The recent view about TGD and applications to condensed matter

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Abstract

The purpose of this article is to give a rough overall view about Topological Geometrodynamics (TGD) and to consider possible applications of TGD to condensed matter physics. The preparation of this summary led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

   Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

2. The understanding of the details of the $M^8 - H$ duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. $M^8 - H$ duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H. Mass shell (its positive and negative energy parts) is mapped to a light-like boundary of CD with size $T = h_{eff}/m$, m the mass associated with momentum.

3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

After a summary of TGD as it is now, some concepts of condensed matter physics are considered from the TGD view. Also concrete questions about condensed matter are discussed. Hydrodynamical turbulence represents one of the unsolved problems of physics and therefore as an excellent test bench for the TGD based vision. Some tests for the proposed vision are considered.

1 Introduction

The purpose of this article is to give a rough overall view about Topological Geometrodynamics (TGD) and to consider its possible applications in condensed matter physics at the general level. It must be emphasized that TGD is only a vision, not a theory able to provide precise rules for calculating scattering amplitudes. A collective theoretical and experimental effort would be needed to achieve this. The proposal for a model of superconductivity [L35] provides a representative example about what TGD could possibly give for condensed matter physics.

It is perhaps good to explain what TGD is not and what it is or hoped to be. The article [L31] gives an overview of various aspects of TGD and is warmly recommended.

1. "Geometro-" refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with sub-manifold geometry. The basic motivation is the loss of classical conservation laws in General Relativity Theory (GRT)(see **Fig. 1**). Also the interpretation as a generalization of string models by replacing string with 3-D surface is natural.

   Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [L2]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely so that TGD is unique [L13, L15](see **Fig. 6**). The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "world of classical worlds" (WCW) - becomes the basic object endowed with Kähler geometry (see **Fig. 7**). General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. Given 3-surface fixes the space-time surface almost completely as analog of Bohr orbit (preferred extremal).This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces.

2. Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$.

   The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 3**).

   Practically any GCI action has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correletes for massless particles. World as a set or particles is replaced with a network having particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

   "Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 10**).

3. Adelic physics fusing real and various p-adic physics are part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical" orders. Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in all scales.

   The concrete realization of the number theoretic vision is based on $M^8 - H$ duality (see **Fig. 8**). The physics in the complexification of M^8 is algebraic - field equations as partial differential equations are replaced with algebraic equations associating to a polynomial with rational coefficients a X^4 mapped to H by $M^8 - H$ duality. The dark matter hierarchy corresponds to a hierarchy of algebraic extensions of rationals inducing that for adeles and has interpretation as an evolutionary hierarchy (see **Fig. 9**).

   $M^8 - H$ duality provides two complementary visions about physics (see **Fig. 2**), and can be seen as a generalization of the q-p duality of wave mechanics, which fails to generalize to quantum field theories (QFTs).

4. In Zero energy ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 12** and **Fig. 13**).

   Quantum jumps occur between these and the basic problem of standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 14**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions.

   In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary of CD and states at it remain unchanged (Zeno effect).

This work led to considerable progress in several aspects of TGD.

1. The mutual entanglement of fermions (bosons) as elementary particles is always maximal so that only fermionic and bosonic degrees can entangle in QFTs. The replacement of point-like particles with 3-surfaces forces us to reconsider the notion of identical particles from the category theoretical point of view. The number theoretic definition of particle identity seems

to be the most natural and implies that the new degrees of freedom make possible geometric entanglement.

Also the notion particle generalizes: also many-particle states can be regarded as particles with the constraint that the operators creating and annihilating them satisfy commutation/anticommutation relations. This leads to a close analogy with the notion of infinite prime.

2. The understanding of the details of the M^8-H duality forces us to modify the earlier view. The notion of causal diamond (CD) central to zero energy ontology (ZEO) emerges as a prediction at the level of H. The pre-image of CD at the level of M^8 is a region bounded by two mass shells rather than CD. M^8-H duality maps the points of cognitive representations as momenta of quarks with fixed mass in M^8 to either boundary of CD in H.

3. Galois confinement at the level of M^8 is understood at the level of momentum space and is found to be necessary. Galois confinement implies that quark momenta in suitable units are algebraic integers but integers for Galois singlet just as in ordinary quantization for a particle in a box replaced by CD. Galois confinement could provide a universal mechanism for the formation of all bound states.

4. There is considerable progress in the understanding of the quantum measurement theory based on ZEO. From the point of view of cognition BSFRs would be like heureka moments and the sequence of SSFRs would correspond to an analysis having as a correlate the decay of 3-surface to smaller 3-surfaces.

The improved vision allows us to develop the TGD interpretation for various condensed matter notions.

1. TGD is analogous to hydrodynamics in the sense that field equations at the level of H reduce to conservation laws for isometry charges. The preferred extremal property meaning that space-time surfaces are simultaneous extremals of volume action and Kähler action allows interpretation in terms of induced gauge fields. The generalized Beltrami property implies the existence of an integrable flow serving as a correlate for quantum coherence. Conserved Beltrami flows currents correspond to gradient flows. At the QFT limit this simplicity would be lost.

2. The fields H, M, B and D, P, E needed in the applications of Maxwell's theory could emerge at the fundamental level in the TGD framework and reflect the deviation between Maxwellian and the TGD based view about gauge fields due to CP_2 topology.

3. The understanding of macroscopic quantum phases improves. The role of the magnetic body carrying dark matter is central. The understanding of the role of WCW degrees of freedom improves considerably in the case of Bose-Einstein condensates of bosonic particles such as polaritons. M^8 picture allows us to understand the notion of skyrmion. The formation of Cooper pairs and analogous states with higher energy would correspond to a formation of Galois singlets liberating energy used to increase h_{eff}. What is new is that energy feed makes possible supra-phases and their analogs above the critical temperature.

4. Fermi surface emerges as a fundamental notion at the level of M^8 but has a counterpart also at the level of H. Galois groups would be crucial for understanding braids, anyons and fractional Quantum Hall effect. Space-time surface could be seen as a curved quasicrystal associated with the lattice of M^8 defined by algebraic integers in an extension of rationals. Also the TGD analogs of condensed matter Majorana fermions emerge.

In section 1 this picture is discussed in more detail. In section 2 some concepts of condensed matter physics are discussed from the TGD view. In section 3 some concrete questions about condensed matter are discussed Hydrodynamical turbulence represents one of the unsolved problems of physics and therefore as an excellent test bench for the TGD based vision and is discussed in the 4^{th} section. The last section lists some tests for the TGD based vision. The approach is rather general: this is the only possible option since I am not a condensed matter specialist.

2 Physics as geometry

The following provides a sketchy representation of TGD based on the vision about physics as geometry which is complementary to the vision of physics as number theory. $M^8 - H$ duality relates these two visions. A longer representation can be found in [L31].

2.1 Space-time as 4-surface in $H = M^4 \times CP_2$

1. The energy problem of GRT means that since space-time is curved, one cannot define Poincare charges as Noether charges (see **Fig. 1**). If space-time X^4 is a surface in $H = M^4 \times CP_2$, the situation changes. Poincare symmetries are lifted to the level of $M^4 \subset H$.

2. Generalization of the notion of particle is in question: point-like particle $\rightarrow$ 3-surface so that TGD can be seen also as a generalization of string model. String $\rightarrow$ 3-surface. String world sheet $\rightarrow X^4$. The notions of the particle and space are unified.

3. Einstein's geometrization program is extended to standard model interactions. CP_2 codes for standard model symmetries and gauge fields. Isometries $\leftrightarrow$ color SU(3). Holonomies of spinor connection $\leftrightarrow$ electroweak U(2) [L2]. Genus-generation correspondence provides a topological explanation of the family replication phenomenon of fermions [K2]: 3 fermion families are predicted.

4. Induction of spinors structure as projection of components of spinor connection from CP_2 to X^4 is central for the geometrization. The projections of Killing vectors of color isometries yield color gauge potentials. Parallel translation at X^4 using spinor connection of H. Also spinor structure is induced and means projection of gamma matrices.

5. Dynamics for X^4 is determined by an action S consisting of Kähler action plus volume term (cosmological constant) following from the twistor lift of TGD [K34, L15].

6. The dynamics for fermions at space-time level is determined by modified Dirac action determined by S being super-symmetrically related to it. Gamma matrices are replaced with modified gamma matrices determined by the S as contractions of canonical momentum currents with gamma matrices. Preferred extremal property follows as a condition of hermiticity for the modified Dirac operator.

   Second quantized H-spinors, whose modes satisfy free massless Dirac equation in H restricted to X^4: this induces second quantization to X^4 and one avoids the usual problems of quantization in a curved background. This picture is consistent with the modified Dirac equation satisfied by the induced spinors in X^4.

   Only quarks are needed if leptons are 3-quark composites in CP_2 scale: this is possible only if one accepts the TGD view about color symmetries. This also provides a new view about matter antimatter asymmetry [L22, L34]. CP violation is forced by the M^4 part of Kähler form forced by the twistor lift.

2.1.1 Basic extremals of classical action

Practically any GCI action allows the same basic extremals (for basic questions related to classical TGD see **Fig. 3**).

1. CP_2 type extremals having light-like geodesic as M^4 projection and Euclidian signature of the induced metric serve as building bricks of elementary particles. If the volume term is absent as it might be at infinite volume limit, the geodesics become light-like curves [L52]. Wormhole contacts connecting two Minkowskian space-time sheets can be regarded as a piece of a deformed CP_2 type extremal. Monopole flux through contact stabilizes the wormhole contact.

2. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow superposition of modes with single direction of ligth-like momentum. Ideal laser beam is a convenient analogy here.

3. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and their thickenings to flux tubes are also a central notion.

2.1.2 QFT limit of TGD

The induced gauge fields and gravitational field are expressible in terms of only 4 H- coordinates. Locally the theory is too simple to be physical.

1. Many-sheeted space-time means that X^4 is topologically extremely complex. CP_2 coordinates are many-valued functions of M^4 coordinates or vice versa or both. In contrast to this, the space-time of EYM theory is topologically extremely simple.

2. Einsteinian space-times have 4-D projection to M^4. Small test particle experiences the sum of the classical gauge potentials associated with various space-time sheets. At QFT limit the sheets are replaced with a single region of M^4 made slightly curved and gauge potentials are defined as the sums of gauge potentials from different space-time sheets having common M^4 projection. Topological complexity and local simplicity are replaced with topological simplicity and local complexity. (see **Fig. 3**).

2.2 World of classical worlds (WCW)

The notion of WCW emerges as one gives up the idea about quantizing by path integral.

2.2.1 The failure of path integral forces WCW geometry

The extreme non-linearity implies that the path integral for surfaces space-time surfaces fails. A possible solution is generalize Einstein's geometrization program to the level of the entire quantum theory.

1. "World of classical worlds" (WCW) can be identified as the space of 3-surfaces endowe with a metric and spinor structure (see **Fig. 7**). Hermitian conjugation must have a geometrization. This requires Kähler structure requiring also complex structure. WCW has Kähler form and metric.

2. WCW spinors are Fock states created by fermionic oscillator operators assignable to spinor modes of H basically [L29]. WCW gamma matrices as linear combinations of fermionic (quark) oscillator operators defining analog of vielbein.

   WCW has also spinor connection and curvature in WCW. correspond The quantum states of world correspond formally to *classical* spinor fields in WCW. Gamma matrices of WCW expressinble in terms of fermionic oscillator operators are also purely classical objects.

2.2.2 Implications of General Coordinate Invariance

General Coordinate Invariance (GCI) in 4-D sense forces to assign to 3-surface X^3 a 4-surface $X^4(X^3)$, which is as unique as possible. This gives rise to Bohr orbitology and quantum classical correspondence (QCC), and holography. Also zero energy ontology (ZEO) emerges.

Quantum states quantum superpositions of space-time surfaces as analogs of Bohr orbits. QCC means that the classical theory is an exact part of quantum theory (QCC).

A solution to the basic paradox of quantum measurement theory emerges [L21]: superposition of deterministic time evolutions is replaced with a new one in state function reduction (SFR): SFR does not force any failure of determinism for individual time evolutions.

2.2.3 WCW Kähler geometry from classical action

WCW geometry is determined by a classical action defining Kähler function $K(X^3)$ for a preferred extremal $X^4(X^3)$ defining the preferred extremal/Bohr orbit [K6] (see **Fig. 7**).

1. QCC suggests that the definition of Kähler function assigns a more or less unique 4-surface $X^4(X^3)$ to 3-surface X^3. Finite non-uniqueness is however possible [L52].

2. $X^4(X^3)$ is identified as a *preferred* extremal of some general coordinate invariant (GCI) action forcing the Bohr orbit property/holography/ZEO. This means a huge reduction of degrees of freedom.

   Remark:: Already the notion of induced gauge field and metric eliminates fields as primary dynamical variables and GCI leaves locally only 4 H-coordinates as dynamical variables.

3. Twistor lift [L13, L15] of TGD geometrizes the twistor Grassmann approach to QFTs. The 6-D extremal X^6 of 6-D Kähler action as a 6- surface in the product $T(M^4) \times T(CP_2)$ of twistor spaces of M^4 and CP_2 represents the twistor space of X^4.

   The condition that X^6 reduces to an S^2 bundle with X^4 as base space, forces a dimensional reduction of 6-D Kähler action to 4-D Kähler action + volume term, whose value for the preferred extremal defines the Kähler function for $X^4(X^3)$.

4. The volume term corresponds to a p-adic length scale dependent cosmological constant Λ approach zero at long p-adic length scale so that a solution of the cosmological constant problem emerges. Preferred extremal/Bohr orbit property means a simultaneous extremal property for *both* Kähler action and volume term. This forces X^4 to have a generalized complex structure (Hamilton-Jacobi structure) so that field equations trivialize and there is no dependence on coupling parameters. Universality of dynamics follows and the TGD Universe is quantum critical. In particular, Kähler coupling strength is analogous to a critical temperature and is quantized [L42].

5. Soap film analogy is extremely useful [L52]: the analogs of soap film frames are singular surfaces of dimension $D < 4$. At the frame the space-time surface fails to be a simultaneous extremal of both actions separately and Kähler and volume actions couple to each other. The corresponding contributions to conserved isometry currents diverge but sum up to a finite contribution. The frames define the geometric analogs for the vertices of Feynman diagrams.

2.2.4 WCW geometry is unique

WCW geometry is fixed by the existence of Riemann connection and requires maximal symmetries.

1. Dan Freed [A2] found that loop space for a given Lie group allows a unique Kähler geometry: maximal isometries needed in order to have a Riemann connection. Same expected to be true now [K3, K16].

2. Twistor lift of TGD [L13, L15] means that one can replace X^4 with its twistor space $X^6(X^4)$ in the product $T(M^4) \times T(CP_2)$ of the 6-D twistor spaces $T(M^4)$ and $T(CP_2)$. $X^6(X^4)$ is 6-surface with the structure of S^2 bundle.

   Dimensionally reduced 6-D Kähler action gives sum of 4-D Kähler action and volume term. Twistor space must however have a Kähler structure and only the twistor spaces of M^4,E^4, and CP_2 have Kähler structure [A5]. TGD is unique both physically and mathematically!

2.2.5 Isometries of WCW

What can one say about the isometries of WCW? Certainly, they should generalize conformal symmetries of string models.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [K3].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

2.2.6 A possible problem related to the twistor lift

The twistor lift strongly suggests that the Kähler form of M^4 exists. The Kähler gauge potential would be the sum of M^4 and CP_2 contributions. The definition of M^4 Kähler structure is however not straightforward [L25, L26]. The naive guess would be that J represents an imaginary unit as the square root of -1 represented by the metric tensor. This would give the condition $J^2 = -g$ for the tensor square but this leads to problems.

To understand the situation, notice that the analogs of symplectic/Kähler structures in $M^4 \subset H$ have a moduli space, whose points correspond to what I have called Hamilton-Jacobi structures defined by integrable distributions of orthogonal decompositions $M^4 = M^2(x) \times E^2(x)$: $M^2(x)$ is analogous to string world sheet and Y^2 to partonic 2-surface. This means the presence of slicing by string world sheets $X^2(x)$, where x labels a point of Y^2. $X^2(x)$ is orthogonal to Y^2 at x. One can interchange the roles X^2 ad Y^2 in the slicing.

The induced Kähler form has an analogous decomposition. The decomposition is completely analogous to the decomposition of polarizations to non-physical time-like ones and physical space-like ones. This decomposition allows a natural modification of the definition of the symplectic structure so that the problem caused by $J^2 = -g$ conditions is avoided.

Consider first the problem. The $E^2(x)$ part of M^4 Kähler metric produces no problems since the signature of the metric is Euclidean. For $M^2(x)$ part, the Minkowskian signature produces problems. If one assumes that the $M^2(x)$ part of the Kähler form is non-vanishing, it should be imaginary in order to satisfy $J^2(M^2(x)) = -g(M^2(x))$. This implies that Kähler gauge potential is imaginary and this spoils the hermiticity of the modified Dirac equation [K12]. Also the electric contribution to the Kähler energy is negative.

The solution of the problem turned out to be ridiculously simple and I should have noticed it a long time ago.

1. $M^2(x)$ has a hypercomplex structure, which means that the imaginary unit e satisfies $e^2 = 1$ rather than $e^2 = -1$. Hamilton-Jacobi structure allows one to decompose J locally into two parts $J = J(M^2(x)) + J(E^2(x))$ such that $J^2 = g(M^2(x)) - g(E^2(x))$. This gives $J^4 = g(M^4)$. The Kähler energy of the canonically embedded M^4 is non-vanishing and positive whereas Kähler action vanishes by self-duality. Situation is identical to that in Maxwell's electrodynamics.

2. Kähler action for the canonically embedded M^4 vanishes and it is possible to define also Lagrangian 2-surfaces as surfaces for which the induced Kähler form vanishes. These are of special interest since they would guarantee small CP violation: string world sheets could be examples of these surfaces. Note that since the magnetic part of J induces violation of CP, the violation is vanishing for CP_2 type extremals and cosmic strings and also small for flux tubes.

If the notion of symplectic/canonical transformation generated by Hamiltonian preserving J generalizes, one could generate an infinite number of slicings.

Consider first ordinary symplectic transformations.

1. For the ordinary symplectic transformations, the closedness of the symplectic for J is essential ($dJ = 0$ corresponds to topological half of Maxwell's equations).

2. Second essential element is that symplectic transformation is generated as a flow for some Hamiltonian H: $j_H = i_{dH}J$ or more explicitly: $j^l_H = J^{kl}\partial_l H$. It is essential that one has $i_{j_H}J = -dH$: having a vanishing exterior derivative. In other words, $J_{kl}j^l_H = -\partial_k H$ is a gradient vector field and has therefore a vanishing curl. Together with $dJ = 0$, this guarantees the vanishing of the Lie derivative of J: $d_{j_H}J = d(i_{j_H}J) + i_{j_H}dJ = ddH + dJ(j_H) = 0$ so that J is preserved.

Could one talk about symplectic transformations in M^4?

1. The analogs of symplectic/canonical transformations should map the Hamilton-Jacobi structure to a new one and leave $J(M^2(x))$ and $J(E^2(x))$ invariant. The induced metrics of X^2 and Y^2 need not be preserved since only the diagonal metric $g^k_l(X^2/Y^2)$ appears in the conditions $J^2 = g(X^2) - g(Y^2)$.

2. The symplectic transformation generated by the Hamiltonian H would be a flow defined by the vector field $j_H = i_{dH}J$ and one would have $i_{j_H}J = -d_1H + d_2H$, where d_1 and d_2 are gradients operators in X^2 and Y^2. Usually one would have $J_{kl}j^l = dH$ satisfying $d^2H = 0$.

   The condition $ddH = 0$ satisfied by the ordinary symplectic transformations is replaced with the condition $d(-d_1H + d_2H) = 0$. This can be written as $-d_1^2H + d_2^2H + [d_2, d_1]H = 0$, and is satisfied. Therefore this part is not a problem.

3. Also the orthogonality of $M^2(x)$ and $E^2(x)$ must be preserved. This is a highly non-trivial condition since the metrics are induced and the symplectic transformations change the slicing and the metrics. An arbitrary Hamiltonian flow f, which depends on the coordinates of Y^2 only, maps Y^2 to itself but takes the tangent space $E^2(x)$ to $E^2(f(x))$. Unless the slicing satisfies special conditions, $E^2(f(x))$ is not orthogonal to $M^2(x)$.

4. The orthogonality is expressed as orthogonality of the projectors $P(X^2)$ and $P(Y^2)$: $P(X^2)P(Y^2) = 0$. This condition must be respected by the Hamiltonian flow. The product involves 4 components giving 4 conditions which turn out to be partial differential equations for Hamiltonian. The naive expectation is that there are very few solutions. The Lie-derivative of the product must therefore vanish:

$$L_{j_H}[P(X^2)P(Y^2)] = L_{j_H}(P(X^2))P(Y^2) + P(X^2))L_{j_H}(P(Y^2)) = 0 \ . \tag{2.1}$$

   The projector $P_{mn}(X^2)$ can be expressed as

$$P^{mn} = g^{\alpha\beta}\partial_\alpha m^k \partial_\beta m^l \ . \tag{2.2}$$

   Here $g_{\alpha\beta} = m_{kl}\partial_\alpha m^k \partial_\beta m^l$ is the induced metric of X^2 or Y^2. m_{kl} is Minkowski metric and one can use linear Minkowski coordinates so that m_{kl} is constant.

   The Lie derivative of $P^{mn}(X^2) \equiv P$ can be written as

$$L_jP^{mn} = L_j(g^{\alpha\beta})\partial_\alpha m^k \partial_\beta m^l + g^{\alpha\beta})(\partial_r j^k \partial_\alpha m^r \partial_\beta m^l + \partial_r j^l k \partial_\alpha m^r \partial_\beta m^k \ . \tag{2.3}$$

   The Lie derivative of the induced metric is

$$\begin{aligned} L_j g^{\alpha\beta} &= g^{\alpha\mu} g^{\beta\nu} L_J g_{\mu\nu} \quad , \\ L_j g_{\alpha\beta} &= m_{kl}(\partial_\alpha j^k \partial_\beta m^l + \partial_\alpha m^k \partial_\beta j^l \quad . \end{aligned} \tag{2.4}$$

Although the existence of symplectic transformations in the general case seems implausible, one can construct special slicings for which symplectic transformations are possible.

1. One can start from a trivial slicing defined by $M^2 \times E^2$ decomposition and perform slicings of M^2 and E^2. The orthogonality is trivially true for all slicings of this kind since $Y^2(y)$ is orthogonal to X^2 not only at y but at every point x. Symplectic transformations of M^2 and Y^2 produce new slicings of this kind. Even symplectic flowqs defined by general Hamiltonians respect the orthogonality.

2. Second example is provided by the slicing of the light-one boundary by light-like 2-surfaces Y^2_v labelled by the value of light-like radial coordinate v with metrics differing by r^2 factor. The surfaces X^2 would be planes $X^2(y)$ orthogonal to Y^2 at y with light-like coordinates u and v. The orthogonality would be preserved by symplectic transformations.

   The open question is whether these slicings are the only possible slicings allowing symplectic transformations. Although the construction of these slicings looks trivial, they are not trivial physically.

2.3 Different manners to understand the "complete integrability" of TGD

There are several ways to see how TGD could be a completely integrable theory.

2.3.1 Preferred extremal property

Preferred extremal property requires Bohr orbit property and holography and is an extremely powerful condition.

1. Twistor lift of TGD implies that X^4 in H is simultaneous extremal of volume action and Kähler action. Minimal surface property is counterpart for massless field equations and extremality for Kähler action gives interpretation for massless field as Kähler form as part of induced electromagnetic field.

   The simultaneous preferred extremal property strongly suggests that 2-D complex structure generalizes for 4-D space-time surfaces and so called Hamilton-Jacobi structure [L18] meaning a decomposition of M^4 to orthogonal slicings by string world sheets and orthogonal partonic 2-surfaces would realize this structure.

2. Generalized Beltrami property [L35] implies that 3-D Lorentz force and dissipation for Kähler form vanish. The Kähler form is analogous to the classical Maxwell field. Energy momentum tensor has vanishing divergence, which makes it plausible that QFT limit is analogous to Einstein-Maxwell theory.

   The condition also implies that the Kähler current defines an integrable flow so that there is global coordinate varying along flow lines. This is a natural classical correlate for quantum coherence. Quantum coherence would be always present but broken only by the finite size of the region of the space-time considered.

   Beltrami property plus current conservation implies gradient flow and an interesting question is whether conserved currents define gradient flows: non-trivial space-time topology would allow this at the fundamental level. Beltrami condition is a very natural classical condition in the models of supraphases.

2.3.2 Supersymplectic symmetry

The third approach is based on the super-symplectic symmetry of WCW. Isometry property would suggest that an infinite number of super-symplectic Noether charges are defined at the boundaries of CD by the action of the theory. They need not be conserved since supersymplectic symmetries cannot be symmetries of the action: if they were, the WCW metric would be trivial.

The gauge conditions for Virasoro algebra and Kac-Moody algebras suggest a generalization. Super-symplectic algebra (SSA) involves only non-negative conformal weights n suggesting extension to a Yangian algebra (this is essential!). Consider the hierarchy of subalgebras SSA_m for which the conformal weights are m-tiples of those of entire algebra. These subalgebras are isomorphic with the entire algebra and form a fractal hierarchy.

Assume that the sub-algebra SSA_m and commutator $[SSA_m, SSA]$ have vanishing classical Noether charges for $m > m_{max}$. These conditions could fix the preferred extremal. One can also assume that the fermionic realizations of these algebras annihilate physical states. The remaining symmetries would be dynamical symmetries.

The generators are Hamiltonians of $\delta M^4_+ \times CP_2$. The symplectic group contains Hamiltonians of the isometries as a normal sub-algebra. Also the Hamiltonians of and one could assume that only the isometry generators correspond to non-trivial classical and quantal Noether charges. Could the actions of SSA and Kac-Moody algebras of isometries be identical if a similar construction applies to Kac-Moody half-algebras associated with the light-like partonic orbits. Super-symplectic symmetry would reduce to a hierarchy of gauge symmetries.

3 Physics as number theory

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L11, L12], and classical number fields [K33].

3.1 p-Adic physics

Motivation for p-adicization came from p-adic mass calculations [K8, K2].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Bolztmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to power of 2: $p \simeq 2^k$. For instance, for electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1 + i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [K15] is interpreted as a correlate for cognition (see **Fig. 10**). Motivation comes from the observation that piecewice constant functions depending a finite number of pinary digts have vanishing derivative. Thefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

3.2 Adelic physics

Adelic physics fuses real and various p-adic physics to a single structure [L12].

1. One can combine real numbers and p-adic number fields to a product: number fields would be like pages of a book intersecting along rationals acting as the back of the book.

2. Each extension of rational induces extensions of p-adic number fields and extension of the basic adele. Points in the extension of rationals are now common to the pages. The infinite hierarchy of adeles defined by the extensions forms an infinite library.

3. This leads to an evolutionary hierarchy (see **Fig. 9**) . The order n of the Galois group as a dimension of extension of rationals is identified as a measure of complexity and of evolutionary level, "IQ". Evolutionary hierarchy is predicted.

4. Also a hierarchy of effective Planck constants interpreted in terms of phases of ordinary matter is predicted. X^4 decomposes to n fundamental regions related by Galois symmetry. Action is n times the action for the fundamental region. Planck constant h is effectively replaced with $h_{eff} = nh$. Quantum coherence scales are typically proportional to h_{eff}. Quantum coherence in arbitrarily long scales is implied. Dark matter at the magnetic body of the system would serve as controller of ordinary matter in the TGD inspired quantum biology [L55].

   $h_{eff} = nh_0$ is a more general hypothesis. Reasons to believe that h/h_0 could be the ratio R^2/L_p^2 for CP_2 length scale R deduced from p-adic mass calculations and Planck length L_P [L42]. The CP_2 radius R could actually correspond to L_P and the value of R deduced from the p-adic mass calculations would correspond to a dark CP_2 radius $\sqrt{h/h_0}l_P$.

3.3 Adelic physics and quantum measurement theory

Adelic physics [L12] forces us to reconsider the notion of entanglement and what happens in state function reductions (SFRs). Let us leaven the question whether the SFR can correspond to SSFR or BSFR or both open for a moment.

1. The natural assumption is that entanglement is a number-theoretically universal concept and therefore makes sense in both real and various p-adic senses. This is guaranteed if the entanglement coefficients are in an extension E of rationals associated with the polynomial Q defining the space-time surface in M^8 and having rational coefficients.

   In the general case, the diagonalized density matrix ρ produced in a state function reduction (SFR) has eigenvalues in an extension E_1 of E. E_1 is defined by the characteristic polynomial P of ρ.

2. Is the selection of one of the eigenstates in SFR possible if E_1 is non-trivial? If not, then one would have a number-theoretic entanglement protection.

3. On the other hand, if the SFR can occur, does it require a phase transition replacing E with its extension by E_1 required by the diagonalization?

Let us consider the option in which E is replaced by an extension coding for the measured entanglement matrix so that something also happens to the space-time surface.

1. Suppose that the observer and measured system correspond to 4-surfaces defined by the polynomials O and S somehow composed to define the composite system and reflecting the asymmetric relationship between O and S. The simplest option is $Q = O \circ S$ but one can also consider as representations of the measurement action deformations of the polynomial $O \times P$ making it irreducible. Composition conforms with the properties of tensor product since the dimension of extension of rationals for the composite is a product of dimensions for factors.

2. The loss of correlations would suggest that a classical correlate for the outcome is a union of uncorrelated surfaces defined by O and S or equivalently by the reducible polynomial defined by the $O \times S$ [L38]. Information would be lost and the dimension for the resulting extension is the sum of dimensions for the composites. O however gains information and quantum classical correspondence (QCC) suggests that the polynomial O is replaced with a new one to realize this.

3. QCC suggests the replacement of the polynomial O the polynomial $P \circ O$, where P is the characteristic polynomial associated with the diagonalization of the density matrix ρ. The final state would be a union of surfaces represented by $P \circ O$ and S: the information about the measured observable would correspond to the increase of complexity of the space-time surface associated with the observer. Information would be transferred from entangled Galois degrees of freedom including also fermionic ones to the geometric degrees of freedom $P \circ O$. The information about the outcome of the measurement would in turn be coded by the Galois groups and fermionic state.

4. This would give a direct quantum classical correspondence between entanglement matrices and polynomials defining space-time surfaces in M^8. The space-time surface of O would store the measurement history as kinds of Akashic records. If the density matrix corresponds to a polynomial P which is a composite of polynomials, the measurement can add several new layers to the Galois hierarchy and gradually increase its height.

   The sequence of SFRs could correspond to a sequence of extensions of extensions of..... This would lead to the space-time analog of chaos as the outcome of iteration if the density matrices associated with entanglement coefficients correspond to a hierarchy of powers P^k [L27, L37].

