TGD as it is towards end of 2024: part II

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Abstract

This article is the second part of the article trying to give a rough overall view about Topological Geometrodynamics (TGD) as it is towards the end of 2024. In the first part of the article the two visions of TGD: physics as geometry and physics as number theory were discussed. The second part is devoted to the details of $M^8 - H$ duality relating these two visions, to zero energy ontology (ZEO), and to a general view about scattering amplitudes.

Classical physics is coded either by the space-time surfaces of H or by 4-surfaces of M^8 with Euclidean signature having associative normal space, which is metrically M^4. $M^8 - H$ duality as the analog of momentum-position duality relates geometric and number theoretic views. The pre-image of causal diamond cd, identified as the intersection of oppositely directed light-cones, at the level of M^8 is a pair of half-light-cones. $M^8 - H$ duality maps the points of cognitive representations as momenta of fermions with fixed mass m in M^8 to hyperboloids of $CD \subset H$ with light-cone proper time $a = h_{eff}/m$.

Holography can be realized in terms of 3-D data in both cases. In H the holographic dynamics is determined by generalized holomorphy leading to an explicit general expression for the preferred extremals, which are analogs of Bohr orbits for particles interpreted as 3-surfaces. At the level of M^8 the dynamics is determined by associativity of the normal space..

Zero energy ontology (ZEO) emerges from the holography and means that instead of 3-surfaces as counterparts of particles their 4-D Bohr orbits, which are not completely deterministic, are the basic dynamical entities. Quantum states would be superpositions of these and this leads to a solution of the basic problem of the quantum measurement theory. It also leads also to a generalization of quantum measurement theory predicting that in the TGD counterpart of the ordinary state function reduction, the arrow of time changes.

A rather detailed connection with the number theoretic vision predicting a hierarchy of Planck constants labelling phases of the ordinary matter behaving like dark matter and ramified primes associated with polynomials determining space-time regions as labels of p-adic length scales. There has been progress also in the understanding of the scattering amplitudes and it is now possible to identify particle creation vertices as singularities of minimal surfaces associated with the partonic orbits and fermion lines at them. Also a connection with exotic smooth structures identifiable as the standard smooth structure with defects identified as vertices emerges.

Contents

1 Introduction

The purpose of this article is to give a rough overall view of the basic ideas of Topological Geometrodynamics (TGD) as it is now (2024). I wrote a similar summary 3 years ago. Several new ideas have emerged during these years, the realization of some ideas has simplified dramatically, and some ideas have turned out to be obsolete.

In the first part of the chapter the two visions of TGD: physics as geometry and physics as number theory are discussed. The second part is devoted to $M^8 - H$ duality relating these two visions, to zero energy ontology (ZEO), and to a general view about scattering amplitudes.

1.1 Zero Energy Ontology

In Zero Energy Ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 4** and **Fig. 5**).

Quantum jumps occur between these superpositions and the basic problem of the standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 6**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions [L12].

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary and the states at it remain unchanged (Zeno effect). The sequence of "small" state function reductions (SSFRs) defined the TGD counterpart of the generalized Zeno effect, would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces and would also give rise to a conscious entity, self.

This means considerable progress in the understanding of the quantum measurement theory based on ZEO [L17, L35, L44] [K8].

The first new result is that Negentropy Maximization Principle [L42] implying evolution follows as an analog of the second law. In the sequence of quantum jumps the algebraic complexity, which is measured as the dimension of extension of rationals associated with the polynomials associated with the singularities, is bound to increase in a statistical sense.

The second new result [L44] is a quantum formulation of the ZEO. Zero energy states within a single CD as an analog of a perceptive field and containing space-time surfaces is generalized so that quantum states also involve a wave function in the space of CDs. The moduli space of CDs is finite-dimensional and maximally symmetric and forms the backbone of WCW in the sense that each space-time surface satisfying holography is within a particular CD. This leads also to a new view of Poincare symmetry allowing to overcome the problems due to the fact that CD itself is not Poincare invariant.

1.2 Progress in the understanding of the relationship of $M^8 - H$ duality and holography = holomorphy vision

TGD develops by explaining what TGD is and also this work led to considerable progress in several aspects of TGD, in particular $M^8 - H$ duality and its relation to holography = holomorphy vision.

1. The understanding of the details of the $M^8 - H$ duality forces us to modify the earlier view [L20, L21, L43] to a much simpler vision [L47]. The notion of causal diamond (CD) is central to zero energy ontology (ZEO) and emerges as a prediction at the level of H. The pre-image of CD under $M^8 - H$ duality in M^8 is a region bounded by two mass shells in the normal space of $y \in Y^4 \subset M^8$, which itself is an Euclidean region.

   $M^8 - H$ duality maps the points of cognitive representations defined by the points of Y^4 with coordinates as algebraic integers in the algebraic extension of rationals and identified as momenta of fermions with a fixed mass squared in M^8 to either boundary of CD in H.

2. The emergence of holography = holomorphy principle [L49] forces a profound modification of the ideas about $M^8 - H$ duality and number theoretical vision and have given a strong motivation for this article. There are tensions between the holography= holomorphy vision and number-theoretic vision and in this article a more precise form of the holography = holomorphy vision resolving the tensions is discussed.

3. Galois confinement for physical states at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that fermion momenta using a suitable unit determined by CD are algebraic integers but integers for Galois singlets just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

1.3 The description of particles and particle interactions in the TGD framework

The TGD based description of particles [L39] and particle interactions [L53, L40] has developed considerably during the last yeaars and the new view deserves a separate section.

Several key ideas of quantum TGD distinguish between TGD and QFTs.

1. The basic problem of QFT is that it involves only an algebraic description of particles. An explicit geometric and topological description is missing but is implicitly present since the algebraic structure of QFTs expresses the point-like character of the particles via commutation and anticommutation relations for the quantum fields assigned to the particles.

   In the string models, the point-like particle is replaced by a string, and in the string field theory, the quantum field Ψ(x) is replaced by the stringy quantum field Ψ(string), where "string" corresponds a point in the infinite-D space of string configurations (say loop space). The interpretation is as a second quantization of string theory. This approach is rather baroque since one must introduce a non-linear action principle in the string space. This however allows to understand M^4 as the configuration space for the positions of a point-like particle.

   In TGD, the quantum field Ψ(x) is replaced by a formally *classical* spinor field Ψ (Bohr orbit). The 4-D Bohr orbits are preferred extremals of classical action satisfying holography forced by general coordinate invariance without path integral and represent points of the "world of classical worlds" (WCW). The components of Ψ correspond to multi-fermion states, which are pairs of ordinary 3-D many-fermion states at the boundaries of causal diamond (CD).

   The gamma matrices of the WCW spinor structure are linear combinations of the fermionic oscillator operators for the second quantized free spinor field of H. They anticommute to the WCW metric, which is uniquely determined by the maximal isometries for WCW guaranteeing the existence of the spinor connection. Physics is unique from its existence, as implied also by the twistor lift and number theoretic vision and of course, by the standard model symmetries and fields.

   Also the notion of induced spinor fields as a restriction of spinor fields to space-time surface is involved and the induced spinor fields satisfy the modified Dirac equation as the analog of massless Dirac equation [L50].

2. In TGD, the notion of a classical particle as a 3-surface moving along 4-D "Bohr orbit" as the counterpart of world-line and string world sheet is an exact aspect of quantum theory at the fundamental level. The notions of classical 3-space and particle are unified. This is not the case in QFT and the notion of a Bohr orbit does not exist in QFTs. TGD view of course conforms with the empirical reality: particle physics is much more than measuring of the correlation functions for quantum fields.

   Quantum TGD is a generalization of wave mechanics defined in the space of Bohr orbits. The Bohr orbit corresponds to holography realized as a generalized holomorphy generalizing 2-D complex structure to its 4-D counterpart, which I call Hamilton-Jacobi structures (see this). Classical physics becomes an exact part of quantum physics in the sense that Bohr orbits are solutions of classical field equations as analogs of complex 4-surfaces efined as roots of two generalized analytic functions in $H = M^4 \times CP_2$ endowed with generalized complex coordinates. The space of these 4-D Bohr orbits gives the WCW (see this), which corresponds to the configuration space of an electron in ordinary wave mechanics.

   There is no need for the second quantization to describe many particle systems as in the case of wave mechanics since the many-particle states are described topologically as unions of disjoint 3-surfaces and unions of partonic orbits inside them.

3. The second quantized spinor fields of H are needed to define the spinor structure in WCW. The spinor fields of H are the free spinor fields in H coupling to its spinor connection of H. The Dirac equation can be solved exactly and second quantization is trivial and one avoids the usual problems caused encountered for curved space-times, such as the non-existence of spinor structure and existence of several spinor structures encountered already in QCD lattice calculations where the periodic boundary conditions effectively replace the topology of the Minkowski space with that of 4-torus.

   This determines the fermionic propagators in H and induces them at the space-time surfaces. The propagation of fermions is thus trivialized. All that remains is to identify the vertices.

4. At the fermion level, all elementary particles, including bosons, can be said to be made up of fermions and antifermions, which at the basic level correspond to light-like world lines on 3-D parton trajectories, which are the light-like 3-D interfaces of Minkowski spacetime sheets and the wormhole contacts connecting them.

   The light-like world lines of fermions are boundaries of 2-D string world sheets and they connect the 3-D light-like partonic orbits bounding different 4-D wormhole contacts to each other. The 2-D surfaces are analogues of the strings of the string models.

5. In TGD, classical boson fields are induced fields and no attempt is made to quantize them. Bosons as elementary particles are bound states of fermions and antifermions. This is extraordinarily elegant since the expressions of the induced gauge fields in terms of embedding space coordinates and their gradients are extremely non-linear as also the action principle. This makes standard quantization of classical boson fields using path integral or operator formalism a hopeless task.

   There is however a problem: how to describe the creation of a pair of fermions and, in a special case, the corresponding bosons, when there are no primary boson fields? Can one avoid the separate conservation of the fermion and the antifermion numbers?

2 $M^8 - H$ duality

There are several observations motivating $M^8 - H$ duality [L20, L21, L43, L47] (see **Fig. 3**).

1. There are four classical number fields: reals, complex numbers, quaternions, and octonions with dimensions $1, 2, 4, 8$. The dimension of the embedding space is $D(H) = 8$, the dimension of octonions. Spacetime surface has dimension $D(X^4) = 4$ of quaternions. String world sheet and partonic 2-surface have dimension $D(X^2) = 2$ of complex numbers. The dimension $D(string) = 1$ of string is that of reals.

2. Isometry group of octonions is a subgroup of automorphism group G_2 of octonions containing $SU(3)$ as a subgroup. $CP_2 = SU(3)/U(2)$ parametrizes quaternionic 4-surfaces containing a fixed complex plane. $SU(3)$ could be interpreted as color group acting as a subgroup of G_2 in M^8 and as an isometry group of CP_2.

3. There was also the intuition that TGD could allow two dual descriptions: physics as geometry and physics as number theory. This vision also conforms with the philosophy behind Langlands duality. Number theoretical description could be based on algebraic concepts, in particular roots of polynomials in some sense.

This raised the question whether M^8 and $H = M^4 \times CP_2$ could provide alternative dual descriptions of physics (see **Fig. 3**)?

2.1 About the evolution of the concept of $M^8 - H$ duality

$M^8 - H$ duality [L20, L21, L43, L47] can be seen as a number-theoretic analog of momentum position duality implied by the replacement of point-like particles with 3-surfaces. $M^8 - H$ duality also relates geometric and number theoretic vision of quantum TGD and could serve as a physical realization of Langlands duality [L38]. Note that all single-fermion states are predicted to be massless in the 8-D sense and therefore their 8-D momenta are expected to reside at the 7-D light-cone of M^8.

The development of M^8-H duality has been a rather tortuous process and has involved several wrong tracks.

2.1.1 The problems with the original forms of $M^8 - H$ duality

The very first form of $M^8 - H$ duality mapped $M^4 \subset H$ to a quaternionic tangent space of $M^4 \subset M^8$: octonions indeed correspond metrically to M^8 with respect to the number theoretic norm $RE(o^2)$ and induce a map of $X^4 \subset H$ to $Y^4 \subset M^8$. The cold shower [L20, L21] was that the distribution of quaternionic tangent spaces is not in general integrable to a 4-surface Y^4 and it seems that only trivial associative surfaces (M^4) are possible. The conclusion was that tangent space of Y^4 cannot be quaternionic.

This forced the conclusion that the normal spaces of Y^4 must be quaternionic, one could call this co-associativity [L20, L21]. The distribution of normal spaces $N(y)$, $y \in Y^4$ is indeed known to be always integrable, meaning that it defines a 4-surface $Y^4 \subset M^8$. The assumption that normal spaces are quatertionic together with the assumption $X^4 \subset H$ is mapped to $Y^4 \subset M^8$ forced the complexification of M^8. I failed to realize that $M^4 \subset H$ could be mapped to the normal space $N(y)$ for each point $y \in Y^4$!

The norm was defined by o^2 and this forced a complexification of M^8 by introducing a commutative imaginary unit i in order to get the Minkowskian signature. It was necessary to restrict the subspaces so that the octonionic coordinates were either real or imaginary (with respect to i). The first problem was that all metric signatures became possible and the question was how to get rid of them. The second problem was that the identification of $M^4 \subset M^8$ was highly non-unique and it it was not clear whether this had some physical meaning.

The form of $M^8 - H$ duality discussed in [L20, L21] was based on idea about algebraization of physics at the M^8 side and assumed the algebraic continuation of polynomials $P(z)$ of complex variable z for which the imaginary unit commutes with octonionic imaginary units to polynomials $P(0)$ of a complexified octonion.

1. $P(z)$ was assumed to have rational (equivalently integer-) coefficients. An even stronger assumptions was that the number of the coefficients is smaller than the degree P. The assumption that either real or imaginary part of P in quaternionic sense vanishes for Y^4, led to the proposal that the roots of $P(t)$ correspond to mass squared values m^2 defining mass shells in $M^4 \subset M^8$ as $p_0^2 - \sum p_i^2 = m^2$.

   This assumption had very nice implications concerning the number theoretic vision. This also led to the notion of associative holography. One identifies the 3-surfaces defining the boundary data of the holography to be mass shells $H^3 \subset M^4 \subset M^8$ and continues this data to associative 4-surfaces. The problem was that these holographic data look quite too simple. Deformations in normal direction would seem to be necessary. The definition of the complex variant of H^3 is problematic idea since the extremely beautiful symmetries of H^3 might be lost.

2. Perhaps the most serious problem of the approach was that complex roots correspond to complex mass squared values so that one must complexity M^8. This led to rather complicated constructions [L20, L21, L43]. The assumption that the roots are real could solve this problem but looks somewhat ad hoc.

3. The image of $M^4 \subset H$ in M^8 was identified as a co-quaternionic sub-space of complexified octonions M^8_c. The octonionic number theoretic norm is defined without the conjugation with respect to i. There are many choices for the subspace for which the octonionic coordinates are real or purely imaginary and all signatures of the number theoretic norm defined by o^2 are possible. This also led to interpretational problems: to what what particular $M^4 \subset M^8$ does $M^4 \subset H$ correspond to? Does this choice have a physical significance?

2.1.2 The recent view of $M^8 - H$ duality

In [L47], I introduced a dramatic simplification of the earlier version of the $M^8 - H$ duality [L20, L21, L43] allowing to get rid of the complexification of M^8.

The trick is to define the number theoretic norm, not as $o\overline{o}$, but as the real part $RE(o^2)$ of o^2 used already in the very first trical.The real part here means the part of o^2 proportional to the octonionic real unit. This definition applies also to complex numbers and quaternions as subspaces of octonions. This norm is Minkowskian and allows the identification of octonions as M^8 in the metric sense.

In this interpretation, the projections of the points of H to $M^4 \subset H = M^4 \times CP_2$ correspond by M^8-H duality to the points of $Y^4 \subset M^8 = M^4 \times E^4$ such that the M^4 projection m of the point of X^4 associated with CP_2 point s is mapped by inversion $I : m^k \rightarrow \hbar_{eff} m^k/m_l m^l$ to a point of the quaternionic normal space $N(y(s))$ as its preferred point and naturally projects to the corresponding point of Y^4. Y^4 is Euclidian with respect to the number theoretic norm already described.

Number theoretic holography would be realized by requiring that the mass squared value assigned to the M^4 point p of the normal space $N(y)$, $y \in Y^4$, is equal Euclidean mass squared value y^2: $p^2 = y^2$. Therefore Y^4 belongs to the 7-D light-cone of octonions with Minkowskian number-theoretic norm: this is in accordance with the 8-D light-likeness realized for the spinor modes of H. This correspondence would define an analogue of Wick rotation and pose a constraint on the number theoretic holography. The boundary data given at these Euclidian mass shells S^3 and associativity (quaternionic normal space) would serve as a dynamical principle.

Consider first a simplified view of M^8-H duality as I understand it now, assuming that the M^4 projection of X^4 is 4-D.

1. The idea is that the points of $X^4 \subset H$ are mapped to the points of $Y^4 \subset M^8$. The M^4 projection of X^4 points are mapped to the normal Minkowskian normal space of Y^4. The CP_2 projection associated with a given point of Y^4 parametrizes the normal spaces $N(y)$ for Y^4. The assumption that the normal spaces are quaternionic and contain a Minkowskian 2-D commutative space implies that they can be parametrized by CP_2. The normal spaces and 2-D subspaces form integrable distributions and determine Y^4 by number theoretic holography using the integrable distribution of Minkowskian normal spaces as holographic data located at 3-D surfaces. The tangent spaces of Y^4 have an Euclidian number theoretic metric. The points of Y^4 are in 1-1 correspondence with those of CP_2 except at the singularities at which the normal space is not unique.

2. The 4-surface $Y^4 \subset M^8$ is Euclidean. The points of Y^4 have quaternionic normal space $N(y)$ isomorphic to $M^4 \subset M^8$ containing $M^2(y)$, which is complex, commutative subspace of octonions.

   $N(y)$ ontains the M^4 point p associated with the point of CP_2 as a preferred point having an interpretation as 4-momentum. This point is "active" if there is a fermion at the corresponding point of X^4. One has a geometric representation of the many-fermion state in Y^4 mapped to the points of M^4 projection of X^4 by M^8-H duality.

