TGD as it is towards end of 2024: part I

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Abstract

This article is the first part of of overall view of Topological Geometrodynamics (TGD) as it is towards the end of 2024. Various views about TGD and their relationship are discussed at the general level.

1. The first view generalizes Einstein's program for the geometrization of physics. Space-time surfaces are 4-surfaces in $H = M^4 \times CP_2$ and general coordinate invariance leads to their identification as preferred extremals of an action principle satisfying holography. This implies zero energy ontology (ZEO) allowing to solve the basic paradox of quantum measurement theory.
2. Holography = holomorphy principle makes it possible to construct the general solution of field equations in terms of generalized analytic functions. This leads to two different views of the construction of space-time surfaces in H, which seem to be mutually consistent.
3. The entire quantum physics is geometrized in terms of the notion of "world of classical worlds" (WCW), which by its infinite dimension has a unique Kähler geometry. Holography = holomorphy vision leads to an explicit general solution of field equations in terms of generalized holomorphy and has induced a dramatic progress in the understanding of TGD.

Second vision reduces physics to number theory.

1. Classical number fields (reals, complex numbers, quaternions, and octonions) are central as also p-adic number fields and extensions of rationals. Octonions with number theoretic norm $RE(o^2)$ is metrically Minkowski space, having an interpretation as an analog of momentum space M^8 for particles identified as 3-surfaces of H, serving as the arena of number theoretical physics.
2. Classical physics is coded either by the space-time surfaces of H or by 4-surfaces of M^8 with Euclidean signature having associative normal space, which is metrically M^4. $M^8 - H$ duality as analog of momentum-position duality relates these views. The pre-image of CD at the level of M^8 is a pair of half-light-cones. $M^8 - H$ duality maps the points of cognitive representations as momenta of fermions with fixed mass m in M^8 to hyperboloids of $CD \subset H$ with light-cone proper time $a = h_{eff}/m$.

   Holography can be realized in terms of 3-D data in both cases. In H the holographic dynamics is determined by generalized holomorphy leading to an explicit general expression for the preferred extremals, which are analogs of Bohr orbits for particles interpreted as 3-surfaces. At the level of M^8 the dynamics is determined by associativity. The 4-D analog of holomorphy implies a deep analogy with analytic functions of complex variables for which holography means that analytic function can be constructed using the data associated with its poles and cuts. Cuts are replaced by fermion lines defining the boundaries of string world sheets as counterparts of cuts.

3. Number theoretical physics means also p-adicization and adelization. This is possible in the number theoretical discretization of both the space-time surface and WCW implying an evolutionary hierarchy in which effective Planck constant identifiable in terms of the dimension of algebraic extension of the base field appearing in the coefficients of polynomials is central.

This summary was motivated by a progress in several aspects of TGD.

1. The notion of causal diamond (CD), central to zero energy ontology (ZEO), emerges as a prediction at the level of H. The moduli space of CDs has emerged as a new notion.
2. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that fermion momenta in suitable units are algebraic integers but integers for Galois singlets just as in the ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.
3. There has been progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs could correspond to an analysis, possibly having the decay of 3-surface to smaller 3-surfaces as a correlate.

In the first part of the article the two visions of TGD: physics as geometry and physics as number theory are discussed. The second part is devoted to M^8-H duality relating these two visions, to zero energy ontology (ZEO), and to a general view about scattering amplitudes.

Contents

1 Introduction

The purpose of this article is to give a rough overall view of the basic ideas of Topological Geometrodynamics (TGD) as it is now (2024). I wrote a similar summary 3 years ago. Several new ideas have emerged during these years, the realization of some ideas has simplified dramatically, and some ideas have turned out to be obsolete.

It must be emphasized that TGD is only a vision, not a theory able to provide precise rules for calculating scattering amplitudes although also in this respect dramatic progress has taken place during the last years. A collective theoretical and experimental effort would be needed to achieve the analogs of Feynman rules if this is possible at all.

Applications have played a key role in the development of TGD. TGD replaces the length scale reductionism of the standard model and string theories with fractality so that the applications range over all scales from QCD type physics, via nuclear and hadron physics, to atomic and molecular physics and biology and eventually to astrophysics and cosmology. In all scales the basic concepts which are new from the perspective of the standard model physics play a key role and lead to non-trivial predictions.

Furthermore, the new view of quantum measurement theory together with number theoretic vision leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory solving the basic problem of the standard quantum measurement theory and also predicts the possibility of quantum coherence in arbitrarily long scales. Applications are not discussed in this article but there are numerous articles and quite a number of books at my homepage as also articles published in the journals founded by Huping Hu, which are devoted to various applications.

It is perhaps good to explain what TGD is not and what it is or hoped to be. The article [L24] gives a slighly out-of-date overview of various aspects of TGD and is warmly recommended.

1.1 What does the attribute "Geometro-" mean?

"Geometro-" refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with sub-manifold geometry. The basic motivation is the loss of classical conservation laws in General Relativity Theory (GRT)(see **Fig. 1**). Also the interpretation as a generalization of string models by replacing string with a 3-D surface is natural.

Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [L56]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely [A2] so that TGD is unique [L7, L9, L38, L39] (see **Fig. 6**). Practically any GCI action has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correlates for massless particles. World as a set or particles is replaced with a network having 3-D particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

During last years it has become clear that holography reduces to the notion of generalized complex structure for both imbedding space and the space-time surface [L52] and space-time surfaces correspond to roots of two functions f_1 and f_2 analytic with respect to the 4 generalized complex coordinates, one of which is hypercomplex coordinate varying along light-like curves. This means a general solution of field equations which is universal in the sense that it the same for any general coordinate invariant action constructible in terms of the induced geometry. The dependence on action comes only from, presumably 2-D, singularities at which the generalized holomorphy and the associated minimal surface property fail. There are good reasons to believe that the singularities contain the information needed to construct the scattering amplitudes.

The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "the world of classical worlds" (WCW) - becomes the basic object endowed with a Kähler geometry (see **Fig. 7**). General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. A Given 3-surface fixes the space-time surface almost completely as an analog of Bohr orbit (preferred extremal [K2]). This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces. Quantum TGD reduces to wave mechanics in the space of these Bohr orbits, the WCW.

1.2 What does the attribute "Topological" mean?

Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$.

1. The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 3**).

"Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 10**).

2. Adelic physics fusing real and various p-adic physics are part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical worlds". Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in all scales.

3. The concrete realization of the number theoretic vision is based on $M^8 - H$ duality (see **Fig. 8**). One motivation for writing this summary is that quite recently the realization of the $M^8 - H$ duality has simplified dramatically [L59]. M^8 can be regarded as octonions with the Minkowskian norm identified as the octonionic real part $RE(o^2)$. As in H, the physics in M^8 is based on holography.

   An integrable distribution of the normal spaces $N(y)$, $y \in Y^4 \subset M^8$, which is Euclidean with respect to the number theoretical metric is assumed to be quaternionic and thus associative and to contain an integrable sub-distribution of commutative and complex sub-spaces. This kind of normal space is parametrized by a point of CP_2. Associative holography determines a 4-surface $Y^4 \subset M^8$ in terms of 3-D holographic data assigned to 3-spheres S^3.

   At the first step, $M^8 - H$ duality maps the point $y \in Y^4$ to a point of its normal space $N(y)$ isomorphic to M^4 by a multiplication by an octonionic imaginary unit e, fixed apart from a local $U(2)$ rotation. The interpretation is in terms of a number theoretic counterpart of the electroweak gauge invariance, realized as holonomies at the level of H whereas olor symmetries correspond to the $SU(3)$ subgroup of octonionic automorphisms at the level of M^8 and isometries of CP_2 at the level of H. At the second step, the point in $N(y)$ is mapped to a point $M^4 \subset M^4 \times CP_2$ by inversion. This realization avoids the shortcomings of the earlier proposal [L18, L19, L54].

   $M^8 - H$ duality provides two complementary visions about physics (see **Fig. 2**), and can be seen as a generalization of the q-p duality of wave mechanics, which fails to generalize to quantum field theories (QFTs).

4. The earlier formulation of $M^8 - H$ duality led to number theoretical universality justifying adelic and p-adic physics. In the recent formulation, the realization requires that the analytic functions f_1 and f_2 reduce to polynomials with integer (or even algebraic) coefficients at the 2-D singularities of the surfaces X^2 at which the holomorphy and the minimal surface property fail. This realization for the quantum criticality of TGD would select rationals and their algebraic extensions from the ocean of complex continuum.

   The hierarchy of algebraic extensions of rationals gives rise to a hierarchy of Planck constants $h_{eff} = nh_0$ and defines extensions of p-adic number fields and adeles allowing an interpretation as an evolutionary hierarchy (see **Fig. 9**) Physically this defines a hierarchy of phases of ordinary matter behaving like dark matter. The interpretation is not in terms of galactic dark matter but in terms of missing baryonic matter identified. The polynomials in turn are characterized by ramified primes having a natural interpretation as the p-adic primes characterizing elementary particles [L47] [K8].

1.3 Zero Energy Ontology

In Zero Energy Ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 12** and **Fig. 13**).

Quantum jumps occur between these superpositions and the basic problem of the standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 14**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions [L10].

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary and the states at it remain unchanged (Zeno effect). The sequence of "small" state function reductions (SSFRs) defined the TGD counterpart of the generalized Zeno effect, would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces and would also give rise to a conscious entity, self.

This means considerable progress in the understanding of the quantum measurement theory based on ZEO [L14, L40, L55] [K22].

The first new result is that Negentropy Maximization Principle [L53] implying evolution follows as an analog of the second law. In the sequence of quantum jumps the algebraic complexity, which is measured as the dimension of extension of rationals associated with the polynomials associated with the singularities, is bound to increase in a statistical sense.

The second new result [L55] is a quantum formulation of the ZEO. Zero energy states within a single CD as an analog of a perceptive field and containing space-time surfaces is generalized so that quantum states also involve a wave function in the space of CDs. The moduli space of CDs is finite-dimensional and maximally symmetric and forms the backbone of WCW in the sense that each space-time surface satisfying holography is within a particular CD. This leads also to a new view of Poincare symmetry allowing to overcome the problems due to the fact that CD itself is not Poincare invariant.

TGD develops by explaining what TGD is and also this work led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can have a dynamic entanglement. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime [K15].

2. The understanding of the details of the M^8-H duality forces us to modify the earlier view [L18, L19, L54] to a much simpler vision [L59]. The notion of causal diamond (CD) is central to zero energy ontology (ZEO) and emerges as a prediction at the level of H. The pre-image of CD under M^8-H duality in M^8 is a region bounded by two mass shells in the normal space of $y \in Y^4 \subset M^8$, which itself is an Euclidean region.

   M^8-H duality maps the points of cognitive representations defined by the points of Y^4 with coordinates as algebraic integers in the algebraic extension of rationals and identified as momenta of fermions with a fixed mass squared in M^8 to either boundary of CD in H.

3. The emergence of holography = holomorphy principle [L62] forces a profound modification of the ideas about M^8-H duality and number theoretical vision and have given a strong motivation for this article. There are tensions between the holography= holomorphy vision and number-theoretic vision and in this article a more precise form of the holography = holomorphy vision resolving the tensions is discussed.

4. Galois confinement for physical states at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that fermion momenta using a suitable unit determined by CD are algebraic integers but integers for Galois singlets just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

1.4 The description of particles and particle interactions in the TGD framework

The TGD based description of particles [L50] and particle interactions [L69, L51] has developed considerably during the last yeaars and the new view deserves a separate section.

Several key ideas of quantum TGD distinguish between TGD and QFTs.

1. The basic problem of QFT is that it involves only an algebraic description of particles. An explicit geometric and topological description is missing but is implicitly present since the algebraic structure of QFTs expresses the point-like character of the particles via commutation and anticommutation relations for the quantum fields assigned to the particles.

   In the string models, the point-like particle is replaced by a string, and in the string field theory, the quantum field Ψ(x) is replaced by the stringy quantum field Ψ(string), where "string" corresponds a point in the infinite-D space of string configurations (say loop space). The interpretation is as a second quantization of string theory. This approach is rather baroque since one must introduce a non-linear action principle in the string space. This however allows to understand M^4 as the configuration space for the positions of a point-like particle.

   In TGD, the quantum field Ψ(x) is replaced by a formally *classical* spinor field Ψ (Bohr orbit). The 4-D Bohr orbits are preferred extremals of classical action satisfying holography forced by general coordinate invariance without path integral and represent points of the "world of classical worlds" (WCW). The components of Ψ correspond to multi-fermion states, which are pairs of ordinary 3-D many-fermion states at the boundaries of causal diamond (CD).

   The gamma matrices of the WCW spinor structure are linear combinations of the fermionic oscillator operators for the second quantized free spinor field of H. They anticommute to the WCW metric, which is uniquely determined by the maximal isometries for WCW guaranteeing the existence of the spinor connection. Physics is unique from its existence, as implied also by the twistor lift and number theoretic vision and of course, by the standard model symmetries and fields.

   Also the notion of induced spinor fields as a restriction of spinor fields to space-time surface is involved and the induced spinor fields satisfy the modified Dirac equation as the analog of massless Dirac equation [L63].

2. In TGD, the notion of a classical particle as a 3-surface moving along 4-D "Bohr orbit" as the counterpart of world-line and string world sheet is an exact aspect of quantum theory at the fundamental level. The notions of classical 3-space and particle are unified. This is not the case in QFT and the notion of a Bohr orbit does not exist in QFTs. TGD view of course conforms with the empirical reality: particle physics is much more than measuring of the correlation functions for quantum fields.

   Quantum TGD is a generalization of wave mechanics defined in the space of Bohr orbits. The Bohr orbit corresponds to holography realized as a generalized holomorphy generalizing 2-D complex structure to its 4-D counterpart, which I call Hamilton-Jacobi structures (see this). Classical physics becomes an exact part of quantum physics in the sense that Bohr orbits are solutions of classical field equations as analogs of complex 4-surfaces efined as roots of two generalized analytic functions in $H = M^4 \times CP_2$ endowed with generalized complex coordinates. The space of these 4-D Bohr orbits gives the WCW (see this), which corresponds to the configuration space of an electron in ordinary wave mechanics.

   There is no need for the second quantization to describe many particle systems as in the case of wave mechanics since the many-particle states are described topologically as unions of disjoint 3-surfaces and unions of partonic orbits inside them.

3. The second quantized spinor fields of H are needed to define the spinor structure in WCW. The spinor fields of H are the free spinor fields in H coupling to its spinor connection of H. The Dirac equation can be solved exactly and second quantization is trivial and one avoids the usual problems caused encountered for curved space-times, such as the non-existence of spinor structure and existence of several spinor structures encountered already in QCD

lattice calculations where the periodic boundary conditions effectively replace the topology of the Minkowski space with that of 4-torus.

This determines the fermionic propagators in H and induces them at the space-time surfaces. The propagation of fermions is thus trivialized. All that remains is to identify the vertices.

4. At the fermion level, all elementary particles, including bosons, can be said to be made up of fermions and antifermions, which at the basic level correspond to light-like world lines on 3-D parton trajectories, which are the light-like 3-D interfaces of Minkowski spacetime sheets and the wormhole contacts connecting them.

   The light-like world lines of fermions are boundaries of 2-D string world sheets and they connect the 3-D light-like partonic orbits bounding different 4-D wormhole contacts to each other. The 2-D surfaces are analogues of the strings of the string models.

5. In TGD, classical boson fields are induced fields and no attempt is made to quantize them. Bosons as elementary particles are bound states of fermions and antifermions. This is extraordinarily elegant since the expressions of the induced gauge fields in terms of embedding space coordinates and their gradients are extremely non-linear as also the action principle. This makes standard quantization of classical boson fields using path integral or operator formalism a hopeless task.

   There is however a problem: how to describe the creation of a pair of fermions and, in a special case, the corresponding bosons, when there are no primary boson fields? Can one avoid the separate conservation of the fermion and the antifermion numbers?

2 Physics as geometry

The following provides a sketchy representation of TGD based on the vision about physics as geometry which is complementary to the vision of physics as number theory. M^8-H duality relates these two visions. A longer representation of the situation as it was in 2021 can be found in [L24]. Representations summarizing aspects of the recent state can be found in [L56, L57, L52, L69, L62].

At the general level one can say that physics as geometry vision is dictated by 3 basic principles: General Coordinate Invariance (GCI) and Equivalence Principle of General Relativity and the Relativity Principle of Special Relativity interpreted as Poincare invariance. Equivalence Principle would be realized at the QFT limit in terms of Einstein's field equations. At quantum level it has taken a long time to understand its realization [L69].

2.1 Space-time as 4-surface in $H = M^4 \times CP_2$

The starting idea is the identification of the space-time as 4-surface in $H = M^4 \times CP_2$.

1. The energy problem of GRT means that since space-time is curved, one cannot define Poincare charges as conserved Noether charges (see **Fig. 1**). If space-time X^4 is a surface in $H = M^4 \times CP_2$, the situation changes. Poincare symmetries are lifted to the level of $M^4 \subset H$.

2. A generalization of the notion of particle is in question: a point-like particle is replaced with a 3-surface so that TGD can be seen also as a generalization of the string model. String is replaced with a 3-surface. String world sheet is replaced with the space-time surface. The notions of the particle and space are unified.

3. Einstein's geometrization program is extended to standard model interactions. CP_2 codes for standard model symmetries and gauge fields. Isometries $\leftrightarrow$ color SU(3). Holonomies of spinor connection $\leftrightarrow$ electroweak U(2) [L56]. Genus-generation correspondence provides a topological explanation of the family replication phenomenon of fermions [K3]: 3 fermion families are predicted.

4. Induction of the spinor structure by projecting the components of spinor connection from CP_2 to X^4 is central for the geometrization. The projections of Killing vectors of color

isometries yield classical color gauge potentials. Parallel translation at X^4 using spinor connection of H.

Also the spinor structure is induced and means that H spinors are restricted to space-time surfaces and the induced gamma matrices are obtained as projections of gamma matrices of H. Here one can also consider a second option in which the modified gamma matrices are defined as contractions of the canonical momentum currents with the gamma matrices of H. For the volume action, one obtains essentially the induced gamma matrices. This seems to be the correct option since the anticommutators of the induced gamma matrices give the induced metric. The induction can be defined also for the second quantized free spinor fields of H and one can define a modified Dirac action and Dirac equation [L63, L69].

5. The twistor lift of TGD strongly suggests that the dynamics of X^4 is determined by an action S consisting of Kähler action plus volume term (cosmological constant) following from the twistor lift of TGD [K17, L9, L38, L39, K19]masterformula, which is dimensional reduction of 6-D Kähler action for the 6-D surfaces X^6 in the 12-D twistor space of H defining the counterpart of the twistor space of X^4. The field equations outside the lower-D singularities do not however depend on action as long as it is a general coordinate invariant constructed in terms of the induced geometry. Since they are purely algebraic involving only contractions of generalized complex tensors with different types.

6. The dynamics for fermions at the space-time level can be determined by the modified Dirac action. For the modified gamma matrices defined by the entire action there are huge super-conformal and supersymplectic symmetries but in the recent formulation scattering amplitudes would be trivial [L69, L62]. In this case the anticommutators of modified gammas are not proportional to the induced metric. For the induced gamma matrices, determined by the volume action as a part of the action, the superconformal and related symmetries fail at 2-D singular surfaces, where the generalized holomorphy and minimal surface property fail. Quantum theory becomes non-trivial so that the violation of symmetries is the price paid for non-trivial dynamics. Second quantized H-spinors, whose modes satisfy free massless Dirac equation in H restricted to X^4: this induces second quantization to X^4 and one avoids the usual problems of quantization in a curved background. This picture is consistent with the modified Dirac equation satisfied by the induced spinors in X^4.

7. The most plausible option is that leptons and quarks correspond to different H-chiralities with different coupling to the induced Kähler gauge potential of CP_2. B and L would be separately conserved. Matter antimatter symmetry could be due to the fact that fermions and antifermions are conjugates of each other with respect to the light-like hypercomplex coordinate and possible also the complex coordinates of CP_2 so that they cannot appear at the space-time space sheet, which can have even astrophysical size if the number theoretical vision predicting the hierarchy of effective Planck constants is accepted.

   Only quarks would be needed if leptons can be identified as 3-quark composites in the CP_2 scale: this option is not excluded if one accepts the TGD view about color symmetries. Protons would not be stable. This would provide a new view about matter antimatter asymmetry [L15, L27]. CP violation could be forced by the M^4 part of Kähler form forced by the twistor lift. It however seems that the first option is more plausible.

2.1.1 Basic extremals of classical action

For a long time it was clear that practically any GCI action allows the same basic extremals (for basic questions related to classical TGD see **Fig. 3**). Now it clear that the solutions outside singularities are universal and satisfy generalized holomorphy [L52, L69, L62]. One can however consider also minimal surface solutions [L68] which do not possess the holomorphy and for which the field equations do not reduce to purely algebraic conditions.

1. CP_2 type extremals having a light-like geodesic as M^4 projection and Euclidean signature of the induced metric serve as building bricks of elementary particles. If the volume term is absent as it might be at an infinite volume limit, the geodesics become light-like curves [L37]. Wormhole contacts connecting two Minkowskian space-time sheets can be regarded as a

piece of a deformed CP_2 type extremal. Monopole flux through contact stabilizes the wormhole contact.

2. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow superposition of modes with a single direction of light-like momentum. Ideal laser beam is a convenient analogy here.

3. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and their thickenings to flux tubes, playing a fundamental role in all scales, are also a central notion.

2.1.2 QFT limit of TGD

The induced gauge fields and gravitational field are expressible in terms of only 4 H- coordinates. Locally the theory is quite too simple to be physical and applies only at the microscopic level.

1. Many-sheeted space-time means that X^4 is topologically extremely complex: this has far reaching implications in all scales, which are not predicted by Einstein's theory. The key point is that connected space-time surfaces define quantum coherence regions and can be arbitrarily large. CP_2 coordinates are many-valued functions of M^4 coordinates or vice versa or both. In contrast to this, the space-time of EYM theory is topologically extremely simple.

2. Einsteinian space-times have 4-D projection to M^4. Small test particle experiences the sum of the classical gauge potentials associated with various space-time sheets. At QFT limit the sheets are replaced with a single region of M^4 made slightly curved and gauge potentials are defined as the sums of gauge potentials from different space-time sheets having common M^4 projection. Topological complexity and local simplicity are replaced with topological simplicity and local complexity. (see **Fig. 3**). Einstein YM equations can be interpreted as a remnant of the isometries of Poincare symmetry.

2.1.3 A possible problem related to the twistor lift

The twistor lift strongly suggests that the Kähler form of M^4 exists. The Kähler gauge potential would be the sum of M^4 and CP_2 contributions. The definition of M^4 Kähler structure is however not straightforward [L18, L19]. The naive guess would be that J represents an imaginary unit as the square root of -1 represented by the metric tensor. This would give the condition $J^2 = -g$ for the tensor square but this seems to leads to problems.

To understand the situation, notice that the analogs of symplectic/Kähler structures in $M^4 \subset H$ have a moduli space, whose points correspond to what I have called Hamilton-Jacobi structures [L52] defined by integrable distributions of orthogonal decompositions $M^4 = M^2(x) \times E^2(x)$: $M^2(x)$ is analogous to string world sheet and Y^2 to partonic 2-surface. This means the presence of slicing by string world sheets $X^2(x)$, where x labels a point of Y^2. $X^2(x)$ is orthogonal to Y^2 at x. One can interchange the roles X^2 ad Y^2 in the slicing.

The induced Kähler form has an analogous decomposition. The decomposition is completely analogous to the decomposition of polarizations to non-physical time-like ones and physical space-like ones. This decomposition allows a natural modification of the definition of the symplectic structure so that the problem caused by $J^2 = -g$ conditions is avoided.

Consider first the problem. The $E^2(x)$ part of M^4 Kähler metric produces no problems since the signature of the metric is Euclidean. For $M^2(x)$ part, the Minkowskian signature produces problems. If one assumes that the $M^2(x)$ part of the Kähler form is non-vanishing, it should be imaginary in order to satisfy $J^2(M^2(x)) = -g(M^2(x))$. This implies that Kähler gauge potential is imaginary and this spoils the hermiticity of the modified Dirac equation [K21]. Also the electric contribution to the Kähler energy is negative.

The solution of the problem turned out to be ridiculously simple and I should have noticed it a long time ago.

1. $M^2(x)$ has a hypercomplex structure, which means that the imaginary unit e satisfies $e^2 = 1$ rather than $e^2 = -1$. Hamilton-Jacobi structure allows one to decompose J locally into two parts $J = J(M^2(x)) + J(E^2(x))$ such that $J^2 = g(M^2(x)) - g(E^2(x))$. This gives

$J^4 = g(M^4)$. The Kähler energy of the canonically embedded M^4 is non-vanishing and positive whereas Kähler action vanishes by self-duality. Situation is identical to that in Maxwell's electrodynamics.

2. Kähler action for the canonically embedded M^4 vanishes and it is possible to define also Lagrangian 2-surfaces as surfaces for which the induced Kähler form vanishes. These are of special interest since they would guarantee small CP violation: string world sheets could be examples of these surfaces. Note that since the magnetic part of J induces violation of CP, the violation is vanishing for CP_2 type extremals and cosmic strings and also small for flux tubes.

If the notion of symplectic/canonical transformation generated by Hamiltonian preserving J generalizes, one could generate an infinite number of slicings.

Consider first ordinary symplectic transformations.

1. For the ordinary symplectic transformations, the closedness of the symplectic for J is essential ($dJ = 0$ corresponds to topological half of Maxwell's equations).

2. Second essential element is that symplectic transformation is generated as a flow for some Hamiltonian H: $j_H = i_{dH}J$ or more explicitly: $j^l_H = J^{kl}\partial_l H$. It is essential that one has $i_{j_H}J = -dH$: having a vanishing exterior derivative. In other words, $J_{kl}j^l_H = -\partial_k H$ is a gradient vector field and has therefore a vanishing curl. Together with $dJ = 0$, this guarantees the vanishing of the Lie derivative of J: $d_{j_H}J = d(i_{j_H}J) + i_{j_H}dJ = ddH + dJ(j_H) = 0$ so that J is preserved.