Does this information transfer take place for both BSFRs and SSFRs? Concerning BSFRs the situation is not quite clear. For SSFRs it would occur naturally and there would be a connection with SSFRs to which I have associated cognitive measurement cascades [?]

1. Consider an extension, which is a sequence of extensions $E_1 \to ..E_k \to E_{k+1}.. \to E_n$ defined by the composite polynomial $P_n \circ .... \circ P_1$. The lowest level corresponds to a simple Galois group having no non-trivial normal subgroups.

2. The state in the group algebra of Galois group $G = G_n$ having G_{n-1} as a normal subgroup can be expressed as an entangled state associated with the factor groups G_n/G_{n-1} and subgroup G_{n-1} and the first cognitive measurement in the cascade would reduce this entanglement. After that the process could but need not to continue down to G_1. Cognitive measurements considerably generalize the usual view about the pair formed by the observer and measured system and it is not clear whether $O - S$ pair can be always represented in this manner as assumed above: also small deformations of the polynomial $O \times S$ can be considered.

   These considerations inspire the proposal the space-time surface assigned to the outcome of cognitive measurement G_k, G_{k-1} corresponds to polynomial the $Q_{k,k-1} \circ P_n$, where $Q_{k,k-1}$ is the characteristic polynomial of the entanglement matrix in question.

3.4 Entanglement paradox and new view about particle identity

A brain teaser that the theoretician sooner or later is bound to encounter, relates to the fermionic and bosonic statistics. This problem was also mentioned in the article of Keimer and Moore [D2] discussing quantum materials `https://cutt.ly/bWdTRj0`. The unavoidable conclusion is that both the fermions and bosons of the entire Universe are maximally entangled. Only the reduction of entanglement between bosonic and fermionic states of freedom would be possible in SFRs. In the QFT framework, gauge boson fields are primary fields and the problem in principle disappears if entanglement is between states formed by elementary bosons and fermions.

In the TGD Universe, all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) so that if Fock space the Fock states of fermions and bosons express everything worth expressing, SFRs would not be possible at all!

Remark: In the TGD Universe all elementary particles are composites of fundamental fermions (quarks in the simplest scenario) localized at the points of space-time surface defining a number theoretic discretization that I call cognitive representation. Besides this there are also degrees of freedom associated with the geometry of 3-surfaces representing particles. These degrees of freedom represent new physics. The quantization of quarks takes place at the level of H so that anticommutations hold true over the entire H.

Obviously, something is entangled and this entanglement is reduced. What these entangled degrees of freedom actually are if Fock space cannot provide them?

1. Mathematically entanglement makes sense also in a purely classical sense. Consider functions $\Psi_i(x)$ and $\Psi_j(y)$) and form the superposition $\Psi(x) = \sum_{ij} c_{ij}\Psi_i(x)\Psi_j x)$. This function is completely analogous to an entangled state.

2. Number theoretical physics implies that the Galois group becomes the symmetry group of physics and quantum states are representations of the Galois group [L32, L33]. For an extension of extension of ...., the Galois group has decomposition by normal subgroups to a hierarchy of coset groups.

   The representation of a Galois group can be decomposed to a tensor product of representations of these coset groups. The states in irreps of the Galois group are entangled and the SFR cascade produces a product of the states as a product of representations of the coset groups. Galois entanglement allows us to express the asymmetric relation between observer and observed very naturally. This cognitive SSFR cascade - as I have called it - could correspond to what happens in at least cognitive SFRs.

   If so, then SFR would in TGD have nothing to do with fermions and bosons (consisting of quarks too) since the maximal fermionic entanglement remains. For instance, when one for instance talks about long range entanglement the entanglement that matters would correspond to entanglement between degrees of freedom, which do not allow Fock space description.

In the TGD framework, the replacement of particles with 3-surfaces brings in an infinite number of non-Fock degrees of freedom. Could it make sense to speak about the reduction of entanglement in WCW degrees of freedom? There is no second quantization at WCW level so that one cannot talk about Fock spaces WCW level but purely classical entanglement is possible as observed.

1. In WCW unions of disjoint 3-surfaces correspond to classical many-particle states. One can form single particle wave functions for 3-surfaces with a single component, products of these single particle wave functions, and also analogs of entangled states as their superposition realized as building bricks of WCW spinor fields.

   If one requires that these wave functions are completely symmetric under the exchange of 3-surfaces, maximal entanglement in this sense would be realized also now and SFR would not be possible. But can one require the symmetry? Under what conditions one can regard two 3-surfaces as identical? For point-like particles one has always identical particles but in TGD the situation changes.

2. Here theoretical physics and category theory meet since the question when two mathematical objects can be said to be identical is the basic question of category theory. The mathematical answer is they are isomorphic in some sense. The physical answer is that the two systems are identical if they cannot be distinguished in the measurement resolution used.

4 $M^8 - H$ duality

There are several observations motivating $M^8 - H$ duality (see **Fig. 8**).

1. There are four classical number fields: reals, complex numbers, quaternions, and octonions with dimensions $1, 2, 4, 8$. The dimension of the embedding space is $D(H) = 8$, the dimension of octonions. Spacetime surface has dimension $D(X^4) = 4$ of quaternions. String world sheet and partonic 2-surface have dimension $D(X^2) = 2$ of: complex numbers. The dimension $D(string) = 1$ of string is that of reals.

2. Isometry group of octonions is a subgroup of automorphism group G_2 of octonions containing $SU(3)$ as a subgroup. $CP_2 = SU(3)/U(2)$ parametrizes quaternionic 4-surfaces containing a fixed complex plane.

Could M^8 and $H = M^4 \times CP_2$ provide alternative dual descriptions of physics (see **Fig. 8**)?

1. Actually a complexification $M^8_c \equiv E^8_c$ by adding an imaginary unit i commuting with octonion units is needed in order to obtain sub-spaces with real number theoretic norm squared. M^8_c fails to be a field since $1/o$ does not exist if the complex valued octonionic norm squared $\sum o_i^2$ vanishes.

2. The four-surfaces $X^4 \subset M^8$ are identified as "real" parts of 8-D complexified 4-surfaces X_c^4 by requiring that $M^4 \subset M^8$ coordinates are either imaginary or real so that the number theoretic metric defined by octonionic norm is real. Note that the imaginary unit defining the complexification commutes with octonionic imaginary units and number theoretical norm squared is given by $\sum_i z_i^2$ which in the general case is complex.

3. The space H would provide a geometric description, classical physics based on Riemann metric, differential geometric structures and partial differential equations deduced from an action principle. M_c^8 would provide a number theoretic description: no partial differential equations, no Riemannian metric, no connections...

   M_c^8 has only the number theoretic norm squared and bilinear form, which are real only if M_c^8 coordinates are real or imaginary. This would define "physicality". One open question is whether all signatures for the number theoretic metric of X^4 should be allowed? Similar problem is encountered in the twistor Grassmannian approach.

4. The basic objection is that the number of algebraic surfaces is very small and they are extremely simple as compared to extremals of action principle. Second problem is that there are no coupling constants at the level of M^8 defined by action.

   Preferred extremal property realizes quantum criticality with universal dynamics with no dependence on coupling constants. This conforms with the disappearance of the coupling constants from the field equations for preferred extremals in H except at singularities, with the Bohr orbitology, holography and ZEO. $X^4 \subset H$ is analogous to a soap film spanned by frame representing singularities and implying a failure of complete universality.

5. In M^8, the dynamics determined by an action principle is replaced with the condition that the *normal* space of X^4 in M^8 is associative/quaternionic. The distribution of normal spaces is always integrable to a 4-surface.

   One cannot exclude the possibility that the normal space is complex 2-space, this would give a 6-D surface [L25, L26]. Also this kind of surfaces are obtained and even 7-D with a real normal space. They are interpreted as analogs of branes and are in central role in TGD inspired biology.

   Could the twistor space of the space-time surface at the level of H have this kind of 6-surface as M^8 counterpart? Could $M^8 - H$ duality relate these spaces in 16-D M_c^8 to the twistor spaces of the space-time surface as 6-surfaces in 12-D $T(M^4) \times T(CP_2)$?

6. Symmetries in M^8 number theoretic: octonionic automorphism group G_2 which is complexified and contains $SO(1,3)$. G_2 contains $SU(3)$ as M^8 counterpart of color $SU(3)$ in H. Contains also $SO(3)$ as automorphisms of quaternionic subspaces. Could this group appear as an (approximate) dynamical gauge group?

   $M^8 = M^4 \times E^4$ as $SO(4)$ as a subgroup. It is not an automorphism group of octonions but leaves the octonion norm squared invariant. Could it be analogous to the holonomy group $U(2)$ of CP_2, which is not an isometry group and indeed is a spontaneously broken symmetry.

   A connection with hadron physics is highly suggestive. $SO(4) = SU(2)_L \times SU(2)_R$ acts as the symmetry group of skyrmions identified as maps from a ball of M^4 to the sphere $S^3 \subset E^4$. Could hadron physics $\leftrightarrow$ quark physics duality correspond to $M^8 - H$ duality. The radius of S^3 is proton mass: this would suggest that M^8 has an interpretation as an analog of momentum space.

7. What is the interpretation of M^8? Massless Dirac equation in M^8 for the octonionic spinors must be algebraic. This would be analogous to the momentum space Dirac equation. Solutions would be discrete points having interpretation as quark momenta! Quarks pick up discrete points of $X^4 \subset M^8$.

   States turn out to be massive in the M^4 sense: solves the basic problem of 4-D twistor approach (it works y for massless states only). Fermi ball is replaced with a region of a mass shell (hyperbolic space H^3).

   M^8 duality would generalize the momentum-position duality of the wave mechanism. QFT does not generalize this duality since momenta and position are not anymore operators.

4.1 Associative dynamics in M_c^8

How to realize the associative dynamics in M_c^8 [L25, L26]?

1. Number theoretical vision requires hierarchy of extensions of rationals and polynomials with rational coefficients would realize them. Rational coefficients make possible the interpretation as a polynomial with p-adic argument and therefore number theoretical universality.

   One cannot exclude the possibility that also real argument is allowed and that number theoretic universality and adelization applies only for the space-time surfaces defined by polynomials with rational coefficients.

2. Algebraic physics suggests that X^4 is in some sense a root of a M_c^8 valued polynomial. One can continue polynomials P with rational coefficients to M_c^8 by replacing the real argument with a complexified octonion.

3. The algebraic conditions should imply that the normal space of X^4 is quaternionic/associative. One can decompose octonions to sums $q_1 + I_4 q_2$, or "real" and "imaginary" parts q_i, which are quaternions and I_4 is octonion unit orthogonal to quaternions. The condition is that the "real" part of the octionic polynomial vanishes. Complexified 4-D surface whose projection to a real section (M^8 coordinates imaginary or real so that complexified octonion norm squared is real) is 4-D.

4. $M^8 - H$ duality requires an additional condition. The normal space contains also a complex plane M^2 which is commutative. This guarantees that normal spaces correspond to a point of CP_2. This is necessary in order to define $M^8 - H$ duality mapping X^4 from M^8 to H. M^2 can be replaced with an integrable distribution of M^2s if the assignment of the CP_2 point to tangent space can be made unique. This is the case if the spaces $M^2(x)$ are obtained from $M^2(y)$ by a unique G_2 automorphism $g(x, y)$.

4.2 Associativity condition at the level of M^8

Associativity condition for polynomials allows to characterize space-time surfaces in terms of polynomials with rational coefficients and possibly also analytic functions with rational Taylor coefficients at M^8 level. $M^8 - H$ duality would map $X^4 \subset M^8$ to $X^4 \subset H$. In M_c^8 the space-time surfaces could be also seen as graphs of local (complex) G_2 gauge transformations.

Remark: Even real coefficients can be considered. In this case polynomials with rational coefficients would define a unique discretion of WCW and allow p-adicization and adelization.

In the generic case the the set of points in the extension of rationals defining cognitive representation is discrete and finite. The surprise was that the "roots"can be solved explicitly and that the discrete cognitive representation is dense so that momentum quantization due to the finite volume of CD must be assumed to obtain finite cognitive representation inside CD. Cognitive representation could be defined by the points which correspond to the 8-momenta solving octonionic Dirac equation. This is excellent news concerning practical applications.

4.3 Uncertainty Principle and $M^8 - H$ duality

The detailed realization of $M^8 - H$ duality involves still uncertainties. The quaternionic normal spaces containing fixed 2-space M^2 (or an integrable distribution of M^2) are parametrized by points of CP_2. One can map the normal space to a point of CP_2.

The tough problem has been the precise correspondence between M^4 points in $M^4 \times E^4$ and $M^4 \times CP_2$ and the identification of the sizes of causal diamonds (CDs) in M^8 and H. The identification is naturally linear if M^8 is analog of space-time but if M^8 is interpreted as momentum space, the situation changes. The option discussed in [L25, L26] maps mass hyperboloids to light-cone proper time =constant hyperboloids and it has turned out that this correspondence does not correspond to the classical picture suggesting that a given momentum in M^8 corresponds in H to a geodesic line emanating from the tip of CD.

4.3.1 $M^8 - H$ duality in M^4 degrees of freedom

The following proposal for $M^8 - H$ duality in M^4 degrees of freedom relies on the intuition provided by UP and to the idea that a particle with momentum p^k corresponds to a geodesic line with this direction emanating from the tip of CD.

1. The first constraint comes from the requirement that the identification of the point $p^k \in X^4 \subset M^8$ should classically correspond to a geodesic line $m^k = p^k\tau/m$ ($p^2 = m^2$) in M^8 which in Big Bang analogy should go through the tip of the CD in H. This geodesic line intersects the opposite boundary of CD at a unique point.

   Therefore the mass hyperboloid H^3 is mapped to the 3-D opposite boundary of $cd \subset M^4 \subset H$. This does not fix the size nor position of the CD ($= cd \times CP_2$) in H. If CD does not depend on m, the opposite light-cone boundary of CD would be covered an infinite number of times.

2. The condition that the map is 1-to-1 requires that the size of the CD in H is determined by the mass hyperboloid M^8. Uncertainty Principle (UP) suggests that one should choose the distance T between the tips of the CD associated with m to be $T = \hbar_{eff}/m$.

   The image point m^k of p^k at the boundary of $CD(m, h_{eff})$ is given as the intersection of the geodesic line $m^k = p^k\tau$ from the origin of $CD(m, h_{eff})$ with the opposite boundary of $CD(m, h_{eff})$:

   $$m^k = \hbar_{eff} X \frac{p^k}{m^2} \quad , X = \frac{1}{1+p_3/p_0} \quad . \tag{4.1}$$

   Here p_3 is the length of 3-momentum.

   The map is non-linear. At the non-relativistic limit ($X \to 1$), one obtains a linear map for a given mass and also a consistency with the naive view about UP. m^k is on the proper time constant mass shell so the analog of the Fermi ball in $H^3 \subset M^8$ is mapped to the light-like boundary of $cd \subset M^4 \subset H$.

3. What about massless particles? Also now the geometric definition should work. How to identify the CD? UP suggests a CD with temporal distance $T = 2\hbar_{eff}/p_0$ between its tips so that the geometric definition gives $p^k = \hbar_{eff} p^k/p_0^2$ as the point at the 2-sphere defining the corner of CD. p-Adic thermodynamics [K8]) strongly suggests that also massless particles generate very small p-adic mass, which is however proportional to $1/p$ rather than $1/\sqrt{p}$. The map is well defined also for massless states as a limit and takes massless momenta to the 3-ball at which upper and lower half-cones meet.

4. What about the position of the CD associated with the mass hyperboloid? It should be possible to map all momenta to geodesic lines going through the 3-ball dividing the largest CD involved with T determined by the smallest mass involved to two half-cones. This is because this 3-ball defines the geometric "Now" in TGD inspired theory of consciousness. Therefore all CDs in H should have a common center and have the same geometric "Now".

   $M^8 - H$ duality maps the slicing of momentum space with positive/negative energy to a Russian doll-like slicing of $t \geq 0$ by the boundaries of half-cones, where t has origin at the bottom of the double-cone. The height of the $CD(m, h_{eff})$ is given by the Compton length $L(m, h_{eff}) = \hbar_{eff}/m$ of quark. Each value of h_{eff} corresponds its own scaled map and for $h_{gr} = GMm/v_0$, the size of $CD(m, h_{eff}) = GM/v_0$ does not depend on m and is macroscopic for macroscopic systems such as Sun.

5. The points of cognitive representation at quark level must have momenta with components, which are algebraic integers for the extension of rationals considered. A natural momentum unit is $m_{Pl} = \hbar_0/R$, h_0 is the minimal value of $h_{eff} = h_0$ and R is CP_2 radius. Only "active" points of $X^4 \subset M^8$ containing quark are included in the cognitive representation. Active points give rise to active CD:s $CD(m, h_{eff})$ with size $L(m, h_{eff})$.

   It is possible to assign $CD(m, h_{eff})$ also to the composites of quarks with given mass. Galois confinement suggest a general mechanism for their formation: bound states as Galois singlets must have a rational total momentum. This gives a hierarchy of bound states of bound states of ..... realized as a hierarchy of CDs containing several CDs.

6. This picture fits nicely with the general properties of the space-time surfaces as associative "roots" of the octonionic continuation of a real polynomial. A second nice feature is that the notion of CD at the level H is forced by this correspondence. "Why CDs?" at the level of H has indeed been a longstanding puzzle. A further nice feature is that the size of the largest CD would be determined by the smallest momentum involved.

7. Positive and negative energy parts of zero energy states would correspond to opposite boundaries of CDs and at the level of M^8 they would correspond to mass hyperboloids with opposite energies.

8. What could be the meaning of the occupied points of M^8 containing fermion (quark)? Could the image of the mass hyperboloid containing occupied points correspond to sub-CD at the level of H containing corresponding points at its light-like boundary? If so, $M^8 - H$ correspondence would also fix the hierarchy of CDs at the level of H.

It is enough to realize the analogs of plane waves only for the actualized momenta corresponding to quarks of the zero energy state. One can assign to CD as total momentum and passive *resp.* active half-cones give total momenta $P_{tot,P}$ *resp.* $P_{tot,A}$, which at the limit of infinite size for CD should have the same magnitude and opposite sign in ZEO.

The above description of $M^8 - H$ duality maps quarks at points of $X^4 \subset M^8$ to states of induced spinor field localized at the 3-D boundaries of CD but necessarily delocalized into the interior of the space-time surface $X^4 \subset H$. This is analogous to a dispersion of a wave packet. One would obtain a wave picture in the interior.

4.3.2 Uncertainty Principle requires also delocalization of X^4 and sub-CDs in H

One can argue that Uncertainty Principle (UP) requires more than the naive condition $T = \hbar_{eff}/m$ on the size of sub-CD. The intuitive idea that a single point in M^8 corresponds to a discretized plane wave in H in a spatial resolution defined by the total mass at the passive boundary of CD. UP requires that this plane wave should be realized at the level of H and also WCW as a superposition of shifted space-time surfaces defined by the above correspondence.

1. The basic observation leading to TGD is that in the TGD framework a particle as a point is replaced with a particle as a 3-surface, which by holography corresponds to 4-surface.

   Momentum eigenstate corresponds to a plane wave. Now planewave could correspond to a delocalized state of 3-surface - and by holography that of 4-surface - associated with a particle.

   A generalized plane wave would be a quantum superposition of shifted space-time surfaces with a phase factor determined by 4-momentum. This suggests that $M^8 - H$ duality should map the point of M^8 containing an object with momentum p to a generalized plane wave in H.

2. This would also provide WCW description. Recent physics relies on the assumption about single background space-time: WCW is effectively replaced with M^4 since 3-surface is replaced with point and CP_2 is forgotten so that one must introduce gauge fields and metric as primary field variables.

4.3.3 Exact ZEO emerges only at the limit of CD with infinite size

At the limit when the volume of CD becomes infinite, the sum of the momenta associated with opposite boundaries of CD should automatically vanish and one would obtain ideal zero energy states. The original assumption that ideal zero energy states are possible for finite size of CD, is not strictly true. The situation is the same for quantization in a finite volume.

1. Denote the sum of the total momenta with positive energy associated with passive boundaries of all CDs by $P_{tot,P} =\equiv P_{tot}$. For finite size of CD, $P_{tot,P}$ need not be the same as the total momentum $P_{tot,A}$ associated with the active boundary which can change during the sequence of SSFRs. Denote the difference $P_{tot,P} - P_{tot,A}$ by ΔP.

   This momentum is P_{tot} is large for large CDs, and naturally defines the spatial resolution. Denote by $M^k = nXh_{eff}P^k_{tot}/\cdot P^2_{tot}$, $X = 1/(1 + P_3/P_0)$, the shift defined by P_{tot}. The analogs of plane waves for the sub-CDs should be discretized with this spatial resolution and at the limit of large total mass the discretization improves.

2. The image of X^4 in H for a given mass hyperboloid H^3 should define a geometric analog of a plane wave in WCW for the total momentum $P^k = \sum_i p^k_i$, $p^2_i = m^2$ of H^3, associated with the CD(M) in M^8. It is also possible to include the momenta with different masses since they have images also at the boundaries of all CDs in the Russian doll hierarchy. For $\hbar_{gr}$ there is a common CD for all particle masses with size Λ_{gr}.

   The WCW plane wave would not be a superposition of points but of shifted space-time surfaces. The argument of the plane wave would correspond to the shift of the $X^4 \subset CD(M) \subset H$.

   Maximal spatial resolution is achieved if one shifts the X^4 and corresponding CD(m) in H inside the large CD by nM^k, $M^k = nh_{eff}XP^k_{tot}/\cdot P^2_{tot}$ and forms the WCW spinor field as a superposition of shifted space-time surfaces $X^4(m)$ with $U_n = exp(i\Delta P \cdot nM)$ appearing as plane wave phase factor.

3. At the limit when the size of the largest CD becomes infinite (the mass M defining Λ_{gr} becomes very large), the sum $\sum_n U_n$ obtained as integral over the identical shifted copies of the space-time surfaces is non-vanishing only for $\Delta P = 0$ and one obtains an momentum conserving ideal zero energy state.

These states would be analogs of single particle states as plane waves, with particle replaced with many-quark state inside $CD(m)$. The generalization is obvious: perform the analog of second quantization by forming N-particle states in which one has N $CD(m)$ plane waves.

4.4 The revised view about $M^8 - H$ duality and the "very special moments in the life of self"

The polynomial equations allow at M^8 level also highly unique brane-like solutions having the topology of 6-sphere S^6 and intersecting M^4 along $p^0 = E = constant$ hyperplane. These quantizd values of energy E correspond to the roots of the polynomial defining the solution and are algebraic numbers and algebraic integers for monic polynomials of form $P(x) = x^n + p_{n-1}x^{n-1} + ...$

The TGD inspired theory of consciousness motivated the interpretation of these hyperplanes as "very special moments in the life of self": this interpretation [L17] emerged before the realization that M^8 corresponds to momentum space. The images of these planes under $M^8 - H$ duality should however allow this interpretation also in the new picture. Is this possible?

To answer the question one must understand what the image of S^6 under $M^8 - H$ duality is.

1. The image must belong to $M^4 \times CP_2$. The 2-D normal space of the point of S^6 is a complex commutative plane of octonions. Since 4-D normal planes of space-time surface containing complex plane correspond to points of CP_2, the natural proposal is that the image now corresponds to point of CP_1 identified as homologically trivial geodesic sub-manifold S^2_G of CP_2 carrying Kähler magnetic charge.

2. The first thing to notice about the H-image of the 3-D $E = constant$ surface $X^3(E) \subset M^4$ is that it is indeed 3-D rather than 4-D. In M^4 the map has the form $m^k = X\hbar_{eff}/m^2$, $X = 1/(1 + p_3/p_0)$ already discussed.

   The value of $m^2 = E^2 - p^2_3$ decreases as p^2_3 increases so that the values of light-cone proper time $a = t^2 - r^2$ for the image are larger than $a_{min} = \hbar_{eff}/m$. "Fermi-spheres" $S^2_F(p_3)$ are mapped to 2-spheres $S^2(r) \subset M^4 \subset H$ with an increasing radius $r(t) = \sqrt{t^2 - a^2_{min}}$. 2-sphere is born at $t = a_{min}$ and starts to increase in size and the expansion velocity approaches light velocity asymptotically. This expanding sphere would be magnetically charged.

The sequence a_n of "very special moments in the life of self" in the life of self would mean the birth of this kind of expanding sphere and a_n would correspond to the roots of the polynomial considered identified as quantized energies. The dispersion relation $E = constant$ means that energy does not depend on the momentum: plasmons provide the condensed matter analogy.

3. There are interesting questions to be answered. Do the surfaces $X^3(E)$ intersect the 4-D space-time surface $X^4 \subset H$? At the level of M^8 the intersections of 4-D and 6-D surfaces are 2-D. The proposal is that these 2-surfaces M^8 are mapped to partonic vertices identified as 2-surfaces $X^2 \subset X^4 \subset H$ at which 4-D surfaces representing particles meet. This should happen also for the new identification of $M^8 - H$ duality.

   However, in the generic case the intersections of 3-surfaces and 4-surfaces in H are empty. The recent situation is however not a generic one since the S^6 solutions are non-generic (one would expect only 4-D solutions) and 4-D and 6-D solutions are determined by the same polynomial. Therefore the points to which the 2-spheres contract for $t = a_{min}$ should be mapped to partonic 2-surfaces in H. Single point should correspond to the geodesic sphere S^2_G.

   Does this conform with the view that 4-D CP_2 type extremals in H correspond to "blow-ups" of 1-D line singularities of $X^4 \subset M^8$ for which the quaternionic tangent spaces at singularity are not unique and define 3-D surface as points of CP_2. Now the 2-D normal spaces of S^2_F would span $S^2_G \subset CP_2$ and at the limit of S^2_F contracting to a point, one would have a 2-D singularity having an interpretation as a partonic vertex.

4. Cosmic strings $X^4 = X^2 \times S^2_G \subset M^4 \times CP_2$ carrying monopole charge are basic solutions of field equations. Could these cosmic strings relate to the images of $X^3(E)$? For instance, could $X^3(E_1)$ and $X^3(E_2)$ correspond to the ends of a cosmic string thickening to a monopole flux tube? Thickening would correspond to the growth of M^4 projection $S^2(r(t))$ of the flux tube having $r(t) = \sqrt{t^2 - a^2_{min}}$. The interpretation would be as a pair of magnetic poles connected by a monopole flux tube. Cosmic strings would be highly dynamical entities if this is the case.

4.5 Space-time surfaces as images of associative surfaces in M^8

$M^8 - H$ duality would provide an explicit construction of space-time surfaces as algebraic surfaces with an associative normal space [L25, L26]. M^8 picture codes space-time surface by a real polynomial with rational coefficients. One cannot exclude coefficients in an extension of rationals and also analytic functions with rational or algebraic coefficients can be considered as well as polynomials of infinite degree obtained by repeated iteration giving rise algebraic numbers as extension and continuum or roots as limits of roots.

$M^8 - H$ duality maps these solutions to H and one can consider several forms of this map. The weak form of the duality relies on holography mapping only 3-D or even 2-D data to H and the strongest form maps entire space-time surfaces to H. The twistor lift of TGD allows to identify the space-time surfaces in H as base spaces of 6-D surfaces representing the twistor space of space-time surface as an S^2 bundle in the product of twistor spaces of M^4 and CP_2. These twistor spaces must have Kähler structure and only the twistor spaces of M^4 and CP_2 have it [A5] so that TGD is unique also mathematically.

An interesting question relates to the possibility that also 6-D commutative space-time surfaces could be allowed. The normal space of the space-time surface would be a commutative subspace of M^8_c and therefore 2-D. Commutative space-time would be a 6-D surface X^6 in M^8.

This raises the following question: Could the inverse image of the 6-D twistor-space of 4-D space-time surface X^4 so that X^6 would be M^8 analog of twistor lift? This requires that $X^6 \subset M^8_c$ has the structure of an S^2 bundle and there exists a bundle projection $X^6 \to X^4$.

The normal space of an associative space-time surface actually contains this kind of commutative normal space! Its existence guarantees that the normal space of X^4 corresponds to a point of CP_2. Could one obtain the M^8_c analog of the twistor space and the bundle bundle projection

$X^6 \rightarrow X^4$ just by dropping the condition of associativity. Space-time surface would be a 4-surface obtained by adding the associativity condition.

One can go even further and consider 7-D surfaces of M^8 with real and therefore well-ordered normal space. This would suggest dimensional hierarchy: $7 \rightarrow 6 \rightarrow 4$.

This leads to a possible interpretation of twistor lift of TGD at the level of M^8 and also about generalization of $M^8 - H$ correspondence to the level of twistor lift. Also the generalization of twistor space to a 7-D space is suggestive. The following arguments representa vision about "how it must be" that emerged during the writing of this article and there are a lot of details to be checked.

4.5.1 Commutative 6-surfaces and twistorial generalization of $M^8 - H$ correspondence

Consider first the twistorial generalization of $M^8 - H$ correspondence.

1. The complex 6-D surface $X_c^6 \subset M_c^8$ has commutative normal space and thus corresponds to complexified octonionic complex numbers $(z_1 + z_2 I)$. X_c^6 has real dimension 12 just as the product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces of M^4 and CP_2. It has a bundle structure with a complex 4-D base space which is mapped $M^4 \times CP_2$ by $M^8 H$ duality. The fiber has complex dimension 2 and corresponds to the dimension for the product of twistor spheres of the twistor spaces of M^4 and CP_2.

2. This suggests that $M^8 - H$ duality generalizes so that it maps $X_c^6 \subset M_c^8$ to $T(M^4) \times T(CP_2)$. It would map the point of X_c^6 to its real projection identified as a point of $T(M^4)$. "Real" means here that the complex continuation of the number theoretical norm squared for octonions is real so that the components of M^8 point are either real or imaginary with respect to the commuting imaginary unit i. The complex 6-D tangent space of X_c^6 would be mapped to a point of $T(CP_2)$.

   The beauty of this picture would be that the entire complex 6-D surface would carry physical information mapped directly to the twistor space.

One can try to guess the form of the map of X_c^6 to the product $T(M^4) \times T(CP_2)$.

The surfaces X^6 have local normal space basis $1 \oplus e_7$. The problem is that this space is invariant under $SU(3)$ for $M^8 - H$ for CP_2. Could one choose the 2-D normal space to be something else without losing the duality. If e_7 and e_1 are permuted, the tangent space basis vector transforms by a phase phase factor under $U(1) \times U(1)$. The 4-D sub-basis of normal space would be now $(1, e_1, e_7, e_2)$. This does not affect the $M^8 - H$-duality map to CP_2. The 6-D space of normal spaces would be the flag manifold $SU(2)/U(1)timesU(1)$, which is nothing but the twistor space $T(CP_2)$.

