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Electromagnetic fields in Maxwell's theory and in Topological Geometrodynamics

M. Pitkänen *
Email: matpitka6@gmail.com

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This article considers the differences between the notions of electromagnetic field according to Maxwell's theory and Topological Geometrodynamics (TGD) (see my homepage `http://www.tgdtheory.fi` and the article `http://tinyurl.com/zrx5mdz`). The discussion actually applies also to electroweak, color, and gravitational fields. All these fields emerge from much more simpler dynamics of space-time surfaces in certain 8-dimensional space-time uniquely fixed to be $H = M^4 \times CP_2$, where M^4 is Minkowski space-time of Special Relativity (SRT) and CP_2 so called complex projective space with 4 real dimensions replacing points of M^4.

Consider first Maxwell's theory.

1. Maxwell's equations consists of two pairs of equations. The first pair does not involve currents and charge densities. The first equation states that there are no magnetic charges. Local form says that the magnetic field has vanishing divergence: $\nabla \cdot B = 0$. Second equation formulates Faraday's law stating that time dependent magnetic field is accompanied by an electric field not expressible as gradient of potential. Local form states $\partial_t B = -\nabla \times E$ ($c = 1$).

   Second pair says that charge density and current serve as sources of electromagnetic fields: $\nabla \cdot E = -\rho$ and $\nabla \times B = j - \partial_t E$. Unlike the first pair, these equations require a model for charged matter.

   Maxwell's equations are simple linear wave equations derivable from minimization of Maxwell action and allowing in absence of currents radiation fields as a general solution. In presence of currents and charge densities also static magnetic and electric fields are possible.

2. Maxwell's equations have two exceedingly important symmetries. The first symmetry is Poincare invariance, which led to the discovery of special relativity theory by Einstein. Translations and Lorentz transformations leaving the 4-dimensional distance function $s^2 = t^2 - x^2 - y^2 - z^2$ unchanged (generalization of law of Pythagoras) leave light velocity invariant as maximal signal velocity. These symmetries form the basics of entire particle physics.

   Second symmetry - gauge invariance - guarantees magnetic charges vanish and electric charge is conserved. Gauge invariance allows to express electric and magnetic fields in terms of scalar potential Φ and vector potential A (the expression is not unique): $(E = -\nabla\Phi, B = \nabla \times A)$. Gauge symmetry is the starting point in the generalizations of Maxwell's field to non-Abelian gauge fields representing electroweak and color gauge fields in standard model.

3. In the relativistic formulation in the 4-D Minkowski space M^4 of special relativity (SRT) with linear coordinates (t, x, y, z) and endowed with above mentioned distance function electric and magnetic fields combine to single antisymmetric tensor $F \leftrightarrow (E, B)$ expressible in terms of 4-vector gauge potential $A \leftrightarrow (\Phi, A)$. Charge density and current are combined to 4-dimensional current.

TGD starts from the dream of Einstein.

*Matti Pitknen is a theoretical physics working as a free researcher in Finland. He has developed Topological Geometrodynamics (TGD) during the last 39 years (PhD 1982). The material related to TGD can be found at homepage (`http://tgdtheory.fi`), in blog (`http://matpitka.blogspot.fi`), in Research Gate, and as various published articles.

1. Einstein constructed two theories: Special Relativity (SRT) and General Relativity (GRT) reducing gravitation to dynamical space-time geometry. STR relies on relativity principle and GRT on General Coordinate Invariance (GCI) and Equivalence Principle (EP). Both theories have been extremely successful. Einstein wanted to extend his successful geometrization of gravitation to electromagnetism but failed.

   To proceed one must identify the problem. The basic problem of General Relativity (GRT) relates to the notions of energy and momentum well-defined in SRT but not in GRT. By Noether's theorem these conservation laws follow from the symmetries of space-time geometry (Poincare invariance) but since translational and rotational symmetries of SRT are lost in the curved space-time of GRT, also conservation laws are also lost. Could one have a variant of GRT for which Poincare invariance is not lost?

2. One can! One must only generalize string models. Poincare symmetry requires 4-D Minkowski space M^4. What if space-time X^4 is a 4-D *surface* in some higher-dimensional space $M^4 \times S$, S some internal space? Geometric symmetries would not be symmetries of space-time surface X^4 but those of empty Minkowski space M^4! The symmetries would not shift points of X^4 but the entire X^4, behaving like 4-D rigid body! One would replace point like particles with 3-D surfaces and world lines with X^4.

3. How to geometrize classical fields in this framework? $H = M^4 \times S$ has Riemannian metric defining length measurement and one can induce this metric to X^4: one just measures distances at X^4 using the metric of H. This is exactly what one does for 2-D surfaces in 3-D space.