3. The new form of M^8-H duality requires a lift of the point $y \subset Y^4$ to a point p of $N(y)$, which is mapped by inversion to $M^4 \subset H$. Multiplication of y with an imaginary unit e of Y^4 would perform the lift but can one cannot choose e uniquely. The uniqueness is however not necessary except at the active points containing a fermion.

   The choice of the basis of quaternionic units for the quaternionic normal space is fixed up to local $U(2)$ rotation. The same is true for the basis of the complement. Could the interpretation be in terms of the M^8 counterpart of the electroweak gauge group? Color symmetries would correspond to the $SU(3) \subset G_2$ of octonionic automorphisms so that the standard model symmetries would be realized number-theoretically. The inverse of the lift would allow to map the points $M^4 \subset H$ to the points of Y^4 and realize the correspondence between points of M^4 and CP_2 for the surfaces representable as graphs of a map $M^4 \rightarrow CP_2$.

   If the point of the normal space is fixed to be the momentum p of a fermion located at y by M^8-H duality, the choice of e is unique. The momenta of the many-fermion state would fix the units e at the Y^4 locations of the fermions. The gauge would be fixed and $U(2)$ gauge invariance would be broken only where the fermions are. This conforms with the view that various symmetries are broken only at the singularities where the minimal surface property fails.

4. The fermionic momenta associated with the active points y assign mass shells $p^2 = m^2$, that is hyperboloid H^3 in the normal space $N(y)$ and points of 3-sphere S^3 in the tangent space $T(y)$ of Y^4. The $M^8 - H$ images of these in H define hyperbolic 3-space $a = h_{eff}/m$ in H with a constant value of light-cone proper time. Earlier I talked of these hyperboloids as "very special moments of time in the life of self" [L13]. This interpretation might make sense, if the time values correspond to the moments of time when the partonic two-surface was created in a splitting of a monopole flux tube (see **Fig. 7**) having interpretation as a creation of fermion pair. The value of a would correspond to the point at which splitting occurs, the "birth day" of the fermion.

   These points correspond to singularities of X^4 as a minimal surface. They could also correspond to defects of an ordinary smooth structure equivalent with an exotic smooth structure [L53, L49]. There would also be non-determinism associated with these moments of time. The touching of parallel flux tubes at opposite Minkowskian space-time sheets would correspond to touching, possibly leading to the splitting.

5. The spinor modes of fermion in CP_2, can be mapped to the spinor modes in Y^4 for each fermionic momentum. Number theoretical vision suggests that the momentum components are real algebraic integers of the extensions of rationals associated with the space-time surface in question.

This picture applies when the M^4 projection of X^4 is 4-D. If this is not the case the situation is more complex.

1. The ideal CP_2 extremal for which M^4 coordinates are constant, corresponds to a singularity, where the normal space at the point of M^8 is not unique: the normal open spaces span entire CP_2 one has what alebraic geometers call blow up and it occurs often for algebraic surfaces. The vertex of a cone is the basic example: in this case tangent and normals spaces are not unique.

   For deformed CP_2 extremals M^4 projection is a 1-D light-like geodesic line in $M^4 \subset H$ and also in M^8. Along this curve, the normal spaces form give rise to a 3-dimensional surfaces of CP_2.

2. Cosmic strings also correspond to such a singularity: in a 2-D string world sheet $X^2 \subset M^4$, the normal spaces at a given point form a 2-D complex manifold of CP_2.

3. At singularities at which the normal space of Y^4 is not unique, there are additional conditions on the allowed spinor modes since the spinor mode must have the same value for all normal spaces involved. The vanishing of the allowed spinor modes at these points would allow to satisfy these condition.

4. The number-theoretic quantization of M^4 momenta requires that the momentum components are real integers in the algebraic extenssion of the rationals related to the region of X^4 considered. The momentum unit is determined by the size scale of the causal diamond (CD).

What could be the physical interpretation of Y^4?

1. Y^4 can be sliced by the images of $r=$ constant 3-spheres $S^3 \subset CP_2$. Could the time evolution in X^4 with respect to the light-cone proper time of $M^4 \subset H$ have as an analog the evolution of the CP_2 projection with respect to the radial coordinate r of CP_2 defining a slicing of Y^4 by $M^8 - H$ duality? Or can one speak of time evolution below the scale of causal diamond CD, which implies temporal non-locality below its scale.

2. Positive and negative energy states at the half-cones of CD would be mapped to the opposite light-cones in the Minkowskian normal space of Y^4. This brings in mind Wick rotation as the Euclidization trick used in quantum field theories. Could $M^8 - H$ duality define Y^4 as a kind of Euclidization of X^4? If so, one would have both M^4 and CP_2 perspective of the dynamics and also mixed perspectives (cosmic strings). The failure of M^4- and/or CP_2 projection to be 4-D would force mixed perspective.

2.1.3 Hamilton-Jacobi structure and Kähler structure of $M^4 \subset H$ and their counterparts in $M^4 \subset M^8$

The Kähler structure of $M^4 \subset H$, forced by the twistor lift of TGD [L33, L34], has deep physical implications and seems to be necessary. It implies that for Dirac equation in H, modes are eigenstates of only the longitudinal momentum and in the 2 transversal degrees of freedom one has essentially harmonic oscillator states [L31, L29, L50], that is Gaussians determined by the 2 longitudinal momentum components. For real longitudinal momentum the exponents of Gaussians are purely imaginary or purely real.

The longitudinal momentum space $M^2 \subset M^4$ and its orthogonal complement E^2 is in a preferred role in gauge theories, string models, and TGD. The localization of this decomposition leads to the notion of Hamilton-Jacobi (HJ) structure of M^4 [L41] and the natural question is how this relates to Kähler structures of M^4. At the level of H spinors fields only the Kähler structure corresponding to constant decomposition $M^2 \oplus E^2$ seems to make sense and this raises the question how the H-J structure and Kähler structure relate.

TGD suggests the existence of two geometric structures in M^4: HJ structure and Kähler structure. It has remained unclear whether HJ structure and Kähler structure with covariantly constant self-dual Kähler form are equivalent notions or whether there are several H-J structures accompanying the Kähler structure. The most economic view supported by the new view of M^8_H duality is that H-J structure is a Kähler structure accompanied by both complex structure and symplectic structure.

Now complex structure would be 4-D generalization of 2-D complex structure to which one can assign hypercomplex coordinate and complex coordinate, which play a key role in the explicit formulation of the general solution of field equations in terms of two analytic functions. Kähler structure and H-J structure would not be "primordial" properties of M^4 but emerge as effective structures of both M^4 and X^4 via the solution ansatz. One can imagine that one can assign to a given space-time topology a moduli space of H-J structures. In the following I will argue that H-J structure, that is generalized Kähler structure defines as a by product symplectic structure and that the properties of $X^4 \subset H$ determined by $M^4 - H$ duality make it natural to choose both generalized complex coordinates and symplectic coordinates as alternative coordinates for M^4.

Consider first what H-J structure and Kähler structure could mean in H.

1. The H-J structure of $M^4 \subset H$ would correspond to an integrable distribution of 2-D Minkowskian sub-spaces of M^4 defining a distribution of string world sheets $X^2(x)$ and orthogonal distribution of partonic 2-surfaces $Y^2(x)$. Could this decomposition correspond to self-dual covariantly Kähler form in M^4?

   What do we mean with covariant constancy now? Does it mean a separate covariant constancy for the choices of $M^2(x)$ and $Y^2(x)$ or only of their sum, which in Minkowski coordinates could correspond to a constant electric and magnetic fields orthogonal to each other?

2. The non-constant choice of $M^2(x)$ ($E^2(x)$) cannot be covariantly constant. One can write $J(M^4) = J(M^2(x)) \oplus J(E^2(x)$ corresponding to decomposition to electric and magnetic parts. Constancy of $J(M^2(x)$ would require that the gradient of $J(M^2(x)$ is compensated by the gradient of an antisymmetric tensor with square equal to the projector to $M^2(x)$. Same condition holds true for $J(E^2(x))$. The gradient of the antisymmetric tensor would be parallel to itself implying that the tensor is constant.

3. H-J structure can only correspond to a transformation acting on J but leaving $J_{kl}dm^kdm^l$ invariant. One should find analogs of local gauge transformations leaving J invariant. In the case of CP_2, these correspond to symplectic transformations and now one has a generalization of the notion. The M^4 analog of the symplectic group would parameterize various decompositions of $J(M^4)$.

   Physically the symplectic transformations define local choices of 2-D space $E^2(x)$ of transversal polarization directions and longitudinal momentum space M^2 emerging in the construction of extremals of Kähler action.

4. For the simplest Kähler form for $M^4 \subset H$, this decomposition in Minkowski coordinates would be constant: orthogonal constant electric and magnetic fields. This Kähler form extends to its number theoretical analog in M^8. The local $SU(3)$ element g would deform M^4 to $g(M^4)$ and define an element of local CP_2 defining $M^8 - H$ duality. g should correspond to a symplectic transformation of M^4.

Consider next the number theoretic counterparts of Hamilton-Jacobi and Kähler structures of $M^4 \subset H$ in $M^4 \subset M^8$.

1. In M^4 coordinates the simplest H-J structure would correspond to a constant $M^2 \times E^2$ decomposition of M^4. In M^4 coordinates Kähler structure would correspond to constant E and B orthogonal to each other. For a general H-J structure the spaces $M^2(x)$ and $E^2(x)$ define integrable distributions. Symplectic transformations give various representations of this structure as H-J structures.

   This would give rise to a number theoretically defined slicing of $X^4 \subset H$ by string world sheets X^2 and partonic 2-surfaces Y^2 orthogonal with respect to the octonionic inner product for complexified octonions.

2. The number theoretic analog of H-J structure makes sense also for $Y^4 \subset M^8$ obtained from the distribution of quaternionic normal spaces $N(y)$, $y \in Y^4$, containing 2-D commutative sub-space at each point. There is a hierarchy of CDs and the choices of these structures are naturally parameterized by G_2.

3. In $M^8 - H$ duality defined by $g(p) \subset SU(3)$ assigns a point of CP_2 to a given point of M^4. $g(p)$ maps the number theoretic H-J to H-J in $M^4 \subset M^8$. The space-time surface itself - that is $g(p)$ - defines these symplectic coordinates and the local $SU(3)$ element g would naturally define this symplectic transformation.

4. For $X^4 \subset M^8$, g reduces to a constant color rotation satisfying the condition that the image point is $U(2)$ invariant. Unit element is the most natural option. This would mean that g is constant at the mass and energy shells corresponding to the roots of P and the mass shell is a mass shell of M^4 rather than some deformed mass shell associated with images under $g(p)$.

   This alone does not yet guarantee that the 4-D tangent space corresponds to M^4. The additional physically very natural condition on g is that the 4-D momentum space at these mass shells is the same. $M^8 - H$ duality maps these mass shells to the mass shells of $cd \subset M^4$ (CD= $cd \times CP_2$). This conforms with the identification of zero energy states as pairs of 3-D states at the boundaries of CD.

2.2 Uncertainty Principle and $M^8 - H$ duality

The detailed realization of $M^8 - H$ duality involves still uncertainties. The quaternionic normal spaces $N^2(y)$ containing complex 2-space $M^2(y)$ are parametrized by points of CP_2. One can map the normal space to a point of CP_2.

The tough problem has been the precise correspondence between M^4 points in M^8 and $M^4 \times CP_2$ and the identification of the sizes of causal diamonds (CDs) in M^8 and H. The identification is naturally linear if M^8 is analog of space-time but if M^8 is interpreted as momentum space, the situation changes.

The option discussed in [L20, L21, L43] maps mass hyperboloids to light-cone proper time $a = constant$ hyperboloids and it has turned out that this correspondence does not correspond to the classical picture suggesting that a given momentum in M^8 corresponds in H to a geodesic line emanating from the tip of CD.

2.2.1 Two alternatives for $M^8 - H$ duality in M^4 degrees of freedom

The following two proposals for $M^8 - H$ duality for the details in M^4 degrees of freedom relies on the intuition provided by UP.

1. Both proposals involve a map of point $y \in Y^4$ to a point of its normal space identifiable as momentum space. This map is a multiplication of y by a co-quaternionic imaginary unit e to give a momentum p in the Minkowskian normal space $N(y)$. This takes y to a point $p \in N(y)$ with mass squared $p = m^2$ equal to the Euclidean mass squared $y^2 = m^2$ (norm is $RE(y^2)$) so that each y would correspond to its mass shell in $N(y)$. This corresponds to 8-D light-likeness.

   The choice of e is unique apart from a local $U(2)$ gauge rotation. If p is momentum of zero energy state, one can say that y is an active point and p is fixed so that e is highly uniquely determined and one can say that gauge invariance is broken. Depending on the sign of energy $E = p^0$, the momentum belongs to the positive energy or negative energy half-cone in $N(y)$.

2. The classical version of UP suggests that the point p is mapped inversion $p \rightarrow h_{eff}p^k/p \cdot p = m^k$ to the hyperboloid H^3 with $a = \hbar_{eff}/m$ $cd \subset H$. The image m^k can be visualized as a vector located at either tip of the CD determined by the sign of the energy $E = p^0$. Light-like momenta would be mapped by the map $m^k = \hbar_{eff}p^k/E^2$, where $E = p^0$ is energy, to the light-cone boundary associated with the *cd*.

These conditions leave two options. Both options are consistent with the constraint that the identification of the point $p^k \in X^4 \subset M^8$ should classically correspond to a geodesic line $m^k = p^k\tau/m$ ($p^2 = m^2$) in H which in Big Bang analogy should go through the tip of the $cd \subset H$.

Global option: The hyperboloids $a = \hbar_{eff}/m$ for fermions present in the state are inside single *cd* or the half-cone of single *cd* assignable to the entire X^4. The size of *cd* must be so large that all momenta are mapped to the half-cone or at least to the entire *cd*. In presence of only massive fermions, the largest value of $a = \hbar_{eff}/m$ determines an upper limit for the size scale L of *cd*. If there are light-like fermionic momenta, the largest value of $T = \hbar_{eff}/E$ gives a second lower limit for L. The presence of nearly massless particles such as gravitons and massless gauge bosons would make possible large sizes for CDs.

The large values of gravitational Planck constant $\hbar_{gr} = GMm/\beta_0$ [E1] would also increase the size scale of *cd* for massive particles and give $a = GM/\beta_0$, which is equal to the gravitational Compton length Λ_{gr}, which does not depend on particle. For the Sun it is roughly one half of the radius of Earth [L36]. For the electric Planck constant $\hbar_{em} = Qze^2/\beta_0$ for a particle with charge ze one as $a = \hbar_{em} = Qze^2/\beta_0 m$. The fact that h_{eff} depends on the product of masses (charges) suggests infinitesimal symmetries affecting several particles simultaneously. Yangian symmetries are such symmetries [B1] [L7].

Local option: The point is mapped to the second light-cone boundary of *cd*. This would determine the size of the *cd*. Each fermion would have its own personal *cd*. If the active fermions correspond to singular points at partonic 2-surfaces defining the vertices as singular 2-surfaces where the minimal surface property fails this option could make sense and would provide each fermion with its personal *cd* as a kind of perceptive field.

Could the values $a = \hbar_{eff}/m$ for massive particles and M^4 times $T = \hbar_{eff}/E$ for light-like momenta defining 3-D hyperplanes of M^4 have a special physical significance? These parameters are analogous to Compton times and could define a lower bound for quantum coherence length and time they do relate to the size scale of *cd*.

The TGD inspired theory of consciousness motivated the interpretation of these times as "very special moments in the life of self". If the values of a correspond to hyperbolic 3-spaces H^3 containing the singular points assignable assignable to the partonic 2-surfaces X^2 identifiable as reaction vertices as analogs of frames for 4-dimensional soap films as minimal surfaces, and if there is indeed quantum criticality and non-determinism associated with X^2 (this is the case of ordinary 2-D minimal surfaces in E^3 [L32]), this interpretation could make sense.

2.2.2 Does Uncertainty Principle require a delocalization in H or in X^4?

One can argue that Uncertainty Principle (UP) requires more than the naive condition $L = \hbar_{eff}/m$ on the size scale of CD. I have already mentioned several approaches to the problem: they could be called inertial and gravitational representations.

1. The inertial representations assigned to the particle as a space-time surface (holography) an analog of plane wave as a superposition of the translates of the space-time surface

related to its M^8 counterpart by $M^8 - H$ duality: this is natural at the level of WCW. This requires a delocalization of space-time surfaces and of the cd of the particle in H. The natural constraint leading to a violation of translational invariance is that the translates are inside the larger cd containing the space-time surface and associated sub-cd.

2. The gravitational representation relies on the analog of the braid representation of isometries in terms of the projections of their flows to the space-time surface. This does not require delocalization in H since it occurs in X^4. It is not clear whether the mass squared spectrum assignable to the gravitational representation coincides with the mass spectrum in the inertial case.

   Another gravitational representation would correspond to partial waves of H-spinors inside, CD which would naturally correspond to the irreducible representation for the Lorentz group leaving either tip of the CD invariance. The mass spectrum would be the same and this might have interpretation as a manifestation of the Equivalence Principle.

3. The moduli space for cds suggests that the quantum states in WCW are characterized by a wave function in the moduli space of cd. These degrees of freedom including scalings, translations, Lorentz boosts for the center point of the CD and for either tip of the CD. Also special conformal transformations can be considered. One can imagine plane waves in CD translational degrees of freedom. In "small" state function reductions (SSFRs)a localition at least in scaling degrees of freedom would occur giving rise to a well-defined value of the geometric time as the distance between the tips of cd or equivalently as the size scale of CD. This representation could be equivalent with the inertial representation.

Consider first the inertial representation. The intuitive idea that a single point in M^8 corresponds to a discretized plane wave in H in a spatial resolution defined by the total mass at the passive boundary of CD. UP requires that this plane wave should be realized at the level of H and also WCW as a superposition of shifted space-time surfaces defined by the above correspondence.

1. The basic observation leading to TGD is that in the TGD framework a particle as a point is replaced with a particle as a 3-surface, which by holography corresponds to 4-surface.

   Momentum eigenstate corresponds to a plane wave. Now planewave could correspond to a delocalized state of 3-surface - and by holography that of 4-surface - associated with a particle.