What about the twistorial counterpart for the map of $M^4 \subset M^8 \rightarrow M^4 \subset M^8$? One can consider several options.

1. At the level of M^8, M^4 is replaced by M^6 at least locally in the sense that one can use M^6 coordinates for the point of X^6. Can one identify the M^6 image of this space as the projective space $C^4/C_\times$ obtained from C^4 by dividing with complex scalings? This would give the twistor space $CP_3 = SU(4)/U(3)$ of M^4. This is not obvious since one has (complexified) octonions rather than C^4 or its hypercomplex analog. This would be analogous to using several (4) coordinate charts glued together as in the case of sphere CP_1.

2. If $M^8 - H$ duality generalizes as such, the points of M^6 could be mapped to the 6-D analog of cd_4 such that the image point is defined as the intersection of a geodesic line with direction given by the 6-D momentum with the 5-D light-like boundary of 6-D counterpart cd_6 of cd? Does the slicing of M^6 by 5-D light-boundaries of cd_6 for various values of 6-D mass squared have interpretation as CP_3? Note that the boundary of cd_6 does not contain origin and the same applies to $CP_3 = C^4/C_\times$.

3. Or could one identify the octonionic analog of the projective space $CP_3 = C^4/C_\times$? Could the octonionic M^8 momenta be scaled down by dividing with the momentum projection in the commutative normal space so that one obtains an analog of projective space? Could one use these as coordinates for M^6?

The scaled 8-momenta would correspond to the points of the octonionic analog of CP_3. The scaled down 8-D mass squared would have a constant value.

A possible problem is that one must divide either from left or right and results are different in the general case. Could one require that the physical states are invariant under the automorphisms generated $o \rightarrow gog^{-1}$, where g is an element of the commutative subalgebra in question?

What about the physical interpretation at the level of M^8_c.

1. The first thing to notice is that in the twistor Grassmann approach twistor space provides an elegant description of spin. Partial waves in the fiber S^2 of twistor space representation of spin as a partial wave. All spin values allow a unified treatment.

   The problem is that this requires massless particles. In the TGD framework 4-D masslessness is replaced with its 8-D variant so that this difficulty is circumvented. This kind of description in terms of partial waves is expected to have a counterpart at the level of the twistor space $T(M(^4) \times T(CP_2)$. At level of M^8 the description is expected to be in terms of discrete points of M^8_c.

2. Consider first the real part of $X^6_c \subset M^8_c$. At the level of M^8 the points of X^4 correspond to points. The same must be true also at the level of X^6. Single point in the fiber space S^2 would be selected. The interpretation could be in terms of the selection of the spin quantization axis.

   Spin quantization axis corresponds to 2 diametrically opposite points of S^2. Could the choice of the point also fix the spin direction. There would be two spin directions and in the general case of a massive particle they must correspond to the values $S_z = \pm 1/2$ of fermion spin. For massless particles in the 4-D sense two helicities are possible and higher spins cannot be excluded. The allowance of only spin 1/2 particles conforms with the idea that all elementary particles are constructed from quarks and antiquarks. Fermionic statistics would mean that for fixed momentum one or both of the diametrically opposite points of S^2 defining the same and therefore unique spin quantization axis can be populated by quarks having opposite spins.

3. For the 6-D tangent space of X^6_c or rather, its real projection, an analogous argument applies. The tangent space would be parametrized by a point of $T(CP_2)$ and mapped to this point. The selection of a point in the fiber S^2 of $T(CP_2)$ would correspond to the choice of the quantization axis of electroweak spin and diametrically opposite points would correspond to opposite values of electroweak spin 1/2 and unique quantization axis allows only single point or pair of diametrically opposite points to be populated.

   Spin 1/2 property would hold true for both ordinary and electroweak spins and this conforms with the properties of $M^4 \times CP_2$ spinors.

4. The points of $X^6_c \subset M^8_c$ would represent geometrically the modes of H-spinor fields with fixed momentum. What about the orbital degrees of freedom associated with CP_2?

   M^4 momenta represent orbital degrees of M^4 spinors so that E^4 parts of E^8 momenta should represent the CP_2 momenta. The eigenvalue of CP_2 Laplacian defining mass squared eigenvalue in H should correspond to the mass squared value in E^4 and to the square of the radius of sphere $S^3 \subset E^4$.

   This would be a concrete realization for the $SO(4) = SU(2)_L \times SU(2)_R \leftrightarrow SU(3)$ duality between hadronic and quark descriptions of strong interaction physics. Proton as skyrmion would correspond to a map S^3 with radius identified as proton mass. The skyrmion picture would generalize to the level of quarks and also to the level of bound states of quarks allowed by the number theoretical hierarchy with Galois confinement. This also includes bosons as Galois confined many quark states.

5. The bound states with higher spin formed by Galois confinement should have the same quantization axis in order that one can say that the spin in the direction of the quantization axis is well-defined. This freezes the S^2 degrees of freedom for the quarks of the composite.

4.5.2 7-surfaces with real normal space and generalization of the notion of twistor space

It would seem that twistorialization could correspond to the introduction of 6-surfaces of M^8, which have commutative normal space. The next step is to ask whether it makes sense to consider 7-surfaces with a real norma space allowing well-ordering? This would give a hierarchy of surfaces of M^8 with dimensions 7, 6, and 4. The 7-D space would have bundle projection to 6-D space having bundle projection to 4-D space.

What could be the physical interpretation of 7-D surfaces of M^8 with real normal space in the octonionic sense and of their H images?

1. The first guess is that the images in H correspond to 7-D surfaces as generalizations of 6-D twistor space in the product of similar 7-D generalization of twistor spaces of M^4 and CP_2. One would have a bundle projection to the twistor space and to the 4-D space-time.

2. $SU(3)/U(1) \times U(1)$ is the twistor space of CP_2. $SU(3)/SU(2) \times U(1)$ is the twistor space of M^4? Could 7-D $SU(3)/U(1)$ *resp.* $SU(4)/SU(3)$ correspond to a generalization of the twistor spaces of M^4 *resp.* CP_2? What could be the interpretation of the additional dimension in the S^2 fiber of the twistor spaces of M^4 and CP_2 and of X^4? What could the interpretation of the fiber be? S^3 isomorphic to $SU(2)$ and having $SO(4)$ as isometries is the obvious candidate.

3. The analog of $M^8 - H$ duality in Minkowskian sector in this case could be to use coordinates for M^7 obtained by dividing M^8 coordinates by the real part of the octonion. Is it possible to identify $RP_7 = M^8/R_\times$ with $SU(4)/SU(3)$ or at least relate these spaces in a natural manner. It should be easy to answer these questions with some knowhow in practical topology.

   A possible source of problems of or understanding is the presence of a commuting imaginary unit implying that complexification is involved in Minkowskian degrees of freedom whereas in CP_2 degrees of freedom it has no effect. RP_7 is complexified to CP_7 and the octonionic analog of CP_3 is replaced with its complexification,

4. Twistorialization takes care of spin and electroweak spin. The remaining standard model quantum numbers are quark number and em charge. Could the additional dimension allow their geometrization as partial waves in the 3-D fiber. Could this fiber be $SU(2)$ locally? This would mean that charge and quark number are correlated for a given space-time surface (region) (U or D quark or charged or neutral lepton).

4.5.3 How do the Grassmannians of standard twistor approach emerge number theoretically?

One can identify the TGD counterparts for various Grassmann manifolds appearing in the standard twistor approach.

Consider first, the various Grassmannians involved with the standard twistor approach (`https://cutt.ly/XE3vDKj`) can be regarded as flag-manifolds of 4-complex dimensional space T.

1. Projective space is FP_{n-1} the Grasmannian $F_1(F^n)$ formed by the k-D planes of V^n where F corresponds to the field of real, complex or quaternionic numbers, are the simplest spaces of this kind. The F-dimension is $d_F = n-1$. In the complex case, this space can be identified as $U(n)/U(n-1) \times U(1) = CP_{n-1}$.

2. More general flag manifolds carry at each point a flag, which carries a flag which carries ... so that one has a hierarchy of flag dimensions $d_0 = 0 < d_1 < d_2 ... d_k = n$. Defining integers $n_i = d_i - d_{i-1}$, this space can in the complex case be expressed as $U(n)/U(n_1) \times .....U(n_k)$. The real dimension of this space is $d_R = n^2 - \sum_i n_i^2$.

3. For $n = 4$ and $F = C$, one has the following important Grassmannians.

   (a) The twistor space CP_3 is projective is of complex planes in $T = C^4$ and given by $CP_3 = U(4)/U(3) \times U(1)$ and has real dimension $d_R = 6$.

(b) $M = F_2$ as the space of complex 2-flags corresponds to $U(4)/U(2) \times U(2)$ and has $d_R = 16 - 8 = 8$. This space is identified as a complexified Minkowski space with $D_C = 4$.

(c) The space $F_{1,2}$ consisting of 2-D complex flags carrying 1-D complex flags has representation $U(4)/U(2) \times U(1) \times U(1)$ and has dimension $D_R = 10$.

$F_{1,2}$ has natural projection ν to the twistor space CP_3 resulting from the symmetry breaking $U(3) \to U(2) \times U(1)$ when one assigns to 2-flag a 1-flag defining a preferred direction. $F_{1,2}$ also has a natural projection μ to the complexified and compactified Minkowski space $M = F_2$ resulting in the similar manner and is assignable to the symmetry breaking $U(2) \times U(2) \to U(1) \times U(1)$ caused by the selection of 1-flag.

These projections give rise to two correspondences known as Penrose transform. The correspondence $\mu \circ \nu_{-1}$ assigns to a point of twistor space CP_3 a point of complexified Minkowski space. The correspondence $\nu \circ \mu_{-1}$ assigns to the point of complexified Minkowski space a point of twistor space CP_3. These maps are obviously not unique without further conditions.

This picture generalizes to TGD and actually generalizes so that also the real Minkowski space is obtained naturally. Also the complexified Minkowski space has a natural interpretation in terms of extensions of rationals forcing complex algebraic integers as momenta. Galois confinement would guarantee that physical states as bound states have real momenta.

1. The basic space is $Q_c = Q^2$ identifiable as a complexified Minkowski space. The idea is that number theoretically preferred flags correspond to fields R, C, Q with real dimensions 1,2,4. One can interpret Q_c as Q^2 and Q as C^2 corresponding to the decomposition of quaternion to 2 complex numbers. C in turn decomposes to $R \times R$.

2. The interpretation $C^2 = C^4$ gives the above described standard spaces. Note that the complexified and compactified Minkowski space is not same as $Q_c = Q^2$ and it seems that in TGD framework Q_c is more natural and the quark momenta in M^4_c indeed are complex numbers as algebraic integers of the extension.

Number theoretic hierarchy $R \to C \to Q$ brings in some new elements.

1. It is natural to define also the quaternionic projective space $Q_c/Q = Q^2/Q$ `https://cutt.ly/LE3vMOG`, which corresponds to real Minkowski space. By non-commutativity this space has two variants corresponding to left and right division by quaternionic scales factor. A natural condition is that the physical states are invariant under automorphisms $q \to hqh^{-1}$ and depend only on the class of the group element. For the rotation group this space is characterized by the direction of the rotation axis and by the rotation angle around it and is therefore 2-D.

   This space is projective space QP_1, quaternionic analog of Riemann sphere CP_1 and also the quaternionic analog of twistor space CP_3 as projective space. Therefore the analog of real Minkowski space emerges naturally in this framework. More generally, quaternionic projective spaces Q^n have dimension $d = 4n$ and are representable as coset spaces of symplectic groups defining the analogs of unitary/orthogonal groups for quaternions as $Sp(n+1)/Sp(n) \times Sp(1)$ as one can guess on basis of complex and real cases. M^4_R would therefore correspond to $Sp(2)/Sp(1) \times SP(1)$.

   QP_1 is homeomorphic to 4-sphere S^4 appearing in the construction of instanton solutions in E^4 effectively compactified to S^4 by the boundary conditions at infinity. For Minkowski signature it would be replaced by 4-D hyperboloid $H^4 = SO(1,4)/SO(3)$ known also as anti-de Sitter space AdS(4,1) (`https://cutt.ly/RRuXIBS`). An interesting question is whether the self-dual Kähler forms in E^4 could give rise to M^4 Kähler structure and could correspond to this kind of self-dual instantons and therefore what I have called Hamilton-Jacobi structures.

2. The complex flags can also contain real flags. For the counterparts of twistor spaces this means the replacement of $U(1)$ with a trivial group in the decompositions.

The twistor space CP_3 would be replaced $U(4)/U(3)$ and has real dimension $d_R = 7$. It has a natural projection to CP_3. The space $F_{1,2}$ is replaced with representation $U(4)/U(2)$ and has dimension $D_R = 12$.

To sum up, the Grassmannians associated with M^4 as 6-D twistor space and its 7-D extension correspond to a complexification by a commutative imaginary unit i - that is "vertical direction". The Grassmannians associated with CP_2 correspond to "horizontal ", octonionic directions and to associative, commutative and well-ordered normal spaces of the space-time surface and its 6-D and 7-D extensions. Geometrization of the basic quantum states/numbers - not only momentum - representing them as points of these spaces is in question.

4.5.4 How could the quark content of the physical state determine the geometry of the space-time surface?

In the standard quantum field theory, fermionic currents serve as sources of the gauge fields. This correlation must have a counterpart in the TGD framework. Somehow the selection of the active points of the cognitive representation containing quarks must determine the 4-surface of M^8 determined by a polynomial P with rational coefficients. $M8-H$ duality would in turn determine the space-time surface.

This requirement gives a motivation for the earlier assumption that the roots of P defining 6-D surfaces fix P. Two kinds of surfaces appear.

1. The special $E = E_n$ roots of P having interpretation as energy have 3-D hyperplanes as M^4 intersections that I have misleadingly called "special moments in the life of self".

   The proposal [L25, L26] was that quarks are associated with the 2-D intersections of 4-D space-time surfaces with these planes. At the level of H, these 2-D intersections were assigned to partonic 2-surfaces serving as vertices of topological Feynman diagrams represented as space-time surfaces. Knowledge of the values of energy E_n defining 3-D complex planes at which the quarks of the quantum state are located in momentum space fixes the minimal polynomial P and therefore also space-time surface.

2. Besides energy hyper-planes there are also complex mass hyperboloids. The general 4-D solution of co-associativity conditions is 4-D (in real sense) intersection of two complex mass shells with mass squared $m^2_{c,odd}$ *resp.* $m^2_{c,even}$ with complex mass squared equal to a root of the odd *resp.* even part of the polynomial P defining the 4-surface [L25]. The real projection of the 4-D intersection is 2-D and might have interpretation as counterpart of a partonic 2-surface.

   This complex surface has complex dimension 4 and 4-D real projection in the sense that the number theoretic quadratic form is real. The 6-surface defined by the root reduces to a 3-D real mass shell if the imaginary part of m^2_c can vanishes: this is possible for real roots only. The 4-D intersection of these complex mass shells provide natural seats for the quark momenta as algebraic integers, which in general are complex. This data can fix the roots of the imaginary part of P as complex mass squared values.

3. Interestingly, also 6-D surfaces having these 4-surfaces as sub-manifolds emerge. A good guess is that these are just the surfaces with commutative normal space and serve as M^8 counterparts of twistor space.

4.5.5 How to understand leptons as bound states of 3 quarks?

A benchmark test for the view about the twistorial aspects of M^8 is the challenge of describing leptons as bound states of 3 quarks assignable to single wormhole contact, single throat, or even single point. The assumption that wormhole contacts correspond to blow-ups of line singularities in M^8 containing quarks favors the strongest option.

1. At the level of H , quarks with different colors (color partial waves in CP_2) could have exactly the same M^4 location inside a single wormhole throat but different CP_2 locations to realize statics. Color can be realized as H partial waves and this would require that the oscillator

operators act at the level of M^8 allowing to put several oscillators at a single M^4 point at the level of H.

2. At the level of M^8 the Fermi statistics would state that only a single quark corresponds to a given point. If one works at the level of 4-surface so that only momentum is taken into account, this is not possible. Could the 3 quarks be at different points in the 7-D extension of the twistor space bringing in quark spin and Kähler magnetic charge?

   The total spin of lepton is 1/2 so that two spins are opposite. Kähler magnetic charges of quarks are proposed to be proportional to color hypercharge (2,-1,-1) for quarks to realize Fermi statistics topologically. The points (p,1/2,-1),(p,1/2,-1) and (p,-1/2,2) and the states obtained by permuting Kähler charges would allow arealization of lepton as a 3 quark state with identical momenta.

4.6 Hierarchies of extensions for rationals and for inclusions of hyperfinite factors

TGD suggests 3 different views of finite measurement resolution.

1. At the space-time level, finite measurement resolution is realized in terms of cognitive representations at the level of M^8 actualized in terms of fermionic momenta with momentum components identifiable as algebraic integers. Galois group has natural action on the momentum components.

2. The inclusion $N \subset M$ of group algebras of Galois groups is proposed to realize finite measurement resolution for which the number theoretic counterpart is Galois singlet property of N with respect to the Galois group of M relative to N identifiable as the coset group of Galois groups of M and N. If the origin serves as a root of all polynomials considered, the composite $P \circ Q$ inherits the roots of Q.

   The idea generalizes to infinite-D Galois groups [L37, L33]. The HFF in question would be infinite-D group algebra of infinite Galois group for a polynomial R obtained as a composite $R = P_{infty} \circ Q$ of an infinite iterate P_{infty} of polynomial P and of some polynomial Q of finite degree (inverse limit construction). The roots of R at the limit correspond to the attractor basin associated with P_∞, which is bounded by the Julia set so that a connection with fractals emerges.

3. The inclusions $N \subset M$ of hyperfinite factors of type II_1 (HFFs) [K37, K26] is a natural candidate for the representation of finite measurement resolution. N would represent the degrees of freedom below measurement resolution mathematically very similar to gauge degrees of freedom except that gauge algebra would be replaced with the super-symplectic algebra and analogs of Kac Moody algebra with non-negative conformal weights and gauge conditions would apply to sub-algebra with conformal weights larger than the weight h_{max} defining the measurement resolution.

For HFFs, the index $[M : N]$ of the inclusion defines the quantum dimension $d(N \subset M) \leq 1$ as a quantum trace of the projector $P(M \to N)$ (the identify operator of M has quantum trace equal to one). $d(N \subset M)$ is defined in terms of quantum phase q and serves as a dimension for the analog of factor space M/N representing the system with N regarded as degrees of freedom below the measurement resolution and integrated out in "quantum algebra" M/N. Quantum groups and quantum spaces are closely related notions [K37, K26].

Galois confinement would suggest that $N \subset M$ corresponds to the algebra creating Galois singlets with respect to the Galois group of N relative to M whereas M includes also operators which are not this kind of singlets. In the above example $R = P \circ Q$, the Galois group of P would be represented trivially and the Galois group of Q or its subgroup would act non-trivially. In the case of hadrons, color degrees of freedom perhaps assignable to the Galois group Z^3 in the case of quarks would correspond to the degrees of freedom below the measurement resolution.

The universality of the quantum dimension and its expressibility in terms of quantum phase suggests that the integer m in $q = rxp(i2\pi/m)$ is closely related to the dimension for the extension of rationals $n = h_{eff}/h_0$ and depends therefore only very weakly on the details of the extension.

The simplest guess is $m = n$. This conforms with the concrete interpretation of charge fractionation as being due to the many-valuedness of the graphs of space-time surfaces as maps from $M^4 \rightarrow CP_2$ or vice versa.

4.7 Galois confinement

The notion of Galois confinement emerged in TGD inspired biology [L55, L30, L33, L39]. Galois group for the extension of rationals determined by the polynomial defining the space-time surface $X^4 \subset M^8$ acts as a number theoretical symmetry group and therefore also as a physical symmetry group.

1. The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L30, L39, L38]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units.

2. The idea generalizes also to nuclear physics and suggests an interpretation for the findings claimed by Eric Reiter [L47] in terms of dark N-gamma rays analogous to BECs and forming Galois singlets. They would be emitted by N-nuclei - also Galois singlets - quantum coherently [L47]. Note that the findings of Reiter are not taken seriously because he makes certain unrealistic claims concerning quantum theory.

4.7.1 Galois confinement as a number theoretically universal manner to form bound states?

It seems that Galois confinement might define a notion much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Why not also hadrons could be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L20, L28, L9].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states. The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action inside the space-time surface is not well-defined. At the level of M^8 octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H. In contrast to this, weak gauge transformations acting as holonomies act in the tangent space of H.

   One can argue that the symmetries of H and even of WCW should/could have a reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (momenta) of M^8_c with coordinates, which are algebraic integers for the extension.

   Momenta as points of M^8_c would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of (in general complex) momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are algebraic, possibly complex, integers.

Also other quantum numbers, in particular color, would correspond to representations of the Galois group. In the case of angular moment Galois confinement would allow algebraic half-integer valued angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential factors which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

4.7.2 Explicit conditions for Galois confinement

It is interesting to look more explicitly at the conditions for the Galois confinement.

Single quark states have momenta, which are algebraic integers generated by so called integral basis (`https://cutt.ly/SRuZySX`) spanning algebraic integers as a lattice and analogous to unit vectors of momentum lattice but for single component of momentum as a vector in extension. There is also a theorem stating that one can form the basis of extension as powers of a single root. It is also known that irreducible monic polynomials have algebraic integers as roots.

1. In its minimal form Galois confinement states that only momenta, which are rational integers are allowed by Galois confinement. Note that for irreducible polynomials with rational coefficients one does not obtain any rational roots. Monic polynomials with integer coefficients can allow integer roots. If one assumes that single particle states can have arbitrary algebraic integer as momentum, one obtain also rational integers for momentum values. These states are not at mass - or energy shell associated with the single particle momenta.

2. A stronger condition would be that also the inner products of the momenta involved are real so that one has $Re(p_i) \cdot Im(p_j) = 0$. For $i = j$ this gives a condition is possible only for the real roots for the real polynomials defining the space-time surface.

To see that real roots are necessary, some facts about the realization of the co-associativity condition [L25] are necessary.

1. The expectation is that that the vanishing condition for the real part (in quaternionic sense) of the octonionic polynomial gives a co-associative surface. By the Lorentz symmetry one actually obtains as a solution a 6-D complex mass shell $m_c^2 \equiv m_{Re}^2 - m_{Im}^2 + 2iRe(p) \cdot Im(p) = r_1$, where the real and imaginary masses are defined are $m_{Re}^2 = Re(p)^2$ and $m_{Im}^2 = Im(p)^2$ and r_1 is some root for the odd part of the polynomial P assumed to determining the 4-surface.

2. This surface can be co-associative but would be also co-commutive. Maximally co-associative surface requires quaternionic normal space. The first proposal is that the space-time surface is the intersection of the surface defined by the polynomial and its conjugate with respect to i. This gives 4-D surface as the intersection of the two 6-D surfaces.

   Second proposal is that the 6-surface having a structure of S^2 bundle defines as its base space quaternionic 4-surface. This space would correspond to a gauge choices selecting point of S^2 at very point of M^4. To a given polynomial one could assign entire family of 4-surfaces mapped to different space-time surfaces in H. A possible interpretation of gauge group would be as quaternionic automorphisms acting on the 2-sphere.

   These proposals are equivalent if the base base is the intersection of the 6-D bundle spaces. One could say that the fibers are conjugates of each other. This might be relevant for ZEO.

Concerning Galois confinement, the basic result is that for complex roots r_1 the conditions $Re(p_i) \cdot Im(p_i) = 0$ cannot be satisfied unless one requires that r_1 is real. Therefore the stronger option makes sense for real roots only.

1. Galois confinement allows the momenta p_i forming the bound state to be in an extension of rationals defined by the polynomial defining the space-time surface. Galois confinement condition states that the total momentum is rational integer when a suitable unit defined by the size of CD is used (periodic boundary conditions).

2. Another natural condition is the vanishing of the inner products between the real part $Re(p)$ and imaginary part $Im(p)$ of p. This guarantees that the number theoretical norm squared for the momentum is real. For time-like p, this means that $Im(p)$ belongs to the 3-D orthogonal complement E^3 of $Re(p)$. For light-like p, $Im(p)$ belongs to 2-D orthogonal complement E^2.

3. Suppose one has several number theoretic momenta p_i such that $\sum p_i = p$ is rational integer and $p_i \propto p$ holds true. Also in this case, the number theoretic inner products must be real. The orthogonality conditions read as

$$Re(p_i) \cdot Im(p_j) = 0 \ . \tag{4.2}$$

   For a given pair (i,j), one has several conditions corresponding to algebraically independent imaginary momentum components and it is quite possible that very few solutions exist besides $Im(p_i) = 0$. If $Re(p_i)$ is not a rational integer, the number of conditions still increases.

4. The proposal for Galois confinement is that the real parts of p_i are parallel or even identical: $Re(p_i) \propto Re(\sum p_i) = p$, which is a rational integer. In this case the conditions reduce to $Re(p) \cdot Im(p_i) = 0$ and their number is much smaller.

5. For a given momentum component, the basis $p_{i,k}$ has the dimension n of extension. The basis contains m complex elements e_k and their conjugates $\overline{e}_k$ plus $n - 2m - 1$ real but algebraically trivial elements r_k besides the real unit 1. The sums $E_k = e_k + \overline{e}_k$ are algebraic integers and give m real basis elements. Note that $F_k = e_k - \overline{e}_k$ are purely imaginary algebraic integers.

   r_k and E_i give $n-m-1$ algebraically non-trivial real momenta. The momentum components $p_{i,k}$ formed as linear combinations of r_k, E_i, and 1 are real. This gives $n-m$-dimensional real subspace and momenta formed in this way satisfy the reality conditions for the inner products.

6. One can also construct complex momenta such that $Im(p_i)$ is a linear combination $Im(p_i) = \sum n_{i,k} F_k$. If $Re(p_i)$ are parallel and rational integers and $p_i \propto p$ holds true, the reality conditions reduce to

$$p \cdot Im(p_i) = \sum_k p^i n_{i,k} F_k = 0 \ . \tag{4.3}$$

   One can construct a maximal set of complex momenta P_K characterized by matrices n_{ik}^K satisfying these conditions. Also linear combinations of P_K satisfy the reality conditions and one obtains a lattice of momenta.

This looks like nice construction but it seems that mere Galois confinement is more realistic.

4.8 Some questions and ideas related to $M^8 - H$ duality

In the following some questions and ideas, which do not quite fit under the titles of the previous sections, are considered.

4.8.1 A connection with Langlands program

Langlands correspondence [A6, A1, A4, A3], which I have tried to understand several times [K27] [L1, L4, L8] relates in an interesting manner to $M^8 - H$ duality and Galois confinement.

1. Global Langlands correspondence (GLC) states that there is connection between representations of continuous groups and Galois groups of extensions of rationals.

2. Local LC states (LLC) states this in the case of p-adics.

There is a nice interpretation for the two LCs in terms of sensory experience and cognition in TGD inspired theory of consciousness.

1. In adelic physics real numbers and p-adic number fields define the adele. Sensory experience corresponds to reals and cognition to p-adics. Cognitive representations are in their discrete intersection and for extensions of rationals belonging to the intersection.

    (a) Sensory world, "real" world corresponds to representation of continuous groups/Galois groups of rationals. GLC.

    (b) "p-Adic" worlds correspond to cognition and representations of p-adic variants of continuous groups and Galois groups over p-adics. Local LLC.

    (c) One could perhaps talk also about Adelic LC: ALC in the TGD framework. Adelic representations would combine real and p-adic representations for all primes and give as complete information about reality as possible.

    TGD provides a geometrization for the identification of Galois groups as discrete subgroups of Lie groups, not only of the isometry (automorphism) groups of H (M^8) but perhaps also as discrete sub-groups of more general Lie groups to which the the action of super-symplectic representations could reduce. A naive guess is that these groups correspond to the ADE groups appearing in the McKay correspondence [L10, L23, L24].

    The representation of real continuous groups assignable to the real numbers as a piece of adele [L11, L12] would be related to the representations of Galois groups GLC. Also p-adic representations of groups are needed to describe cognition and these p-adic group representations and representations of p-adic Galois groups would be related by LLC.

4.8.2 Could the notion of emergence of space-time have some analog in the TGD Universe?

The idea about the emergence of space-time from entanglement is as such not relevant for TGD. One can however ask whath the emergence of *observed* space-time could mean in TGD. Space-time surface as a continuum exists in TGD but they are not directly observable due to a finite measurement resolution. One can ask what a body with an outer boundary means physically. The space-time regions defined by solid bodies have boundaries. What makes the boundaries of the bodies "hard"?

1. In momentum space Fermi statistics does not allow fermions to get through the boundary of Fermi ball. This is a good guideline.

2. Second feature of a spatial object such as an atom is that it is a bound state quantum mechanically. If it has parts they stay together. In QFT theory the notion of a bound state is however poorly understood.

3. Quantum coherence is a further property considered in the article. Spatial objects correspond to quantum coherent structures. Quantum coherence reduces to entanglement. Quantum coherence length and time determine the size of a quantum object. Somehow one must have stable entanglement in long scales.

Let us see what these guidelines could give in the framework of $M^8 - H$ duality which generalizes the wave particle duality of wave mechanics.

1. In adelic physics space-times can be seen as either surfaces in M^8 or $H = M^4 \times CP_2$. $X^4 \subset M^8$ is analogous to momentum space cognitive representations consist of points of $X^4 \subset M^8$, whose points are algebraic integers in the extension of rationals defined by the polynomial defining the space-time surface and are algebraic integers as roots of monic polynomials of form $x^n + ...$ This defines a unique discretization of the space-time surface. The discretization guarantees number theoretical universality: the cognitive representation makes sense also p-adically and space-time has also p-adic variants.

   Cognitive representations give rise to "cognitive emergence" of the space-time in cognitive sense and since cognitive representations are intersection of reality and p-adicities they must closely related to the "sensory emergence"".

2. $X^4 \subset M^8$ is mapped to H by $M^8 - H$ duality determined by the condition that it momentum is mapped to a geodesic with a direction of momentum and starting from either tip of CD: the image point is its intersection with the opposite light-like boundary of CD and selects a point of space-time surface. The size of CD is $T = h_{eff}/m$ for quark with mass m to satisfy Uncertainty Principle. The map generalizes to bound states of quarks (whatever they are).

Consider the problem of "sensory emergence" in this framework.

1. What makes a point of a cognitive representation "hard"? Quarks are associated with points (not necessarily all) of a cognitive representation: one can say that the point is activated when there is a quark at it. Fermi ball corresponds to a discrete set of activated points at the level of momentum space. These points define activated points also in $X^4 \subset H$ by $M^8 - H$ duality. One could perhaps say that these activated points in M^8 and their H-image containing fermions define the spatial objects as something "hard" and having a boundary. Another fermion knows that there is a space-time point there because it cannot get to this point. The presence of a fermion (quark) would make a space-time point "hard".

