Comparing the Berry phase model of super-conductivity with the TGD based model

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Abstract

Hiroyasu Koizumi has proposed a new theory of superconductivity (SC) based on the notion of Berry phase related with an effective magnetic field assignable to adiabatically evolving systems. The model shares similarities with the TGD inspired view about SC. The article also mentioned anomalies that were new to me. This motivated a fresh look in the TGD inspired model. The outcome was an integration of two separate ideas about supraphases.

1. Space-time surfaces as preferred extremals with CP_2 projection of dimension $D = 2$ or $D = 3$ would naturally correspond to 4-D generalizations of so called Beltrami flows, which are integrable flows defined by the flow lines of the induced Kähler field. The existence of a global coordinate z varying along flow lines requires the integrability of the flow. Classical dissipation is absent so that these surfaces are excellent candidates for the space-time correlates of supra flows. The exponential of z gives a phase factor associated with the complex order parameter of a coherent state of Cooper pairs as a counterpart of the Berry phase. Kähler magnetic monopole flux defines the TGD counterpart of "novel" magnetic field.
2. The identification of supra phases as dark matter as $h_{eff} > h$ phases at magnetic flux quanta (tubes and sheets) implies that Cooper pairs correspond to dark fermions associated with the members of flux tube pair, which actually combine to form a closed flux tube. Also single electrons can define supraflow.
3. The Cooper pairs must be created by bosonic oscillator operators constructed from fermionic oscillator operators by bosonization. This is possible only in 1+1-dimensional situations. Thanks to the Beltrami flow the situation is effectively 1+1-dimensional. Bosonization makes it possible to identify SU(2) Kac-Moody algebra, which has an interpretation in the TGD framework.

The assumption that Cooper pairs reside at the magnetic flux quanta solves the 4 problems of standard framework mentioned by Koizumi: high-Tc SCs have two transition temperatures; electron mass m_e instead of its effective mass m_e^* appears in Thomson moment; the reversible phase transition in an external magnetic field inducing a splitting of Cooper pairs does not involve dissipation; why the erratic calculation of the Josephson frequencies in standard model neglecting the chemical potentials gives a correct result?.

The formation of the Cooper pairs appears as a condition stabilizing the space-time sheets carrying dark matter and all preferred extremals could satisfy the conditions guaranteeing integrable flow and existence of a phase factor varying along flow lines. Could supra phases exist in all scales? Could the breaking of supra phases be only due to the finite size of the space-time sheets? Could even hydrodynamic flow involve super-fluidity of some kind - perhaps based on neutrino Cooper pairs as speculated earlier?

Contents

1 Introduction

Hiroyasu Koizumi has proposed a new theory of superconductivity (SC) based on the notion of Berry phase related with an effective magnetic field assignable to adiababically evolving systems. I learned about the theory from a popular article published in Scitechdaily (`https://cutt.ly/LmS4tOl`).

A more technical description of the model can be found in an article [D7])(`https://cutt.ly/WmBkIsp`) by Koizumi. The article has title "Superconductivity by Berry connection from many-body wave functions: revisit to Andreev-Saint-James reflection and Josephson effect".

1.1 Summary of Berry phase model

The Berry phase model (BPM) explains SC as an implication of a collective phase for which the Berry phase would be a prerequisite. Berry connection acting on the space of quantum states rather than in the space of gauge fields. My interpretation about the basic aspects of the Berry phase theory formed on basis of abstract of [D7] is following:

1. In standard model of super-conductity Cooper pairs form a coherent states which is not an eigenstate of electron number. In the new theory fermion number is conserved for Cooper pairs in collective phase and electrons in single electron phase.

2. If Berry connection is non-trivial, it gives rise to a collective mode that generates super-current. This collective mode creates number-changing operators for particles participating in this mode, and these number-changing operators stabilize the superconducting state by exploiting the Cooper instability.

   In the new theory, the role of the electron-pairing is to stabilize the nontrivial Berry connection; it is not the cause of SC: also ordinary electrons in collective phase flow without dissipation.

3. In BCS SCs the simultaneous appearance of the nontrivial Berry connection and the electron-pairing occurs. Therefore, the electron-pairing amplitude can be used as an order parameter for the super- conducting state and corresponds to Berry phase. In high-Tc SCs the temperatures for the formation of Cooper pairs and for the appearance of SC are different.

4. Andreev-Saint-James reflection [D3] and Josephson effect are explained as consequences of the presence of the Berry connection. Bogoliubov quasiparticles are created by superpositions of creation and annihilation operators and utilized also in the BCS model as a convenient tool to diagonalize the kinetic part of the Hamiltonian. In Berry phase model they are replaced by particle-number conserving Bogoliubov excitations that describe the transfer of electrons between the collective and single particle modes.

The assumption of the model for Josephson effect inducing critics is that the the current in the junctions consists of electrons rather than Cooper pairs.

1. The model treats Josephson junction as an insulator rather than piece of a super-conductor. The model predicts two distinct cases corresponding depending on whether junction is a) thin or b) thick. For a) the Bogoliubov excitations for the two SCs are assumed to be identical. For b) they are not identified. For a) the effect is a first order effect and for b) a second order effect.

2. In BPM a) explains the AC Josephson effect as first order effect when chemical potential difference is taken into account. The supercurrent would be a flow of electrons brought about by the non-trivial Berry connection, which provides an additional U(1) gauge field besides the electromagnetic one.

   This conclusion is due to the presence of chemical potential difference equal to Coulomb energy in equilibrium, otherwise the Josephson frequency spectrum would come as even integer multiples of Josephson frequency.

3. Case b) is the one considered in the standard theory. The effect is second order effect also in the BCS model. If the chemical potential difference between the two SCs is neglected, the model gives BCS prediction for Josephson frequencies.

   If the the chemical potential difference is taken into account in BCS model, the Josephson frequency spectrum would come as half integer multiples of Josephson frequency. The same prediction follows for b) also in the BPM. Some evidence for this kind of half-odd integer spectrum has been repoported.

Berry phase theory is highly interesting from TGD view point and the comparison with TGD based view about SC is well-motivated.

1. In TGD space-times can be regarded as surfaces in $M^4 \times CP_2$. The effective magnetic field related to the Berry phase has as its TGD counterpart the monopole flux part of the ordinary magnetic field made possible by the non-trivial homology of CP_2 and having no Maxwellian counterpart. Monopole flux assignable magnetic flux tubes carrying also dark matter as $h_{eff} = n_0 > h$ phases of ordinary matter.

2. The existence of Berry phase corresponds to an existence of a phase defined by an angle like coordinate varying along flow lines of an integrable flow associated with induced Kähler field. Integrability is the geometric condition for the existence of the flow and thus superfluid flow or supracurrent.

   The integrable flow is a 4-D generalization [K9, K10, K3] of the notion of 3-D Beltrami (magnetic) field [B1, B5, B3, B4]. There is no classical counterpart of dissipation as quantum-classical correspondence suggests and the surfaces in question are minimal surfaces as one expects [L25]. Generalized Beltrami flows are possible if the dimension D of CP_2 projection of the space-time surface satisfies $D \leq 4$.

3. The members of Cooper pairs in TGD picture would be associated with parallel flux tubes, which form a closed flux tube in a long enough scale. This kind of connections by flux tube pairs can be formed by reconnection of U-shaped flux tube tentacles between two systems and play crucial role in the TGD based model of quantum biology [L31].

4. The formation of Cooper pairs accurs at the level of ordinary matter and the liberation of binding energy in their formation allows their transfer to the flux tubes, where dissipation is absent or at least slower by the large value of h_{eff}. Ordinary macroscopic SC requires high enough value of h_{eff} making possible long enough U-shaped flux loops and thus flux tube pairs.

   As in BPM, the creation of Cooper pairs stabilizes the flux tubes and makes possible non-dissipative currents of both electrons and their Cooper pairs so that SC as non-dissipative current flow is possible also for electrons.

5. Cooper pairs are in a coherent state at the flux tubes and this gives very simple effective Hamiltonian describing the interactions with ordinary matter. The outcome has a lot of common with the standard theory of SC. Both Cooper pairs and electrons are possible supracurrent carriers. The assumption Cooper pairs are at magnetic flux tubes allows however to circumvent the anomalies of the standard models.

6. The Cooper pairs must be created by bosonic oscillator operators constructed from fermionic oscillator operators by bosonization. This is possible only in 1+1-dimensional situations. Thanks to the Beltrami flow the situation is effectively 1+1-dimensional. Bosonization makes it possible to identify SU(2) Kac-Moody algebra, which has an interpretation in the TGD framework.

1.2 BPM, TGD based model, and the anomalies

The BPM is claimed to solve several basic problems of the standard model of SC. Also TGD based model suggests a solution to these anomalies based on the assumption that electrons and Cooper pairs are dark in TGD sense and reside at magnetic flux tubes.

1. High-Tc-superconductivity remains poorly understood. My understanding of the BPM is too limited to allow how it could increase the understanding in this respect.

2. The presence of 2 transition temperatures means that Cooper pairs emerge at higher critical temperature and SC at a lower critical temperature remains poorly understood. BPM predicts that the presence of Cooper pairs stabilizes the collective phase and is a prerequisite for the Berry phase in turn making possible non-dissipative flow. To me this would suggest that these two transition temperatures are identical.

   TGD: The flux tube pairs forming closed flux tubes serve as carriers of Cooper pairs. The first transition temperature would give rise to rather short flux tubes and SC in short scales and second transition temperature to rather long flux tube and SC in long scales with a larger value of h_{eff} (the scale of quantum coherence scales like h_{eff}).

3. The experimental finding is that London magnetic moment depends on the real mass m_e of electron rather than effective mass m_e^* . If supracurrent flows at the level of ordinary matter, one would expect the appearance of m_e^*. BPM explains this if the collective phase is separate from the ordinary phase.

   TGD: The dark electrons at magnetic flux tubes would not interact directly with condensed matter so that the real mass would appear in the expression of London moment caused by rotation of dark electrons.

4. It has found that the phase transition from SC to ordinary phase in an external magnetic field does not cause dissipation although one would expect this if Cooper pairs split to ordinary electrons. This can be understood in BPM if electrons in collective phase do not dissipate and thus do not interact with ordinary matter.

   TGD: The dissipation would be absent if the dark electrons from the split Cooper pairs do not dissipate. This is indeed true. One could thus talk about analogs of supra currents for electrons.

