A Possible Explanation for Shnoll Effect

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Contents

Abstract

Shnoll and collaborators have discovered strange repeating patterns of random fluctuations of physical observables such as the number n of nuclear decays in a given time interval. Periodically occurring peaks for the distribution of the number $N(n)$ of measurements producing n events in a series of measurements as a function of n is observed instead of a single peak. The positions of the peaks are not random and the patterns depend on position and time varying periodically in time scales possibly assignable to Earth-Sun and Earth-Moon gravitational interaction.

These observations suggest a modification of the expected probability distributions but it is very difficult to imagine any physical mechanism in the standard physics framework. Rather, a universal deformation of predicted probability distributions would be in question requiring something analogous to the transition from classical physics to quantum physics.

The hint about the nature of the modification comes from the TGD inspired quantum measurement theory proposing a description of the notion of finite measurement resolution in terms of inclusions of so called hyper-finite factors of type II_1 (HFFs) and closely related quantum groups. Also p-adic physics -another key element of TGD- is expected to be involved. A modification of a given probability distribution $P(n|\lambda_i)$ for a positive integer valued variable n characterized by rational-valued parameters λ_i is obtained by replacing n and the integers characterizing λ_i with so called quantum integers depending on the quantum phase $q_m = exp(i2\pi/m)$. Quantum integer n_q must be defined as the product of quantum counterparts p_q of the primes p appearing in the prime decomposition of n. One has $p_q = sin(2\pi p/m)/sin(2\pi/m)$ for $p \neq P$ and $p_q = P$ for $p = P$. m must satisfy $m \geq 3$, $m \neq p$, and $m \neq 2p$.

The quantum counterparts of positive integers can be negative. Therefore quantum distribution is defined first as p-adic valued distribution and then mapped by so called canonical identification I to a real distribution by the map taking p-adic -1 to P and powers P^n to P^{-n} and other quantum primes to themselves and requiring that the mean value of n is for distribution and its quantum variant. The map I satisfies $I(\sum P_n) = \sum I(P_n)$. The resulting distribution has peaks located periodically with periods coming as powers of P. Also periodicities with peaks corresponding to $n = n^+n^-$, $n_q^+ > 0$ with fixed $n_q^- < 0$, are predicted. These predictions are universal and easily testable. The prime P and integer m characterizing the quantum variant of distribution can be identified from data. The shapes of the distributions obtained are qualitatively consistent with the findings of Shnoll but detailed tests are required to see whether the number theoretic predictions are correct.

The periodic dependence of the distributions would be most naturally assignable to the gravitational interaction of Earth with Sun and Moon and therefore to the periodic variation of Earth-Sun and Earth-Moon distances. The TGD inspired proposal is that the p-dic prime P and integer m characterizing the quantum distribution are determined by a process analogous to a state function reduction and their most probably values depend on the deviation of the distance R through the formulas $\Delta p/p \simeq k_p \Delta R/R$ and $\Delta m/m \simeq k_m \Delta R/R$. The p-adic primes assignable to elementary particles are very large unlike the primes which could characterize the empirical distributions. The hierarchy of Planck constants allows the gravitational Planck constant assignable to the space-time sheets mediating gravitational interactions to have gigantic values and this allows p-adicity with small values of the p-adic prime P.

1 Introduction

Usually one is not interested in detailed patterns of the fluctuations of physical variables, and assumes that possible deviations from the predicted spetrum are due to the random character of the phenomena studied. Shnoll and his collaborators have however studied during last four decades the patterns associated with random fluctuations and have discovered a strange effect described in detail in [1] , [1, 5, 4, 2, 7, 3] . The examples of [1] , [1] give the reader a clear picture about what is involved.

1. Some examples studied by Shnoll and collaborators are fluctuations of chemical and nuclear decay rates, of particle velocity in external electric field, of discharge time delay in a neon lamp RC oscillator, of relaxation time of water protons using the spin echo technique, of amplitude of concentration fluctuations in the Belousov-Zhabotinsky reaction. Shnoll effect appears also in financial time series [8] which gives additional support for its universality. Often the measurement reduces to a measurement of a number of events in a given time interval τ. More generally, it is plausible that in all measurement situations one divides the value range of the studied observable to intervals of fixed length and counts the number of events in each interval to get a histogram representing the distribution $N(n)$, where n is the number of events in a given interval and $N(n)$ is the number of intervals with n events. These histograms allow to estimate the probability distribution $P(n)$, which can be compared with theoretical predictions for the spectrum of fluctuations of n. Typical theoretical expectations for the fluctuation spectrum are characterized by Gaussian and Poisson distributions.

2. Contrary to the expectations, the histograms describing the distribution of $N(n)$ has a distribution having several maxima and minima (see the figures in the article of Shnoll and collaborators). Typically -say for Poisson distribution - one expects single peak. As the duration of the measurement period increases, this structure becomes gets more pronounced: standard intuituin would suggest just the opposite to take place. The peaks also tend to be located periodically. According to [1] , [1] the smoothed out distribution is consistent with the expected distribution in the case that it can be predicted reliably.

3. There are also other strange features involved with the effect. The anomalous distribution for the number n of events per fixed time interval (or more general value interval of measured observable) seems to be universal as the experiments carried out with biological, chemical, and nuclear physics systems demonstrate. The distribution seems also to be same at laboratories located far away from each other. The comparison of consecutive histograms shows that the

histogram shape is likely to be similar to the shape of its nearest temporal neighbors. The shapes of histograms tend to recur with periods of 24 hours, 27 days, or 365 days. The regular time variation of consecutive histograms, the similarity of histograms for simultaneous independent processes of different nature and occurring in different geographical positions, and the above mentioned periods, suggest a common reason for the phenomenon possibility related to gravitational interactions in Sun-Earth and Earth-Moon system.

In the case that the observable is number n of events per given time interval, theoretical considerations predict a distribution characterized by some parameters. For instance, for Poisson distribution the probabilities $P(n)$ are given by the expression

$$P(n|\lambda) = exp(-\lambda)\frac{\lambda^n}{n!} . \quad (1.1)$$

The mean value of n is $\lambda > 0$ and also variance equals to λ. The replacement of distribution with a many-peaked one means that the probabilities $P(n|\lambda)$ are modified so that several maxima and minima result. This can occur of course by the randomness of the events but for large enough samples the effect should disappear.