2. What about the role of entanglement? The size and duration of the space-time surface (inside a causal diamond CD) defines quantum coherence length and time. Fermionic statistics makes fundamental fermions - to be distinguished from elementary fermions - maximally entangled. One cannot reduce fermionic entanglement in SFR and quantum measurements would be impossible. The entanglement in the WCW degrees of freedom comes to the rescue. This entanglement can be reduced in SFRs since the particles as surfaces are identical under very special - naturally number theoretical - conditions.

   Negentropy Maximization Principle and hierarchy of $h_{eff} = n \times h_0$ phases favor the generation of stable entanglement in the TGD Universe. Also, if the coefficients of the entanglement matrix belong to extension of rationals, entanglement probabilities in general belong to its extension and the density matrix is not diagonalizable without going to a larger extension. This might require "big" SFR increasing the extension: only after this state function reduction to an eigenstate could occur. This leads to a concrete proposal for how the information about the diagonal form of the density matrix expressed by its characteristic polynomial is coded into the geometry of the space-time surface [L33].

3. Bound state formation is third essential element. Momenta are points of the space-time surface $X^4 \subset M^8$ with components which are algebraic integers. Physical momenta are however ordinary integers for a particle in a finite volume defined by causal diamond (CD). This means that one can allow only composites of quarks with rational integer valued momenta which correspond to Galois singlets.

   Galois confinement would be the universal mechanism behind formation of all bound states and also give rise to stable entanglement. One would obtain a hierarchy of bound states corresponding to a hierarchy of polynomials and corresponding Galois groups and extensions of rationals. By $M^8 - H$ duality, bound states of quarks and higher structures formed from them in M^8 would give rise to spatial objects.

5 Zero energy ontology (ZEO)

ZEO [K38] forms the cornerstone of the TGD inspired quantum theory extending to a theory of consciousness. ZEO has so far reaching consequences that it would have deserved a separate section. Since it involves in an essential manner the notion of CD, it is natural to include it to the section discussing $M^8 - H$ duality.

5.1 The basic view about ZEO and causal diamonds

The following list those ideas and concepts behind ZEO that seem to be rather stable.

1. GCI in WCW implies holography, Bohr orbitology and ZEO [L21, K38].

2. X^3 is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. Finite failure of determinism is however possible [L52]. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD) defined as intersection of future and past directed light cones ($\times CP_2$) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. 12 Fig. 13**).

4. The opposite light-like boundaries of CD correspond for fermions dual vacuums (bra and ket) annihilated by fermion annihilation *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD. This seems to be the correct view in H. At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing would be by mass shells (hyperboloids) at the level of M^8 and by CDs with same center point at the level of H.

5. Zeno effect can be understood if the states at either cone of CD do not change in "small" state function reductions (SSFRs). SSFRs are analogs of weak measurements. One could call this half-cone call as a passive half-cone. I have earlier used a somewhat misleading term passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate - geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change. The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like deterministic time evolution leading to the final state [L16] as observed by Minev et al [L16].

7. h_{eff} hierarchy [K23, K24, K25] implied by the number theoretic vision [L25, L26] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

8. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. Provides also a mechanism of self-organization [L19]: dissipation with reversed arrow of time looks like self-organization [L55].

5.2 Open questions related to ZEO

There are many unclear details related to the time evolution in the sequence of SSRs. Before discussing these unclear details let us make the following assumptions.

1. The size of CDs increases at least in a statistical sense in the sequence of CD and the second boundary remains stationary apart from scaling (note that one can also consider the possibility that the entire CD is scaled and temporal shift occurs in both directions).
2. Mental mentals (say after images) are in kind of Karma's cycle: they are born and die roughly periodically.
3. I do not experience directly mental images with the opposite arrow of time.
4. I can have memories only about states of consciousness with the same arrow of time that I have. This explains why I do not have memories about periods of sleep if sleep is interpreted as a time reversed state of some subself of me responsible for self-ness.

One can use three empirical inputs in an attempt to fix the model.

1. After images appear and disappear roughly periodically. Also I fall asleep and wake up with a standard arrow of time roughly periodically.
   (a) The first interpretation is that as a sequence of wake up-sleep periods I am a time crystal-like structure consisting of nearly copies of the mental image, such that each mental image - including me as mental images of higher level self - continues Karma's cycle in my geometric past. How "me" is transferred to a new almost copy of my biological body? Does my MB just redirect its attention?
   (b) The second interpretation is that me and my mental images somehow drift towards my geometric future, while performing the Karma's cycle so that my mental images follow me in my time travel. This would require that the sub-CDs of mental images drift towards the geometric future.

       Also sleep could be a "small" death at some layer of the personal hierarchy of MBs. I do not however wake-up in BSFR at the moment of geometric time defined by the moment of falling asleep but later. So it seems that my CD must drift to the geometric future with the same speed that those of other living beings in the biosphere.
2. There is however an objection. In cosmology the observation of stars older than the Universe would have a nice solution if the stars evolve forth and back in time in our distant geometric past rather than drifting towards the future so that they could age by continuing their Karma's cycle with a constant center of mass value of time. Can these three observations be consistent?

Suppose that the sub-CDs within a bigger CD "follow the flow". How the dynamics of the bigger CD could induce this flow?

1. The scalings of bigger CD in unitary evolutions between SSFRs i induce the scaling of sub-CS. This would not be shifting but scaling and the distance between given CD and larger CDs would gradually scale up.

   This would remove the objection. The astrophysical objects in distant geometric past would move towards the geometric future but with much smaller velocity as the objects with cosmic scale so that the temporal distance to future observers would increase. These objects would be ageing in their personal Karma's cycle, and the paradox would disappear.
2. The flow would be defined by the scalings of a larger CD containing our CDs and those of others at my level. Each CD would define a shared time for its sub-CDs. If the CDs form a hierarchy structure with a common center, this is indeed true of the time evolutions as scalings of CDs. There would be scalings induced by scalings at higher levels and "personal" scalings.

3. It however seems that the common center is too strong an assumption and shifted positions for the sub-CDs and associated hierarchy inside a given CD are indeed possible for the proposed realization of $M^8 - H$ duality and actually required by Uncertainty Principle.

A further open question is what happens to the size of CD in the BSFR. Does it remain the same so that the size of the CD would increase indefinitely? Or is the size reduced in the sense that there would be scaling, reducing the size of the CD in which the passive boundary of the CD would be shifted towards the active one. After every BSFR, the self would experience a "childhood".

5.3 What happens in quantum measurement?

According to the proposed TGD view about particle identity, the systems for which mutual entanglement can be reduced in SFR must be non-identical in the category theoretical sense.

When SFR corresponds to quantum measurement, it involves the asymmetric observer-system $O-S$ relationship. One cannot exclude SFRs without this asymmetry. Some kind of hierarchy is suggestive.

The extensions of rationals realize this kind of $O-S$ hierarchy naturally. The notion of finite measurement resolution strongly suggests discretization, which favors number theoretical realization. The hierarchies of effective Planck constants and p-adic length scale hierarchies reflect this hierarchy. What about the topological situation: can one order topologies to a hierarchy by their complexity and could this correspond to $O-S$ relationship?

The intuitive picture about many-sheeted space-time is as a hierarchical structure consisting of sheets condensed at larger sheets by wormhole contacts, whose throats carry fermion number. Intuitively, the larger sheet serves as an observer. p-Adic primes assignable to the space-time sheet could arrange them hierarchically and one could have entanglement between wavefunctions for the Minkowskian regions of the space-time sheets and the surface with a larger value for p would be in the role of O

Quantum measurement involves also a measurement interaction. There must be an interaction between two different levels O and S of the hierarchy.

One can look at the measurement interaction from number theoretic point of view.

1. For cognitive measurements the step forming the composite $O \circ S$ of polynomials would represent the measurement interaction. Before measurement interaction systems would be represented by O and S and measurement interaction would form $O \circ S$ and after the measurement the situation would be as proposed.

   Could one think that in BSFR the pair of uncorrelated surface defined by $O \times S$ with degree $n_O + n_S$ (analog for the additivity of classical degrees of freedom) is replaced with $O \circ S$ with degree $n_O \times n_S$ (analog for multiplicativity of degrees of freedom in tensor product) in BSFR? This would mean that the formation of $O \circ S$ is like a formation of a intermediate state in particle reaction or in chemical reaction.

   Could the subsequent SSFR cascade define a cascade of cognitive measurements [L32]. I have proposed that this occurs in all particle reactions. For instance, nuclear reactions involving tunnelling would involve formation of dark nuclei with $h_{eff} > h$ in BSFR and a sequence of SSFRs in opposite time direction performing cognitive quantum measurement cascade [L20] and also the TGD based model for "cold fusion" relies on this picture [L9, L28]. After the SSFR cascade second BSFR would occur and bring back the original arrow of time and lead to the final state of the nuclear reaction.

   From the point of view of cognition, BSFR would correspond to the heureka moment and the sequences of SSFRs to the cognitive analysis decomposing the space-time surface defined by $O \circ S$ to pieces.

2. One can also consider small perturbations of the polynomials $O \circ S$ as a measurement interaction. For instance, quantum superpositions of space-time surfaces determined by polynomials depending on rational valued parameters are possible. The Galois groups for two polynomials with parameters which are near to each other are the same but for some critical values of the parameters the polynomials separate into products. This would reduce the Galois

group effectively to a product of Galois groups. Quantum measurement could be seen as a localization in the parameter space [L33].

The measurement interaction can be also considered from the topological point of view.

1. Wormhole contacts are Euclidean regions of $X^4 \subset H$ couples two parallel space-time regions with Minkowskian signature and could give rise to measurement interaction. Wormhole contact carries a monopole flux and there must be a second monopole contact to make flux loop possible. This structure has an interpretation as an elementary particle, for instance a boson. The measurement interaction could correspond to the formation of this structure and splitting by reconnection to flux loops associated with the space-time sheets after the interaction has ceased.

   Remark: Wormhole contacts for $X^4 \subset H$ correspond in $M^8 - H$ duality images of singularities of $X^4 \subset M^8$. The quaternionic normal space at a given point is not unique but has all possible directions, which correspond to all points of CP_2. This is like the monopole singularity of an electric or magnetic field. At the level of CP_2 wormhole contact is the "blow-up" of this singularity.

2. Flux tube pairs connecting two systems serve also as a good candidate for the measurement interaction. U-shaped monopole flux tubes are like tentacles and their reconnection creates a flux tube pair connecting two systems. SFR would correspond geometrically to the splitting of the flux tube pair by inverse re-connection.

5.4 Negentropy Maximization Principle

Negentropy Maximization Principle (NMP) [L41] is the basic variational principle of TGD based quantum measurement theory giving rise to a theory of consciousness.

1. The adelic entanglement entropy is the sum of the real entanglement entropy and p-adic entropies. The adelic negentropy is its negative.

   The real part of adelic entropy is non-negative but p-adic negentropies can be positive. The sum of p-adic negentropies can be larger than the real entropy for non-trivial extensions of rationals. NMP is expected to take care that this is indeed the case. Second law for the real entropy would still hold true and guarantee NMP.

2. NMP states that SFRs cannot reduce the *overall* entanglement entropy although this can happen to subsystems. In SFRs this local reduction of negentropy would happen. Entanglement is not destroyed in SFRs in general and new entanglement negentropy can be generated.

3. Although real entanglement entropy tends to increase, the positive p-adic negentropies assignable to the cognition would do the same so that net negentropy would increase. This would not mean only entanglement protection, but entanglement generation and cognitive evolution. This picture is consistent with the paradoxical proposal of Jeremy England [I2] [L6] that biological evolution involves an increase of entropy.

4. It should be noticed that the increase of real entanglement entropy as such does not imply the second law. The reduction of real entropy transforms it to ensemble entropy since the outcome of the measurement is random. This entropy is entropy of fermions at space-time sheets. The fermionic entanglement would be reduced but transformed to Galois entanglement.

6 Some notions of condensed matter physics from the TGD point of view

Before continuing I must emphasize that I am not a condensed matter physicist and have no practical experience about experimental physics. Therefore I cannot propose any experimental protocols. I dare to hope that the new vision about space-time and quantum theory could inspire people who are doing real condensed matter physics.

6.1 The notion of Brillouin zone from the TGD viewpoint

In condensed matter physics the notions of lattice, reciprocal lattice, unit cell and Brillouin zone at its counterpart in reciprocal lattices are central notions.

The reciprocal lattice in momentum space is the dual of the lattice in 3-space. This follows automatically from the periodicity of properties of wave functions in the lattice : they force wave vectors to be in the reciprocal lattice. The diffraction amplitude has peaks at the photon momenta in the reciprocal lattice.

$M^8 - H$ duality can be seen as the counterpart of position-momentum duality. Therefore it is interesting to look at these notions from the point of view of M^8_H duality. Recall that 4-surfaces in $H = M^4 \times CP_2$ is identified as space-time whereas the 4-surface in $M^8 = M^4 \times E^4$ is analogous to momentum space with slicing induced by the mass shells (hyperboloids) of M^4. In H the corresponding slicing is by CDs inside CDs with size given by the Compton length associated with mass m.

1. At the level of H, periodic minimal surfaces would nicely produce lattice-like structures and the momenta associated with the peaks of Fourier transforms would belong to the reciprocal lattice. I have considered the construction of also more general structures in [L52].

2. At the level of M^8, the allowed momenta as points of $X^4 \subset M^8$ belong to cognitive representations: the momentum components are algebraic integers in the extension defined by the polynomial defined the 4-surface in M^8. This guarantees the theoretical universality of the adelic physics [L11, L12]) so that the points make sense also as points of the p-adic variants of space-time surface defining geometric correlates of cognition.

   Lattice-like structures are naturally associated with the lattice of algebraic integers and one obtains a hierarchy of lattices. The lattices can be seen as products of ordinary lattices in E^3 and lattices in the extension of rationals having dimension n: this feature is completely new.

6.1.1 Construction of bound states

Number theoretic vision suggests a universal way to construct bound states as Galois confined states. This would mean that many quark states in M^8 consisting of points of cognitive representation carrying quark are Galois singlets. In the case of momentum degrees of freedom this would mean that the total momentum is (rational) integer.

The physical motivation for Galois confinement is that periodic boundary conditions require integer value 4-momenta which are rational integers using a suitable momentum unit determined by the size scale of CD (Compton length $\hbar_{eff}/m$ for some particle would be in question for $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ the gravitational Compton length $\Lambda_{gr} = GM/v_0 = r_s(M)/2v_0$ would not depend at all on mass of the particle.

1. The condition that the total 4-momentum is integer-valued poses a strong condition on the bound states.

2. Second condition is that the inner products of the momenta (algebraic integers which can have an imaginary part) defining number theoretical metric are real valued. This poses strong quantization conditions, and one obtains also lattice structures in the lattice defined by the unit vectors of extension and by 3-space. These lattice structures are sublattices of lattice E^3, whose points are n-D number theoretical lattices defined by the unit vectors of the extension of rationals.

3. The fundamental entities are quarks and the construction gives a hierarchy of increasingly complex bound states of them. One obtains also atoms and their lattices. Quasi-crystals are obtained as cut and project construction and it is feasible that number theoretical lattices makes them possible also now.

4. The lattices in M^8 involving particles with the same mass are actually lattices in 3-D hyperbolic space and called tessellations. In good approximation they are lattices in E^3 since H^3

can be approximated by E^3 below length scale given by h_{eff}/m which is Λ_{gr} for $\hbar_{gr}$ (.9 cm for Earth and of the order of radius of Earth for Sun).

The structure of tessellations is extremely rich and perhaps the simplest tessellations known as icosa-tetrahedral tessellations involve all basic Platonic solids and are proposed to give rise to universal realization of genetic code having chemical realization only as a special case and having besides DNA also higher dimensional realizations [L39].

M^8 picture allows also universal 6-D brane-like solutions with a topology of 6-sphere, whose projection to CD is its intersection with 3-D hyperplane E^3 of constant energy. This plane would allow many quarks states with an ordinary lattice structure. There both hyperbolic tesselations and Euclidian lattices would be allowed.

5. Even the lattice formed by atoms would be a bound state of this kind. The reciprocal lattice in M^8 has an interpretation in terms of cognitive representation in M^8 mapped to H by M^8-H duality defined by particle momenta, which are basically bound states of quarks (also leptons).

6.1.2 M^8-H duality and the relation between lattices and reciprocal lattices

M^8 and H descriptions are related by M^8-H duality as an analog for momentum-position duality. Uncertainty Principle (UP) must be respected but what does this really require is not quite clear. The map of $X^4 \subset M^8$ to $X^4 \subset H$ is certainly involved. This would be the M^8-H duality for space-time surfaces. This description is not enough: M^8-H duality is required also at the level of "world of classical worlds" (WCW).

1. M^8-H duality at the level of 4-surfaces

Consider first the M^8-H duality at space-time level.

1. Uncertainty Principle (UP) is the basic constraint on M^8-H duality and fixes the form of M^8-H duality at the space-time level.

   One takes the momentum projection p in M^4 - an algebraic integer for cognitive representations and quarks are at these points, not all - and maps it to a point of $M^4 \subset M^4 \times CP_2$ that is to a point of $X^4 \subset H$. One assigns to p a geodesic line in the direction of momentum beginning at the common center of all CDs. In this way the slicing by mass shells of $M^4 \subset M^8$ is mapped to a slicing by CDs inside CDs (Russian doll-like structure).

2. p is mapped to the intersection of this geodesic line with the boundary of CD. One obtains the analog of the pattern produced by diffraction from the lattice. In particular, the intersections of the geodesics with the $t=T$ plane above the center point of CD form a reciprocal lattice, whose projection to the 2-D surface of a large 2-sphere corresponds to the standard diffraction pattern. One would be happy if one would obtain a lattice, rather than its reciprocal.

   As if there were a lattice around the center of the ball producing the diffraction pattern as a projection of the reciprocal lattice to the heavenly sphere. Intuition would suggest that this must be the case but one must be very cautious.

3. The momenta of quarks (or atoms) are therefore mapped to the light-cone boundaries of CD and basically define boundary values for the induced quark fields for quarks composing both proton, nuclei, and even electrons. These fields would be localized at these points at the boundary of CD and disperse in the interior. Induced spinor fields are second quantized H-spinor fields restricted to space-time surface and obeying modified Dirac equation for induced geometry and determined by variational principle.

   One can assign to the points at the boundary of CD corresponding to the image of the reciprocal lattice localized states of atoms of the lattice (many-quark states). At quark level this corresponds to a superposition of spinor harmonics of H localized to the point of the boundary (this corresponds to so-called light-cone quantization). This would dictate the time evolution of the induced spinor field inside the space-time surface and it would reflect the data coding for the reciprocal lattice.

4. Does this mean the emergence of lattice (as desired) or of reciprocal lattice in the interior? Since the lattice points by definition would correspond to peaks of plane waves generated by the reciprocal lattice at the boundary of CD would expect that the peak positions define the lattice.

One can also wonder whether one could one define $M^8 - H$ duality so that it would take momentum lattice in M^8 to its dual in H? The notion of dual lattice makes sense for the lattice defined by the extension. If one defines the cognitive representation in M^8 by selecting a tessellation at the mass shell of M^8 (this might follow the conditions for bound states), one could map the momenta of tessellations to their duals and would obtain the desired result in H. It is however not clear whether the map of tesselation to its dual (if it exists) can be completed to a continuous map of H^3 to itself.

2. $M^8 - H$ duality at the level of WCW

It seems that the proposed description need not be enough to realize UP at the level of H and the "world of classical worlds" (WCW) . The objection is that localized states in M^8 correspond to delocalized states at the level of H.

The above description maps quarks at points of $X^4 \subset M^8$ to states of induced spinor field localized at the 3-D boundaries of CD but necessarily delocalized into the interior of the space-time surface $X^4 \subset H$. This is analogous to a dispersion of a wave packet. One would obtain a wave picture in the interior and the lattice should emerge.

1. The basic observation leading to TGD is that in the TGD framework a particle as a point is replaced with a particle as a 3-surface, which by holography corresponds to 4-surface.

   Momentum eigenstate corresponds to a plane wave. Now planewave could correspond to a delocalized state of 3-surface associated with a particle in M^4 and by holography that of 4-surface.

2. A generalized plane wave would be a quantum superposition of shifted space-time surfaces with a phase factor determined by 4-momentum. This suggests that $M^8 - H$ duality should map the point of M^8 containing an object with momentum p to a generalized plane wave and this is assumed.

   This would also define WCW description. Recent physics relies on the assumption about single background space-time: WCW is effectively replaced with M^4 since 3-surface is replaced with point and CP_2 is forgotten so that one must introduce gauge fields and metric as primary field variables.

3. For cognitive representations, momenta are given by algebraic integers. Lattice plane waves can be idealized as waves in a discrete lattice. This would suggest that the plane wave is replaced by a discretized plane wave corresponding to the points of H at which the plane wave has the same value. One can say that one counts only the wave crests and thus only the information about wavelength and frequency.

4. For reciprocal momenta, one obtains a wave function in H for the shifted images of the 3-surface/4-surface labelled by a vector of the reciprocal lattice in H and this wave function can be regarded as a wave function with the periodicities of lattice.

The WCW picture is necessary if one wants to take into account WCW degrees of freedom. In the approximate description of phenomena involving only elementary particles constructible from quarks, WCW is not absolutely necessary.

6.2 Topological condensed matter physics and TGD

Topological considerations have become an essential part of condensed matter physics. In condensed matter physics the topology of patterns of order parameters and of Fermi surface play a key role. In the TGD framework the topology of space-time surface in X^4 and the dual 4-surface in M^8 having an interpretation as an analog of momentum space are non-trivial and the question how this could reflect itself in condensed matter physics.

6.2.1 Topology of the energy bands in solids

The notions of 2-D face states, edge states, and corner states seem to be behind many topological states. It is interesting to see what they could correspond to in the TGD framework.

One can imagine two alternative guesses.

1. At H level 4-surfaces as analogous to 4-D complexified momentum space are algebraic surfaces, that is 4-D "roots" of polynomials.nThese algebraic surfaces have singularities at the level of H mapped to singularities at the level of H. They can have corners, edges, and intersection points, 2-D singular surfaces. At the level of H they correspond to strings, string world sheets, and light-like orbits of partonic 2-surfaces: in this case the line singularity is blown up to a 3-D singularity.

2. These singularities need not however correspond as such to the above listed singularities since the active points of cognitive representation defined by mometa which are algebraic integers do not correspond as such to the physical states. Rather, physical states are Galois confined bound states of quarks for a given extension of rationals and it is the energy and momentum spectrum of these states which is relevant.

The second guess is based on the idea that the energy bands correspond to substructures formed by discrete 4-momenta of Galois confined states.

1. Cognitive representation consists of momenta for which momentum components are algebraic integers. Some of these points are occupied by quarks, they are "active" (this brings in mind Bohm's notion of active information).

   Physical states must have total momentum which is rational integer using the unit defined by the largest CD involved defining IR cutoff. Smallest CD defines the UV cutoff. This means Galois confinement in momentum degrees of freedom. Same happens also in spinorial degrees of freedom.

2. Bloch waves are of the form $exp(ikx)u(x)$ where u is a periodic function with the periods of lattices and k is continuous pseudo-momentum. k can be restricted to the first Brillouin zone defined as the counterpart of a lattice cell in momentum space. For Bloch states the translational symmetry is broken down to a discrete subgroup of the translation group acting as symmetries of lattices and therefore of u.

   For Bloch waves, the wave vectors and also energies would be quantized by periodic boundary conditions which would mean in the TGD framework that the momenta are integer valued using a suitable unit. The phase factors $epx(iknL)$ would be roots of unity and therefore number theoretically universal. This requires that $kL = m$ is a rational integer.

3. Mass shells as hyperboloids $H^3(m)$ are of special interest as are also the 3-D M^4 projections of 6-D universal brane-like entities. The latter are 3-surfaces $E = E_n$ where E_n is the root of the polynomial defining the 4-surface in M^8. Hyperboloid allows tessellations and the Euclidean 3-space E_3 defined $E = E_n$ surfaces inside light-cone allows lattices expected to emerge naturally from Galois confinement.

4. This picture suggests that each $E = E_n$ shell gives rise to real energy shells with rational integer valued energy and momentum components as sums of the multiples of algebraic integers for quarks. The allowed momenta for given total energy would correspond to states assignable to a given total energy analogous to a given $E = constant$ 2-surface of an energy band. The singular topologies could correspond to intersections or touchings of these bands.

One cannot exclude the possibility that the states with quarks with momenta at the singular pieces of 4-surfaces (touching along 0,1, or 2-D surface) could correspond to these singularities. For instance, the touching of two energy bands could correspond to this kind of singularity.

The article of Carpentier [D5] gives a nice introduction to the topology of bands in solids and it is interesting to see the situation from the TGD point of view. Topological insulators, semimetals, so called Majorana fermions, etc. involve singular situations in which energy bands touch each other and the question is what this means at the level of M^8.

Can one have a situation in which different energy bands touch each other at a single point or possibly along 1-D or 2-D (discrete) surfaces? The discussion is very similar for mass shells $H^3(m)$ and energy bands $E^3(E_n)$ so that only the case of E^3 is discussed.

1. Consider first energy bands E_n. For a given mass m, one obtains a set of energies E_n corresponding to the roots of P. When two roots co-incide, entire energy bands coincide. This would be however the situation for single quark states which are not possible by Galois confinement for irreducible polynomials with rational coefficients.

2. Two Galois confined states belonging to different energy bands E_n have energies, which are sums of the integer combinations of rational parts of energies E_n of single particle states. These sums are identical for some states associated with E_n and E_n.

   One can imagine that these bound states energies are the same for two different values of E_n so that bands formed by bound states can touch. Even higher-dimensional intersections can be considered. Similar situation might occur for the Galois confined states associated with different mass hyperboloids.

3. In condensed matter situation momenta are defined only modulo the addition of lattice momentum, which is multiple of $\hbar_{eff}/a = N\hbar_{eff}/L$ where a and L are UV and IR length scale cutoffs defined by the smallest and largest CDs. This condition would loosen the conditions for touching.

6.2.2 Discrete symmetries at the level of M^8

Discrete symmetries T, PT, and CP and their violations are closely involved with the phenomena of topological condensed matter physics. The challenge is to understand T, PT, and CP violations at the level of M^8.

The definition of discrete symmetries in $H = M^4 \times CP_2$ was discussed already in my thesis [K1] [L3] about TGD. In particular, geometrically C corresponds to a complex conjugation in CP_2. At the level of M^8, these discrete symmetries should allow a realization as symmetries of the polynomials defining the space-time surface.

P changes the direction of 3-momenta. The counterpart of the Fermi surface should therefore become reflection asymmetric in the violation of P. The reflections are with respect to the middle point of the CD. T changes the sign of energy and half cones of CD in H and mass shells with opposite sign in M^8 are permuted. Also the time reversed classical time evolutions are different if T is violated. One can ask whether the violation of P implies a compensating violation of T (by CPT)?

Both M^4 and CP_2 contributions to Kähler magnetic field could induce T violation and M^4 contribution could do this in long scales. If T violation takes place at the fundamental level, topological instanton term which is divergence of axial current appearing in Kähler action could induce it. The analogs of instantons induce a violation of the conservation of monopole charge. This is possible only if the M^4 projection of the space-time surface is 4-dimensional. Analogous statement applies in the case of CP_2 and CP_2 type extremals have indeed 4-D CP_2 projection.

C involves a complex conjugation and changes the signs of charges. What does this mean in M^8? The normal spaces of 4-surface in M^8 containing a preferred complex plane or having integrable distribution of them are labelled by CP_2 coordinates. They are mapped to their complex conjugates.

What happens to the polynomial defining the space-time surface? Polynomial itself is real and cannot change but its algebraic continuation to an octonionic polynomial can be different. Indeed, real function can be algebraically continued to a complex function or its conjugate.

1. The complexified octonions involve a commutative imaginary unit i. Complex conjugation with respect to i leaving the real polynomial invariant but leading to a complex conjugate of the 4-surface looks like a reasonable first guess. One can however argue that the conjugation with respect to i is associated with T.

   Recall, that the proposal [L25] that co-associative 4-surfaces in M^8_c, having an interpretation as an analog of momentum space, correspond to 4-surfaces identifiable as roots of complexified octonionic polynomials yielded a cold shower. Due to Lorentz symmetry, naive counting of dimensions fails and one obtains 2 polynomial equations with complexified mass as argument stating that the mass squared is a complex root of the polynomial. The solutions correspond to common roots and are 6-D.

The solution of the problem would be that 4-surface is the intersection of 6-surface and its complex conjugate with respect to the commuting imaginary unit i. The common root must be real but the points in the intersection can be complex. Hence the action of T on X^4 is in general non-trivial and a spontaneous violation of T is possible at momentum space level.

2. Also octonions allow conjugation. In M^4 sector conjugations for octonionic units this would give rise to P and T. In the complement E^4 the conjugations for 2-D subspaces are also possible.

   Could C relate to the commutative normal spaces of 6-D surfaces labelled by points of the CP_2 twistor space $SU(3)/U(1)\times U(1)$. Could the complex conjugation in the 2-D $U(1)\times U(1)$ fiber of this space, correspond to C. The complex conjugation would therefore act on the (integrable distribution of) 2-D normal spaces of these 6-D surfaces and would not act in $M^4 \subset M^8$.

3. At the level of H, C and P are violated for the Dirac equation for a fixed H-chirality of quarks spinors and also for the modified Dirac equation, which corresponds to the octonionic Dirac equation in M^8. Also CP is violated for the modified Dirac equation in H if the action contains topological Kähler instanton terms. This violation should have a counterpart for the octonionic Dirac equation. Since this equation selects a single point at 4-surface, the CP violation for the 4-surface could induce CP violation.

6.2.3 Instantons in the TGD framework

Instantons induce violations of CP and therefore of T in gauge theories such as QCD.

It is interesting to consider the interpretation of Q as an instanton number.

1. Montonen-Olive duality (`https://cutt.ly/HE6gMX6`) is associated with a gauge theory in which magnetic and electric charges are rotated so that the coefficient of YM action in the action exponential is replaced with the quantity $\tau = \theta/2\pi + 4\pi i/g^2$.

2. τ is invariant under modular transformations $SL(2,Z)$ generated by a shift $\tau \rightarrow \tau+1$ and $\tau \rightarrow 1/\tau$. The inversion symmetry has strong implications for the understanding of the strong coupling phases of quantum field theories, in which magnetic monopoles replace particles as elementary objects.

3. In the gauge theory θ is analogous to momentum. The vacuum state is plane-wave like superposition $\sum_N exp(iN\theta/2pi))|N\rangle$ of vacuum states differing by a topologically non-trivial gauge transformation as a map $S^3 \rightarrow G$. Note that ball B^3 is effectively S^3 if the gauge transformations are trivial at its boundary. The homotopy equivalence classes of gauge transformations are labelled by the winding number N. N characterizes instantons changing the magnetic charge by N units so that the ground state is a superposition of states with varying values of N transforming by a phase factor under a topologically non-trivial gauge phase transformation.