The proposed view encourages several questions. The formation of the Cooper pairs appears as a condition stabilizing the space-time sheets carrying dark matter and all preferred extremals could satisfy the conditions guaranteeing integrable flow and existence of a phase factor varying along flow lines. Could supra phases as non-dissipative phases of also fermionic states exist in all scales? Could the breaking of supra phase property be only due to the finite size of the space-time sheets? Could even hydrodynamic flow involve super-fluidity of some kind - perhaps based on neutrinos or neutrino Cooper pairs as speculated earlier?

2 TGD based model of superconductivity

TGD inspired model of super-conducitivity has developed slowly during years [K9, K10] [L5] [L10].

2.1 Brief summary of TGD based model

The breakthrough came around 2005 with the emergence of the idea about a hierarchy of phases of ordinary matter having non-standard value $h_{eff} = nh_0$ of Planck constant having arbitrarly large values and behaving in many respects like dark matter. Super-conducting phases would reside at the magnetic flux tubes carrying monopole flux and large value of h_{eff} would be crucial for their stability even at high temperatures.

2.1.1 General mechanism of superconductivity in TGD framework

The ideas about high temperature SC have evolved gradually as a reaction to experimental input and evelution in the understanding of TGD.

1. The many-sheeted space-time concept suggests a very general mechanism of SC based on a transfer of charged particles from atomic space-time sheets to larger space-time sheets. Later these space-time sheets were identified as magnetic flux tubes carrying as $h_{eff} = nh_0$ phases behaving like dark matter.

   The first guess was that larger space-time sheets are very dry, cool and silent so that the necessary conditions for the formation of high T_c macroscopic quantum phases are met. The criticism against this model was that particles touch all space-time sheets having non-empty Minkowski space projection to the region where the particle is so that thermal equilibrium is generated. Darkness as $h_{eff} > h$ property would allow even same temperature since various energy scales would typically scale like h_{eff} implying thermal stability.

   One must however take the assumption about thermal equilibrium with a grain of salt. The TGD based model for the aging of a living system [L32] assumes that the space-time sheets carrying dark matter slowly approach thermal equilibrium with the space-time sheets carrying ordinary matter [L30]. The slow approach to thermal equilibrium would be due to a small amount of dissipation at flux tubes.

2. The possibility of large h_{eff} quantum coherent phases makes the assumption about thermal isolation between space-time sheets un-necessary. In the model to be discussed in this article Cooper pairs are created at the level of ordinary matter by standard mechanisms and transferred to flux tubes.

3. It became clear quantum criticality predicting a new kind of SC explaining the strange features of high T_c SC is essential. Two kinds of Cooper pairs, or rather flux tubes are assumed. They correspond to a different values of $h_{eff} > h$. Either the Cooper pairs or flux tubes with smaller value of h_{eff} have shorter life time (proportional to h_{eff}). Both Cooper pairs and flux tubes correspond to super-conductivity but in different time and length scales. In the transition to SC in long scales the closed but short flux tubes looking like flux tube pairs reconnect to long flux tubes.

   Below temperature $T_{c_1} > T_c$ only the Cooper pairs with smaller value of h_{eff} are present and their short lifetime implies that SC is broken to ordinary conductivity in longer scales satisfying scaling laws characteristic for criticality. At T_c Cooper pairs and flux tubes with longer lifetime become possible and have considerably longer life time.

   These two superconducting phases compete in a certain narrow interval around critical temperature T_c for which body temperature of endotherms is a good candidate in the case of living matter.

4. Magnetic flux tubes would be carriers of dark particles and according to the findings about high temperature SC magnetic fields would be crucial for SC. Two parallel flux tubes carrying magnetic fluxes in opposite directions is the simplest candidate for a super-conducting system. This conforms with the observation that antiferromagnetism is somehow crucial for high temperature SC. The spin interaction energy is proportional to h_{eff} and can be above thermal energy: if the hypothesis that dark cyclotron energy spectrum is universal is accepted, then the energies would be in bio-photon range and high temperature SC is obtained. If fluxes are parallel spin $S = 1$ Cooper pairs are stable. $L = 2$ states are in question since the members of the pair are at different flux tubes. These two kinds of Cooper pairs could correspond to BCS type and exotic Cooper pairs.

   The fact that the critical magnetic fields can be very weak or large values of $\hbar$ is in accordance with the idea that various almost topological quantum numbers characterizing induced magnetic fields provide a storage mechanism of bio-information.

5. This mechanism of high temperature SC is extremely general and in principle works for electrons, protons, bosonic ions and Cooper pairs of fermionic ions, charged molecules and even neutrinos and an entire zoo of high T_c bio-SCs, super-fluids and Bose-Einstein condensates is predicted. The variant of the model to be discussed in this article predicts that also charged fermionic states give rise to non-dissipative currents and that the formation of Cooper pairs a prerequisite for the $h_{eff} > h$ phase.

6. For gravitational flux tubes the generalization of Nottale hypothesis [E1] states that $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ is very large and to the particle mass. Therefore the binding energy of Cooper pairs identifiable as spin-spin interaction energy and does not depend on the mass of the Cooper pair. Supraphases would be universal in this case. This form of superconductivity is proposed to be crucial for living matter.

2.1.2 Quantitative model of high-T_c SC and bio-SC

I have developed already earlier [K6, K7, K9, K10] a rough model for high T_c super conductivity [D4, D5, D6, D2, D1, D8]. The members of Cooper pairs are assigned with parallel flux tubes carrying fluxes which have either same or opposite directions. The essential element of the model is hierarchy of Planck constants defining a hierarchy of dark matters.

1. In the case of ordinary high T_c SC bound states of charge carriers at parallel short flux tubes become stable as spin-spin interaction energy becomes higher than thermal energy.

   The transition to SC is known to occur in two steps: as if two competing mechanisms were at work. A possible interpretation is that at higher critical temperature Cooper pairs become stable but that the flux tubes are stable only below rather short scale: perhaps because the spin-flux interaction energy for current carriers is below thermal energy. At the lower critical temperature the stability would is achieved and supra-currents can flow in long length scales.

2. The phase transition to SC is analogous to a percolation process in which flux tube pairs fuse by a reconnection to form longer super-conducting pairs at the lower critical temperature. This requires that flux tubes carry anti-parallel fluxes: this is in accordance with the anti-ferro-magnetic character of high T_c super conductivity. The stability of flux tubes very probably correlates with the stability of Cooper pairs: coherence length could dictate the typical length of the flux tube.

3. A non-standard value of h_{eff} for the current carrying magnetic flux tubes is necessary since otherwise the interaction energy of spin with the magnetic field associated with the flux tube is much below the thermal energy.

There are two energies involved.

1. The spin-spin-interaction energy should give rise to the formation of Cooper pairs with members at parallel flux tubes at higher critical temperature. Both spin triplet and spin singlet pairs are possible and also their mixture is possible.

2. The interaction energy of spins with magnetic fluxes, which can be parallel or antiparallel contributes also to the gap energy of Cooper pair and gives rise to mixing of spin singlet and spin triplet. In TGD based model of quantum biology antiparallel fluxes are of special importance since U-shaped flux tubes serve as kind of tentacles allow magnetic bodies form pairs of antiparallel flux tubes connecting them and carrying supra-currents. The possibility of parallel fluxes suggests that also ferro-magnetic systems could allow SC.

   One can wonder whether the interaction of spins with magnetic field of flux tube could give rise to a dark magnetization and generate analogs of spin currents known to be coherent in long length scales and used for this reason in spintronics (`http://tinyurl.com/5cu3qh`). One can also ask whether the spin current carrying flux tubes could become stable at the lower critical temperature and make SC possible via the formation of Cooper pairs. This option does not seem to be realistic.

2.2 TGD counterparts of the collective phase, novel magnetic field, and Berry's phase

In the standard model of superconductivity SC is characterized by a complex order parameter for which the Berry phase would serves as an analog in BPM. Berry phase is a consequence of adiabaticity and characterizes collective phase. One can assign to the Berry phase effective $U(1)$ gauge field which reduces to magnetic field in a static situation. What are the TGD counterparts of these notions?

2.2.1 Beltrami flow as space-time correlate for non-dissipative flow

TGD provides the geometrization of classical physics in terms of space-time surfaces carrying gravitational and standard model field as induced fields so that both the supra current and the phase should have geometric intepretation. This serves as a powerful constraint on the model.

1. Supra current must correspond to a flow. The flow must be integrable in the sense that the coordinate defined along flow lines defines a global coordinate at flux tubes. One can indeed argue that an operational defition of a coordinate system requires that coordinates correspond to coordinates varying along flow lines of some physical flow. The exponential

of the coordinate would define the phase factor of the complex order parameter such that its gradient defines the direction of the supracurrent.

If the motion of particles is random one cannot talk of a hydrodynamic flow but something analogous to the motion of gas particles or Brownian motion. In the TGD framework this situation corresponds to disjoint space-time sheets as a representation of particle orbits. The flow property could however hold true inside the "pieces" of space-time. The coherence scales of flow would become short.

2. One must make it clear that here an approximation is made. Elementary particles have as building bricks wormhole contacts defining light-like partonic orbits to which one can assign light-like curves as M^4 projections. For a vanishing value $\Lambda = 0$ of cosmological constant (real analytic functions at M^8 level), these curves are light-like (light-likeness condition reduces to Virasoro conditions) whereas for $\Lambda > 0$ (real polynomials) at M^8 level the projections consist of pieces which are light-like geodesics somewhat like in the twistor diagrams [L25]. Smooth curve is replaced with its approximation.

   For massive particles, this orbit would be analogous to zitterbewegung orbit and the motion in the long scales would occur with velocity $v < c$: this provides a geometric description of particle massiation. The supracurrent would not actually correspond to the flow as such but to CP_2 type extremals along the flow lines.

3. In the Appendix appearing also in [K3], I have briefly discussed a decades old proposal that the 4-D generalization of so called Beltrami flow [B1, B5, B3, B4], which defines an integrable flow in terms of flow lines of magnetic field, could be central in TGD. Superfluid flows and supra currents could be along flux lines of Beltrami flows defined by the Kähler magnetic field [K5, K2].