The universality and position independence of the patterns suggest that the modification changes slowly as a function of geographic position and time. The interpretation of the periodicities as periods assignable to gravitational interactions in Sun-Earth system is highly suggestive. It is however very difficult to imagine any concrete physical models for the effect since distributions look the same even for processes of different nature. It would seem that the very notion of probability somehow differs from the ordinary probability based on real numbers and that this deformation of the notion of probability concept somehow relates to gravitation.

In the following the possibility that direct p-adic variants of real distribution functions such as Poisson distribution could allow to understand the findings is discussed. It turns out that this is not the case but that the replacement of integers with quantum integers [2] n_q identified as the product of quantum integers associated with their prime factors with quantum phase $q = exp(i\pi/m)$, where $m \geq 2$ is not of form $m = p$, p prime, leads to a well-defined correspondence between p-adic probabilities $P(n)$ and real probabilities conserving the sum of probabilities.

There is however a difficulty, which was not fully realized in the original version of this article. Quantum primes l_q are non-negative only for $l < m$ and this could lead to non-negative probabilities (consider for instance the counterparts of $n!$ in Poisson distribution). The solution of the problem is provided by what I call quantum arithmetics [7, 8] providing a more rigorous formulation of quantum integers. The recipe is following. To define quantum integer n_q decompose first n to its prime factors l. For $l < m$ one has $l_q > 0$ but not necessarily for $l > m$. Express $l > m$ as a q-adic expansion in powers of m with coefficients smaller than m and thus expressible as products of quantum primes l_q for $l < m$ so that the resulting quantum q-adic integers for $q = m$ is non-negative. For $m = p$ one obtains what one might call quantum p-adics. For quantum p-adics one can

Usually quantum groups are assigned with exotic phenomena in Planck length scale. In TGD they are assignable to a finite measurement resolution [6] . TGD inspired quantum measurement theory describes finite measurement resolution in terms of inclusions of hyper-finite factors of type II_1 (HFFs) and quantum groups related closely to the inclusions and appear also in the models of topological quantum computation [1] based on topological quantum field theories [5] .

The universal modification of probability distributions $P(n|\lambda_i)$ characterized by rational numbers predicts patterns analogous to the ones observed by Shnoll. The parameters P and m characterize the deformation of the probability distribution and the periodic slow variation of the p-adic prime P and explain the periodically occurring peaks of the histograms for $N(n)$ as function of n. Also the dependence of the distribution of $N(n)$ on the direction of the momentum of alpha particle [2, 7] can be understood in terms of the effect of the measurement apparatus on many-sheeted space-time topology and geometry.

The p-adic primes P in question are small. This makes sense in TGD framework only if one accepts that a very large value of Planck constant is involved. TGD indeed predicts a hierarchy of Planck constants and identifies dark matter as phases with a large value of Planck constant. The Planck constant associated with the space-time sheets mediating gravitational interaction is predicted to be gigantic meaning macroscopic quantum coherence in astrophysical scales. This modification allows also to formulate a general correspondence principle between real and p-adic physics as a rule stating that all primes p except the p-adic prime P itself appearing in various formulas are replaced with their quantum counterparts and P is mapped to its inverse in the modified distribution.

For the reader not familiar with TGD the article series in Prespacetime journal [2, 3, 6, 7, 4, 1, 5, 11] and the two articles about TGD inspired theory of consciousness and of quantum biology in Journal of Consciousness Research and Exploration [10, 8, 9] are recommended. Also the online books at my homepage provide the needed background.

2 p-Adic topology and the notion of canonical identification

p-Adic physics has become gradually a central part of quantum TGD [5] and the notion of p-adic probability has already demonstrated its explanatory power in the understanding of elementary particles masses using p-adic thermodynamics [2] . This encourages the attempt to understand Shnoll effect in terms of an appropriate modification of probability concept based on p-adic numbers.

p-Adic topology [3] is characterized by p-adic norm given by $|x|_p = p^{-k}$ for $x = p^k(x_0+\sum_{k>0} x_k p^k)$, $x_0 > 0$. This notion of nearness differs radically from its real counterpart. For instance, numbers differing bya large power of p are p-adically near to each other. Therefore p-adic continuity means short range chaos and long range correlations in real sense. One might hope that p-adic notion of nearness allow the existence of p-adic variants of standard probability distributions characterized by rational valued parameters and transcendental numbers existing also p-adically such that these distributions can be mapped to their real counterparts by canonical identification mapping sum of probabilities to the sum of the images of the probabilities.

2.1 Canonical identification

In the case of p-adic thermodynamics [2] the map of real integers to p-adic integers and vice versa relies on canonical identification and its various generalizations and canonical identification is also now a natural starting point.

1. The basic formula for tge canonical identification for given prime p characterizing p-adic number field Q_p is obtained by using for a real number x pinary expansion $x = \sum x_n p^{-n}$, $x_n \in \{0, p-1\}$ analogous to decimal expansion. The map is very simple and given by

$$\sum_n x_n p^{-n} \rightarrow I(x) = \sum_n x_n p^n \quad . \tag{2.1}$$

   The map from reals to p-adics is two-valued in the case of real numbers since pinary expansion itself is non-unique ($p = (p-1)\sum_{k\geq 0} p^{-k}$ as the analog of 1=.99999.. for decimal expansion). The inverse of the canonical identification has exactly the same form. Canonical identification maps p-adic numbers to reals in a continuous manner and also the inverse map is continuous apart from the 2-valuedness eliminated if one introduces pinary cutoff which is indeed natural when finite measurement resolution is assumed.