Consider now the situation in the TGD framework.

1. There are differences between TGD and gauge theory context. Gauge group is replaced with $U(1)$ having a trivial third homotopy group.

   Could a localized version of the quaternionic automorphism group $SO(3)$ serve as a counterpart of a gauge group. The surfaces in M^8 can be indeed thought of as maps from M^4 to the quaternionic automorphism group G_2.

2. The non-trivial gauge transformations - $U(1)$ instantons - are clearly possible. The non-trivial gauge transformation could correspond to a tological non-trivial gauge transformation $A+nd\phi$, where ϕ is angle coordinate around axis going through a line singularity as a puncture

in 3-space associated with the time-like line connecting the tips of CD. Note however that color gauge action reduces to the Kähler action so that both interpretations might make sense.

3. Kähler action generalizes to

$$S_K = \frac{1}{\alpha_K}\int J\wedge *J\sqrt{g} - \frac{(}{i}\theta/2\pi)\int J\wedge J\sqrt{g} \ . \quad (6.1)$$

Since only the exponent of S_K matters in the vacuum functional, I contributes a non-trivial phase factor to the Kähler function only for $exp(i\theta/2\pi)\neq 1$ ($\theta\neq n2\pi$). One can assign θ to both M^4 and CP_2 parts of Kähler action. The value of instanton term characterizes the non-conservation of the axial (monopole) current having instanton term as divergence.

If one assumes self-duality of the gauge field true for instantons interpreted as gauge fields in S^4, the action reduces to ordinary Kähler action with coefficient proportional to τ. Interestingly, the quaternionic projective space M^4/Q can be regarded as S^4 so that Hamilton-Jacobi structures of M^4 proposed to serve as moduli space for the self-dual Kähler fields in M^4 could appear naturally.

4. $I(CP_2)$ is non-trivial due to the non-trivial homology of CP_2. $I(CP_2)$ gives a 3-D contribution, which appears at the boundaries between Minkowskian and Euclidean regions of the space-time surface as a topological Chern-Simons term and affecting the boundary conditions at the light-like orbits of partonic 2-surfaces in this way. These boundaries have interpretations as light-like parton orbits carrying quarks lines.

5. If CD contains a time-like "hole" along the axis connecting the tips of CD, also $I(M^4)$ is non-trivial. One can imagine extremals for which a genuine hole in the metric sense is generated along the M^4 time axis. What is required is that the induced metric using M^4 coordinates is of the form $dt^2 - dr^2 - (r^2 + r_0^2)d\Omega^2$. These holes should correspond to "blow-ups" of singularities of the algebraic surface in M^8. Now the 3-D tangent spaces would have no special direction at the singular points. For CP_2 type extremals the same would hold true at the level of M^8. Could this "hole" be the TGD counterpart of the blackhole of GRT and could it serve as a signature of CD?

6. J^J is non-vanishing only if the M^4 *resp.* CP_2 projection is 4-D. This does not guarantee self-duality unless also the induced metric reduces to the metric of M^4 *resp.* CP_2. This is true for the canonical embedding of M^4 and for CP_2 type extremals having light-like M^4 projection. Self-duality is true for the Kähler forms of M^4 and CP_2 but not for the induced Kähler forms $J(M^4)$ and $J(CP_2)$. Therefore classical gravitation breaks the self-duality and Montonen-Olive duality in the TGD framework. The possibility of extremals with M^4 and CP_2 projections smaller than $D = 4$ implies that θ is effectively vanishing for them.

$\theta(M^4)$ and $\theta(CP_2)$ as fundamental parameters obeying number theoretical coupling constant evolution would imply a violation of CP symmetry in both M^4 and CP_2 sector. Are the instanton terms present at the fundamental level or are they present only at the QFT limit and induced as a description of spontaneous violation of CP and T? Indeed, as in the condensed matter systems, CP violation could be caused by the magnetic part of the generalized Kähler action even without instanton term.

1. The strong CP problem of QCD is due to instanton inducing an instanton term in effective color YM action. The parameter characterizing the violations should be very small.

In the TGD framework, a proposal for a solution of this problem could be that the counterpart of the color gauge field does not allow instantons. Here one must be cautious however. The components of the proposed classical color gauge field are proportional to the products of Hamiltonians of color isometries and Kähler form and instanton terms for the induced Kähler form would induce a CP violation. Indeed, Kähler action can be also regarded as a color gauge action and therefore instanton term makes sense for it.

2. Could $\theta(M^4)$ and $\theta(CP_2)$ induce a CP violation consistent with the observed CP violation in hadron physics or does one encounter the strong CP problem also in the TGD framework?

   If hadrons are string-like objects, they correspond to flux tubes as deformations of strings. For deformations with dimension $D < 4$, instanton term vanishes. Could this be the reason for the small violation of CP at the level of M^4? For CP_2 type extremals, $I(CP_2)$ is non-vanishing but equal to the Kähler action and non-dynamical for the basic CP_2 extremals since dynamics in in M^4 degrees of freedom with CP_2 taking the role of arena of physics. Could these effects make the hadronic CP violation small?

3. Matter-antimatter asymmetry is a CP violation, which does not look small at all. If the mechanism is actually a small CP violation implying that rate for the condensation of anti-quarks to leptons is slightly larger than that for the condensation of quarks to antileptons, the matter antimatter symmetry could emerge during a very early period of the cosmic evolution when leptons were formed.

4. There are also further questions. Could the QCD instantons have TGD counterparts as Hamilton-Jacobi structures and also as analogs of S^4 instantons in the quaternionic projective space of octonions which would be 4-D mass hyperboloid H^4 as Minkowski analog of S^4 but with space-like signature. Could the parameter θ in the instanton term of Kähler action induce the formation of the ground state (θ vacuum) as a superposition of space-time surfaces with various instanton numbers in the sector of WCW consting space-time surface with 4-D M^4 and/or CP_2 projection?

6.3 The new view about classical fields

The TGD view about classical gauge fields differs in many aspects from the Maxwellian and gauge theory view since the classical fields associated with the system define a geometric what I call its field body (magnetic body (MB)) is the term that I have used. MB can carry also electric fields very closely related to magnetic fields unless the corresponding space-time surface is static. MB consists of flux tubes and flux sheets.

There are 2 kinds of cosmic strings: with monopole flux (see **Fig. 15**) or without it. The simplest cases correspond to Y^2, which is either a homologically non-trivial or trivial geodesic sphere of CP_2.

This predicts two kinds of magnetic flux tubes and two kinds of magnetic and electric fields. This suggests a possible interpretation for the fields H, M, B appearing in Maxwell's theory as field H carrying monopole flux requiring no current as source, magnetization M as non-monopole part induced by H , and $B = H + M$ as their sum experienced by test particle in many-sheeted space-time. The same would apply to D, P and E. If this interpretation is correct, TGD would have been secretly present in Maxwell's theory from the beginning.

The proposal that MB serves as a seat for dark matter as $h_{eff} = nh_0$ phases is central in the TGD inspired theory of consciousness and living matter. MB would be the boss and receive sensory input from ordinary biomatter and control it. This would happen in terms of dark photons with frequencies in EEG range and also in other ranges. The energies would be in the visible and UV range assigned to biophotons to which the dark photons would transform.

Magnetic flux tubes could accompty quantum vortices appearing in various macroscopic quantum phases. Even the hydrodynamical vortices in macroscopic scales could correspond to quantum coherent magnetic flux tubes with a large value of h_{eff} acting as a master forcing the coherent dynamics or ordinary matter. In hydrodynamics the classical Z^0 magnetic field, which in situations allowing skyrmions, is proportional to the induced Kähler form, could be important. Large parity breaking effects would be the prediction.

Also the view about radiation fields changes. Massless extremals (MEs)/topological light rays are counterparts for massless modes. They allow a superposition of modes with a single direction of massless momentum. The ordinary superposition of gauge potentials in gauge theory is replaced with union of space-time surfaces with common M^4 projection. The test particle experiences the sum of gauge potentials associated with various space-time sheets so that the gauge potentials effectively superpose. Ideal laser beam is a convenient analogy.

MEs are ideal for precisely targeted communications without dispersion and dissipation. MEs are soliton-like entities and one can ask whether MEs could provide a model for solitons or accompany solitons. TGD based model for nerve pulse involves Sine-Gordon solitons with large h_{eff} assigned to the cell membrane and dark Josephson radiation would have MEs as space-time correlate [K10, K4, K11].

MEs do not allow standing waves possible in Maxwell theory but a set theoretic union of parallel MEs can effectively give rise to standing waves. Lorentz transformations give rise to waves moving with arbitrary sub-luminal velocity. Even a superposition in which fields effectively sum up to zero but there is a non-vanishing energy density as sum of energy densities for the two MEs, is possible.

6.4 About quantum criticality in TGD

In TGD number theoretical vision about physics brings a new view about quantum criticality.

1. Quantum criticality is actually the basic assumption of TGD: the Kähler coupling strength α_K appearing in the classical action principle of TGD would be analogous to a critical temperature and have a discrete spectrum. This would make the theory unique. All space-time sheets are quantum critical but at QFT limit this is of course masked by the replacement of sheets with a single region of M^4 made curved.

2. At the number theoretical M^8 side there is no action principle. The universality of the dynamics could be seen as a manifestation of quantum criticality. Can α_K emerge at M^8 level somehow from scattering amplitudes in M^8 and have a number theoretical origin [L42].

   At the level of H coupling constants are visible only at the level of frames defining the space-time as an analog of soap film. The parts of the frame are images of singularities for the X^4 in M^8. The challenge is to understand how the singularities of the space-time surfaces determine α_K already at the level of M^8?

   p-adic thermodynamics for mass squared predicts a spectrum of temperatures with values coming as inverse integers [K8, K2]. Also this temperature quantization could be seen as a counterpart for the quantum criticality.

3. Quantum criticality involves long range correlations and the hierarchy of Planck constants characterizing them [K23, K24, K25]. h_{eff} corresponds to a dimension of extension of rationals characterizing the space-time surfaces. At criticality there is quantum superposition of space-time surfaces with various values of h_{eff} corresponding to polynomials defining the X^4 and one value of h_{eff} is selected in state function reduction.

6.5 What infinite-volume limit could mean in TGD?

Infinite volume limit corresponds to both thermodynamic and QFT limit and should be understood in the TGD framework. The questions are what it means if the infinite volume limit is actually realized and whether this has practical consequences.

1. At the level of ZEO infinite volume limit means that the size of causal diamond (CD) as an analog of Nature given quantization volume becomes infinite. The scattering amplitudes coded by zero energy states conserve Poincare quantum numbers at this limit.

2. At the level of H the volume action vanishes since the p-adic length scale dependent cosmological constant $\Lambda \propto 1/L_p^2$ approaches zero at the limit when the p-adic length scale L_p characterizing the X^4 becomes infinitely large.

   If $\Lambda = 0$ phase is real, the action would reduce to mere Kähler action containing both M^4 contribution and CP_2. In this case, one would also have extremals of form $X^2 \times Y^2$ for which CP_2 projection if the Lagrangian manifold with vanishing induced Kähler form. These extremals receive a negative contribution to energy from M^2. Could the preferred extremal property exclude these solutions?

   Remark: If the sign of M^4 Kähler action is changed, the electric contribution to energy is positive and magnetic contribution negative. For string- like objects this would guarantee positive contribution.

3. In the number theoretic picture infinite volume limit in H could mean that polynomials defining $X^4 \subset M^8$ mapped to H are replaced with analytic functions with rational coefficients.

   Polynomials are assumed to vanish at origin (this guarantees that roots are "inherited" in their functional composition) and so should also the analytic functions. The inverse 1/f is infinite at origin and does not belong to the set so that one does not have a function field. Since one has only multiplication, one can speak about functional primes as in the case of polynomials.

   One can ask whether they should satisfy conditions guaranteeing that they can be regarded as polynomials of infinite order. Could one speak about polynomials of infinite degree as the limit of functional composites of polynomials with finite degree. As a matter of fact, infinite Galois groups are profinite groups and this requires this kind of inverse limit definition [L37].

   A concrete example is provided by the iteration of a polynomial of finite degree [L37]. In this case the spectrum of roots contains a continuous part at the limit so that complex numbers as completion of rationals would emerge at the infinite volume limit much like the continuum spectrum of momenta emerges from a discrete spectrum.

6.6 The notions of geometric phase, Berry curvature, and fidelity in TGD?

Non-contractible ground state Berry phase in the loop over the parameter space is associated with QPTs and is associated Berry curvature defining non-trivial U(1) holonomy (`https://cutt.ly/RWy7Deq`) Geometric phase (`https://cutt.ly/6Wy7GIT`) is a more general notion. It can be associated with homotopically non-trivial loops. For homotopically trivial loop geometric phase is due to non-trivial holonomy manifesting itself as Berry curvature. The Aharonov-Bohm effect represents an example about non-trivial holonomy. Electrons pass along paths closing together a region containing a magnetic field, which vanishes at the paths. Berry phase can be associated with loops in the parameter space for the Hamiltonian modelling the system.

Fidelity [D22] `https://cutt.ly/VWy5sVj`) defines a metric in the space of parameter dependent quantum states. It could be induced from metric of the parameter space. The abrupt changes of fidelity serve as a signature of quantum criticality.

Is this possible at the level of WCW?

1. WCW is a Kähler manifold [K16, K6]. Finite-dimensional Kähler manifolds have a trivial homotopy group. Complex coordinates of WCW contributing to Kähler form and metric correspond to complex coordinates. In these degrees there should be no homotopically trivial loops so that topological phase is not possible. The curvature of the Kähler form can however have effects.

2. The remaining degrees of freedom are zero modes and define the analog of the base space in bundle theory. They appear as parameters - essentially classical background fields - in the Kähler metric and Kähler form. The topology in the zero modes can have non-trivial homotopy. Geometric phase could be assigned with homotopically trivial loops in the zero modes.

   At the infinite-volume limit the sub-WCW defined by the degenerate ground states with a Lagrangian manifold Y^2 as CP_2 projection (vanishing Kähler form and color gauge fields but non-vanishing weak gauge fields) is highly interesting. The preferred extremal property could exclude these space-time surfaces.

It seems that TGD could provide a unified description of all these exotic quantum coherent phases.

6.6.1 How the description in terms of Berry phase and fidelity could relate to TGD?

Consider first the identification of the TGD counterparts of Berry phase and fidelity.

1. In TGD the ground states are defined as space-time surfaces/3-surfaces and quantum states are their superpositions. The Kähler metric defines the analog of the quantum metric and the Kähler form corresponds to Berry curvature.

   The fidelity of two quantum states $\Psi(\lambda)$ and $\Psi(\lambda+\delta\lambda)$ is defined as the overlap $\langle\Psi(\lambda))\Psi(\lambda+\delta\lambda)\rangle$ in parameter space. The fidelity for nearby states is expected to change dramatically at singularity.

   Fidelity at the level of WCW - rather than WCW spinor fields representing quantum states - would mean disappearance of appearance of quantal WCW degrees of freedom as zero models transform to dynamical quantal degrees of freedom or vice versa. This change would make itself visible at the level of quantum states whose inner product depends on the WCW Kähler metric.

2. WCW also allows spinor connection with some gauge group acting as non-abelian holonomies. This corresponds to non-Abelian Berry phase Kac-Moody algebras of H isometries are an excellent candidate in this respect. WCW allows super-symplectic group as isometries.

3. WCW metric has also zero modes, which do not contribute to the WCW metric. Any symplectic invariant associated with X^4 defines such an invariant and the induced CP_2 Kähler form is invariant under the symplectic transformations of CP_2 and can be said to define a continuum of this kind of invariants. This could induce a geometric phase, which is not due to a holonomy but non-trivial homotopy.

   Kähler magnetic fluxes over 2-surfaces define such invariants. For closed surfaces these invariants reduce to quantized magnetic fluxes. Also M^4 Kähler form defines such invariants. At the boundary of CD the sphere S^2 (light-like radial coordinate =constant) has symplectic structure and also this defines solid angles assignable to 3-surfaces as seen from the tip of the CD as invariants.

6.6.2 Could the singularity of the quantum metric relate to number theoretical physics?

The singularity of the quantum metric would mean a reduction of the number of the dynamical quantum degrees of freedom contributing to the WCW metric meaning that the rank of the WCW metric tensor decreases. At criticality complex coordinates would transform to zero modes. Some complex coordinates of WCW would reduce to real coordinates. This would correspond to quantum criticality. In a concrete mechanical system some eigen modes would vanish and corresponding frequencies would become zero.

Since the TGD Universe is quantum critical and this is expected to be a generic phenomenon. Quantum criticality involves long range fluctuations which would correspond to large values of h_{eff} and therefore space-time surfaces which are algebraically complex. Could these long range fluctuations relate to almost zero modes with small frequencies and large wave lengths?

These phase transitions could be number theoretic. They would change the polynomial defining the X^4 (recall that quantum state is the superposition of space-time surfaces in ZEO). The dimension n for the extension of rationals is equal to the order of the Galois group and would change. Galois symmetries would act as zero mode symmetries. The dimensions of the representations of the Galois group in terms of quarks would also change. The change in the number of degrees of freedom would change the fidelity.

n defines also the algebraic dimension of the integers extended to algebraic integers for extension as a space regarded as a ring of integers. If algebraic integers can define components of the momenta, the dimension of the momentum space with integer components of momentum increases from 3 to $3n$ as the dimension of the Galois group increases by factor n. This increase occurs in the transitions in which the polynomial Q defining the space-time region is replaced with $P \circ Q$ such that P defines n-dimensional extension.

This would have rather dramatic effects since the radius of the Fermi ball with radius would be reduced by factor $1/n$ and could contain the same number of states as ordinary Fermi ball: this would mean an increase of density by factor n^3 corresponding to n sheets. Quasicrystal structure in both $X^4 \subset M^8$ and its images in $X^4 \subset H$ is also suggestive.

6.6.3 Does infinite volume limit have spin-glass type degeneracy?

One can look at the situation also at the infinite volume limit. At the infinite volume limit the action is expected to reduce to Kähler action. Whether this implies ground state degeneracy depends on whether preferred extremal property allows it.

1. In the original picture there was only CP_2 contribution to Kähler action. This implies huge vacuum degeneracy of CP_2 Kähler action. Any X^4 with CP_2 projection which is 2-D Lagrangian manifold is a vacuum extremal. WCW metric becomes singular if its inverse does not exist: this means singularity and the existence of zero modes. 4-D spin variant of glass degeneracy (`https://cutt.ly/0RuZfgu`) and classical non-determinism emerge. Classical non-determinism does however not look physically acceptable.

2. The twistor lift forces the Kähler action to have also an M^4 part obtained by analytical continuation from E^4. Does the resulting Kähler action have ground state degeneracy at infinite volume limit?

   The simplest extremals are of the form $X^4 = X^2 \times Y^2$, X^2 a minimal surface in M^4 and Y^2 a Lagrangian manifold in CP_2. Symplectic transformations in CP_2 degrees act like $U(1)$ gauge transformations on CP_2 Kähler gauge potential and do not affect either Kähler form nor the Lagrangian manifold property.

   Only the induced metric is affected so that the effects are purely gravitational. This gives rise to the ground state degeneracy. The area of CP_2 projection is not changed and the action is affected only by the change of the induced metric. Conserved quantities are modified only by gravitational effects and are non-vanishing. The extremals are deterministic and apart from gravitational effects one has a huge ground state degeneracy analogous to spin glass degeneracy.

   Apart from gravitation, the WCW Kähler metric receives contributions only from M^4 degrees of freedom, which are not affected under these deformations. Could one say that CP_2 degrees have transformed to zero modes?

3. One can also have surfaces $X^2 \times Y^2 \subset M^4 \times CP_2$ such that both X^2 and Y^2 are Lagrangian manifolds at infinite volume limit. These would be vacuum extremals. Preferred extremal property should exclude them. Could the interpretation be that all quantum degrees of freedom have transformed to zero modes?

4. One can invent objections against this proposal.

   (a) Negative energies might emerge from the electric energy in M^4 degrees of freedom. Electric field gives a negative contribution to energy density. Signature is Minkowskian for M^2 subset $M^2 \times E^2$. The M^2 part of Kähler form is obtained from its E^2 variant by multiplication with factor i. This might cause problems.

   (b) These surfaces are extremals but the preferred extremal property could fail since the needed 4-D analog of complex structure is missing since Y^2 as a Lagrangian manifold is not a complex surface of CP_2.

   (c) There is however also an argument in favor of this picture. Ordinary Maxwellian magnetic fields correspond to a homologically trivial geodesic sphere of CP_2 and they are Lagrangian submanifolds. Therefore one cannot exclude the proposal.

6.6.4 The parameters of the effective Hamiltonian from the TGD point of view

Could the parameters of effective Hamiltonians have counterparts at the level of WCW?

1. 4-surfaces as WCW points define parameters in the analogs of eigenvalues of observables. Both supersymplectic and Kac-Moody algebras have as parameters the parameters coding the point of WCW and Kac-Moody algebra. Number theoretic coding of ground states based on the Galois group as a symmetry group and p-adic primes defining p-adic length scale is what comes to mind.

The preferred 4-surfaces would naturally correspond to the maxima of Kähler function. It is quite possible that Kähler coupling constant is complex so that the complex number defining the exponent of Kähler function has phase $\pm pi/2$. The phase of the exponent is different and maxima are also stationary points. This would make possible interference effects central in QFTs. This is implied by the condition that classical conserved charges are apart from a phase factor real and can therefore be made real.

If M^8 space-time sheets are defined as "roots" of polynomials with rational coefficients [L25, L26], WCW becomes discrete and has the coefficients of polynomials as coordinates of a given point (X^4). An open question is why the maxima of Kähler function should correspond to rational polynomials with rational coefficients.

2. Super-symplectic transformations [K3, K16] as isometries of WCW are symmetries and can be regarded as a generalization of Kac-Moody type symmetries. The complex coordinate z and light-like radial coordinate r of the light-cone boundary are in the role of parameters. Analog of 3-D gauge group but gauge group replaced with the symplectic group of $S^2 \times CP_2$ is in question. The light-like orbits of partonic surfaces could naturally carry Kac-Moody algebra representations of isometries - at least at infinite volume limit.

   Non-negative conformal weights parameterize the representations of this algebra. The construction of states would be as follows. A sub-algebra $SCA_{n_{max}}$ with conformal weight larger than n_{max} and its commutator with the entire algebra annihilate states. Only the states with conformal weight smaller than n_{max} remain. Other degrees of freedom are effectively gauge degrees of freedom. n_{max} is expected to depend on the polynomial, its Galois group and degree. A huge reduction of degrees of freedom takes place. The remnant of the super-symplectic group would act as dynamical symmetries.

   Same could occur in the symplectic degrees of freedom labelled by Hamiltonians which are products of S^2 and CP_2 Hamiltonians. The only non-trivial normal subalgebra corresponds to isometries and states would be annihilated by the generators in the complement of this algebra.

   Rational coefficients of a polynomial defining the X^4 serve as the parameters characterizing the ground state. Higher level description is in terms of the Galois group which depends only weakly on the polynomial.

3. What about the description at the level of X^4? The solutions of modified Dirac action for induced spinor fields depend on the parameters characterizing the space-time surface.

6.7 Quantum hydrodynamics in TGD context

In the standard picture quantum hydrodynamics is obtained from the hydrodynamic interpretation of the Schrödinger equation. Bohm theory involves this interpretation. (`https://cutt.ly/cWy3O9Ts`).

1. Quantum hydrodynamics appears in TGD as an *exact* classical correlate of quantum theory [K19]. Modified Dirac equation forces as a consistency condition classical field equations for X^4. Actually, a TGD variant of the supersymmetry, which is very different from the standard SUSY, is in question.

2. TGD itself has the structure of hydrodynamics. Field equations for a single space-time sheet are conservation laws. Minimal surfaces as counterparts of massless fields emerge as solutions satisfying simultaneously analogs of Maxwell equations [L52]. Beltrami flow for classical Kähler field defines an integrable flow [L35]. There is no dissipation classically and this can be interpreted as a correlate for a quantum coherent phase.

3. Induced Kähler form J is the fundamental field variable. Classical em and Z^0 fields have it as a part. For $S^3 \subset CP_2$ em and Z^0 fields are proportional to J: which suggests large parity breaking effects. Hydrodynamic flow would naturally correspond to a generalized Beltrami flow and flow lines would integrate to a hydrodynamic flow.

4. The condition that Kähler magnetic field defines an integrable flow demands that one can define a coordinate along the flow line. This would suggest non-dissipating generalized Beltrami flows as a solution to the field equations and justifies the expectation that Einstein's equations are obtained at QFT limit.

5. If one assumes that a given conserved current defines an integrable flow, the current is a gradient. The strongest condition is that this is true for all conserved currents. The non-triviality of the first homotopy group could allow gradient flows at the fundamental level. The situation changes at the QFT limit.

6. Beltrami conditions make sense also for fermionic conserved currents as purely algebraic linear conditions stating that fermionic current is a gradient of some function bilear in oscillator operators. Whether they are actually implied by the classical Beltrami conditions, is an interesting question.

7. Minimal surfaces as analogs of solutions of massless field equations and their additional property of being extremals of Kähler action gives a very concrete connection with Maxwell's theory [L52].

6.8 Length scale hierarchies

The length scale hierarchy associated with the hierarchy of Planck constants and p-adic length scale hierarchy lead to the proposal that one has quantum coherence and supra phase always realized in some scale and the loss of say superconductivity means only the reduction of this scale.

Also dark variants of valence electrons make sense and there is evidence for them. When looking at the definition of say exciton, one cannot avoid the impression that something is missing. Electrons and holes are assumed to have incredibly small effective masses. The very notion of effective mass is in conflict with the idea that one has a fundamental quantum theory description.

One also introduces in the Schrödinger equation dielectric constant which comes from macroscopic description. Why doesn't one do the same in the case of ordinary atoms. This kind of mixing of phenomenological descriptions with a fundamental description is to me a deadly sin.

One cannot avoid the crazy looking question whether exciton could be a valence electron which is dark with $h_{eff} = k \times h$ and binds with an atom. It would be automatically accompanied by a hole. The binding energies would be scaled like $1/k^2$ and one would obtain the energies which can be 3 orders of magnitude smaller than those for hydrogen.

6.9 A general model of macroscopic quantum phases

6.9.1 Hierarchy of quantizations at the level of WCW

Before saying anything about macroscopic quantum phases, one must define what many-particle states correspond at the level of WCW.

1. The combination of UP with $M^8 - H$ duality leads to the view that many particle states at the level correspond to many-fermion (quarks actually) such that the momenta of quarks correspond to momenta as points of $X^4 \subset M^8$ with components, which are algebraic integers. In TGD framework, where all particles, also bosons, are composites of fermions. At M^8 level Cooper pairs would correspond to pairs of occupied points of a mass shell $H^3 \subset M^8$. The image of the region of momentum space in H corresponds for quarks of given mass m corresponds to a region at the boundary of sub-CD with size given by Compton length $L = \hbar_{eff}/m$.

2. At the level of WCW, the analog of the many-quark state associated with a given quark mass corresponds to the analog of plane wave inside a large $CD \subset H$ defined by the smallest mass involve but with point-like particle replaces with space-time surface inside sub-CD (CD(m)) carrying zero energy state characterized by quark momenta at opposite boundaries of CD(m) having opposite sign of energy.

3. The entanglement between these states due to Fermi statistics is however maximal and SFRs are not possible. How can one construct entangled states. The answer is simple perform the analog of second quantization at the level of WCW. One can form the analogs of 2-particle states by taking two CDs with specified quark content and assign to both the analogs of plane waves. If the CDs correspond to different extensions of rationals so that the effective Planck constants are different, one can entangle these states in WCW degrees of freedom. One can construction N-particle states by using the same recipe.

4. To each many quark state one can assign odd or even boson number and regard this state as analog of elementary fermion or boson. This is what is indeed done quite generally. Could this operation have deeper meaning. Could one require that the many-quark operators indeed commute or anticommute mutually. This condition cannot hold true generally but could be posed as an additional condition to the physical states: the commutator/anticommutation would be proportional $h_{eff}I$, I identity matrix.

   This construction would be third quantization. And nothing prevents from performing also fourth quantization within even larger CD. This hierarchy of quantizations brings in mind the basic hierarchical structures of the TGD Universe: many-sheeted space-time characterized by p-adic and dark length scale hierarchies, and also the hierarchy of infinite primes which corresponds to a repeated second quantization of supersymmetric arithmetic QFT [K32] conjecture to correspond to the hierarchy of space-time sheets.

6.9.2 WCW description of BECs and their excitations as analogs of particles

Fermi statistics requires that the BEC correspond to a distribution of correlated momentum pairs with the sum of the momenta equal to the momentum of the boson. Cooper pairs also have binding energy so that the mass of the pairs is slightly smaller than the particle mass so that the Cooper pairs belong to different $H^3 \subset M^8$ than the free fermions.

For the excitations of BEC condensate giving rise to supracurrents and superflows, some momenta of fermions are different from the common momentum of BEC, usually larger than the common momentum of BEC. The image of excitation of BEC in H would be a pair at proper time=constant hyperboloid in H and the map of momentum to position would be linear inside $CD(m)$. BEC would look very much the same at both M^8 and H side of duality.

The space-time surface $X^4 \subset CD(m)$ should correspond to a minimal surface and to a generalized Beltrami flow defining an integrable coordinate along the flux lines. In the case of conserved current gradient flow (vortex flow is an example of this). All many-particle states would be of this kind in the scale of $CD(m)$. These multi-BEC states would be analogs of many-particle states and one would have many-particle states of BECs and their condensates, which could entangle in WCW degrees of freedom. For instance, the entanglement between geometric representations of Galois groups is possible. In the TGD inspired quantum biology the multi-BEC like states are proposed to play a key role [L30, L39].

6.9.3 Superconductivity and superfluidity in TGD framework

The TGD based view about superconductivity and fluidity [L35] differs in many respects from BCS theory.

1. In the BCS theory superconducting state does not have a well defined fermion number and this leads to a somewhat questionable notion of coherent state of Cooper pairs. The Bogobliubov transformation creates the diagonalizable oscillator operator basis by mixing creation and annihilation operators. The resulting operators create superpositions of electrons and holes.

   In the TGD framework, the interpretation would be that the hole actually corresponds to dark fermion with $h_{eff} > h$ at dark space-time sheet so that fermion number conservation is not lost. Bogoliubov operators would be replaced with superpositions of creation/annihilation operators associated with different space-time sheets and create states which are superpositions of state at the two space-time sheets. Effective Hamiltonian would include diagonalizable kinetic parts assignable to both space-time sheets, and the terms quadratic in creation/annihilation operators breaking fermion number conservation would be replaced with pairs of creation and annihilation operators associated with different space-time sheets describing the transfer of electron between the space-time sheets.