   If the Beltrami property is universal, one must ask whether even the ordinary hydrodynamics flow could represent Beltrami flow with flow lines interpreted in terms of flow lines Kähler magnetic field appearing as a a part of classical Z^0 field. Could hydrodynamical flow be stabilized by a superfluid made of neutrino Cooper pairs. h_{eff} hierarchy of dark matters in turn inspires the question whether weak length scale could be scaled up to say cellular length scales (neutrino mass corresponds to a length scale of large neuron).

4. The integrability condition

$$j \wedge dj = 0 \tag{2.1}$$

   of the Beltrami flow states that the flow is of form

$$j = \Psi d\Phi \ , \tag{2.2}$$

   where Φ and Ψ are scalar functions, which means that Ψ defines a global coordinate varying along the flow lines.

5. Beltrami property means that the classical dissipation characterized by the contraction of the Kähler current

$$j^{\alpha} = D_{\beta} J^{\alpha\beta} \tag{2.3}$$

   with Kähler form $J_{\alpha\beta}$ is absent:

$$j^{\beta} J_{\alpha\beta} = 0 \ . \tag{2.4}$$

In absence of Kähler electric field (stationary situation), this condition states the 3-D current is parallel with the magnetic field that it creates.

In 4-D case, the orthogonality condition guarantees the vanishing of the covariant divergence of the energy momentum tensor associated with the Kähler form. This condition is automatically true for the volume part of the energy momentum tensor but not for the Kähler part, which is essentially energy momentum tensor for Maxwell's field in the induced metric. As far as energetics is considered, the system would be similar to Maxwell's equations.

The vanishing of the divergence of the energy momentum tensor would support Einstein's equations expected at QFT limit of TGD when many-sheeted space-time is approximated with a slightly curved region of M^4 and gauge and gravitational fields are defined as the sums of correspond induced fields (experienced by test particles touching all space-time sheets).

6. An interesting question is whether Beltrami condition holds true for all preferred extremals [K2] [L25], which have been conjectured to be minimal surfaces analogous to soap films outside the dynamically generated analogs of frames at which the minimal surface property fails but the divergences of isometry currents for volume term and Kähler action have delta function divergences cancelling each other. The Beltrami conditions would be satisfied for the minimal surfaces.

   If the preferred extremals are minimal surfaces and simultaneous extremals of both the volume term and the Kähler action, one expects that they possess a 4-D analog of complex structure [L25]: the identification of this structure would be as Hamilton-Jacobi structure [K2] to be discussed below.

7. Earlier I have also proposed that preferred extremals involving light-like local direction as direction of the Kähler current and orthogonal local polarization direction. This conforms with the fact that Kähler action is a non-linear generalization of Maxwell action and minimal surface equations generalize massless field equations. Locally the solutions would look like photon like entities.

   This inspires the question whether all preferred extremals except CP_2 type extremals defining basic building bricks of space-time surfaces in H have a 2-D or 3-D CP_2 projection and allow interpretation as thickening of flux tubes? CP_2 type extremals have 4-D CP_2 projection and light-like M^4 projection and an induced metric with an Euclidean signature.

2.2.2 Could all conserved currents define integrable flows?

In the TGD framework, the classical field equations for the space-time surface can be regarded as hydrodynamical in the sense that they express the conservation of the currents associated with the isometries of $H = M^4 \times CP_2$ [K11]. The classical field equations for the preferred extremals follow as consistency conditions for the modified Dirac equation obeyed by a second quantized induced spinor field [K16], whose second quantization is induced by the quantization of free spinor fields of H [L17].

An attractive conjecture is that all isometry currents or at least part of them (depending on the situation) are also Beltrami currents. For $j^A = \Psi^A d\Phi^A$ this implies $d\Psi^Q \wedge d\Phi^A = 0$ so that Ψ is function of Φ and j^A is expressible as a gradient of a scalar function: $j^A = d\chi^A$ and defines an integrable flow with a global coordinate varying along the flow lines of the current.

This prediction is obviously very powerful. This condition is linear and could make sense also for the fermionic currents involving bilinears of the oscillator operators. One would have genuine quantum hydrodynamics. In the TGD framework, the classical field equations for the space-time surface can be regarded as hydrodynamical in the sense that they express the conservation of the currents associated with the isometries of $H = M^4 \times CP_2$. The field equations follow as consistency conditions for the modified Dirac equation for a second quantized induced spinor field whose second quantization is induced by the quantization of free spinor fields of H.

The obvious objection against the strong form of the conjecture is that gradient currents are irrotational. This is true in Euclidean space but if the first homology group of the 3-surface is non-trivial. Gradient current can be rotational in the same sense as the vortices of supraflow, which have a quantized circulation concentrated at the axis of rotation.

2.2.3 Some examples

Some special cases help to get some perspective.

1. For $j^\alpha = 0$ condition he condition is trivially true: this is true for CP_2 type extremals. For massless extremals (MEs) the condition is true because of light-likeness of j^α. MEs are proposed to have a generalization with 3-D CP_2 projection.

2. In [K5, K3] it is found that for non-trivial solutions the dimension of the CP_2 projection of the space-time surface is $D = 2$ or $D = 3$. $D = 2$ would include string-like objects $X^2 \times Y^2 \subset M^4 \times CP_2$ having a 2-D string world sheet X^2 as an M^4 projection: in this case $j^\alpha = 0$ would hold true so these extremals cannot describe SC. This phase would be highly ordered.

3. $D = 3$ phase would be between order and chaos and extremely complex: in this case j^α could be non-vanishing. The topologies of the flux lines for magnetic fields satisfying the Beltrami condition gives an idea about the complexity. SC would correspond to this situation.

2.2.4 Does the M^4 part of Kähler form produce problems?

The auml; of TGD [K12, K15] suggests that the Kähler form of H has also M^4 part. M^4 part could give rise to observed small CP breaking and be relevant also for matter antimatter asymmetry [L13, L19].

The M^4 Kähler form corresponds to an analog of a self-dual instanton field for which E and B are constant, orthogonal and have the same strength so that the action vanishes for the canonically imbedded M^4. Physically M^2 is characterized by light-like direction and E^2 complex coordinate. This field selects a preferred decomposition $M^2 \times E^2$ of M^4 and breaks Lorentz invariance. How can one save Lorentz invariance?

One can also consider local selections of polarization and light-like momentum directions. I call Hamilton-Jacobi structures [K2] and they provide a concrete realization of the analog of complex structure in the case of M^4.

Hamilton-Jacobi structure is an integrable distributions of M^2 and E^2 defining slicings of M^4 by string world sheets having an orthogonal Euclidian 2-surface at each point. The moduli space for the Hamilton-Jacobi structures serves as the analog of the moduli space of complex structures for 2-D surface. Hamilton-Jacobi structure should not be God-given but be dynamically determined and part of WCW.

The existence of $M^8 - H$ duality implies this. The construction of $X^4 \subset M^8$ assigns to it $M^4 \subset M^8$. Space-time surface $X^4 \subset M^8$ as a "root" of an octonionic polynomial associates to M^4 a Hamilton-Jacobi structure. This makes it possible to parametrize the tangent spaces of $X^4 \subset M^8$ by CP_2 coordinates and therefore $M^8 - H$ duality as a map $X^4 \subset M^8 \rightarrow X^4 \subset H$.

Could the contribution of M^4 Kähler action to the total Kähler current spoil the minimal surface property by spoiling the analytic structure? Could the existence of a 4-D analog of the complex structure and implying minimal surface property prevent this? Note that Beltrami flow property is not lost if the contribution $j^\alpha(M^4)$ to Kähler current vanishes.

2.3 Coherent states and the problem with fermion number conservation

The number of electrons and Cooper pairs are ill-defined for SC. This is required by the existence of an order parameter ψ having a well-defined phase. In the phase space picture of the harmonic oscillator phase angle is a conjugate of the radial phase space coordinate, whose quantized value in the Bohr model characterized by an integer n characterizing the energy eigenvalues of the harmonic oscillator. In quantum field theory n has interpretation as the number of particles in a given mode. Phase is well-defined for coherent states for Cooper pairs, which are eigenstates of annihilation operators of Cooper pairs. In QFTs the eigenvalues of annihilation operators define analogs of Fourier components of classical fields.

One can argue that the assumption of ill-defined fermion number and energy is unphysical. In the TGD framework one can consider two solutions to the problem.

1. Zero energy ontology (ZEO) provides the first candidate for a solution. In ZEO quantum state is a superposition of deterministic time evolutions and by holography equivalent to a superposition of pairs of ordinary 3-D quantum states located at the boundaries of causal diamond (CD) identified as intersection of future and past directed light-cones. These 3-D states have the same total quantum numbers and for keeping purposes their quantum numbers can be taken to be opposite so that the entire state has zero quantum numbers. Zero energy state can be a superposition of states for which the 3-D states at either boundary with varying quantum numbers such as energy and fermion number. There are no problems with the conservation of fermion number and energy. The density matrix describing the entanglement between the 3-D states at the opposite boundaries of CD is non-trivial for these states and the interpretation in terms of a thermal state is attractive.

2. Second solution is that the system is not closed. The total number of electrons and total energy are well-defined only for the system consisting of ordinary matter and dark matter at magnetic flux tubes. Superconductivity would be direct proof of the reality of dark matter. The transition to super-conductivity would transfer Cooper pairs formed at the level of ordinary matter to the magnetic flux tubes as dark phase.

   The collective phase proposed in BPM is analogous to the dark matter at flux tubes. The novel magnetic field as an effective magnetic field assigned with the Berry phase would correspond in TGD framework to Kähler magnetic field at flux tubes carrying monopole flux not possible in Maxwelian world.

   This option seems to be the realistic one.

2.3.1 Bosonization requires effective 1+1-dimensionality

It is convenient to denote the oscillator operators for electrons at the level of ordinary matter by $b_k^\dagger$ and b_k and oscillator operators for Cooper pairs at flux tubes by $c_m^\dagger$ and c_m. They are assumed to satisfy standard anticommutation/commutation relations.

1. The bosonic oscillator operators c_N creating Cooper pairs must be representable as superpositions of electron pairs. An even stronger condition is that a subset of fermionic oscillator operator pairs are representable as bosonic oscillator operators. This requires what is known as bosonization. Bosonization was discovered independently by particle physicists Sidney Coleman and Stanley Mandelstam and condensed matter physicists Daniel C. Mattis and Alan Luther. Unfortunately the Wikipedia article about bosonization (`https://cutt.ly/HmGYPnM`) is very confusing and it is better to read the article [B2] (`https://cutt.ly/BmGNzeA`) about bosonization. Remarkably, bosonization is possible only when the system is effectively 1+1-dimensional.