2. The first modification of canonical identification replaces pinary expansion of real number in powers of p with expansion in powers of p^k: $x = \sum x_n p^{-nk}$, $x_n \in \{0, p^k - 1\}$ and reads as

$$\sum_n x_n p^{-nk} \rightarrow I_k(x) = \sum_n x_n p^{nk} \quad . \tag{2.2}$$

3. A further variant applies to rational numbers. By using the unique representation $q = r/s$ of given rational number as ratio of co-prime integers one has

$$I_k(q = \frac{r}{s}) = \frac{I_k(r)}{I_k(s)} \quad . \tag{2.3}$$

2.2 Estimate for the p-adic norm of factorial

In the p-adic variant of Poisson distribution canonical images of the factorial $n!$ appear and the the basic properties of $I(n!)$ as function of n will be needed in the sequel.

1. Given integer n can be written as $n = p^{k(n)}m(n)$ such that $m(n)$ has unit norm p-adically. $n!$ in turn can be written as

$$n! = \prod_{r=1}^{n} p^{k(r)}m(r) = p^{K(n)} \times \prod_r m(r) \ , \ K(n) = \sum_r k(r) \ . \tag{2.4}$$

2. The p-adic norm of $n!$ is given by

$$N_p(n!) \ = \ p^{-K(n)} \ . \tag{2.5}$$

$\prod_r m(r)$ has unit norm p-adically and its p-adic canonical image satisfies the upper bound

$$I_k(\prod_r m(r)) \leq p^k \ . \tag{2.6}$$

3. $N_p(n!)$ is reduced by the power $p^{k(r)}$ in the step $n = r-1 \rightarrow r$. Therefore $I(n!) \equiv I_{k=1}(n!)$ is a decreasing function with discontinuous drops of the value which are especially large when n is proportional to a large power of p. The peaks corresponding to given value k of $k(r)$ occur periodically and one has fractal pattern with periodicities define by powers of p. Similar consideration applies to $I_k(n!)$: now the periodicities correspond to powers of p^k rather than p. In both cases one has local chaos and long range correlations due to the fact that in p-adic topology nearby points differing by a large power p^n are far away in real sense. The natural question is whether the periodicity of peaks in histograms of [1] , [1] could represent a special case of of these periodicities.

In the sequel an estimate for the maximal power of p dividing $n!$ defining the norm $N_p(n!)$ is needed. The following estimate gives $N_p(n!) \simeq p^{-n}$ for $n \gg p$.

1. What is needed is an estimate for the number $N(n,k)$ of for the number of integers $k(r)$ with given value of $k \geq 1$. If this estimate is available for large values of n, one obtains for the exponent defined associated with the p-adic norm of $n!$ the formula

$$K(n) \ \equiv \ \sum(k_r) = \sum N(k)k \ . \tag{2.7}$$

2. By studying the 2-adic numbers one finds that the formula

$$K(n=2^m) \ = \ \sum N(k)k \ , \ N(k) = \frac{2^m}{2^k} = 2^{m-k} \tag{2.8}$$

holds true.

3. The generalization of the this formula to for $p > 2$ reads as

$$K(n=p^m) \ = \ \sum N(k)k \ , \ N(k) = (p-1)\frac{p^m}{p^{k+1}} = p^{m-k} \ . \tag{2.9}$$

This would give at the limit $n \rightarrow \infty$

$$K(n=p^m) = \frac{p^{m+1}}{p-1} \simeq p^m = n \ . \tag{2.10}$$

There one has $K(n) = n$ in this special case.

4. For a general value of n the approximate formula would be

$$K(n) \leq \sum N(k)k \ , \ N(k) \simeq (p-1)\frac{n}{p^k} \ . \tag{2.11}$$

Also now one would have $K(n) \simeq n$ so that the p-adic norm of $n!$ would be approximately p^{-n} . The justification for this formula comes by noticing that the number of integers smaller than n with p-adic norm p^k is roughly $(p-1)n/p^k$ since the numbers $kp^k + X$ with $N_p(X) \leq p^{-k-1}$ and k running from $1, ..., p-1$ satisfy the required conditions.

3 Arguments leading to the identification of the deformed Poisson distribution

The following argument represents a trial and error procedure to a unique identificaiton of deformed Poisson distribution $P(n|\lambda)$ with a rational value of λ and more generally, to a modication of any distribution $P(n, \lambda_i)$ characterized by rational parameters λ_i.

3.1 The naive modification of Poisson distribution based on canonical identification fails

To gain some intuition it is instructive to study the possible variants of Poisson distribution based on canonical identification. The discussion generalizes to more general distributions for probabilities of integer valued observables provided the parameters of the distribution exist p-adically. The idea is to start from a p-adic variant of probability theory [4] , assume that the p-adic valued probability distributions are mappable to their real counterparts using canonical identification, and to look whether this procedure yields something consistent with the findings of Shnoll.

To begin with, assume that the notion of p-adic valued probability makes sense. This requires that the probabilities exist as p-adic numbers. This is true if probabilities are rational numbers which can be regarded as being common to reals and padic numbers. Also the sum of probabilities must make sense p-adically so that it can be normalized to to unity. In absence of cutoff to the values of N this condition is highly non-trivial.

The condition that the canonical identification commutes with the summation of probabilities is especially strong and would state

$$\sum (P(n))_R = (\sum P_n)_R \ . \tag{3.1}$$

Here x_R denotes the image of x under canonical identification. For ordinary p-adic numbers this condition requires that the probabilities are just powers of p. If one allows algebraic extensions of p-adic numbers defined by quantum phases defined by roots of unity mapped to real numbers as such, the probabilities can be of form Xp^n where X is function of these phases. This condition excludes automatically the naivest attempts to define canonical image of p-adic variant of Poisson distribution. This is due to the presence of $1/n!$ and possible rational appearing in λ.

Optimist could give up the normalization condition and consider instead of probabilities rational numbers. There are problems also now.

1. The first problem is that normalization factor is defined only up to a multiplication with a rational and each choice of the normalization factor gives different real counterpart of the p-adic distribution irrespective of the manner how the real probabilities are defined.

