E_8 theory of Garrett Lisi and TGD

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Abstract

Recently (towards end of the year 2007) there has been a lot of fuss about the E_8 theory proposed by Garrett Lisi in physics blogs, in media, and even New Scientist wrote about the topic. There are serious objections against Lisi's theory and it is interesting to find whether the theory could be modified so that it would survive the basic objections. Although it seems that Lisi's theory cannot be saved, one achieves further insights about HO-H ($M^8 - H$) duality. Number theoretical spontaneous compactification can be formulated in terms of the Kac-Moody algebra assignable to Poincare group and standard model gauge group having also rank 8. The representation can be constructed in standard manner using quantized M^8 coordinates at partonic 2-surfaces. Also E_8 representations are in principle possible and the question concerns their physical interpretation.

1 Introduction

Recently (towards end of the year 2007) there has been a lot of fuss about the E_8 theory proposed by Garrett Lisi [B3] in physics blogs, in media, and even New Scientist [B2] wrote about the topic. There are serious objections against Lisi's theory and it is interesting to find whether the theory could be modified so that it would survive the basic objections. Although it seems that Lisi's theory cannot be saved, one achieves further insights about HO-H ($M^8 - H$) duality. Number theoretical spontaneous compactification can be formulated in terms of the Kac-Moody algebra assignable to Poincare group and standard model gauge group having also rank 8. The representation can be constructed in standard manner using quantized M^8 coordinates at partonic 2-surfaces. Also E_8 representations are in principle possible and the question concerns their physical interpretation.

1.1 Objections against Lisi's theory

The basic claim of Lisi is that one can understand the particle spectrum of standard model in terms of the adjoint representation of a non-compact version E_8 group [B1] . There are several objections against E_8 gauge theory interpretation of Lisi.

1. Statistics does not allow to put fermions and bosons in the same gauge multiplet. Also the identification of graviton as a part of a gauge multiplet seems very strange if not wrong since there are no roots corresponding to a spin 2 two state.

2. Gauge couplings come out wrong for fermions and one must replace YM action with an ad hoc action.
3. Poincare invariance is a problem. There is no clear relationship with the space-time geometry so that the interpretation of spin as E_8 quantum numbers is not really justified.
4. Finite-dimensional representations of non-compact E_8 are non-unitary. Non-compact gauge groups are however not possible since one would need unitary infinite-dimensional representations which would change the physical interpretation completely. Note that also Lorentz group has only infinite-D unitary representations and only the extension to Poincare group allows to have fields transforming according to finite-D representations.
5. The prediction of three fermion families is nice but one can question the whole idea of putting particles with mass scales differing by a factor of order 10^{12} (top and neutrinos) into same multiplet. For some reason colleagues stubbornly continue to see fundamental gauge symmetries where there seems to be no such symmetry. Accepting the existence of a hierarchy of mass scales seems to be impossible for a theoretical physicist in main main stream although fractals have been here for decades.
6. Also some exotic particles not present in standard model are predicted: these carry weak hyper charge and color (6-plet representation) and are arranged in three families.

2 Three attempts to save Lisi's theory

To my opinion, the shortcomings of E_8 theory as a gauge theory are fatal but the possibility to put gauge bosons and fermions of the standard model to E_8 multiplets is intriguing and motivates the question whether the model could be somehow saved by replacing gauge theory with a theory based on extended fundamental objects possessing conformal invariance.

1. In TGD framework H-HO duality allows to consider Super-Kac Moody algebra with rank 8 with Cartan algebra assigned with the quantized coordinates of partonic 2-surface in 8-D Minkowski space M^8 (identifiable as hyper-octonions HO). The standard construction for the representations of simply laced Kac-Moody algebras allows quite a number of possibilities concerning the choice of Kac-Moody algebra and the non-compact E_8 would be the maximal choice.
2. The first attempt to rescue the situation would be the identification of the weird spin 1/2 bosons in terms of supersymmetry involving addition of right-handed neutrino to the state giving it spin 1. This options does not seem to work.
3. The construction of representations of non-simply laced Kac-Moody algebras (performed by Goddard and Olive at eighties [A2]) leads naturally to the introduction of fermionic fields for algebras of type B, C, and F: I do not know whether the construction has been made for G_2. E_6, E_7, and E_8 are however simply laced Lie groups with single root length 2 so that one does not obtain fermions in this manner.
4. The third resuscitation attempt is based on fractional statistics. Since the partonic 2-surfaces are 2-dimensional and because one has a hierarchy of Planck constants, one can have also fractional statistics. Spin 1/2 gauge bosons could perhaps be interpreted as anyonic gauge bosons meaning that particle exchange as permutation is replaced with braiding homotopy. If so, E_8 would not describe standard model particles and the possibility of states transforming according to its representations would reflect the ability of TGD to emulate any gauge or Kac-Moody symmetry.