2. One considers chiral fermions for which the spinor fields with different helicities are decomposed to parts ψ with wave vectors $k>0$ and $\overline{\psi}$ with wave vectors $k<0$.

$$\begin{aligned} \psi &= \int_{k>0} \frac{dk}{2\pi}[exp(ikx)\alpha(k)+exp(-ikx)\beta^\dagger(k)] \ , \\ \overline{\psi} &= \int_{k<0} \frac{dk}{2\pi}[exp(ikx)\alpha(k)+exp(-ikx)\beta^\dagger(k)] \ . \end{aligned} \tag{2.5}$$

   It should be noticed that the definition of $\overline{\psi}_+$ does not involve hermitian conjugation as usually.

3. ψ is expressed in terms of bosonic field ϕ as

$$\psi =: exp(i\int_\infty \partial_{++}\phi) : \ , \quad \overline{\psi} =: exp(-i\int_\infty \partial_{++}\phi) : \ . \tag{2.6}$$

   The subscript $\pm$ refers to either light-like coordinate.

   The bosonic and fermionic currents are related by

$$\partial_{++}\phi =: \overline{\psi}\psi : \ . \tag{2.7}$$

Note that the right hand side has fermion number 2. The condition $\partial_{+-}\phi = 0$ is satisfied and corresponds to massless d'Alembert equation in 1+1 dimensions. Coherent state is an eigenstate of ϕ and therefore of $\partial_{++}\phi$) and thus an eigenstate of the supracurrent.

4. The explicit formulas for the bosonization are given in the book "Field Theories of Condensed Matter Physics" by Eduardo Fradkin [B2] in the chapter about Luttinger liquid (page 164). Although this model does not apply as such in the TGD framework, it gives an idea about the construction.

   The bosonized expression fermionic oscillator operators with opposite spins and chiralities are mapped to the bosonized variants by the rule

$$\begin{array}{ll} \psi^{\dagger}_{R,\uparrow} \rightarrow exp(i\sqrt{2\pi}\theta_c) \ , & \psi^{\dagger}_{R,\downarrow} \rightarrow exp(-i\sqrt{2\pi}\theta_c) \ , \\ \psi^{\dagger}_{L,\uparrow} \rightarrow exp(i\sqrt{2\pi}\Phi_s) \ , & \psi^{\dagger}_{L,\downarrow} \rightarrow exp(-i\sqrt{2\pi}\Phi_s) \ . \end{array} \quad (2.8)$$

   In the singlet case, these rules give the correspondences

$$O_{SS} = \psi^{\dagger}_{R\uparrow}\psi^{\dagger}_{L\downarrow} \quad \rightarrow \quad exp(i\sqrt{2\pi}\theta_c)exp(-i\sqrt{2\pi}\Phi_s) \ . \quad (2.9)$$

   In the triplet case, one obtains

$$\begin{array}{lll} O^{1}_{TS} = \psi^{\dagger}_{R\uparrow}\psi^{\dagger}_{R\uparrow} & \rightarrow & exp(i\sqrt{2\pi}\theta_c)exp(i\sqrt{2\pi}\phi_s) \ , \\ O^{-1}_{TS} = \psi^{\dagger}_{R\downarrow}\psi^{\dagger}_{R\downarrow} & \rightarrow & exp(-i\sqrt{2\pi}\theta_c)exp(-i\sqrt{2\pi}\phi_s) \ 0. \end{array} \quad (2.10)$$

One must generalize these formulas to the TGD framework for a given flux tube. It is important to notice that there is quantum superposition over different flux tube configurations in the "world of classical worlds" (WCW) so that the inclusion of WCW degrees of freedom not present in QFT description is unavoidable.

1. The fermionic modes of opposite spin are defined at the same closed flux tube. Whether one should restrict the fields with opposite chirality to different flux tube portions is an open question.

2. The fermionic oscillator operators at space-time surface are labelled by a longitudinal momentum like quantum number, which in suitable units for closed flux tube allowing in a good approximation as a straight flux tube locally becomes integer valued momentum locally parallel to the flow line - the momentum scale is determined by Fermi momentum.

   The members of pairs have momenta $P_{\pm} = p_{cm}/2 \pm k$, where k has magnitude of order Fermi momentum, and p_{cm} is the total longitudinal momentum, which has an upper bound below Fermi momentum. The transversal quantum numbers are integer valued using as a basic unit $p_{min} = \hbar_{eff}/L$, where L of the order of the length of the flux tube.

   Since one has $p_{cm} = \hbar_{eff}/\lambda$ for Cooper pairs, their wavelengths are scaled up by the ratio $\hbar_{eff}/\hbar$ from their normal values. Also the length L of the flux tube is scaled up in this way from that for $h_{eff} = h$.

3. Additional quantum numbers are angular momentum eigenvalue m in the local flux tube direction and harmonic oscillator quantum number n labelling cyclotron states. The most plausible option is that one has phases characterized by the values of n and n.

   The breaking of rotational symmetry caused by the magnetic field takes place for a given space-time surface in the superposition. For $m > 0$ the angular momentum eigenvalues of cyclotron states contribute and one obtains Cooper pairs with relative angular momentum. Fermi statistics allows only even integer valued total angular momentum.

2.3.2 Kac-Moody symmetry associated with the bosonization

In [B2] it is mentioned that the bosonization gives rise to SU(2) Kac-Moody algebra such that I_3 generators is generated by the ϕ and generators $I_\pm$ by normal order exponentials of ϕ. This construction is applied in string models by extending the Cartan algebra represented by scalar fields to the entire algebra.

TGD predicts that the isometries of H give rise to an extended Kac-Moody algebra assignable to the 3-D light-like orbits of the partonic 2-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian.

This algebra is localized not only with respect to the complex coordinate z of the partonic 2-surface but also with respect to the light-like coordinate r varying along the partonic orbit. The extension is possible because light-likeness implies metric 2-dimensionality.

General coordinate invariance motivates the question whether this Kac-Moody algebra extends to a slicing by light-like 3-surfaces parallel to the partonic orbits.

2.3.3 Bosonization requires Beltrami property

Bosonization requires effective 1+1-dimensionality. This is guaranteed by the Beltrami flow property of supra currents. In TGD all fermionic oscillator operators at space-time surface are representable in terms of oscillator operators associated with the spinor harmonics of $H = M^4 \times CP_2$. The existence of Beltrami flow implies the existence of single preferred coordinate assignable to the flux tubes and if the transversal degrees of freedom are frozen for the Cooper pairs in given phase of SC, the system is effectively 1-D.

One can consider a variety of phases in which the cyclotron excitations assignable to the transversal degrees of freedom assignable are present. These cyclotron states and transitions between them play a key role in TGD inspired view about quantum biology.

2.3.4 Why the formation of Cooper pairs is necessary for the formation of $h_{eff} > h$ dark phase?

Why would the formation of Cooper pairs be necessary for the formation of the dark phase? Here the understanding of the energetics $h_{eff} > h$ phases helps.

1. Quite generally, the energy of the quantum state increases with h_{eff} so that the creation of the dark electrons requires energy.

   This energy would be provided in the formation of the Cooper pair as the liberated binding energy. Cooper pairs would be formed already at the level of the ordinary matter. The bosonic field modelling Cooper pairs would couple to 2-electron bilinear characterizing the quantum state of the Cooper pair.

   Since Cooper pairs are formed at the level of ordinary matter, the view of the formation of Cooper pairs is consistent with the conventional picture involving photons and effective attractive interaction generated by the attractive interaction between electrons and atoms.

2. In this process fermion number decreases by 2 units (in the recombination of electron and hole it would decrease by 1 unit). This process is analogous to Andreev-Saint-James reflection [D3] (`https://cutt.ly/AmDYDTG`), which could therefore be seen as direct evidence for the transfer of electron pairs to magnetic flux tubes. Andreev-Saint-James reflection occurs at the normal metal-SC interface and gives rise to lower energy states at the surface of unconventional SC.

2.4 The general form of the effective Hamiltonian

Consider now the general form of the effective Hamiltonian H_{eff} obtained from a quartic Hamiltonian in oscillator operators of H spinor field at space-time surface.

1. The effective Hamiltonian operator H_{eff} modelling the system would be formed as a linear in the oscillator operators $c_k^\dagger$ (c_k) creating (annihilating) Cooper pairs at flux tubes and products $b_k b_l$ ($b_k^\dagger b_l^\dagger$) of annihilation (creation) operators for ordinary electrons.

If also free electrons are possible at flux tubes, H_{eff} contains also a part, which is bilinear both in the electronic oscillator operators at flux tubes and at the level of ordinary matter.

2. H_{eff} contains a term of form $H_1 = H_2 + H_2^\dagger$

$$H_2 = C^{Nkl} c_N b_k^\dagger b_l^\dagger \quad . \tag{2.11}$$

3. For coherent states of Cooper pairs the action of the annihilation operators c_N reduces to a multiplication with a complex number C_N so that H_2 reduces to a kinetic term

$$H_2 = B_{kl} b_k^\dagger b_l^\dagger \quad , \quad B_{kl} = C^{Nkl} C_N \quad . \tag{2.12}$$

The kinetic part has the same form as the kinetic term of H_{eff} in the standard model for SC. One can diagonalize this part of Hamiltonian by a Bogoliubov transformation `https://cutt.ly/DmDcbC7` mixing the creation and annihilation operators for electrons with different quantum numbers. Bogololiubov transformation can be regarded as a symplectic transformations at the level of phase space.

4. The remaining terms can be treated as a perturbation. $H_2^\dagger$ is of form

$$H_2^\dagger = \overline{C}^{Nkl} c_N^\dagger b_k b_l \quad . \tag{2.13}$$

$H_2^\dagger$ makes possible the transfer of electron pairs to the flux tubes as Cooper pairs. It also makes possible the Andreev-Saint-James reflection regarded as the reflection of the electron as a hole from the boundary of SC.

5. H_{eff} contains also a quartic term quadratic in electronic oscillator operators both at the flux tubes and at the level of ordinary matter. This term makes possible the transfer of electrons to electron pairs not forming Cooper pairs at flux tubes.

   The oscillator operators $b_k(tube)$ at flux tube creating single fermion states should correspond to oscillator operators not appearing in $\partial\phi_{++} =: \overline{\psi}\psi :$.