2. The normalization factor $exp(-\lambda)$ is p-adic number only if λ is proportional to a positive power of p. This condition also implies that the powers $\lambda^k/k!$ approach to zero with respect to p-adic norm since the p-adic norm of λ^k is always small than that of $k!$. The naive guess for the canonical identification map of p-adic probabilities to their real counterparts is given by the formula

$$\lambda^n \rightarrow I(\lambda^n)/I(n!)$$

One can consider also other other variants but for the purposes of argument one can restrict the consideration to this one. The problem is that $I(\lambda^n)$ does not increase but decreases like p^{-n} so that $\lambda_R < 1$ would hold true. The decrease of the factor $1/n!$ guarantees the convergence of probabilities for Poisson distribution. The canonical image $I(1/n!) = 1/I(n!)$ however increases. The same result is obtained irrespective of the detailed definition of canonical identification. Therefore the first guess for the canonical image of the proposed p-adic variant of Poisson distribution has very little to do with ordinary Poisson distribution. The attempts to cure the situation by modifying the map from p-adics to reals fail. This suggests that one must modify the p-adic variant of the Poisson distribution itself.

3.2 Quantum integers as a solution of the problems

The problems associated with the naive generalization of the Poisson distribution relate to the behavior of canonical identification when applied to integers other than powers of p. This suggests that one should replace the integers systematically with some of kind of deformations of integers guaranteeing also that canonical identification maps sum of probabilities the sum of their images. The notion of quantum integer [2] is what comes first in mind.

TGD based motivation for the notion of quantum integer comes from the fact that the so called hyper-finite factors of type II_1 (HFFs) play a key role in quantum TGD and allow to formulate the notion of finite measurement resolution in terms of inclusions of HFFs [6] to which the quantum groups assignable to roots of unity are closely related. The findings of Shnoll would therefore relate to the delicacies of quantum measurement theory with finite measurement resolution.

The quantum groups based on quantum phases

$$q = U_m = exp(i\phi_m) \ \ , \ \ \phi_m = \frac{\pi}{m} \ \ . \ \ m \geq 3 \tag{3.2}$$

appear in TGD framework and the long standing intuitive expectation has been that there might exist a deep connection between p-adic length scale hypothesis and quantum phases defined by roots of unity defining algebraic extensions of p-adic numbers.

3.2.1 The standard definition of quantum integer does not help

The first thing to do is to see whether the standard notions of quantum integer and quantum factorial [2] could allow to get rid of the problems.

1. Quantum integers for $q = U_m$ are given by

$$n_{U_m} = \frac{U_m^n - \overline{U}_m^n}{U_m - \overline{U}_m} = \frac{sin(n\phi_m)}{sin(\phi_m)} \ \ . \tag{3.3}$$

   For $n \ll m$ one has

$$n_{U_m} \simeq n \ \ . \tag{3.4}$$

   This property makes quantum integers a good candidate if one wants to generalize the notion of Poisson distribution and more generally, any probability distribution $P(n|\lambda_i)$ parametrized by rationals. The rule would be very simple: replace all integers by their quantum counterparts: $n \rightarrow n_q$.

This proposal has however some problematic features.

1. n_q is negative for $n \ mod \ 2m > m$ so that in the case of Poisson distribution one would have negative probabilities in real context. In the p-adic context there is no well-defined notion of negative number so that one might avoid this difficulty if one can map p-adic probabilities to positive real probabilities. Quantum integers have unit norm p-adically so that p-adic Poisson distribution makes sense for $N_p(\lambda) < 1$.

2. n_{U_m} vanishes for $n = m$ always. Therefore $n_q!$ defined as a product of quantum integers smaller than n vanishes for all $n > m$. One way out is to restrict the values of n to satisfy $n < m$. This number theoretic cutoff would mean in the p-adic case that the sum of p-adic probabilities is finite without the condition $N_p(\lambda) < 1$.

3. Quantum integers defined in the standard manner are periodic with period m so that quantum factorial obtained by dropping the vanishing terms would behave like a product of factorial associated with $m - 1$ times quantum factorial of $k \leq m - 1$. Ordinary factorial $n!$ increases much faster. It seems that the standard definition of quantum integer is not correct.

3.2.2 Quantum integers must allow factorization to quantum primes

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Physics as a generalized number theory vision [5] suggests a manner to circumvent above described problems.

1. Quantum integers defined in the standard manner do not respect the decomposition of integers to a product of factors- that is one does not have

$$(mn)_q = m_q n_q \ . \tag{3.5}$$

The preferred nature of the quantum phases associated with primes in TGD context however suggests that one should guarantee this property by hand by simply defining the quantum integer as a product of quantum integers associated with its prime factors:

$$n_q \equiv \prod (p_i)_q^{n_i} \ \text{for} \ n = \prod p_i^{n_i} \ . \tag{3.6}$$

This would guarantee that the notion of primeness and related notions crucial for p-adic physics would make sense also for quantum integers. Note that this deformation would not be made for the exponents of integers for which sum is the natural operation.

2. If $q = U_m$ is such that m is not prime, the quantum phases associated with primes are always non-vanishing and quantum integers and therefore also quantum factorials $n_q!$ defined using the proposed definition of quantum integers are non-vanishing for all values of n. In p-adic context this would mean that the probabilities associated with Poisson distribution are finite and for $N_p(\lambda_p) < 1$ sum up to a finite value.

The number theoretic definition of quantum integers does not automatically solve the problem of negative quantum integers obtained when integer contains prime factors $p > m$ and vanishing problem when integer is divisible by p.

1. If the number N_- of prime factors of n satisfying $p \ mod \ 2m > m$ is odd, the product of minus signs coming from them is odd and the over all quantum integer is negative. Since the p-adic probabilities are well defined in p-adic context, one could consider the mapping of these probabilities to real probabilities by the basic form of canonical identification. If also λ is expressed in terms of quantum primes only the real image of overall minus sign must be determined. p-Adically -1 corresponds to a positive p-adic integer $(p - 1)(1 + p + p^2 + ...)$ for which one has $I(-1) = p$ from the basic definition of canonical identification. Hence the p-adic and real quantum variants of Poisson distribution would be unique.

This prescription would predict peaks of Poisson distribution for $n = n_+n_-$, such that $(n_+)_q$ is positive and has only prime factors $p_+ \ mod\ 2m < m$ and $(n_-)_q$ is having therefore odd number of negative prime factors $(p_-)_q$ satisfying $p_- \ mod\ 2m > m$. These peaks would occur periodically with period n_-. Large number of this kind of periods would be present. It might be possible to identify the periodicities of the peaks of the histograms of Shnoll in this manner.

