symmetries of standard approach and would correspond to dark matter.

   In principle these symmetries could act also in parameter space instead of space-time. Maybe this could allow to understand the gauge fields assignable to Berry phases in TGD framework at the fundamental level. ZEO inspires the postulate is that the phase transitions transforming matters with different vales of h_{eff} occur at quantum criticality or even criticality [K7].

2. Ground state degeneracy is one of the macroscopic characteristics of topological order. In TGD framework 4-D spin glass degeneracy meaning huge number of vacuum extremals of Kähler action would give rise to this kind of ground state degeneracy.

   The twistor lift of Kähler action however brings in volume term multiplied by extremely small (in recent cosmological scales) cosmological constant, which removes the degeneracy by replacing vacuum extremals with minimal surfaces. The approximate vacuum degeneracy however implies almost degeneracy since the volume depends very weakly on preferred extremal. This implies the failure of the perturbation theory around the extremals for which Kähler action vanishes due to the extremely large value of the inverse of the cosmological constant analogous to inverse of a coupling strength.

   Nature itself would solve the problem using the hierarchy of (effective) Planck constants $h_{eff} = n\times$. The space-time surfaces are replaced by singular coverings with n sheets such that sheets co-incide at the 3-D ends of space-time surface at boundaries of causal diamond (CD). Each sheet give the same contribution to the action so that the total action for ordinary Planck constant becomes n-fold. One can also say that effectively the coupling constant strength - call it α - is effectively replaced with α/n due to the introduction of effective Planck constant replacement $h_{eff} = n \times h$ so that perturbation theory converges. Note that the large N limit of gauge theories at which the topological orders appear has also $g^2 \rightarrow g^2/N$. The interpretation are however completely different.

   At QFT-GRT limit of TGD one lumps the essentially identical n sheets together and speaks about effective Planck constant $h_{eff} = n \times h$. In ore fundamental description using many-sheeted space-time one keeps the sheets as separate and uses h.

3. Chern-Simons action is essential in the description of topological orders. For ground states gauge fields vanish but gauge potentials cannot be gauge transformed away and the non-integrable phase factors characterize the state.

   The vision about TGD as almost topological QFT involves the reduction of Kähler action from the Minkowskian regions to Abelian Chern-Simons term at the boundaries separating

Minkowskian region and Euclidian regions. The presence of Euclidian regions is something new as compared to GRT space-time, which emerges at the QFT limit of TGD lumping together the sheets of many-sheeted space-time and replacing classical fields with the sums of contributions from different sheets. Euclidian regions or equivalently the light-like orbits of partonic surfaces identifiable as boundaries between Euclidian and Minkowskian regions have interpretation as "lines" of scattering diagrams.

For twistor lift of TGD cosmological constant and volume term appear in the dimensionally reduced 6-D Kähler action. The reduction to Chern-Simons term takes place only in the approximation that cosmological constant vanishes. The smallness of the cosmological constant forces $h_{eff} = n \times h$ phases and quantum entanglement in long length scales and also long range quantum coherence.

Kähler action is Abelian (it could be also identified as classical color action since classical color fields are proportional to the induced Kähler form). Electroweak gauge and color fields are however present in even long scales due to the hierarchy of Planck constants giving rise to scaled up Compton lengths.

The hierarchy of dynamically generated gauge symmetries (ADE type gauge groups or Kac-Moody algebras) could have description in terms of topological QFTs at QFT limit of TGD. These gauge symmetries could also act in parameter spaces instead of space-time.

4. In TGD framework SH allows to interpret macroscopic description in terms of geometry and topology of space-time surface whereas microscopic description would be in terms of states assignable to partonic 2-surfaces which can entangle if flux tubes connect the 2-surfaces. A concrete description would be in terms of tensor networks consisting of possibly braided flux tubes connecting partonic 2-surfaces. These networks can have large number of topologies and the states at partonic 2-surfaces connected by flux tube can entangle. Different connection patterns with various braidings determined topological ground states.

   Topological order could be a central element in quantum biology according to TGD. For instance, in the model for cell as a topological quantum computer [K2] one assumes that the DNA codons and/or tubulins of microtubules are connected to lipids of the lipid layer of cell membrane by flux tubes. The flow of the lipids (liquid crystal) would mean braiding in time direction inducing spatial braiding of flux tubes (dancers with feet connected to wall is a good metaphor here) allowing to store the lipid induced by nerve pulse pattern to the spatial braiding - one form of memory.

5. Gap and edge states have analogs at the fundamental level in TGD Universe. By SH 2-D partonic surfaces are the carriers of quantum numbers but the entanglement between states at different partonic 2-surfaces having magnetic flux tubes containing fermionic string as topological correlate (this is TGD variant of ER-EPR correspondence) brings in 3-dimensionality. The light-like orbits of partonic 2-surfaces are 2+1-D systems containing the light-like boundaries of string world sheets associated with orbits of flux tubes and the fermionic lines define counterparts of edge states. Therefore the attribute "topological" in TGD seems to be well-deserved also in this respect.

   The many-sheeted coveringse allow also to understand charge fractionization [K3] in terms of h_{eff}. One can say that the charge for single sheet of covering is fraction $1/n$ of the entire charge. In case of spin the ordinary 2π rotation in M^4 must be made n times before one reaches the starting point (think in terms of covering associated with $z^{1/n}$). Charge fractionizaton would be a direct evidence for the TGD view about space-time.

6. According to Wikipedia article, topological orders have a category theoretical description in 2+1 dimensions: in TGD framework light-like orbits of partonic 2-surfaces are excellent candidates in this respect and the braiding of fermion lines identifiable as string boundaries is one of the key aspects of quantum TGD at fundamental level. The new element is light-likeness giving rise to a huge extension of conformal symmetries at both boundary of CD and at partonic orbits. Light-likeness of the orbit of partonic surface requires that the boundary term to which Kähler action from Minkoskian region reduces is independent of the induced metric and hence reduces to Chern-Simons term for Kähler action. Note however that non-Abelian dynamically generated gauge fields can be present.

> A further new element would be p-adic scale hierarchies associated with CDs. Dark matter hierarchy associated with the hierarchy of Planck constants might be essential for the understanding of the long range entanglement. Quantum criticality at which phase transitions changing h_{eff} happen and generate long range fluctuations is also a new element and could finally lead to the understanding of dark matter. For instance, one can have quantum superpositions of different tensor net topologies realized in terms of flux tubes connecting partonic 2-surfaces.

Topological materials have become a hot topics of condensed matter physics. One of the key ideas is that one can have topological analog of condensed matter. One has topological objects with discrete spectrum of energies and boundary states assignable to lattice like structures could possess energy gaps and energy bands. One example might be cell membrane consisting of lipids. For instance, topological insulators (see `http://tinyurl.com/hr9b3z6`) are insulators in 3-D bulk but have surface states, which are conducting.

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