Could one talk about symplectic transformations in M^4?

1. The analogs of symplectic/canonical transformations should map the Hamilton-Jacobi structure to a new one and leave $J(M^2(x))$ and $J(E^2(x))$ invariant. The induced metrics of X^2 and Y^2 need not be preserved since only the diagonal metric $g^k_l(X^2/Y^2)$ appears in the conditions $J^2 = g(X^2) - g(Y^2)$.

2. The symplectic transformation generated by the Hamiltonian H would be a flow defined by the vector field $j_H = i_{dH}J$ and one would have $i_{j_H}J = -d_1H + d_2H$, where d_1 and d_2 are gradients operators in X^2 and Y^2. Usually one would have $J_{kl}j^l = dH$ satisfying $d^2H = 0$.

   The condition $ddH = 0$ satisfied by the ordinary symplectic transformations is replaced with the condition $d(-d_1H + d_2H) = 0$. This can be written as $-d_1^2H + d_2^2H + [d_2, d_1]H = 0$, and is satisfied. Therefore this part is not a problem.

2.2 World of classical worlds (WCW)

The notion of WCW emerges as one gives up the idea about quantizing by path integral, which in TGD framework is a mission impossible due to the extreme non-linearity of the theory.

2.2.1 The failure of path integral forces WCW geometry

The extreme non-linearity implies that the path integral for the space-time surfaces fails. A possible solution of the problem is to generalize Einstein's geometrization program to the level of the entire quantum theory.

1. "World of classical worlds" (WCW) can be identified as the space of 3-surfaces endowed with a metric and spinor structure (see **Fig. 7**). Hermitian conjugation must have a geometrization. This requires Kähler structure requiring also complex structure. WCW has Kähler form and metric.

2. WCW spinors are Fock states created by fermionic oscillator operators assignable to spinor modes of H basically [L22]. WCW gamma matrices as linear combinations of fermionic (quark) oscillator operators defining analog of vielbein.

WCW has also spinor connection and curvature in WCW. The quantum states of world correspond formally to *classical* spinor fields in WCW. Also the gamma matrices of WCW expressinble in terms of fermionic oscillator operators in a formal sense also purely classical objects.

One can however ask whether the analog path integral could emerge as a discrete variant. 4-D Bohr orbits as minimal surfaces are not strictly deterministic: already the 2-D minimal surfaces fail to be deterministic. The failure of determinism would naturally take place at the singularities identifiable as particle vertices, where also holomorphy and minimal surface property would fail. Vertices would correspond to increased quantum criticality. Transition amplitude would be a sum over Bohr orbits: could this sum define a discrete analog of the path integral.

2.2.2 Implications of General Coordinate Invariance

General Coordinate Invariance (GCI) in 4-D sense forces to assign to 3-surface X^3 a 4-surface $X^4(X^3)$, which is as unique as possible. This means slightly non-deterministic holography and WCW must be identified as the space of 4-D Bohr orbits of 3-surfaces. This leads to zero energy ontology (ZEO). This also gives rise quantum classical correspondence (QCC) meaning that the classical theory is an exact part of quantum theory (QCC).

A solution to the basic paradox of quantum measurement theory emerges [L14]: superposition of deterministic time evolutions is replaced with a new one in state function reduction (SFR): SFR does not force any failure of determinism for individual time evolutions. The singularities of the minimimal surfaces as analogs of particle vertices [L69, L62] serve as seats of non-determinism and a discrete analog of path integral emerges.

2.2.3 WCW Kähler geometry from classical action

WCW geometry is determined by a classical action defining Kähler function $K(X^3)$ for a preferred extremal $X^4(X^3)$ defining the preferred extremal/Bohr orbit [K6] (see **Fig. 7**). But any general coordinate invariant action can be considered if holography=holomorphy vision is accepted.

1. QCC suggests that the definition of Kähler function assigns a more or less unique 4-surface $X^4(X^3)$ to 3-surface X^3. Finite non-uniqueness is however possible and highly plausible by the experience with 2-D minimal surfaces [L37].

2. $X^4(X^3)$ is identified as a *preferred* extremal of some general coordinate invariant (GCI) action forcing the Bohr orbit property/holography/ZEO. This means a huge reduction of degrees of freedom.

   Remark:: Already the notion of induced gauge field and metric eliminates fields as primary dynamical variables and GCI leaves locally only 4 H-coordinates as dynamical variables.

3. The twistor lift [L7, L9] of TGD geometrizes the twistor Grassmann approach to QFTs. The 6-D extremal X^6 of 6-D Kähler action as a 6-surface in the product $T(M^4)\times T(CP_2)$ of twistor spaces of M^4 and CP_2 represents the twistor space of X^4.

   The condition that X^6 reduces to an S^2 bundle with X^4 as base space, forces a dimensional reduction of 6-D Kähler action to 4-D Kähler action + volume term, whose value for the preferred extremal defines the Kähler function for $X^4(X^3)$.

4. The volume term corresponds to a p-adic length scale dependent cosmological constant Λ approach zero at long p-adic length scale so that a solution of the cosmological constant problem emerges. Preferred extremal/Bohr orbit property means a simultaneous extremal property for *both* Kähler action and volume term. This forces X^4 to have a generalized complex structure (Hamilton-Jacobi structure) [L52] so that field equations trivialize and there is no dependence on coupling parameters. Universality of dynamics follows and the TGD Universe is quantum critical. In particular, Kähler coupling strength is analogous to a critical temperature and is quantized [L34].

5. Soap film analogy is extremely useful [L37]: the analogs of soap film frames are singular surfaces of dimension $D < 4$. At the frame the space-time surface fails to be a simultaneous extremal of both actions separately and Kähler and volume actions couple to each other. The corresponding contributions to conserved isometry currents diverge but sum up to a finite contribution. The frames define the geometric analogs for the vertices of Feynman diagrams.

2.2.4 WCW geometry is highly unique

WCW geometry is fixed to a high degree by the existence of Riemann connection and requires maximal symmetries.

1. Dan Freed [A1] found that loop space for a given Lie group allows a unique Kähler geometry: maximal isometries are needed in order to have a Riemann connection. Same expected to be true now [K4, K13] [L57]. Supersymplectic symmetries and generalized super conformal symmetries allow conserved Noether currents and their super counterparts outside the singularities, where the minimal surface property fails, define candidates for the isometries of WCW.

2. Twistor lift of TGD [L7, L9] means that one can replace X^4 with its twistor space $X^6(X^4)$ in the product $T(M^4) \times T(CP_2)$ of the 6-D twistor spaces $T(M^4)$ and $T(CP_2)$. $X^6(X^4)$ is a 6-surface with the structure of S^2 bundle.

   Dimensionally reduced 6-D Kähler action gives the sum of 4-D Kähler action and volume term. Twistor space must however have a Kähler structure and only the twistor spaces of M^4,E^4, and CP_2 have Kähler structure [A2]. TGD is unique both physically and mathematically!

How unique the Kähler geometry of WCW is? Symmetric space property without any zero modes would mean that all 3-surfaces are isometrically equivalent. This cannot make sense. The proposal has been that there are zero modes which do not contribute to the line element of WCW but that the components of the metric depend on the zero modes. For instance, the the induced Kähler form, in particular its fluxes, would represent such degrees of freedom and be identifiable as moduli of the WCW metrics. Also the moduli space of generalized complex structures of the space-time surface [L52] could correspond to zero modes. Also moduli space for the hierarchy of CDs [L55] would define zero modes.

Holography=holomorphy vision implies that the space-time surfaces and the scattering amplitudes depend on the action only at the singularities where the minimal surface property and holography fail. Could this mean that different choices for the action could code for the zero modes and also parametrize coupling constant evolution as in quantum field theories.

2.2.5 Isometries of WCW

What can one say about the isometries of WCW? Certainly, they should generalize the conformal symmetries of string models.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group of symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r as a candidate for the isometries of WCW [K4].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

A new element in this picture are generalized conformal transformations acting as a dynamical symmetry group. They do not leave action invariant but the Noether currents are conserved. The bosonic field equations are true also at the singularities at which the minimal surface property fails. In the fermionic sector, the assumption that the modified Dirac action involves induced gamma matrices defined by the volume term of the action rather than the entire action, implies that supercurrents are not conserved at the singularities meaning a failure of supersymmetry. Also the breaking of generalized conformal symmetries could take place in this way. This seems necessary for the non-triviality of the scattering amplitudes [L69, L62].

Conformal transformations in 2 dimensions preserve orthogonality. The restrictions of 4-D conformal transformation of M^4 to the boundary of the light-cone preserved the angles on the light-cone and also the corresponding restrictions at the light-like partonic orbits. These transformations can be also chosen to be isometries. This might be true for all for the slicings by surfaces parallel to the light-like orbits defined by the light-like Hamilton-Jacobi (hypercomplex) coordinate u and its dual?

It seems should be true in the 4-dimensional case in the sense that angles between vectors of $M^2(x)$ and $E^2(x)$ are preserved. It $M^2(x)$ and $E^2(x)$ are not orthogonal, the induced metric has mixed components and this is consistent with the holomorphy.

2.3 About Dirac equation in TGD framework

Quantum TGD involves three levels of geometry corresponding to H, to space-time surfaces in H and to WCW. Also the spinor structure appears at 3 levels.

2.3.1 Three Dirac equations

In TGD spinors appear at 3 levels:

1. At the level of embedding space $H = M^4 \times CP_2$ the spinor field embedding space $M^4 \times CP_2$spinor fields (quark field) is a superposition of the harmonics of the Dirac operator. In the complexified M^8 having interpretation as complexified octonions, spinors are octonionic spinors. In accordance with the fact that M^8 is analogous to momentum space, the Dirac equation is purely algebraic and its solutions correspond to discrete points analogous to occupied points of Fermi ball.

2. The spinors at the level of 4-surfaces $X^4 \subset H$ are restrictions of the second quantized embedding space spinor field in X^4 so that the problematic second quantization in curved background is avoided. At the level of M^8 the restriction selects the points of M^8 belonging to 4-surface and carrying quark. The simplest manner to realize Fermi statistics is to assume that there is at most a single quark at a given point.

3. The third realization is at the level of the "world of classical worlds" (WCW) assigned to H consisting of 4-surfaces as preferred extremals of the action. Gamma matrices of WCW are expressible as superpositions of quark oscillator operators so that anti-commutation relations are geometrized. The conditions stating super-symplectic symmetry are a generalization of super-Kac-Moody symmetry and of super-conformal symmetry and give rise to the WCW counterpart of the Dirac equation [K13] [L24].

4. What the realization of WCW at the level of M^8 is, has remained unclear. The notion of WCW geometry does not generalize to his level and should be replaced with an essentially number theoretic notion.

   Adelic physics as a fusion of real and p-adic physics suggests a possible realization. Given extension of rationals induces extensions of various p-adic number fields. These can be glued to a book-like structure having as pages real numbers and the extensions of p-adic number fields.

The pages would intersect along points with coordinates in the extension of rationals. These points form a cognitive representation. The additional condition that the active points are occupied by quarks guarantees that this makes sense also for octonions, quaternions and 4-surface in M^8. The p-adic sector could consist of discrete and finite cognitive representations continued to the p-adic surface and define the counterpart of WCW at the level of M^8?

2.3.2 The relationship between Dirac operator of H and modified Dirac operator

At the level of $X^4 \subset H$, the proposal is that modified Dirac action for the induced spinor fields defines the dynamics somehow. Modified Dirac equation or operator should be also consistent with the second quantization of induced spinor fields performed at the level of H and inducing the second quantization at the level of X^4.

1. The modified gamma matrices Γ^α are defined by the contractions of H gamma matrices Γ_k and canonical momentum currents $T^{k\alpha}$ associated with the action defining space-time surface. The modified Dirac operator $D = \Gamma^\alpha D_\alpha$, where D_α is X^4 projection of the vector defined by the covariant derivative operators of H ($D_\alpha = \partial_\alpha h^k D_k$). Hermiticity requires $D_\alpha \Gamma^\alpha = 0$ implying that classical field equations are satisfied.

2. Can one assume that the modified Dirac equation is satisfied? Or is it enough to assume that this is not the case so that the modified Dirac operator defines the propagator as its inverse as the QFT picture would suggest?

   In fact, the propagators in H allow to compute N-point functions involving quarks and at the level of H the theory is free and the restriction to the space-time surface brings in the interactions. Therefore the notion of space-time propagator is not absolutely necessary. One can however ask whether some weaker condition could be satisfied and provide new insights.

   One can also ask whether the solutions of the modified Dirac equation correspond to external particles, which correspond to space-time surfaces for which the solution of the modified Dirac equation is consistent with the solution of the Dirac equation in H. Are these kinds of space-time surfaces possible?

3. The intuitive picture is that the solutions of the modified Dirac equation correspond to the external particles of a scattering diagram having an interpretation on mass shell states and are possible only for a very special kind of preferred extremals. Intuitively they should correspond to singular surfaces in M^8 and their mapping to H would involve blow-up due to the non-uniqueness of the normal space along lower than 4-D surface. String like objects and CP_2 type extremals would be basic entities of this kind. Could the modified Dirac equation or its weakened form hold true for these surfaces.

The strong form of equivalence of modified Dirac equation and ordinary Dirac equation would mean the equivalence of the actions of two Dirac operators acting on the second quantized induced spinor field.

1. The modified Dirac operator is given by $\Gamma_k T^{\alpha k} \partial_\alpha h^k D_k$ and its action should be same as H Dirac operator $\Gamma^k D_k$. This would require

$$\Gamma_k T^{\alpha k} \partial_\alpha h^k D_k \Psi = \Gamma^k D_k \Psi \quad . \tag{2.1}$$

   Not surprisingly, it turns out that this condition is too strong.

2. One can express Γ_k using an overcomplete basis defined by the Killing vector fields j_A^k for H isometries. In the case of M^4 it is enough to use translations by using the identity $\sum_A j_A^k j_A^l = h^{kl}$. This allows to define gamma matrices $\Gamma_A = \Gamma_k j_A^k$ and to write the equation in the form

$$\Gamma_A T^{A\alpha} \partial_\alpha h^k D_k \Psi = \Gamma_A j_A^k D_k \Psi \quad . \tag{2.2}$$

Here $T^{A\alpha}$ is the conserved isometry current associated with the Killing vector j_A^k. Is it possible to satisfy the condition

$$T^{A\alpha}\partial_\alpha h^k = j_A^k \tag{2.3}$$

or its suitably weakened form?

The strong form of the condition cannot be satisfied. The left hand side of the equation is determined by the gradients of H coordinates and parallel to X^4 whereas the right hand side also involves the component normal to X^4. Therefore the condition cannot be satisfied in the general case.

3. By projecting the condition to the tangent space, one obtains a weaker condition stating that the tangential parts of two Dirac operators are proportional to each other with a position dependent proportionality factor $\Lambda(x)$:

$$\begin{aligned} T^{A\alpha} &= \Lambda(x) j_A^\alpha \\ j_A^\alpha &= j_A^k \partial^\alpha h_k = j_A^k h_{kl} g^{\alpha\beta}\partial_\beta h^l \ . \end{aligned} \tag{2.4}$$

The conserved isometry current is proportional to the projection of the Killing vector to the tangent space of X^4. $\Lambda(x)$ is proportionality constant depending on the point of X^4. Isometry current is analogous to a Hamiltonian vector field being parallel to the Killing vector field.

4. If the action were a mere cosmological volume term, the isometry currents would be proportional to j^α so that the conditions would be automatically satisfied. The contribution to $\Lambda(x)$ is proportional to the p-adic length scale dependent cosmological constant.

Kähler action receives contributions from both M^4 and CP_2. Both add to $T^{A\alpha}$ a term of form $T^{\alpha\beta}j_{A\beta}$ coming from the variation of the Kähler action with respect to $g_{\alpha\beta}$. $T^{\alpha\beta}$ is the energy momentum tensor with a form similar to that for Maxwell action.

Besides this, M^4 *resp.* CP_2 contribute a term proportional to $J^{\alpha\beta}J_{kl}\partial_\beta h^k j_A^k$ coming from the variation of the Kähler action with respect to $J_{\alpha\beta}$ contributing only to M^4 *resp.* CP_2 isometries. These contributions make the conditions non-trivial. The Kähler contribution to $\Lambda(x)$ need not be constant. Note that the Kähler contributions to the energy momentum tensor vanish if X^4 is (minimal) surface of form $X^2 \times Y^2 \subset M^4 \times CP_2$ so that both X^2 and Y^2 are Lagrangian.

5. The vanishing of the divergence of $T^{A\alpha}$ using the Killing property $D_l j_{Ak} + D_k j_{Al} =$ of j_{Ak} gives

$$j^{A\alpha}\partial_\alpha\Lambda = 0 \ . \tag{2.5}$$

Λ is constant along the flow lines of $j^{A\alpha}$ and is therefore analogous to a Hamiltonian. The constant contribution from the cosmological term to Λ does not contribute to this condition.

6. An attractive hypothesis, consistent with the hydrodynamic interpretation, is that the proposed condition is true for all preferred extremals. The conserved isometry current along the X^4 projection of the flow line is proportional to the projection of Killing vector: this conservation law is analogous to the conservation of energy density $\rho v^2/2 + p$ along the flow line). One can say that isometries as flows in the embedding space are projected to flows along the space-time surface. One could speak of projected or lifted representation.

7. The projection to the normal space does not vanish in the general case. One could however ask whether a weaker condition stating that the second fundamental form $H^k_{\alpha\beta} = D_\alpha h^k$, which is normal to X^4, defines the notion of the normal space in terms of data provided by space-time surface. If X^4 is a geodesic submanifold of H, in particular a product of geodesic submanifolds of M^4 and CP_2, one has $H^k_{\alpha\beta} = 0$.

2.3.3 Gravitational and inertial representations of isometries

The lift/projection of the isometry flows to X^4 strongly suggests a new kind of representation of isometries as analog of the braid representation considered earlier.

1. Projected/lifted representation would clarify the role of the classical conserved charges and currents and generalize hydrodynamical conservation laws along the flow lines of isometries. In particular, quark lines would naturally correspond to time-like flow lines of time translations. In the case of CP_2 type extremals, quark momenta for the lifted representations would be light-like.

2. The conservation conditions along the flow lines are very strong, and one can wonder if they might provide a new formulation of the preferred extremal property. It is quite possible that the conditions apply only to a sub-algebra. Quantum classical correspondence (QCC) suggests Cartan algebra for which the quantum charges can have well-defined eigen values simultaneously. In accordance with QCC, the choice of the quantization axes would affect the space-time surfaces considered and could be interpreted as a higher level quantum measurement.

3. Projected/lifted representation provides a new insight also to the Equivalence Principle (EP) stating that gravitational and inertial masses are identical. At the level of scattering amplitudes involving isometry charges defined at the level of H, the isometries affect the entire space-time surface, and one could see EP as an almost trivial statement. QCC however forces us to consider EP more seriously.

   I have proposed that QCC could be seen as the identification of the eigenvalues of Cartan algebra isometry charges for quantum states with the classical charges associated with the preferred extremals. EP would follow from QCC: gravitational charges would correspond to the representation of the flows defined by isometries as their projections/lifts to X^4 whereas inertial charges would correspond to the representation at the level of H with isometries affecting the entire space-time surfaces.

4. The lifted/projected/gravitational representation of isometries, which seems possible in 4-D situation, is analogous to braid group representation making sense only in 2-D situation. Indeed, for the many-sheeted space-time surfaces assignable to $h_{eff} > h_0$, it can happen that rotation by 2π leads to a new space-time sheet and that the $SO(2)$ subgroup of the rotation group associated with the Cartan algebra is lifted to n-fold covering. Same can can happen in the case of color rotations. This leads to a fractionation of quantum numbers usually assigned with quantum group representations suggested to correspond to $h_{eff} > h$ [K12].

   Also for the quantum groups, Cartan algebra plays a special role. In the case of the Poincare group, the 2-D nature of braid group representations would correspond to the selection $M^2 \times SO(2)$ as a Cartan subgroup implying effective 2-dimensionality in the case rotation group. Gravitational representations could therefore correspond to quantum group representations.

5. The gravitational representation provides also a new insight on $M^8 - H$ duality. The source of worries has been whether Uncertainty Principle (UP) is realized if a given 4-surface in M^8 is mapped to a single space-time surface in M^8. It seems that UP can be realized both in terms of inertial and gravitational representations.

   (a) In the case of the "inertial" representation of H-isometries at the level of H, one must regard $X^4 \subset H$ representing images of particle-like 4-surface in M^8 analog of Bohr orbit (holography) and map it to an analog of plane wave define as superposition of its translates and by the total momentum associated with the either boundary of CD associated with the particle. The same applies to the transforms to other Cartan algebra generators.

   In a cognitive representation based on extension of rationals, the shifts for Cartan algebra would be discrete: the values of the plane wave would be roots of unity belonging to the extension and satisfy periodic boundary conditions at the boundary of larger CD.

   Periodic boundary conditions pose rather strong conditions on the time evolution by scaling between two SSFRs. The scaling must respect the boundary conditions. If the momenta assignable to the plane waves of massive particles are conserved and h_{eff} is conserved, the scaling must multiply CD size by integers. The iterations of integer scalings, in particular $n = 2$ scalings (period doubling), are in a preferred position.

(b) If one replaces the inertial representation of isometries with the gravitational representation, the quantum states can be realized at the level of a single space-time surface. One would have two representations: gravitational and inertial -subjective and objective, one might say.

(c) Gravitational representations make also sense for the super-symplectic group acting at the boundary of light-cone as well as for the Kac-Moody type algebra associated with the isometries of H realized the light-like orbits of partonic 2-surfaces.

2.4 Different ways to understand the "complete integrability" of TGD

Complete integrability has profound consequences for the computablity of the theory. One can imagine several ways for how TGD could be a completely integrable theory.

2.4.1 Preferred extremal property

Preferred extremal property requires Bohr orbit property and holography and is an extremely powerful condition.

1. Twistor lift of TGD implies that X^4 in H is simultaneous extremal of volume action and Kähler action. Minimal surface property is counterpart for massless field equations and extremality for Kähler action gives interpretation for massless field as Kähler form as part of induced electromagnetic field.

   The simultaneous preferred extremal property strongly suggests that 2-D complex structure generalizes for 4-D space-time surfaces and so called Hamilton-Jacobi structure [L52] meaning a decomposition of M^4 to orthogonal slicings by string world sheets and orthogonal partonic 2-surfaces would realize this structure.

2. Generalized Beltrami property [L28] implies that 3-D Lorentz force and dissipation for Kähler form vanish. The Kähler form is analogous to the classical Maxwell field. Energy momentum tensor has vanishing divergence, which makes it plausible that QFT limit is analogous to Einstein-Maxwell theory.

   The condition also implies that the Kähler current defines an integrable flow so that there is global coordinate varying along flow lines. This is a natural classical correlate for quantum coherence. Quantum coherence would be always present but broken only by the finite size of the region of the space-time considered.

   Beltrami property plus current conservation implies gradient flow and an interesting question is whether conserved currents define gradient flows: non-trivial space-time topology would allow this at the fundamental level. Beltrami condition is a very natural classical condition in the models of supraphases.

3. The condition that the isometry currents for the Cartan algebra of isometries are proportional to the projections of the corresponding Killing vectors is a strong condition and could also be at least an important aspect of the preferred extremal property.

2.4.2 Supersymplectic symmetry

The third approach is based on the super-symplectic symmetry of WCW. Isometry property would suggest that an infinite number of super-symplectic Noether charges are defined at the boundaries of CD by the action of the theory. They need not be conserved since supersymplectic symmetries cannot be symmetries of the action: if they were, the WCW metric would be trivial.

The gauge conditions for Virasoro algebra and Kac-Moody algebras suggest a generalization. Super-symplectic algebra (SSA) involves only non-negative conformal weights n suggesting extension to a Yangian algebra (this is essential!). Consider the hierarchy of subalgebras SSA_m for which the conformal weights are m-tiples of those of entire algebra. These subalgebras are isomorphic with the entire algebra and form a fractal hierarchy.

Assume that the sub-algebra SSA_m and commutator $[SSA_m, SSA]$ have vanishing classical Noether charges for $m > m_{max}$. These conditions could fix the preferred extremal. One can also assume that the fermionic realizations of these algebras annihilate physical states. The remaining symmetries would be dynamical symmetries.

The generators are Hamiltonians of $\delta M^4_+ \times CP_2$. The symplectic group contains Hamiltonians of the isometries as a normal sub-algebra. Also the Hamiltonians of and one could assume that only the isometry generators correspond to non-trivial classical and quantal Noether charges. Could the actions of SSA and Kac-Moody algebras of isometries be identical if a similar construction applies to Kac-Moody half-algebras associated with the light-like partonic orbits. Super-symplectic symmetry would reduce to a hierarchy of gauge symmetries.

2.4.3 Holography=holomorphy vision

Just like 2-D conformal theories can be seen as a realization of (quantum) criticality, the generalized complex structure for space-time surfaces induced from that for H can be seen as a realization of the quantum criticality of the TGD Universe [L62]. The generalized complex structure of the space-time surface, or Hamilton-Jacobi structure as I call it, combines hypercomplex and complex structures into a 4-D structure [L52]. Hamilton-Jacobi structure involves an integral distribution of the local tangent space-decompositions $M^2(x) \times E^2(x)$ allowing to assign a pair (u, v) of coordinates with light-like coordinate curves to the distribution of $M^2(x)$ and complex coordinates $w, \overline{w}$ to the distribution transversal spaces $E^2(x)$. This structure generalizes to H by introducing complex coordinates (ξ^1, ξ^2) for CP_2.

I have have considered two options for the realization of the holography=holomorphy vision [L62].

1. The earlier guess was that the spacetime surfaces can be identified as roots $(f^1, f^2) = 0$ of two generalized analytic functions $f_1(u, w, \xi^1, \xi^2)$ and $f_2(u, w, \xi^1, \xi^2)$ defined in H and are minimal surfaces apart from possible lower-dimensional singularities at which the minimal surface property and holomorphy fail. Field equations are trivially true by generalized holomorphy and for any general coordinate action constructed in terms of the induced geometry. Singularities have an interpretation as interaction vertices.