The standard construction for simply laced Kac-Moody algebras might be generalized considerably to allow also more general algebras and fractionization of spin and other quantum numbers would suggest fractionization of roots. In stringy picture the symmetry group would be reduced considerably since longitudinal degrees of freedom (time and one spatial direction) are non-physical. This would suggest a symmetry breaking to $SO(1,1) \times E_6$ representations with ground states created by tachyonic Lie allebra generators and carrying mass squared 2 in suitable units. In TGD framework the tachyonic conformal weight can be compensated by super-symplectic conformal weight so that massless states getting their masses via Higgs mechanism and p-adic thermodynamics would be obtained.

2.1 Could super-symmetry rescue the situation?

E_8 is unique among Lie algebras in that its adjoint rather than fundamental representation has the smallest dimension. One can decompose the 240 roots of E_8 to 112 roots for which two components of SO(7,1) root vector are $\pm$ 1 and to 128 vectors for which all components are $\pm$ 1/2 such that the sum of components is even. The latter roots Lisi assigns to fermionic states. This is not consistent with spin and statistics although SO(3,1) spin is half-integer in M^8 picture.

The first idea which comes in mind is that these states correspond to super-partners of the ordinary fermions. In TGD framework they might be obtained by just adding covariantly constant right-handed neutrino or antineutrino state to a given particle state. The simplest option is that fermionic super-partners are complex scalar fields and sbosons are spin 1/2 fermions. It however seems that the super-conformal symmetries associated with the right-handed neutrino are strictly local in the sense that global super-generators vanish. This would mean that super-conformal super-symmetries change the color and angular momentum quantum numbers of states. This is a pity if indeed true since super-symmetry could be broken by different p-adic mass scale for super partners so that no explicit breaking would be needed.

2.2 Could Kac Moody variant of E_8 make sense in TGD?

One can leave gauge theory framework and consider stringy picture and its generalization in TGD framework obtained by replacing string orbits with 3-D light-like surfaces allowing a generalization of conformal symmetries.

H-HO duality is one of the speculative aspects of TGD. The duality states that one can either regard imbedding space as $H = M^4 \times CP_2$ or as 8-D Minkowski space M^8 identifiable as the space HO of hyper-octonions which is a subspace of complexified octonions. Spontaneous compactification for M^8 described as a phenomenon occurring at the level of Kac-Moody algebra would relate HO-picture to H-picture which is definitely the fundamental picture. For instance, standard model symmetries have purely number theoretic meaning in the resulting picture.

The question is whether the non-compact E_8 could be replaced with the corresponding Kac Moody algebra and act as a stringy symmetry. Note that this would be by no means anything new. The Kac-Moody analogs of E_{10} and E_{11} algebras appear in M-theory speculations. Very little is known about these algebras. Already $E < sub > n < /sub >, n > 8$ is infinite-dimensional as an analog of Lie algebra. The following argument shows that E_8 representations do not work in TGD context unless one allows anyonic statistics.

1. In TGD framework space-time dimension is D=8. The speculative hypothesis of HO-H duality inspired by string model dualities states that the descriptions based on the two choices of imbedding space are dual. One can start from 8-D Cartan algebra defined by quantized M^8 coordinates regarded as fields at string orbit just as in string model. A natural constraint is that the symmetries act as isometries or holonomies of the effectively compactified M^8. The article "The Octonions" [A1] of John Baez discusses exceptional Lie groups and shows that compact form of E_8 appears as isometry group of 16-dimensional octo-octonionic projective plane $E_8/(Spin(16)/Z_2)$: the analog of CP_2 for complexified octonions. There is no 8-D space allowing E_8 as an isometry group. Only SO(1,7) can be realized as the maximal Lorentz group with 8-D translational invariance.

2. In HO picture some Kac Moody algebra with rank 8 acting on quantized M^8 coordinates defining stringy fields is natural. The charged generators of this algebra are constructible using the standard recipe involving operators creating coherent states and their conjugates obtained as operator counterparts of plane waves with momenta replaced by roots of the simply laced algebra in question and by normal ordering.

3. Poincare group has 4-D maximal Cartan algebra and this means that only 4 Euclidian dimensions remain. Lorentz generators can be constructed in standard manner in terms of Kac-Moody generators as Noether currents.

4. The natural Kac-Moody counterpart for spontaneous compactification to CP_2 would be that these dimensions give rise to the generators of electro-weak gauge group identifiable as a product of isometry and holonomy groups of CP_2 in the dual H-picture based on $M^4 \times CP_2$. Note that

in this picture electro-weak symmetries would act geometrically in E^4 whereas in CP_2 picture they would act only as holonomies.