   A generalized plane wave would be a quantum superposition of shifted space-time surfaces inside a larger CD with a phase factor determined by the 4-momentum. $M^8 - H$ duality would map the point of M^8 containing an object with momentum p to a generalized plane wave in H. Periodic boundary conditions are natural and would force the quantization of momenta as multiples of momentum defined by the larger CD. Number theoretic vision requires that the superposition is discrete such that the values of the phase factor are roots of unity belonging to the extension of rationals associated with the space-time sheet. If momentum is conserved, the time evolutions for massive particles are scalings of CD between SSFRs are integer scalings. Also iterated integer scalings, say by 2 are possible.

2. This would also provide WCW description. Recent physics relies on the assumption about single background space-time: WCW is effectively replaced with M^4 since 3-surface is replaced with point and CP_2 is forgotten so that one must introduce gauge fields and metric as primary field variables.

As already discussed, the gravitational representation would rely on the lift/projection of the flows defined by the isometry generators to the space-time surface and could be regarded as a "subjective" representation of the symmetries. The gravitational representation would generalize braid group and quantum group representations.

The condition that the "projection" of the Dirac operator in H is equal to the modified Dirac operator, implies a hydrodynamic picture. In particular, the projections of isometry generators are conserved along the lifted flow lines of isometries and are proportional to the projections of Killing vectors. QCC suggests that only Cartan algebra isometries allow this lift so that each choice of quantization axis would also select a space-time surface and would be a higher level quantum measurement.

2.3 Octonionic Dirac equation predicts the presence of both quarks and leptons

M^8-H duality should be generalized also to fermionic sector. The octonionic Dirac equation allows a second perspective on associativity [L21]. The recent view of the M^8-H duality [L47] forces however to modify the earlier proposal. Whether both leptons and quarks appear as fundamental fermion in TGD or whether leptons could be identified as bound states of quarks has been a long standing open question [L26], which remained open in [L31]. The recent view answers this question.

Consider first the general vision.

1. Free field theory is algebraic at the level of momentum space and $O = M^8$ represents 8-D momentum space. Therefore also the octonionic Dirac equation should be algebraic as an analog of the momentum space variant of the ordinary Dirac equation. The modes of spinor fields of H (quarks and leptons) should correspond to the modes of spinor fields of M^8 having the same 4-D mass spectrum. In the 8-D sense the spinor modes of both H and M^8 are massless spinors. Also the induction of the spinor structure to the 4-surface and the Dirac equation for the induced spinors in X^4 should have a counterpart for $Y^4 \subset M^8$.

2. At the level of H there are two different chiralities for H spinors identified in terms of quarks and leptons. How the notion of chirality could be realized at the level of M^8? Quarks and leptons have different couplings to CP_2 Kähler gauge potential: what does this mean at the level of M^8?

One could use ordinary M^8 spinors also at the level of O but the notion of octonionic spinor structure looks like a very natural idea and could bring additional insights. Should one construct only sigma matrices as the idea about masslessness in the 8-D sense would suggest or should one construct gamma matrices? Consider first the sigma matrix option.

1. 8-component octonionic sigma matrices and spinors can be realized in terms of octonionic units just like Pauli spin matrices can be identified as quaternionic units. Octonionic units do not have a matrix representation. The projection of octonionic spinors to the quaternionic normal sub-space of Y^4 would make them quaternionic so that a matrix representation becomes possible.

2. How to identify octonionic spinors? The first guess is that they are simply complexified octonions Ψ. The $\Psi = p^kE_k\Psi_0$ gives a solution of the octonionic Dirac equation $p^kE_k\Psi = 0$ if the 8-D masslessness condition $p_kp^k = 0$ is satisfied.

   The projection to a quaternionic subspace of point $y \in Y^4$ gives the 4-D mass shell condition $p^2 = m^2$ and one obtains a massive fermion in the 4-D sense. M^8-H duality suggests that the mass spectrum is that for H spinor modes this would assign to the mass shells $H^3 \subset M^4$ in $M^4 \subset H$ mass shells in $E^4 \subset M^8$ as 3-spheres $S^3 \subset E^4$ if one requires that the point of Y^4 has length squared give by the m^2. This would code for the electroweak symmetry breaking at the level of H. Note that H spinor modes correspond to color partial waves in CP_2 and these could induce partial waves in Y^4.

3. The quaternionic projection of the octonionic massless spinor can be represented in terms of Pauli sigma matrices representing quaternionic units. This gives 2 2-component spinors corresponding to the two columns of Ψ, which corresponds to single M^4 chirality for say electrons.

4. One can consider several identifications for this degeneracy. Could it correspond to the second M^4 chirality correlating with electroweak chirality when the H-chirality is fixed. Could it correspond to electroweak spin so that different electroweak spins for the modes of H spinor fields would correspond to these two spinor columns? Or could it correspond to quark and lepton chiralities. Since sigma matrices do not mix these two columns it would seem that there are no interactions transforming them to each other. This would suggest that the identification in terms of leptons and quarks makes sense. The number theoretic vision would answer the question whether both leptons and quarks are present or whether leptons could be identified as bound states of quarks. The quark and lepton spinor modes in

H would be mapped to these two spinor modes by $M^8 - H$ duality. CP_2 partial wave would correspond to a partial wave in Y^4 and M^4 momentum would correspond to a momentum vector in the quaternionic normal space of $y \in Y^4$.

If one accepts the interpretation in terms of quarks and leptons, electroweak spin and second M^4 chirality are missing. How could one obtain these additional states? Are they somehow hidden to the additional components of the octonionic spinor but make themselves visible by the presence of 4-D mass shells corresponding to all solutions of H Dirac equation. Or could the introduction of octonionic gamma matrices solve the problem?

1. At the level of H one has gamma matrices rather than sigma matrices. Should one introduce octonionic gamma matrices? One can form 4-D gamma matrices from 2-D sigma matrices as tensor products $\gamma_0 = \sigma_x \otimes 1_{2\times 2}$ and $\gamma_i = i\sigma_y \otimes \sigma_i$, $\sigma_i^2 = 1$. This gives two M^4 chiralities instead of one. In the 8-D case, exactly the same formula applies at the level of octonionic gammas which are replaced with tensor products of sigma matrices and octonionic units.

2. How to identify the octonionic spinors now? Could they correspond to the octonionic gamma matrix γ_0 and γ_i giving a 4-fold increase of degrees of freedom due to the tensor products with sigma matrices. The projection to the quaternionic normal space would give 4-additional degrees of freedom giving the second M^4 chirality and electroweak chirality correlating with the M^4 chirality.

2.4 $M^8 - H$ duality and twistor lift of TGD

$M^8 - H$ duality would provide an explicit construction of space-time surfaces as M^8 images of 4-surfaces $Y^4 \subset M^8 = O$ with an associative normal space. The recent picture [L47] discussed above is however considerably more complex than the original naive vision [L20, L21, L43]. Polynomials are involved also now but they are associated with 2-D partonic 2-surfaces. The two polynomials defining the space-time surfaces can be polynomials but one cannot exclude coefficients in an extension of rationals and also analytic functions with rational or algebraic coefficients can be considered.

$M^8 - H$ duality maps these solutions to H and one can consider several forms of this map. The weak form of the duality relies on holography mapping only 3-D or even 2-D data to H and the strongest form maps entire space-time surfaces to H.

The twistor lift of TGD allows to identify the space-time surfaces in H as base spaces of 6-D surfaces representing the twistor space of the space-time surface as an S^2 bundle in the product of twistor spaces of M^4 and CP_2. These twistor spaces must have Kähler structure and only the twistor spaces of M^4 and CP_2 have it [A5] so that TGD is unique also mathematically. Can one generalize $M^8 - H$ duality so that it would apply at the level of twistor spaces? This question has been already discussed in [L20, L21, L43] but the new view of $M^8 - H$ duality forces us to reconsider this question.

As already found, an attractive realization of holography=holomorphy vision is by assuming that the twistor space X^6 determined as a preferred extremal X^6 of the Kähler action for 12-D twistor space $T(M^4) \times T(CP_2)$ allows space-time surfaces as a holomorphic sections $P(z,u,w) = 0$ determined by a holomorphic polynomial P and that partonic orbits correspond to critical surfaces at which two roots coincide so that one has $(P(z,u,w), dP(z,u,w)/dz) = (0,0)$. One might hope that $P(z,u,w)$ is highly unique.

2.4.1 Does the twistor space of the space-time surface have M^8 counterpart?

Could the $M^8 - H$ dual of the 6-D twistor-space X^6 of the 4-D space-time surface X^4 makes sense and give Y^6 as the M^8 analog of the twistor lift? The twistor space $X^6 \subset M^8$ has the structure of an S^2 bundle and there exists a bundle projection $X^6 \to X^4$. The same be should be true for Y^6.

1. The normal spaces $N(y)$ of an associative space-time surface Y^4 contain a 2-D commutative normal space $M^2(y)$ with Minkowski signature so that one has an integrable distribution of

Minkowskian subspaces $M^2(y) \subset N(y)$. The distribution of these 2-D commutative normal spaces defines a 6-D surface Y^6 in M^8, which has a bundle projection to Y^4.

Could Y^6 be regarded as an S^2 bundle over Y^4 analogous to a twistor bundle. For the twistor space of M^4, S^2 corresponds to light-like vectors emanating from a given point of M^4. In M^8, M^4 is represented as a normal space $N(y)$ of Y^4. Could the fiber of Y^6 be S^2 as a set of light-like normal vectors in $N(y)$?

The selection of a quaternion imaginary unit defining a commutative complex sub-space of quaternions corresponds to a selection of direction in the 3-space defined by the imaginary units, therefore a selection of a point of 2-sphere S^2. Therefore the fiber of Y^6 must be a sphere. One would obtain Y^6 as the analog of the M^4 twistor space and the bundle bundle projection $Y^6 \to Y^4$ just by replacing the condition of associativity of the normal space with its commutativity. The bundle projection would correspond to loosening commutativity to mere associativity.

2. How does this relate to the proposed realization of the twistor space $X^6 \subset T(M^4) \times T(CP_2)$, where the twistor spaces $T(M^4)$ *resp.* $T(CP_2)$ are identified as $T(M^4) = CP_3 = SU(4)/SU(3) \times U(1)$ *resp.* $T(CP_2) = SU(3)/U(1) \times U(1)$. The twistorial counterpart of $M^8 - H$ duality should map Y^6 to $X^6 \subset T(M^4) \times T(CP_2)$ realized as preferred extremal of Kähler action in $T(M^4) \times T(CP_2)$,.

   As already described, the realization of $M^8 - H$ duality requires a multiplication of points of Y^4 by an octonionic imaginary unit e transforming them to points of $N(y) = M^4$. The choice of e is determined only apart from a local $U(2)$ rotation interpreted in terms of electroweak gauge symmetry, which fails only at the active points of Y^4 populated by fermion. $U(2)$ is 4-D should and brings in 4- additional dimensions could these dimensions correspond to M^4.

   What is the counterpart of this gauge invariance correspond in the case of Y^6? $M^8 - H$ duality at the level of twistor spaces would map the points of $T(CP_2) = SU(3)/U(1) \times U(1)$ to $Y^6 \subset M^8$ for which the normal space of Y^6 is hypercomplex sub-space of octonions identifiable as $M^2(y)$. The fiber of $Y^6 \to Y^4$ would be the sphere S^2 defined by the light-like directions characterizing the choices of $M^2(y)$ in $N(y)$ and the base would be Euclidian Y^4.

   The normal space for a point of Y^6 at a given would be $M^2(y)$. The counterpart for multiplication by e should rotate the point of Y^6 to a point of $M^2(x) \subset M^4(x) = N(y)$ defining the imaginary unit and would correspond to an $U(1) \times U(1)$ rotation transforming the octonionic unit to the imaginary unit characterizing M^2. One would have an analog of $U(1) \times U(1)$ gauge invariance. The Cartan group of electroweak group would be in question.

   $M^8 - H$ duality would require that $X^4 \subset M^4 \times T(CP_2)$ contains an integrable distribution of tangent spaces $M^2(x)$ as a counterpart of the distribution of normal sub-spaces $M^2(y)$ on M^8 side. This has been conjectured on basis of physical motivations and $M^2(x)$ corresponds to the local longitudinal degrees of freedom. For Hamilton-Jacobi structure it corresponds to the local hypercomplex sub-space transversal to the complex subspace.

3. Could one regard $Y^6 = T(Y^4)$ as the counterpart of $X^6 = T(X^4)$ so that $M^8 - H$ duality would map $T(Y^4)$ to $T(CP_2)$. This kind of identification map occurs also in construction of $X^6 \subset T(M^4) \times T(CP_2)$ by dimensional reduction to 6-dimensional X^6, which identifies the twistor spheres of $T(M^4)$ and $T(CP_2)$ and assigns S^2 to X^4 as a fiber. Could the twistor space $T(M^4)$ correspond to Y^6 as $M^8 - H$ duals of $T(CP_2)$? Could $M^8 - H$ duality do the same as the construction of X^6 as preferred extremal of 6-D Kähler action in $X^6 \subset T(M^4) \times T(CP_2)$?

4. Could this mean that there is no need to postulate $T(M^4) \times T(CP_2)$ and that 10-D $M^4 \times T(CP_2)$ is enough? X^6 would correspond to $X^6 \subset M^4 \times T(CP_2)$ with S^2 fiber assigned to CP_2. $T(CP_2)$ allows Kähler action and the construction of solutions by dimensional reduction reduces to that in $M^4 \times T(CP_2)$ rather than $T(M^4) \times T(CP_2)$. The problem is that one would not obtain variable cosmological constant due to the possibility that the local map identifying S^2 fibers of $T(M^4)$ and $T(CP_2)$ can have an integer valued winding number.

5. The fact that the identification of the M^4 and CP_2 fibers effectively reduces $T(M^4) \times T(CP_2)$ to $M^4 \times T(CP_2)$ serves as a clue. Where does the winding originate in the description based on $M^8 - H$ duality? The counterpart of $T(M^4)$ at the M^8 side as $M^8 - H$ image Y^6 of $T(CP_2)$. Could the associated 4-D $U(1) \times U(1)$ local gauge invariance assign the missing S^2 fiber to $M^4 \subset M^4 \times T(CP_2)$ to give $T(M^4) \subset T(M^4) \times T(CP_2)$. Could the division of electroweak group $U(2)$ by $U(1) \times U(1)$ factor giving S^2 as additional degrees of freedom at H side and could it also correspond to the division of $SU(3)$ with $U(1) \times U(1)$ as the fact that $U(2)$ holonomies correspond to $U(2) \subset SU(3)$? Electroweak interactions would lead to the winding.

2.4.2 Physical interpretation of the counterparts of twistors at the level of M^8

What about the physical interpretation of the counterparts of twistors at the level of M^8? The twistor space allows a geometrization of fermionic spin so that momentum and spin combine to a purely geometric entity with 6 components. The "active" points of the normal space $N(y)$ would correspond to fermions with a given momentum and spin. How to obtain the electroweak spin?

1. The first thing to notice is that in the twistor Grassmannian approach twistor space provides an elegant description of spin. Partial waves in the fiber S^2 of twistor space representation of spin as a partial wave. All spin values allow a unified treatment.

   The problem is that this requires massless particles. In the TGD framework 4-D masslessness is replaced with its 8-D variant so that this difficulty is circumvented. This kind of description in terms of partial waves is expected to have a counterpart at the level of the twistor space $T(M(^4) \times T(CP_2)$. At level of M^8 the description is expected to be in terms of discrete points of Y^6.

2. Consider first the real part of $Y^6 \subset M^8$ having the structure of S^2 bundle. The points of X^4 correspond to points of Y^4. In the normal space of X^4 momentum value would be selected if the point is populated by a fermion. The same must be true also at the level of X^6. A single point in the fiber space S^2 would be selected besides the point representing momentum. The interpretation could be in terms of the selection of the spin quantization axis.

   Spin quantization axis corresponds to 2 diametrically opposite points of S^2. Could the choice of the point also fix the spin direction? There would be two spin directions and in the general case of a massive particle they must correspond to the values $S_z = \pm 1/2$ of fermion spin. For massless particles in the 4-D sense two helicities are possible and higher spins cannot be excluded. The allowance of only spin 1/2 particles conforms with the idea that all elementary particles are constructed from fermions and antifermions. Fermionic statistics would mean that for a fixed momentum one or both of the diametrically opposite points of S^2 defining the same and therefore unique spin quantization axis can be populated by fermions having opposite spins.

   The non-local version of $M^8 - H$ duality could map the points of Y^6 to partial waves in the the $T(M^4)$ so that point of S^2 would be mapped to a partial wave in CP_2.

3. The selection of a point in the fiber S^2 of $T(CP_2)$ would correspond to the choice of the quantization axis of electroweak spin and diametrically opposite points would correspond to opposite values of electroweak spin 1/2 and unique quantization axis allows only single point or pair of diametrically opposite points to be populated.

   Spin 1/2 property would hold true for both ordinary and electroweak spins and this conforms with the properties of $M^4 \times CP_2$ spinors. This also conforms with the identification of the twistor spheres of $T(M^4)$ and $T(CP_2)$.

4. The point of a normal space of $N(y)$ of $y \in Y^6 \subset M^8$ would represent geometrically the modes of H-spinor fields with fixed momentum and spin and electroweak spin. The CP_2 partial waves representing the orbital degrees of freedom associated with CP_2 would correspond to partial waves in Y^4 as images under $M^8 - H$ duality.

   M^4 momenta represent orbital degrees of M^4 spinors so that E^4 parts of E^8 momenta should represent the CP_2 momenta. The eigenvalue of CP_2 Laplacian defining mass squared eigenvalue in H should correspond to the mass squared value in E^4 and to the square of the radius of sphere $S^3 \subset E^4$.

This would be a concrete realization for the $SO(4) = SU(2)_L \times SU(2)_R \leftrightarrow SU(3)$ duality between hadronic and quark descriptions of strong interaction physics. Proton as skyrmion would correspond to a map S^3 with radius identified as proton mass. The skyrmion picture would generalize to the level of fermions and also to the level of bound states of fermions allowed by the number theoretical hierarchy with Galois confinement. This also includes bosons as Galois confined many fermion states.

5. The bound states with higher spin formed by Galois confinement should have the same quantization axis in order that one can say that the spin in the direction of the quantization axis is well-defined. This freezes the S^2 degrees of freedom for the quarks of the composite.