2. In the BSC theory Cooper pairs are carriers of supra current. In the TGD framework, dark electrons at dark spacetime-sheets could be the carriers. The binding energy of Cooper pairs liberated in their formation would provide the energy needed to transform ordinary electrons to dark electrons (the energies of particle states typically increase with h_{eff}). This makes possible superconductivity driven by energy feed possible also above critical temperature.

3. Can one describe supra currents and supra flows in terms of a single space-time surface as the classical space-time view based on Beltrami currents would suggest? This would mean that supracurrent would correspond to a collection of momenta of dark electrons at $H^3 \subset M^8$ in the proposed TGD based model or collection of Cooper pairs with $h_{eff} = h$ as in the standard description. The current carriers would have fixed momenta at the two boundaries of $CD(m)$ corresponding to the analogs of initial and final state momenta. Is this all that one can say at the quantum level and is the description as a flow only a classical description. At quantum level one could only deduce the change of the positions for the group of particles defining the flow. This indeed conforms with the UP.

6.9.4 WCW level is necessary for the description for purely geometric bosonic excitations

The quantum description of sound requires WCW description since the phonons as oscillations of relative position of particles cannot be described in terms of quark-antiquark pairs. The description of exotic supra flows like that associated with magnon BEC in say ^3He supra fluid allowing orbital magnetization requires WCW. A good manner to clarify thoughts is to look at what this means in the case of magnons.

1. Standard classical description (`https://cutt.ly/HRuZh53`) suggests a direction of magnetization M which has changed due to the presence of external field H. This leads to the Landau-Lifschitz equation for the magnetization.

   The Fock space picture about magnons is as a plane wave for which the argument is the position of spin whose direction has changed. The quantization is described by introducing a Hamiltonian for spins. The relationship between these descriptions is somewhat obscure.

2. In TGD the fermionic Fock space description is not possible. Bosonic creation and annihilation operators would be needed but one cannot construct bosonic operators with a vanishing fermion number from quarks. Therefore magnons should correspond to WCW degrees of freedom.

3. In the TGD description, M would correspond at space-time level to the magnetic field at a non-monopole flux tube and H possibly at a monopole flux tube inducing the magnetization. Magnons would correspond to magnetization waves, as kinks propagating along magnetic flux tubes for M. Magnon should correspond to space-time surface H and this would determine its M^8 pre-image. If these excitations behave like identical particles, one can assign to them wave vectors and classical momenta.

4. Also the notion of BEC makes sense at WCW level since one can construct the counterparts of genuine bosonic oscillator operators. Super-symplectic and Kac-Moody algebras of WCW acting at the boundaries of CD indeed include purely bosonic operators. Similar description at WCW level applies also to phonons as quanta.

   Cooper pair BECs allow approximate description in terms of fermion pairs with given total momentum but with members having different momenta. One cannot however exclude the possibility that there purely bosonic BEC at WCW level such that each Cooper pair is associated with a bosonic excitation of space-time surface.

6.9.5 Exciton-polariton BECs

The claimed room temperature superconductivity for exciton-polariton Bose-Einstein condensate in quasi-crystals suggests that the TGD based model for superconductivity could generalize to a unified description of quantum coherent phases. In this case the energy feed is crucial and would serve in TGD framework as "metabolic energy feed" taking care that the distribution of $h_{eff} > h$ is preserved.

Also WCW level might be needed to describe the bosonic aspects of exciton-polariton BECs although exciton polariton states involve only photons excitons and electron-hole bound states. The description of plasmons involves oscillations of the relative position of electron and atomic nucleus and this requires the counterparts of the bosonic creation operators at the level of WCW.

1. Polaritons and excitons could correspond to dark valence electrons in $h_{eff} > h$ phase and the value of h_{eff} would determine in which scale the phase appears. Beltrami fields would provide a quantum hydrodynamical description as an exact classical description of these phases. In principle also fermionic Beltrami currents could make sense and provide genuine quantum hydrodynamical description.

   By the way, I saw an article about empirical verification of BvK vortex street in exciton-polariton BE condensate. TGD would provide at the level of principle a universal description as minimal surfaces also for this kind of system.

2. One of the basic predictions is induced supraphases possible above the critical temperature where the formation of Cooper pairs of ordinary fermions cannot liberate the needed metabolic energy. h_{eff} can be however increased and its reduction can be prevented by energy feed. In living matter this would be metabolic energy feed. I learned that exciton-polariton condensate is an open system involving an energy feed!

3. What about ordinary hydrodynamics? The basic prediction of TGD is that quantum coherence appears in all length scales at the level of magnetic bodies of the systems. Zero energy ontology predicts that state function reductions change the arrow of time and have a classical description in terms of time reverse solutions of classical field equations so that the Universe is bound to look classical for the observed with standard arrow of time.

   Could it be that ordinary hydrodynamics corresponds to large values of h_{eff} at the level of MB controlling the dynamics at the lower levels?

7 Some concrete questions and problems

7.1 Could one assign quantum hydrodynamics to photonic quasi-crystalline structures?

Photons and polaritons are analogous to conduction electrons in metals. Again I can only ask questions.

1. Could they have as a classical correlate classical induced gauge fields such that the induced Kähler form defines a Beltrami flow with periodic properties? Flow lines are light-like locally but there would be a zitterbewegung involved.

2. What does the quasicrystal structure mean? Photonic quasicrystal should have a description as a quasiperiodic X^4. The identification of quasicrystals in terms of algebraic extensions of the ordinary lattices has been already considered. As a matter of fact, space-time surface X^4 defines a curved generalization of a quasicrystal obtained as points of X^4 belong to the set of points of $M^4 \subset M^8$ for which the M^4 coordinates are algebraic integers in the extension of rationals. In the "cut and project" construction (`https://cutt.ly/IWjxpLv`) one only replaces the low-dimensional plane in higher-D space containing ordinary crystal with the curved space-time surface. One can also define in M^8 crystal lattices tilted with respect to the chosen $M^4 \times E^4$ and obtain quasi-crystals and M^4 projections.

7.1.1 Bernard-von Karman (BvK) vortex streets in TGD framework?

Bernard–von Karman (BvK) vortex streets are observed in an exciton-polariton superfluid [D21] (https://cutt.ly/FWy3cNw). The formation of BvK vortex streets (https://cutt.ly/YWy3mjC and https://cutt.ly/JWy3WYP) is a hydrodynamical phenomenon due to dissipation.

Some facts about classical BvK are in order.

1. The flow past obstacle is laminar or turbulent. Turbulence occurs above critical Reynolds number this corresponds to a critical velocity of supraflow. Turbulence gives rise to BvK vortex streets observed in various macroscopically coherent phases analogous to hydrodynamic flows.

2. BvK involves a periodic emission of vortices from opposite sides of the body, say cylinder, occurring alternately. This means long range coherence in the scale of the body. Vortices grow after leaving the body. Boundary layer is at rest.

3. The role of pressure increase caused by velocity decrease. Change of the direction of velocity gives rise to vortices. Separation and formation of vortices occurs at critical fluid velocity at the thickest part of the obstacle.

7.1.2 Is BvK for supra flows basically quantum phase transition increasing h_{eff}?

One can ask whether BvK for supra flows could be quantum phase transition creating MBs of vortices with $h_{eff} > h_{eff,flow}$.

1. TGD suggests that hydrodynamic vortices at the fundamental level correspond to Z^0 magnetic vortices. If the CP_2 projection of the X^4 is U(2) invariant sphere of S^3, both em and Z^0 field are proportional to Kähler form and long range weak interactions are possible.

2. The picture based on minimal surfaces would suggest that dissipation occurs at the frames and elsewhere there is no classical dissipation. Obstacles of the flow would serve as analogs of frames. Vortices have singular cores: do they correspond to frames?

3. Separation and formation of vortices is a critical phenomenon. In the TGD framework, it could relate to quantum criticality at some level of dark matter hierarchy and lead to the formation of phases with a large value of h_{eff}. The "metabolic energy" needed to increase h_{eff} would come from dissipation.

4. Even ordinary hydrodynamical vortices would be accompanied by quantum coherent structures at the level of their MBs.

What could happen in the process? One can only ask questions.

1. The velocity pattern of the vortex has radial velocity gradient zero and means absence of dissipation. The reason for the formation of vortices are the facts that near the obstacle velocity gradient becomes too large and dissipation starts and flow separation occurs.

2. Quantum criticality would appear when the flow velocity is above critical value so that dissipation near the obstacle begins. Could it give rise to a metabolic energy feed driving generation of $h_{eff} > h_{eff,flow}$ phases? Above this the dissipating flow would serve as an energy source making possible the increase of complexity and self-organization and generation of vortices with $h_{eff} > h_{eff,flow}$.

3. Could the formation of vortices correspond to a formation of new MBs with a different value of h_{eff} expected to occur at quantum criticality? Metabolic energy feed would generate the MBs of the vortices as additional layers in the hierarchy of dark matter. Although the values of h_{eff} could be even smaller than for the entire MB, the complexity would increase since the number of levels would increase.

4. Could the integer value quantized vortices correspond to the values of $h_{eff}/h = n$?

7.2 Skyrmions in TGD framework

In hadron physics skyrmions (https://cutt.ly/qRuXYMX) appear at the level of momentum space. Proton as a skyrmion corresponds to a map of a 3-ball B^3 to $S^3 \subset E^4$ with non-trivial winding number. The points at the boundary are mapped to a single point so that B^3 effectively behaves like S^3. The map thus represents an element of third homotopy group and if this element is non-trivial one has skyrmions whose winding number has interpretation as number of protons. The radius of S^3 is the proton mass so that S^3 indeed lives in momentum space. $SO(4) = SU(2)_L \times SU(2)_R$ assigned to the current algebra picture of hadron physics acting as isometries of S^3 serves as the field space of skyrmions.

Skyrmions appear as topological defects also in condensed matter physics and correspond to 3-D magnetic field configurations inside B^3 and vanishing at the boundary of B^3 so that they define a map to S^3. In this case, the winding number of the map can correspond to the number of electron pairs. They appear in superconductivity, quantum Hall systems, liquid crystals, magnetic systems, and Bose-Einstein condensates (BECs). One example corresponds to ferromagnetic spin-1 Bose-Einstein condensates [D15](https://cutt.ly/MWy3S5J). Their universal appearance suggests that they could appear at fundamental level.

What TGD view would be following.

1. The proposal is that $M^8 - H$ duality allows to understand skyrmions as duality between the $SO(4)$ description of hadrons and $SO(4)$ symmetry group at M^8 level and QCD description in terms quarks and gluons and color $SU(3)$ at the level of H.

   In TGD framework skyrmions are associated with space-time surfaces in M^8 and skyrmion means a maps from a ball $B^3 \subset M^4$ to the sphere $S^3 \subset E^4$.The radius of S^3 is proton mass squared: this conforms with the interpretation of M^8 as momentum space.

2. Skyrmion in as a map $B^3 \to S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ is mapped to a map $B^3 \to S^3 \subset CP_2 \subset H$ by $M^8 - H$ duality. The map $B_3 \to B^3$ is by inversion (Uncertainty Principle). The map would have a non-trivial winding number.

   What does the skyrmion sphere S^3 subset E^4 correspond to in CP_2. Recall that normal space of X^4 is mapped to a point of CP_2. The image of the Skyrmion looks like a graph for the normal space of $X^4 \subset M^8$ as a function of the point of X^4. How does the normal space correlate with the E^4 point at S^3? Continuity and single-valuedness look natural. The 3-sphere in X^4 is mapped to a D$\leq$ 3 surface.

   Essentially homotopy associating normal space characterized by a point of CP_2 to $S^3 \subset CP_2$ is in question. CP_2 has a trivial third homotopy group. The homotopy equivalence class is trivial unless one fixes the radius as is done also in the original model by fixing the mass to correspond to the radius of $S^3 \subset E^4$.

   Could $S^3 \subset E^4$ containing the octonionic real axis be mapped to a sphere $S^3 \subset CP_2$ invariant under $U(2)$. At S^3 Z^0 gauge field is proportional to Kähler form J as is also the electromagnetic field [L2]. Therefore the long range correlations for Kähler form J are associated also with Z^0. Large parity breaking effects would become possible and indeed appear in living matter (chirality selection for biomolecules).

3. Could the sphere $S^3 \subset M^8$ mapped to $S^3 \subset CP_2$ related by $M^8 - H$ duality define a common denominator of several exotic condensed matter phenomena? $S^3 \subset M^8$ define a quaternionic 3-sphere and the automorphism group of quaternions. One can assign to skyrmions a flat $SO(3)$ gauge potential [D18] (https://arxiv.org/abs/1812.07974). Could this relate to the speculated emergence of $SO(3)$ as a synthetic gauge group [D8])(https://cutt.ly/qWy3H9M?

7.3 Dark matter and condensed matter physics

7.3.1 Could one make dark matter visible?

Dark matter in TGD sense could make itself visible in many manners.

1. One can imagine diffraction by generating a dark photon or (dark) polariton beam using a laser beam providing the energy feed increasing h_{eff}. Dark photon beam would diffract from an analog of hole: the ordinary laser beam could represent the hole as a source of dark photons. The structure of dark matter at flux tubes involving flux tubes and their geometric patterns could become visibille in this manner.

   For instance, the braids formed by flux tubes could become visible. Here braid entropy is a central notion and central in TGD based view of hydrodynamics involving braiding in both time-like and space-like braiding [K18, K17, K36].

2. In quantum biology dark matter at magnetic body with large h_{eff} as measure for complexity and intelligence, serves as the boss controlling ordinary biomatter, and its quantum coherence forces ordinary coherence of ordinary biomatter, which cannot be understood in physics and chemistry based on ordinary quantum physics [L55].

Solids are either in crystal or amorphous phase. Long range order in crystals is lacking and this is visible in the X-ray diffraction pattern. The diffraction pattern [D20] (`https://cutt.ly/ZWyLgjk`) for a hyperuniform amorphous material is very different and is called highly exotic (see **Fig. 16**). Apart from forward scattering peak, the diffraction pattern involves no scattering for a considerable range of scattering angles. I cannot avoid the temptation to speculate.

1. Suppose that the proposed dark looking phases with $h_{eff} > h$ by their higher algebraic complexity (larger extension of rationals, larger Galois symmetries) control the lower levels in master slave hierarchy, in particular ordinary matter (now the amorphous film.

   Suppose that the scattering of say laser light feeding energy and increasing the value of h_{eff} creates dark photons or polaritons at this higher level. Suppose that polaritons scatter at flux tubes or flux sheets structures at higher level and eventually a transformation to ordinary photons occurs spontaneously. Could the interference of the scattered beam with incoming beam make the geometry of dark matter level visible as the example about scattering in hyperuniform matter would suggest?

2. This high level would have longer quantum coherence length and perhaps range order since h_{eff} is larger. The long range order would be visible in the scattering pattern. Could just this happen when laser light generates a polariton-exciton condensate [D14](`https://cutt.ly/4Wy8zi9`). Could one think of polariton vortex lattices [D12] (`https://cutt.ly/qWy8Zqf`) as counterparts of crystal lattices and could their presence become visible so that one could see dark matter.

   The polariton could correspond at flux tubes superposition of dark photon and of dark exciton identifiable as dark electron paired with ordinary hole formed when the electron was transferred to the flux tube. The photon component of the outgoing polariton beam formed by the transformation of dark photon to ordinary photon would reflect the structure of dark matter and flux tubes and leave the system as ordinary photons and generate the scattering pattern by interference.

7.3.2 Polaritons and excitons in TGD

It is better to represent the ideas as questions.

1. Could one take the number theoretical model of macroscopic quantum phases as a guideline in attempts to understand polariton superfluidity and other quantum coherent phases involved.

2. The increase of h_{eff} and preservation of its values requires energy feed to prevent dissipation if. Could the formation of quasiparticles provide the metabolic energy for $h_{eff} > h$ phases at MB responsible for the long range order? In the case of superconductivity the formation of the Cooper pairs would provide this energy for dark electrons at MB.

   In BCS theory of superconductivity superconducting state does not have a well defined fermion number and this leads to somewhat questionable notion of coherent state of Cooper pairs. The Bogobliubov transformation creates the diagonalizable oscillator operator basis by mixing creation and annihilation operors. The resulting operators creat superpositions of electron and hole.

In the TGD framework, the interpretation would be that the hole actually corresponds to dark fermion at another space-time sheets so that fermion number conservation is not lost. Bogoliubov operators would correspond to superpositions of creation/annilation operators associated with different space-time sheets and create states which are superpositions of state at the two space-time sheets. Effective Hamiltonian would include parts assignable to both space-time sheets, and the terms quadratic in creation/annihilation operators breaking fermion number conservation would be replaced with pairs of creation and hilation operators associated with different space-time sheets describing the transfer of electron between the space-time sheets.

3. Is the polariton condensate actually a macroscopic quantum phase? Could the polariton BE condensate only provide the energy feed making possible a macroscopic quantum phase at the level of MB, which would then induce ordinary (non-quantum) coherence of the polariton condensate.

7.3.3 Braids, anyons, and Galois groups

Braids and anyons in the TGD framework are discussed in [K29]. Braid statistics has an interpretation in terms of rotations as homotopies at a 2-D plane of the space-time surfaces instead of rotations in M^4. One can use M^4 coordinates for the M^4 projection of the space-time surface.

As a matter of fact, arbitrary isometry induced flows of H can be lifted to rotations as flows along the lifted curve at the space-time surface and for many-sheeted space-time the flows which correspond to identity in H can lead to a different space-time sheet so that the braid groups structure emerges naturally. Is the 2-dimensionality, which makes possible non-trivial and non-Abelian homotopy groups, really necessary for the notion of the braid group in the TGD framework?

This view has some nice consequences.

1. If the space-time surface is n-sheeted, the rotation of 2π can take the particle to a different space-time sheet, and only n fold-rotation brings it back to its original position. The formula for fractional Hall conductivity is the same as in the case of integer Hall effect except that the $1/\hbar$-proportionality is replaced with $1/\hbar_{eff}$-proportionality in TGD framework [K29].

2. Degeneracy of fermion states also makes non-Abelian braid statistics possible. Since the Galois group acts as a symmetry group, the degeneracy would be naturally associated with the representations of the Galois group. Galois singletness of the many-anyon states guarantees reduces braid statistics to ordinary statistics for these. Galois confinement is proposed to be a central element of quantum biology [L55, L38].

Braid statistics could also relate to the problem created by Bose-Einstein and Fermi statistics.

1. The problem is that many-boson and many-fermion states are maximally entangled so that state function reduction is in the QFT framework possible only for the entanglement between fermions and bosons.

   In the TGD framework the situation is even more difficult since all elementary particles can be constructed from quarks. The replacement of point-like particles with 3-surfaces however forces us to re-consider the notion of particle identity. Number theoretic definition of identity applying to cognitive representations is attractive.

2. The intuitive proposal is that Galois representations can entangle and that the reduction of entanglement is possible. In particular, the decomposition of extension to a hierarchy of extensions with Galois groups forming a hierarchy of normal subgroups allows the notions of cognitive measurement cascade [L32].

3. A more rigorous basis for the intuition emerges from the TGD view about braiding. The Galois group can be always represented as a subgroup of a suitable symmetric group S_n. S_n allows braidings and therefore induces a braiding of the Galois group. The discrete subgroups

of symmetry groups of TGD could allow representation as a Galois group of the space-time surface. They could also allow braiding defined by the lift of the continuous isometry flow to the space-time surface. This suggests that the notion of a quantum group could allow a geometric interpretation in terms of the braiding based on the many-sheeted sub-manifold geometry.

4. The Galois group is in general non-Abelian and the braided Galois group would define braid statistics allowing higher-D representations. This would also make possible a non-maximal entanglement and the reduction of entanglement for the tensor products would be possible..

7.3.4 Fractons and TGD

In Quanta Magazine there was a highly interesting article about entities known as fractons (`https://cutt.ly/kQPph8n`).

There seems to be two different views about fractons as one learns by going to Wikipedia. Fracton can be regarded as a as self-similar particle-like entity (`https://cutt.ly/KQPadQL` or as "sub-dimensional" particle unable to move in isolation (`https://cutt.ly/yQPayJt`). I do not understand the motivation for "sub-dimensional". It is also unclear whether the two notions are related. The popular article assigns to the fractons both the fractal character and the inability to move in isolation.

The basic idea shared by both definitions is however that discrete translational symmetry is replaced with a discrete scaling invariance. The analog of lattice which is invariant under discrete translations is fractal invariant under discrete scalings.

One can also consider the possibility that the time evolution operator acts as a scaling rather than translation. At classical level this would produce scaled versions of the system in discrete steps. This is something totally new from quantum field theory (QFT) point of view and it is not clear whether QFT can provide a description of fractons. In QFTs energy corresponds to time translational symmetry and Hamiltonian generates infinitesimal translations. In string models the analog of stringy Hamiltonian is the infinitesimal scaling operator, Virasoro generator L_0. Energy eigenstates would be replaced by scaling eigenstates with energy replaced with conformal weight.

In TGD the extension of physics to adelic physics provides number theoretic and geometric descriptions as dual descriptions of physics [L12, L25, L26, L40]. This approach also provides insights about what fractons as scale invariant (or covariant) entities might be.

1. The extension of conformal invariance to its 4-D analog is key element of TGD and leads to the notion of super-symplectic invariance and to an extension of conformal and Kac-Moody symmetries with two coordinates analogous to the complex coordinate z for ordinary conformal symmetry. Second coordinate is light-like and the fact that light-like 3-surfaces are effectively 2-dimensional is absolutely essential for this approach. The existence of extended conformal symmetries makes the space-time dimension $D = 4$ unique whereas the twistor lift of TGD fixes H to be $H = M^4 \times CP_2$.

2. The predicted cosmological expansion is not smooth but occurs by discrete scalings as rapid jerks in which the size scale of 3-space as 3-surface increases. Actually they would correspond to discrete quantum jumps but in zero energy ontology (ZEO) in which quantum state are superpositions of space-time surfaces, their classical correlates are smooth time evolutions.

   Scalings by power of 2 are p-adically preferred [K5] [L51]. M^8-H duality allows us to imagine what this means at M^8-level [L52]. This proposal conforms with the puzzling observation that also astrophysical objects participate in cosmological expansion by comoving with it, they do not expand themselves.

3. The analog of a unitary time evolution between "small" state function reductions (SSFRs) as the TGD counterparts of weak measurements, is generated by the exponential of the infinitesimal scaling operator, Virasoro generator L_0. One could imagine fractals as states invariant under discrete scalings defined by the exponential of L_0. They could be counterparts of lattices but realized at the level of space-time surfaces having quite concrete fractal structure.

4. In p-adic mass calculations the p-adic analog of thermodynamics for infinitesimal scaling generator L_0 proportional to mass squared operator M^2 replaces energy. This approach is the counterpart of the Higgs mechanism which allows only to reproduce masses but does not predict them. I carried out the calculations already around 1995 and the predictions were amazingly successful and eventually led to adelic physics fusing real and various p-adic physics [K15].

5. Long range coherence and absence of thermal equilibrium are also mentioned as properties of fractons (at least those of the first kind). Long range coherence could be due to the predicted hierarchy of Planck constants $h_{eff} = n \times h_0$ assigned with dark matter and predicting quantum coherence in arbitrarily long scales and associated with what I called magnetic bodies.

   If translations are replaced by discrete scalings, the analogs of thermodynamic equilibria would be possible for L_0 rather than energy. Fractals would be the analogs of thermodynamic equilibria. In p-adic thermodynamics, elementary particles are thermodynamic equilibria for L_0 but it is not clear whether the fractal analogy with a plane wave in lattice makes sense.

   An attractive identification of the fractal counterpart of an energy eigenstate created in the unitary evolution preceding SSFR is as a scaling eigenstate defined as a superposition of scaled variants of space-time surface obtained by discrete scalings. Energy eigenvalue would be replaced with conformal weight. In zero energy ontology (ZEO), the counterpart of a fractal quantum state could be a superposition over zero energy states located inside the scaled variants of a causal diamond (CD).

   The ZEO based proposal is that each unitary evolution preceding SSFR creates a superposition of scaled variants of CD and that the SSFR induces a localization to single CD [L21, L32, L38]. The interpretation would be as a time measurement determined by the scale of the CD.

Second definition assumes that fractons are able to move only in combinations. This need not relate to the scaling invariance. Color confinement comes to mind as an analogy. Quarks are unable to exist as isolated entities, not only to move as in isolated entities.

In the TGD framework, the number theoretical vision leads to the notion of Galois confinement analogous to color confinement [L33]. The Galois group of a given extension of rationals indeed acts as a symmetry at the space-time level. In the TGD inspired biology Galois groups would play a fundamental role [L38]. For instance, dark analogs of genetic codons, codon pairs, and genes would be singlets (invariant) under an appropriate Galois group and therefore behave as a single quantum coherent dynamical and informational unit [L55, L39].

Suppose that one has a system - say a fractal analog of a lattice consisting of Galois singlets. Could fracton be identified as a state which is analogous to quark or gluon and therefore not invariant under the Galois group. The physical states could be formed from these as Galois singlets and are like hadrons.

7.3.5 Could dark matter as $h_{eff} = nh_0$ phases, quasicrystals, and the empirical absence of hyperon stars relate to each other?

How could the dark matter make itself at the level of the fermionic states?

1. Consider the momentum space, which by (anti-)periodic boundary conditions corresponds to a 3-D space with integer coordinates with a momentum unit defined by the quantization volume.

2. In the TGD framework, fermionic momenta are realized as points of X^4 for which coordinates belong to the extension of rationals for the polynomial P defining the X^4.

   For $n - D$ algebraic extension of rationals, the integers labelling the momentum components are replaced with points of an algebraically n-dimensional space with n integer coordinates. n basic vectors correspond to the roots of P. The Galois group acts as symmetries of this discrete space. Momentum vectors have $3n$ components.

3. If one assumes that momenta are real, the real momenta would be projections of these $3n$-dimensional vectors to a real section of X^4 for which M^8_c coordinates are real or purely imaginary.

   This projection from an algebraically 3n-D space to 3-D real space is analogous to the projection from higher dimensional space used to realize quasicrystals and the outcome is quasicrystal-like structure defined by the momentum components. This structure can be mapped from M^8 to H and since quasicrystals are observed at space-time level this suggests that the linear version of $M^8 - H$ duality is its correct version.

   Structures analogous to aperiodic crystals (quasicrystals) might be seen as a direct support for dark matter in the TGD sense. The quasicrystals could be realized at the level of the magnetic body (MB) or MB could induce their formation.

4. Algebraic extension increases the effective dimension of the discrete momentum space from 3 to $3n$ and the number of fermions inside the Fermi surface is increased by factor n^3. This prediction looks non-sensible and supports the view about Galois confinement, which means that physical states, now configurations of some number of neutrons, are Galois singlets. This implies that the total momentum for the singlet is integer valued as usual and also that the rational valued part is same for all neutrons of the singlet. Ordinary neutrons would be automatically Galois singlets.

   Neutrons could have momenta in an extension of rationals but form Galois confined K-neutron states such that the sum of the momenta is ordinary integer valued lattice momentum. Cooper pairs with $K = 2$ is one possible option The mass of the state would be Km_n and the number of states with the same Fermi momentum would be the number of Galois states from from K neutrons with momenta which are algebraic integers. One can assume that the real part of momentum is just the same integer for all neutrons of the composite and the non-rational part is one of the units defining the extension if the representation is the representation defined by roots of the polynomial.

   The formation of Galois singlets implies reduction of the translational degrees of freedom of K neutrons to those of a single particle with K-fold mass. This also explains the reduction of the Fermi energy. Galois degrees of freedom would replace the momentum degrees of freedom so that Fermi statistics can be realized.

   K-neutron states would have same momentum component k_i so that the density of states in the 3-D case would be reduced $d^3n/dk^3 \rightarrow K^{-3}d^3n/dk^3 = K^{-3}(2\pi/L)^3$, L the side of quantization cube. On the other hand, there would be a degeneracy $D(K, n)$ depending on extension and its dimension n so that one would have $d^3n/dk^3 \rightarrow (D(K, n)/K)^3(2\pi)^3/V$. The N/V number of states per volume would scale as $N/V \rightarrow (D(K, n)/K)^3N/V$ and Fermi energy $E_F \propto (N/V)^{2/3}/m$ would scale as $E_F \rightarrow (D(K, n)/K)^2E_F/K$ by $m \rightarrow Km$. For $(D(K, n)^2/K^3 < 1$, E_F would be reduced and the formation of a dark Galois confined state would be energetically favourable. For dark Cooper pairs with $K = 2$, the condition would be $D(2, n)/8 < 1$.

   In the TGD inspired quantum biology genetic code is realized by triplets of dark protons at magnetic flux tubes parallel to DNA strands are assumed to be be Galois singlets and genes in turn would be Galois singlest for a Galois group at larger space-time sheet [L30, L39]. Also dark photon triplets would be Galois singlets.

   Ordinary superconductors could have as a current carrier either i) a single dark fermion or ii) dark Cooper pair. For option i), Cooper pairs of ordinary fermions provide the energy needed to increase h_{eff} to get the dark electron. For option ii), Galois confinement would generate dark Cooper pairs. The energy liberated in the formation of the Cooper pair would be used to increase h_{eff} of the pair.

   A possible application is provided by the hyperon puzzle of neutron stars (`https://cutt.ly/jWy3Cnf`). The problem is that the core should suffer a transformation to a hyperon star because the Fermi energy is inversely proportional to the mass of the fermion and would therefore be reduced. There is however no evidence for hyperon stars or hyperon cores. Could part of neutrons transform to dark phase with $h \rightarrow nh$ forming Galois singlets of K neutrons (dark Cooper pairs (neutron superfluidity) or dark triplets) so that the Fermi energy would be reduced in the way explained. Dark Cooper pairs is the second option meaning neutron superfluidity.

7.3.6 Periodic self-organization patterns, minimal surfaces, and time crystals

Periodic self-organization patterns which die and are reborn appear in biology. Even after images, which die and reincarnate, form this kind of periodic pattern. Presumably these patterns would relate to the magnetic body (MB), which carries dark matter in the TGD sense and controls the biological body (BB) consisting of ordinary matter. The periodic patterns of MB represented as minimal surfaces would induce corresponding biological patterns.

The notion of time crystal [B3] (`https://cutt.ly/2n65x0k`) as a temporal analog of ordinary crystals in the sense that there is temporal periodicity, was proposed by Frank Wilczeck in 2012. Experimental realization was demonstrated in 2016-2017 [D13] but not in the way theorized by Wilczek. Soon also a no-go theorem against the original form of the time crystal emerged [B4] and motivated generalizations of the Wilzeck's proposal.

Temporal lattice-like structures defined by 4-D minimal surfaces as preferred extremals of action which sum of volume term and Kähler action [L52] would be obvious candidates for the space-time correlates of time crystals.