2. Second manner to solve the sign problem has been already mentioned and relies on the notion of quantum arithmetics [7, 8]. The construction recipe for quantum integers is following. To define quantum integer n_q decompose first n into its prime factors l. This guarantees the quantum integers respect prime factorization for ordinary integers. For $l < m$ one has $l_q > 0$ but not necessarily for $l > m$. Express $l > m$ as a q-adic expansion $l = \sum l_k m^k$, with $l_k < m$ and thus expressible as products of quantum primes l_q for $l < m$ so that the resulting quantum q-adic integers for $q = exp(i\pi/m$ are non-negative.

   For $m = p$ one obtains what one might call quantum p-adics and in this case $p_q = 0$ holds true so that one must assume cutoff $n < p$ or exclude integers n divisible by p. Note that q-adicity is consistent with p-adicity for prime factors of m.

   One can consider also a more general recipe for quantum p-adic integers (and also quantum m-adic integers) [8]. One allows all expansions $l = \sum l_n p^n$ of primes $l > p$ in powers of p with coefficients l_n also now having only prime factors $l < p$ but giving up the constraint $l_n < p$ so that given p-adic integer corresponds to several quantum p-adic integers. This gives quantum q-adic integers for $q = m$ which in well-defined sense forms a covering of q-adic integers and one can assign to it what might be called quantum Galois group.

3.2.3 The most general choice of λ

Consider next the most general choice of λ consistent with the constraint that canonical identification conserves probabilities. Denote by P the p-adic prime characterizing the deformed Poisson distribution and by p a generic prime.

1. If one assumes the following product representation

$$\lambda_q = P^n Q_{U_m} \ , \tag{3.7}$$

   where P is the p-adic prime and Q_{U_m} is quantum rational in the proposed sense, p-adic probabilities $P(n)$ are finite for positive values of n and m satisfying the proposed constraints. The expression for the real counterpart $\lambda_R$of λ_q is given by

$$\lambda_R = \frac{Q_{U_m}}{P^{-n}} \ . \tag{3.8}$$

   With a proper choice of Q_{U_m} arbitrary large values are possible for λ_R and standard form of of canonical identification for a well-defined p-adic probability distribution produces a real variant of quantum Poisson distribution which is in a well-defined sense a small deformation of the Poisson distribution.

2. The value of the parameter λ assignable to the ordinary Poisson distribution giving rise to q-Poisson does not correspond to λ_R as such. For given λ_q the value of λ can be determined from the condition that the average values of n are same for the two distributions:

$$\lambda = \langle n \rangle_P = \langle n \rangle_{qP} \ . \tag{3.9}$$

3. For $m = P$ the vanishing of P_{U_P} would require a cutoff $n < P$ in Poisson distribution. One could however argue that all values of m must be allowed. The manner to circumvent the difficulty is to to treat prime $p = P$ as an exception and *define* in the most general case

$$P_q \equiv P \ . \qquad (3.10)$$

A stronger condition would be that P appears as a factor of m and it might well be that there could exist a number theoretical justification for this. Canonical identification would introduce to $P(n)$ a factor $P^{K(n)}$ defined by the largest power $P^{K(n)}$ dividing $n!$. By the rough estimate $n!$ of Eq. 2.11 one has $K(n) \sim n$. This would introduce additional peaks to the distribution coming with periodicities defined by p^m besides those coming with periodicities defined by integers n_-, which involve odd number of integers $p \ mod \ m > m/2$. This requires

$$\lambda_q = P^n Q_{U_m} \ , \ n > 1 \qquad (3.11)$$

in order that the sum of p-adic probabilities is well-defined. The sum of real probabilities converges due to the properties of quantum factorial defined in the manner respecting the decomposition of integer to a product of primes.

4. This definition of quantum Poisson satisfies also the strongest possible constraint on the map of p-adic probabilities to real ones. One can indeed include the p-adic normalization factor to the distribution and rational canonical identification commutes with the normalization factor in the sense that one has $\sum(P(n))_R = (\sum P_n)_R$. This is due to the fact that the canonical image of the sum of probabilities is by definition a sum of images of probabilities since only numbers expressible in terms of roots of unity and not allowing expression as ordinary p-adic number multiplied by powers of p and p-adic -1 appear in the sum.

5. Fig. 1 represents a comparison of q-Poisson distribution characterized by $(p = 7, m = 300, \lambda_0 = 100, k = 1)$ giving $\lambda_q = p^k \times \lambda_0 = 700$ and $\lambda_R = 14.229$ with the corresponding ordinary Poisson distribution characterized by $\lambda = 25.256$ which is almost twice the value of λ_R. The presence of peaks with periodicity $p = 7$ due to the identification $p_q = p$ for the prime defining p-adicity and mapped to $1/p$ in canonical identication is clearly visible in the distribution.

These considerations are for Poisson distribution but they generalize in an obvious manner to any distribution $P(n|\lambda_i)$ for which parameters $\lambda_i)$ are rational numbers.

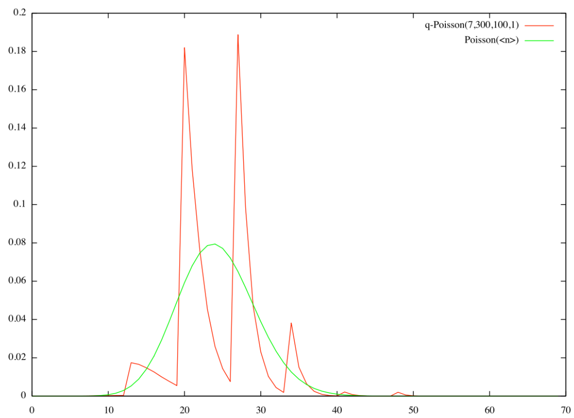

Figure 1: A comparison of q-Poisson distribution with Poisson distribution with the same mean value of n assuming $p_q = p$ and that p is mapped to $1/p$ and -1 in numerator is mapped to p in canonical identification. The values of quantum parameters are $(p = 7, m = 300, k = 1, \lambda_0 = 100)$ giving $\lambda_q = p^k \times \lambda_0 = 700$ and $\lambda_R = 14.229$. The mean value of Poisson distribution turns out to be $\lambda = \langle n \rangle_q = 25.256$.