   Number theoretical vision strongly suggests that f_i are polynomials P_i of degree m_i with coefficients, which are rationals or belong to an extension E of rationals. If P_i has degree m_i, it has m_i roots as 6-surface multisheeted 6-D surface X^6_i and the intersection $X^6_1 \cap X^6_2$ would give rise to a space-time surface X^4. The lower-dimensional regions of X^4 at which some roots coincide and corresponding space-time sheets meet. There is an analogy with catastrophe theory. These regions define a hierarchy: the larger the number of coindicing roots, the higher the criticality.

   This proposal need not be wrong but it leads to problems with the realization of the number theoretic vision based on the Galois groups for polynomials of single variable with coefficients which are rational or in extension F of rationals and on the identification of ramified primes as p-adic primes central in the applications of TGD and for the adelic physics [?]

2. This raises the question whether the space-time surfaces could be represented as roots of a single polynomial by adding one more complex variable. During writing the idea that space-time surfaces are representable as holomorphic sections of the 6-D twistor space X^6 identified as a holomorphic extremal of 6-D Kähler action in the product $T(M^4) \times T(CP_2)$ of twistor spaces of M^4 and CP_2 [L62].

   The complex coordinate z for the twistor sphere of X^6 would be the additional coordinate and space-time surfaces would corresponds to the roots of the polynomial $P_{u,t}(z)$, where u is the light-like hyper-complex coordinate of M^4 and t is one of the 3 complex coordinates of $M^4 \times CP_2$. This gives three polynomial conditions corresponding to various choices of t. Fermionic lines correspond to the conditions $(P, dP/dz) = (0, 0)$ defining the loci at which two roots of $P = 0$ coincide. This option solves the problems of the first option and reproduces the results of the earlier approach based on a single polynomial.

The fractal hierarchy of symmetry breakings defined by the isomorphic sub-algebras is expected to make sense also for the generalized conformal algebra with non-negative conformal weights. The representations for the algebra of generalized conformal symmetries involve two conformal weights, which are related to hypercomplex and complex structures. This provides a solution to a longstanding problem of p-adic mass calculations, which was that the vacuum state had to possess negative conformal weight [L47].

2.5 Surfaceology, twistors, and TGD

The inspiration coming from the work of Nima Arkani-Hamed and colleagues concerning the twistor Grassmannian approach [B3, B6, B4, B2, B7, B1, B5] provided a strong boost for the development of TGD. I started from the problems of the twistor approach and ended up with a geometrization of the twistor space in terms of sub-manifold geometry with twistor space represented as a 6-surface. Also the twistor space of CP_2 played a key role.

This led to rather dramatic results. Most importantly, the twistor lift of TGD is possible only for H=$M^4 \times CP_2$ since only M^4 and CP_2 allow twistor space with Kähler structure [A2]: TGD is unique. The most recent result [L60] is that one can formulate the twistor-lift in terms of 6-surfaces of H (rather than 6-surfaces in the product of the twistor spaces of M^4 and CP_2). These twistor surfaces represent twistor spaces of M^4 and CP_2 or rather their generalizations, their intersection would define the space-time surface. Therefore one can formulate the twistor lift without the the 12-D product of twistor spaces of M^4 and CP_2.

During last years I have not followed the work of Nima and others since our ways went in very different directions: Nima was ready to give up space-time altogether and I wanted to replace it with 4-surfaces. I was also very worried about giving up space-time since twistor is basically a notion related to a flat 4-D Minkowski space.

However, in Quanta Magazine there there was recently a popular article telling about the recent work of Nima Arkani Hamed and his collaborators (see this). The title of the article was "Physicists Reveal a Quantum Geometry That Exists Outside of Space and Time". The article discusses the notions of amplituhedron and associahedron [L4] which together with the twistor Grassmann approach led to considerable insights about theories with $\mathcal{N} = 4$ supersymmetry. These theories are however rather limited and do not describe physical reality. In the fall of 2022, a Princeton University graduate student named Carolina Figueiredo realized that three types of particles lead to very similar scattering amplitudes. Some kind of universality seems to be involved. This leads to developments which allow to generalize the approach based on $\mathcal{N} = 4$ SUSY.

This approach, called surfaceology, still starts from the QFT picture, which has profound problems. On the other hand, it suggests that the calculational algorithms of QFT lead universally to the same result and are analogous to iteration of a dynamics defined in a theory space leading to the same result irrespective of the theory from which one starts from: this is understandable since the renormalization of coupling constants means motion in theory space.

2.5.1 Surfaceology and TGD

How does the surfaceology relate to TGD?

1. What one wants are the amplitudes, not all possible ways to end up them. The basic obstacle here is the belief in path integral approach. In TGD, general coordinate invariance forces holography forcing to give up path integral as something completely unnecessary.

2. Surfaceology and brings strongly in mind TGD. I have talked for almost 47 years about space-time as surfaces without any attention from colleagues (unless one regards the crackpot label and the loss of all support as such). Now I can congratulate myself: the battle that has lasted 47 years has ended in a victory. TGD is a more or less mature theory.

   It did not take many years to realize that space-times must be 4-surfaces in H=$M^4\times$CP$_2$, which is forced by both the standard model symmetries including Poincare invariance and by the mathematical existence of the theory. Point-like particles are replaced with 3-surfaces or rather the 4-D analogs of their Bohr orbits which are almost deterministic. These 4-surfaces contain 3-D light-like partonic orbits containing fermion lines. Space-time surfaces can in turn be seen as analogs of Feynman graphs with lines thickened to orbits of particles as 3-surfaces as analogs of Bohr orbits.

3. In holography=holomorphy vision space-time surfaces are minimal surfaces realized as roots of function pairs (f_1, f_2) of 4 generalized complex coordinates of H (the hypercomplex coordinate has light-like coordinate curves) [L60]. The roots of f_1 and f_2 are 6-D surfaces analogous to twistor spaces of M^4 and CP_2 and their intersection gives the space-time surface. The condition $f_2 = 0$ defines a map between the twistor spheres of M^4 and CP_2 and identifies the twistor spheres of M^4 and CP_2 [L7]. f_2 defines a slowly varying background whereas f_1 determines the fast dynamics. Outside the 3-D light-like partonic orbits appearing as singularities and carrying fermionic lines, these surfaces are extremals of any general coordinate invariant action constructible in terms of the induced geometry. In accordance with quantum criticality, the dynamics is therefore universal.

   Holography=holomorphy [L66, L67] vision generalizes ordinary holomorphy, which is the prerequisite of twistorialization. Now light-like 4-D momenta are replaced with 8-momenta which means that the generalized twistorialization applies also to particles massive in 4-D sense.

This strongly resembles what the popular article talks about surfaceology: the lines of Feynman diagrams are thickened to surfaces and lines are drawn to the surfaces which are however not space-time surfaces. Also Nima Arkani-Hamed admits that it would be important to have the notion of space-time.

The TGD view is crystallized in Geometric Langlands correspondence [L60] is realized naturally in TGD and implying correspondence between geometric and number theoretic views of TGD.

1. Space-time surfaces form an algebra decomposing to number fields so that one can multiply, divide, sum and subtract them. By holography= holomorphy vision, space-time surfaces are holomorphic minimal surfaces with singularities to which the holographic data defining scattering amplitudes can be assigned.

2. What is marvellous is that the minimal surfaces emerge irrespective of the classical action as long as it is general coordinate invariant and constructed in terms of induced geometry: action makes itself visible only at the partonic orbits and vacuum functional. This corresponds to the mysterious looking finding of Figueiredo.

   There is however a unique action and it corresponds to Kähler action for 6-D generalization of twistor space as surface in the product of twistor spaces of M^4 and CP_2. These twistor spaces of M^4 and CP_2 must allow Kähler structure and this is only possible for them. TGD is completely unique. Also number theoretic vision as dual of geometric vision implies uniqueness. A further source of uniqueness is that non-trivial fermionic scattering amplitudes exist only for 4-D space-time surfaces and 8-D embedding space.

3. Scattering amplitudes reduce at fermionic level to n-point functions of free field theory expressible using fermionic propagators for free leptonic and quark-like spinor fields in H with arguments restricted to the discrete set of self-intersections of the space-time surfaces and in more general case to intersections of several space-time surfaces. This works only for 4-D space-time surfaces and 8-dimensional H. Also pair creation is possible and is made possible by the existence of exotic smooth structures [L67, L69], which are ordinary smooth structures with defects identifiable as the intersection points. Therefore there is a direct correspondence with 4-D homology and intersection form. One can say that TGD in its recent form provides an exact construction recipe for the scattering amplitudes.

4. There is no special need to construct scattering amplitudes in terms of twistors as proposed in [L38, L39] although this is possible since the classical realization of twistorialization is enough and only fermions with spin 1/2 and isospin 1/2 are present as fundamental particles. Since all particles are bound states of fundamental fermions propagating along fermion lines associated with the partonic orbits, all amplitudes involve only propagators for free fermions of H. The analog of twistor diagrams correspond to diagrams, whose vertices correspond to the intersections and self-intersections for space-time surfaces.

2.5.2 Could quantum field theories be universal

The findings of Nima Arkani Hamed and his collaborators, in particular Carolina Figueiredo, suggest a universality for the scattering amplitudes predicted quantum field theories. Is it possible to understand this universality mathematically and what could its physical meaning be?

The background for these considerations comes from TGD, where holography = holomorphy principle and $M^8 - H$ duality relating geometric and number theoretic visions fixing the theory to a high degree.

enumerate

Space-time surfaces are holomorphic surfaces in $H = M^4 \times CP_2$ and therefore minimal surfaces satisfying nonlinear analogs of massless field equations and representing generalizations of light-like geodesics. Therefore generalized conformal invariance seems to be central and also the Hamilton-Jacobi structures [L52] realizing this conformal invariance in M^4 in terms of a pair formed by complex and hypercomplex coordinate, which has light-like coordinate curves.

Quantum criticality means that minima as attractors and maxima as repulsors are replaced with saddle points having both stable and unstable directions. A particle at a saddle point tends to fall in unstable directions and end up to a second saddle point, which is attractive with respect to the degrees of freedom considered.

Zero energy ontology (ZEO) predicts that the arrow of time is changed in "big" state function reductions (BSFRs). BSFRs make it possible to stay near the saddle point. This is proposed to be a key element of homeostasis. Particles can end up to a second saddle point by this kind of quantum transition.

Quantum criticality has conformal invariance as a correlate. This implies long range correlations and vanishing of dimensional parameters for degrees of freedom considered. This is the case in QFTs, which describe massless fields.

Could one think that the S-matrix of a massless QFT actually serves as a model for transition between two quantum critical states located near saddle points in future and past infinity? The particle states at these temporal infinities would correspond to incoming and outgoing states and the S-matrix would be indeed non-trivial. Note that masslessness means that mass squared as the analog of harmonic oscillator coupling vanishes so that one has quantum criticality.

What can one say of the massless theories as models for the quantum transitions between two quantum critical states?

1. Are these theories free theories in the sense that both dimensional and dimensionless coupling parameters associated with the critical degrees of freedom vanish at quantum criticality. If the TGD inspired proposal is correct, it might be possible to have a non-trivial and universal S-matrix connecting two saddle points even if the theories are free.

2. A weaker condition would be that dimensionless coupling parameters approach fixed points at quantum criticality. This option looks more realistic but can it be realized in the QFT framework?

QFTs can be solved by an iteration of type $DX_{n+1} = f(X_n)$ and it is interesting to see what this allows to say about these two options.

1. In the classical gauge theory situation, X_{n+1} would correspond to an $n+1$:th iterate for a massless boson or spinor field whereas D would correspond to the free d'Alembertian for bosons and free Dirac operator for fermions. $f(X_n)$ would define the source term. For bosons it would be proportional to a fermionic or bosonic gauge current multiplied by coupling constant. For a spinor field it would correspond to the coupling of the spinor field to gauge potential or scalar field multiplied by a dimensional coupling constant.

2. Convergence requires that $f(X_n)$ approaches zero. This is not possible if the coupling parameters remain nonvanishing or the currents become non-vanishing in physical states. This could occur for gauge currents and gauge boson couplings of fermions in low enough resolution and would correspond to confinement.

3. In the quantum situation, bosonic and fermionic fields are operators. Radiative corrections bring in local divergences and their elimination leads to renormalization theory. Each step in the iteration requires the renormalization of the coupling parameters and this also requires empirical input. $f(X_n)$ approaches zero if the renormalized coupling parameters approach zero. This could be interpreted in terms of the length scale dependence of the coupling parameters.

4. Many things could go wrong in the iteration. Already, the iteration of polynomials of a complex variable need not converge to a fixed point but can approach a limit cycle and even chaos. In more general situations, the system can approach a strange attractor. In the case of QFT, the situation is much more complex and this kind of catastrophe could take place. One might hope that the renormalization of coupling parameters and possible approach to zero could save the situation.

It is interesting to compare the situation to TGD? First some general observations are in order.

1. Coupling constants are absorbed in the definition of induced gauge potentials and there is no sense in decomposing the classical field equations to free and interaction terms. At the QFT limit the situation of course changes.

2. There are no primary boson fields since bosons are identified as bound states of fermions and antifermions and fermion fields are induced from the free second quantized spinor fields of H to the space-time surfaces. Therefore the iterative procedure is not needed in TGD.

3. CP_2 size defines the only dimensional parameter and has geometric meaning unlike the dimensional couplings of QFTs and string tension of superstring models. Planck length scale and various p-adic length scales would be proportional to CP_2 size. These parameters can be made dimensionless using CP_2 size as a geometric length unit.

The counterpart of the coupling constant evolution emerges at the QFT limit of TGD.

1. Coupling constant evolution is determined by number theory and is discrete. Different fixed points as quantum critical points correspond to extensions of rationals and p-adic length scales associated with ramified primes in the approximation when polynomials with coefficients in an extension of rationals determine space-time surfaces as their roots.

2. The values of the dimensionless coupling parameters appearing in the action determining geometrically the space-time surface (Kähler coupling strength and cosmological constant) are fixed by the conditions that the exponential of the action, which depends n coupling parameters, equals to its number theoretic counterparts determined by number theoretic considerations alone as a product of discriminants associated with the partonic 2-surfaces [L60, L65]. These couplings determine the other gauge couplings since all induced gauge fields are expressible in terms of H coordinates and their gradients.

3. Any general coordinate invariant action constructible in terms of the induced geometry satisfies the general holomorphic ansats giving minimal surfaces as solutions. The form of the classical action can affect the partonic surfaces only via boundary conditions, which in turn affects the values of the discriminants. Could the partonic 2-surfaces adapt in such a way that the discriminant does not depend on the form of the classical action? The modified Dirac action containing couplings to the induced gauge potentials and metric would determine the fermioni scattering amplitudes.

4. In TGD the induction of metric, spinor connection and second quantized spinor fields of H solves the problems of QFT approach due to the condition that coupling parameters should approach zero at the limit of an infinite number of iterations. Minimal surfaces geometrizes gauge dynamics. Space-time surfaces satisfying holography = holomorphy condition correspond to quantum critical situations and the iteration leading from one critical point to another one is replaced with quantum transition.

3 Physics as number theory

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L5, L6],rationals and their algebraic extensions, and classical number fields [K16].

3.1 p-Adic physics and its problems

The motivation for p-adicization came from p-adic mass calculations [K8, K3].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass [K8, K3, L47].

2. The Boltzmann weights are replaced with their p-adic counterparts existing p-adically if e is replaced by p: $exp(-H/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$, where L_0 is Virasoro generator with an integer valued spectrum (apart from a possible additive constant) of conformal weights necessary for the existence of the exponential. Conformal invariance guarantees integer spectrum. Partition function is given by p^{L_0/T_p}. p-Adic valued mass squared value is mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$.

3. p-Adic length scale $L_p \propto \sqrt{p}$ follows from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical Mersenne length scale. Also Gaussian Mersenne primes $M_{G,n} = (1+i)^n - 1$ seem to be realized. Nuclear p-adic length scale corresponds to $n = 113$. The number theoretical miracle is that there are as many as 4 biologically important length scales in the biologically important range 10 nm,2.5 μm corresponding to $n = 151, 157, 163, 167$.

4. p-Adic physics [K10] is interpreted as a correlate for cognition. One motivation comes from the observation that piecewise constant functions depending on a finite number of the pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination. The number theoretic interpretation of p-adic pimes is as ramified primes associated with a polynomial defining an algebraic extension. The ramified primes can be different although the extension can be the same [L56, L47].

Unlike the Higgs mechanism, p-adic thermodynamics as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra. [K8] provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry which guarantees the existence of p-adic Boltzmann weights. Holography= holomorphy vision generalizes complex structure from 2 dimensions to 4 dimensions and therefore also the p-adic thermodynamics and could allow to solve its problems.

One cannot however exclude the possibility that the TGD counterpart of the Higgs mechanism might have some meaning.

1. From the beginning it was clear that the CP_2 part of the trace of the second fundamental form defining the CP_2 part of generalized acceleration behaves group theoretically like the Higgs field. It holography=holomorphy vision it vanishes everywhere except at the lower-dimensional singularities, where the minimal surface property and the generalized holomorphy fail [L69]. These singularities are identifiable as vertices. Maybe Higgs expectation located to vertices could provide an alternative description of massivation.

2. In the original description based on the 2-D conformal invariance, the massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-conformal representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Holography=holomorphy vision makes it possible to overcome the problem due to the fact that in the earlier version based on 2-D conformal invariance, the vacuum conformal weight had to be negative meaning tachyonicity.

Ordinary superconformal algebras in the TGD framework half-algebra are half-algebras having only non-negative conformal weights. Kac-Moody type algebras assignable to the partonic orbits have two conformal weights associated with the radial light-like coordinate and to the partonic 2-surfaces. Also the super-symplectic algebra [L12, L57] has two conformal weights assignable to the complex coordinate of the sphere S^2 and to the light-like radial coordinate of light-cone boundary.

3. In the new description based on 4-D superconformal invariance, there are two conformal weights h_1 *resp.* h_2 corresponding to hypercomplex *resp.* complex degrees of freedom and having non-negative *resp.* non-positive values so that tachyonic states are possible as ground states.

   The following assumptions look physically attractive.

   (a) Physical states have non-negative conformal weights h determining the value of mass squared as $h = h_1 + h_2$. For massless ground states, the conformal weights sum up to zero. p-Adic thermodynamics applies only to the conformal weight h_1 having positive values such that $h \geq 0$ is true. $h_2 \leq 0$ would correspond to the tachyonic contribution to h.

   (b) Thermodynamics applies only to h_1 assignable assignable to 2-D conformal sub-algebra and the second conformal algebra assignable to the hypercomplex part of the algebra would determine only the vacuum conformal weight h_2. For the massless ground states one would have $h_1 + h_2 = 0$ This conforms with the gauge theoretic idea that longitudinal degrees of freedom are not dynamical.

What does the *thermodynamic* description mean?

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and approximately describes the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors.

2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment. The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state is massless.

3. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

What does one mean with the environment?

1. Could the effective environment for fermions consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons)? If the wormhole contact contributes to the mass squared at the H level, tachyonicity could be interpreted in terms of the Euclidean signature of the induced metric for a wormhole contact. One can however argue that the Euclidean wormhole contact cannot contribute to the mass squared.

2. A natural proposal is that only the fermionic entanglement contributes. But can one speak of entanglement between the fermionic degrees of freedom of the partonic 2-surface and those inside the wormhole contact? This does not seem to be possible if one assigns fermion lines only with the partonic orbits.

3. Could the quantum entanglement be between fermionic degrees of freedom only? It is natural to assume that the modes of the Dirac operator of H define the ground states for superconformal representations. The mass squared spectrum for the ordinary Dirac equation in H is non-negative since it is massless in the 8-D sense: no tachyons.

If the fermionic mass spectrum is all that one has, the basic question is how can one assign tachyonic conformal weights to single fermion states.

1. In [L33] I have considered a modification of the Dirac equation in H by introducing the coupling to the *cd* ($CD = cd \times CP_2$) Kähler form identifiable in terms of self-dual $U(1)$ gauge field with electric and magnetic parts which have the same same strength. This would make tachyonic states possible as modes of the right-handed neutrino.

   If $J(cd)$ is present, the masses of the left-handed mode and corresponding right-handed mode differ by the $S = J^{kl}(cd)\Sigma_{kl}$, whose eigenvalues define the vacuum conformal weight $\pm h_{vac}$. If S is non-vanishing for the right-handed mode, the number of right-handed modes with tachyonic mass squared would be the number of CP_2 modes with mass squared smaller than h_{vac}. Covariantly constant neutrino would certainly define this kind of state.

   Note that this interpretation requires that J and S give an additional contribution to the M^4 mass squared besides the CP_2 contribution.

2. A possible interpretation is that the Dirac equation in CD involves M^4 Kähler form and the corresponding mass squared values correspond to the gravitational mass whereas the Dirac equation in H corresponds to the inertial mass. The two mass spectra would be identical for the physical states with non-tachyonic masses. The modes of the Dirac equation in CD would not be plane waves but to unitary representations of a subgroup of $SO(1,3)$ consistent with the symmetries of $J(cd)$ at the second half-cone of cd.

What objections can one invent against this idea?

1. Poincare invariance and Lorentz invariance are lost at the level of H if M^4 Kähler form characterizes the entire $M^4 \subset H$. One can argue that it is enough to have Poincare symmetry only in the moduli space of CDs [L55].

   The only physically convinging option seems to be that the Hamilton-Jacobi structure [L52] is a dynamically generated effective structure of cd defining also an analog of symplectic structure. By holography=holomorphy hypothesis space-time surface X^4 would Hamilton-Jacobi structure, which would effectively induce the same structure to *cd* in the regions where the M^4 projection of X^4 is 4-D.

2. The Hamilton-Jacobi structure in this dynamical sense need not be unique but analogous to the ordinary complex structure [L52]. *cd* would be still flat but could have an infinite variety of effective H-J structures manifesting physically via the choice of the generalized complex coordinates of M^4 for the holomorphic solution ansatz. The Hamilton-Jacobi structure would be consistent with the symmetries of the light-cone boundary so that it would define a slicing of the light-cone boundary by 2-spheres and radial light-like geodesics.

Accepting the 4-D superconformal symmetry, the most plausible option is the following.

1. The hierarchy of the symmetry breakings of generalized conformal invariance to a subalgebra for which the conformal weights are above some minimal value, would allow p-adic thermodynamics in the sub-algebra which does not annihilate the physical states. The complement of this algebra and its commutator with the entire algebra would annihilate the physical states. This would make p-adic thermodynamics possible in the finite-dimensional sub-algebra.

2. Thermal mass squared corresponds to a total mass squared for the fermions. Only the fermionic degrees of freedom at the wormhole throat would interact with the external world defined by the Minkowskian space-time sheets. The Euclidean region would not interact with the external world and its contribution to the conformal weight would vanish. The presence of two conformal weights with opposite signs would be essential and make possible the effective tachyonicity.

The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural. This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K11] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and M^8-H duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L38, L39].

3.2 Adelic physics

Adelic physics fuses real and various p-adic physics to a single structure [L6, L5].

1. One can combine real numbers and p-adic number fields to what is essentially like a Cartesian product: number fields would be like pages of a book intersecting along rationals acting as the back of the book.

   Recently a slightly different view of how to fuse various p-adic physics to an analog of adeles has emerged [L58]. One can glue two p-adic number fields together along p-adic numbers, which have expansions in terms of integers having both primes as factors. Excluding the expansions which are not in powers of prime, one obtains a structure looking like a Cartesian product of subsets of p-adic number fields, which contain only expansions in powers of the p-adic prime in question.

   The nice feature of this variant is that the transitions changing the value of the p-adic prime of the p-adic space-time surface might become possible. They would be due the presence of the regions in which the expansion of p-adic numbers defining the coordinates are with respect to an integer having both primes as factors. A phase transition changing the p-adic prime could start from a seed at which the pinary expansion has this property which then grows and transforms so that the new p-adic primes becomes dominant.

   In p-adic mass calculations [L47] p-adic primes assumed to characterize elementary particles, in particular their mass scales. The p-adic prime would correspond to a ramified prime associated with a polynomial characterizing the particle as partonic 2-surface. These transitions might be relevant for the p-adic description of a transition changing the p-adic prime of the particle. The phase transition would be restricted to 2-D singularities of the 3-D light-like partonic orbit associated with the particles and affect the polynomial characterizing the partonic 2-surface and therefore also the spectrum of corresponding ramified primes. A quantum tunnelling between polynomials with different spectrum of ramified primes would be in question and is allowed by the holography=holomorphy vision.

2. Each extension of rational induces extensions of p-adic number fields and extension of the basic adele. Points in the extension of rationals are now common to the pages. The infinite hierarchy of adeles defined by the extensions forms an infinite library, one might say.

3. This leads to an evolutionary hierarchy (see **Fig. 9**). The order n of the Galois group as a dimension of extension of rationals is identified as a measure of complexity and of evolutionary level, "IQ". Evolutionary hierarchy is predicted.

4. Also a hierarchy of effective Planck constants interpreted in terms of phases of ordinary matter is predicted. X^4 decomposes to n fundamental regions related by Galois symmetry. Action is n times the action for the fundamental region. Planck constant h is effectively replaced with $h_{eff}=nh_0$, where $h_<h0$ is the minimal value of h_{eff}. Quantum coherence scales are typically proportional to h_{eff}. Quantum coherence in arbitrarily long scales is implied. Dark matter at the magnetic body of the system would serve as controller of ordinary matter in the TGD inspired quantum biology [L70].

   There are reasons to as whether h/h_0 could be the ratio R^2/L_p^2 for CP_2 length scale R deduced from p-adic mass calculations and Planck length L_P [L34]. The CP_2 radius R could actually correspond to L_P and the value of R deduced from the p-adic mass calculations would correspond to a dark CP_2 radius $\sqrt{h/h_0}l_P$.