   Classical gravitational field emerges from the dynamics of space-time surface X^4 determined by some action principle. By general coordinate invariance (GCI) telling that 4 suitably chosen coordinates of H serve as primary dynamical field variables - a huge simplification.

   How to geometrize electroweak and color fields? The choice $S = CP_2$ allows to achieve this. Classical electroweak gauge potentials would be induced from the spinor connection of CP_2 and would have correct coupling structure. One performs parallel translation using the spinor connection of H. One can identify color gauge potentials as projections of so called Killing vectors of CP_2 representing infinitesimal isometries. One can also induce the generalized spinor structure of $H = M^4 \times CP_2$. This explains standard model quantum numbers. Twistorial considerations fix $M^4 \times CP_2$ as the only possible option.

Electromagnetic field (all classical fields) is geometrized and inherits the dynamics of X^4. The first guess for the action principle was so called Kähler action S_K, a non-linear geometric analog of Maxwell action. S_K is not quite enough: also 4-D volume term V forced by so called twistor lift of TGD and interpretable in terms of cosmological constant is also needed.

The known non-vacuum extremals of S_K are also minimal surfaces extremizing the volume term V. This is expected to be true generally. Minimal surface property means that non-linear variant of massless d'Alembert equation with gravitational self-coupling and generalizing Laplacian equation for Newtonian gravity is satisfied. Extremal property for Kähler action defines the analog for the Maxwell's equations. Gravitational dynamics and Maxwell dynamics decouple but only apparently.

One can study space-time surfaces as solutions of the field equations.

1. 4 primary field like variables is certainly not enough to describe the physics as we know it. One can however use space-time sheets as building bricks to engineer more complex space-time surfaces - many-sheeted space-time with fractal hierarchical structure. Given sheet of many-sheeted space-time carries smaller sheets glued to it by topological sum contacts (wormhole contacts) and is in turn glued to larger sheet: particles consist of particles! We "see" these sheets and identify them as matter in background space-time: the wild topology of many-sheeted space-time is directly visible as "matter" but we do not realize this!

2. 3-surfaces are quite generally bounded. Either 3-surface develops an outer boundary - or more plausibly, is a covering space obtained by gluing two 3-surfaces along their outer boundaries to form a single 3-surface without boundary. The reason is that Maxwell gauge potentials defining a linear field are effectively replaced with the 4 coordinates of compact space CP_2. By

compactness global imbeddings of arbitrary gauge field fail. Space-time surface decomposes to topological field quanta.

Topological quantization occurs for both sources and fields. Elementary particles have regions of space-time with Euclidian signature of induced metric (something new!) as building bricks; the TGD counterpart of classical radiation field decomposes to topological light-rays analogous to laser beams; magnetic field decomposes to flux quanta, flux tubes and flux sheets, and so on.

3. The notions of field body and magnetic body emerge. Every system creates classical fields giving rise to field body - field identity of system. This is not possible in Maxwell's theory, where the fields of all systems interfere. The notion of magnetic body (MB) has become central in TGD inspired quantum biology. MB can be seen as intentional agent using biological body as a motor instrument and sensory receptor. This also allows to define the notion of coherence: only fields at the same sheet can interfere.

4. There is objection against this picture. The linear superposition of Maxwell's theory and classical field theories is lost and applies only for modes of topological light rays representing radiation moving in fixed direction. This is not a catastrophe. The physical motivation for the linear superposition is that the forces caused by different systems on particle sum up in a good approximation. One introduces fields and expresses forces in terms of them and thus reduces superposition of forces to that for fields.

   In TGD framework one can look what happens for a particle - small 3-surface in many-sheeted space-time. Space-time sheets can be envisaged as 4-D analogs of slightly deformed planes extremely near to each other (the distance cannot be larger than CP_2 size scale). Particle 3-surface necessarily touches these sheets and experiences the sum of forces caused by the induced fields at sheets. Superposition for classical fields is replaced with set theoretic union of corresponding space-time sheets implying superposition of their effects.

   This allows to understand GRT-QFT limit of TGD. In long length scales the many-sheeted space-time is replaced with single slightly curved region of M^4. Classical gravitational field as deviation of metric from flat M^4 metric is identified as sum of corresponding deviations of the induced metrics for space-time sheets. The induced gauge potentials are identified as sums of those for space-time sheets. Since the number of space-time sheets can be very large, the complexity of GRT-QFT emerges. The classical dynamics of single sheet is extremely simple.

Many-sheetedness and the notion of field body mean deviations from GRT-QFT picture manifesting as anomalies. I have discussed these anomalies in various books and online books about TGD.