3.2.4 Quantum integers and correspondence between real and p-adic physics

The understanding of the relationship between real and p-adic physics has been plagued by the fact that canonical identification and its variants do not make sense when applied to say energy levels characterized by integers. In this case the correspondence via common rationals is assumed or I_k for large enough k is used.

The replacement of ordinary integers with their q-counterparts using the proposed rules provides much more general correspondence principle relating p-adic and real quantum physics to each other in the case that the formulas of real physics involve only rationals. For instance, in p-adic mass calculations [2] the integers characterizing conformal weights would be replaced by their quantum counterparts defined in the proposed manner mapping products to products. This does not affect p-adic mass calculations if the exceptional prime corresponds to p-adic prime and m which is equal to p or contains p as a factor. One can also define p-adic harmonic oscillator and p-adic hydrogen atom and for $n > m$ is large exotic effects become possible. For large values of p-adic prime P and for $m \gg P$ these effects are not detectable.

For the p-adic variants of the wave functions the natural space-time coordinates would be discretized to integers to guarantee that the wave functions exist p-adically for $p = P$. For hydrogen atom (/harmonic oscillator) one would obtain the formal analog of q-Poisson (/q-Gaussian) in the radial coordinate discretized to integer. In angle degrees of freedom the form of discretized wave functions would be same as in real context obtained by replacing $exp(i\phi)$ and $cos(\theta)$ and $sin(\theta)$ with their discretized versions in an algebraic extension of p-adic numbers containing appropriate roots of unity for $p = P$. If the integer m defines the algebraic extension it should be divisible by the integers defining the angular momentum projections M up to some cutoff.

This correspondence might apply even at space-time level and imbedding space-level when preferred coordinates are introduced for imbedding space. This would allow to map the rational imbedding space points of a real space-time surface to their p-adic counterparts by canonical identification. For $(p, m) \rightarrow (\infty, \infty)$ this map would effectively reduce to the identification along common rationals but with respect to p-adic norm it would have totally different behavior.

4 Explanation for the findings of Shnoll

One should be able to undertand both the many-peaked character of the distributions as well as their spatial and temporal variation involving correlations with the gravitational physics of Sun-Earth and Earth-Moon systems.

4.1 The basic characteristics of the distributions

The properties of the deformed distributions might allow to explain the findings of Shnoll at least qualitatively. The testing of numerical predictions would require detailed numerical data. It is assumed that the p-adic probabilities can be formally negative with $-1 = (P-1)/(1-P) = (P-1)\sum P^k$ mapped to real number by canonical identification to give a positive number. There are also other options to overcome negativity problem not considered here. The integer m characterizing quantum phase m is not prime but can be assumed to be proportional to P to avoid vanishing quantum integers. This corresponds to m-adicity consistent with P-adicity.

1. The presence of maxima and minima due to canonical identification mapping p-adic distribution function to its p-adic counterpart is consistent with the basic property of the fluctuation distributions as expressed by the histograms for the number $N(n)$, where n is the fluctuating number n of events per fixed time unit or discretization interval for the values of some observable.

2. The basic predictions are following. Modified distributions are characterized by a relatively small prime defining the p-adicity - call it P- and integer m which is not prime but could be divisible by P. The peaks in histogram for $N(n)$ should appear with periods in n giving rise to short range chaos and long range order in variable n. Periods of first kind come as powers of P. A small change of P corresponds to a small change of periodicities. The periods for second kind correspond to integers n_- which contain an odd number of primes l in the ranges $((2r+1)m, (2r+2)m)$, $r = 0, 1, 2, ..$ (quantum phase and thus l_q is same for l and $l + 2rm$).

The spectrum of integers $n_{\pm}$ changes as m changes but if the change is small, the new spectrum contains integers in old spectrum. For instance, if n_- corresponds to single prime which is in the middle region of interval $(m, 2m)$ a change $|\Delta m| < m/2$ does not remove n_- from spectrum.

3. For instance, in one of the experiments (Fig.1 of [1] , [1]) the histogram for $N(n)$ has peaks, which seem to occur periodically with a separation Δn of about 100 units. If these periods correspond to P, its value must be smaller than 100. The nearest primes are $P = 89, 97, 101, 113$. In Fig. 2 of same reference one has also periodicity and P must be near 10. Hence there are good hopes that the proposed model might be able to explain the findings.

4. According to the earlier proposal the selection of p-adic prime is outcome of a process analogous to quantum measurement. This interpretation would suggests that there is a sequence of quantum measurements in which various p-adic primes are selected with some probability each and that the probability distribution for the primes depends on external astrophysical parameters varying periodically. One can also consider the possibility that P and m behave as classical variables.

4.2 The temporal and spatial dependence of the distributions

One should also understand the variation of the shape of the distribution with time and its spatial variation.

1. The situation is sensitive to the values of P and m. The changes should be such that the parameters of the smoothed out real probability distribution are not affected much. For instance, in the case of q-Poisson distribution the values of P and m should change in such a manner that $\langle n \rangle = \lambda$ is not unaffected much. The change of P would affect the positions of the peaks but small changes of P would not mean too dramatic changes. Periodic time dependence of these parameters would explain the findings of Shnoll. Gravitational interactions in Sun-Earth-Moon system and therefore the periodic variations of Sun-Earth and Earth-Moon distances is the first guess for the cause of the periodic variations.