Also the notions of gravitational Planck constant [K1] [L43, L41], proposed first by Nottale [E1], and electric Planck constant [L49] emerge in the TGD framework. Gravitational (electric) Planck constant would characterize pairs of two masses (charges) and whereas ordinary Planck constant is usually regarded as a universal constant.

3.3 Adelic physics and quantum measurement theory

Adelic physics [L6] forces us to reconsider the notion of entanglement and what happens in state function reductions (SFRs). Let us leave the question whether the SFR can correspond to SSFR or BSFR or both open for a moment.

1. The natural assumption is that entanglement is a number-theoretically universal concept and therefore makes sense in both real and various p-adic senses. This is guaranteed if the entanglement coefficients are in an extension E of rationals associated with the polynomial Q defining the space-time surface in M^8 and having rational coefficients.

   In the general case, the diagonalized density matrix ρ produced in a state function reduction (SFR) has eigenvalues in an extension E_1 of E. E_1 is defined by the characteristic polynomial P of ρ.

2. Is the selection of one of the eigenstates in SFR possible if E_1 is non-trivial? If not, then one would have a number-theoretic entanglement protection.

3. On the other hand, if the SFR can occur, does it require a phase transition replacing E with its extension by E_1 required by the diagonalization?

Let us consider the option in which E is replaced by an extension coding for the measured entanglement matrix so that something also happens to the space-time surface.

1. Suppose that the observer and measured system correspond to 4-surfaces defined by the polynomials O and S somehow composed to define the composite system and reflecting the asymmetric relationship between O and S. The simplest option is $Q = O \circ S$ but one can also consider as representations of the measurement action deformations of the polynomial $O \times P$ making it irreducible. Composition conforms with the properties of tensor product since the dimension of extension of rationals for the composite is a product of dimensions for factors.

2. The loss of correlations would suggest that a classical correlate for the outcome is a union of uncorrelated surfaces defined by O and S or equivalently by the reducible polynomial defined by the $O \times S$ [L30]. Information would be lost and the dimension for the resulting extension is the sum of dimensions for the composites. O however gains information and quantum classical correspondence (QCC) suggests that the polynomial O is replaced with a new one to realize this.

3. QCC suggests the replacement of the polynomial O the polynomial $P \circ O$, where P is the characteristic polynomial associated with the diagonalization of the density matrix ρ. The final state would be a union of surfaces represented by $P \circ O$ and S: the information about the measured observable would correspond to the increase of complexity of the space-time surface associated with the observer. Information would be transferred from entangled Galois degrees of freedom including also fermionic ones to the geometric degrees of freedom $P \circ O$. The information about the outcome of the measurement would in turn be coded by the Galois groups and fermionic state.

4. This would give a direct quantum classical correspondence between entanglement matrices and polynomials defining space-time surfaces in M^8. The space-time surface of O would store the measurement history as kinds of Akashic records. If the density matrix corresponds to a polynomial P which is a composite of polynomials, the measurement can add several new layers to the Galois hierarchy and gradually increase its height.

The sequence of SFRs could correspond to a sequence of extensions of extensions of..... This would lead to the space-time analog of chaos as the outcome of iteration if the density matrices associated with entanglement coefficients correspond to a hierarchy of powers P^k [L20, L29].

Does this information transfer take place for both BSFRs and SSFRs? Concerning BSFRs the situation is not quite clear. For SSFRs it would occur naturally and there would be a connection with SSFRs to which I have associated cognitive measurement cascades [?]

1. Consider an extension, which is a sequence of extensions $E_1 \rightarrow ..E_k \rightarrow E_{k+1}.. \rightarrow E_n$ defined by the composite polynomial $P_n \circ .... \circ P_1$. The lowest level corresponds to a simple Galois group having no non-trivial normal subgroups.

2. The state in the group algebra of Galois group $G = G_n$ having G_{n-1} as a normal subgroup can be expressed as an entangled state associated with the factor groups G_n/G_{n-1} and subgroup G_{n-1} and the first cognitive measurement in the cascade would reduce this entanglement. After that the process could but need not to continue down to G_1. Cognitive measurements considerably generalize the usual view about the pair formed by the observer and measured system and it is not clear whether $O - S$ pair can be always represented in this way as assumed above: also small deformations of the polynomial $O \times S$ can be considered.

   These considerations inspire the proposal the space-time surface assigned to the outcome of cognitive measurement G_k, G_{k-1} corresponds to polynomial the $Q_{k,k-1} \circ P_n$, where $Q_{k,k-1}$ is the characteristic polynomial of the entanglement matrix in question.

3.4 The tension between the holography=holomorphy vision and number-theoretic vision

Number theoretical quantum criticality states that rationals and algebraic numbers correspond to islands in the ocean of complex continuum unstable under perturbations selected by quantum criticality which is the basic principle of TGD. This already implies holography=holomorphy principle but does not fix its details completely. p-Adic primes would characterize elementary particles rather than space-time regions. This suggests that the number theoretic quantum criticality is reduced to single fermion level and allows to identify light-like fermion lines at the light-like orbits of partonic 2-surfaces and assign the ramified primes and $h_{eff} = nh_0$ to them.

3.4.1 $(P_1, P_2) = (0, 0)$ option or $P = 0$ option or both?

In the earlier version of $M^8 - H$ duality [L18, L19, L54] a single polynomial P of a single complex variable z with coefficients in the field of rationals (or its extension), continued to a polynomial in a complexification of octonions, defined the holographic data in turn defining the space-time surface.

Although this approach had shortcomings it also had very nice features. The dimension of the algebraic extension determined by the roots of P defined effective Planck constant and the spectrum of ramified primes of P as factors of its discriminant had interpretation as p-adic primes. The applications of TGD rely on these notions.

In the new approach forced by holography=holomorphy vision, there are many tensions to be resolved. One must reconsider both the earlier view of $M^8 - H$ duality and the number theoretical vision. The existing number theoretical vision in turn challenges the detailed realization of the holography=holomorphy vision.

1. For the most obvious guess for the realization of the holography=holomorphy vision, the pair (P_1, P_2) of polynomials replaces single polynomial $P(z)$. Is it possible to reduce the conditions $(P_1, P_2) = (0, 0)$ to a single condition $P(z) = 0$ for some choice of P and z? In the recent case z would correspond to a complex coordinate at the light-like partonic 2-surface as a slice of the partonic orbit and there are very many choices.

2. Putting the lightlike-coordinate u to constant (restriction of fermion line at a light-like partonic orbit) one has polynomials P_i of w, ξ^1, and ξ^2 and one can choose any w, ξ^1, or ξ^2 as dependent variable z. The degree of P_i as a polynomial of z depends on the choice of z. One

can find the common 6-D roots of P_i for each choice and they correspond to the intersection of 4-D surfaces $P_1 = 0$ and $P_2 = 0$. If the argument w, ξ^1, or ξ^2 is an algebraic number, the roots are algebraic numbers. This leaves a lot of freedom and it is very difficult to figure out the general picture!

Therefore it is far from clear how to identify fermionic lines represented as points of X^2 such that they are roots of a polynomial with coefficients in some extension of rationals. However, if can identify a unique extension E of rationals, and a unique polynomial $P(z)$ of a highly unique variable z independent of the variables w, ξ^1, and ξ^2, its ramified primes would determine the spectrum of p-adic length scales and $h_{eff} = nh_0$ would corresponds the degree of its Galois group.

3. This problem does not mean that the $(P_1, P_2) = (0, 0)$ approach is wrong but that it might exists an alternative way to represent the space-time surface so that the basic elements of the number theoretical vision emerge naturally.

How could one solve the problem?

1. The quantum criticality of TGD suggests that there is a catastrophe theoretic hierarchy of criticalities corresponding to the surfaces $P(z) = 0$ giving the space-time surface and as special case partonic 2-surface X^2 as $v = 0$ constant section of the partonic orbit X^3. Criticality corresponds to the coincidence of two roots so that one would have $P(z) = 0$ and $dP(dz) = 0$ at criticality. The roots would give the intersections of the fermionic lines with the partonic 2-surface. The roots of P would define an extension of the coefficient field F of P as an extension of rationals and the ramified primes of P belonging to F.

2. The twistor lift of TGD [L7] [L38, L39] suggests a natural identification of the coordinate z. Twistor lift replaces the space-time surface X^4 with a twistor space X^6 as a S^2 bundle over X^4. X^6 would be determined by the 6-D Kähler action. z could be identified as a complex coordinate of S^2 determined up to holomorphies. Suppose that X^6 is known. A natural identification of X^4 is a section of twistor bundle X^6 can be identifiable as a root of a polynomial $P_{u,t}(z)$, where t can be take to be one of the coordinates (w, ξ^1, ξ^2) (note that $u = constant$ at X^3. The conditions $P = 0$ and $dP/dz = 0$ at fermionic lines would fix the value of z and if t belongs to F, one obtains an algebraic extension of F.

   The fermion line would be identified sufficiently uniquely if the choice P defining the section is sufficiently unique. In fact, different sections could define different physics. The optimistic expectation is that there is a finite number of sections or at least a finite-dimensional moduli space of sections for a given twistor-surface X^6.

There are 3 obvious choices for the coordinate t corresponding the set $\{t_1, t_2, t_3\} \equiv \{w, \xi^1, \xi^2\}$. Can one identify the complex coordinate t uniquely or does one obtain 3 kinds of roots also now and what could this mean?

1. If the partonic 2-surface is regarded as a Riemann surface, the natural local coordinate is t_k, and the polynomials $P_{u,t_k}(z)$ are uniquely determined. The choice of t_k is determined apart from holomorphic bijection and Hamilton-Jacobi structure [L52] dictates the choice of w a high degree and in CP_2 Eguchi-Hanson coordinates, favoured by their group theoretical properties, are natural.

2. $P_{u,t_k}(z)$ is not a linear polynomial, one obtains several roots. Can one accept this or should on require that the section is single valued? Many-valuedness is not in conflict with the geometric vision and physical intuition suggests that one should allow it.

   If all 3 choices of t_k are possible, one obtains 3 kinds of roots. If the roots $z_{k_1,i}$ and $z_{k_2,j}$ coincide, they can correspond to the same point of X^4 but need not do so. For instance, different roots $z_k(w)$ of $P_{u,t_k}(w)$ would correspond to different points of CP_2 since the variables do not appear in P so that multi-sheetedness would reflect the many-valuedness of CP_2 coordinates as a function of w.

   The partonic 2-surface is many-sheeted with respect to both CP_2 and M^4. Different roots $z_k(w)$ of $P_{u,t_k}(w)$ would correspond to the multi-sheetedness of X^6 with respect to CP_2 so that different roots of $z_k(w)$ would correspond to different points of CP_2. Different sheets could be assigned with parallel monopole flux tubes (see **Fig. 15**) going through the w-plane. The physical intuition suggests that, due to the small size of CP_2, the number of roots $z_k(\xi^i)$ of $P_{u,\xi^i}(z)$ in CP_2 direction is small CP_2 for a given M^4 point whereas in the direction of M^4 the number of w-roots can be very large giving rise to a large value of $h_{eff}/h_0 = n$.

3. In the case of the standard twistor bundle over M^4, S^2 represents the directions of light-like geodesics emanating from a point of M^4. The twistor fibers S^2 at different M^4 points have a common point if there is a light-like geodesic connecting them. This is expected to be a reasonable guess also now.

   Could the roots $z_k(w)$ correspond to intersecting twistor spheres for which the points with a different w coordinate are connected by a light-like geodesic of M^4? Since the light-like coordinate u is constant and v is fixed, this is not plausible. This is however the situation at the light-like partonic orbits, where points of the partonic 2-surfaces with different values of u are connected by a light-like geodesic.

4. The same question can be posed in the case of CP_2 for which the light-like geodesics are replaced with geodesics, say the radial geodesics from the origin of Eguchi-Hanson coordinates directed to the homologically non-trivial geodesic sphere at $r = \infty$. This sphere is a concrete representation for the twistor sphere of CP_2 and two such geodesic spheres always intersect since their middle points are connected by a geodesic of CP_2. The points pairs at these geodesics have intersecting twistor spheres of CP_2. The section of X^6 goes through the intersection point of CP_2 geodesic spheres associated with CP_2 points associated with to the points w_1 and w_2 only if the values of a root $z_k(w)$ are identical for w_1 and w_2.

One can argue that there is a problem with number theoretic general coordinate invariance (GCI) since the form of the P can change in a generalized holomorphism of H expected to have no physical effect. Is there a unique choice of coordinates allowing to avoid the problem?

1. For the (P_1, P_2) option, the X^4 is identified as an intersection $X_1^6 \cap X_2^6$ of 6-surfaces X_i^6 as roots of P_i. Could X_i^6 be identified as twistor surfaces as counterparts of the twistor spaces $T(M^4)$ and $T(CP_2)$ with different twistor spheres but the same base space?

   If so,the complex coordinates of the twistor spheres of $T(M^4)$ and $T(CP_2)$ should correspond to the complex coordinates of the twistor sphere of light-like geodesics of the light-cone boundary and of radial geodesics of CP_2 directed from origin to homologically nontrivial sphere of CP_2 "at infinity".

2. The construction of X^6 as an extremal of the 6-D Kähler action for $X^6 \subset T(M^4) \times T(CP_2)$ [L7] identifies the twistor spheres of $T(M^4)$ and $T(CP_2)$. Does this mean X_i^6 as twistor bundles are related by the mapping of the space of light-like geodesics of light-cone boundary and/or light-like partonit orbit to the space of radial geodesics of CP_2?

3. This would make the situation highly unique. Holomorphies for a given choice of w *resp.* ξ^1 or ξ^2 would correspond to $SO(3)$ and $U(2)$ acting linearly on the complex coordinate. These groups reduce to $SO(2)$ and $U(1) \times U(1)$ by the choice of the quantization axes. The coordinate w would reduce to the complex coordinate of the twistor sphere of $T(M^4)$ at the light-cone boundary (at least). At partonic orbits the complex coordinate ξ of the geodesic sphere of CP_2 and w would be related by a map characterized by a winding number.

4. Could one find realize concretely the analogy with the solution ansatz based on the 6-D Kähler action [L7] characterized by a winding number for the map of CP_2 twistor sphere to M^4 twistor sphere or vice versa determining the value of the cosmological constant.

   Assume that P_i are polynomials with coefficients in an extension F of rationals. Consider a slicing of cd by light-cone boundaries δM^4_+ parallel to the light-like boundary of cs and identify w as the complex coordinate of the M^4 twistor sphere $S^2(M^4)$ associated with δM^4_+ and x^1 as the coordinate of CP_2 twistor sphere $S^2(CP_2)$. Take $P_1 = P_1(u, w, \xi^1, x^2)$. Take $P_2 = \xi^1 - Q(w)$ or $P_2 = w - Q(\xi^1)$, where Q is a polynomial. $P_2 = 0$ maps the $S^2(M^4)$ to

$S^2(CP_2)$ or vice versa with a winding number determined by the degree of P_2. These two options correspond to multi-sheetedness with respect to M^4 or CP_2.

The roots P_2 at algebraic algebraic points of S^2 with respect to E are algebraic and for a generic algebraic point the extension of F is trivial. For rational points the extension is maximal so that these points are of special physical interest. Fermion lines could correspond to this kind of points or, in a complete analogy with the $P = 0$ option, to the points at which $dP_2/dz = 0$ at which two roots as sheets meet (analog with the cusp catastrophe).

Some comments on the physical interpretation are in order.

1. p-Adic primes are rather large, $M_{127} = 2^{127} - 1$ for electrons. I have proposed that one could pose constraints on the size of the polynomial coefficients, say that they are smaller than the degree of the polynomial. In this case it is not clear how to obtain such large ramified primes unless the degree of P is very large. The degree of polynomial increases exponentially in repeated iteration giving rise to an analog for the approach to chaos [L20]. This would increase the dimension of the extension.

2. As found, the polynomial P can be identified as a polynomial of Minkowski-coordinate w, or of CP_2 coordinate ξ^1 or ξ^2. CP_2 is rather small and one expects that in CP_2 directions the number of sheets is rather small so that P as a polynomial of ξ^1 or ξ^2 should have a rather small degree and corresponding h_{eff}/h_0 should be rather small.

   In M^4 there is a lot of room and the degree of P as a polynomial of w can be rather large and therefore also the value of h_{eff}/h_0 for these fermion lines is large and their number can be large. Therefore the corresponding ramified primes and associated p-adic length scales can be rather large in this case. It would seem that the p-adic length scale is naturally assignable to $P_{u,w}(z)$. If p-adic length scale is assignable to $P_{u,\xi^i}(z)$, it should be smaller than CP_2 scale and could correspond to excitations of superconformal and supersymplectic algebras with mass scales which are higher than CP_2 mass scale.

It must be emphasized that $P = 0$ option assumes that X^6 is known and (P_1, P_2) option could fix X^6. $P = 0$ option reduces the pair of polynomials (P_1, P_2) to a single polynomial P, allows to interpret the space-time surface as a section of its twistor space determined by 6-D Kähler action and to identify fermion lines as surfaces $(P = 0, dP/dz = 0)$. This view implies the notions of effective Planck constant and ramified primes, and allows to understand number theoretical evolution in two ways: as evolution of the extensions of rationals F appearing as the coefficient field of P and as the evolution of the polynomial P as the increase of its complexity. Also connections with chaos theory and catastrophe theory emerge and space-time surfaces are analogous to complexifications of the cusp catastrophe.

3.4.2 A detailed comparison of $(P_1, P_2) = (0, 0)$ and $P = 0$ options

It is interesting to compare the $(P_1, P_2) = (0, 0)$ option, which is problematic since 2 polynomials are involved, with the P option. Note again that these options represent different approaches can be consistent as already noticed.

1. The condition $(P_1, P_2) = (0, 0)$ allows to assign h_{eff} and p-adic prime only to fermion lines if there is a simple rule for their identification. Assume a restriction to a light-like partonic orbit so that the situation is effectively 2-D corresponding to a partonic 2-surface X^2. At the algebraic points of X^2, the polynomials P_i have coefficients in F. Their roots are algebraic and in an extension E of F, which depends on the algebraic point.

   The condition that the algebraic roots of P_i coincide means that P_1 and P_2 have a common algebraic root. This condition would select some fermionic lines. It is extremely difficult to say anything general about how the roots are selected, if there are such common roots, and what their number is. This could be seen as the basic difficulty of the $(P_1, P_2) = (0, 0)$ option. The common roots of P_i would define an extension of rationals. The product of the monomials vanishing for the common roots would naturally define the polynomial defining the extension E of F. One could assign to it h_{eff} also the ramified primes.

2. For the $P=0$ option, the fermion lines are selected by the conditions $(P, dP/dz)=(0,0)$ and correspond to coinciding roots z_{u,t_k} of P so that the Galois group is reduced. If the degree of $P(z)$ is n, there are $n(n-1)/2$ co-inciding pairs of coinciding roots. Each root $z_r(t_k)$ corresponds to one particular fermion line. The action of the reduced Galois group Gal/Z_2 permutes these fermion lines since the condition $z_k = z_l$ is invariant under the Galois group.

   The condition $dP/dz = (0,0)$ for a selected root z_r gives a condition for the parameter t_k in $P_k \equiv P_{u,t_k}(z)$. If the degree of dP_k/dz as a polynomial of t_k is m_k, there are m_k roots $t_{k,i}$ as a solution to $dP_k/dz = 0$.

   For instance, for $t_k = w$, the roots w_i correspond to the same value of z for the points of CP_2 so that these points are connected by a geodesic of CP_2 and their CP_2 twistor spheres intersect.

   For $t_k = \xi^1$, the roots ξ^1_i, the M^4 twistor spheres intersect. Could this mean that corresponding points are connected by a light-like geodesic of M^4 or by a CP_2 geodesic. Since w corresponds to inherently Euclidian coordinate, the first option looks impossible to satisfy.

3. In the twistor Grassmannian approach the intersections of the twistor spheres associated with $T(M^4)$ connected by light-like geodesic play a central role in twistor diagrams. In the recent case, the fermion lines at light-like orbits of partonic 2-surface would take their role. Also the roots w_n to the condition $dP_{u,w}(z_r = z_s)/dz = 0$ for a pair z_r, z_s correspond to the intersection point of the CP_2 twistor spheres associated with w_n, are expected to be important.

4. The condition $(P, dP/dz) = (0,0)$ has an interpretation in terms of quantum criticality and there is a connection with the catastrophe theory. At the fermion line the coefficients of P must be assumed to be in F and two roots co-coincide: the criticality condition selects a set of fermion lines with a fixed value of the coordinate t_k and the reduced Galois group permutes them points. These fermion lines correspond to intersections of different twistor spheres as analogs of points of M^4 connected by a light-like geodesic.

   In this case, the value of $h_{eff} = nh_0$ corresponds to the dimension of algebraic extension $F \to E$ assignable to P with coefficients in F characterizing the entire space-time surface. n would characterize the dimension of $F \to E$, which can be also regarded as an extension of rationals. One of the ramified primes p assignable to the fermion line would determine the mass scale of the particle as a p-adic length scale L_p in turn giving a lower bound for the size scale of the $cd \subset M^4$.

The options, which are by no means mutually exclusive, also have common features.

1. In both cases there is a background extension F of rationals associated with a connected space-time surface X^4 or even the entire CD containing X^4. One expects that the hierarchy of space-time surfaces corresponds to a hierarchy of extensions F such that topological condensation to a larger space-time surface means an inclusion of extensions. F would be therefore common to all fermionic lines and partonic orbits associated with the space-time surface. This hierarchy would naturally define an evolutionary hierarchy. Second hierarchy is defined by the extensions of F.

   The Galois group of E would characterize the space-time surface. The Galois group of F would characterize the fermion line since the polynomials P are obtained for each point of the space-time surface using the discretization in which coordinates have values in F using the scale of cd as a unit.

2. For both options, there are three kinds of polynomials involved since the polynomials can be identified as a polynomial of Minkowski-coordinate w, or of CP_2 coordinate ξ^1 or ξ^2. They would correspond to different fermionic lines. Since CP_2 is rather small, one expects that in CP_2 directions the number of sheets is not large so that P as a polynomial of ξ^1 or ξ^2 should have a rather small degree. In M^4 there is a lot of room that the degree as a polynomial of w can be rather large AS also the value of h_{eff}/h_0 characterizing these fermion lines is large.

   What might look like a problem of $(P_1, P_2) = (0,0)$ option is the number theoretic asymmetry between different choices of the dependent argument of P. The dimensions of extensions E

of F would be different for the 3 H coordinates appearing as a dependent coordinate and it is not clear whether one of the coordinates could serve as a preferred coordinate. For $P = 0$ option z would be the complex coordinate of the twistor sphere and the asymmetry is not encountered.

3. At least formally, one could assign also to the background extension F a monic polynomial having as roots the powers of the root generating F and also ramified primes having an interpretation as possible p-adic scales for the space-time surface or a CD containing it. This might give rise to gravitational and electric Planck constants [L41, L49] in macroscopic or even astrophysical p-adic length scales.

4. $cd \subset M^4$ would contain the partonic 2-surface X^3 as an analog of a perceptive field. By $M^8 - H$ duality this cd would correspond to a pair of oppositely directed light-cones in the normal spaces $N(y)$ of the $M^8 - H$ images of the points of X^2 in M^8.

   The size scale defining the mass scale of $cd \subset N(y)$ would correspond to one of the ramified primes p of P. The size scale of $cd \subset N(y)$ would not depend on the dimension $n = h_{eff}/h_0$. It would however characterize the length scale of the $cd \subset M^4$ assignable to the M^4 image X^2.

5. Both options have problems with general coordinate invariance unless the choice of complex coordinates is unique enough.

3.4.3 The description of the twistor lift at the level of M^8

What could be the description of the twistor lift at the level of M^8? IT is good to summarize first the essentials of the H picture.

1. The description of 6-D twistor spaces X_i^6 is as roots of P_1 and P_2 such that their intersection gives the common base space $X^4 \subset X_i^6$ should generalize. In the case of H, the simplest view is that one has the conditions $P_1 = 0$ and $P_2 = \xi - P(w)$ or $P_2 = w - P(\xi)$, where w is the complex coordinate for the light-cone boundary and ξ is the geodesic coordinate of geodesic sphere of CP_2. w corresponds to the twistor sphere assignable to the light-like boundaries forming a slicing of M^4 and ξ corresponds to the twistor sphere of CP_2 of a given CP_2 identifiable as homologically non-trivial geodesic sphere S^2 of CP_2 defined by the radial geodesics directed to the S^2 points.

2. Does this description have a counterpart at the level of M^8? The counterpart of X_i^6 would be Y_i^6 as S_i^2 bundle over Y^4 such that S_i^2 is the counterpart twistor sphere of M^4 *resp.* CP_2.

   The earlier picture based on variational principle simply assumes that Y_i^6 corresponds to the twistor spaces $T(M^4)$ and $T(CP_2)$ and X^6 is surface in $T(M^4) \times T(CP_2)$ with the twistor spheres of $T(M^4)$ and $T(CP_2)$ identified by the analog of the map P_2 characterized by a winding number. This picture corresponds to the above holomorphic map. One has twistor surfaces X^6 with the same base X^4 but different twistor spheres S_i^2 and these are identified by a map defined by the $P_2 = 0$ condition. These identifications give many-sheetedness with respect to M^4 or CP_2.

   The representation of the fibers S_i^2 as concrete twistor spheres of H assignable to points of $M^4 \times CP_2$ gives rise to the 8-D description.

Can one have the analog of this picture at the level of M^8? Can one do the same for the twistor space $T(M^8) = T(M^4) \times T(E^4)$ and also now identify the twistor spheres S_i^2 by $P_2 = 0$ condition.

The possibility to project this description to M^8 in P_1, P_2 approach gives both twistor spaces and the common base space X^4. What could be the counterparts of the twistor spheres S_i^2 of M^4 and CP_2 for M^4 and E^4?

1. For M^4 defining the normal space of Y^4 the counterpart of twistor space and twistor sphere is obvious and the same as in the above situation.

2. What about Y^4. The space of tangential radial geodesics of Y^4 point is 3-sphere S^3: 2-sphere S^2 is required. How to overcome the problem? One should be able to distinguish a preferred direction in the local tangent space E^4 of Y^4 so that S^2 would be the space of radial geodesics in the orthogonal complement E^3.