What does the map of the twistor space $T(M^4)$ to $T(CP_2)$ mean physically? Fermions are doublets with both spin and electroweak spin but spin and electroweak spin values are independent quantum numbers and one can select the quantization axis of spin freely. Somehow the $M^8 - H$ duality should involve a relative rotation of the quantization axes for spin and electroweak.

1. The direction of the latter should define a fixed direction of the quantization axis since superpositions of different isospin states is not possible. The $U(2)$ gauge invariance associated with the choice of co-quaternionic imaginary unit e allows to rotate the point $y \in Y^4$ to a point of quaternionic $N(y)$ representing momentum of a fermion associated with y if the mass squared corresponds to the radius squared of 3-surface S^3 to which Y^4 projects. At other points one has $U(2)$ gauge invariance.

2. $U(1)\times$ gauge invariance is in turn associated with the choice of the quaternionic unit e_1 allowing to rotate the momentum vector of $N(y)$ to the direction of the imaginary axis of a complex sub-space of of $N(y)$. Could e_1 characterize the rotation relating the spin axis parallel to 3-momentum and the isospin axes defining the imaginary unit of the complex sub-space?

2.5 $M^8 - H$ duality at the level of WCW

The key observation is that any functional or path integral with integrand defined as exponent of action, can be *formally* calculated as an analog of Gaussian integral over the extrema of the action exponential $exp(S)$. The configuration space of fields would be effectively discretized. Unfortunately, this holds true only for the so called integrable quantum field theories and there are very few of them and they have huge symmetries.

TGD is indeed completely integrable in the sense that one can write explicit general solutions in the realization of holography as generalized holomorphy. Without maximal symmetries WCW Kähler metric does not exist. This is the situation already for loop spaces [A2]. There are super-symplectic symmetries and generalized conformal symmetries associated with the generalized conformal invariance [L45]. Is the WCW functional integral well-defined thanks to the maximal symmetries of the WCW metric?

2.5.1 How could adelization take place at the level of WCW

WCW emerges in the geometric view of quantum TGD. $M^8 - H$ duality should also work for WCW. What is the number theoretic counterpart of WCW? What is the number theoretic counterpart of the discretization in terms of the maxima of Kähler function?

For the Kähler function K, its maxima (or maybe extrema) would define a natural effective discretization of the sector of WCW. If one can assign a discretization to the space-time surfaces or at least to the holographic data defining the space-time surfaces, there are hopes that this discretization induces a number theoretic discretization of WCW.

Discretization is possible both at the level of WCW and H (and space-time surfaces).

1. The coefficients of polynomials (P_1, P_2) and P can be in any extension F of rationals and this gives rise to a kind of collective number theoretic evolution at the level of imbedding space,

space-time surface, and WCW. For the $P = 0$ option, the space-time surface is identified as a zero section of the twistor surface $X^6 \subset T(M^4) \times T(CP_2)$.

One obtains an extension E of F at the level of the twistor sphere for the fermion lines. This gives a number theoretic evolution at single fermion level. This discretization could give rise to p-adic primes and ramified primes (and their generalizations) characterizing elementary particles. WCW decomposes to a union of sub-WCWs with polynomial coefficients in various fields F and inclusion of the extensions correspond to inclusion of sub-WCWs. At the level of WCW, this discretization is very elegant and also necessary if one wants p-adization and adelization at the level of WCW.

2. Both the embedding space H and M^8 can be discretized number-theoretically and this induces a discretization of the space-time surfaces. This in turn induces a discretization of WCW. The coordinates of points in preferred coordinates defined by a subset of generalized complex coordinates of H are algebraic rationals or even algebraic integers for an extension F of rationals defining the background extension of X^4 therefore making sense in all p-adic number fields extended by F. This would make possible adelization at the level of M^8 and H. It is however not obvious whether $M^8 - H$ duality is consistent with it in CP_2 degrees of freedom and whether it should be restricted to the linear M^4 degrees of freedom.

   The effective discretization of H could make computations easy. At the level of M^8, the assumption that M^8 point points or at least the points in the normal space of Y^4, having interpretation as 4-momenta, are in F could facilitate the calculations dramatically. This is necessary for the notion of Galois confinement. This induces discretization of WCW.

One can consider the number theoretic discretization from the global and local perspectives. The first question is how the h_{eff} and ramified primes are determined.

1. Global option is the simpler one. F would define both $h_{eff}/h_0 = n$ and ramified primes. One can assign $h_{eff} = nh_0$ to the extension F of rationals serving as a background number field for the coefficients of various polynomials. Could one also assign ramified primes to it so that a given CD and the space-time surfaces inside it could be characterized by a p-adic length scale?

   One can identify a unique monic polynomial defining E if one assumes that its roots correspond to the powers of a generating algebraic prime for E. This procedure is somewhat formal and looks ad hoc but is general coordinate invariant and conforms with the generalization of p-adicity to allow algebraic primes. One could assign the discretization induced by algebraic integers of F to $H^3(a)$. A further problem is that the p-adic primes in p-adic mass calculations are assigned with rationals. Global option is not enough.

2. The local option and assigns an extension of $F \to E$ to fermion lines satisfying $(P, dP/dz) = (0, 0)$. The coefficients of $P_{u,t}(z)$ have values in F depend on a light-like coordinate u (having constant value at the partonic orbit and along fermion line) and a generalized complex coordinate t of H and would characterize the entire space-time surface. At fermion lines $P_{u,t}(z)$ should reduce to a polynomial in F. This would correspond to number theoretical criticality making possible adelization.

   $n = h_{eff}/h_0$ could correspond to the dimension of $F \to E$ for the co-inciding roots z_k of P at the fermion line $(P_{u,t}, dP_{u,t}/dz) = (0, 0)$, which correspond to different points of the twistor sphere permuted by the Galois group of P at the fermion line. The criticality conditions determine possible values of t defining a set of fermion lines, which would be related by the Galois group. Here the graph of the cusp catastrophe (see) visualizes the situation.

   At the criticality, the order of the Galois group of P is reduced to a normal subgroup Gal/H, where H is the subgroup (Z_2 for two degenerate roots) leaving the degenerate root invariant. One can also assign to the extension $F \to E$ a Galois group and identify p-adic prime as a ramified prime of P for a particular fermion line. Without additional conditions, each degenerate pair of roots defines its own extension $F \to E$ characterized by dimension and by ramified primes. If the F consist of rationals, p-adic primes are rational primes. This is enough for the physical interpretation.

3. The Galois degrees of freedom would provide new spin-like degrees of freedom represented by the orbit of the fermion line since one can have discrete wave functions at the orbit of a degenerate root. Could color quantum numbers at the level of the space-time surface (color quantum numbers are not spin-like in TGD) could be represented as Galois quantum numbers? At the level of H, the components of four-momenta of fermions would be algebraic integers in F and for the strongest form of Galois confinement the components of the total momenta of many-fermion states would be integers.

4. The mass of the fermion associated with a fermion line of the light-like partonic orbit could select $a = \hbar_{eff}/m$ hyperboloid $H^3(a)$ as a preferred M^4 projection of X^4. m characterized a mass shell of a normal space of Y^4.

The big picture could be as follows.

1. The TGD Universe is analogous to the spin glass phase [L30]. The discretized WCW corresponds to the energy landscape of spin glass having an ultrametric topology consisting of preferred extremals defined by polynomial roots and forming algebraic hierarchies of sub-WCWs in terms of polynomials and extensions F of rationals. The extensions of $F \rightarrow E$ would define associated hierarchies.

   M^8 and H and corresponding 4-surfaces would be replaced by their discretization defined by the background extension. Ultrametric topology of WCW means that the space-time surfaces in discretized WCW are characterized by a set of discretized $a = \hbar_{eff}/m$ hyperboloids H^3. F could define discretizations of the hyperboloids in M^8 and H and the collection of ramified primes of $F \rightarrow E$ would define a subspace of adele associated with a fermion.

2. Space-time surface would be characterized by F, by various polynomials and the extensions $F \rightarrow E$ associated with the critical fermion lines and their orbits under reduced Galois group of $F \rightarrow E$.

3. In the adelization, one can assign discriminant to the Galois orbit of a fermion line as a product of the non-vanishing differences of the roots of P. This generalized discriminant could assign to the fermion line algebraic ramified primes as preferred p-adic primes. The collection of algebraically extended adele's assignable to these collections would characterize the space-time surfaces in a given sector of WCW.

2.5.2 What could be the M^8 counterpart of the vacuum functional?

The vacuum functional as an exponent $exp(-K)$ of the Kähler function K determines the physics at WCW level at the level of H. $M^8 - H$ duality suggests that it should have a counterpart at the level of M^8 and appear as a weight function in the summation. Adelic physics suggests that the weight function is a product of powers of discriminants of the algebraic extensions associated with the number theoretically critical light-like fermion lines at the light-like partonic orbits assignable to the space-time surface. The discriminant is expressible as products of the ramified primes of the extension definnes natural candidates for the p-adic primes characterizing the fermion. This would guarantee that the $exp(-K)$ makes sense p-adically for all ramified primes associated with X_i^2.

$M^8 - H$ duality suggests that the exponent $exp(-K)$ of Kähler function has an M^8 counterpart with a purely number theoretic interpretation. The discriminants $D(P(X_i^2))$ of the polynomials P associated with the 2-surfaces X_i^2 is the natural guess for the building blocks. For monic polynomials D is integer having ramified primes as factors. The product $Y = \prod_i D(P(X_i^2))$ would be a natural building block of $exp(-K)$.

There are two options for the correspondence between $exp(-K)$ at its maximum and Y assuming that $P(X_i^2)$ is a monic polynomial.

1. In the real topology, one would naturally have $exp(-K) = 1/Y$. For monic polynomials of form $P(x) = x^n + ..$ (also more general polynomials with integer coefficients can be considered) with high degree, Y becomes large so that $exp(-K)$ is small. The sum of the exponentials $exp(-K)$ is expected to appear as a renormalization factor when several maxima of K contribute to the functional integral so that this effect could be compensated.

2. In a p-adic topology defined by p-adic prime p identified as a ramified prime of $D(X_i^2)$ for some values of i, one can write $Y = p^k E$, where E is p-adic integer of unit norm with respect to p. The canonical identification $I(x) = \sum x_n p^n = \sum x_n p^{-n}$ gives $I(Y) = p^{-k} \times I(E)$. For $I(1/Y)$ one would have $I(Y) = \prod_i D(P(X_i^2))) = p^k \times I(1/E)$.

3. Is the identification $exp(-K) = I(Y)$ or $exp(-K) = I(1/Y)$ more realistics? The assumption that K is non-negative would suggest the first option for which $I(Y)$ is not larger than 1. The first identification would not favor large p-adic primes unlike the second identification. These effects could be compensated by the normalization factor of the functional integral also now.

Are some ramified primes favoured? The p-adic primes associated with elementary particles are rather large which suggests that the largest ramified prime P associated with $D(X(2_i)$ could be favored. P would correspond to a unique p-adic length scale L_p and a given L_p would correspond to all polynomials P for which the largest ramified prime is p.

This might provide some understanding concerning the p-adic length scale hypothesis stating that p-adic primes tend to be near powers of integer. In particular, understanding of why Mersenne primes are favored might emerge. For instance, Mersennes could correspond to primes for which the number of polynomials having them as the largest ramified prime is especially large. The quantization condition $exp(-K) = D(p)$ could define which p-adic primes are the fittest ones.

The condition that $exp(-K)$ at its maximum equals to D via canonical identification gives a powerful number theoretic quantization condition.

2.6 Some questions and ideas related to $M^8 - H$ duality

In the following some questions and ideas, which do not quite fit under the titles of the previous sections, are considered.

2.6.1 A connection with Langlands program

Langlands correspondence [A6, K5, A4, A3], which I have tried to understand several times [K5] [L1, L2, L5] relates in an interesting manner to $M^8 - H$ duality and Galois confinement.

1. Global Langlands correspondence (GLC) states that there is connection between representations of continuous groups and Galois groups of extensions of rationals.

2. Local LC correspondence states (LLC) states this in the case of p-adics.

There is a nice interpretation for the two LCs in terms of sensory experience and cognition in TGD inspired theory of consciousness.

1. In adelic physics real numbers and p-adic number fields define the adele. Sensory experience corresponds to reals and cognition to p-adics. Cognitive representations are in their discrete intersection and for extensions of rationals belonging to the intersection.

   (a) Sensory world, "real" world corresponds to representation of continuous groups/Galois groups of rationals (GLC).

   (b) "p-Adic" worlds correspond to cognition and representations of p-adic variants of continuous groups and Galois groups over p-adics (LLC).

   (c) One could perhaps talk also about Adelic LC: ALC in the TGD framework. Adelic representations would combine real and p-adic representations for all primes and give as complete information about reality as possible.

   TGD provides a geometrization for the identification of Galois groups as discrete subgroups of Lie groups, not only of the isometry (automorphism) groups of H (M^8) but perhaps also as discrete sub-groups of more general Lie groups to which the action of super-symplectic representations could reduce. A naive guess is that these groups correspond to the ADE groups appearing in the McKay correspondence [L8, L18, L19].

The representation of real continuous groups assignable to the real numbers as a piece of adele [L9, L10] would be related to the representations of Galois groups GLC. Also p-adic representations of groups are needed to describe cognition and these p-adic group representations and representations of p-adic Galois groups would be related by LLC.

2.6.2 Could the notion of emergence of space-time have some analog in the TGD Universe?

The idea about the emergence of space-time from entanglement is as such not relevant for TGD. One can however ask whath the emergence of *observed* space-time could mean in TGD. Space-time surface as a continuum exists in TGD but they are not directly observable due to a finite measurement resolution. One can ask what a body with an outer boundary means physically. The space-time regions defined by solid bodies have boundaries. What makes the boundaries of the bodies "hard"?

1. In momentum space Fermi statistics does not allow fermions to get through the boundary of Fermi ball. This is a good guideline.
2. Second feature of a spatial object such as an atom is that it is a bound state quantum mechanically. If it has parts they stay together. In QFT theory the notion of a bound state is however poorly understood.
3. Quantum coherence is a further property considered in the article. Spatial objects correspond to quantum coherent structures. Quantum coherence reduces to entanglement. Quantum coherence length and time determine the size of a quantum object. Somehow one must have stable entanglement in long scales.

Let us see what these guidelines could give in the framework of M^8-H duality which generalizes the wave particle duality of wave mechanics.

1. In adelic physics space-times can be seen as either surfaces in M^8 or $H = M^4 \times CP_2$. $X^4 \subset M^8$ is analogous to momentum space cognitive representations consist of points of $X^4 \subset M^8$, whose points are algebraic integers in the extension of rationals defined by the polynomial defining the space-time surface and are algebraic integers as roots of monic polynomials of form $x^n + ...$ This defines a unique discretization of the space-time surface. The discretization guarantees number theoretical universality: the cognitive representation makes sense also p-adically and space-time has also p-adic variants.

   Cognitive representations give rise to "cognitive emergence" of the space-time in cognitive sense and since cognitive representations are intersection of reality and p-adicities they must closely related to the "sensory emergence"".
2. $X^4 \subset M^8$ is mapped to H by $M^8 - H$ duality determined by the condition that it momentum is mapped to a geodesic with a direction of momentum and starting from either tip of CD: the image point is its intersection with the opposite light-like boundary of CD and selects a point of space-time surface. The size of CD is $T = h_{eff}/m$ for quark with mass m to satisfy Uncertainty Principle. The map generalizes to bound states of quarks (whatever they are).

Consider the problem of "sensory emergence" in this framework.

1. What makes a point of a cognitive representation "hard"? Fermions are associated with points (not necessarily all) of a cognitive representation: one can say that the point is activated when there is a quark at it. Fermi ball corresponds to a discrete set of activated points at the level of momentum space. These points define activated points also in $X^4 \subset H$ by $M^8 - H$ duality. One could perhaps say that these activated points in M^8 and their H-image containing fermions define the spatial objects as something "hard" and having a boundary. Another fermion knows that there is a space-time point there because it cannot get to this point. The presence of a fermion (quark) would make a space-time point "hard".

2. What about the role of entanglement? The size and duration of the space-time surface (inside a causal diamond CD) defines quantum coherence length and time. Fermionic statistics makes fundamental fermions - to be distinguished from elementary fermions - maximally entangled. One cannot reduce fermionic entanglement in SFR and quantum measurements would be impossible. The entanglement in the WCW degrees of freedom comes to the rescue. This entanglement can be reduced in SFRs since the particles as surfaces are identical under very special - naturally number theoretical - conditions.

   Negentropy Maximization Principle and hierarchy of $h_{eff} = n \times h_0$ phases favor the generation of stable entanglement in the TGD Universe. Also, if the coefficients of the entanglement matrix belong to extension of rationals, entanglement probabilities in general belong to its extension and the density matrix is not diagonalizable without going to a larger extension. This might require "big" SFR increasing the extension: only after this state function reduction to an eigenstate could occur. This leads to a concrete proposal for how the information about the diagonal form of the density matrix expressed by its characteristic polynomial is coded into the geometry of the space-time surface [L25].

3. Bound state formation is the third essential element. Momenta are points of the space-time surface $X^4 \subset M^8$ with components which are algebraic integers. Physical momenta are however ordinary integers for a particle in a finite volume defined by causal diamond (CD). This means that one can allow only composites of quarks with rational integer valued momenta which correspond to Galois singlets.

   Galois confinement would be the universal mechanism behind formation of all bound states and also give rise to stable entanglement. One would obtain a hierarchy of bound states corresponding to a hierarchy of polynomials and corresponding Galois groups and extensions of rationals. By $M^8 - H$ duality, bound states of quarks and higher structures formed from them in M^8 would give rise to spatial objects.

3 Zero energy ontology (ZEO)

ZEO [K8] forms the cornerstone of the TGD inspired quantum theory extending to a theory of consciousness. ZEO has so far reaching consequences that it deservedd a separate section.

3.1 The basic view about ZEO and causal diamonds

The following list those ideas and concepts behind ZEO that seem to be rather stable.

1. General Coordinate Invariance for the geometry of WCW without path integral producing infinite hierarchy of divergences implies holography, Bohr orbitology and ZEO [L17] [K8].