2. The correlation of the fluctuation periods with astrophysical periods assignable to Earth-Sun system (diurnal period and period of Earth's orbit) suggests that the gravitational interaction of the measurement apparatus with Sun is involved. Also the period 27.28 days which corresponds to sidereal period of Moon measured in the system defined by distant star. In [1] , [1] this period is somewhat confusingly referred to as synodic period of Sun with respect to Earth (recall that synodic period corresponds to a period for the appearance of third object (say Moon) in the same position relative to two other objects (say Earth and Moon)). Therefore also Moon-Earth gravitational force seems to be involved. Moon-Earth and Earth-Sun gravitational accelerations indeed have roughly the same order of magnitude. That gravitational accelerations would determine the effect conforms with Equivalence Principle. The most natural dimensionless parameter characterizing the situation is $|\Delta \mathbf{a_{gr}}|/\mathbf{a_{gr}}$ expressible in terms of $\Delta R/R$ and $\Delta r/r$, where R *resp* r denotes the distance between Earth and Sun *resp.* Earth and Moon, and the ratio R/r and cosine for the angle θ between the direction vectors for the positions of Moon and Sun from Earth. The observed palindrome effect [3] is consistent with the assumed dependence of the effect on the distances of Earth from Sun and Moon. Also the smallness of the effect as one approaches North Pole conforms with the fact that the variations of distances fro Sun and Moon become small at this limit .

3. In 24 hour time scale it is enough to take into account only the Earth-Sun gravitational interaction. One could perform experiments at different positions at Earth's surface to see whether the the variation of distributions correlates with the variation of the gravitational potential. The maximal amplitude of $\Delta R/R$ is $2R_E/R \simeq .04$ so that for $\Delta p/p = k\Delta R/R$ one would have $\Delta p/p = .04k$. Already for $p \sim 100$ the variation range would be rather small. For $\Delta m/m$ one expects that analogous estimate holds true.

4. One observes in alpha decay rates periodicities which correspond to both sidereal and solar day [2] . The periodicity with respect to solar day can be understood in terms of the periodic variation of Sun-Earth distance. The periodicity with respect to sidereal day would be due to

the diurnal variation of the Earth-Moon distance. Similar doubling of periodicities are predicted in other relevant time scales.

In the case of alpha decay the effect reveals intricacies not explained by the simplest model [2, 7] . In this case one studies random fluctuations random fluctuations for the numbers of alpha particles emitted in a fixed direction. Collimators are used to select the alpha particles in a given direction and this is important for what follows. Two especially interesting situations correspond to a detector which is located to North, East, or West from the sample. What is observed that the effect is different for East and West directions and there is a phase shift of 12 hours between East and West. In Northern direction the effect vanishes. Also other experiments reveal East-West asymmetry called local time effect by the authors [5, 4] .

1. What the findings mean is that P and m characterizing the distribution for the counts of alpha particles in a given angle depend on time and and the time dependence sensitive to the direction angle of the alpha particle. This might be however only apparent since collimators are used to select alpha particles in given direction. The authors speak about anisotropy of space-time and Finsler geometry [1] could be considered as a possible model. In this approach the geometry of space-time would be something totally independent of measurement apparatus.

   In TGD framework the space-time is topologically non-trivial in macroscopic scales and the presence of collimators making possible to select alpha particles in a given direction affect the geometry of many-sheeted space-time sheets describing the measurement apparatus and therefore the details of the interaction with the gravitational fields of Earth, Sun, and Moon. As a consequence, the values of P and m should reflect the geometry of the measurement apparatus and depend only apparently on the direction of v_α. If this interpretation is correct, a selection of events from a sample without collimators should yield distributions without any dependence on the direction of v_α.

2. At quantitative level the distribution for counts in a given direction can depend on angles defined by the vectors formed from relevant quantities. These include at least the tangential velocity $v = \omega \times r$ of the laboratory, the direction of the velocity v_α of alpha particle with respect to sample actually reflecting the geometry of collimators, the net gravitational acceleration a_{net}, and the direction of Earth's gravitational acceleration g.

3. The first task is to construct from these vectors a scalar or a pseudo-scalar (if one is ready to allow large parity breaking effects), which vanishes for North-East direction, has opposite signs for East and West direction and has at least approximately a behavior consistent with the phase shift of 12 hours between East and West. The constraints are satisfied by the scalar

$$X = E \cdot a_{net} \ , \ E = \frac{(v \times g) \times v_\alpha}{|(v \times g) \times v_\alpha|} \ . \tag{4.1}$$

   Unit vector E changes sign in East-West permutation and also with a period of 12 hours meaning the change of the roles of East and West with this period in the approximation that the net acceleration vector is same at the opposite sides of Earth. The approximation makes sense if the change of sign induces much large variation than the change of the Earth-Sun and Earth-Moon distances. Unless P and m are even functions of X, the predicted effect can be consistent with the experimental findings in the approximation that a_{net} is constant in 24 hour time scale.

5 Hierarchy of Planck constants allows small-p p-adicity

In particle physics applications of p-adic physics [2] the values of p-adic primes are very large and favor p-adic primes near powers of two. For instance, electron is characterized by a p-adic prime $M_{127} = 2^{127} - 1$. Small p-adic primes correspond to very short time and length scales, which are not plausible in the recent situation. Biological systems however suggest the possibility of small values of p. This is consistent with p-adic length scale hypothesis if one accepts the hypothesis that dark matter corresponds to a hierarchy of Planck constants coming as integer multiples of the ordinary Planck constant $\hbar_0$: $\hbar/\hbar_0 = r$, r integer.

5.1 Estimate for the value of Planck constant

In the recent formulation of quantum TGD the hierarchy of Planck constants there is an argument reducing the hierarchy of Planck constants to the basic quantum TGD and one can say that scaled up values of Planck constant are effective values of Planck constant. The scaling of the p-adic prime scales up the secondary time scale assignable with the particle characterized by prime p as $T_k = 2^k T_{CP_2} \rightarrow rT_k$. Here T_{CP_2} denotes CP_2 time expressible as $T_{CP_2} = 2^{-127}T(2,127) \simeq 5.877 \times 10^{-40}$ seconds. There $T(2,127) \simeq .1$ seconds is secondary p-adic time scale assignable to Mersenne prime M_{127} characterizing electron. T_{CP_2} is 1.0902×10^4 times Planck time $T_{Pl} = 5.391 \times 10^{-44}$ s.

To obtain small-p p-adicity one must have very large value of r. The proposed quantum model for dark matter in astrophysical scales indeed predicts gigantic values of gravitational Planck constant of order GMm for a system of two masses. This would suggests that gravitational interaction allows large values of Planck constant and small-p p-adicity in macroscopic time scales.