3. Could octonion structure make this possible? In the quaternionic and Minkowskian normal spaces of Y^4 points, the points with the real quaternion coordinate define a time direction for the rest system. The octonionic imaginary unit J of Y^4 tangent space, needed in M^8-H duality, could define the local distinguished direction. One could assign to each point of Y^4 twistor sphere $S^2(E^3)$.

4. The map between $S^2(M^4)$ and $S^2(E^3)$ would correspond to the identification defined by the condition $P_2=0$. An interesting question is whether the M^8-H duality, generalized so that it applies to the suraces Y_i^6, is consistent with the assumption that also at the level of M^8 one has polynomials, call them Q_1, Q_2. The question boils down to the question whether a polynomial Q_1 could define the inverse image of X_1^6 as its root. It is probably too much to hope that the polynomials at the two sides are identical.

3.5 Do local Galois group and ramified primes make sense as general coordinate invariant notions?

Space-time surface can be regarded as a 4-D root for a pair P_1, P_2 of polynomials. Each gives rise to a 6-D surface proposed to be identifiable as analog of twistor space and their intersection defines space-time surface as a common base of these twistor spaces as S^2. One can also think of the space-time surface X^4 as a base space of a twistor surface X^6 in the product $T(M^4)\times T(CP_2)$ of the twistor spaces of M^4 and H. One can represent X^4 as a section of this twistor space as a root of a single polynomial P. The number roots of a polynomial does not depend on the point chosen. One considers polynomials with rational coefficients but also analytic functions can be considered and general coordinate invariance would suggest that they should be allowed.

Could one generalize the notion of the Galois group so that one could speak of a Galois group acting on 4-surface X^4 permuting its sheets as roots of the polynomial? Could one speak of a local Galois group with local groups $Gal(x)$ assigned with each point x of the space-time surface. Could one have a general coordinate invariant definition for the generalized Galois group, perhaps working even when one considers analytic functions f_1, f_2 instead of polynomials. Also a general coordinate invariant definition of ramified primes would be desirable.

3.5.1 The standard notion of Galois group in TGD framework

Consider first the standard definition of the Galois group as an automorphisms of the extension generated by the roots of P and permuting the roots and leaving the coefficient field of P invariant.

1. For simplicity, restrict the consideration to a section of $X^6\subset T(M^4)\times T(CP_2)$ as a root of a single polynomial P. Almost all points of H and space-time surface X^4 are algebraic for a given choice of the generalized complex coordinates but this property is not general coordinate invariant.

   For a given algebraic point, the coordinate values of the point generate an extension of rationals, call it E. At an algebraic point of X^4, one can find the extension F of this extension generated by the roots of the P. If E is large enough, the polynomial factorizes into a product of monomials in this extension and one has $F=E$ and the Galois group is trivial.

   Clearly, the order of the Galois group decreases as the algebraicity of the point increases and at most algebraic points the Galois group is expected to be trivial. The local Galois group would be trivial only at a discrete set of points with algebraic coordinates. These points could be in a physically preferred role and one could speak of number theoretic criticality.

2. What about the coefficients in transcendental extensions E of rationals as a coefficient field of P? For instance, the extension generated by Neper number e is infinite-dimensional in the real sense but has a finite dimension for p-adic numbers since e^p exists as an ordinary

p-adic number. Transcendental extension can be finitely generated. The number of generators is indeed finite when the coefficients of polynomials and space-time coordinates at a given point generate the extension. Could one define the Galois group in this case as automorphisms of the extension of transcendental extension leaving the transcendental extension invariant?

It seems that the standard notion of Galois group should be generalized so that one can speak of a local Galois group as a general coordinate invariant notion. Could one modify the definition of the Galois group so that one could speak about a local Galois group of, say, TGD variant X^6 of the twistor space permuting the sheets of the space-time surface as a section of this bundle? It would be also highly desirable to have a general coordinate invariant notion of ramified primes identified as p-adic primes.

3.5.2 Could on modify the definition of the Galois group

There is also a second approach concerning the definition of the Galois group (see this).

1. One can return to the roots and start from the discriminant D defined as the product of the root differences. D is a symmetric function of roots and the symmetric group S_n permutes the n roots acting like the standard Galois group. The naive approach, based on the idea that the Galois group permutes the roots of the polynomial, has the nice feature that it generalizes also to the case, when the roots are not algebraic numbers. The permutation group has a subgroup leaving the roots invariant.

   The roots can be regarded as the discrete space $S_n/S_{n-1} = Z_n$ and a given root remains invariant under S_{n-1}. The Galois group defined in the standard way is a subgroup of S_n. If it is maximal, the Galois group is S_n.

2. Galois group in the naive sense just permutes the roots. The action of the standard Galois group is in the extension and leaves the coefficient field invariant. The problem is that there seems to exist no obvious geometric realization for the action of the Galois group acting on the arguments of the polynomial.

3. Could the action of the Galois group as permutations of space-time sheets be represented in the TGD framework geometrically as an isometry or as a discrete generalized conformal transformation of H permuting the space-time sheets as roots of the polynomials P_1 and P_2. Or could these permutations act as gauge transformations acting on the twistor sphere permuting the roots at a given point of X^4 as points of S^2?

   In the case of a real line, one can find a holomorphy of the plane compactified to the Riemann sphere permuting 3 points on the real axis, that is Möbius transformation. In the case of a general polynomial of a single argument this is in general not possible using holomorphy but more general complex transformations can do this.

4. The braid group B_n is a covering group of the permutation group S_n and emerges very naturally in quantum TGD. I have considered the possibility [L45] that the braid counterpart of Galois group acts as a flow, which permutes the space-time sheets (, which could be n flux tubes as space-time sheets with respect to CP_2) and acts as subgroup of the braid group B_n. In this case one would have a connection with quantum groups and the inclusions of hyperfinite factors of type II_1.

3.5.3 Local Galois group for the space-time surface as a section in twistor space X^6

Consider first X^4 as a section of X^6. One should permute the spacetime sheets as roots of P having X^4 coordinates as parameters and the complex coordinate z of the twistor sphere S^2 as behaviour variable defining the space-time sheets as roots for $P = 0$. Could the action of the local Galois group be a discrete local gauge transformation in the S^2 fiber permuting the local roots?

1. Could the permutation group be a discrete *isometry* subgroup of the local $SO(3)$ or $U(2)$ permuting the roots as points of the section representing X^4? Disappointingly, only

octahedron and cube allow permutation groups as discrete subgroups of $SO(3)$ acting as isometries. The alternating groups $A_n = S_n/Z_2$ of degree 4 *resp.* 5 appear as isometries of tetrahedron *resp.* icosahedron and dodecahedron. There are also infinite discrete subgroups realized as subgroups of $SO(3)$.

Note that $U(2)$ gauge transformations, local with respect to the space-time surface, emerge also in the formulation of the M^8-H duality. This $U(2)$ gauge symmetry having interpretation as electroweak gauge symmetry fails in all points except the "active" points carrying a fermion. The remaining genuinely dynamical degree of freedom would be a discrete Galois group permuting the S^2 points as roots.

2. There is no deep reason for the condition that the full permutation group for the sheets is realized as isometries. What if one gives up this condition? Also the condition that a discrete subgroup of S_n is realized as *isometries* implies powerful constraints. The strong variant of this condition would be true for the entire X^4 so that one would have local isometry. The weak condition would be that this is true at discrete points, say at fermion lines and allow to identify them.

   The finite discrete subgroups of $SO(3)$ and $U(2)$ are known and correspond to the famous hierarchy of inclusions of hyperfinite factors of type II_1 and McKay correspondence [A3] [L17, L44, L16]. There are only 4 finite groups with genuinely 3-dimensional action and they correspond to the symmetries of the Platonic solids.

   The remaining groups in the hierarchy act in a plane as groups Z_n or in a pair of parallel planes as Z_n with a vertical reflection added. This would allow to realize only these subgroups of permutation groups as isometries and possibly only at fermion lines. The Galois group in the proposed sense could correspond to a subgroup of the Galois group in the standard sense. An interesting question is what are the polynomials associated with the subgroups of the rotation group. Note that the icosa tetrahedral tessellation of the hyperbolic 3-space is completely exceptional and appears in the TGD based model of the genetic code and would define a universal genetic code [L48].

   The mysterious origin of these groups has been a continual source of inspiration also in the TGD framework. Now they would be here but somehow it feels like a disappointment that a very limited set of Galois groups would be possible in the proposed sense. The representability as roots of a polynomial alone does not have such strong implications.

3. If the local local transformations of S^2 correspond to transformations, which are more general transformations than isometries, most naturally restrictions of conformal transformations, which are not global holomorphies, the generic Galois group in the TGD sense might allow a representation at each point of X^4 as a local Galois group. However, the Galois groups allowing a representation of *discrete* isometries would be in a physically preferred position and could select fermion lines as singularities giving rise to number theoric criticality. This has very powerful implications and might also closely relate to the breaking of generalized conformal invariance. The generalization of holography=holomorphy vision indeed suggests that holomorphies of X^6 respecting the bundle structure act dynamical symmetries.

3.5.4 Local Galois group for the space-time surface as a root for a pair of polynomials

A more general situation would correspond to the roots for a pair P_1, P_2 of polynomials. There are in general $n_1 n_2$ roots as space-time sheets. Can one identify the counterpart of the generalized Galois group also now?

1. One has two 6-surfaces X^6_1 *resp.* X^6_2 satisfying $P_1 = 0$ *resp.* $P_2 = 0$ having S^2_1 *resp.* S^2_2 as fiber and could have an interpretation as analogs of twistor bundles with X^4 as a common base space. One would have 2 analogs of 6-D twistor spaces with X^4 as a base represented as a S^2_i valued section. Also now one might perform discrete gauge transformation to permute the n_1 *resp.* n_2 roots as space-time sheets of S^2. Galois group would be a product of the Galois groups for S^2_1 *resp.* S^2_2 permuting the sheets with respect to CP_2 *resp.* M^4. This is just the original intuitive picture [K12].

2. The definition of the Galois group might also generalize to the roots of analytic functions f_1, f_2. It would be general coordinate invariant in the restricted sense that general coordinate transformations in the twistor sphere S^2 be restricted to holomorphies. What restrictions one must pose to the allowed holomorphies depends on what wants.

3.5.5 About the generalization of the holography=holomorphy ansatz to general analytic functions

The general ansatz works also for analytic functions with poles since $(f_1 = 0, f_2 = 0)$ implies that the poles do not belong to the space-time surface. What is required is that the roots are not essential singularities. For rational functions $R_i = P_i/Q_i$ the vanishing conditions reduce to those for the polynomials P_i.

The generalization Rieman zeta to polyzeta $S_n(s_1, ..., s_n)$ is s function of n complex variables [L46] and satisfies identities analogous to those satisfied by Riemann zeta. This generalization is extremely interesting from the point of view of physics of chaotic and quantum critical systems. Polyzeta S_4 with four complex arguments would define as its roots a 6-D analog of the twistor space of the space-time surface expected to have an infinite number of 6-D roots having interpretation as a generalization of zeros of Riemann zeta.

One could have $f_1 = S_4$ so that its roots would correspond to 6-D zeros of polyzeta $S_4(s_1, ..., s_4)$ defining the counterparts of twistor surfaces! $f_2 = 0$ could define a map from the M^4 twistor sphere S_1^2 to CP_2 twistor sphere S_2^2 characterized by a winding number or vice versa.

A further extremely nice feature is that the space-time surfaces form a number field in the sense that one can sum, multiply and divide the members of f_i and g_i of (f_1, f_2) and (g_1, g_2) elementwise. Also functional composition is possible. One could say that the space-time surface is a number. One can also consider polynomials and polynomials with prime order behave like multiplicative primes. It is also possible to identify prime polynomials with respect to functional composition [L42].

3.5.6 Can one identify ramified primes in a general coordinate invariant way?

Ramified primes seem to be something physical and should be a general coordinate invariant notion.

Can one identify ramified primes or their generalization to algebraic primes in a general coordinate invariant way? They correspond to the prime factors of the discriminant defined by the root differences with roots identified as points of S^2 and make sense only if the roots of $f_1 = 0$ and $f_2 = 0$ (or of $P = 0$ or even $f = 0$ for the section of X^6) are algebraic numbers. Also algebraic primes can be considered. Ramified primes would be associated with the points at which the fermion lines intersect X^2. For suitable coordinates of X^2 they could be associated with the algebraic points for the partonic 2-surface X^2 but there is no need to specify the coordinates of X^2: the situation is general coordinate invariant with respect to X^4 if the fermion lines can be identified in a general coordinate invariant way. Note that general coordinate invariant definition of the notion of polynomial would be as function as a polynomial-like which can be expressed as a polynomial in suitable coordinates. If the roots of the function are algebraic numbers, this is the case. In the recent case the restriction of coordinates to be generalized complex coordinates is physically appropriate. A possible interpretation is in terms of number theoretical criticality meaning that fermion lines for which the polynomial P defining the section has rational or algebraic coefficients. This is the case if the points of H have coordinates in an extension E of rationals. A different choice of generalized complex coordinates of X^4 can transform the polynomial to a more general analytic function. This choice does not however affect the roots so that a general coordinate invariance is achieved at the level of X^4. What is remarkable that due to hypercomplex analyticity only the S^2 coordinate associated with either the light-cone sphere or CP_2 geodesic sphere appears in the polynomial P_i, when one restricts the consideration to the partonic 2-surface X^2. The possible loss of the general coordinate invariance relates to the choice of the complex coordinate of the twistor sphere S^2 or its representative in H as a sphere associated with the light-cone boundary or as a homologically nontrivial geodesic sphere of CP_2. Which could be the preferred complex coordinate of the twistor sphere S^2?

Linear Möbius transformations $z \rightarrow az + b$ correspond to holomorphies are a good candidate in this respect. The root differences are scaled since the root differences are scaled by a. These transformations could be also restricted so that they would map the real axis to itself but this is not necessary. The restriction of a and b rational is also natural. Ramified primes remain invariant in the scaling of the discriminant. This preferred coordinate z would exist for its representative in H for $f_1, f_2 = 0$ option. The complex coordinate defining the roots for f_1 *resp.* f_2 would correspond to the light-cone sphere *resp.* CP_2 geodesic sphere.

3.6 How do the hierarchies of effective Planck constants and p-adic mass- and energy scales emerge?

In the following subsections the phenomenological aspects of the number theoretic vision will be considered.

3.6.1 A phenomenological view about p-adic length scales

Consider now the detailed definition of the p-adic mass and energy scales.

1. For massive particles p-adic length scales L_p are given in the original p-adic mass calculations given by $M^2 = \hbar/L_p^2$, where L_p is the p-adic length scale $L_p = k\sqrt{p}R$, where R corresponds to the CP_2 length scale and k is some numerical constant. For massless particles this formula does not make sense. Instead of mass scale one has energy scale $E = k\hbar/L_p$.

2. In the original calculations, the value of the CP_2 scale turned out to be roughly $10^4 l_P$, where l_P is Planck length, from the condition that the mass of electron assumed to correspond to Mersenne prime M_{127} (the largest Mersenne prime which does not correspond to super-astronomical p-adic length scale).

3. If one assumes that CP_2 scale corresponds to Planck length l_p, one must replace h with h_0, which in the number theoretic vision would be the minimal value of effective Planck constant. One would have $h \simeq 10^4 h_0$. The hierarchy of Planck lengths would be given as $h_{eff} = nh_0$, where n is the dimension of the extension of rationals associated with the particle. Also the values $h_{eff} < h$ a are possible and there is some evidence for them [L2].

4. The minimal value of h_{eff}/h_0 is 2 and corresponds to 2-D irreducible algebraic extensions assignable to polynomials of degree 2. In the simplest situation there is only a single ramified prime determining the p-adic mass scale $M(p) \propto 1/\sqrt{p}$. The condition that the polynomial $P(x) = ax^2 + bx + c$ with integer coefficients as only single ramified prime boils is that the discriminant defined as the square for the product of non-vanishing root differences is prime. This gives the condition $ac = (b^2 - p)/4$ and the condition $b^2 - p \ mod \ 4 = 0$. For odd b one has $b^2 \ mod \ 4 = 1$ so that one must have $p \ mod \ 4 = 3$ in order that the solution exists. For instance, Mersenne primes satisfy the condition.

Number-theoretical vision suggests that coupling constant evolution reduces to a discrete p-adic mass scale evolution in terms of the p-adic prime identifiable as a ramified prime of the polynomial defining the extension of rationals. There can also be a dependence of h_{eff} visible at the level of H. The p-adic mass scale would characterize the size of cd in the normal space of $y \in Y^4$.

1. The point $y \in Y^4$ is determined by the CP_2 projection of X^4 defining the normal space $N(y)$ and assigning to the point y a point of $N(y)$ as an image of M^4 point. The scaling of $M^4 \subset H$ induces opposite scaling for the points of the normal spaces but leaves the integrable distribution of normal spaces parametrized by the points of X^4 invariant. Does this mean that Y^4 is invariant under M^4 scalings of X^4?

2. The properties of Y^4 should determine the coupling constant evolution as a function of the p-adic length scale. The logarithmic coupling constant evolution could reflect the logarithmic dependence of the size scale of Y^4 on the p-adic length scale L_p.

3. p-Adic length scale hypothesis states that p-adic length scales L_p with $p \simeq 2^k$, k prime, are physically favored. The proposal has been that L_k corresponds to the size of the wormhole throat X^2 assignable to a wormhole contact connecting two Minkowskian space-time sheets with size scale L_p. This hypothesis could also mean that Y^4 has a size scale determined by p-adic p-adic length scale L_k. The recent view forces to challenge this view.

3.6.2 An attempt to build an overall view

The goal is to produce the intuitive picture supported by the p-adic mass calculations. This poses several challenges. One can consider the situation at the level of X^4 and at the level of fermion lines.

Consider first the situation at the level of X^4.

1. Concerning the hierarchy of Planck constants, it seems that h_{eff}/h_0 as a dimension of algebraic extension and p-adic length scales are independent. The problem is that if one assigns both h_{eff} and p-adic primes as ramified primes to fermion lines, a very strong correlation between the two notions is forced.

2. The recent view suggests a general solution to the problem. h_{eff}/h_0 does not characterize fermion lines but to the dimension of the extension E of rational assignable to the X^4. There is no need to assign $h_{eff}/h_0 = n$ to a single polynomial P and there is no need to assign p-adic length scales to E. This is something new as compared to the earlier view.

3. The notion of cognitive measurement cascade is highly attractive and would be realized in the hierarchy of Galois groups associated with an extension of rationals, in which the Galois group has the included Galois as a normal sub-group. The cascade would reduce the entanglement between the Galois group and normal sub-group at each level of the hierarchy and eventually effectively reduce the Galois group to a Cartesian product of normal subgroups. This cascade could take place at the level of X^4 for the Galois group of E and could relate to the hierarchy of field bodies. The cascade would reduce the entanglement between the field bodies in the hierarchy.

One can also consider the situation at the level of fermion lines.

1. In the proposed framework, the p-adic length scales characterizing the elementary particles would be associated with the extensions of rationals assignable to the fermion lines at which P is expected to be a polynomial in a small extension of rationals to which E reduce, perhaps real or complex rationals.

   According to the p-adic mass calculations, all elementary particles except muon correspond to p-adic length scales characterized by rational ramified primes. Muon corresponds to $k = 113$ assignable to a Gaussian Mersenne prime so that for muon the coefficients of P would be Gaussian integers rather than real integers. This picture would conform with the notion of elementarity in the theoretical sense.

2. The biologically important p-adic length scales associated with the four Gaussian primes $M_{G,k}$, $k = 151, 157, 163, 167$ in the range between 10 nm and 2.5 μm could correspond to the values of $\hbar_{em}/h > 1$ but the fermion lines could still correspond to the usual rational primes and have the standard masses. An alternative interpretation is that the elementary particles involved, say quarks, really correspond to Gaussian primes and to k-fold iterations of a complex polynomial with degree 2 and are very light.

   I have indeed proposed that scaled up variants of elementary particles are possible in the TGD Universe [K7] and that scaled up variant of QCD makes sense in biological length scales and that there is a resonant coupling between the dark variants of particles and the p-adically scaled particles with the same p-adic length scale. These particles would have very different mass scales. This would make possible the scaling up of the color confinement scale to biological scales.

3. One should also understand the origin of the p-adic length scale hypothesis. A possible interpretation of the p-adic length scale hypothesis $p \simeq 2^k$ (there is also evidence for powers of 3) is as follows. The earlier proposal was that, in the case that k is a prime, it could characterize the p-adic length scale of the wormhole throat. However, other than prime values of k are possible in p-adic mass calculations. The recent view suggests that p characterizes fermion lines and the size scale of wormhole throats and p-adic length scale corresponds to Compton length of the particle for $h_{eff} = h$.

   This raises the question whether a fermion with p-adic length scale $L(k) \propto 2^{k/2}$ could correspond to a k-fold iterate of a second degree polynomial P? This would conform with the idea of elementarity and give a direct connection with the generalization of the chaos theory and period doubling naturally associated with the iteration. The iteration of polynomials of degree 3 would give rise to the hierarchy in powers of 3 for which there is also evidence. Larger primes such as $k = 5$ are distinguished from $k = 2$ and 3 since only for these primes the polynomial of degree k can be solved in a closed form.

   Quite generally, the iteration hierarchies $P \rightarrow P \circ P \rightarrow ...$ could give rise to the counterpart of chaos theory at the space-time level realized as sections of the twistor bundle X^6. $P(0) = 0$ guarantees that the roots of P are also roots of iterates and one could speak of a cognitive hierarchy of states of elementary particles for which the number of roots increases exponentially. I have proposed that cognitive measurement cascades reducing the entanglement between the normal subgroups of the Galois group of the composite $P_1 \circ \circ .... \circ P_n$ are fundamental for cognition, maybe even at the elementary particle level.

   Especially interesting are prime polynomials [L42], which are not expressible as composites of polynomials with a lower degree. Since the order of composite is a product of orders of factors, polynomials of prime order are prime polynomials.

4. One can assign also to the fermion line an extension of rationals with dimension $n_f = h_{eff,f}/h_0$ as the analog of $n = h_{eff}/h_0$. E should reduce to rationals or complex rationals at the fermion line. The roots of P at the fermion line giving rise to its copies characterizing the partonic orbit would be a physical manifestation of this hierarchy and h_{eff} would characterize it. $n_f = h_{eff,f}/h_0$ would define a measure of algebraic complexity to a fermion line as a kind of IQ whereas $n = h_{eff}/h$ would characterize the complexity of the entire X^4.

One should also understand the notions of gravitational and electric Planck constant.

1. Gravitational Planck constant $\hbar_{gr}$ and electric Planck constant h_{em} differ from the standard Planck constant h believed to be a universal constant of Nature in that they characterize a pair of systems, typically with large mass or charge and a particle: also identical masses and charges might make sense [L61]. $\hbar_{gr}$ or $\hbar_{em}$ would typically characterize the pair formed by X^4 and the fermion line. This gives rise to gravitational Compton length as a universal Compton length, which does not depend on the mass of the particle and also to electric Compton length. These parameters could be still a single particle characteristic assignable to the fermion lines and scaled up by $\hbar_{gr}/h$ or $\hbar_{em}/h$.

   The problem is that the values $\hbar_{gr}/h$ or $\hbar_{em}/h$ change when particle characteristics change. This would suggest that they correspond to different space-time surfaces for different particles? Could these parameters characterize parts of the field body as 4-surfaces connecting the two systems? This would suggest a very refined theoretical organization of the space-time sheets, somewhat like books at book shelves: something very different from thermal chaos.

   I have considered the possibility that the realization of Yangian symmetries in terms of multilocal infinitesimal transformations could justify this dependence. $\hbar_{gr}$ and $\hbar_{em}$ could relate to interactions between space-time sheets with different length scales.

One should also understand the value of $h = n_0 h_0$. In the recent framework it should correspond to the dimension of extension E for X^4. The proposal that the ratio of CP_2 length scale squared to Planck length scale squared corresponds to n_0 is consistent with the recent view.

3.7 p-Adicization, assuming holography = holomorphy principle, produces p-adic fractals and holograms

The recent chat with Tuomas Sorakivi, a member of our Zoom group, was about the concrete graphical representations of the spacetime surfaces as animations. The construction of the representations is shockingly straightforward, because the partial differential equations reduce to algebraic equations that are easy to solve numerically. For the first time, it seems that GPT has created a program without obvious bugs. The challenges relate to how to represent time=constant 2-D sections of the 4-surface most conveniently and how to build animations about the evolution of these sections.

Tuomas asked how to construct p-adic counterparts for space-time surfaces in $H = M^4 \times CP_2$. I have been thinking about the details of this presentation over the years. Here is my current vision of the construction.

1. By holography = holomorphy principle, space-time surfaces in H correspond to roots $(f_1, f_2) = (0, 0)$ for two analytic (holomorphic) functions f_i of of 3 complex coordinates and one hypercomplex coordinate of H. The Taylor coefficients of f_i are assumed to be rational or in an algebraic extension of rationals but even more general situations are possible. A very important special case are polynomials $f_i = P_i$.

2. If we are talking about polynomials or analytic functions with coefficients that are rational or in algebraic extension to rationals, then a purely formal p-adic equivalent can be associated with every real surface with the same equations.

3. However, there are some delicate points involved.

   (a) The imaginary unit sqrt(-1) is in algebraic expansion if p modulo 4=3. What about p modulo 4=1. In this case, sqrt(-1) can be multiplied as an ordinary p-adic number by the square root of an integer that is only in algebraic expansion. So the problem is solved.

   (b) In p-adic topology, large powers of p correspond to small p-adic numbers, unlike in real topology. This eventually led to the canonical concept of identification. Let's translate the powers of p in the expansion of a real number into powers of p (the equivalent of the decimal expansion).

   $$\sum x_n p^n \leftrightarrow \sum x_n p^{-n} \quad .$$

   This map of p-adic numbers to real numbers is continuous, but not vice versa. In this way, real points can be mapped to p-adic points or vice versa. In p-adic mass calculations, the map of p-adic points to real points is very natural. One can imagine different variants of the canonical correspondence by introducing, for example, a pinery cutoff analogous to the truncation of decimal numbers. This kind of cutoff is unavoidable.