6. The assumption that a closed flux tube forming effectively a flux tube pair is involved suggests that the members of the Cooper pair are at different flux tubes. If this is the case the fermionic oscillator operators at different flux tubes anticommute and the commutator of the bosonic oscillator operators does not involve bi-local terms.

2.5 A more precise formulation of TGD based theory by starting from BCS theory

It is instructive to see whether BCS theory could allow a more detailed formulation of the TGD inspired theory. The Wikipedia article (`https://cutt.ly/4mBkA5i`) gives a good summary of BCS theory.

1. Electrons of the lattice are treated as free Fermi gas and at zero temperature electrons are below the Fermi surface. In the simplest situation, Fermi surface is a sphere defined by Fermi energy (`https://cutt.ly/UmBkFhb`)

$$E_F = \frac{p_F^2}{2m_e} = \frac{\hbar^2}{2m_e}(3\pi^2 n_e)^{2/3} \quad . \tag{2.14}$$

Here n_e is the density of conduction electrons. Fermi temperature is equal to Fermi energy in the natural units. Examples of the values of the Fermi energy, Fermi temperature, Fermi velocity, and electron number density can be found in `https://cutt.ly/zmBkHt7`.

2. Fermi statistics implies that the transition to super-conductivity involving formation of Cooper pairs occurs for electrons near the Fermi surface. Any attractive interaction between electrons can cause the creation of Cooper pairs and the mechanism based on the interaction with phonons is the mechanism in BCS theory.

2.5.1 Critical temperature as Hagedorn temperature for magnetic flux tubes

The transition to super-conductivity involves an exponential increase in the heat capacity. This could be seen as a support for the flux tube picture. Flux tubes are string like objects and have an infinite number of degrees of freedom and the feed of energy excites these degrees of freedom so that temperature increases very slowly.

This implies a maximal temperature known as Hagedorn temperature T_H in string model context. The identification $T_c = T_H$ is suggestive. Living matter can remain functional in a rather narrow range of temperatures. I have proposed that the critical temperature corresponds to Hagedorn temperature [L30] for the magnetic body of the system receiving information from and controlling the biological body.

If the identification of T_c as Hagedorn temperature for the magnetic flux tubes is correct, the spectrum of the critical temperatures could be universal. On the other hand, in the general BCS model of SC, critical temperature depends on the mechanism for the formation of Cooper pairs.

1. For attractive interaction caused by phonon vibrations one has

$$T_c = 1.134 \times E_D \times exp(-\frac{1}{N(0)}) \quad , \quad N(0) = n(0)V \quad . \tag{2.15}$$

   Here $N(0)$ is the total number of conduction electrons at $T = 0$ and $E_D = \hbar\omega_D$ is Debye energy defined as the maximal value of frequency $f_D = c_s/\lambda_D$ for sound wave defined by the minimal wavelength λ_D by the minimal size of objects involved in the oscillations. The size of the lattice cell gives an order of magnitude estimate for λ_D (https://cutt.ly/RmBkKKI.

2. The Debye frequencies of 1-D chain, 2-D square lattice, and 3-D cubic lattices are given by $\omega_D = k_n c_s/a$. $k_1 = \pi$, $k_2 = 2\sqrt{\pi}$, $k_3 = (6\pi^2)^{1/3}$. One obtains an idea about the range of the sound velocities at https://cutt.ly/hmBkZGX, which are typically by two orders of magnitude large than the sound velocity in air.

$T_c = T_H$ requires an interaction between condensed matter and magnetic flux tubes carrying dark matter.

1. In the TGD inspired model of living matter [L31, L18], the magnetic body (MB) receives information from the biological body (BB) and controls it. For instance, biophotons would be dark photons transformed to ordinary photons.

2. Communication and control would use energy conserving resonant interaction between dark matter associated with the flux tubes and ordinary matter. In particular, sound waves with $h_{eff} = h$ can be transformed to dark photons with $h_{eff} = h_{gr}$ satisfying $E = hf_{high} = \hbar_{eff} f_{low}$, could be example of energy resonance. Living matter is ferroelectric and the transformation of acoustic waves to dark em waves is possible.

3. The resonant transformation of photons to dark photons and back to phonons could give rise to the interaction usually interpreted as a phonon exchange. In the model of cell membrane and EEG, cell membrane sends dark Josephson photons to MB and MB responds by sending dark cyclotron photons absorbed by dark variant of DNA central in TGD inspired model of genetic code [L18]. Also acoustic oscillations of cell membrane and DNA are important and also these could participate in the resonance.

2.5.2 TGD based interpretation of the gap energy

The decrease of some kind of binding energy as one approaches T_c from below is highly suggestive. Some kind of binding energy - gap energy ΔE - seems to be involved. At $T = 0$, BCS theory predicts the universal relationship between ΔE and critical temperature T_c

$$\Delta E(T = 0) = 1.1764 T_c \ . \tag{2.16}$$

As one approaches T_c, the gap energy obeys the formula

$$\Delta E(T) = 3.06 \sqrt{1 - \frac{T}{T_c}} \ . \tag{2.17}$$

Consider now the TGD inspired interpretation of the gap energy.

1. In the TGD framework ΔE represents the difference $\Delta E = E_B - E_{dark}$ of the binding energy E_B liberated in the formation of the Cooper pair the additional energy of the dark Cooper pair due to $h_{eff} > h$ property (the energy of state as function of h_{eff} increases with h_{eff}). The decrease of E_B with increasing T, which could be caused by weakening of phonon-electron interactions, implies critical temperature.

   For temperatures below T_c, $\Delta E = E_B - E_{dark} < 0$ implies that $h \rightarrow h_{eff}$ transition is possible. The surplus energy can be realized as kinetic energy of supra currents. At T_c ΔE vanishes and above T_c $h \rightarrow h_{eff}$ transition is impossible.

2. One can however consider a situation in which external energy feed could provide the needed energy. There are situations in which this kind of transitions might be induced thermally or by external energy feed. Indeed, in biology metabolic energy feed would make possible high-Tc superconductivity above Tc.

3. From the gap energy, BCS model predicts the maximal momentum of the Cooper pair as $p^{1)}_{max} = 2m_e \Delta E/p_F$ (in units $c = 1$) allowing to estimate the velocity range for the Cooper pairs of supracurrent.

   In TGD ΔE would go to the longitudinal energy of cyclotron state and transversal cyclotron energy due to the magnetic field of the flux tube. If all energy goes to the momentum, one has $p^{2)}_{max} = \sqrt{4 m_e \Delta E}$. This gives for the ratio $p^{2)}_{max}/p^{1)}_{max}$

   $$\frac{p^{2)}_{max}}{p^{1)}_{max}} = \sqrt{E_F}\Delta = \sqrt{\frac{E_F}{1.764 T_c}} \ .$$

   For conventional SCs, this ratio is of order $10^3 - 10^4$ since the value of E_F varies in the range 2-10 eV and the value of T_c is in the range $.1 - 1$ meV. This would suggest that cyclotron energy of the Cooper pair with the scale $\hbar_{eff} qB/m_e$ takes most of the energy.

2.5.3 What could be the values of the monopole flux magnetic field and h_{eff}?

In order to say something about the value of B and h_{eff}, some assumptions are needed.

1. The generalization of the Nottale hypothesis [E1] to TGD context [K13, K8] makes sense. Nottale hypothesis introduces gravitational Planck constant $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where M in the recent situation is the Earth's mass M_E and v_0 is velocity parameter.

2. The monopole part of the Earth's magnetic field corresponds to the endogenous magnetic field $B_{end} \simeq 2B_E/5 = .2 \times 10^{-4}$ Tesla [K9, K10] [L31] deduced from the effects of ELF em fields on mammal brain by Blackman and others [J1]. The spectrum of B_{end} is assumed to contain also other values, in particular a representation of 12-note scale [L2, L18, L20] but this particular value seems to be of special importance.

3. The monopole flux tubes, which carry the field B_{end} are identifiable as gravitational flux tubes mediating gravitational interaction. Whether this is the case or not has remained an open question.

4. Assume that gravitational flux tubes are essential for SC so that quantum gravitation in the TGD sense would be a central element of SC. Therefore SC would not be a mere local condensed matter phenomenon but depend also on M_E and the Earth's gravitational field. Life is also a phenomenon of this kind and the TGD based quantum model for living systems indeed involves high-Tc superconductivity.

Consider now the consequences of these assumptions.

1. For $\hbar_{gr}$, cyclotron energies

$$E_c(\hbar_{gr}) = \hbar_{gr}\frac{qB_{end}}{m} = \frac{GM}{v_0}qB_{end} = \frac{\hbar_{gr}}{\hbar}E_c(\hbar) \tag{2.18}$$

2. are independent of the mass m of the charged particle. This universality reflects Equivalence Principle.

   Second consequence is that the gravitational Compton length

$$\lambda_{gr} = \frac{\hbar_{gr}}{m} = \frac{GM}{v_0} \tag{2.19}$$

   is also universal.

3. The model of fountain effect of super-fluidity suggests $v_0 = c/2$ near the surface of Earth. This predicts that λ_{gr} equals to Scwartschild radius r_S which is 9 mm for Earth. All particles would have this gravitational Compton length.

   Also smaller values of v_0 are possible: for instance, $v_0/c \simeq 2^{-11}$ would be true for the 4 inner planets of the Sun [L24, L23, L22].

4. $v_0 = c/2$ predicts that in the case of electron

$$\frac{\hbar_{gr}(m)}{\hbar} = \frac{2GMm}{v_0\hbar} = \frac{r_S}{L_c(m)} \quad . \tag{2.20}$$

   From the value $r_S = 9$ mm for the Earth's Schwartschild radius and the value of electron Compton length $L_c(e) = 2.4 \times 10^{-12}$ m, one obtains

$$\frac{\hbar_{gr}(e)}{\hbar} \simeq .4 \times 10^{10} \quad .$$

   The cyclotron frequency f_c of electron in the endogenous magnetic field B_{end} is $f_c \simeq 6 \times 10^5$ Hz giving for cyclotron energy $E_c(\hbar) = 2.48 \times 10^{-9}$ eV. This gives for

$$E_c(\hbar_{gr}) \simeq 9.3 \text{ eV} \quad ,$$

   which is near to the upper bound of the Fermi energies $E_F(T = 0)$ for electrons in condensed matter.