2. X^3 as extended particle is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. Finite failure of determinism is however possible [L32]. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD) defined as intersection of future and past directed light cones ($\times CP_2$) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. 4 Fig. 5**). The introduction of the moduli space of CDs [L44] is necessary to solve some interpretational problems of ZEO. In particular, the problem due to the failure of Poincare invariance for single CD in quantum theory finds a solution.

4. The opposite light-like boundaries of CD correspond to dual fermionic vacuums (bra and ket) annihilated by fermion annihilation *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD (the observed positive energies are of course associated with these 3-D states). A more general option, suggested by the violation of the exact classical determinism, is that the two kinds of states correspond to states at the $a = constant$ mass shells $M^4 \subset H$ inside lower *resp.* upper half-cone of CD.

At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing of the normal spaces of $Y^4 \subset M^8$ would be by mass shells (hyperboloids) mapped by $M^8 - H$ duality to a slicing by light-cone proper time=constant hyperboloids.

5. Zeno effect can be understood if the states associated with either half-cone of CD do not change in "small" state function reductions (SSFRs) but change at the opposite half-cone. SSFRs are analogs of weak measurements. One could call this half-cone call as a passive half-cone. I have earlier used a somewhat misleading term passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate - geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. The size of CDs increases at least in a statistical sense in the sequence of SSFRs and in the first approximation CD would be scaled rather than translated. In conformal theories, scalings indeed induce the counterpart of time evolution. For the first option, the scaling leaves the center point of the CD invariant. For the second option, the second tip of the CD remains invariant: in this case the center point is shifted and the passive boundary remains invariant. For both options, the states at the "passive" boundary are unaffected although the scaling affects the passive boundary. The considerations related to TGD inspired theory of consciousness favor the option for which center point remains unaffected in the scalings [L17, L27]. The notion of moduli space of CDs leads to a more general picture in which SSFRs correspond to dispersion in this space followed by localizations associated with SSFRs.

7. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change (see **Fig. 6 Fig. 5**). The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like a deterministic time evolution leading to the final state [L12] as observed by Minev *et al* [L12].

8. h_{eff} hierarchy [K1, K2, K3, K4] implied by the number theoretic vision [L20, L21, L43, L47] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

9. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. This provides also a mechanism of self-organization [L14]: dissipation with reversed arrow of time looks like self-organization [L54].

3.2 Open questions related to ZEO

There are many unclear details related to the time evolution in the sequence of SSRs. Before discussing these unclear details it is useful to summarize the basic ideas related to the interpretation in the framework of TGD inspired theory of consciousness.

1. I do not experience directly mental images (subselves) with the opposite arrow of time. The reason is that the classical signals travel to the direction of my geometric past and their receival requires BSFR. In particular, I cannot have memories from the period of sleep induced by classical signals if it involves an opposite arrow of time.

In ZEO, mental mentals as sub-selves (say after images) are in a kind of Karma's cycle: they are born and die roughly periodically corresponding to a sequence of BSFRs meaning a reincarnation with an opposite geometric arrow of time [L27].

2. I can have memories only about states of consciousness with the same arrow of time that I have. This explains why I do not have memories about periods of sleep if sleep is interpreted as a time reversed state of some subself of me responsible for self-ness.

One can use three empirical inputs in an attempt to fix the model.

1. After images appear and disappear roughly periodically. Also I fall asleep and wake up with a standard arrow of time roughly periodically.
   (a) The first interpretation is that as a sequence of wake up-sleep periods I am a time crystal-like structure consisting of nearly copies of the mental image, such that each mental image - including me as mental images of higher level self - continues Karma's cycle in my geometric past. How "me" is transferred to a new almost copy of my biological body? Does my MB just redirect its attention?
   (b) The second interpretation is that me and my mental images somehow drift towards my geometric future, while performing the Karma's cycle so that my mental images follow me in my time travel. This would require that the sub-CDs of mental images drift towards the geometric future.

       Also sleep could be a "small" death at some layer of the personal hierarchy of MBs. I do not however wake-up in BSFR at the moment of geometric time defined by the moment of falling asleep but later. So it seems that my CD must drift to the geometric future with the same speed that those of other living beings in the biosphere.
2. There is however an objection. In cosmology the observation of stars older than the Universe [L46, L48] would have a nice solution if the stars evolve forth and back in time in our distant geometric past rather than drifting towards the future so that they could age by continuing their Karma's cycle with a constant center of mass value of time. Can these three observations be consistent?

3.2.1 Could the scaling dynamics CD induce the temporal shifting of sub-CDs as 4-D perceptive fields?

Suppose that the sub-CDs within a bigger CD "follow the flow". How the dynamics of the bigger CD could induce this flow?

1. The scalings of bigger CD in unitary evolutions between SSFRs induce the scaling of sub-CS. This would not be shifting but scaling and the distance between given CD and larger CDs would gradually scale up.

   This would remove the objection. The astrophysical objects in distant geometric past would move towards the geometric future but with much smaller velocity as the objects with cosmic scale so that the temporal distance to future observers would increase. These objects would be aging in their personal Karma's cycle, and the paradox would disappear.
2. The flow would be defined by the scalings of a larger CD containing our CDs and those of others at my level. Each CD would define a shared time for its sub-CDs. If the CDs form a hierarchy structure with a common center, this is indeed true of the time evolutions as scalings of CDs. There would be scalings induced by scalings at higher levels and "personal" scalings.
3. It however seems that the common center is too strong an assumption and shifted positions for the sub-CDs and associated hierarchy inside a given CD are indeed possible for the proposed realization of $M^8 - H$ duality and actually required by Uncertainty Principle.

A further open question is what happens to the size of CD in the BSFR. Does it remain the same so that the size of the CD would increase indefinitely? Or is the size reduced in the sense that there would be scaling, reducing the size of the CD in which the passive boundary of the CD would be shifted towards the active one. After every BSFR, the self would experience a "childhood". To justify this option one should explain why the time evolution as dispersion in the moduli space of CDs applies only to SSFRs.

3.2.2 Are we sure about what really occurs in BSFR?

It has been assumed hitherto that a time reversal occurs in BSFR. The assumption that SSFRs correspond to a sequence of time evolutions identified as scalings, forces to challenge this assumption. Could BSFR involve a time reflection T natural for time translations or inversion $I : T \to 1/T$ natural for the scalings or their combination TI?

I would change the scalings increasing the size of CD to scalings reducing it. Could any of these options: time reversal T, inversion I, or their combination TI take place in BSFRs whereas arrow would remain as such in SSFRs? T (TI) would mean that the active boundary of CD is frozen and CD starts to increase/decrease in size.

There is considerable evidence for T in BSFRs identified as counterparts of ordinary SFRs but could it be accompanied by I?

1. Mere I in BSFR would mean that CD starts to decrease but the arrow of time is not changed and passive boundary remains passive boundary. What comes to mind is blackhole collapse.

   I have asked whether the decrease in size could take place in BSFR and make it possible for the self to get rid of negative subjective memories from the last moments of life, start from scratch and live a "childhood". Could this somewhat ad hoc looking reduction of size actually take place by a sequence of SSFRs? This brings into mind the big bang and big crunch. Could this period be followed by a BSFR involving inversion giving rise to increase of the size of CD as in the picture considered hitherto?

2. If BSFR involves TI, the CD would shift towards a fixed time direction like a worm, and one would have a fixed arrow of time from the point of view of the outsider although the arrow of time would change for sub-CD. This modified option does not seem to be in conflict with the recent picture, in particular with the findings made in the experiments of Minev *et al* [L12] [L12].

   This kind of shifting must be assumed in the TGD inspired theory of consciousness. For instance, after images as a sequence of time reversed lives of sub-self, do not remain in the geometric past but follow the self in travel through time and appear periodically (when their arrow of time is the same as of self). The same applies to sleep: it could be a period with a reversed arrow of time but the self would shift towards the geometric future during this period: this could be interpreted as a shift of attention towards the geometric future. Also this option makes it possible for the self to have a "childhood"".

3. However, the idea about a single arrow of time does not look attractive. Perhaps the following observation is of relevance. If the arrow of time for sub-CD correlates with that of sub-CD, the change of the arrow of time for CD, would induce its change for sub-CDs and now the sub-CDs would increase in the opposite direction of time rather than decrease.

To sum up, TI or T can be considered as competing options for what happens in BSFR. T should however be able to explain why sub-selves (sub- CDs) drift to the direction of the future. If the time evolutions between SSFRs correspond to scalings rather than time translations, and if the scalings occur also for sub-CDs this can be understood. The dynamics of spin glasses strongly suggests that SSFRs correspond to scalings [L30].

3.3 What happens in quantum measurement?

According to the proposed TGD view about particle identity, the systems for which mutual entanglement can be reduced in SFR must be non-identical in the category theoretical sense. When SFR corresponds to quantum measurement, it involves the asymmetric observer-system $O-S$ relationship. One cannot exclude SFRs without this asymmetry. Some kind of hierarchy is suggestive.

The extensions of rationals realize this kind of $O-S$ hierarchy naturally. The notion of finite measurement resolution strongly suggests discretization, which favors number theoretical realization. The hierarchies of effective Planck constants and p-adic length scale hierarchies reflect this hierarchy. What about the topological situation: can one order topologies to a hierarchy by their complexity and could this correspond to $O-S$ relationship?

The intuitive picture about many-sheeted space-time is as a hierarchical structure consisting of sheets condensed at larger sheets by wormhole contacts, whose throats carry fermion number. Intuitively, the larger sheet serves as an observer. p-Adic primes assignable to the space-time sheet could arrange them hierarchically and one could have entanglement between wavefunctions for the Minkowskian regions of the space-time sheets and the surface with a larger value for p would be in the role of O

3.3.1 Number theoretic view about measurement interaction

Quantum measurement involves also a measurement interaction. There must be an interaction between two different levels O and S of the hierarchy.

One can look at the measurement interaction from a number theoretic point of view.

1. For cognitive measurements the step forming the composite $O \circ S$ of polynomials would represent the measurement interaction. Before measurement interaction systems would be represented by O and S and measurement interaction would form $O \circ S$ and after the measurement the situation would be as proposed.

   Could one think that in BSFR the pair of uncorrelated surface defined by $O \times S$ with degree $n_O + n_S$ (analog for the additivity of classical degrees of freedom) is replaced with $O \circ S$ with degree $n_O \times n_S$ (analog for multiplicativity of degrees of freedom in tensor product) in BSFR? This would mean that the formation of $O \circ S$ is like a formation of an intermediate state in particle reaction or in chemical reaction.

   Could the subsequent SSFR cascade define a cascade of cognitive measurements [L23]. I have proposed that this occurs in all particle reactions. For instance, nuclear reactions involving tunneling would involve formation of dark nuclei with $h_{eff} > h$ in BSFR and a sequence of SSFRs in opposite time direction performing cognitive quantum measurement cascade [L16] and also the TGD based model for "cold fusion" relies on this picture [L6, L22]. After the SSFR cascade, a second BSFR would occur and bring back the original arrow of time and lead to the final state of the nuclear reaction.

   From the point of view of cognition, BSFR would correspond to the heureka moment and the sequences of SSFRs to the cognitive analysis decomposing the space-time surface defined by $O \circ S$ to pieces.

2. One can also consider small perturbations of the polynomials $O \circ S$ as a measurement interaction. For instance, quantum superpositions of space-time surfaces determined by polynomials depending on rational valued parameters are possible. The Galois groups for two polynomials with parameters which are near to each other are the same but for some critical values of the parameters the polynomials separate into products. This would reduce the Galois group effectively to a product of Galois groups. Quantum measurement could be seen as a localization in the parameter space [L25].

3.3.2 Topological point of view about measurement interaction

The measurement interaction can be also considered from the topological point of view.

1. Wormhole contacts are Euclidean regions of $X^4 \subset H$ couples two parallel space-time regions with Minkowskian signature and could give rise to measurement interaction. Wormhole contact carries a monopole flux and there must be a second monopole contact to make flux loop possible. This structure has an interpretation as an elementary particle, for instance a boson. The measurement interaction could correspond to the formation of this structure

and splitting by reconnection to flux loops associated with the space-time sheets after the interaction has ceased.

Remark: Wormhole contacts for $X^4 \subset H$ correspond in $M^8 - H$ duality images of singularities of $X^4 \subset M^8$. The quaternionic normal space at a given point is not unique but has all possible directions, which correspond to all points of CP_2. This is like the monopole singularity of an electric or magnetic field. At the level of CP_2 wormhole contact is the "blow-up" of this singularity.

2. Flux tube pairs connecting two systems serve also as a good candidate for the measurement interaction. U-shaped monopole flux tubes (see **Fig. 7**) are like tentacles and their reconnection creates a flux tube pair connecting two systems. SFR would correspond geometrically to the splitting of the flux tube pair by inverse re-connection.

3.3.3 Geometric view about SSFR

The considerations of [L24] strongly suggest the following picture about SSFRs.

1. In the measurement interaction a quantum superposition of functional composites of polynomials P_i defining the space-time surfaces of external states as Galois singlets is formed. A priori all orders for the composites in the superposition are allowed but if one requires that the same SSFR cascade can occur for all of them simultaneously, only single ordering and its cyclic permutations can be allowed.

   The SSFR cascade can of course begin with a reduction selection single permutation and its cyclic permutations: localization in S_n/Z_n would take place.

2. Incoming states at passive boundary of CD correspond to prepared states and outgoing states at active boundary to state function reduced states. The external states could correspond to products of polynomials as number theoretical correlates for the absence of correlations in unentangled states.

   Number theoretic existence for the scattering amplitudes [L33, L34] require that the p-adic primes characterizing the external states correspond to maximal ramified primes of the corresponding polynomials and therefore also to unique p-adic length scales L_p. In the interaction regions this ramified prime is the largest p-adic (that is ramified) prime for particles participating in the reaction. This correlation between polynomial and p-adic length scale allows a rather concrete geometric vision about what happens in the cascade.

3. SSFR cascade begins with a reduction of the state to a superposition of single composite with its cyclic variants for positive and negative energy parts separately: this kind of cyclic superpositions appear also in the twistor Grassmann picture [L24] and in string models. In the recent situation this makes possible a well-defined state preparation and SFR cascades at the two sides of CD. In ZEO, the cascade could take place for positive energy states only during SSFR.

4. A number theoretic SFR cascade would take place and decompose the Galois state group of the composite having decomposition to normal sub-groups to a product of states for the relative Galois groups for the composite.

   A given step of the cascade would be a measurement of a density matrix ρ producing information coded by its reduction probabilities as its eigenvalues in turn coded by the characteristic polynomial P_M of the density matrix.

The simplest guess is that the final state polynomial is simply the product $\prod P_{i_-}$ of the polynomials P_{i_-} for the passive boundary of CD and product $\prod P_{i_+}$ for the active boundary.

3.3.4 ZEO codes subjective memory as information about SSFR to the final state of SSFR

We have memories about the conscious experiences of the past. How are these memories formed? Zero energy ontology (ZEO) [L17] [K8] suggests a rather concrete model for the representations of the memories in terms of the geometry of the space-time surface.

Consider first a brief summary of ZEO.

1. The basic notions of ZEO are causal diamond (CD), zero energy state, and state function reduction (SFR). There are two kinds of SFRs: "small" SFRs (SSFRs) and "big" SFRs (BSFRs).

2. A sequence of SSFRs is the TGD counterpart for a sequence of repeated measurements of the same observables: in wave mechanics they leave the state unaffected (Zeno effect). Already in quantum optics, one must loosen this assumption and one speaks of weak measurements. In the TGD framework, SSFRs give rise to a flow of consciousness, self.

3. BSFR is the counterpart of the ordinary SFR. In the BSFR the arrow of the geometric time changes and BSFR means the death of self and to a reincarnation with an opposite arrow of geometric time. Death and birth as reincarnation with an opposite arrow of time are universal notions in the TGD Universe.

Consider now this view in more detail.

1. Causal diamond $CD = cd \times CP_2$ [L44] is the intersection of future and past directed light-cones of M^4. In the simplest picture, zero energy states are pairs of 3-D many-fermion states at the opposite light-like boundaries of the CD.

2. Zero energy states are superpositions of space-time surfaces connecting the boundaries of CD. These space-time surfaces obey holography, which is almost deterministic. holography=holomorphy principle allows their explicit construction as minimal surfaces and they are analogous to Bohr orbits when one interprets 3-surface as a generalization of a point-like particle. Already 2-D minimal surfaces fail to be completely deterministic (a given frame can span several minimal surfaces). This non-determinism forces ZEO: in absence of it one could have ordinary ontology with 3-D objects as basic geometric entities.

   The failure of complete determinism makes 4-dimensional Bohr orbits dynamical objects by giving them additional discrete degrees of freedom. They are absolutely essential for the understanding of memory and one can speak of a 4-dimensional brain.

3. The 3-D many-fermion states and the restriction of the wave function in WCW to a wave function to the space-of 3-surfaces as the ends of Bohr orbits at the passive boundary of CD are unaffected by the sequence of SSFRs. This is the counterpart for the Zeno effect. This requires that a given SSFR must correspond to a measurement of observables commuting with the observables which define the state basis at the passive boundary.

   The states at the opposite, active, boundary of CD are however affected in SSFRs and this gives rise to self and flow of consciousness. Also the size of CD increases in a statistical sense. The sequence of SSFRs gives rise to subjective time correlating with the increase of geometric time identifiable as the temporal distance between the tips of the CD. The arrow of time depends on which boundary of CD is passive and the time increases in the direction of the active boundary.

4. Ordinary SFRs correspond in TGD to BSFRs. Both BSFRs and SSFRs are possible in arbitrarily long scales since the h_{eff} hierarchy makes possible quantum coherence in arbitrary long scales.

   The new element is that the arrow of geometric time changes in BSFR since the roles of the active and passive boundaries of CD change. BSFR occurs when the set of observables measured at the active boundary no longer commutes with the set of observables associated with the passive boundary.

   The density matrix of the 3-D system characterizing the interaction of the 3-surface at the active boundary with its complement is a fundamental observable and if it ceases to commute with the observables at the active boundary, BSFR must take place.

Consider now what memory and memory recall could mean in this framework.

1. The view has been that active memory recall requires what might be regarded as communications with the geometric past. This requires sending a signal to the geometric past propagating in the non-standard time direction and absorbed by a system representing the memory (part of the brain or of its magnetic /field body). In the ZEO this is possible since BSFRs change the arrow of the geometric time.