1. One must first specify what one means with time crystals. If the time crystal is a system in thermo-dynamic equilibrium, the basic thermodynamics denies periodic thermal equilibrium. A thermodynamical non-equilibrium state must be in question and for the experimentally realized time crystals periodic energy feed is necessary.

   Electrons constrained on a ring in an external magnetic field with fractional flux posed to an energy feed form a time crystal in the sense that due to the repulsive Coulomb interaction electrons form a crystal-like structure which rotates. This example serves as an illustration of what time crystal is.

2. Breaking of a discrete time translation symmetry of the energy feed takes place and the period of the time crystal is a multiple of the period of the energy feed. The periodic energy feed guarantees that the system never reaches thermal equilibrium. According to the Wikipedia article, there is no energy associated with the oscillation of the system. In rotating coordinates the state becomes time-independent as is clear from the example. What comes to mind is a dynamical generation of Galilean invariance applied to an angle variable instead of linear spatial coordinate.

3. Also the existence of isolated time crystals has been proposed assuming unusual long range interactions but have not been realized in laboratory.

Time crystals are highly interesting from the TGD perspective.

1. The periodic minimal surfaces constructed by gluing together unit cells would be time crystals in geometric sense (no thermodynamics) and would provide geometric correlates for plane waves as momentum eigenstates and for periodic self-organization patterns induced by the periodic minimal surfaces realized at the level of the magnetic body. It is difficult to avoid the idea that geometric analogs of time crystals are in question.

2. The hierarchy of effective Planck constants $h_{eff} = nh_0$ is realized at the level of MB. To preserve the values of h_{eff} energy feed is needed since h_{eff} tends to be reduced spontaneously. Therefore energy feed would be necessary for this kind of time crystals. In living systems, the energy feed has an interpretation as a metabolic energy feed.

   The breaking of the discrete time translation symmetry could mean that the period at MB becomes a multiple of the period of the energy feed. The periodic minimal surfaces related to ordinary matter and dark matter interact and this requires con-measurability of the periods to achieve resonance.

3. Zero energy ontology (ZEO) predicts that ordinary ("big") state function reduction (BSFR) involves time reversal [L21, L43]. The experiments of Minev et al [L16] [?] give impressive

experimental support for the notion in atomic scales, and that SFR looks completely classical deterministic smooth time evolution for the observer with opposite arrow of time. Macroscopic quantum jump can occur in all scales but ZEO together with h_{eff} hierarchy takes care that the world looks classical! The endless debate about the scale in which quantum world becomes classical would be solely due to complete misunderstanding of the notion of time.

4. Time reversed dissipation looks like self-organization from the point of view of the external observer. A sub-system with non-standard arrow of time apparently extracts energy from the environment [L19]. Could this mechanism make possible systems in which periodic oscillations take place almost without external energy feed?

Could periodic minimal surfaces provide a model for this kind of system?

1. Suppose that one has a basic unit consisting of the piece $[t_1, .., t_k]$ and its time reversal glued together. One can form a sequence of these units.

   Could the members of these pairs be in states, which are time reversals of each other? The first unit would be in a self-organizing phase and the second unit in a dissipative phase. During the self-organizing period the system would extract part of the dissipated energy from the environment. This kind of state would be "breathing" [L54].

   There is certainly a loss of energy from the system so that a metabolic energy feed is required but it could be small. Could living systems be systems of this kind?

2. One can consider also more general non-periodic minimal surfaces constructed from basic building bricks fitting together like legos or pieces of a puzzle. These minimal surfaces could serve as models for thinking and language and behaviors consisting of fixed temporal patterns.

7.4 Spin ice and quantum spin ice from TGD viewpoint

In this section the notions of spin glass, spin ice and quantum spin ice are considered from TGD point of view.

7.4.1 Spin ice

There is a Wikipedia article (`https://cutt.ly/eEDTIwp`) about spin ice as a system in which magnetic moments, that is spins, form a lattice-like state. The basic property of spin glasses, and therefore also of spin ice, is that there is ground state degeneracy that is several states with the same energy giving rise to what is called frustration: the term comes from the obvious social analogy. Two examples of these compounds are dysprosium titanate $Dy_2Ti_2O_7$ and holmium titanate $Ho_2Ti_2O_7$.

Spin ice has properties resembling those of crystalline water ice. For spin ice, the sum of the outward pointing moments and inward point magnetic moments is zero for a tetrahedron forming a basic unit. The rule holds true only in ground state configuration analogous to ferromagnetic state but with non-constant direction of magnetization and need not be the situation in general. Its violation gives rise to analogs of magnetic monopoles analogous to charges for which there is evidence.

When the rule holds true, it is possible to formally define a conserved current, which is locally in the direction of the magnetic moment. It is divergenceless like a magnetic field and can be said to carry an analog of magnetic or electric charge as long as the rule is satisfied. Thermal fluctuations can change the direction of say one spin in the volume: this means formally creation of an analog of magnetic monopole. This system of pseudo-monopoles could be described by a theory resembling electromagnetism with an effective fine structure constant ten times larger than α [D11, D9]. This leads to ask whether this implies especially strong interaction with electromagnetic radiation.

7.4.2 Quantum spin ice

The special feature of certain spin ice systems is that the directions of spins can be random down to zero temperature since the energies of the frustrated configurations are the same and no energy is needed to change the configurations. This suggests that quantum fluctuations are involved and the system is actually quantum spin glass rather than a thermodynamical one.

It has been proposed that the interactions of the effective monopoles [D3, D17, D11, D9] (for a popular article see `https://cutt.ly/vED27e5`) can be described by an analog of QED. The value of the emergent fine structure constant assignable to the interaction with electromagnetic radiation would be 10 times larger than the real α.

In [D17] quantum tunnelling as transitions between degenerate configurations involving in the simplest situation 4 tetrahedrons and differing by an orientation of a loop formed by the imagined flux lines of the magnetization field analogous to magnetic field and connecting the 4 tetrahedrons is proposed as an essential element of the emergent lattice QED. The tunnelling makes possible long range correlations and makes implies large value of effective α.

This should be visible as a large enhancement of the low energy scattering of neutrons from the quantum spin ice materials. Low energy quasi-elastic neutron scattering would measure the 2-point momentum space correlation function of spins of the quantum spin glass. This correlation function would become long ranged in the real space. Lattice photons having linear dispersion relation $\omega \propto k$ but much smaller propagation velocity than ordinary photons would cause this behavior. This lattice photon would be visible in inelastic neutron scattering.

The effective magnetic monopoles that play the role of em charges are identified as spinons in [D11]. Electrons are proposed to consist of spinon, orbiton, and holon carrying spin, orbital quantum numbers and charge and in some cases they can behave like independent quasiparticles. I don't quite understand what this is supposed to mean. In the case of decomposition to spinon and holon which can occur in 1-D systems, spin waves and charge waves would propagate as independent waves.

If I understand correctly, charge waves would represent an oscillatory variation in the charge density of electrons and spin waves in the spin direction. They could have different wavelengths and phases.

7.4.3 TGD biew about quantum spin ice

What about the TGD based description of the quantum spin ice?

1. In the TGD framework, magnetic field corresponds to flux tubes which can be either monopole flux tubes or carry normal flux caused by currents. Monopole flux tubes require no currents and this has powerful implications in astrophysics and cosmology. This suggests that the pseudo-monopoles could be "real" in some sense. Note however that TGD does not however allow free monopoles but only closed monopole flux tubes.

2. Long range interactions are required to create a spin glass phase and one can realize the basic rule of spin ice ground state as a special case. In the TGD framework the large values of h_{eff} could make this possible even at high temperatures. This rule allows frustration as the existence of several configurations with the same interaction energy. The transitions between these configurations would lead to the emergence of large effective α. In [D17] the transitions between degenerate configurations involving four tetrahedrons and differing by an orientation of a loop which in the TGD picture corresponds to a closed flux tube are mentioned as simplest transitions.

3. The spine of spin ice would be a flux tube network formed by monopole flux tubes and that magnetic moments associated with flux tubes have suffered spontaneous magnetization, which is locally in the direction of the local flux tube. If the numbers of the incoming and outgoing flux tubes in a given volume unit are the same and magnetic moments are parallel to the magnetic fluxes, the sum of magnetic moments is zero for a ferromagnetic situation. The formal current would be realized with real and quantized monopole flux which is conserved.

   Spin ice would be analogous to ferromagnet, a spaghetti of flux tubes accompanied by spontaneously magnetized spins such that the directions of magnetization at flux tubes can carry. Neutron scattering has demonstrated that the aligned spins indeed form intertwined tube-like bundles.

4. What could the TGD counterparts of the effective monopoles be? There are two options to consider.

(a) In the many-sheeted space-time of TGD, the monopole fluxes can go to parallel space-time sheets via wormhole contact and return back at a rather long distance. The wormhole contact looks like a pair of throats behaving like magnetic monopoles. The throats have an extremely short distance. This option does not look attractive since at the QFT limit the many-sheetedness and the monopole pairs formed by the throats of the wormhole contact become invisible.

What remains are flux tubes and the spin ice phase can make directly visible the underlying network of monopole flux tubes as it indeed does.

(b) Thermal and quantum fluctuations can however change spin direction and spin is formally like a magnetic monopole or charge and it seems that this is enough also in the TGD framework. This could also happen at the zero temperature limit as quantal rather than thermal fluctuations of the flux tube structure inducing the long range correlates between spins. The quantum fluctuations of spin ice would correspond to the long range quantum fluctuations of the dark flux tubes with $h_{eff} \geq h$.

5. TGD predicts the existence of two kinds of flux tubes corresponding to monopole flux tubes having a closed surface rather than disk as a cross section and requiring no currents to generate the magnetic field and Maxwellian non-monopole flux tubes for which the induced Kähler field can vanish. The Maxwellian flux tubes have a Lagrangian 2-manifold as a CP_2 projection, and the action reduces to a mere volume term proportional to length scale dependent cosmological constant approaching zero in long scales.

   At a long length scale limit, the deviation of the Kähler function from the ground state value becomes very small which has interpretation in terms of a strongly interacting phase. One expects large fluctuations, which give rise to the quantum spin glass phase. The two kinds of flux tubes could correspond to vortex-like entities with a monopole flux tube associated with the vortex core and the Lagrangian non-monopole part with its exterior.

6. Since very large values of h_{eff} are involved, the findings about the role of solar mass inspire the good guess $\hbar_{gr} = GM_{Sun}/v_0$, $\beta_0 \simeq 2^{-11}$. The size of the throat would be scaled from about CP_2 size for $h_{gr}(Sun)/hbar \sim 2 \times 10^{20}$. The size scale of the dark wormhole throat would be about 10 nm, which is the thickness of the neuronal membrane so that a connection with biology is highly suggestive.

   Remark: If the huge values $\hbar_{gr}$ of $\hbar_{eff}$ are possible, the size of leptonic wormhole throat could be of order .9 cm for M_E! Leptons consist of 3 antiquarks in TGD framework [**?**] Could this mean that it might be possible to detect free quark?

The emergence of the strong interactions can be understood at the general level in the TGD framework.

1. Quantum spin glass is a strongly interacting quantum system since the quantum fluctuations are large even at the temperature zero limit. Quite concretely, the deviations of the Kähler function from the value for the ground state are very small.

   Using the language of QFTs, one has a very large number of almost degenerate configurations in the path integral with the same value of the action. This is achieved if the coupling strength is very large so that the action exponential appearing in the path integral is analogous to Gaussian with very large width.

   In the TGD picture, one says that the Kähler function for 3-surfaces (by holography for 4-surfaces) has the same value for a large class of 3-surfaces and is therefore slowly varying as a function of 3-surface. This picture is mathematically very much like the thermodynamic picture with Hamiltonian replaced by the Kähler function.

2. The original TGD based prediction based on the huge vacuum degeneracy of the Kähler action was that TGD allows 4-D analogs of spin glasses a as vacuum extremals with 2-D Lagrangian sub-manifold as CP_2 projection, meaning huge non-determinism. This however leads to problems.

The inclusion of the M^4 contribution to Kähler form removes the vacuum degeneracy since one must have Lagrangian projection also in M^4 so that string-like entities, which are minimal surfaces, are in question.

3. The recent picture implied by twistor lift involves an additional volume term in the action leaving only finite non-determinism analogous to that for soap films. At the long length scale limit spin glass type behavior is suggestive when the Kähler action vanishes (Lagrangian property in CP_2 degrees of freedom for Maxwellian flux tubes). The volume term is very small.

   The basic reason would be the smallness of the volume term, that is the smallness of length scale dependent cosmological constant Λ [L13] giving rise to cosmological p-adic length scale $L_{cosmo} \sim 1/\sqrt{\Lambda}$ and a relatively short p-adic length scale L_{short} as geometric mean $L_{short} = \sqrt{L_{cosmo}L_{Pl}}$ of the Planck length and L_{cosmo}. L_{short} is of order 10^{-4} m and defines a biological length scale.

   Smallness of the volume action means large fluctuations in the functional integral characteristic for strongly interacting systems. Quite concretely, the flux tubes have very small string tension and their shapes fluctuate wildly. Long flux tube-like objects have a small volume and small string tension and would be very loose strings having very many configurations with the same energy. Quantum spin glass property would correspond to the existence of a large number of spaghetti-like configurations with the same value of the Kähler function.

4. The assumption that velocity field is proportional to Kähler gauge potential implies that it is not only Beltrami field but also gradient for the Lagrangian situation prevailing outside the vortex cores. There would be no classical dissipation at the level of Kähler action.

   Cores would have non-vanishing Kähler field and action. What about the Beltrami property in the vortex core? If the projection of the vortex core is 2-D complex surface, the Kähler gauge potential is Beltrami field. For instance, for a projection with is geodesic sphere S^2, the Kähler gauge potential is proportional to $A = cos(\Theta)d\Phi$ in the spherical coordinates and Φ defines the global coordinate along flow lines. $D > 2$-D deformations spoil the Beltrami property.

   Same is true for the M^4 projection: when the projection as a string world sheet is deformed to $D > 2$-D surface, Beltrami property is lost and classically there is dissipation meaning that Kähler 4-force is non-vanishing.

   Whether the dissipative option is realized at all for preferred extremals is not at all clear. Dissipative effects might be solely due to the finite sizes of space-time surfaces, which are proportional to h_{eff}.

5. There is a further delicacy involved. The assumption that both M^4 and CP_2 projections are at most 2-D is not enough for Beltrami or gradient flow. This condition alone would give a Kähler gauge potential, which is the sum $A(M^4) + A(CP_2)$ of two contributions $A(M^4) = \Psi_1 d\Phi_1$ and $A(CP_2) = \Psi_2 d\Phi_2$ satisfying the conditions separately. Besides this, the gradients $d\Psi_1$ and $d\Psi_2$ must be proportional to each other so that Ψ_1 and Ψ_2 are functionally dependent.

   Is this condition satisfied for all preferred extremals in which case classical dissipation would be absent or in special cases only.

6. The Lagrangian flux tubes associated with the exteriors of vortex cores would give rise to quantum spin glass property if they have a large value of $\hbar_{eff}$. In some situations even $\hbar_{eff} = \hbar_{gr}$ can be considered. This would give rise to long range quantum fluctuations and correlations and also to the absence of dissipation.

How to understand the predicted strong interaction of quantum spin glass phases with the electromagnetic radiation predicted by the emergent QED [D3, D17, D11, D9] to give rise to a strong enchancement of neutron scattering cross section?

1. Spin glasses could correspond to dark flux tube spaghettis so that the spins would be locally magnetized in the direction of the magnetic field of the dark flux tube playing the role of H field.

2. $h_{eff} > h$ would imply long range correlations but would also mean a reduction of the value of fine structure constant $\alpha \propto 1/h_{eff}$. This is just the opposite for the proposal of [D3, D17, D11, D9] that the analog of the fine structure constant emerging in the analog of lattice QED is larger than α.

   Paradox disappears as one realizes that the transition $h \rightarrow h_{eff}$ is Nature's manner to guarantee that perturbation theory converges. This requires the change of the nature of the quantum states and Galois confinement would be the underlying mechanism and also behind color confinement. Quantum spin glass would be analogous to hadron.

   At the level of M^8 (analogous to momentum space) this implies the increase of the dimension of the extension of rationals determining the space-time region at the level of M^8. This also means the increase of complexity.

3. Spin glass degeneracy, realized as degeneracy of Galois confined states, suggests that the neutron scattering rate is enhanced since the transitions between degenerate states become possible. The same happens in the case of hadrons since the number of color confined final states is large.

7.5 Condensed matter Majorana fermions in the TGD framework

Condensed matter Majorana fermions are not genuine Majorana fermions, which have not been found in Nature and are impossible also in TGD as fundamental particles. Condensed matter Majorana quasiparticles could however have a TGD counterpart.

Majorana fermions (`https://cutt.ly/FWdXK4s`) are quasiparticles created by superpositions of fermionic creation and annihilation operators invariant under charge conjugation. This motivates the term Majorana particle. Majorana particles are also zero energy excitations and therefore can be created at topological defects as pairs with degenerate energies. This is due to the fact that momenta $p = G/2$ and $p = -G/2$, where G is a lattice momentum, correspond to the same energy.

The valence and conduction bands for a topological insulator must intersect at its boundary: this is the topological singularity at the level of the momentum space. This can happen at boundaries of insulators and at topological defects. The single point intersection of Fermi bands at a single point looks locally like a double cone and at the tip the normal space is non-unique and the normal normal spaces span a circle in 3-D momentum space.

7.5.1 TGD counterpart for the notion of Majorana quasiparticle

Consider now the situation in the TGD framework.

1. The counterparts of Majorana fermions should correspond to superpositions of ordinary and dark fermions at different energy bands - just like the Boboliuv particles of superconductors in the BCS model. These states cannot be C invariant. Kind of half dark-half visible, perhaps gray - fermions would be in question.

2. The momenta of the occupied fermion states of the momentum space of fermion (mass shell $H^3 \subset M^8$) define what I call cognitive representation consisting of a discrete set of points in an extension of rationals) $M^8 - H$ duality maps the points of $H^3 \subset M^4 \subset M^8$ to the points of the boundary δcd of 4-D causal diamond $cd \subset M^4 \subset H$ and therefore to the points of space-time surface. In particular, the boundaries of energy bands in M^8 are mapped to boundaries of the image in $\delta cd \subset H$ and define 2-D surfaces containing the edge states. In M^8, the touching of two bands corresponds to a single point intersection of algebraic surfaces. These surfaces can be continued to the interior of X^4 by the flow defined by qv generalized Beltrami field.

3. The direction of the quaternionic normal spaces in M^8 at the tip should have all directions parametrized by a circle. This suggests that the tip is not be mapped to a single point, but to a circle formed by the set of CP_2 points. The conical topological singularity in M^8 would correspond to a closed circle $S^1 \subset CP_2$.

4. If Majorana particles have a counterpart in TGD, they should correspond to superpositions of ordinary and dark fermion with the special property that the fermions have identical energies i.e. momenta are $G/2$ and $-G/2$. This condition guarantees that these states have identical energies as required by the condition $E^2 - p^2 = m^2$ holding true in H^3.

   At the level of M^8 the polynomial defining the space-time surface would characterize topological defects as singularities. Various lower-D surfaces in momentum space and position space should be isometric surfaces as surfaces of H^3, which looks a rather non-trivial prediction.

   Remark: Note that the product of polynomials defines a disjoint set of spacetime surfaces [L33]. Also a single irreducible polynomial can have several space-time surfaces as roots and possibly intersecting at a discrete set of points in the generic situation.

7.5.2 Majorana quasiparticles and topological quantum computations

TGD leads to a general vision about topological quantum computation TQC [**?**]ased on braids formed by magnetic flux tubes. The reconnection of flux tubes brings in a new topological element and corresponds to the formation of 2-knots. The proposal is that TQC in this sense is a basic aspect of living matter. Also the hierarchy of effective Planck constants making possible long range quantum coherence and ZEO making possible time reversals of TQCs represent new elements.

The bound states of Majorana quasiparticles located at the ends of superconducting wire are analogous to Cooper pairs entangling non-locally and have been proposed by Kitaev to make possible TQC without a need for massive error correction procedure [D24]. The association with the ends of wire would give rise to non-locality and long range quantum entanglement making it difficult to destroy entanglement by local measurements.

In an effectively 2-D system, the braid group defines non-standard statistics. The braid group must be non-Abelian so that higher than 1-D representations are possible and can be utilized in TQC. $SU(2)$ is the minimal option. The states of braid group representation are robust against perturbations destroying the entanglement

If I have understood correctly, the two energy degenerate states of the bound state of Majoranas would correspond to SU(2) doublet with energy degeneracy, which vanishes when the zero of energy corresponds to the middle point of the band gap.

1. In the TGD framework, the Majorana property does not seem to be absolutely essential. It is essential to have non-commutativity and energy degeneracy. Galois groups act as number theoretical symmetries and all non-trivial representations of the Galois groups allow this degeneracy. One might therefore speak of a hybrid of number theoretic and topological quantum computation. There seems to be no reason preventing the representations of discrete subgroups of the braid group defined by some Lie group acting in the cognitive representations defined by algebraic integer valued momenta at the intersection of mass shell and $X^4 \subset M^8$, that is at the level of M^8 on the cognitive representations. The quantum variants Gal_q of Galois groups could be involved.

2. $SU(2)$ has an interpretation as automorphisms of quaternions and acts in E^4 factor of M^8, could be in a special role physically in TGD and also because its discrete subgroups appear in the hierarchy of hyper-finite factors of type II_1 (HFFs). The discrete subgroups E_6, E_7 and E_8 (tetrahedral, octahedral and icosahedral groups). These groups could have representations as Galois groups. Momenta as algebraic integers correspond to the vertices of corresponding Platonic solids and total momenta for many-quark states vanish for the states. Also spinor representations are involved bringing in spin and electroweak degrees of freedom. Galois confinement requires that the states as a whole are Galois singlets. TQC would also be a basic process of quantum cognition.

3. In TGD superpositions of fermion and hole correspond to superpositions of fermion states at the ordinary and dark space-time sheet. Could the entanglement between dark and ordinary fermions (more generally, with different values of h_{eff}) with the same energy give rise to the analogs of Majorana quasiparticles?

7.6 Condensate of electron quadruplets as a new phase of condensed matter

Formation of fermion quadruplet condensates [D23] (https://cutt.ly/TRcxQtz) is a new exotic condensed matter phenomenon discovered by Prof. Egor Babaev almost 20 years ago and 8 years after publishing a paper predicting it. Recently Babaev and collaborators presented in Nature Physics evidence of fermion quadrupling in a series of experimental measurements on the iron-based material, $Ba_{1-x}K_xFe_2As_2$.

The abstract of the article summarizes the finding.

> The most well-known example of an ordered quantum state—superconductivity—is caused by the formation and condensation of pairs of electrons. Fundamentally, what distinguishes a superconducting state from a normal state is a spontaneously broken symmetry corresponding to the long-range coherence of pairs of electrons, leading to zero resistivity and diamagnetism.
>
> Here we report a set of experimental observations in hole-doped $Ba_{1-x}K_xFe_2As_2$. Our specific-heat measurements indicate the formation of fermionic bound states when the temperature is lowered from the normal state. However, when the doping level is $x \sim 0.8$, instead of the characteristic onset of diamagnetic screening and zero resistance expected below the superconducting phase transition, we observe the opposite effect: the generation of self-induced magnetic fields in the resistive state, measured by spontaneous Nernst effect and muon spin rotation experiments. This combined evidence indicates the existence of a bosonic metal state in which Cooper pairs of electrons lack coherence, but the system spontaneously breaks time-reversal symmetry. The observations are consistent with the theory of a state with fermionic quadrupling, in which long-range order exists not between Cooper pairs but only between pairs of pairs.

Fermion quadruplets are proposed to be formed as pairs of Cooper pairs are formed somewhat above the critical temperature T_c for a transition to superconductivity. Breaking of the time reversal symmetry T is involved.

The question is why quadruplets are stable against thermal noise above the critical temperature. Superconductivity is thought to be lost by the thermal noise making the bound states of electrons in Cooper pair unstable. Is the binding energy for quadruplets larger than for Cooper pairs so that quadruplet condensate is possible below higher critical temperature. What is the mechanism of binding?

The discovery is highly interesting from the TGD point of view.

1. TGD leads to a model of super-conductivity involving new physics predicted by TGD.
2. Adelic physics number theoretic view about dark matter as $h_{eff} > h$ phases h_{eff} proportional to the order of the Galois group. This leads to the notion of Galois confinement. Galois confinement could serve as a universal mechanism for the formation of bound states including also Cooper pairs and even quadruplets. In quantum biology triplets of protons representing genetic codons and even their sequences representing genes would be formed by Galois confinement.
3. The finding also allows to develop more preices view of TGD view concerning discrete symmetries and their violation.

7.6.1 Time reversal symmetry in TGD

What do time reversal symmetry and its violation mean in TGD.

1. The presence of magnetic field causes violation of T in condensed matter systems.
2. Second, not necessarily independent, manner to violate T in TGD framework is analogous to that in strong CP breaking but different from it many crucial aspects. Vacuum functional is exponent of Kähler function but exponent can contain also an instanton term I, which is

equal to a divergence of topological instant current which is axial. so that non-vanishing I suggests parity violation. The fact that exponent of I is imaginary while exponent of Kähler action is real, means C violation. If instanton current is proportional to conserved Kähler current its divergence is vanishing and M^4 projection is less than 4-D.

I is non-vanishing only if the space-time sheet in $X^4 \subset M^4 \times CP_2$ has 4-D CP_2 or M^4 projection. The first case corresponds to CP_2 instanton term $I(CP_2)$ and second case to $I(M^4)$ present since twistor lift forces also M^4 to have an analog of Kähler structure. The two Kähler currents are separately conserved.

3. These two mechanisms of T violation might be actually equivalent if the T violation is caused by the M^4 part of Kähler action. Consider a space-time surface with 2-D string world sheet as M^4 projection carrying Kähler electric field but necessarily vanishing Kähler magnetic field B_K. If it is deformed to make M^4 projection 4-D, B_K is generated and T is violated. Therefore generation of B_K in M^4 can lead to a T violation.

7.6.2 Generalized Beltrami currents

Generalized Beltrami currents are nother key notion in TGD based view about superconductivity [L35].

1. The existence of a generalized Beltrami current $j = \Psi d\Phi$ implies the existence of global coordinate Φ varying along the flow lines of the current. Also the condition $dj \wedge j = 0$ follows. The 4-D generalization states that Lorentz force and electric force vanish. In effectively 3-D situation, j could correspond to magnetic field B and dj to current as its rotor and the Beltrami condition fof B implies that Lorentz force vanishes.

2. The proposal of [K20] is that for the preferred extremals CP_2 *resp.* M^4 Kähler current is proportional to instanton current $I(CP_2)$ *resp.* $I(M^4)$ and therefore topological for $D(CP_2) = 3$ *resp.* $D(M^4) = 3$. For $D = 2$ the contribution to instanton current vanishes. In this case the Lorentz force vanishes so that the divergence of the energy momentum tensor is proportional to I and vanishes so that dissipation is absent. One can verify this result using the effective 3-dimensionality of the projection and using 3-D notations [K20]: in this formulation the vanishing of Lorentz force reduces to Beltrami property for B as 3-D vector. With this assumption, dissipation for the preferred extremals of Kähler action just as it is absent in Maxwell's theory. An open question is whether this situation is true always so that dissipation and the observed loss of quantum coherence would be due to the finite size of space-time sheet of the system considered.

3. Beltrami property would serve as a classical space-time correlate for the absence of dissipation and presence of quantum coherence. Beltrami property allows defining of a supra current like quantity in terms of Ψ and Φ. Usually the superconducting order parameter Ψ is actually not an order parameter for a coherent state as a superposition of states with a varying number of Cooper pairs. Now the geometry of the space-time sheets (magnetic flux tube carrying dark Cooper pairs) allows the identification of this order parameter below the quantum coherence scale. The TGD interpretation is that the coherent state is an approximation, which does not take into account the fact that the system is not closed. There is exchange of electron pairs between ordinary and dark space-time sheets with $h_{eff} > h$ [L35]. Dark Cooper pairs would form bound states by Galois confinement.

4. In the superconducting state space-time regions would have at most 3-D M^4 projection at fundamental level and T would not be violated. There is no dissipation and pairs are possible below critical temperature.

One can also understand the Meissner effect. According to the TGD view, the monopole flux tubes generate the analog of the field H perhaps serving as an approximate average description for the field of monopole flux tubes. This field induces the analog of magnetization M involving non-monopole flux tubes. Also M would be an average field. For superconductors in the diamagnetic phase, the sum would be zero: $B = H + M = 0$. If the Cooper pairs have spin, the supracurrents of Cooper pairs at monopole flux tubes could generate the compensating magnetization.

7.6.3 TGD view about quadruplet condensate

How could one understand quadruplet condensate in the TGD framework?

1. T violation could be accompanied by the presence of Kähler instanton term $I(M^4)$ or $I(CP_2)$ requiring 4-D M^4 or CP_2 projection: this would also generate M^4 magnetic fields. The M^4 option would bring in new physics for which also the Magnus effect of hydrodynamics suggesting Lorentz forces serves as an indication [L45].

   For 4-D M^4 projection, the divergence of the axial instanton current would be non-vanishing and the proportionality of Kähler current and instanton current implying a vanishing classical dissipation would be impossible. The instanton number can be expressed as instanton flux over 3-D surfaces, which would be "holes".

2. For the quadruplet condensate M^4 projection is 4-D and T is violated. Kähler magnetic fields originating from M^4 part of Kähler action would be present as also dissipation. For quadruplet condensate M would not compensate for H so that net magnetic fields B would be generated and correspond to space-time sheets with 4-D M^4 projection.

3. Dark matter as phases with $h_{eff} > h$ would however be present and quadruplets would correspond to bound states of 4 electrons formed by Galois confinement [L46, L44] stating that the total momentum of the bound state as sum of momenta, which are algebraic - possibly complex - integers, is a rational integer in accordance with the periodic boundary conditions.

4. What prevents the formation of Cooper pairs? Above T_c thermal energy exceeds the gap energy so that Cooper pairs are thermally stable. If the binding energy for quadruplets is larger, they are stable.

5. In what sense the quadruplets could be regarded as bound states of Cooper pairs? Since the ordinary Cooper pairs are Galois singlets, bound state formation does not look plausible since Cooper pairs themselves are unstable. A more plausible option is that Cooper pairs involved are "off-mass-shell" in that they have momenta, which are non-trivial algebraic integers and that the sum of these momenta is a rational integer in the bound state.

Remark: Four-momenta as algebraic integers are in general complex. Usual charge conjugation involves complex conjugation in CP_2 degrees of freedom. Is it accompanied by conjugation of the complex 4-momenta. Kähler currents of M^4 and CP_2 are separately conserved: should one regard complex conjugations in M^4 and CP_2 as independent charge conjugation like symmetries. $C(M^4)$ would however leav Galois singlets invariant.