   (c) As such, this correspondence from reals to p-adics is not realistic at the level of H because the symmetries of the real H do not correspond to those of p-adic H. Note that the correspondence at the level of spacetime surfaces is induced from that at the level of the embedding space.

4. is forces number theoretical discretization, i.e. cognitive representations (p-adic and more generally adelic physics is assumed to provide the correlates of cognition). The symmetries of the real world correspond to symmetries restricted to the discretization. The lattice structure for which continuous translational and rotational symmetries are broken to a discrete subgroup is a typical example.

   Let us consider a given algebraic extension of rationals.

(a) Algebraic rationals can be interpreted as both real and p-adic numbers in an extension induced by the extension of rationals. The points of the cognitive representations correspond to the algebraic points allowed by the extension and correspond to the intersection points of reality as a real space-time surface and p-adicity as p-adic space-time surface.

(b) These algebraic points are a series of powers of p, but there are only a *finite* number of powers so that the interpretation as algebraic integers makes sense. One can also consider rations of algebraic integers if canonical identification is suitably modified. These discrete points are mapped by the canonical identification or its modification to the rational case from the real side to the p-adic side to obtain a cognitive representation. The cognitive representation gives a discrete skeleton that spans the spacetime surface on both the real and p-adic sides.

Let's see what this means for the concrete construction of p-adic spacetime surfaces.

1. Take the same equations on the p-adic side as on the real side, that is $(f_1, f_2 = (0,0)$, and solve them around each discrete point of the cognitive representation in some p-adic sphere with radius p^{-n}.

   The origin of the generalized complex coordinates of H is **not** taken to be the origin of p-adic H, but this canonical identification gives a discrete algebraic point on the p-adic side. So, around each such point, we get a p-adic scaled version of the surface $(f_1, f_2 = (0,0)$ inside the p-adic sphere. This only means moving the surface to another location and symmetries allow it.

2. How to glue the versions associated with different points together? This is not necessary and not even possible!

   The p-adic concept of differentiability and continuity allows fractality and holography. These are closely related to the p-adic non-determinism meaning that any function depending on finite number of pinary digits has a vanishing derivative. In differential and partial differential equations this implies non-determinism, which I have assumed corresponds to the real side of the complete violation of classical determinism for holography.

   The definition of algebraic surfaces does not involve derivatives but also for algebraic surfaces the roots of $(f_1, f_2) = (0,0)$ can develop branching singularities at which several roots as space-time regions meet and one must choose one representative [L64].

   (a) Assume that the initial surface is defined inside the p-adic sphere, whose radius as the p-adic norm for the points is p^{-n}, n integer. One can even assume that a p-adic counterpart has been constructed only for the spherical shell with radius p^{-n}.

   The essential thing here is that the interior points of a p-adic sphere cannot be distinguished from the points on its surface. The surface of a p-adic sphere is therefore more like a shell. How do you proceed from the shell to the "interiors" of a p-adic sphere?

   (b) The basic property of two p-adic spheres is that they are either point strangers or one of the two is inside the other. A p-adic sphere with radius p^{-n} is divided into point strangers p-adic spheres with radius p^{-n-1} and in each such sphere one can construct a p-adic 4-surface corresponding to the equations $(f_1, f_2) = (0,0)$. This can be continued as far as desired, always to some value n=N. It corresponds to the shortest scale on the real side and defines the measurement resolution/cognitive resolution physically.

   (c) This gives a fractal for which the same $(f_1, f_2) = (0,0)$ structure repeats at different scales. We can also go the other way, i.e. to longer scales in the real sense.

   (d) Also a hologram emerges. All the way down to the smallest scale, the same structure repeats and an arbitrarily small sphere represents the entire structure. This strongly brings to mind biology and genes, which represent the entire organism. Could this correspondence at the p-adic level be similar to the one above or a suitable generalization of it?

3. Many kinds of generalizations can be obtained from this basic fractal. Endless repetition of the same structure is not very interesting. p-Adic surfaces do not have to be represented by the same pair of functions at different p-adic scales.

   Of particular interest are the 4-D counterparts to fractals, to which the names Feigenbaum, Mandelbrot and Julia are attached. They can be constructed by iteration

$$(f_1, f_2) \rightarrow G(f_1, f_2) = (g_1(f_1, f_2), g_2(f_1, f_2)) \rightarrow G(G(f_1, f_2)) \rightarrow ...$$

   so that at each step the scale increases by a factor p. At the smallest scale p^{-n} one has $(f_1, f_2) = (0, 0)$. At the next, longer scale p^{-N+1} one has $G(f_1, f_2) = (0, 0)$, etc.... One can assign to this kind of hierarchy a hierarchy of extensions of rationals and associated Galois groups whose dimension increases exponentially meaning that algebraic complexity, serving as a measure for the level of conscious intelligence and scale of quantum coherence also increases in the same way.

   The iteration proceeds with the increasing scale and the number-theoretic complexity measured the dimension of the algebraic extension increases exponentially. Cognition becomes more and more complex. Could this serve as a possible model for biological and cognitive evolution as the length scale increases?

   The fundamental question is whether many-sheeted spacetime allows for a corresponding hierarchy at the real side? Could the violation of classical determinism interpreted as p-adic non-determinism for holography allow this?

3.8 p-Adic primes as ramified primes, effective Planck constant, and evolutionary hierarchy of extensions of rationals

Consider now the number theoretic vision in detail.

3.8.1 What Galois confinement could mean?

The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by its dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L70, L26, L30, L23, L31, L48]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units. The idea has been applied also in the TGD inspired view of condensed matter physics [L36].

In the most plausible variant of the holography=holomorphy vision, the space-time surface is determined as a root a polynomial P that is as as a zero section of the twistor bundle X^6 in defined by the Kähler action in the 12-D twistor space $T(M^4) \times T(CP_2)$. Fermion lines are determined by the criticality conditions $(P, dP/dz) = (0, 0)$. At the number theoretical criticality the coefficients of the polynomial P are algebraic integers in some extension F of rationals and Galois group and ramified primes can be assigned to the extension of F as algebraic primes of F identifiable as p-adic primes. The entire space-time surface is number-theoretically critical. and fermions correspond to even higher criticality. The space-time surface is clearly a complexification of cusp catastrophe topologically. The extension F of rationals as the coefficient field makes possible adelization for polynomials [L5, L6].

his leaves several options for what Galois confinement could mean.

1. The minimal form of Galois confinement would apply to the extension of F defined by the conditions $(P, dP/dz) = (0, 0)$ polynomial having coefficients in F and is restricted to the fermion lines of a single partonic orbit. Galois confinement would mean that the components of 4-momentum are ordinary integers at M^8 level using the natural momentum unit defined by cd. A more general option is that the momentum components are algebraic integers in F rather than ordinary integers.

One can consider a hierarchy of Galois confinements for a hierarchy of extensions of extensions $Q \rightarrow F_1 \rightarrow F_2 ... \rightarrow F_n$ and hierarchy of Galois confinements in which the number of confined particles increases with the level. At the bottom there would be F_n confinement, at the next level F_{n-1} confinement and at the highest level rational Galois confinement. At the fermion lines one could have a hierarchy of extensions E_i of F_i for the same polynomials P. A connection with the hierarchies of the bound states (quarks, hadrons, nuclei, atoms, molecules,...) is suggestive.

Also the functional composition of polynomials as $P \rightarrow (Q \circ P)$ gives rise to a hierarchy of extensions associated with the fermionic lines and one can consider iteration of the functional composition giving rise to the number theoretic analog for the transition to chaos.

2. The maximal number of fermion lines would correspond to the degree of dP/dz as a polynomial of w or ξ^i and multi-fermion states are necessary for Galois confinement if the extension is rational and the momentum components are algebraic integers in E_i as extension of F_i.

   As already proposed, the polynomial P as polynomial $P(w)$ of M^4 coordinate w at fermion lines could have rather high degree but as polynomial of CP_2 coordinates ξ^i a rather low degree. There could be a very large number of fermionic lines giving rise to a 2-D lattice like structure.

   Maybe this option could be realized for partonic orbits of quarks inside hadrons with discretization of the color group represented as the Galois group. The idea that the quarks (say valence quarks) inside hadrons could correspond to a single partonic orbit defining a multisheeted structure with respect to CP_2 looks admittedly rather weird.

3. If the extensions are identical for the fermion lines assignable to different partonic orbits one could also consider the option that there is Galois confinement for the states assignable to the collection of these orbits with respect to the shared Galois group. This option could provide an alternative view of hadrons.

The earlier rather complicated realization of Galois confinement, was based on complexified M^8 [L35] but trivializes when M^8 is real [L59].

1. If F corresponds to rationals, Galois confinement represents a number-theoretic analog of the periodic boundary conditions associated with the causal diamond CD. For irreducible polynomials with rational coefficients one does not obtain any rational roots so that Galois singlest are bound to be many-fermion states. Mass squared values for the physical states are integers and there is an analogy with stringy mass spectrum.

2. Single fermion states have quaternionic 4- momenta, which are algebraic integers and can be expressed by using so called integral basis (`https://cutt.ly/SRuZySX`) spanning algebraic integers as a lattice and analogous to unit vectors of momentum lattice but for single component of momentum as a vector in extension. The real algebraic momentum components are expressed in the basis consisting of the sums of roots and their conjugates (with respect to the complex unit i commuting with octonionic units) in the extension of rationals. There is also a theorem stating that one can form the basis of extension as powers of a single root. It is also known that irreducible monic polynomials have algebraic integers as roots.

3.8.2 Galois confinement as a number theoretically universal way to form bound states

The Galois group would act also at the mass shells of the normal spaces of $Y^4 \subset M^8$ by permuting the momenta of fermions. For instance, it could happen that there is a lattice of fermion states in the mass shell of the normal space and the Galois group permutes the fermions inside the unit cell of the lattice.

It seems that Galois confinement might define a notion much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Could also hadrons be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to

its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L13, L21, L3].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states. The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action on the points of space-time surface is not in general well-defined. At the level of M^8, octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H.

   In contrast to this, weak gauge transformations act as holonomies act in the tangent space of H. The M^8 counterparts of electroweak gauge symmetries relate to $M^8 - H$ duality. A map of a point $y \in Y^4$ to its quaternionic normal space $N(y)$ identifiable as M^4 involves a multiplication of y with a co-quaternionic unit e. The choice of e is determined apart froma $U(2) \subset U(3)$ transformation except at the points which contain fermion with momentum whose value fixes the point of $N(y)$.

   One can argue that the symmetries of H and even of WCW should/could have a reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (4-momenta) of the normal space (y) of $y \int Y^4 \subset M^8$ with coordinates, which are algebraic integers for the extension.

   4-momenta as points of the normal space of Y^4 would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are real algebraic integers.

   Also other quantum numbers, in particular color, would correspond to representations of the Galois group. In the case of angular moment Galois confinement would allow algebraic half-integer valued angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential factors which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

How the extension E associated with the polynomials (P_1, P_2) is determined? (P_1, P_2) characterizes particles represented as a set of fermion line(s) and E characterizes the background space-time and is therefore somewhat analogous to the vacuum expectation of the Higgs field. The character of the many-fermion states associated with the space-time surface is determined by the extension E. NMP states that state function reductions unavoidably lead to a number theoretic evolution [L53]. The algebraic complexity of the coefficient field of P_i increases so that also the complexity of the many-fermion states associated with partonic orbits increases, in particular the upper bound for the number of fermions in a state of this kind increases. Also the algebraic complexity of single-fermion states increases as the polynomials P_i become more complex. Functional composition $(P_1, P_2) \rightarrow (Q_1 \circ P_1, Q_2 \circ P_2)$ is the simplest evolutionary step that one can imagine.

3.8.3 Hierarchies of extensions for rationals and of inclusions of hyperfinite factors

TGD suggests 3 different views of finite measurement resolution.

1. At the space-time level, finite measurement resolution is realized in terms of cognitive representations actualized at the mass shells of normal space of $Y^4 \subset M^8$ in terms of fermionic momenta with momentum components identifiable as algebraic integers. Galois group has natural action on the momentum components. $M^8 - H$ duality maps this repesentation to $a = h_{eff}/m$ hyperbolic spaces $H^3 \subset CD \subset H$.

2. The inclusion $N \subset M$ of group algebras of Galois groups is proposed to realize finite measurement resolution for which the number theoretic counterpart is Galois singlet property of N with respect to the Galois group of M relative to N identifiable as the coset group of Galois groups of M and N. If the origin serves as a root of all polynomials considered, the composite $P \circ Q$ inherits the roots of Q.

   The idea generalizes to infinite-D Galois groups [L29, L26]. The HFF in question would be infinite-D group algebra of infinite Galois group for a polynomial R obtained as a composite $R = P_{infty} \circ Q$ of an infinite iterate P_{infty} of polynomial P and of some polynomial Q of finite degree (inverse limit construction). The roots of R at the limit correspond to the attractor basin associated with P_{∞}, which is bounded by the Julia set so that a connection with fractals emerges.

3. The inclusions $N \subset M$ of hyperfinite factors of type II_1 (HFFs) [K20, K5] is a natural candidate for the representation of finite measurement resolution. N would represent the degrees of freedom below measurement resolution mathematically very similar to gauge degrees of freedom except that gauge algebra would be replaced with the super-symplectic algebra and analogs of Kac Moody algebra with non-negative conformal weights and gauge conditions would apply to sub-algebra with conformal weights larger than the weight h_{max} defining the measurement resolution.

4. The hierarchies of extensions E of rationals are associated with the space-time sheets X^4 are naturally associated with the inclusions of HFFs. Now there is no concrete representation as a polynomial. One can consider them also at the level of fermion lines.

For HFFs, the index $[M : N]$ of the inclusion defines the quantum dimension $d(N \subset M) \leq 1$ as a quantum trace of the projector $P(M \rightarrow N)$ (the identify operator of M has quantum trace equal to one). $d(N \subset M)$ is defined in terms of quantum phase q and serves as a dimension for the analog of factor space M/N representing the system with N regarded as degrees of freedom below the measurement resolution and integrated out in "quantum algebra" M/N. Quantum groups and quantum spaces are closely related notions [K20, K5].

Galois confinement would suggest that $N \subset M$ corresponds to the algebra creating Galois singlets with respect to the Galois group of N relative to M whereas M includes also operators which are not this kind of singlets. In the above example $R = P \circ Q$, the Galois group of P would be represented trivially and the Galois group of Q or its subgroup would act non-trivially. In the case of hadrons, color degrees of freedom perhaps assignable to the Galois group Z^3 in the case of quarks would correspond to the degrees of freedom below the measurement resolution.

The universality of the quantum dimension and its expressibility in terms of quantum phase suggests that the integer m in $q = rexp(i2\pi/m)$ is closely related to the dimension for the extension of rationals $n = h_{eff}/h_0$ and depends therefore only very weakly on the details of the extension. The simplest guess is $m = n$. This conforms with the concrete interpretation of charge fractionation as being due to the many-valuedness of the graphs of space-time surfaces as maps from $M^4 \rightarrow CP_2$ or vice versa.

3.9 Does the universality of the holomorphy-holography principle make the notion of action un-necessary in the TGD framework?

It is gradually becoming clear that in the TGD framework the holography-holomorphy principle could make the notion of action defining the space-time surfaces un-necessary at the fundamental level. Only the Dirac action for the second quantized free spinors of H and the induced Dirac action would be needed.

The geometrization of physics would reduce to its algebraic geometrization and number theoretical universality would allow to describe correlates of cognition. The four-dimensionality of space-time surfaces would be essential in making the theory non-trivial by allowing to identify vertices for fermion pair creation in terms of defects of the standard smooth structure of the space-time surface making it an exotic smooth structure.

3.9.1 Holography=holomorphy as the basic principle

Holography=holomorphy principle allows to solve the field equations for the space-time surfaces exactly by reducing them to algebraic equations.

1. Two functions f_1 and f_2 that depend on the generalized complex coordinates of H=$M^4 \times CP_2$ are needed to solve the field equations. These functions depend on the two complex coordinates ξ_1 and ξ_2 of CP_2 and the complex coordinate w of M^4 and the hypercomplex coordinate u for which the coordinate curves are light-like. If the functions are polynomials, denote them $f_1 \equiv P_1$ and $f_2 \equiv P_2$.

   Assume that the Taylor coefficients of these functions are rational or in the expansion of rational numbers, although this is not necessary either.

2. The condition $f_1 = 0$ defines a 6-D surface in H and so does $f_2 = 0$. This is because the condition gives two conditions (both real and imaginary parts for f_i vanish). These 6-D surfaces are interpreted as analogs of the twistor bundles corresponding to M^4 and CP_2. They have fiber which is 2-sphere. This is the physically motivated assumption, which might require an additional condition stating that ξ_1 and ξ_2 are functions of w as analogs of the twistor bundles corresponding to M^4 and CP_2. This would define the map mapping the twistor sphere of the twistor space of M^4 to the twistor sphere of the twistor space of CP_2 or vice versa. The map need not be a bijection but would be single valued.

   The conditions $f_1 = 0$ and $f_2 = 0$ give a 4-D spacetime surface as the intersection of these surfaces, identifiable as the base space of both twistor bundle analogies.

3. The equations obtained in this way are algebraic equations rather than partial differential equations. Solving them numerically is child's play because they are completely local. TGD is solvable both analytically and numerically. The importance of this property cannot be overstated.

4. However, a discretization is needed, which can be number-theoretic and defined by the expansion of rationals. This is however not necessary if one is interested only in geometry and forgets the aspects related to algebraic geometry and number theory.

5. Once these algebraic equations have been solved at the discretization points, a discretization for the spacetime surface has been obtained.

   The task is to assign a spacetime surface to this discretization as a differentiable surface. Standard methods can be found here. A method that produces a surface for which the second partial derivatives exist because they appear in the curvature tensor.

   An analogy is the graph of a function for which the (y, x)´ pairs are known in a discrete set. One can connect these points, for example, with straight line segments to obtain a continuous curve. Polynomial fit gives rise to a smooth curve.

6. It is good to start with, for example, second-degree polynomials P_1 and P_2 of the generalized complex coordinates of H.

3.9.2 How could the solution be constructed in practice?

For simplicity, let's assume that $f_1 \equiv P_1$ and $f_2 \equiv P_2$ are polynomials.

1. First, one can solve for instance the equation $P_2(u, w, \xi_1, \xi_2) = 0$ giving for example $\xi_2(u, w, \xi_1)$ as its root. Any complex coordinates w, ξ_1 or ξ_2 is a possible choice and these choices can correspond to different roots as space-time regions and all must be considered to get the full picture. A completely local ordinary algebraic equation is in question so that the situation is infinitely simpler than for second order partial differential equations. This miracle is a consequence of holomorphy.

2. Substitute $\xi_2(u, w, \xi_1)$ in P_1 to obtain the algebraic function $P_1(u, w, \xi_1, \xi_2(u, w, \xi_1)) = Q_1(u, w, \xi_1)$.

3. Solve ξ_1 from the condition $Q_1 = 0$. Now we are dealing with the root of the algebraic function, but the standard numerical solution is still infinitely easier than for partial differential equations.

   After this, the discretization must be completed to get a space-time surface using some method that produces a surface for which the second partial derivatives are continuous.

3.9.3 Algebraic universality

What is so remarkable is that the solutions of $(f_1, f_2) = (0, 0)$ to the variation of any action if the action is general coordinate invariant and depends only on the induced geometry. Metric and the tensors like curvature tensor associated with it and induced gauge fields and tensors associated with them.

The reason is that complex analyticity implies that in the equations of motion there appears only contractions of complex tensors of different types. The second fundamental form (external curvature) defined by the trace of the tensor with respect to the induced metric defined by the covariant derivatives of the tangent vectors of the space-time surfaces is as a complex tensor of type (2,0)+(0,2) and the tensors contracted with it are of type (1,1). The result is identically zero.

The holography-holomorphy principle provides a nonlinear analogy of massless field equations and the four surfaces can be interpreted as trajectories for particles that are 3-surfaces instead of point particles, i.e. as generalizations of geodesics. Geodesics are indeed 1-D minimal surfaces. We obtain a geometric version of the field-particle duality.

3.9.4 Number-theoretical universality

If the coefficients of the function f_1 and f_2 are in an extension of rationals, number-theoretical universality is obtained. The solution in the real case can also be interpreted as a solution in the p-adic cases $p = 2, 3, 5, 7, ...$ when we allow the expansion of the p-adic number system as induced by the rational expansions.

p-adic variants of space-time surfaces are cognitive representations for the real surfaces. The so-called ramified primes are selected for a special position, which can be associated with the discriminant as its prime factors. A prime number is now a prime number of an algebraic expansion. This makes possible adelic physics as a geometric correlate of cognition. Cognition itself is assignable to quantum jumps.

3.9.5 Is the notion of action needed at all at the fundamental level?

The universality of the space-time surfaces solving the field equations determined by holography=holomorphy principle forces us to ask whether the notion of action is completely unnecessary. Does restricting geometry to algebraic geometry and number theory replace the principle of action completely? This could be the case.

1. The vacuum functional $exp(K)$, where the Kähler function corresponds to the classical action , could be identified as the discriminant D associated with a polynomial. It would therefore be determined entirely by number theory as a product of differences of the roots of a polynomial P or in fact, of any analytic function. The problem is that the space-time surfaces are determined as roots of two analytic functions f_1 and f_2, rather than only one.

2. Could one define the 2-surfaces by allowing a third analytic function f_3 so that the roots of $(f_1, f_2, f_3) = (0, 0, 0)$ would be 2-D surfaces. One can solve 3 complex coordinates of $M^4 \times CP_2$ as functions of the hypercomplex coordinate u whereas its dual remains free. One

would have a string world sheet with a discrete set of roots for the 3 complex coordinates whose values depend on time. By adding a fourth function f_4 and substituting the 3 complex coordinates, $f_4 = 0$ would allow as roots values of the coordinate u. Only real roots would be allowed. A possible interpretation of these points of the space-time surface would be as loci of singularities at which the minimal surface property, i.e. holomorphy, fails.

Note that for quadratic equations $ax^2 + bx + c = 0$, the discriminant is $D = b^2 - 4ac$ and more generally the product of the differences of the roots. This formula also holds when f_1 and f_2 are not polynomials.

The assumptions that some power of D corresponds to $exp(K)$ and that K corresponds to the action imply additional conditions for the coupling constants appearing in the action , i.e. the coupling constant evolution.

3. This is not yet quite enough. The basic question concerns the construction of the interaction vertices for fermions. These vertices reduce to the analogs of gauge theory vertices in which induced fermion current assignable to the volume action is contracted with the induced gauge boson.

   The volume action is a unique choice in the sense that in this case the modified gamma matrices defined as contractions of the canonical momentum currents of the action with the gamma matrices of H reduce to induced gamma matrices, which anticommute to the induced metric. For a general action this is not the case.

   The vertex for fermion pair creation corresponds to a defect of the standard smooth structure for the space-time surface and means that it becomes exotic smooth structure. These defects emerge in dimension D=4 and make it unique. In TGD, bosons are bound states of fermions and antifermions so that this also gives the vertices for the emission of bosons.

   For graviton emission one obtains an analogous vertex involving the second fundamental form at the partonic orbit. The second fundamental form would have delta function singularity at the vertex and vanish elsewhere. If field equations are true also in the vertex, the action must contain an additional term, say Kähler action. Could the singularity of the second fundamental form correspond to the defect of the standard smooth structure?

4. If this view is correct, number theory and algebraic geometry combined with the geometric vision would make the notion of action un-necessary at the fundamental level. Geometrization of physics would be replaced by its algebraic geometrization. Action would however be a useful tool at the QFT limit of TGD.

3.10 Entanglement paradox and new view about particle identity

A brain teaser that the theoretician sooner or later is bound to encounter, relates to the fermionic and bosonic statistics. This problem was also mentioned in the article of Keimer and Moore [D1] discussing quantum materials. The unavoidable conclusion is that both the fermions and bosons of the entire Universe are maximally entangled. Only the reduction of entanglement between bosonic and fermionic states of freedom would be possible in SFRs. In the QFT framework, gauge boson fields are primary fields and the problem in principle disappears if entanglement is between states formed by elementary bosons and fermions.

In the TGD Universe, all elementary particles are composites of fundamental fermions so that if Fock space the Fock states of fermions and bosons express everything worth expressing, SFRs would not be possible at all!

Remark: In the TGD Universe fundamental fermions localized at the points of space-time surface define a number theoretic discretization that I call cognitive representation. Besides this there are also degrees of freedom associated with the geometry of 3-surfaces representing particles. These degrees of freedom represent new physics. The quantization of fermions takes place at the level of H so that anticommutations hold true over the entire H.

Obviously, something is entangled and this entanglement is reduced. What these entangled degrees of freedom actually are if Fock space cannot provide them?

1. Mathematically entanglement makes sense also in a purely classical sense. Consider functions $\Psi_i(x)$ and $\Psi_j(y)$) and form the superposition $\Psi(x) = \sum_{ij} c_{ij}\Psi_i(x)\Psi_j x)$. This function is completely analogous to an entangled state.

2. Number theoretical physics implies that the Galois group becomes the symmetry group of physics and quantum states are representations of the Galois group [L25, L26]. For an extension of extension of ...., the Galois group has decomposition by normal subgroups to a hierarchy of coset groups.

   The representation of a Galois group can be decomposed to a tensor product of representations of these coset groups. The states in irreps of the Galois group are entangled and the SFR cascade produces a product of the states as a product of representations of the coset groups. Galois entanglement allows us to express the asymmetric relation between observer and observed very naturally. This cognitive SSFR cascade - as I have called it - could correspond to what happens in at least cognitive SFRs.

   If so, then SFR would in TGD have nothing to do with fermions and bosons (consisting of fermions too) since the maximal fermionic entanglement remains. For instance, when one for instance talks about long range entanglement the entanglement that matters would correspond to entanglement between degrees of freedom, which do not allow Fock space description.