2. The signal must be received by a system of geometric past representing the memory. This requires that 4-D space-time surfaces are not completely deterministic: Bohr orbits as 4-D minimal surfaces must have analogs of frames spanning the 2-D soap film, at which determinism fails. The seats of memories correspond to the seats of non-determinism as singularities of the space-time surface as a minimal surface.

3. How are the memories coded geometrically? This can be understood by asking what happens in SSFR. What happens is that from a set of 3-D final states at the active boundary some state is selected. This means a localization in the "world of classical worlds" (WCW) as the space of Bohr orbits. The zero energy state is localized to the outcome of quantum measurement. In ZEO the outcome therefore also represents the quantum transition to the final state! This is not possible in the standard ontology.

   The findings of Minev et al [L12] [L12] that in quantum optics quantum jumps correspond too smooth classical time evolutions leading from the initial state to the final state provide a direct support for this picture.

   ZEO therefore gives a geometric representation of a subjective experience associated with the SSFR. One obtains conscious information of this representation either by passive or active memory recall by waking up the locus of non-determinism assignable to the original conscious event. The slight failure of determinism for BSFRS is necessary for this. The sequence of SSFRs is coded to a sequence of geometric representations of memories about conscious events.

   This is how the Universe gradually develops representations of its earlier quantum jumps to its own state. Since the algebraic complexity of the Universe can only increase in a statistical sense the quantum hopping of the Universe in the quantum Platonia defined by the spinor fields of WCW implies evolution.

3.4 Negentropy Maximization Principle

Negentropy Maximization Principle (NMP) [L28] has been assumed to serve as the basic variational principle of TGD based quantum measurement theory giving rise to a theory of consciousness. It is now clear that the number theoretic vision implies NMP automatically [L42] as a mathematical analog of the second law. NMP reflects the unavoidable increase of the algebraic complexity of the Universe in the sequences of SSFRs, if it is determined by the holography=holomorphy principle. This increase of complexity is due to the fact that for a given extension of rationals, the number of extensions including it is infinitely larger than those included by it.

1. The adelic entanglement entropy is the sum of the real entanglement entropy and p-adic entropies. The adelic negentropy is its negative.

   The real part of adelic entropy is non-negative but p-adic negentropies can be positive. The sum of p-adic negentropies can be larger than the real entropy for non-trivial extensions of rationals. NMP is expected to take care that this is indeed the case. Second law for the real entropy would still hold true and guarantee NMP.

2. NMP states that SFRs cannot reduce the *overall* entanglement entropy although this can happen to subsystems. In SFRs this local reduction of negentropy would happen. Entanglement is not destroyed in SFRs in general and new entanglement negentropy can be generated.

3. Although real entanglement entropy tends to increase, the positive p-adic negentropies assignable to the cognition would do the same so that net negentropy would increase. This would not mean only entanglement protection, but entanglement generation and cognitive evolution. This picture is consistent with the paradoxical proposal of Jeremy England [I1] [L4] that biological evolution involves an increase of entropy.

4. It should be noticed that the increase of real entanglement entropy as such does not imply the second law. The reduction of real entropy transforms it to ensemble entropy since the outcome of the measurement is random. This entropy is entropy of fermions at space-time sheets. The fermionic entanglement would be reduced but transformed to Galois entanglement.

3.5 About TGD based description of entanglement

The general classification of possible quantum entanglements is an interesting challenge and there are many approaches (`https://cutt.ly/iREIg1u`). One interesting approach relies on the irreducible representations of the unitary group $U(n)$ acting as the isometry group of n-D Hilbert space (`https://cutt.ly/ZREIEAT`). The assumption about irreducibility is however not essential for what follows.

1. A system with n-D state space H_n identified as a sub-system of a larger system with N-D state space H_N can entangle with its $M = N - n$-D complement H_M. Suppose $n \leq M$. Entanglement implies that the n-D state space or its sub-space is embedded isometrically into a subspace of the M-D state space. For a non -trivial subspace one can replace H_n with this subspace H_m in what follows. The diagonal form of the density matrix describes this correspondence explicitly. If the subspace is 1-D one has an unentangled situation.

2. $U(n)$ and its subgroups act as automorphism groups of H_n This inspires the idea that the irreducible representations of $U(n)$ define physically very special entanglements $H_n \subset H_M$. The isometric inclusions $H_n \subset H_M$ are parametrized by a flag-manifold $F_{n,M} = U(M)/U(n) \times U(M-n)$. If one allows second quantization in the sense that the wave functions in the space of entanglements make sense, this flag manifold represents additional degrees of freedom for entanglements $H_n \subset H_M$. If the entanglement does not have maximal dimension, the product of flag manifolds $F_{n,M}$ and $F_{m,n}$ characterizes the space of entanglements.

3. Flag manifold has a geometric interpretation as the space of n-D spaces C^n (flags) embedded in C^M. Interestingly,twistor spaces and more general spaces of twistor Grassmannian approach are flag manifolds and twistor spaces are also related to Minkowski space.

4. I have not been personally enthusiastic about the notion of emergence of 3-space or space-time from entanglement but one can wonder whether flag manifolds related naturally to entanglement could lead to the emergence of Minkowski space. Or perhaps better, whether the notion of entanglement and Minkowski space could be natural aspects of a more general description.

5. One can also have flags inside flags inside leading to more complex flag manifolds $F(n_1, n_2, .., n_k = M) = U(M)/U(m_1) \times ... \times U(m_k)$, $m_k = n_k - n_{k-1}$ assuming $n_0 = 0$. In consciousness theories, the challenge is to understand the quantum correlates of attention. Entanglement is the most obvious candidate in this respect. Attention seems to be something with a directed arrow. This is difficult to understand in terms of the ordinary entanglement. Flag hierarchy would suggest a hierarchical structure of entanglement in which the system entangles with a higher-D system, which entangles with a higher-D system. In this picture the state function reduction would be replaced by a cascade starting from the top.

6. The analog of flags inside flags is what happens in what I call number theoretic measurement cascades for wavefunctions [L23] in the Galois groups which are associated with extension of extensions of..... The already mentioned cognitive measurement cascade corresponds to a hierarchy of normal subgroups of Galois group and one can perhaps say that discret Galois group replaces the unitary group. Each normal subgroup in the hierarchy is the Galois group of the extension of the extension below it. This automatically realizes the hierarchical entanglement as an attentional hierarchy. The cognitive measurement cascade can actually start at any level of the hierarchy of extensions of extensions and if it starts from the top all factors are reduced to a pure state.

   If the polynomials defining the 4-surfaces in M^8 satisfy $P(0) = 0$, the the composite polynomial $P_n \circ P_{n-1} ... \circ P_1$ has the roots of $P_1, ..., P_{n-1}$ as its roots. In this case the inclusion of state spaces are unique so that flag manifolds are not needed.

4 Description of particle interactions in TGD

The key ideas of quantum TGD distinguish between TGD and QFTs were already described in the introduction. Many-particle interactions have two aspects: the classical geometric description, which QFTs do not allow, and the description in terms of fermions (bosons do not appear as primary quantum fields in TGD).

1. At the classical level, particle reactions correspond to topological reactions, where the 3-surface splits to, say, into two pieces. This is exactly what we see in particle experiments quite concretely. For instance, a closed monopole flux tube (see **Fig. ??**) representing an elementary particle decomposes to into two in a 3-particle vertex.
2. There is a field-particle duality realized geometrically. The minimal surface as a holomorphic solution of the field equations defines the generalization of the light-like world line of a massless particle as a Bohr orbit as a 4-surface. The equations of the minimal surface in turn state the vanishing of the generalized acceleration of a 3-D particle identified as 3-surface. At the field level, minimal surfaces satisfy the analogs of the field equations of a massless free field. They are valid everywhere except at 2-D singularities associated with 3-D light-like parton trajectories. At singularities the minimal surface equation fails since the generalized acceleration becomes infinite rather than vanishing. Also holomorphy fails.

What can one say about the singularity? Does the singularity correspond to an entire light-like partonic orbit, to a 2-D partonic surface as a section of partonic orbit, to mere light-like fermion line along it, to string world sheet, or to a point-like particle reaction vertex as has been assumed hitherto?

1. The number theoretical considerations related to holography=holomorphy vision realized using $P = 0$ option imply that the light-like fermion lines along a light-like partonic orbit are theoretically very special and analogous to the lines of criticality for cusp catastrophe.

   The fermion line is a static structure and one cannot assign to it acceleration in M^4 degrees of freedom as the trace of the second fundamental form defining a generalized acceleration. What could diverge is the CP_2 part of the generalized acceleration having an interpretation as an analog of the Higgs field.

   The light-like coordinate along the light-like fermion line is constant so that metrically it is analogous to a pole of an analytic function. On the other hand, topologically the fermion line is analogous to a cut and it would form part of a boundary of a string world sheet having interpretation as a generalization of cut. The analogy of the fermion line as the critical line of the catastrophe graph of cusp catastrophe suggested that the trace of the second fundamental form in CP_2 degrees of freedom indeed diverges. There would be a fold in the CP_2 direction. A possible interpretation is that the fermion line serves as a source of various gauge fields.
2. There is also another kind of singularity: in a creation of a fermion pair in the splitting of closed monopole flux tube, the point at which the fermion and antifermion lines begin would correspond to singularity as an analog for an edge at which fermion turns backwards in time. This singularity is analogous to the vertex of a cusp catastrophe at which 2 folds meet.

   This kind of turning point is analogous to an edge on the path of a Brownian particle. A monopole flux tube would decay to two so that the topological viewpoint of a particle emission would be in question. Also at this singularity one expects the holomorphy and minimal surface property to fail. In fact this could be true inside the entire intermediate wormhole contact formed in the process.
3. String world sheets are assumed to connect the 3-D partonic orbits of different monopole flux tubes. The interpretation of string as an analog of a cut is suggestive. For the cuts of analytic function $z^{1/n}$, the real axis is a seat of discontinuity. By introducing the notion of n-fold covering space of the complex plane one gets rid of the discontinuity. Could the string world sheets correspond to the orbits of stringy singularities such that 2π rotation around the singularities in the generalized complex coordinates of H lead to a point at a

different space-time sheet and one obtains n-fold covering. The analogy with line charges suggests that it makes sense to assign dynamics to these objects.

n-fold covering suggests an n-fold value of the effective Planck constant and the interpretation as a dark phase as n-fold covering of M^4 or subspace M^2. Also n-fold coverings of CP_2 or its geodesic sphere S^2 are possible and would look like copies of M^4 regions inside which the map from CP_2 to M^4 is 1-valued. Now the sheets of the covering would correspond to different copies, for instance flux tubes.

Consider now what happens in singularities from the perspective of field equations.

1. At singularities defined as loci where the minimal surface property fails, the field equations for the *entire* action are valid, but are not separately true for various parts of the action. Generalized holomorphy breaks down. The fermion lines as singularities are completely analogous to the poles of an analytic function in 2-D case and there is analogy with the 2-D electrostatics, where the poles of analytic function correspond to point charges. The fermion lines are boundaries of string world sheets and these are analogous to cuts as line charges. The cut disappears as a singularity when the complex plane is replaced with its covering and the same occurs at the space-time level.

2. The field equations at the singularity give the TGD counterparts of Einstein's equations, analogs of geodesic equations, and also the analogy Newton's F=ma (also in CP_2 degrees of freedom). The generalized 8-D acceleration H^k defined by the trace of the second fundamental form, is localized on the singularities. Singularities can be seen as analogs for the sources of gauge fields. This interpretation could apply to fermion lines analogous to line charges, which correspond to cuts of an analytic function in 2-D electrostatics. At these singularities the CP_2 part of the generalized acceleration H^k could diverge.

   Singularities in a stronger sense can be interpreted as sources for various fermion currents and supercurrents implying their apparent non-conservation at the singularity. This kind of singularity could correspond to a vertex at which a fermion-antifermion pair is created and fermion number conservation is apparently violated since the fermion line ends. Actually only the separate conservation of fermion and antifermion numbers is broken. Here also the M^4 part of H^k would diverge.

4.1 Interaction vertices

How to get the TGD counterparts of the QFT vertices? I have discussed this in [L40, L53]. In this discussion I assumed that the singularities correspond to 2-D minimal surfaces at light-like partonic orbits or to point-like singularities associated with them. It is now clear [L49] that vertices naturally correspond to point-like singularities at which the closed monopole flux tube splits into two. Fermion lines can correspond to static singularities as counterparts of point-like charges.

1. Vertices typically contain a fermion-antifermion pair and the gauge potential, which is second quantized. Now, classical gauge potentials are not second quantized and there is no path integral. How to obtain the basic gauge theory vertices?

   This is where the standard approximation of QFTs helps intuition: replace the quantized boson field with a classical one. Also in path integral formalism classical boson fields appear in the vertices. This gives the vertex corresponding to the creation of a pair of fermions. Thanks to that, only the fermion and the sum of the antifermion numbers are conserved and the theory does not reduce to a free field theory. One should be able to do the same now. However, the precise formulation of this vision is far from trivial.

2. The modified Dirac action should give elementary particle vertices for a given 4-D Bohr trajectory as orbit of 3-surface representing particle. There are two options:

   (a) Modified gamma matrices are defined as contractions of the ordinary gamma matrices of the embedding space H with the canonical momentum currents associated with the classical action defining the space-time surface. Supersymmetry is now exact: besides color and Poincare super generators there is an infinite number of conserved super symplectic generators and infinitesimal generalized superholomorphisms.

This option does not work: the modified Dirac equation implies that the Dirac action and also vertices vanish identically. Although one has partonic 2-surfaces as singularities of minimal surfaces defining vertices, the theory is trivial because the usual perturbation theory, treating gauge potentials, as a small perturbation fails.

(b) Modified gamma matrices are replaced by the induced gamma matrices defined by the volume term (cosmological term of the classical action). Supersymmetry is broken but only at the 2-D vertices. The anticommutator of the induced gammas gives the induced metric. This is not true for the modified gammas defined by the entire action: in this case the anticommutators are rather complex, being bilinear in the canonical momentum currents. Is it possible to have a non-trivial theory despite the breaking of supersymmetry at vertices or or does the supersymmetry breaking make possible a non-trivial theory?

It seems that induced gamma matrices predicting supersymmetry breaking allow a non-trivial theory.

1. In 2-D vertices, the generalized acceleration field H^k is proportional to the 2-D delta function and gives rise to the graviton and Higgs vertices. One obtains also the vertices related to gauge bosons from the coupling of the induced spinor field to induced spinor connection. Only the couplings to electroweak gauge potentials and $U(1)$ Kähler gauge potential of M^4 are obtained. The failure of the generalized holomorphy is absolutely essential.

2. Color degrees of freedom are completely analogous to translational degrees of freedom since color quantum numbers are not spin-like in TGD. Strong interactions are vectorial and correspond to Kähler gauge potentials.

3. Generalized Brownian motion gives the vertices. One obtains the equivalents of Einstein's and Newton's equations at the vertices. The M^4 part M^k of the generalized acceleration is related to the gravitons and the CP_2 part S^k to the Higgs field. Spin $J = 2$ for graviton is due to the rotational motion of the closed monopole flux tube associated with the gravitation giving an additional unit of spin besides the spin of M^k, which is $S = 1$ [L53].

4.2 How to obtain pair creation?

Consider now the description of fermion pair creation.

1. Intuitively, the creation of a fermion pair (and thus also a boson) corresponds to the fermion turning backwards in time. At the level of the geometry of the space-time surface, this corresponds to the partonic 2-surface turning backwards in time, and the same happens to the corresponding fermion line. Turning back in time means that effectively the fermion current is not conserved: if one does not take into account that the parton surface turns in the other direction of time, the fermion disappears effectively and the current must has a singular divergence. This is what the divergence of the generalized acceleration means.

2. This implies that the separate conservation is lost for fermion and antifermion numbers. This means breaking of supersymmetry, of masslessness, of generalized holomorphy and also the generation of the analog of Higgs vacuum excitation as CP_2 part S^k of the generalized acceleration H^k. The Higgs vacuum expectation is only at the vertices. But this is exactly what is actually wanted! No separate symmetry breaking mechanism is needed!

3. The failure of the generalized holomorphy at the 2-D vertex means that the holomorphic partonic orbit turns at the singularity to an antiholomorphic one. For the annihilation vertex it could occur only for the hypercomplex part of the generalized complex structure. Note that bosons would be pairs of wormhole throats associated with parallel Minkowskian space-time sheets, which correspond to analytic and anti-analytics surfaces.

(a) Could the generalized complex conjugation of the space-time surface by replacing analytic imbedding by antianalytic imbedding [$(z \rightarrow f(z)) \rightarrow (z \rightarrow \overline{f(z)})$ is a 2-D example]. f_1 and f_2 would be replaced with their conjugates but X^4 would still correspond to a subset of generalized complex coordinates: otherwise the space-time surface is not affected. If the functions involved are obtained from real-analytic functions, this means that H coordinates are conjugated while X^4 coordinates remain as such. General coordinate invariance allows however to use complex conjugates as variables so that the space-time surface is not affected.

(b) One can also consider partial conjugations acting in both H and in X^4 applied to function f_i. Hypercomplex conjugation is certainly involved and could relate to time reflection T. Complex conjugation of the M^4 complex coordinate w could relate to reflection P and the conjugation of the CP_2 coordinates could relate to charge conjugation C. All these conjugations together, that is CPT does not affect the space-time surface if the polynomials have real coefficients or if also their coefficients are conjugated.

The reversal of the arrow of time occurring in a "big" state function reduction changes the role of fermionic creation and annihilation operators. For these states the classical signals propagate backwards in geometric time. The CP conjugates cannot correspond to quantum states with a reversed arrow of time so that CP does not exchange the roles of annihilation and creation operators. It is not quite clear whether this exchange should be included with T and therefore also with CPT.

4. Remarkably, the states associated with connected 4-surfaces consist of either fermions or antifermions but not both. This could explain matter antimatter asymmetry if quantum coherence is possible in arbitrarily long scales. In TGD, space-time surfaces decompose to regions containing either matter or antimatter and, by the presence of quantum coherence even in cosmological scales, these regions can be very large. The quantum coherence in large scales is implied by the number theoretic vision predicting a hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter [L47]. Note however that bosons as bound states of fermions involve partonic orbits, which differ by the conjugation of the hypercomplex coordinate.