In the experiments described in [1] , [1] one studies the number of events per fixed time interval τ. This time interval is macroscopic in the measurements studied. One has $\tau = 36$ seconds ($\tau = 6$ seconds) in the experiment whose histogram is represented by Fig. 1 (Fig. 2) of [1] , [1] . One could argue that the secondary p-adic time scale $T_P(2) = rPT_{CP_2}$ for scaled up Planck constant $\hbar = r\hbar_0$ should of the same order of magnitude as τ. This gives the condition

$$r \sim \frac{\tau}{PT_{CP_2}} < \frac{\tau}{T_{CP_2}} \quad .$$

For $\tau = 36$ seconds one has $\frac{\tau}{T_{CP_2}} \simeq 360 \times M_{127}$. For $r = 2^{127}$ this would give $P \sim 360$. The value of P estimated from the distribution of Fig.1 of [1] , [1] is about $P \sim 100$ which is about 3.5 times smaller than the upper bound. This suggests that one p-adic time scale must be shorter than τ but of same order of magnitude. For the second experiment (Fig. 2 of [1] , [1]) one would obtain $P \leq 50$ which is 5 times larger than the estimate for $P \sim 10$ from periodicity.

$r = 2^{127}$ might make sense since M_{127} defines the secondary p-adic length scale of electron which is .1 seconds, a fundamental bio-rhythm, and corresponds to photon wavelength which is of order of circumference of Earth. This would also suggest that the modification of distributions could correspond to same value of P and m for laboratories at different sides of globe. Whether this is the case is easy to test in principle.

The notion of causal diamond (intersection of future and past directed lightcones central for the notion of zero energy ontology. The proper time distance between its tips is given by $2^k T_{CP_2}$ and assign to each elementary particle a macroscopic time scale identifiable as secondary p-adic time scale characterizing the particle. $T(127) = 2^{127}T_{CP_2}$ characterizes the causal diamond of electron, which in turn corresponds to the length scale assigned with $P = 2$ and $r = 2^{126}$. Could $r = 2^{126}$ be in preferred role that the findings of Shnoll would reflect new physics associated with electron, possibly with its gravitational interactions?

5.2 Is dark matter at the space-time sheets mediating gravitational interaction involved?

The periodic variation of the distributions in time scales assignable to gravitation encourages to ask whether the gigantic value of Planck constant could correspond to gravitational Planck constant introduced originally by Nottale [6] and assumed in TGD Universe to characterize space-time sheets mediating gravitational interaction and carrying dark manner -at least gravitons- with gigantic value of Planck constant implying quantum coherence in astrophysical scales [4, 3] .

The formula proposed by Nottale [6] for the gravitational Planck constant is dictated by Equivalence Principle and reads as

$$r_{gr} = \frac{\hbar_{gr}}{\hbar_0} = \frac{GMm}{v_0} \quad . \tag{5.1}$$

Here v_0 is a parameter with dimensions of velocity and one has $v_0/c \simeq 2^{-11}$ for the inner planets in the model of Nottale and 5 times smaller for outer planets. As a matter fact, the order of magnitude of the rotation velocity of planet around Sun is related to v_0 by numerical constant of order unity by Bohr rules, which in TGD Universe are an exact part of quantum theory.

If the large value of $\hbar_{gr}$ is associated with the gravitational interaction of smaller system with Earth with mass $M_E = 5.9737 \times 10^{24}$ kg, the mass of the system in question should be estimated from the condition

$$r = M^{127} = \frac{GM_E m}{v_0 \hbar_0} \quad . \tag{5.2}$$

This gives $m \simeq 135 \times \frac{v_0}{c}$ kg. For $v_0 = 2^{-11}$ this would give mass about $m = .05$ g which might represent mass for some part of measurement apparatus. The mass of Sun is $M_{Sun} \simeq .333 \times 10^6 M_E$ and similar estimate gives a mass $m = .15 \times 10^{-9}$ kg to be compared with Planck mass $m_{Pl} = 4.3 \times 10^{-9}$ kg. For $c/v_0 = 70$ the estimate would give Planck mass. Note however that it is difficult to relate this value of v_0 to any velocity in Earth-Sun system. For the density of water Planck mass corresponds to a size scale 10^{-4} m assignable to a large cell.

Maybe dark matter systems representing the quanta of gravitational flux equal to Planck mass analogous to quanta of electric flux are involved and are important also for biological systems. The interaction of Planck mass with Earth's gravitational field would correspond to $r = 3 \times 2^{107}$: M_{107} defines the p-adic length scale assignable to hadrons.

6 Conclusions

The proposed model has the potential of explaining the findings of Shnoll but detailed numerical work is required to find whether the model works also at the level of details.

1. The universality of the modified distributions would reduce to the replacement of various rational numbers characterizing the probability distribution with their quantum variants defined in a manner respecting the decomposition of integers to primes. p-Adic counterparts of probability distributions are essential for understanding how to avoid the difficulties resulting from negative values of quantum integers. The model makes very detailed predictions about the periodically occurring positions of the peaks of the probability distribution as function of P and m based on number theoretical considerations and in principle allows to determined these parameters for a given distribution.

2. If the value of P is outcome of state function process, it is not determined by deterministic dynamics but should have a distribution. If this distribution is peaked around one particular value, one can understand the findings of Shnoll.

3. The slow variation of the p-adic prime P and integer m characterizing quantum integers would explain the slow variation of the distributions with position and time. The periodic variations occurings withboth solar and sidereal periods can be understood if the values of P and m are characterized by the sum of gravitational accelerations assignable to Earth-Sun and Earth-Moon systems.

4. Various effects such as the dependence of the probability distributions on the direction of alpha particles selected using collimators and 12 hour phase shift between the directions associated with East and West direction can be understood as direct evidence for the effects of measurement apparatus on the many-sheeted space-time affecting the values of P and m.

5. The small value of p-adic prime P involved can be understood in TGD framework in terms of hierarchy of Planck constants [1] . The value of Planck constant could correspond to Mersenne prime M_{127} characterizing electron but this is not required by any deep principle. Gravitational Planck constant can indeed have gigantic values and for the interaction of a system with mass of order Planck mass with Sun the gravitational Planck constant is of the required order of magnitude.

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