5. For the Cooper pairs of ions suggested to be crucial for living matter, the same prediction holds true. The same prediction for E_c holds true for bosonic ions. What is interesting is that the prediction would have the same scale as for electrons for neutrinos and their possibly existing Cooper pairs. For neutrinos and neutrons E_c would be replaced by the cyclotron energy in classical Z^0 magnetic fields necessarily accompanying induced Kähler fields at monopole flux tubes.

   The large parity breaking effects in living matter have no convincing explanation in the standard physics framework in living matter. This supports the view about large h_{eff} scaling up also the weak Compton scale. $\hbar_{eff} = \hbar_{gr} = GM_Em/v_0$ with $v_0 = c/2$, the gravitational Compton length would be $\lambda_{gr} = r_S = .9$ cm for all particles including neutrinos and weak bosons. Since dark weak bosons would be effectively massless below this scale, large weak interaction induced parity breaking effects would take place below Λ_{gr}. It is of course not clear whether there exists a mechanism for the formation of neutrino Cooper pairs.

   For Sun and inner planets one has $v_0 \simeq 2^{-11}$ and $r_S = 3$ km. This gives $\lambda_{gr} \simeq 2^{10} r_S = 6$ Mm to be compared with the radius $r_S = 6.37$ Mm of Earth. Does this mean that there is a quantum coherent phase in this scale associated with the Earth?

2.5.4 TGD based model of Josephson effect

The basic assumption is that the flux tube connection carries a quantum coherent superconducting phase at Josephson junction. In the simplest description, one can apply the Schrödinger equation for the Scrödinger amplitude of Cooper pairs. Therefore the situation reduces to that already considered by Josephson. Is the Hamiltonian just a single particle Hamiltonian for Cooper pairs.

The kinetic part of the Cooper pair Hamiltonian is quadratic in Cooper pair oscillator operators at both sides. The kinetic part becomes linear by coherent state property. Coupling to the vector potential is with charge 2e as in the standard model. Cooper pairs and free electrons move in an external voltage plus helical magnetic field carrying monopole flux giving rise to the "novel" magnetic field.

The covariant constancy condition

$$(\partial_\mu - 2eA_\mu)\psi = 0 \tag{2.21}$$

is satisfied for two coordinates: for the time coordinate (or possibly light-like coordinate) and for the longitudinal coordinate varying along the flow lines of the Beltrami flow. A_μ reduces inb 1-D situation to gradient. Covariant constancy is satisfied at the flux tubes along the helical flux lines and gives Josephson effect in standard manner. The phase is essentially the integral of voltage.

By coherent state property H becomes linear perturbation just like a perturbation of a harmonic oscillator by a periodic force. The effect is non-trivial only in second order perturbation theory. Chemical potential term is not needed at the level of MB.

2.5.5 The 4 anomalies in TGD framework

The article of Koizumi [D7] mentions 4 anomalies of the BCS model (no generally accepted model of high-Tc SC exists). Besides the absence of the difference of chemical potentials in the condition defining Josephson frequencies, 3 other anomalies are mentioned. These anomalies do not plague the TGD based model. The basic reason is that Cooper pairs reside at the magnetic flux tubes.

1. There is only one transition temperature in the BCS model of SC whereas high-T_c superconductivity involves 2 transition temperatures. In the TGD framework the first transition temperature leads to a superconductivity but in spatial and time scales (proportional to h_{eff}), which are so short that macroscopic super-conductivity is not possible. In the lower transition temperature h_{eff} increases and the flux tubes reconnect in a stable manner to longer flux tubes. The instability of this phase at critical temperature would be due to the geometric instability of the flux tubes.

2. London moment depends on the real electron mass m_e rather than the effective mass m_e^* of the electron. This effect relates to a rotating magnet. There is a supra current in the boundary

region creating the magnetic moment. The explanation is that the electrons resulting from the splitting of Cooper pairs at the flux tubes of magnetic field do not interact with the ordinary condensed matter so that the mass is m_e-

3. For SCs of type I, the reversible phase transition from SC to ordinary phase in an external magnetic field does not cause dissipation. One would expect that the splitting of Cooper pairs produces electrons, which continue to flow and dissipate in collisions with the ordinary condensed matter. The reversibility of the phase transition can be understood if the electrons continue to flow at the flux tubes as supracurrents.

4. Magnetic flux tubes also solve the anomaly related to chemical potential: chemical potentials are present but not at the level of magnetic flux tubes so that the erratic calculation gives a correct result in the standard approach.

2.5.6 The basic objection against the TGD based proposal

The basic objection against the TGD based model of superconductivity is that supercurrents flow along monopole flux tubes but an experimental fact is that magnetic field destroys superconductivity. The problem disappears by analyzing the anatomy of magnetic fields in the TGD framework.

1. TGD predicts two kinds of flux tubes carrying Earth's magnetic field B_E with a nominal value of .5 Gauss. This prediciton is quite general. The flux tubes have a closed cross section - this is possible only in TGD Universe, where the space-time is 4-surface in $M^4 \times CP_2$. The flux tubes can have a vanishing Kähler magnetic flux or non-vanishing quantized monopole flux: this has no counterpart in Maxwellian electrodynamics.

   For Earth, the monopole part would correspond to about .2 Gauss - 2/5 of the full strength of B_E.

2. Monopole part needs no currents to maintain it and this makes it possible to understand how the Earth's magnetic field has not disappeared a long time ago. This also explains the existence of magnetic fields in cosmological scales.

   The orientation of the Earth's magnetic field is varying. In the TGD based model, the monopole part plays the role of master. When the non-monopole part becomes too weak, the magnetic body defined by the monopole part changes its orientation. This induced currents refresh the non-monopole part [L4]. The standard dynamo model is part of this model.

3. There is an interesting (perhaps more than) analogy with the standard phenomenological description of magnetism in condensed matter. One has $B = H + M$. H field is analogous to the monopole part and the non-monopole part is analogous to the magnetization M induced by H. $B = H + M$ would represent the total field. If this description corresponds to the presence of two kinds of flux tubes, the TGD view about magnetic fields would have been part of electromagnetism from the beginning!

   Flux tubes can also carry electric fields and also for them this kind of decomposition makes sense. Could also the fields D, P, and E have a similar interpretation?

   In the linear model of magnetism, one has $M = \chi H$ and $B = \mu H = (1+\chi)H$. For diamagnets one has $\chi \leq 0$ and for paramagnets $\chi \geq 0$. Earth would be paramagnetic with $\chi \simeq 3/2$ if the linear model works. χ is a tensor in the general case so that B and H can have different directions.

4. Superconducting phase is a perfect diamagnet so that $B = H + M = 0$. Supra currents generate M, which effectively cancels H. This happens for the interaction of the test particle with the fields H and M, which are at different space-time sheets. In the interaction the test particle touches these space-time sheets and the effects superpose linearly. At the QFT limit this corresponds to the vanishing of B. B does not destroy superconductivity but superconductivity destroys B. In the Meissner effect superconductivity is lost and B is weakened and monopole field H and possible flux tubes of the external field become visible.

3 Summary and conclusions

TGD suggests that superconducting charge carriers Cooper pairs of them. In this article I have compared this view with the view represented in [D7]. In the following I will will summarize this article and conclude with the recent TGD based view of high Tc superconductivity as it is now (year 2024).

3.1 Comparison of BPM and TGD inspired model of SC

Consider now the relation of BPM to TGD inspired model of SC.

1. In TGD, the phase factor of complex order parameter would be an exponential of a longitudinal coordinate Φ related to a helical flux along a flux tube serving as a longitudinal coordinate. For closed flux tubes with the shape of a long flattened square, the phase factors at the two flux tubes would be exponentials of the same longitudinal cyclic coordinate. There is no obvious reason for the interpretation as Berry's phase although this interpretation cannot be excluded.

2. In TGD, the "novel magnetism" associated with the Berry phase would correspond to the monopole part of the magnetic field not present in Maxwellian theory. The monopole part plays a central role in TGD inspired quantum biology and also in the model of galaxies and stars [L11, L12]. They appear also in the models of hadrons and nuclei and their dark variants leading to a new physics about hadrons and nuclei.

   The flux tubes have closed transversal cross sections and are therefore not possible in Minkowski space. These flux tubes appear in all scales and form a fractal hierarchy.

   Also flux tubes with closed cross section with 2-D homologically trivial projection are possible and carry vanishing magnetic flux as also half flux tubes glued to background 3-surfaces as representation of ordinary flux tubes for which cross section as the topology of disk.

3. The decay of the Beltrami phases could correspond to the decay of a flux tube carrying a Beltrami flow to thinner flux tubes parallel to the original flow. SC would reduce to SC in a shorter scale. The two transitions for high Tc cuprate SCs could correspond to reverse transitions in which flux tubes fused to thicker and longer flux tubes. Low temperature would stabilize longer and thicker flux tubes against splitting to shorter and thinner ones.

It is useful to list the basic differences between BPM and the TGD based model.

1. The authors identify Josephson junction as an insulator. In the TGD framework the junction would consist of superconducting flux tubes accompanied by a parallel structure at the level of ordinary matter, which can be an insulator.

2. In TGD there is a supracurrent of Cooper pairs but it occurs at magnetic flux tubes. Also a supracurrent of electrons is possible.

3. A pair of flux tubes is present in the junction. A reconnection of U-shaped tentacles gives rise to the junctions. Flux tube junctions stabilized have $h_{eff} > h$ and the states have higher energy. The energy liberated in the formation of Cooper pairs provides the energy needed to increase h_{eff}.

4. BPM produces Josephson effect using first order Hamiltonian for thin junctions. For thick junctions a second kind of Josephson effect would result for long junctions.

   In TGD JE does not depend on the length of JE assuming that the junction is accompanied by a magnetic flux tube pair. JE results as a second order effect from the effective Hamiltonian for Cooper pairs which is by coherent state property linear in oscillator operators of Cooper pairs. Situation is essentially the same as in the standard model. Also the mechanism for the formation of Cooper pairs remains the same.

5. BPM predicts chemical potential term. In the TGD framework this is neither predicted nor needed since chemical potential is not needed at the flux tube level. Standard calculation gives a correct result although it is not logically consistent.

3.2 Speculations, questions, and conclusion

The only way to make progress is to speculate and then challenge the speculations by making critical questions. The following represents a list of such speculations and critical questions.