4.3 About the geometric description of fermion pair creation vertex

The intuitive idea is that vertices somehow correspond to partonic 2-surfaces X^2 on one hand and to the points of X^2 at which fermion lines turn back in time. Is the fundamental vertex a 2-D parton surface or is it point-like or are both aspects involved? This is the basic question. One must clarify what happens topologically in the creation of a fermion pair. A non-vanishing monopole flux means homological non-triviality of the partonic 2-surfaces.

1. The catastrophe theory picture suggests that there is a hierarchy of singularities such that the number of co-incinding roots and criticality increases. In the case of cusp catastrophe, the tip of the cusp would correspond to the situation in which all 3 roots of the third order catastrophe polynomial coincide. There are good reasons to expect that also now this is the generic situation. This would suggest that a point-like vertex initiates a process leading to a creation of a pair of wormhole contacts with opposite monopole fluxes.

2. What happens that a single closed monopole flux tube splits two two monopole flux tubes. This flux tube has portions at parallel Minkowskian space-time sheets with distance of order CP_2 radius and they carry opposite monopole fluxes.

   First these two monopole flux tubes touch each other at some point. At this point the signature of the induced metric must be between Minkowskian and Euclidean signatures so that the induced 4 metric is degenerate and the point can belong to a light-like partonic orbit. This point serves as the point-like vertex for the creation of a fermion pair at which the fermion turns backwards in time.

3. The touching point starts to grow to an Euclidean 3-D region and gives gradually rise to a wormhole contact, which involves two opposite monopole flux fluxes summing up to zero.

This makes it possible for the wormhole contact to split to two wormhole contacts with opposite monopole fluxes serving as the "ends" of two closed and separate closed monopole fluxes carrying fermion *resp.* antifermion at the corresponding parton orbits.

4. One can also think that the monopole flux tube split instantaneously in analogy with the instantaneous joining of two cylinders along their ends. This is however not the generic situation.

4.4 Exotic smooth structures make possible fermion pair creation

What is the precise mathematical formulation of the proposed vision? This is where a completely unique feature of 4-dimensional manifolds comes in: they allow exotic smooth structures. Exotic smooth structure is the standard smooth structure with lower-dimensional defects. In TGD, the defects correspond in TGD to 2-D parton vertices as "edges" of Brownian motion. In the exotic smooth structure, the edge disappears and everything is soft. Pair creation and non-trivial theory is possible only in dimension $D = 4$ [L37, L53].

4.5 Challenging some details of the recent view of TGD

The development of the mathematical TGD has been a sequence of simplifications and generalizations. Holography = holomorphy vision removes path integral from quantum physics and together with the number theoretic vision might make the bosonic action unnecessary. This means that this vision allows us to solve field equations explicitly and the solution does not depend on the bosonic action. TGD allows to get rid of primary bosonic fields and fermions are free free fermions at the level of the imbedding space and their localization to space-time surfaces makes them interaction. Pair creation is made possible by the presence of exotic smooth structures possible only in 4-D space-time. This however leads to a problem with the sign of energy. This problem disappears when one realizes that fundamental fermions can have tachyonic momenta and that only the physical l states as their bound states, which are Galois singlets, have non-negative mass squared and positive energy.

4.5.1 Could the classical bosonic action completely disappear from TGD?

Number theoretic vision of TGD and holography = holomorphy principle [L51, L52] forces to challenge the necessity of the classical bosonic action.

1. Any general coordinate action defining the Kähler function K and constructible in terms of the induced geometry gives the same minimal space-time surfaces as extremals and only the boundaries and partonic orbits depend on the action since the boundary conditions stating conservation laws depend on the action. Spinor lift suggests Kähler action for the 6-D twistor surfaces as a unique action principle. But is it necessary?

2. The conjecture $exp(K) \propto D^n$, n an integer, or its generalization to $exp(K) \propto D\overline{D}^n$, where D is a product of discriminants for the polynomials assignable to partonic 2-surfaces define a discrete set of points as their roots, would allow to express vacuum functional completely in terms of number theory. Coupling parameters would be present but evolve in such a way that the condition would hold true.

3. The discriminant D is defined also when the roots assignable to the partonic 2-surfaces are real or even complex numbers. This would conform with the strong form of holography. One could get completely rid of the bosonic action principle. The holomorphy = holography principle would automatically give the non-linear counterpart of massless fields satisfied by the space-time surfaces as minimal surfaces. Could the classical action completely disappear from the theory?

4.5.2 Could the fermionic interaction vertices be independent of the bosonic action principle

Could the interaction vertices for fermions be independent of the bosonic action principle?

1. The long-held idea is [L45, L53, L40], the vertices appearing in the scattering amplitudes are determined by the modified Dirac equation [L50] determined by the bosonic action associated with the partonic orbits as couplings to the induced gauge potentials. Twistor lift suggests that this action contains volume term and Kähler action.

   But is the modified Dirac action necessary or even physically plausible? The problem is that for a general bosonic action the modified gamma matrices, defined in terms of canonical momentum currents, do not commute to the induced metric unlike the modified Dirac action determined by the mere volume term of the bosonic action. This led to the proposal that this option, consistent also with the fact that, irrespective of the bosonic action, space-time surfaces are minimal surfaces outside singularities at which generalized holomorphy fails, is more plausible.

2. Fermion pair creation (and emission of bosons as Galois singlet bound states of fermions and antifermions is possible only for 4-D space-time surfaces. The existence of exotic smooth structures in dimension $D = 4$ [L15] makes possible pair creation vertices [L53, L40]. A given exotic smooth structure corresponds to the unique standard ordinary smooth structure with defects and vertices would correspond to defects at which the fermion line turns backwards in time. The defects would be associated with partonic 2-surfaces at which the generalized holomorphy of the function pair (f_1, f_2) with respect to generalized complex coordinates of H (one of them is hypercomplex coordinate) fails, perhaps only at the defect.

3. There is an objection against this proposal. The creation of fermion pairs with opposite sign of single fermionic energy suggests that a given light-like boundary of CD can contain fermions with both signs of energy. This does not conform with the assumption that the sign of the *single* particle energy is fixed and opposite for the opposite boundaries of CD. Should one only require that the total energy has a fixed sign at a given boundary of the CD?

   Could one only require that the sign of the energy is fixed only for physical states formed as many-fermions states and identified as Galois singlets and that the physical states can also contain negative energy tachyonic fermions or antifermions. Could this make sense mathematically?

4.5.3 Extension of the fermionic state space to include tachyonic fundamental fermions as analogs of virtual fermions

I recently received from Paul Kirsch a link to an interesting article about the possibility to describing tachyons in a mathematically consistent way [B3] (see this). The basic problem is that for tachyons the number of positive energy particles is not well-defined since Lorentz transformation can change positive energy tachyons to negative energy tachyons and vice versa. The proposed solution of the problem is the doubling of the Hilbert space which includes both incoming and outgoing states. To me this looks like a mathematically sensible idea and might make sense also physically.

Surprisingly, this proposal has a rather concrete connection with zero energy ontology (ZEO).

1. In the simplest formulation of ZEO, the fermionic vacua at the passive *resp.* active boundaries of CD correspond to the fermionic vacua annihilated by annihilation operators *resp.* creation operators as their hermitian conjugates. In the standard QFT only the second vacuum is accepted and this allows only a single arrow of geometric time.

2. ZEO allows both arrows and a given zero energy state is a state pair for which the fermionic state at the passive boundary of CD remains fixed during the sequence of small state function reductions (SSFRs) and corresponding time evolution which lead to the increase of CD in a statistical sense. The state at the active boundary changes and this corresponds to the subjective time evolution of a conscious entity, self. SSFRs are the TGD counterparts of repeated measurements for observables which commute with the observables whose eigenstates the states at the passive boundary are.

3. The doubled state space is highly analogous to the space of fermionic states in ZEO involving positive and negative energy physical particles at the opposite boundaries of CD. If one also

allows single fermion tachyonic states then one could have fermions with wrong sign of energy at a given boundary of CD. If bosons correspond to fermion-antifermion pairs such that either fermion or antifermion is tachyonic, one obtains boson emission and physical bosons can have correct sign of mass squared. In the vertex identified as a defect of the standard spinor structure, either fermion or antifermion would be tachyonic. Since several vertices involving the change of the sign of the fermion or antifermion momentum are possible, outgoing physical fermions and antifermions with a correct sign or energy can be produced. Recall that both the physical leptons and quarks involve fermion-antifermion pairs in the recent picture based on closed monopole flux tubes associated with a pair of Minkowskian space-time sheets.

4. Tachyonic single fundamental fermion states (quarks or leptons) are natural in the number theoretic vision of TGD. The components of the fermionic momenta for a given extension of rationals are algebraic integers and mass squared for them can be tachyonic. These states are analogs of virtual fermions of the standard QFT which also can have tachyonic momenta. Physical states are assumed to be Galois singlets so that the total momentum for a bound state of fermions and antifermions has integer valued components and mass squared is integer. The condition that mass squared energy have a fixed sign for the physical states at a given boundary of the CD is natural and has been made.

5 Appendix

5.1 Glossary and figures

The following glossary explains some basic concepts of TGD and TGD inspired biology.

- ***Space-time as surface***. Space-times can be regarded as 4-D surfaces in an 8-D space $M^4 \times CP_2$ obtained from empty Minkowski space (M^4) by adding four small dimensions (CP_2). The study of field equations characterizing space-time surfaces as "orbits" of 3-surfaces (3-D generalization of strings) forces the conclusion that the topology of space-time is non-trivial in all length scales.

- ***Geometrization of classical fields***. Both weak, electromagnetic, gluonic, and gravitational fields are known once the space-time surface in H as a solution of field equations is known.

- ***Many-sheeted space-time*** (see **Fig. 1**) consists of space-time sheets with various length scales with smaller sheets being glued to larger ones by ***wormhole contacts*** (see **Fig. 2**) identified as the building bricks of elementary particles. The sizes of wormhole contacts vary but are at least of CP_2 size (about 10^4 Planck lengths) and thus extremely small.

  Many-sheeted space-time replaces reductionism with ***fractality*** . The existence of scaled variants of physics of strong and weak interactions in various length scales is implied, and biology is especially interesting in this respect.

- ***Topological field quantization (TFQ)*** . TFQ replaces classical fields with space-time quanta. For instance, magnetic fields decompose into space-time surfaces of finite size representing flux tubes or -sheets. Field configurations are like Bohr orbits carrying "archetypal" classical field patterns. Radiation fields correspond to topological light rays or massless extremals (MEs), magnetic fields to magnetic flux quanta (flux tubes and sheets) having as primordial representatives "cosmic strings", electric fields correspond to electric flux quanta (e.g. cell membrane), and fundamental particles to CP_2 type vacuum extremals.

- ***Field body*** (FB) and ***magnetic body*** (MB). Any physical system has field identity - FB or MB - in the sense that a given topological field quantum corresponds to a particular source (or several of them - e.g. in the case of the flux tube connecting two systems).

Maxwellian electrodynamics cannot have this kind of identification since the fields created by different sources superpose. Superposition is replaced with a set theoretic union: only the *effects* of the fields assignable to different sources on test particle superpose. This makes it possible to define the QFT limit of TGD.

- ***p-Adic physics*** [K7] as a physics of cognition and intention and the fusion of p-adic physics with real number based physics are new elements.
- ***Adelic physics*** [L9, L11] is a fusion of real physics of sensory experience and various p-adic physics of cognition.
- ***p-Adic length scale hypothesis*** states that preferred p-adic length scales correspond to primes p near powers of two: $p \simeq 2^k$, k positive integer.
- A ***Dark matter hierarchy*** realized in terms of a hierarchy of values of effective Planck constant $h_{eff} = nh_0$ as integers using $h_0 = h/6$ as a unit. Large value of h_{eff} makes possible macroscopic quantum coherence which is crucial in living matter.
- ***MB as an intentional agent using biological body (BB) as a sensory receptor and motor instrument*** . The personal MB associated with the living body - as opposed to larger MBs assignable with collective levels of consciousness - has a hierarchical onion-like layered structure and several MBs can use the same BB making possible remote mental interactions such as hypnosis [L3].
- ***Cosmic strings Magnetic flux tubes*** belong to the basic extremals of practically any general coordinate invariant action principle. Cosmic strings are surfaces of form $X^2 \times Y^2 \subset M^4 \times CP_2$. X^2 is analogous to string world sheet. Cosmic strings come in two varieties and both seem to have a deep role in TGD.

  Y^2 is either a complex or Lagrangian 2-manifold of CP_2. Complex 2-manifold carries monopole flux. For Lagrangian sub-manifold the Kähler form and magnetic flux and Kähler action vanishes. Both types of cosmic strings are are simultaneous extremals of both Kähler action and volume action: this holds true quite generally for preferred extremals.

  Cosmic strings are unstable against perturbations thickening the 2-D M^4 projection to 3-D or 4-D: this gives rise to monopole (see **Fig. 7**) and non-monopole magnetic flux tubes. Using $M^2 \times Y^2$ coordinates, the thickening corresponds to the deformation for which $E^2 \subset M^4$ coordinates are not constant anymore but depend on Y^2 coordinates.
- ***Magnetic flux tubes and sheets*** serve as "body parts" of MB (analogous to body parts of BB), and one can speak about magnetic motor actions. Besides concrete motion of flux quanta/tubes analogous to ordinary motor activity, basic motor actions include the contraction of magnetic flux tubes by a phase transition possibly reducing Planck constant, and the change in thickness of the magnetic flux tube, thus changing the value of the magnetic field, and in turn the cyclotron frequency. Transversal oscillatory motions of flux tubes and oscillatory variations of the thickness of the flux tubes serve as counterparts for Alfwen waves.

  Reconnections of the U-shaped flux tubes allow two MBs to get in contact based on a pair of flux tubes connecting the systems and temporal variations of magnetic fields inducing motor actions of MBs favor the formation of reconnections.

  In hydrodynamics and magnetohydrodynamics reconnections would be essential for the generation of turbulence by the generation of vortices having monopole flux tube at core and Lagrangian flux tube as its exterior.

  Flux tube connections at the molecular level bring a new element to biochemistry making it possible to understand bio-catalysis. Flux tube connections serve as a space-time correlates for attention in the TGD inspired theory of consciousness.
- ***Cyclotron Bose-Einstein condensates (BECs)*** of various charged particles can accompany MBs. Cyclotron energy $E_c = hZeB/m$ is much below thermal energy at physiological temperatures for magnetic fields possible in living matter. In the transition $h \rightarrow h_{eff}$

E_c is scaled up by a fractor $h_{eff}/h = n$. For sufficiently high value of h_{eff} cyclotron energy is above thermal energy $E = h_{eff}\ ZeB/m$. Cyclotron Bose-Einstein condensates at MBs of basic biomolecules and of cell membrane proteins - play a key role in TGD based biology.

- ***Josephson junctions*** exist between two superconductors. In TGD framework, ***generalized Josephson junctions*** accompany membrane proteins such as ion channels and pumps. A voltage between the two super-conductors implies a ***Josephson current***. For a constant voltage the current is oscillating with the ***Josephson frequency*** . The Josephson current emits ***Josephson radiation***. The energies come as multiples of ***Josephson energy***.

  In TGD generalized Josephson radiation consisting of dark photons makes communication of sensory input to MB possible. The signal is coded to the modulation of Josephson frequency depending on the membrane voltage. The cyclotron BEC at MB receives the radiation producing a sequence of resonance peaks.

- ***Negentropy Maximization Principle*** (NMP). NMP [K6] [L28] is the variational principle of consciousness and generalizes SL. NMP states that the negentropy gain in SFR is non-negative and maximal. NMP implies SL for ordinary matter.

- ***Negentropic entanglement*** (NE). NE is possible in adelic physics and NMP does not allow its reduction. NMP implies a connection between NE, the dark matter hierarchy, p-adic physics, and quantum criticality. NE is a prerequisite for an experience defining abstraction as a rule having as instances the state pairs appearing in the entangled state.

- ***Zero energy ontology (ZEO)*** In ZEO physical states are pairs of positive and negative energy parts having opposite net quantum numbers and identifiable as counterparts of initial and final states of a physical event in the ordinary ontology. Positive and negative energy parts of the zero energy state are at the opposite boundaries of a ***causal diamond*** (CD, see **Fig. 4**)) defined as a double-pyramid-like intersection of future and past directed light-cones of Minkowski space.

  CD defines the "spot-light of consciousness": the contents of conscious experience associated with a given CD is determined by the space-time sheets in the embedding space region spanned by CD.

- ***SFR*** is an acronym for state function reduction. The measurement interaction is universal and defined by the entanglement of the subsystem considered with the external world [L17] [K8]. What is measured is the density matrix characterizing entanglement and the outcome is an eigenstate of the density matrix with eigenvalue giving the probability of this particular outcome. SFR can in principle occur for any pair of systems.

  SFR in ZEO solves the basic problem of quantum measurement theory since the zero energy state as a superposition of classical deterministic time evolutions (preferred extremals) is replaced with a new one. Individual time evolutions are not made non-deterministic.

  One must however notice that the reduction of entanglement between fermions is not possible since Fermi- and als Bose statistics predicts a maximal entanglement. Entanglement reduction must occur in WCW degrees of freedom and they are present because point-like particles are replaced with 3-surfaces. They can correspond to the number theoretical degrees of freedom assignable to the Galois group - actually its decomposition in terms of its normal subgroups - and to topological degrees of freedom.

- ***SSFR*** is an acronym for "small" SFR as the TGD counterpart of weak measurement of quantum optics and resembles classical measurement since the change of the state is small [L17] [K8]. SSFR is preceded by the TGD counterpart of unitary time evolution replacing the state associated with CD with a quantum superposition of CDs and zero energy states associated with them. SSFR performs a localization of CD and corresponds to time measurement with time identifiable as the temporal distance between the tips of CD. CD is scaled up in size - at least in statistical sense and this gives rise to the arrow of time.