In the TGD framework, the replacement of particles with 3-surfaces brings in an infinite number of non-Fock degrees of freedom. Could it make sense to speak about the reduction of entanglement in WCW degrees of freedom? There is no second quantization at WCW level so that one cannot talk about Fock spaces WCW level but purely classical entanglement is possible as observed.

1. In WCW unions of disjoint 3-surfaces correspond to classical many-particle states. One can form single particle wave functions for 3-surfaces with a single component, products of these single particle wave functions, and also analogs of entangled states as their superposition realized as building bricks of WCW spinor fields.

   If one requires that these wave functions are completely symmetric under the exchange of 3-surfaces, maximal entanglement in this sense would be realized also now and SFR would not be possible. But can one require the symmetry? Under what conditions one can regard two 3-surfaces as identical? For point-like particles one has always identical particles but in TGD the situation changes.

2. Here theoretical physics and category theory meet since the question when two mathematical objects can be said to be identical is the basic question of category theory. The mathematical answer is they are isomorphic in some sense. The physical answer is that the two systems are identical if they cannot be distinguished in the measurement resolution used.

4 Appendix

4.1 Comparison of TGD with other theories

Table 1 compares GRT and TGD and **Table 2** compares standard model and TGD.

4.2 Glossary and figures

The following glossary explains some basic concepts of TGD and TGD inspired biology.

- ***Space-time as surface***. Space-times can be regarded as 4-D surfaces in an 8-D space $M^4 \times CP_2$ obtained from empty Minkowski space (M^4) by adding four small dimensions (CP_2). The study of field equations characterizing space-time surfaces as "orbits" of 3-surfaces (3-D generalization of strings) forces the conclusion that the topology of space-time is non-trivial in all length scales.

	GRT	**TGD**
Scope of geometrization	classical gravitation	all interactions and quantum theory
Spacetime		
Geometry	abstract 4-geometry	sub-manifold geometry
Topology	trivial in long length scales	many-sheeted space-time
Signature	Minkowskian everywhere	also Euclidian
Fields		
classical	primary dynamical variables	induced from the geometry of H
Quantum fields	primary dynamical variables	modes of WCW spinor fields
Particles	point-like	3-surfaces
Symmetries		
Poincare symmetry	lost	Exact
GCI	true	true - leads to SH and ZEO
	Problem in the identication of coordinates	$H = M^4 \times CP_2$ provides preferred coordinates
Super-symmetry	super-gravitation	super variant of H: super-surfaces
Dynamics		
Equivalence Principle	true	true
Newton's laws and		
notion of force	lost	generalized
Einstein's equations	from GCI and EP	remnant of Poincare invariance at QFT limit of TGD
Bosonic action	EYM action	Kähler action + volume term
Cosmological constant	suggested by dark energy	length scale dependent coefficient of volume term
Fermionic action	Dirac action	Modified Dirac action for induced spinors
Newton's constant	given	predicted
Quantization	fails	Quantum states as modes of WCW spinor field

Table 1: Differences and similarities between GRT and TGD

	SM	TGD
Symmetries		
Origin	from empiria	reduction to CP_2 geometry
Color symmetry	gauge symmetry	isometries of CP_2
Color	analogous to spin	analogous to angular momentum
Ew symmetry	gauge symmery	holonomies of CP_2
Symmetry breaking	Higgs mechanism	CP_2 geometry
Spectrum		
Elementary particles	fundamental	consist of fundamental fermions
Bosons	gauge bosons, Higgs	gauge bosons, Higgs, pseudo-scalar
Fundamental	quarks and leptons	quarks and leptons
Dynamics		
Degrees of freedom	gauge fields, Higgs, and fermions	3-D surface geometry and spinors
Classical fields	gauge fields, Higgs	induced spinor connection
	SU(3) Killing vectors of CP_2	
Quantal degrees of freedom	gauge bosons,Higgs,	quantized induced spinor fields
Massivation	Higgs mechanism	p-adic thermodynamics with superconformal symmetry

Table 2: Differences and similarities between standard model and TGD

- ***Geometrization of classical fields***. Both weak, electromagnetic, gluonic, and gravitational fields are known once the space-time surface in H as a solution of field equations is known.

- ***Many-sheeted space-time*** (see **Fig. 4**) consists of space-time sheets with various length scales with smaller sheets being glued to larger ones by ***wormhole contacts*** (see **Fig. ??**) identified as the building bricks of elementary particles. The sizes of wormhole contacts vary but are at least of CP_2 size (about 10^4 Planck lengths) and thus extremely small.

  Many-sheeted space-time replaces reductionism with ***fractality*** . The existence of scaled variants of physics of strong and weak interactions in various length scales is implied, and biology is especially interesting in this respect.

- ***Topological field quantization (TFQ)*** . TFQ replaces classical fields with space-time quanta. For instance, magnetic fields decompose into space-time surfaces of finite size representing flux tubes or -sheets. Field configurations are like Bohr orbits carrying "archetypal" classical field patterns. Radiation fields correspond to topological light rays or massless extremals (MEs), magnetic fields to magnetic flux quanta (flux tubes and sheets) having as primordial representatives "cosmic strings", electric fields correspond to electric flux quanta (e.g. cell membrane), and fundamental particles to CP_2 type vacuum extremals.

- ***Field body*** (FB) and ***magnetic body*** (MB). Any physical system has field identity - FB or MB - in the sense that a given topological field quantum corresponds to a particular source (or several of them - e.g. in the case of the flux tube connecting two systems).

  Maxwellian electrodynamics cannot have this kind of identification since the fields created by different sources superpose. Superposition is replaced with a set theoretic union: only the *effects* of the fields assignable to different sources on test particle superpose. This makes it possible to define the QFT limit of TGD.

- ***p-Adic physics*** [K10] as a physics of cognition and intention and the fusion of p-adic physics with real number based physics are new elements.

- ***Adelic physics*** [L5, L8] is a fusion of real physics of sensory experience and various p-adic physics of cognition.
- ***p-Adic length scale hypothesis*** states that preferred p-adic length scales correspond to primes p near powers of two: $p \simeq 2^k$, k positive integer.
- A ***Dark matter hierarchy*** realized in terms of a hierarchy of values of effective Planck constant $h_{eff} = nh_0$ as integers using $h_0 = h/6$ as a unit. Large value of h_{eff} makes possible macroscopic quantum coherence which is crucial in living matter.
- ***MB as an intentional agent using biological body (BB) as a sensory receptor and motor instrument*** . The personal MB associated with the living body - as opposed to larger MBs assignable with collective levels of consciousness - has a hierarchical onion-like layered structure and several MBs can use the same BB making possible remote mental interactions such as hypnosis [L1].
- ***Cosmic strings Magnetic flux tubes*** belong to the basic extremals of practically any general coordinate invariant action principle. Cosmic strings are surfaces of form $X^2 \times Y^2 \subset M^4 \times CP_2$. X^2 is analogous to string world sheet. Cosmic strings come in two varieties and both seem to have a deep role in TGD.

  Y^2 is either a complex or Lagrangian 2-manifold of CP_2. Complex 2-manifold carries monopole flux. For Lagrangian sub-manifold the Kähler form and magnetic flux and Kähler action vanishes. Both types of cosmic strings are are simultaneous extremals of both Kähler action and volume action: this holds true quite generally for preferred extremals.

  Cosmic strings are unstable against perturbations thickening the 2-D M^4 projection to 3-D or 4-D: this gives rise to monopole (see **Fig. 15**) and non-monopole magnetic flux tubes. Using $M^2 \times Y^2$ coordinates, the thickening corresponds to the deformation for which $E^2 \subset M^4$ coordinates are not constant anymore but depend on Y^2 coordinates.
- ***Magnetic flux tubes and sheets*** serve as “body parts” of MB (analogous to body parts of BB), and one can speak about magnetic motor actions. Besides concrete motion of flux quanta/tubes analogous to ordinary motor activity, basic motor actions include the contraction of magnetic flux tubes by a phase transition possibly reducing Planck constant, and the change in thickness of the magnetic flux tube, thus changing the value of the magnetic field, and in turn the cyclotron frequency. Transversal oscillatory motions of flux tubes and oscillatory variations of the thickness of the flux tubes serve as counterparts for Alfwen waves.

  Reconnections of the U-shaped flux tubes allow two MBs to get in contact based on a pair of flux tubes connecting the systems and temporal variations of magnetic fields inducing motor actions of MBs favor the formation of reconnections.

  In hydrodynamics and magnetohydrodynamics reconnections would be essential for the generation of turbulence by the generation of vortices having monopole flux tube at core and Lagrangian flux tube as its exterior.

  Flux tube connections at the molecular level bring a new element to biochemistry making it possible to understand bio-catalysis. Flux tube connections serve as a space-time correlates for attention in the TGD inspired theory of consciousness.
- ***Cyclotron Bose-Einstein condensates (BECs)*** of various charged particles can accompany MBs. Cyclotron energy $E_c = hZeB/m$ is much below thermal energy at physiological temperatures for magnetic fields possible in living matter. In the transition $h \rightarrow h_{eff}$ E_c is scaled up by a fractor $h_{eff}/h = n$. For sufficiently high value of h_{eff} cyclotron energy is above thermal energy $E = h_{eff}\ ZeB/m$. Cyclotron Bose-Einstein condensates at MBs of basic biomolecules and of cell membrane proteins - play a key role in TGD based biology.
- ***Josephson junctions*** exist between two superconductors. In TGD framework, ***generalized Josephson junctions*** accompany membrane proteins such as ion channels and pumps. A voltage between the two super-conductors implies a ***Josephson current***. For a

constant voltage the current is oscillating with the ***Josephson frequency*** . The Josephson current emits ***Josephson radiation***. The energies come as multiples of ***Josephson energy***.

In TGD generalized Josephson radiation consisting of dark photons makes communication of sensory input to MB possible. The signal is coded to the modulation of Josephson frequency depending on the membrane voltage. The cyclotron BEC at MB receives the radiation producing a sequence of resonance peaks.

- ***Negentropy Maximization Principle*** (NMP). NMP [K9] [L32] is the variational principle of consciousness and generalizes SL. NMP states that the negentropy gain in SFR is non-negative and maximal. NMP implies SL for ordinary matter.

- ***Negentropic entanglement*** (NE). NE is possible in adelic physics and NMP does not allow its reduction. NMP implies a connection between NE, the dark matter hierarchy, p-adic physics, and quantum criticality. NE is a prerequisite for an experience defining abstraction as a rule having as instances the state pairs appearing in the entangled state.

- ***Zero energy ontology (ZEO)*** In ZEO physical states are pairs of positive and negative energy parts having opposite net quantum numbers and identifiable as counterparts of initial and final states of a physical event in the ordinary ontology. Positive and negative energy parts of the zero energy state are at the opposite boundaries of a ***causal diamond*** (CD, see **Fig. 12**)) defined as a double-pyramid-like intersection of future and past directed light-cones of Minkowski space.

  CD defines the "spot-light of consciousness": the contents of conscious experience associated with a given CD is determined by the space-time sheets in the embedding space region spanned by CD.

- ***SFR*** is an acronym for state function reduction. The measurement interaction is universal and defined by the entanglement of the subsystem considered with the external world [L14] [K22]. What is measured is the density matrix characterizing entanglement and the outcome is an eigenstate of the density matrix with eigenvalue giving the probability of this particular outcome. SFR can in principle occur for any pair of systems.

  SFR in ZEO solves the basic problem of quantum measurement theory since the zero energy state as a superposition of classical deterministic time evolutions (preferred extremals) is replaced with a new one. Individual time evolutions are not made non-deterministic.

  One must however notice that the reduction of entanglement between fermions is not possible since Fermi- and als Bose statistics predicts a maximal entanglement. Entanglement reduction must occur in WCW degrees of freedom and they are present because point-like particles are replaced with 3-surfaces. They can correspond to the number theoretical degrees of freedom assignable to the Galois group - actually its decomposition in terms of its normal subgroups - and to topological degrees of freedom.

- ***SSFR*** is an acronym for "small" SFR as the TGD counterpart of weak measurement of quantum optics and resembles classical measurement since the change of the state is small [L14] [K22]. SSFR is preceded by the TGD counterpart of unitary time evolution replacing the state associated with CD with a quantum superposition of CDs and zero energy states associated with them. SSFR performs a localization of CD and corresponds to time measurement with time identifiable as the temporal distance between the tips of CD. CD is scaled up in size - at least in statistical sense and this gives rise to the arrow of time.

  The unitary process and SSFR represent also the counterpart for Zeno effect in the sense that the passive boundary of CD as also CD is only scaled up but is not shifted. The states remain unchanged apart from the addition of new fermions contained by the added part of the passive boundary. One can say that the size of the CD as analogous to the perceptive field means that more and more of the zero energy state at the passive boundary becomes visible. The active boundary is however both scaled and shifted in SSFR and states at it change. This gives rise to the experience of time flow and SSFRs as moments of subjective time correspond to geometric time as a distance between the tips of CD. The analog of unitary time evolution corresponds to "time" evolution induced by the exponential of the scaling generator L_0. Time translation is thus replaced by scaling. This is the case also in p-adic thermodynamics. The idea of time evolution by scalings has emerged also in condensed matter physics.

- ***BSFR*** is an acronym for "big" SFR, which is the TGD counterpart of ordinary state function reduction with the standard probabilistic rules [L14] [K22]. What is new is that the arrow of time changes since the roles of passive and active boundaries change and CD starts to increase in an opposite time direction.

  This has profound thermodynamic implications. Second law must be generalized and the time corresponds to dissipation with a reversed arrow of time looking like self-organization for an observed with opposite arrow of time [L11]. The interpretation of BSFR is as analog of biological death and the time reversed period is analogous to re-incarnation but with non-standard arrow of time. The findings of Minev *et al* [L10] give support for BSFR at atomic level. Together with h_{eff} hierarchy BSFR predicts that the world looks classical in all scales for an observer with the opposite arrow of time.

4.3 Figures

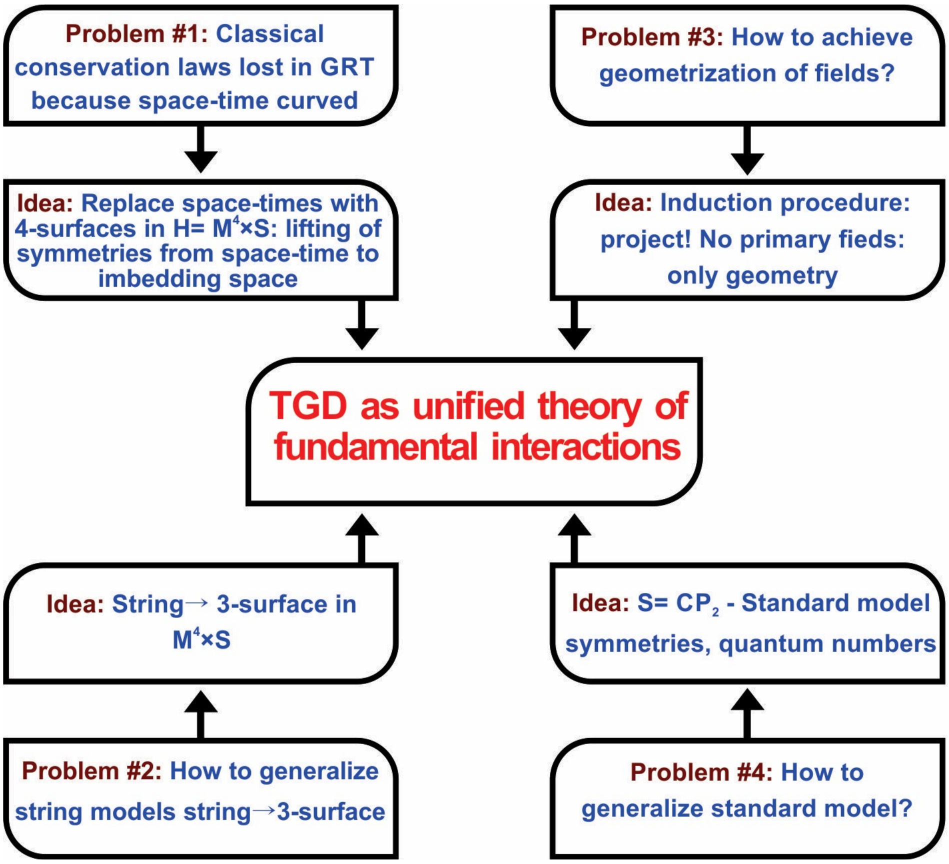

Figure 1: The problems leading to TGD as their solution.

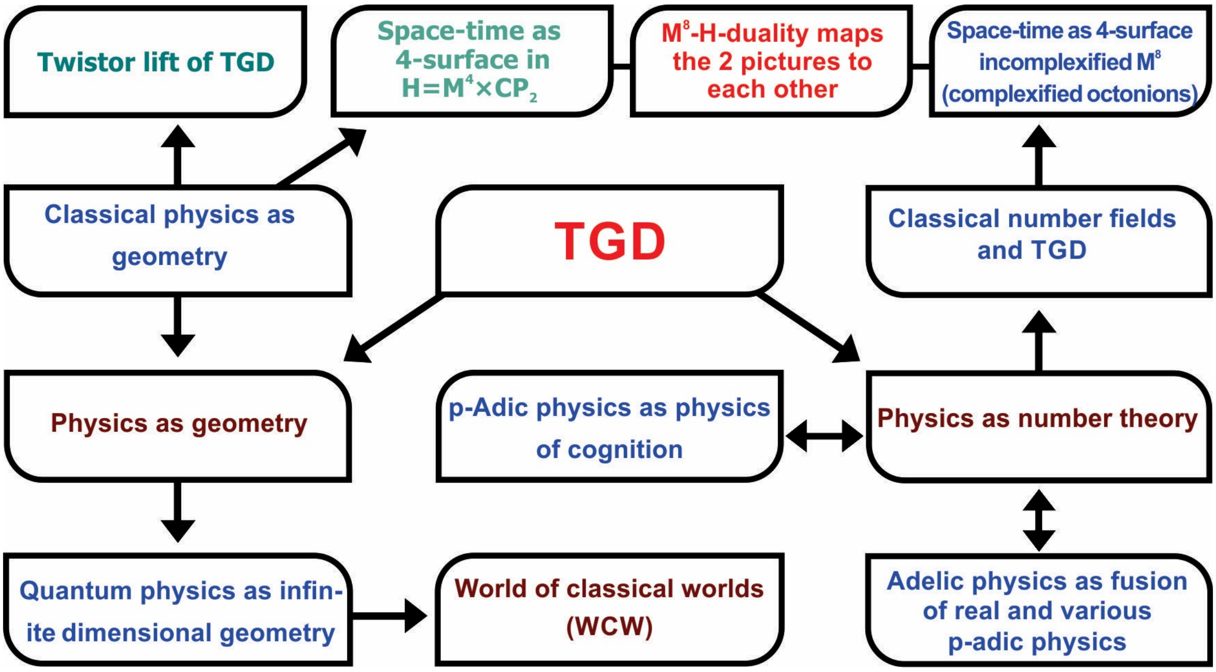

Figure 2: TGD is based on two complementary visions: physics as geometry and physics as number theory.

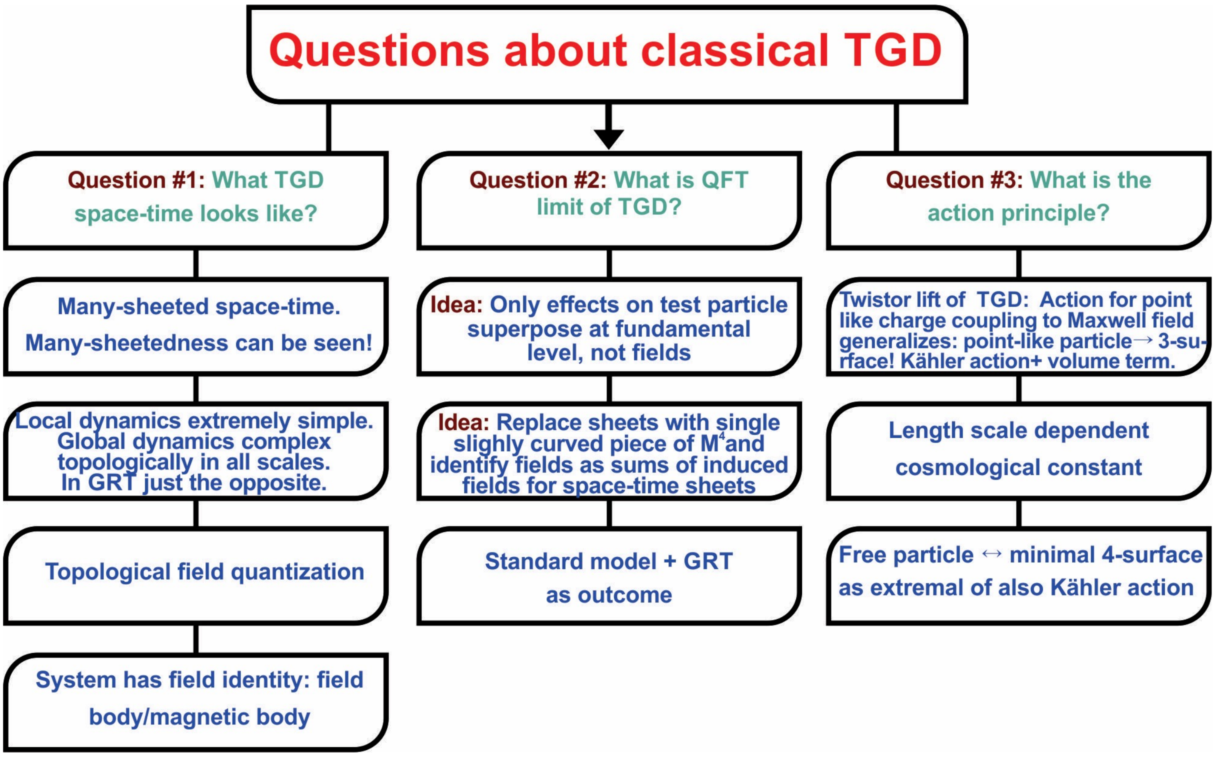

Figure 3: Questions about classical TGD.

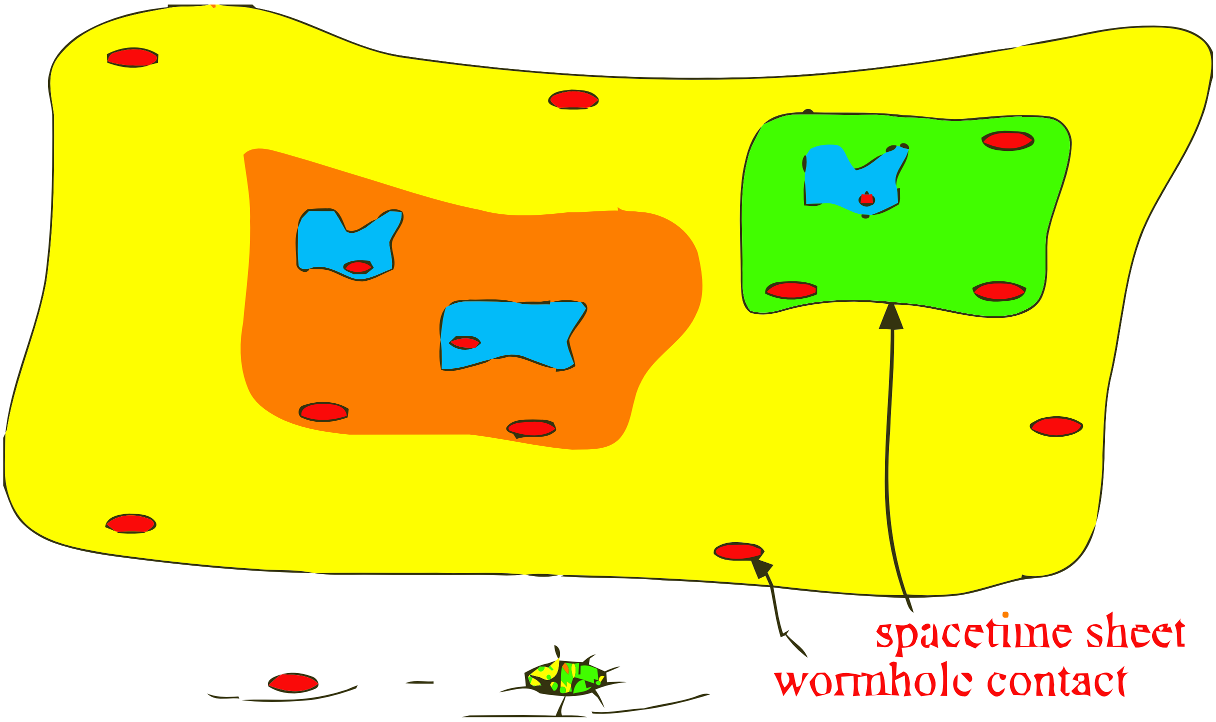

Figure 4: Many-sheeted space-time.

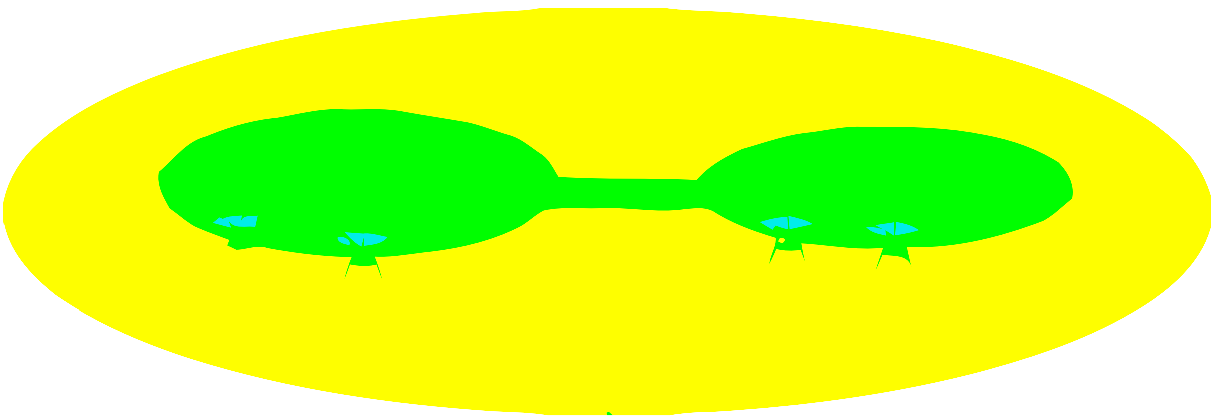

Figure 5: Wormhole contacts.