Could one weaken the assumption that Kac-Moody generators act as symmetries and that spin-statistics relation would be satisfied?

1. The hierarchy of Planck constants relying on the generalization of the notion of imbedding space breaks Poincare symmetry to Lorentz symmetry for a given sector of the world of classical worlds for which one considers light-like 3-surfaces inside future and past directed light cones. Translational invariance is obtained from the wave function for the position of the tip of the light cone in M^4. In this kind of situation one could consider even E_8 symmetry as a dynamical symmetry.

2. The hierarchy of Planck constants involves a hierarchy of groups and fractional statistics at the partonic 2-surface with rotations interpreted as braiding homotopies. The fractionization of spin allows anyonic statistics and could allow bosons with anyonic half-odd integer spin. Also more general fractional spins are possible so that one can consider also more general algebras than Kac-Moody algebras by allowing roots to have more general values. Quantum versions of Kac-Moody algebras would be in question. This picture would be consistent with the view that TGD can emulate any gauge algebra with 8-D Cartan algebra and Kac-Moody algebra dynamically. This vision was originally inspired by the study of the inclusions of hyper-finite factors of type II¡sub¿1¡/sub¿. Even higher dimensional Kac-Moody algebras are predicted to be possible.

3. It must be emphasized that these considerations relate in TGD framework to Super-Kac Moody algebra only. The so called super-symplectic algebra is the second quintessential part of the story. In particular, color is not spin-like quantum number for quarks and quark color corresponds to color partial waves in the world of classical worlds or more concretely, to the rotational degrees of freedom in CP_2 analogous to ordinary rotational degrees of freedom of rigid body. Arbitrarily high color partial waves are possible and also leptons can move in triality zero color partial waves and there is a considerable experimental evidence for color octet excitations of electron and muon but put under the rug.

2.3 Can one interpret three fermion families in terms of E_8 in TGD framework?

The prediction of three fermion generations by E_8 picture must be taken very seriously. In TGD three fermion generations correspond to three lowest genera $g = 0, 1, 2$ (handle number) for which all 2-surfaces have Z_2 as global conformal symmetry (hyper-ellipticity [K1, K2]). One can assign to the three genera a dynamical SU(3) symmetry. They are related by SU(3) triality which brings in mind the triality symmetry acting on fermion generations in E_8 model. SU(3) octet and singlet bosons correspond to pairs of light-like 3-surfaces defining the throats of a wormhole contact and since their genera can be different one has color singlet and octet bosons. Singlet corresponds to ordinary bosons. Color octet bosons must be heavy since they define neutral currents between fermion families.

The three E_8 anyonic boson families cannot represent family replication since these symmetries are not local conformal symmetries: it obviously does not make sense to assign a handle number to a given point of partonic 2-surface! Also bosonic octet would be missing in E_8 picture.

One could of course say that in E_8 picture based on fractional statistics, anyonic gauge bosons can mimic the dynamical symmetry associated with the family replication. This is in spirit with the idea that TGD Universe is able to emulate practically any gauge - or Kac-Moody symmetry and that TGD Universe is busily mimicking also itself.

To sum up, the rank 8 Kac-Moody algebra - emerging naturally if one takes HO-H duality seriously - corresponds very naturally to Kac-Moody representations in terms of free stringy fields for Poincare-, color-, and electro-weak symmetries. One can however consider the possibility of anyonic symmetries and the emergence of non-compact version of E_8 as a dynamical symmetry, and TGD suggests much more general dynamical symmetries if TGD Universe is able to act as the physics analog of the Universal Turing machine.

REFERENCES

Mathematics

[A1] John C. Baez. The Octonions. *Bull. Amer. Math. Soc.*. *http://math.ucr.edu/home/baez/Octonions/octonions.html*, 39(2002), 2001.

[A2] A. Kent P. Goddard and D. Olive. Unitary representations of the Virasoro and super-Virasoro algebras. *Comm. Math. Phys.*, 103(1), 1986.

Theoretical Physics

[B1] E_8. http://en.wikipedia.org/wiki/E8_(mathematics).

[B2] Is mathematical pattern the theory of everything? *New Scientist.* *http://www.newscientist.com/channel/fundamentals/mg19626303.900-is-mathematical-pattern-the-theory-of-everything.html*, 2630.

[B3] G. Lisi. An exceptionally simple theory of everything. http://www.arxiv.org/pdf/0711.0770, 2007.

Books related to TGD

[K1] M. Pitkänen. Construction of elementary particle vacuum functionals. In *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.* Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#elvafu, 2006.

[K2] M. Pitkänen. Massless states and particle massivation. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy.* Onlinebook. http://tgdtheory.fi/public_html/paddark/paddark.html#mless, 2006.