  The unitary process and SSFR represent also the counterpart for Zeno effect in the sense that the passive boundary of CD as also CD is only scaled up but is not shifted. The states remain unchanged apart from the addition of new fermions contained by the added part of the passive boundary. One can say that the size of the CD as analogous to the perceptive field means that more and more of the zero energy state at the passive boundary becomes visible. The active boundary is however both scaled and shifted in SSFR and states at it change. This gives rise to the experience of time flow and SSFRs as moments of subjective time correspond to geometric time as a distance between the tips of CD. The analog of unitary time evolution corresponds to "time" evolution induced by the exponential of the scaling generator L_0. Time translation is thus replaced by scaling. This is the case also in p-adic thermodynamics. The idea of time evolution by scalings has emerged also in condensed matter physics.

- ***BSFR*** is an acronym for "big" SFR, which is the TGD counterpart of ordinary state function reduction with the standard probabilistic rules [L17] [K8]. What is new is that the arrow of time changes since the roles of passive and active boundaries change and CD starts to increase in an opposite time direction.

  This has profound thermodynamic implications. Second law must be generalized and the time corresponds to dissipation with a reversed arrow of time looking like self-organization for an observed with opposite arrow of time [L14]. The interpretation of BSFR is as analog of biological death and the time reversed period is analogous to re-incarnation but with non-standard arrow of time. The findings of Minev *et al* [L12] give support for BSFR at atomic level. Together with h_{eff} hierarchy BSFR predicts that the world looks classical in all scales for an observer with the opposite arrow of time.

5.2 Figures

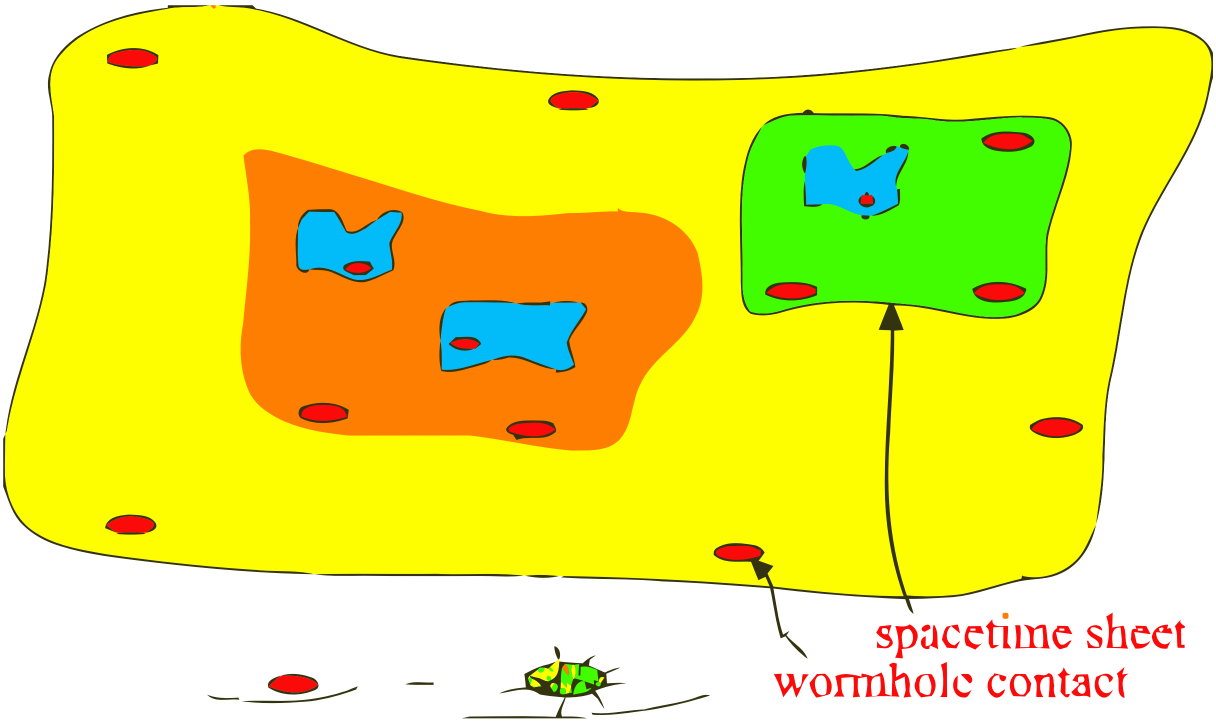

Figure 1: Many-sheeted space-time.

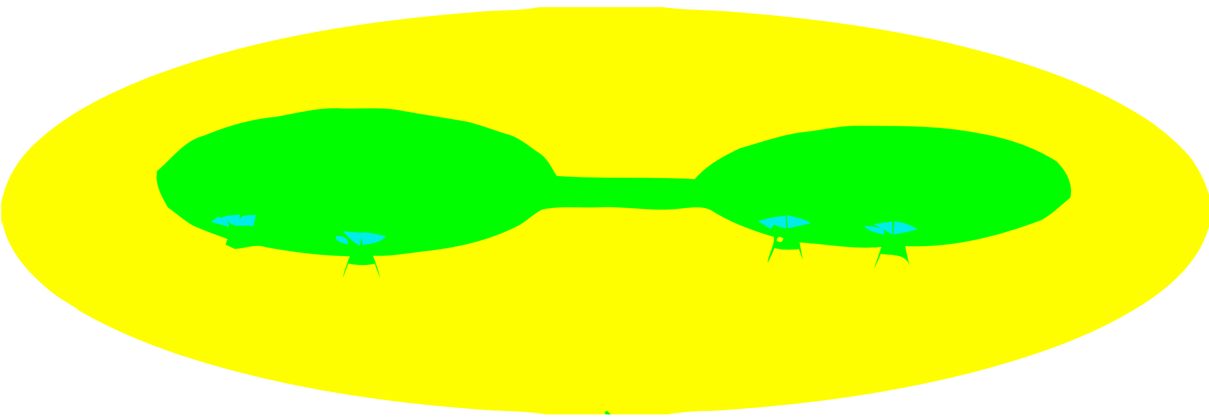

Figure 2: Wormhole contacts.

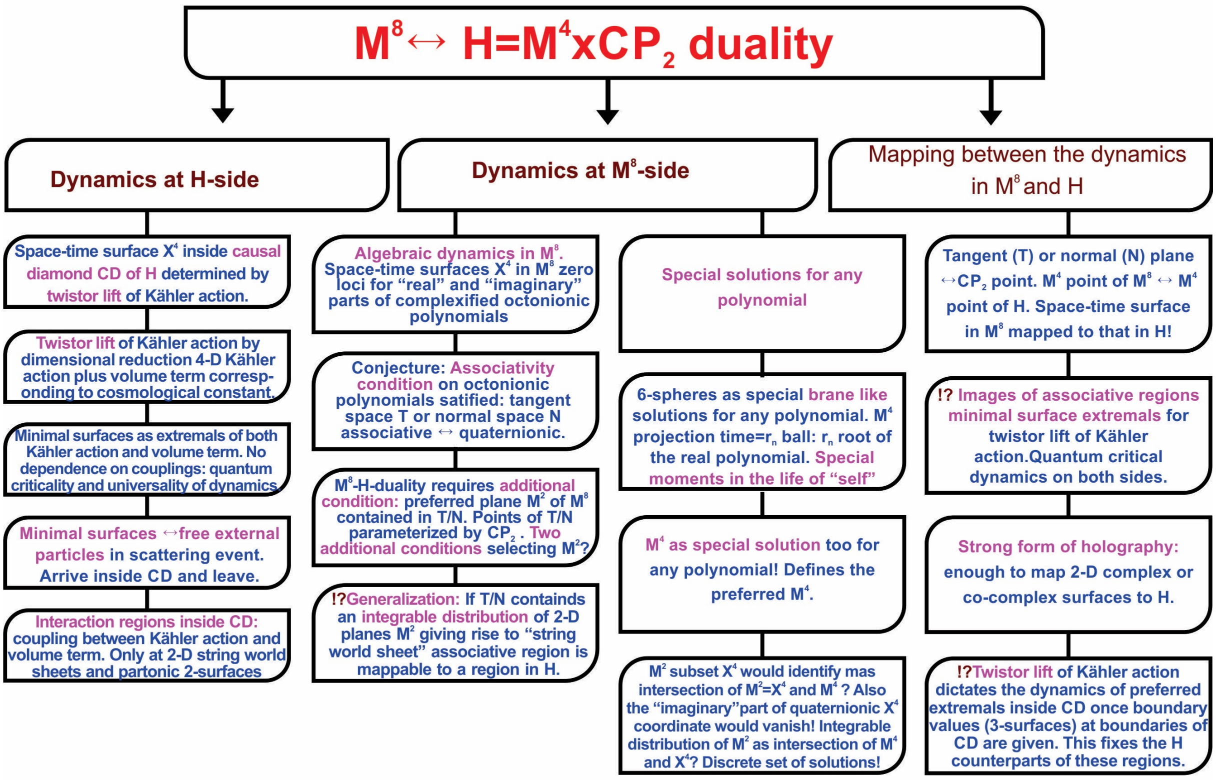

Figure 3: $M^8 - H$ duality

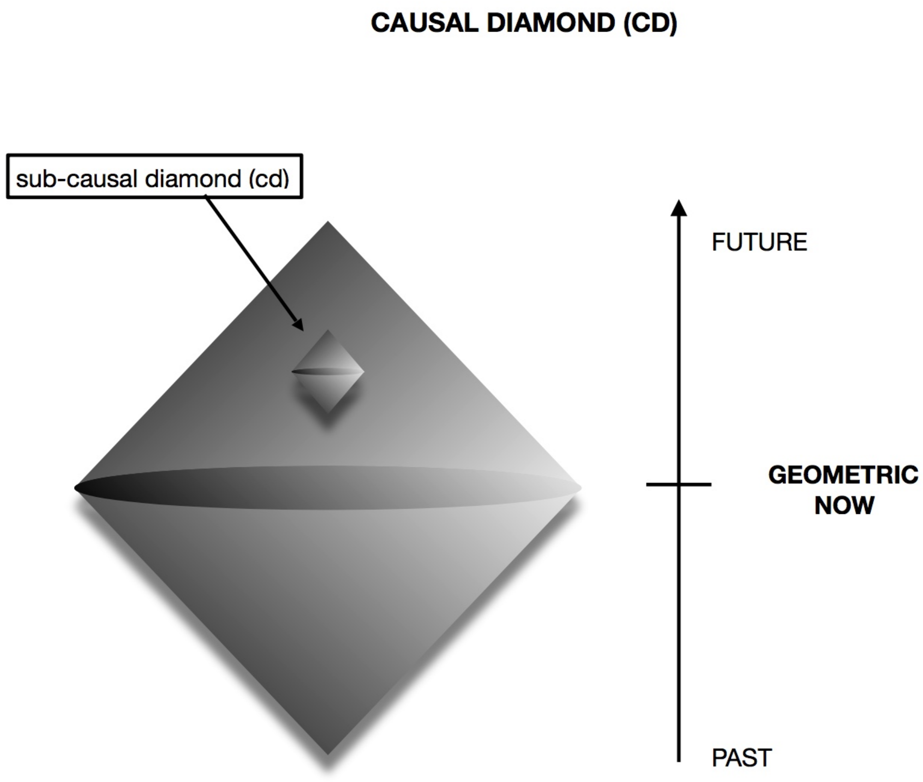

Figure 4: Causal diamond

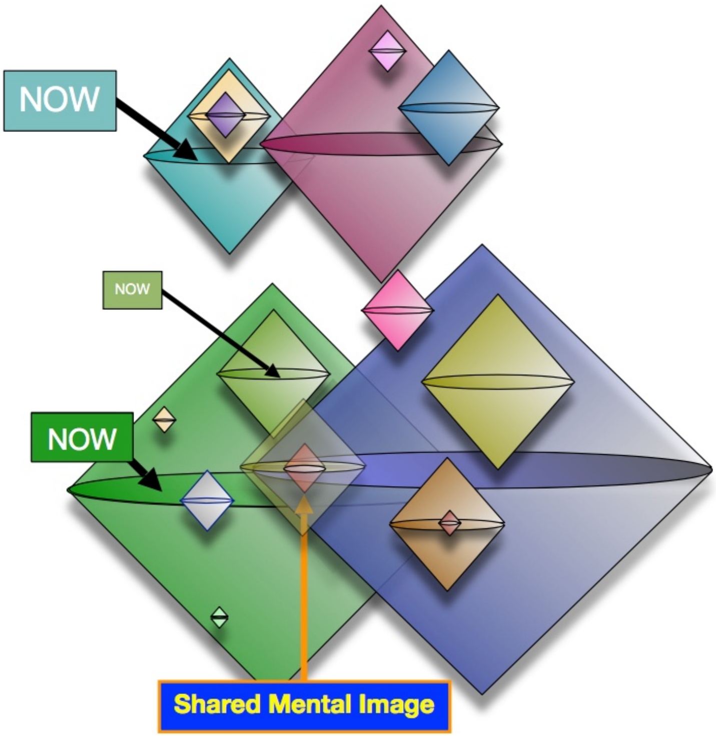

Figure 5: CDs define a fractal "conscious atlas"

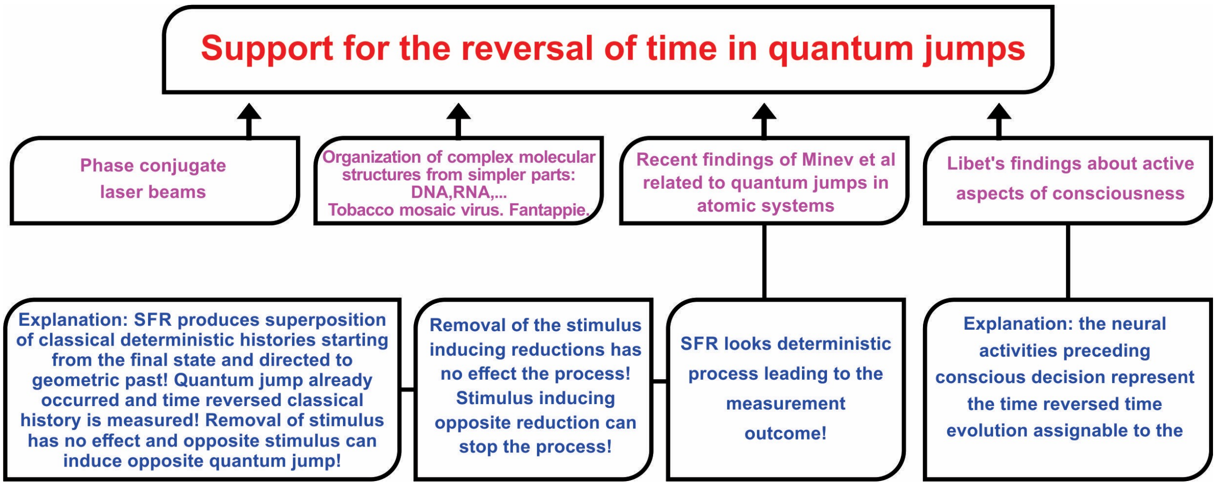

Figure 6: Time reversal occurs in BSFR

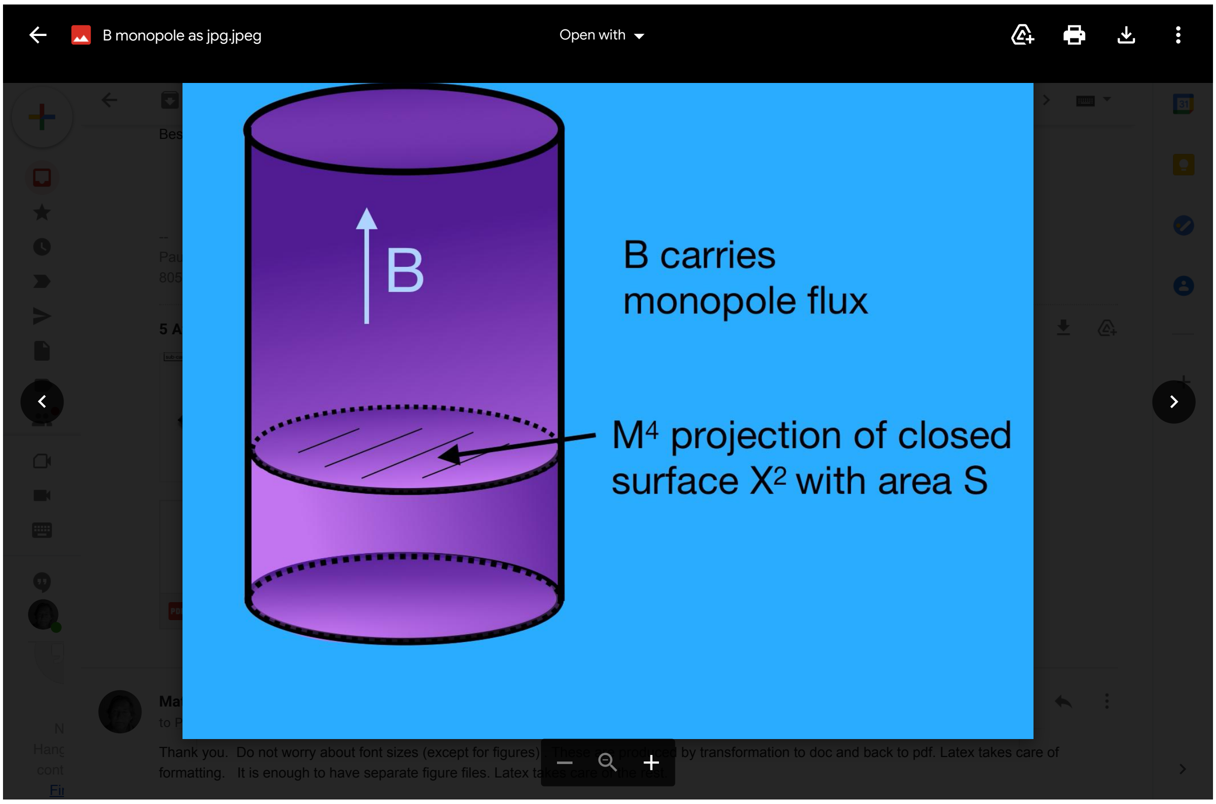

Figure 7: The M^4 projection of a closed surface X^2 with area S defining the cross section for monopole flux tube. Flux quantization $e\oint B\cdot dS = eBS = kh$ at single sheet of n-sheeted flux tube gives for cyclotron frequency $f_c = ZeB/2\pi m = khZ/2\pi mS$. The variation of S implies frequency modulation.

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