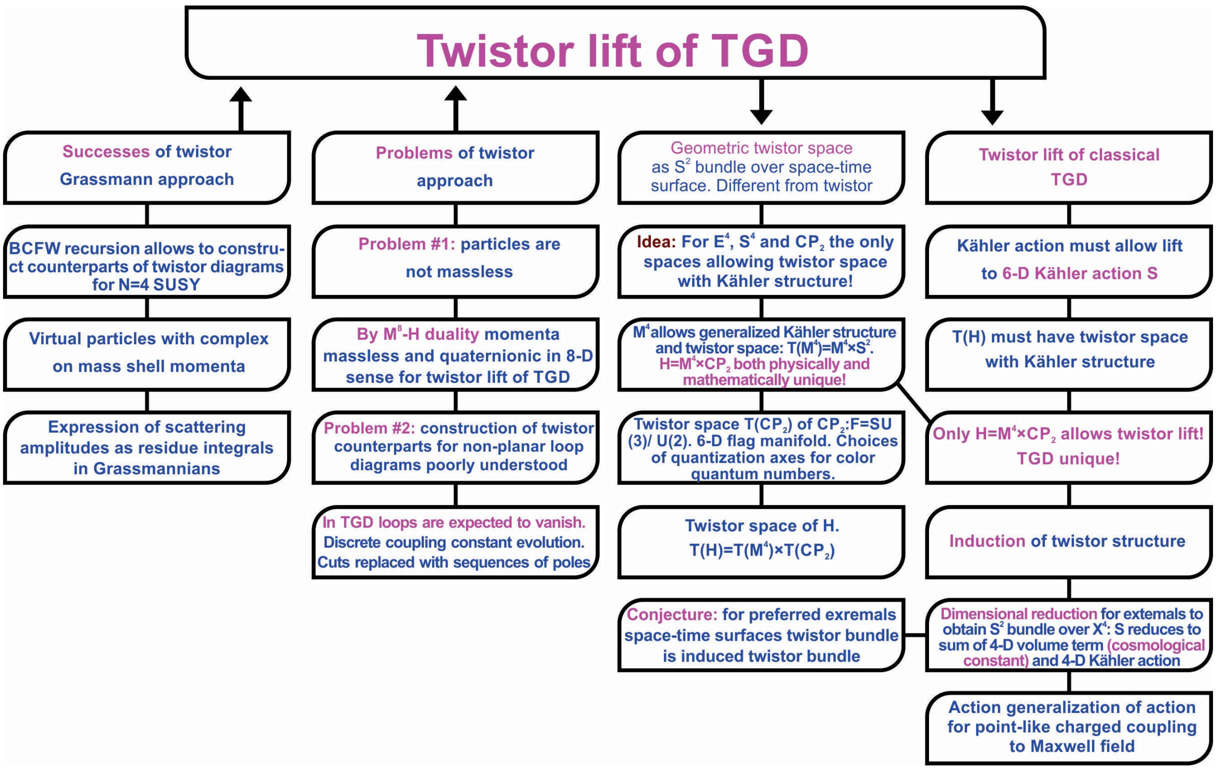

Figure 6: Twistor lift

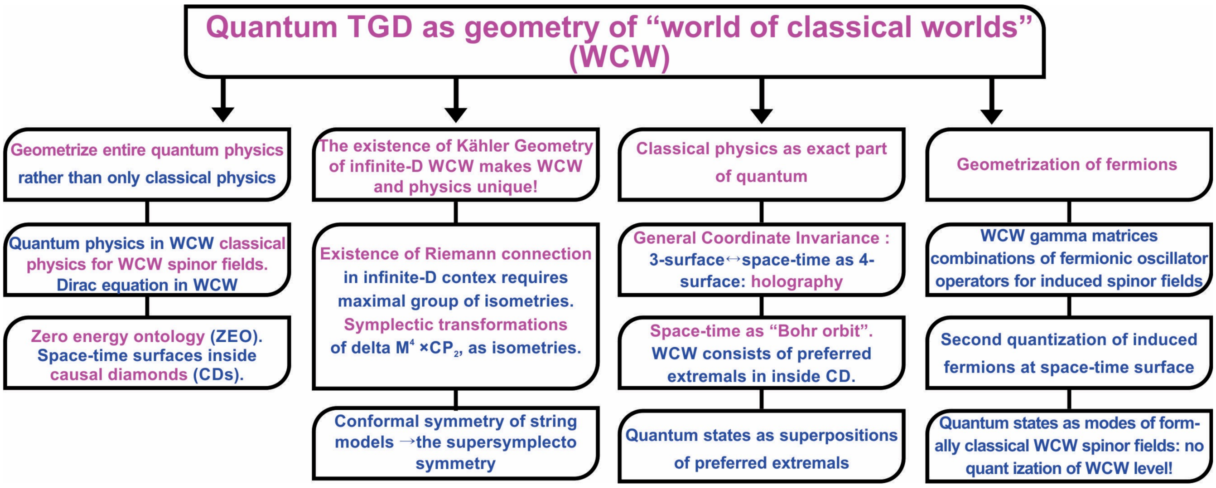

Figure 7: Geometrization of quantum physics in terms of WCW

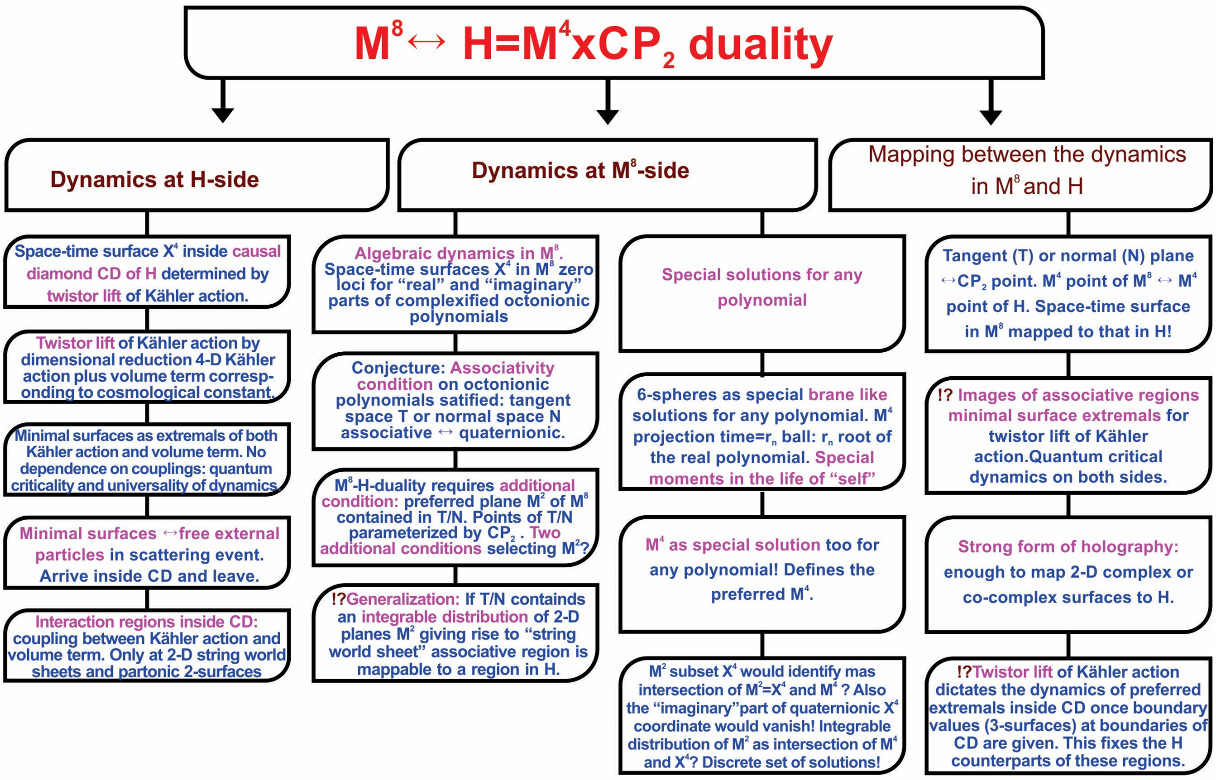

Figure 8: $M^8 - H$ duality

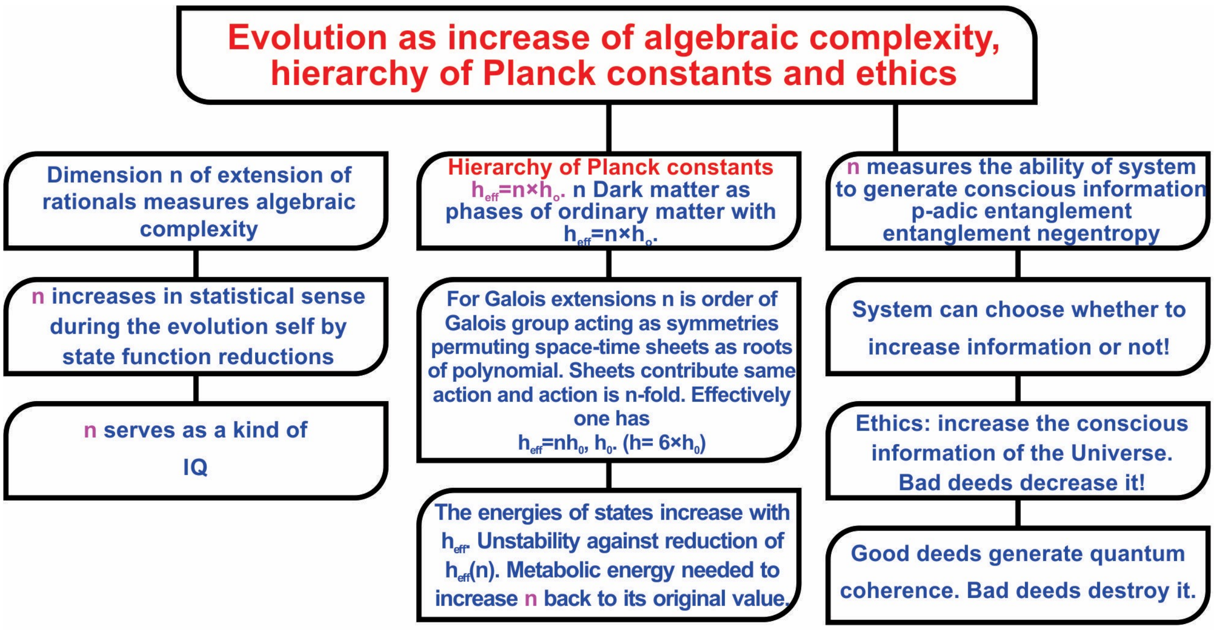

Figure 9: Number theoretic view of evolution

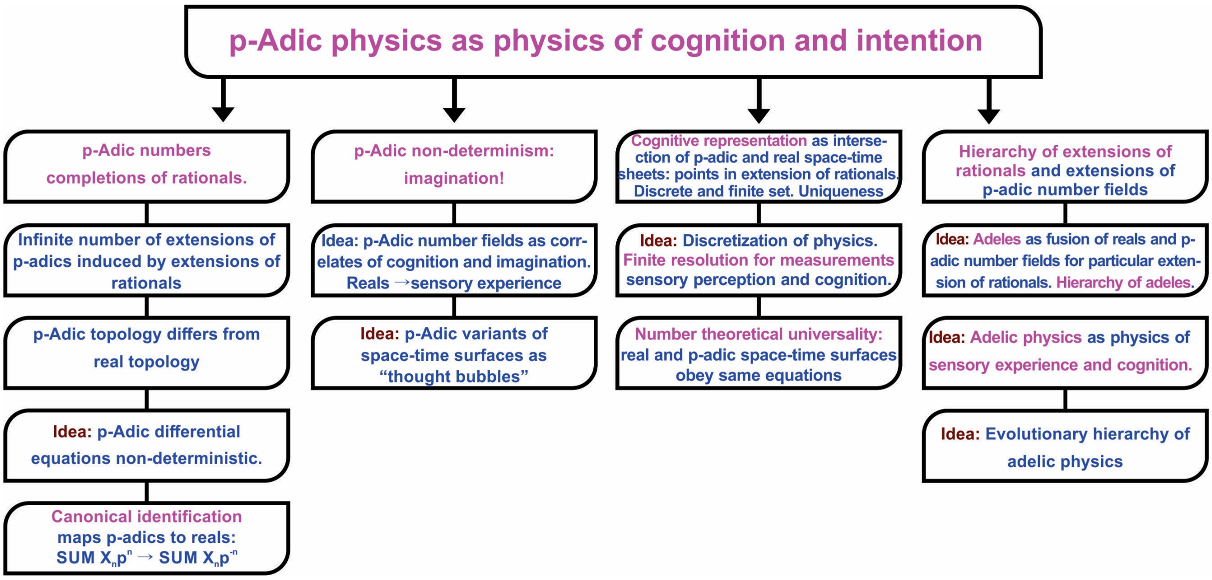

Figure 10: p-Adic physics as physics of cognition and imagination.

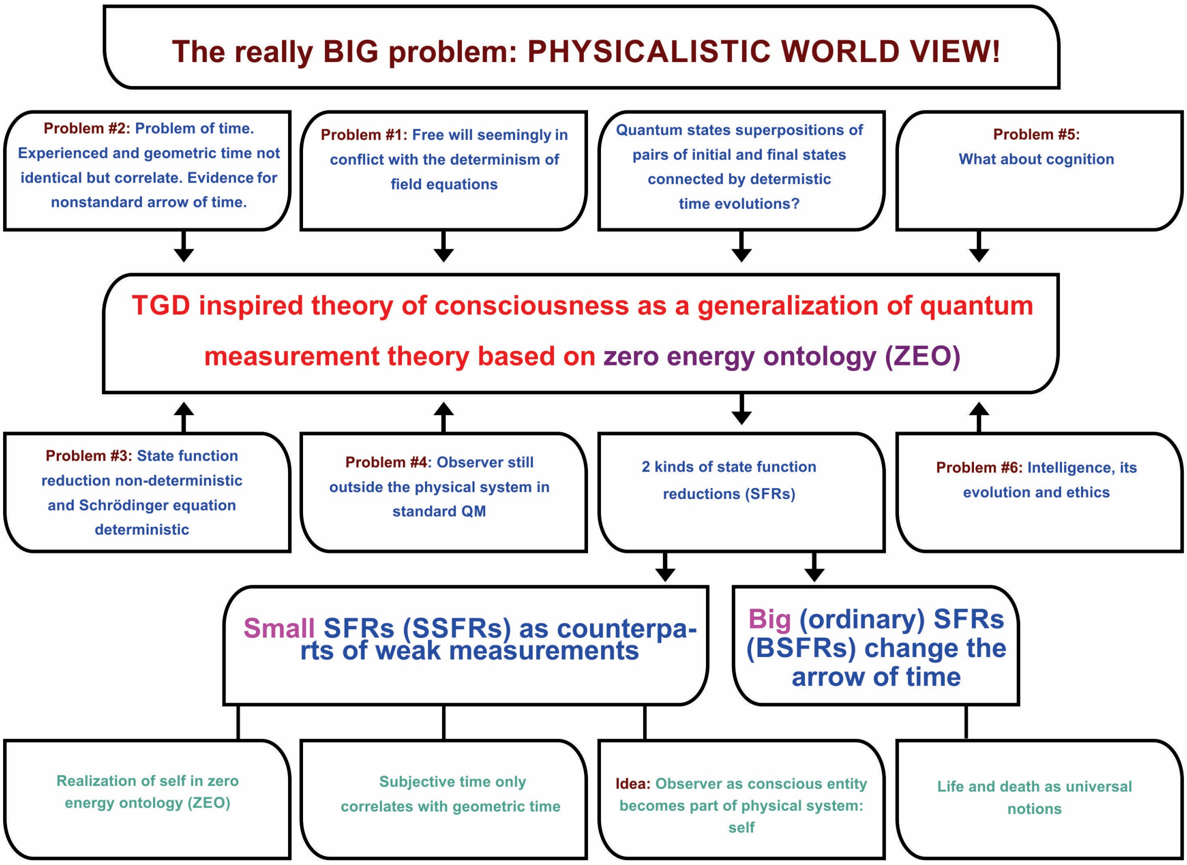

Figure 11: Consciousness theory from quantum measurement theory

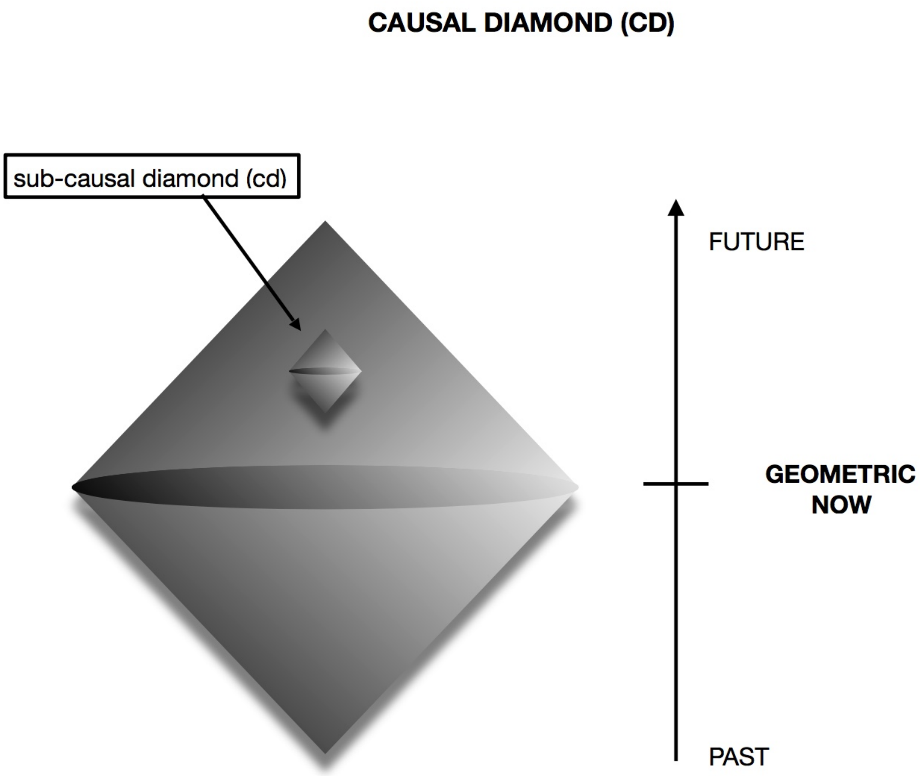

Figure 12: Causal diamond

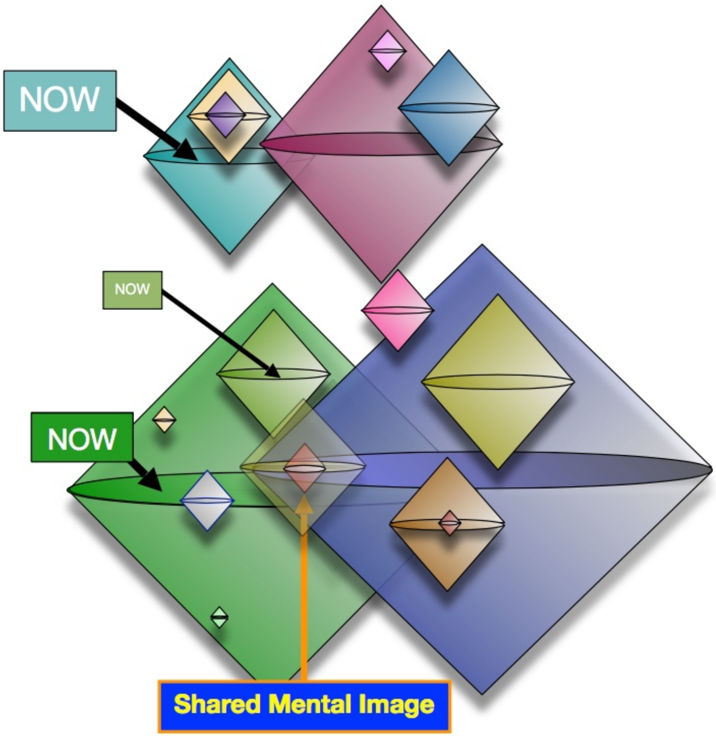

Figure 13: CDs define a fractal "conscious atlas"

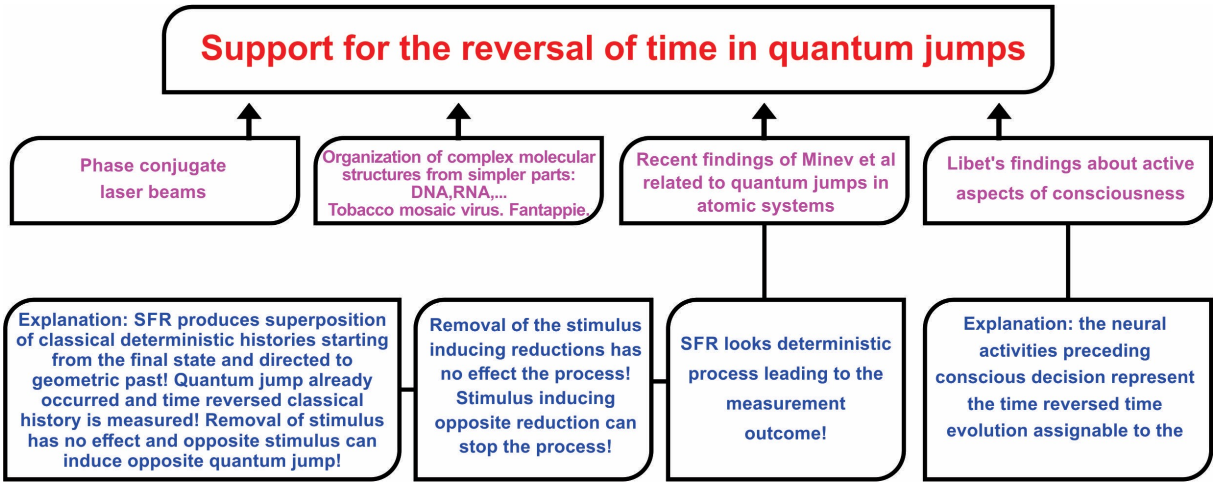

Figure 14: Time reversal occurs in BSFR

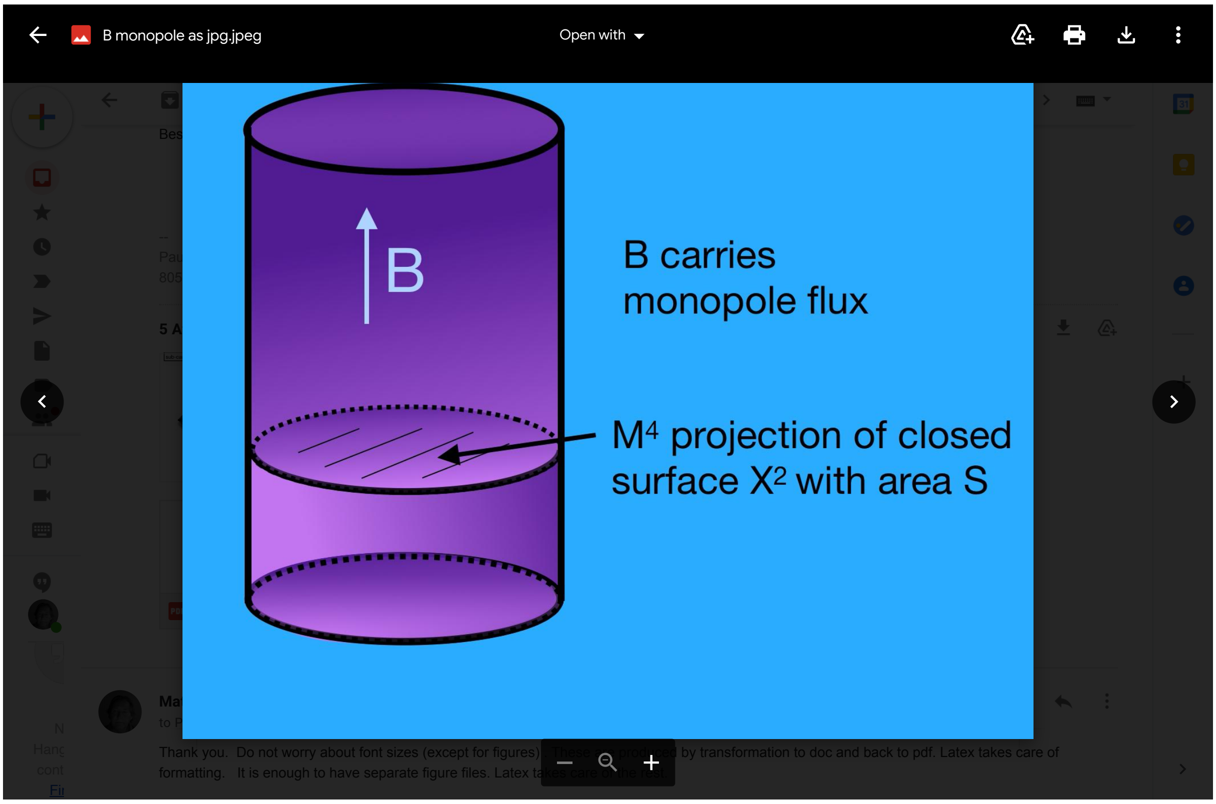

Figure 15: The M^4 projection of a closed surface X^2 with area S defining the cross section for monopole flux tube. Flux quantization $e\oint B\cdot dS = eBS = kh$ at single sheet of n-sheeted flux tube gives for cyclotron frequency $f_c = ZeB/2\pi m = khZ/2\pi mS$. The variation of S implies frequency modulation.

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