3.2.1 Speculations

Consider first some speculations.

1. The TGD inspired model suggests that SC could be possible also above T_c by using energy feed providing the energy needed to increase the value of h_{eff}. This would be the basic role of metabolism. This could have far reaching technological consequences and also profound implications concerning the creation of artificial life.

   Furthermore, the TGD based model for "cold fusion" [L3, L6, L16] led to a reformulation of nuclear physics [L12] in which phase transition to dark phase of nuclei has a key role also in the ordinary nuclear reactions as a description of tunnelling phenomenon.

2. In the TGD inspired quantum biology, the cell membrane is identified as a generalized Josephson junction between superconductors assignable to lipid layers of the cell membrane (actually decomposing in a better resolution to membrane proteins acting as Josephson junctions). One can ask what a straightforward application of the basic formulas gives in the case of neuronal membrane.

   One can estimate the gap energy Δ from the formula $\Delta = \hbar\omega_D$ using the already discussed formula $\omega_D = k_n c_s/a$, where k_n depends on the effective dimension of the lattice like system and has values $k_n \in \{3.14, 3.54, 2.66\}$ for $n = 1, 2, 3$. Sound velocity c_s can be replaced with the conduction velocity v of nerve pulses varying in the range $v/c \in [.1, 1] \times 10^6$. The formula would give for $n = 2$ and maximal value $v/c = 10^{-6}$ $E_D = .044$ eV which is in the range of neuronal membrane potentials.

3. The role of $\hbar_{gr}$ and B_{end} in the model would suggest that the SC observed in laboratories is not a mere local condensed matter phenomenon. What happens to SC on Mars? Is the Earth mass replaced with that of Mars and the monopole part B_{end} with its value in Mars? There is evidence that B_{end} is non-vanishing: for instance, Mars has auroras.

4. If the monopole flux tube indeed mediates graviton exchanges, one can wonder whether SC itself is an essentially quantum gravitational phenomenon. Could the attractive interaction between electrons of the Cooper pair be somehow due to gravitation?

   The extremely weak direct gravitational interaction between electrons and nucleons cannot be responsible for the formation of Cooper pairs. One can however argue that Earth takes the role of atomic nuclei in the proposed description. Earth attracts the electrons and causes an effective attraction between them. Could this interaction force the wave functions of the electrons of the Cooper pair with wavelength $\Lambda_{gr} = r_S = 2GM \simeq 9$ mm to overlap and form a quantum coherent state.

   The proposed duality between gauge theories and gravitation, in particular AdS/CFT duality, has a TGD counterpart. The dynamics for the orbits of partonic 2-surfaces and lower-dimensional surface defining a frame for the space-time surface as an analog of soap film [L25] would be dual to the dynamics in the interior of the space-time surfaces.

   Could the descriptions in terms of cyclotron photon exchanges and graviton exchanges be dual to each other? Note also that at the fundamental level classical TGD are expressible using only 4 classical field-like variables as a selected subset of embedding space coordinates. This implies extremely strong constraints between fundamental interactions.

3.2.2 Critical questions

Consider now some critical questions.

1. Suppose that Cooper pairs are formed at the level of ordinary matter by interaction with phonons (say) and transferred to MB.

   Q: How can the Cooper pairs survive at MB, where acoustic oscillations mediating interaction with atoms are not present?

   A: The presence of the resonant interaction between photons and dark photons would make possible the survival of the Cooper pairs at MB.

   Second option is that the Cooper pairs remain in the ordinary matter and only the electrons are transferred to the flux tubes and the energy liberated in the formation of Cooper pairs makes the transfer energetically possible. Supracurrents would indeed consist of electrons as proposed in [D7].

2. I have routinely used the statement "particle resides at magnetic body".

   Q: What does this really mean?

   A: In many-sheeted space-time, the space-time sheets with common 4-D M^4 projection are extremely near to each other and the test particle touches all the sheets. The conclusion in the case of gravitational flux tubes has been that the particle touches all sheets of the many-sheeted magnetic body rather than resides at it.

   Here one must remember that also many-sheetedness with respect to CP_2 is predicted and leads to the proposal that coherent flux tube bundles in M^4 as many-sheeted space-time with respect to CP_2 explain the value of G in terms of CP_2 length squared following from TGD as a prediction [K4]. In this case, the test particle does not touch all the sheets unless it has a large value of h_{eff}: $h_{eff} = h_{gr}$ could imply this.

   Q: But doesn't this mean that the particle touches all space-time sheets for all flux tubes?

   A: The generalization of Beltrami hypothesis might actually prevent this. If the M^4 part of the Kähler current is proportional to instanton current and conserved, the M^4 projection of flux tube is 3-D so instanton density vanishes. In this case space-time surfaces have 3-D M^4 projection and are like orbits of membranes and the above argument fails.

   It can of course, also happen that only the sum of M^4 and CP_2 currents has vanishing divergence: in this case the M^4 and CP_2 projections would be 4-D.

   Clearly, the situation is unclear but it is now possible to formulate questions and possible answers precisely.

3. The isotope effect of superconductivity means the proportionality $T_c \propto M^p$, where M is the mass of the isotope. The values of p are near $p = -1/2$. This implies the proportionality $\Delta \propto M^p$.

   Q: Can the $T_c = T_H$ hypothesis be consistent with the isotope effect?

   A: Assume that the scale of dark cyclotron energies determines to a high extent the value of Δ. Cyclotron energies are of the form $E_c = \hbar_{gr} qB/m = (GM/v_0)qB_{end}$. Nottale hypothesis implies that v_0 takes the role of a dimensionless coupling constant strength for gravitation and very probably does not vary [L22].

   The local value of B_{end} can however vary and depend on M. This would mean a local variation of the thickness of the flux tube as a response to the contact of the isotope with the isotope. This in turn would cause the local change of the string tension as a sum of densities of the volume and Kähler magnetic energies per unit length.

3.2.3 Which of the TGD based views of superconductivity is correct?

I have considered several TGD inspired views of superconductivity.

1. The key assumption is the presence of phases of ordinary matter with effective Planck constant which can be rather large. The original assumption was that Cooper pairs consist of pairs of dark particles at magnetic monopole flux tubes [K9, **?**, **?**, **?**].

2. One can however ask whether the Cooper pairs are at the level of ordinary matter and whether their formation liberates the binding energy allowing to transform electrons to their dark cariants with large h_{eff}. The TGD view of the Pollack effect as a way to create dark protons at gravitational monopole flux tubes using the energy of solar photons has been considerably generalized [L26] and in principle also the creation of dark electrons is possible [L29]. The energy needed could come from formation of bound states of atoms.

   This view of the Pollack effect suggests that also the formation of Cooper pairs of ordinary electrons could provide the energy needed to generate dark electrons. For this option one could consider dark electrons as charge carriers but also dark Cooper pairs are possible. The splitting of this kind Cooper pairs however creates electrons, which dissipate in conflict with the experimental findings discussed in the introduction. Therefore this option must be given up.

3. Also the possibility that charge carriers are dark electrons rather than Cooper pairs is excluded. Dark electrons could however correlate with the holes that they have left behind: one would have a "half-Cooper pair". Could these "half-Cooper pairs" be present in the temperature range $[T_c, T_{c1}]$, where there is no super-conductivity. The earlier proposal was that the flux tube pairs are so short that the nanoscopic or macroscopic supra currents are not possible.

The recent view about TGD (2024) allows us to conclude that the original view is nearer to the truth. The following represents the recent perspective to high Tc- and bio-superconductivity.

Number theoretic view of TGD predicts a hierarchy of phases of ordinary matter labelled by the value of effective Planck constant $h_{eff} = nh_0$. The simplest assumption is that n is the dimension of algebraic extension of rationals. For a more complex option it is a product of dimensions of two algebraic extensions.

These phases behave like dark matter and would be located at monopole magnetic flux tubes and also electric flux tubes. They would not be galactic dark matter but correspond to the missing baryonic matter whose fraction has been increasing during the cosmological evolution. Galactic dark matter would correspond to the energy of cosmic strings (space-time surfaces with 2-D M^4 and CP_2 projections). The unavoidable number theoretical evolution implies the increase of the number theoretical complexity and therefore increase of n. The larger the value of n the longer the quantum coherence scale of the system.

1. The predicted huge values of h_{eff} assignable to classical gravitational and electric fields of astrophysical objects [L26] mean that weak interactions become as strong as em interactions below the scale up Compton length of weak bosons, which, being proportional to h_{eff}, can be as large as cell size. This amplifies parity violation effects visible for instance in hydrodynamics [K1].

2. Large h_{eff} phases behave like dark matter: they do not however explain the galactic dark matter, which in the TGD framework is dark energy assignable to cosmic strings (no halo and an automatic prediction of the flat velocity spectrum). Instead, large h_{eff} phases solve the missing baryon problem. The density of baryons has decreased in cosmic evolution (having biological evolution as a particular aspect) and the explanation is that evolution as unavoidable increase of algebraic complexity measured by h_{eff} has transformed them to $h_{eff} \geq h$ phases at the magnetic bodies (thickened cosmic string world sheets, 4-D objects), in particular those involved with living matter.

3. The large value of h_{eff} has besides number theoretical interpretation [L14, L15, L27, L28] also a geometric interpretation. Space-time surface can be regarded as many-sheeted over both M^4 and CP_2. In the first case the CP_2 coordinates are many-valued functions of M^4 coordinates. In the latter case M^4 coordinates are many-valued functions of CP_2 coordinates so that QFT type description fails. This case is highly interesting in the case of quantum biology. Since a connected space-time surface defines the quantum coherence region, an ensemble of, say, monopole flux tubes can define a quantum coherent region in the latter case: one simply has an analog of Bose-Einstein condensate of monopole flux tubes.

The flux tube condensate as a covering of CP_2 means a dramatic deviation from the QFT picture and is a central notion in the applications of quantum TGD to biology. Therefore some examples are in order.

1. Fermi liquid description of electrons relies on the notion of a quasiparticle as an electron plus excitations of various kinds created by its propagation in the lattice. In some systems this description fails and these systems would. have a natural description in terms of space-time surfaces which are multiple coverings of CP_2, say flux tube condensates.

2. In high Tc superconductors and bio-superconductors [K9, K10] the space-time surface could correspond to this kind of flux tube condensates and Cooper pairs would be fermion pairs with members at separate flux tubes. The connectedness of the space-time surface having about $h_{eff}/h = n$ flux tubes would correlate the fermions.

3. Bogoliubov quasiparticles related to superconductors are regarded as superpositions of electron excitation and hole. The problem is that they have an ill-defined fermion number. In TGD, they would correspond to superpositions of a dark electron accompanied by a hole which it has left behind and therefore having a well-defined fermion number. Bogoliubov quasiparticle is indeed what can be seen using the existing experimental tools and physical understanding.

4. Strange metals would be an example of a system having no description using quasiparticles, as the linear dependence of the resistance at low temperatures demonstrates. I have considered a description of them in terms of Cooper pairs at short closed flux tubes [K9, K14]: this would however suggest a vanishing resistance in an ideal situation. Something seems to go wrong.

   An alternative description could be in terms of superpositions of dark electrons and holes assignable to the flux tube condensate. Strange metal is between Fermi liquid and superconductor: this conforms with the fact that strange metals are quantum critical systems. The transition to high Tc superconductivity is preceded by a transition to a phase in which something resembling Cooper pairs is present.

   A natural looking interpretation would be in terms of a flux tube condensate and pairs of dark and ordinary electrons. Also now the flux tubes could be short. In this chapter I have considered the possibility that high Tc superconductors could be this kind of "half-superconductors" but this option seems to be wrong: for high Tc superconductors this phase could however appear in the temperature range $[T_c, T_{c1}]$ where superconductivity is not present. Note that the phase transitions between "half-superconductivity" and superconductivity could play a central role also in living matter.

4 Appendix: General considerations related to Beltrami flows

The following text is based on the updated view about the material from the appendix of [K3]. More details can be found in [K2, K5, K3].

4.1 Beltrami ansatz and minimal surface ansatz for the preferred extremals of Kähler action

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium.

1. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all extremals or at least the absolute minima of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. Its vanishing encourages the proposal that Einstein's equations hold true at the Yang-Mills-Einstein limit of TGD.

The absence of the classical dissipation is highly attractive in the case of supra phases. This condition could be universal and be satisfied below the scales defined by the space-time surface.

The corresponding condition is implied by Maxwell-Einstein's equations in General Relativity.

2. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields defining integrable flows serving as candidates for flow lines of the superfluid flow and supracurrents.

3. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current

$$j_I^\alpha = \epsilon^{\alpha\beta\gamma\delta} A_\beta J_{\gamma\delta} \quad (4.1)$$

solves the 4-D Beltrami condition and reduces to it when electric field vanishes.

Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes since the contractions of the gradients of $D < 4$ CP_2 coordinates with a 4-D permutations symbol are involved. Instanton current vanishes for $D = 2$. Note that massless extremals having 2-D CP_2 projection carry non-vanishing light-like current and vanishing instanton current.

The condition implies that Kähler current can be non-vanishing only if the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$ (say CP_2 type extremals) Maxwell's vacuum equations are satisfied by the Kähler form.

4. Beltrami fields are known to be extremely complex but highly organized structures and the same is expected to be true for their generalizations (it is not clear whether Kähler current j is always vanishing or light-like).

An interesting conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their 4-D generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

5. Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current (massless extremals). On the basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

Minimal surface ansatz [L25] for the preferred extremals based on 4-D generalization of complex structure to Hamilton-Jacobi structure. It emerges naturally at M^8 level as a prerequisite of M^8-H duality, and is induce at the level of H by M^8-H duality [L7, L8, L9, L14, L15, L21].

The twistor twistor lift of TGD [K12, K15] leads to a concrete action principle at the level of H involving volume term and Kähler action obtained by a dimensional reduction of the Kähler action at the level of twistor space of H. The Kähler action for twistor space exists only in case of H. Therefore there are 3 different views about preferred extremals and they are proposed to be equivalent.

4.2 The dimension of CP_2 projection as classifier for the fundamental phases of matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. This phase is chaotic and analogous to a demagnetized phase. It might be that only CP_2 type extremals with Euclidean signature of the induced metric and 1-D light-like M^4 projection correspond to this phase.

$D_{CP_2} = 2$ phase is analogous to the ferromagnetic phase: highly ordered and relatively simple. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase is the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life. Both these phases could correspond to SC.

4.3 Connection of Beltrami flows with PCAC hypothesis, massivation, and CP violation

Conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) are essential elements of old-fashioned hadron physics and hold true also in the standard model.

1. The simplest ansatz, which realizes the Beltrami hypothesis, states that the vectorial Kähler current J equals apart from sign $c = \pm 1$ to instanton current I, which is axial current:

$$J = \pm I \ c \ . \tag{4.2}$$

The condition states that only the left or right handed current chiral defined as

$$J_{L/R} = J \pm I \tag{4.3}$$

is non-vanishing. For $c \neq 1$, both J_L and J_R are non-vanishing. Since both right- and left-handed weak currents exist, $c \neq 1$ seems to be a plausible option.

By quantum classical correspondence, these currents serve as space-time correlates for the left- and right-handed fermion currents of the standard model. Note however that induced gamma matrices differ from those of M^4: for instance, they are not covariantly constant but define by field equations a current with a vanishing covariant divergence. Field equations serve as a consistency condition for the modified Dirac action.

2. A more general condition allows c to depend on space-time coordinates. The conservation of J forces conservation of I if the condition $\partial_\alpha c I^\alpha = 0$ is true. This gives a non-trivial condition only in regions with 4-D CP_2 and M^4 projections.

3. The twistor lift of TGD [K12, K15] requires that also M^4 has Kähler structure. Therefore J and I and corresponding Kähler gauge potential A have both M^4 part and CP_2 parts and Kähler action K, A_K, J_K, J and I are sums of M^4 and CP_2 parts:

$$\begin{array}{ll} A_K = A_K(M^4) + A_K(CP_2) \ , & J_K = J(M^4) + J_K(CP_2) \ , \\ K = K(M^4) + K(CP_2) \ , & J = J(M^4) + J(CP_2) \ . \\ I = I(M^4) + I(CP_2) \ . & \end{array} \tag{4.4}$$

Only the divergence of I must vanish:

$$\partial_\alpha I^\alpha = 0 \ . \tag{4.5}$$

A possible interpretation is in terms of the 8-D variant of twistorialization by twistor lift [K12, K15] requiring masslessness in 8-D sense.

PCAC states that the divergence of the axial current is non-vanishing. This is not in conflict with the conservation of the total instanton current I. PCAC corresponds to the non-conservation $I(CP_2)$, whose non-conservation is compensated by that of $I(M^4)$.

4. For regions with at most 3-D M^4- and CP_2 projections, the M^4- and CP_2 instanton currents have identically vanishing divergence. In these regions the conservation of I is not lost if c has both signs. c could be also position dependent and even differ for $I(M^4)$ and $I(CP_2)$ in these regions.

   $D_\alpha I^\alpha$ vanishes for the known extremals. For the simplest CP_2 type extremals and for extremals with 2-D CP_2 projection, I itself vanishes. Therefore parity violation is not possible in these regions. This would suggest that these regions correspond to a massless phase.

5. $D_\alpha I^\alpha \neq 0$ is possible only if both M^4 and CP_2 projections are 4-D. This phase is interpreted as a chaotic phase and by the non-conservation of electroweak axial currents could correspond to a massive phase.

   CP_2 type extremals have 4-D projection and for them Kähler current and instanton current vanish identically so that also they correspond to massless phase (M^4 projection is light-like). Could CP_2 type extremals allow deformations with 4-D M^4 projection (DEs)?

   The wormhole throat between space-time region with Minkowskian signature of the induced metric and CP_2 type extremal (wormhole contact) with Euclidian signature is light-like and the 4-metric is effectively 3-D. It is not clear whether this allows 4-D M^4 projection in the interior of DE.

The geometric model for massivation based on zitterbewegung of DE provides additional insight [L25].

1. $M^8 - H$ duality allows to assign a light-like curve also to DE. For space-time surfaces determined by polynomials (cosmological constant $\Lambda > 0$), this curve consists of pieces which are light-like geodesics.

   Also real analytic functions ($\Lambda = 0$) can be considered and they would allow a continuous light-like curve, whose definition boils down to Virasoro conditions. In both cases, the zigzag motion with light-velocity would give rise to velocity $v < c$ in long length scales having interpretation in terms of massivation.

2. The interaction with $J(M^4)$ would be essential for the generation of momentum due to the M^4 Chern-Simons term assigned with the 3-D light-like partonic orbit. M^4 Chern-Simons term can be interpreted as a boundary term due to the non-vanishing divergence of $I(M^4)$ so that a connection with two views about massivation is obtained. Does the Chern-Simons term come from the Euclidean or Minkowskian region?

I have proposed two models for the generation of matter-antimatter asymmetry. In both models, CP breaking by M^4 Kähler form is essential. Classical electric field induces CP breaking. CP takes self-dual (E, B) to anti-self-dual $(-E, B)$ and self-duality of $J(M^4)$ does not allow CP as a symmetry.

1. In the first model the electric part of $J(M^4)$ would induce a small CP breaking inside cosmic strings thickened to flux tubes inducing in turn small matter-antimatter asymmetry outside cosmic strings. After annihilation this would leave only matter outside the cosmic strings.

2. In the simplest variant of TGD only quarks are fundamental particles and leptons are their local composites in CP_2 scale [L13, L19].

   Both quarks and antiquarks are possible but antiquarks would combine leptons as almost local 3-quark composites and presumably realized CP_2 type extremals with the 3 antiquarks associated with the partonic orbit. I should vanish identically for the DEs representing quarks and leptons but not for antiquarks and antileptons.

   Could the number of DEs with vanishing I be smaller for antiquarks than for quarks by CP breaking and could this induce leptonization of antiquarks and favor baryons instead of antileptons? Could matter-antimatter asymmetry be induced by the interior of DE alone or by its interaction with the Minkowskian space-time region outside DE.

In the standard model also charged weak currents are allowed. Does TGD allow their space-time counterparts? CP_2 allows quaternionic structure in the sense that the conformally invariant Weyl tensor has besides $W_3 = J(CP_2)$ also charged components $W_\pm$, which are however not covariantly constant [L1]. One can assign to $W_\pm$ analogs of Kähler currents as covariant divergences and also the analogs of instanton currents. These currents could realize a classical space-time analog of current algebra.

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