7.7 Possible connections with quantum biology

The flux tube networks assignable spin ice and spin glass phase in general are in the central role in the TGD based vision about quantum biology [L55, L39] [K14, K13, K7, K4, K11].

7.7.1 TGD view about bio-catalysis

TGD leads to a new view about biocatalysis, which is one of the mysteries of standard biology. The general TGD inspired model for bio-catalysis involves the following elements.

1. Reconnections of U-shaped flux tubes of reactants and catalyst make it possible for them to find each other. Cyclotron resonance for flux tubes of same thickness and therefore having the same Kähler magnetic field and the same cyclotron frequency allows reactants and catalyst is an essential element. Both frequency and energy resonance would occur between systems with the same h_{eff} whereas energy resonance would be possible between systems with different values of h_{eff}. This resonance would be the quintessence of what it is to be alive and all communications between various levels of MB having an onion-like hierarchical structure and also between MBs and ordinary biomatter would take place in this manner.

2. A reduction of h_{eff}, leading to a shortening of the flux tubes and bringing catalyst particles and reactants connected by flux tubes together would be also a natural step of the catalytic process.

3. The energy liberated in the reduction of h_{eff} would be used to kick the reactants over the potential energy wall preventing the reaction.

The spin glass type systems formed by flux tubes would be ideal for realizing bio-catalysis and the TGD based view about living matter indeed relies on hierarchical flux tube networks.

7.7.2 Pollack effect and ZEO

The formation of negatively charged regions in the Pollack effect leads to a similar phenomenon. Pollack effect would be behind formation of cells, DNA etc which are indeed negatively charged. Protons would transform to dark protons as magnetic flux tubes and realize genetic codons as Galois confined states of dark protons forming triplets. Genes would be Galois confined sequences of these triplets. These tubes would be parallel to DNA and chemical realization of the genetic code would be only a secondary one.

The regions called exclusion zones (EZs) self-clean themselves. This is in a sharp conflict with the second law. The explanation is that at MB time has a non-standard arrow and self-cleaning is actually dissipation but in a reversed time direction. What would be remarkable would be the long duration of the classical counterpart of BSFR as a deterministic time evolution leading to the final 3-D state of BSFR.

Quite generally, the self-cleaning property would serve as a signature of systems for which the MB stays for long times in a time reversed state making possible self-organization as time reversed dissipation. Large values of h_{eff} would be involved and the largest candidate in the solar system is $\hbar_{gr}(Sun)$.

One must of course also consider the possibility of the Milky Way blackhole with a mass about $4.6 \times 10^6 M_{Sun}$. This would correspond to the scaling up of dark wormhole throat size given by CP_2 size to the scale of 4.6 cm! The Milky Way with a mass of $10^{12} M_{Sun}$ would give a dark wormhole throat with size about 4.6×10^4 km!

This raises spin-ice type systems to a preferred role. They are indeed ideal for the demands of living systems since the ground state degeneracy makes it possible to represent the state of the external world as the state of the system. Also quantum computation requires large degeneracy of states possibly realized in terms of Galois representations and flux tube spaghettis would provide this degeneracy.

8 TGD view about the generation of turbulence

Hydrodynamical turbulence represents one of the unsolved problems of classical physics and therefore as an excellent test bench for the TGD based vision.

Turbulence is generated in many other systems besides hydrodynamical flow. Exotic systems consisting of quasiparticles of a condensed matter system (supra phases, atomic BECs, exciton-polariton BECs, magnon BECs, etc...) involve generation of vortices as the basic element of turbulence. Turbulence appears also in astrophysical systems such as neutron stars. All this suggests the generation of vortices as a universal mechanism in the generation of turbulence.

The understanding of the generation of turbulence is usually regarded as a problem of classical physics. TGD however predicts quantum coherence in all scales so that this assumption must be challenged. Both the new view about space-time and of classical fields (the notion of magnetic body (MB), the hierarchy of effective Planck constants predicting the possibility of quantum coherence in all scales, and the zero energy ontology (ZEO) predicting time reversal in ordinary ("big") state function reductions (BSFRs) could be involved. Even quantum physics in its recent form would not be enough to understand the generation of turbulence.

8.1 The problems of the existing theories of turbulence

The best starting point is to look for the problems of the existing theories. The many problems of the classical theories of turbulence are described in the article of Chaoqun Liu and Shuhyi Chen [D4] (https://cutt.ly/xWMiMV3). As the authors notice, a single wrong prediction in principle kills theory but the theories of turbulence make numerous wrong predictions. Also a general vision of Liu based on empirical facts is discussed.

The phase transition leading to turbulence involves a generation of vortices.

1. Vortex consists of a core region, where the flow has non-vanishing vorticity $\nabla \times v$ and an outer region, where the rotational flow is gradient flow and characterized by a conserved circulation. The gradient flow outside the core is a special case of a Beltrami flow: there is current conservation besides the existence of a global coordinate along the flow lines.

   Rigid body motion with a constant angular velocity is a reasonable approximation allowing to avoid singularity (infinite rotational velocity at the axis of the vortex).

   There are many vortex anatomies. The ends of hair-pin vortices are attached to the boundary and they tend to move with the flow. Λ vortices deserve their name from their shape. There are also circular vortices.

2. No-slippage boundary condition (velocity vanishes at the boundary) for a flow past a body or other medium forces a transversal gradient of the velocity, which is parallel to the boundary and this generates vorticity $\nabla \times v \neq 0$.

   The flow past a body with an over-critical Reynolds number R leads to a generation of vortices. Vortices are coherent structures and clearly separate units and one cannot superpose them as one can superpose eddes. Hairpin vortices are the simplest vortices (https://cutt.ly/nWMiHrJ). It would seem that Nature tends to avoid too large shears (velocity gradients) implying large dissipation and achieves this by generating vortices.

3. This mechanism can be used to generate vortex rings so that one can study the collisions of vortex rings demonstrating the basically topological dynamics of vortices (see the beautiful video at https://cutt.ly/DWMiK3f). The thesis of Ali Dasouqi [D1] (https://cutt.ly/aWMiXWt) gives an overall view about the formation of gas jets and vortex rings in various situations. In particular, collisions of vortex rings and the formation of vortex rings in the bursting of bubbles are discussed.

4. The proposal of Chaoqun Liu [D4] (https://cutt.ly/kWMiVbj) is that the vorticity near the boundary is transferred to the vorticity of the vortex cores. A separation of the flow from the boundary seems to take place. This allows it to avoid large shears and minimize dissipation.

   The generation of turbulence could be regarded as a self-organization process made possible by the energy feed from the flow and not a dissipative process.

5. Turbulence as the decay of vortices is a dissipative process - in a well-defined sense it looks like a reversal of the self-organization process.

   The proposal of Kolmogorov is that the decay of turbulence involves the decay of vortices to smaller ones. The authors argue that this process has not been observed for a single vortex. Presumably it is meant that a linear vortex tube should split into thinner parallel parallel flux tubes. In principle there is no obvious reason why conservation of circulation would prevent this process but this process is highly non-local and does not look plausible.

   It is however possible that a single vortex reconnects and emits a closed vortex ring. This has been observed in the collisions of two vortex rings. The decay process can also involve the reconnection of two vortices as happens in the collision of two vortex rings. This can lead to the decay of larger vortices to smaller vortices such as vortex rings and eventually to so small vortices that they are below measurement resolution.

8.2 Superflow as a starting point

TGD predicts quantum coherence at MB in arbitrarily long length scales. Hence one can motivate the TGD based model by starting from an observation related to the notion of conserved vorticity and its quantization in superfluid flow.

1. For supra flows the conserved vorticity $\Gamma = \oint v\cdot dl$ as integral over a closed flux line associated around the vortex axis in vorticity free region, is quantized as a multiple of $\hbar/m$, where m is the mass of the particle of flow.

2. A possible quantum interpretation could be in terms of a covariant constancy of the Schrödinger amplitude or of spinor field stating $(p_t - qA_t)\Psi = 0$ along flow lines. Here A_t is a projection of an effective $U(1)$ gauge potential, not necessarily electromagnetic.

   The condition $p = mv_t = qA_t$ effectively, where v_t is well-defined for a generalized Beltrami flow as a classical space-time counterpart of quantum coherence, could hold true as a classical correlate of the covariant constancy condition.

   The velocity projection $v_t = A_t/m$ would be proportional to a component of an effective $U(1)$ gauge potential quite generally along flow lines of Beltrami flows and their 4-D time dependent generalizations applicable to non-stationary flows.

3. $B = dA$ would define an effective $U(1)$ magnetic field and could be assigned to any flow. For a gradient flow, one would have $B = dA = 0$ and B would be non-vanishing only inside the vortex core. By Stokes theorem the circulation $\oint v \cdot dl$ would reduce to a conserved magnetic flux $\int BdA$ over the cross section of the vortex core.

   The quantization of the velocity circulation $\oint p \cdot dl = \oint v \circ dl = n\hbar$ is obtained from flux quantization $exp(iq \oint Adl/\hbar) = exp(i \oint d\Phi) = 1$ required by the existence of proper gauge structure. Apart from a gradient $\nabla\psi$ of a single valued function Φ is a multiple of angular coordinate ϕ changing by $n2\pi$ in 2π rotation.

4. It is important to notice that one cannot have a genuine gauge invariance. The gauge transform $A \rightarrow A+d\phi$ gives a new flow with the same circulation. Therefore the identification of A as a standard model gauge field, say U(1) part of the em field does not make sense in the standard model framework but could be sensible in TGD.

5. In Maxwellian electrodynamics B should have some current j as a source: $\nabla \times B = j$, which gives $D^2A \equiv \nabla^2 A - \nabla(\nabla \cdot A) = j$.

   The simplest assumption is that B is constant inside the core and in the direction of the vortex, and can be therefore generated by a current rotating around the vortex axis at the surface of the core. The current would be parallel to A. Vortex core would act like a current coil. The vector potential is effectively massive at the surface of the core since D^2A is proportional to A: mass is formally infinite due to delta-function singularity. This is analogous to the "massivation" of the electromagnetic field in superconductivity for the vortex core inside which the super-conductivity fails.

6. The situation would be essentially quantum mechanical. If the commutator of covariant derivatives $D_i = p_i - qA_i$ given by $[D_i, D_j] = qJ_{ij} = q(\partial_i A_j - \partial_j A_i)$, is non-vanishing, spinors can be eigenstates of only a linear combination D_i, which acts along the flow lines of the integrable Beltrami flow. The classical condition $v_i = qA_i/m$ makes sense only for these components of velocity and about the other components one cannot say anything unless J vanishes or is degenerate. If J vanishes or is degenerate, one can say that some other components of the velocity vanish. This means genuine quantum hydrodynamics. One could perhaps say that $J = 0$ corresponds to classical hydrodynamics.

8.3 Is velocity field proportional to Kähler gauge potential of M^4, of CP_2 or to the sum of both?

The assumption that velocity field is proportional to Kähler gauge potential implies that it is gradient for the Lagrangian situation prevailing outside the vortex cores.

Cores would have non-vanishing Kähler field and Kähler action. What about the Beltrami property in the vortex core? If the CP_2 projection of the vortex core is 2-D complex surface, $A(CP_2)$ is Beltrami field. For instance, for a projection with is geodesic sphere S^2, the Kähler gauge potential is proportional to $A = cos(\Theta)d\Phi$ in the spherical coordinates and Φ defines the global coordinate along flow lines. $D >$ 2-D deformations spoil the Beltrami property. Similar situation is true for the M^4 projection: when the projection as a string world sheet is deformed to a $D > 2$-dimensional surface, the Beltrami property of $A(M^4)$ is lost.

It took some time to realize that the velocity field, and in the compressible case generally mass current, could be proportional

1. to the Kähler gauge potential $A(M^4)$ of M^4,
2. to the Kähler gauge potentia $A(CP_2)$ of CP_2,
3. or to the sum $A(M^4) + (CP_2)$, which at first looks natural if Kähler covariant constancy along flow lines is the basic condition.

These options lead to dramatically different physical pictures, especially so for incompressible flows.

1. For option 1 *resp.* 2, Beltrami or gradient flow in M^4 *resp.* CP_2 is enough. Furthermore, if the velocity field is proportional to $A(M^4)$, there is no need to assume large h_{eff} implying that Z^0 field is massless below scaled up weak length scale and electroweak symmetry breaking is absent in long scales.
2. For option 3, the assumption that both M^4 and CP_2 projections are at most 2-D is a necessary condition and looks unrealistic. But this is not enough for Beltrami or gradient flow. These conditions alone would give a Kähler gauge potential, which is the sum $A(M^4)+A(CP_2)$ of two contributions $A(M^4) = \Psi_1 d\Phi_1$ and $A(CP_2) = \Psi_2 d\Phi_2$ satisfying the conditions separately.

   Besides this, the gradients $d\Psi_1$ and $d\Psi_2$ must be proportional to each other so that Ψ_1 and Ψ_2 are functionally dependent. This however implies that the space-time surface is actually 3-dimensional: the conditions can hold only for effectively 2-D flows at surfaces.

   For incompressible flow velocity and mass flow are proportional and this leads to the unrealistically strong conditions. For incompressible flow the situation changes. If the mass current is proportional to the sum of Z^0 currents of nucleons and neutrinos with same density guaranteeing local neutralization and having velocities proportional to each other, Beltrami/gradient property is possible. One would obtain essentially neutral Z^0 plasma formed by nucleons and neutrinos.

   A possible objection is that the required density of neutrinos is too large as compared to their estimated average density of 10^{-22} Angstrom^{-3}. However, the average density of nuclei is equivalent to nucleon density of 5×10^{-30} Angstrom^{-3}.

   Could one give up the assumption of incompressibility and require that the flow lines of the mass current are globally defined and the mass flow is proportional to Kähler current containing separately conserved contributions from M^4 and CP_2? The mass flow would vanish if both M^4 and CP_2 contributions are Lagrangian. This leaves only $A(M^4)$ and $A(CP_2)$ options.

How does this relate to dissipation? The first naive guess was that the classical dissipation is present if Beltrami property fails? One must however look at the situation more carefully.

1. It is is Kähler current, not Kähler gauge potential, which is proposed to have the generalized Beltrami property guaranteeing that the Kähler 4-force vanishes so that ordinary Lorentz forces and electric force compensate each other and there is no power consumption.
2. This condition does not require the strong conditions posed on the velocity field and Kähler gauge potential. The two conditions are equivalent only if Kähler gauge potential is proportional to current which would be analogous to the massivation of Kähler field. For instance, Kähler current can be vanishing although Kähler gauge potential is non-vanishing.
3. Whether the dissipative option is realized at all for preferred extremals is not at all clear. Dissipative effects might be solely due to the finite sizes of space-time surfaces, which are proportional to h_{eff}. What is however clear is that the loss of Beltrami property for the velocity field does not imply dissipation.

8.4 Could the velocity field be proportional to Kähler gauge potential of CP_2?

What could be the counterpart of the vector potential A in the TGD framework? It was found that there are 3 options corresponding to the proportionality of the velocity field v to $A(M^4)$, $A(CP_2)$ or $A(M^4) + A(CP_2)$. In this section only the option $A(CP_2)$ is considered.

1. A natural identification of A would be as Kähler gauge potential for CP_2. The symplectic transformations of CP_2 act like $U(1)$ gauge transformations and are isometries of WCW but do not (can not) leave Kähler action invariant since the induced metric changes. One can say that classical gravitation breaks the genuine gauge symmetry but the breaking is very small.

   Note in particular that both induced electromagnetic and Z^0 fields can be non-vanishing even if the Kähler form vanishes.

   At the level of fluid flows this means that addition of global gradient to the velocity field indeed gives a new flow but leaves the topology of the flow invariant. Preferred extremal property however restricts strongly the allowed symplectic transformations: one possibility is that they must act as Galois transformations in the cognitive representation so that the Galois images of the space-time surface would be identical in the measurement resolution defined by the cognitive representation. Note that the zero modes characterized by induced Kähler form and not contributing to Kähler metric of WCW remain invariant.

2. Single space-time sheet is certainly not a realistic approximation for a physical situation, and one has actually many-sheeted space-time. Standard model and general relativity would be obtained as an approximation as one replaces the space-time sheets with a single region of M^4 and identifies standard model gauge potentials with the sum over the induced gauge potentials for the space-time sheets. Same applies to the induced metric. This conforms with the idea that a small test particle of CP_2 size necessarily touches all space-time sheets and experiences the sum of the forces.

   If one assumes that various sheets in the experimental situations considered correspond to the same induced Kähler form J defining a symplectic invariant, i.e. have same values of zero modes, then the sum of the induced Kähler forms is a multiple of Kähler form since the sum of global gradients give no contribution: there would be no destructive interference. Both em and Z^0 gauge fields contain a part proportional to J.

What about the contributions from $SU(2)_L$ and $U(1)_R$ parts of the induced gauge fields to the sum [L2]. For the induced W boson fields the contributions are affected by symplectic transformations and the physics inspired guess is that they sum up to zero. This would conform with the short range of the charged weak fields. Note however that the dark weak scale is proportional to h_{eff} and p-adic length scales longer than weak scale in standard model can be considered, in particular in biological systems [K7].

What about the contributions to induced em and Z^0 fields?

1. Conserved vector current hypothesis is the starting point of the standard model. Induced em field γ is sum of $U(1)$ part proportional to J and part proportional to vectorial isospin generator Σ_{12}. Both contributions must be non-vanishing. Z^0 contributions should sum up to zero (note that Z^0 contains both left-handed and vectorial contributions).

2. Using the formulas of [L2], one can express the neutral part F_{nc} of the induced electroweak gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \qquad (8.1)$$

   $n_+ = 1$ and $n_- = 3$ refer to quark and lepton chiralities: both were assumed to be present in the original view about fermions. If only quarks are fundamental spinors [L22, L34], one must drop the $n+ = 3$ contribution. Leptons as composites of 3 antiquarks however effectively behave like opposite H-chirality.

3. The axial part R_{03}, vectorial part R_{12} and $U(1)$ part are

$$\begin{array}{lll} R_{03} & = & 2(2e^0\wedge e^3+e^1\wedge e^2) \ , \\ R_{12} & = & 2(e^0\wedge e^3+2e^1\wedge e^2) \ , \\ J & = & 2(e^0\wedge e^3+e^1\wedge e^2) \ , \end{array} \quad (8.2)$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc}=\gamma Q_{em}+Z^0(I_L^3-pQ_{em}) \quad p=sin^2(\theta_W) \ . \quad (8.3)$$

4. Here θ_W is Weinberg angle. Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{array}{lll} \gamma & = & 3J-pR_{12} \ , \\ Z^0 & = & 2R_{03} \ . \end{array} \quad (8.4)$$

Note that for $p=sin^2(\theta_W)=0$ one has $\gamma=3J$ and Z^0 has purely left handed coupling.

What condition should one pose on Z^0 and γ magnetic fields at the monopole flux tubes in hydrodynamics?

1. If one assumes that there are practically no parity breaking effects in long length scales as the standard model predicts, $\sum Z^0=0$ looks natural but implies that $\sum_\gamma$ is non-vanishing. Since no em currents are needed to generate the monopole magnetic field this might make sense.
2. $\sum_\gamma=0$ looks however more natural and implies $\sum_{sheets} Z^0\neq 0$. Also now one can argue that this makes sense since no currents carrying Z^0 charges are needed to generate Z^0 magnetic monopole fields. This would imply parity violation, which should be observable for vortices. In biology the chirality selection for the basic biomolecules is assumed to be induced by magnetic flux tubes.

This inspires the question whether ordinary hydrodynamics could be magnetohydrodynamics (MHD) for Z^0 magnetic fields at monopole flux tubes and whether MHD in the usual sense could be HD replacing Z^0 fields with ordinary magnetic fields. This question was also motivated a nice lecture about MHD of Alexander Schekochihin (`https://cutt.ly/RW24bTN`) suggesting that the generation of MHD is very similar to the generation of hydrodynamic turbulence in the TGD picture.

Could the basic difference between HD and MHD be that plasma flow replaces mass flow and Z^0 monopole flux tubes are replaced by electromagnetic monopole flux tubes? One can also consider the possibility that both kinds of flux tubes are present in MHD in the usual sense.

With this question in mind, one can consider the condition for the vanishing of $\sum Z^0$ and $\sum\gamma=0$ at monopole flux tubes. It is important to notice that the induced Kähler form is given by $\sum(J_{M^4}+J_{CP_2})$ and weak fields receive contributions only from CP_2.

1. The condition $\sum Z^0=0$ perhaps relevant to MHD implies

$$\sum_{sheets} 2(2Y+X)=0 \ , \quad Y=e^0\wedge e^3 \ , \quad X=e^1\wedge e^2 \ . \quad (8.5)$$

There is no obvious reason for why this should be the case automatically.

This would give

$$\sum_{sheets} e^1 \wedge e^2 = \sum_{sheets} J_{CP_2} \quad . \tag{8.6}$$

This implies

$$\begin{aligned} &\sum_{sheets} e^1 \wedge e^2 = \sum_{sheets} J_{CP_2} \quad , \\ &\sum_{sheets} R_{12} = \sum_{sheets} 3J_{CP_2} \quad , \\ &\sum_{sheets} \gamma = \sum_{sheets} 3(1-p))J_{CP_2} + 3JM^4 \quad . \end{aligned} \tag{8.7}$$

The vanishing of $\sum J_{CP_2}$ (Lagrangian surface in CP_2) implies $\sum Z^0 = 0$ and $\sum \gamma = 3J_{M^4}$.

2. The condition $\sum \gamma = 0$ perhaps relevant for ordinary hydrodynamics can be treated in a similar manner. One obtains

   This gives

$$\sum_{sheets} 2(2X+Y) = 0 \quad . \tag{8.8}$$

   From this one obtains

$$\begin{aligned} &\sum_{sheets} X = -aY - bJ_{M^4} \quad , \quad a = -\frac{3-p}{3-2p} \quad , \qquad b = -\frac{3}{2(3-2p)} \quad . \\ &J_{CP_2} = 2(cY + dJ_{M^4}) \qquad c = -\frac{2p}{6-4p} - \frac{6}{6-4p} \quad . \end{aligned} \tag{8.9}$$

   From the latter equation one can solve Y in terms of J_{CP_2} but at the limit $p=0$, Y diverges unless one has $J = J_{CP_2} + J_{M^4} = 0$. For $p=0, J=0, \gamma=0$ case, one has

$$Z^0 = 2(-Y - J_{M^4}) = 2(-Y + J_{CP_2}) \quad . \tag{8.10}$$

   If this case corresponds to a Lagrange manifold of CP_2 it also corresponds to Lagrange manifold of M^4. This case might be interesting from the hydrodynamics point of view.

   The $\gamma = 0$ condition quite generally implies parity violation and an interesting question is whether the large parity violation in living matter could be due to the long range classical Z^0 field. Could parity violation be present at MB and become chemically visible via the chiral molecules assignable to the helical monopole flux tubes serving as the templates for the formation of these molecules?

3. One can also argue that the sum vanishes for the part of $R_{03} = 2(2e^0 \wedge e^3 + e^1 \wedge e^2)$ orthogonal to J since it is not a symplectic invariant. The natural inner product is the one in which $e^0 \wedge e^3$ and $e^1 \wedge e^2$ are orthogonal and have norm $1/N = 1/8$ implying $(J,J) = 8/N = 1$. This would give $\sum Z^0 = \sum R_{03} = \sum R_{12} = (3/2)\sum J_{CP_2}$ and $\sum \gamma = 3(1-p/2)\sum J_{CP_2} + 3J_{M^4}$. This would imply parity violation. Could this condition be relevant for MHD?

4. If one poses only the condition $\sum J_{CP_2} = 0$, both $\sum Z^0$ and $\sum \gamma$ are non-vanishing , and one has $\sum \gamma = -p \sum Z^0 + 3J_{M^4}$. Magnetohydrodynamics could correspond to this situation but does $\sum \gamma \neq 0$ make any sense in hydrodynamics?

   Could the value of Weinberg angle in hydrodynamical scales differ from its value in particle physics? For $p=0$ Z^0 would be massless like γ suggesting that electroweak symmetry breaking is absent. For Lagrangian flux tubes $\sum Z^0$ would be non-vanishing and $\sum \gamma$ could vanish as one might expect.

Large value of h_{eff} means scaling up of the weak scale and the proposal has been that in living matter the weak scale can be as large as the cell scale. This would be allowed if one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$. The expectation is that below the scaled-up weak scale weak bosons are massless, electroweak symmetry is not broken, and $p = 0$ holds true.

It must be however emphasized that the identification as v in terms of $A(M^4)$ or $A(M^4) + A(CP_2)$ can be also considered.

8.5 Description in terms of monopole- and non-monopole flux tubes

In a condensed matter system the classical em field and weak fields should vanish in long length scales.

8.5.1 Kähler gauge potential is not associated with gauge invariance

In many-sheeted space-time, the standard model counterpart of em field is in the above model proportional to J so that the space-time surfaces in question should have at most 2-D Lagrangian manifold as CP_2 projection with the property that induced J vanishes. Kähler action would vanish and the space-time surface would be a minimal surface.

What is of central importance, is that $J = 0$ does not imply the vanishing of the induced Kähler gauge potential A. Since one does not have a genuine $U(1)$ gauge invariance, the situations corresponding to different Kähler potentials are physically different and correspond to space-time surfaces related by symplectic transformation and also to different hydrodynamical flows. Not all symplectic transformations are possible since symplectic transformations are not volume preserving.

8.5.2 Kähler magnetic structure of the vortices

Outside the core regions, A would be a gradient field but inside the core region J would be non-vanishing. The notion of many-sheeted space-time suggests a description in terms of two kinds of cosmic strings and their deformations giving rise to flux tubes is highly suggestive. Both cosmic strings are of the form $X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is a minimal surface. M^4 projection is 2-D but for the flux tubes as deformations it becomes at least 3-dimensional.

1. For the first option Y^2 is a complex submanifold of CP_2 and the cosmic string carries a monopole flux (see glossary at A.2 and **Fig. ??**) . Homologically non-trivial geodesic sphere represents the simplest example. Monopole flux tubes distinguish TGD from Maxwell's theory and for instance explain why the magnetic field of Earth has not disappeared long time ago and how magnetic fields in cosmic scales are possible. They play a crucial role in TGD inspired quantum biology as carriers of dark matter as $h_{eff} = nh_0$ phases controlling ordinary biomatter.

2. For the second option Y^2 is a Lagrangian manifold of CP_2 with a vanishing Kähler form. The simplest example corresponds to a homologically trivial geodesic sphere.

One can assign to MB consisting of monopole flux tubes the role of external controlling field H, which can induce magnetization M assignable to the controlled magnetic flux tubes of non-monopole type so that one has at the standard model limit $B = H + M$. Monopole flux tubes could have a similar role in condensed matter physics.

The core of the vortex would be associated with a monopole flux tube and the exterior of the core would be associated with the non-monopole flux tube. The monopole flux tube needs no current to generate its magnetic field. The cross section is a closed 2-surface rather than a 2-surface with a boundary (say disk).

The current at the surface of the vortex core creating the magnetic field B inside the core in Maxwellian framework would be replaced with a non-trivial topology of 3-space. If monopole flux tubes with larger h_{eff} control the space-time sheets carrying ordinary matter, the latter space-time sheets could contain a current creating magnetic field with non-mopole flux.

8.5.3 Magnus force as a direct evidence for the classical Z^0 force or for M^4 Kähler force?

Magnus force (https://cutt.ly/MEGn3TQ) means that a spinning object moving in fluid suffers a force, which tends to lift in a direction orthogonal to the spin axis and the direction of motion. Boomerang effect is the most dramatic example of Magnus effect and the effect is utilized in various ball games.

One manner to intuitively understand the Magnus force is in terms of friction at the surface of the spinning object. The drag of the liquid implies that the velocities of the liquid at the opposite sides of the spinning object differ and the conservation of the energy density $p + \rho v^2/2$ along the flow lines of the fluid flow, causes a pressure difference inducing the force. Actually, Magnus force is the sum of several effects and even its sign can change.

Here an example of the Magnus force known as a Kutta-Joukowski lift is considered. The idealized situation involves a long cylinder spinning in the liquid. The lift involves also the generation of a turbulent wake which also contributes to the effect. This situation could also apply to linear vortices.

The force per length of the cylinder is

$$\frac{F}{L} = \rho v \Gamma \quad , \quad \Gamma = \oint v \cdot dl = \int (\nabla \times v) \cdot dA \ . \tag{8.11}$$

Here ρ and v are the density and velocity of the liquid at the cylindrical surface containing the cylindrical object.

The form of the expression brings in mind the Z^0 Lorentz force with Z^0 force proportional to Kähler force affecting vortex cores in hydrodynamics as Z^0 MHD.

The second option is that $A(M^4)$ or $A(M^4) + A(CP_2)$ gives rise to the Magnus force. Since the M^4 Kähler charges of leptons and quarks are opposite if leptons are composites of 3 antiquarks, the total charge density could vanish, and one would have a neutral plasma like state and the analog of MHD would describe hydrodynamics.

1. Z^0 option

In the following, only the Z^0 option is considered in detail since the discussion is similar for the M^4 case.

1. The first thing to notice is that the density is that of the fluid. This suggests that one must look at the situation using linear superposition property and regard the lack of the fluid inside the spinning cylinder as the effective presence of fluid with Z^0 current compensating that of the fluid. Z^0 current would reduce to Kähler current at the QFT limit.

   The Z^0 and Kähler charge densities would be opposite for nuclei and neutrinos but flow velocities would be of opposite sign. to that of the fluid and having inertial mass density of the object. The spinning object effectively would correspond to a fluid with a Z^0 charge density opposite to that of the fluid.

   Remark: One cannot exclude the possibility that also the spinning object carries Z^0 current. This would give rise to a force which would depend on the mass of the object since nuclear Z^0 charge is proportional to mass.

2. Suppose that the liquid particles have Z^0 charges of the same sign and average charge q_z so that the Z^0 charge density ρ_Z is given by $\rho_Z = (q_Z/m)\rho$. This assumption can be challenged. At which length scale do dark neutrinos neutralize the nuclear Z^0 charges and is also the nuclear Z^0 charge dark?

3. Suppose that the assumption $v = q_Z A_Z/m$ inspired by super-fluidity holds true at the MB. This implies that the vorticity is given by $\nabla \times v = q_Z B_Z/m$. This gives $\Gamma = (m/q_Z)\Phi_Z = (m/q_Z)\oint A_Z \cdot dl$. On the other hand, the Z^0 Lorentz force per unit length is $F/L = q_Z \rho_Z v \times B_Z dA = \int \rho v \times (\nabla \times v) dA$. Since v can be taken spatially constant inside the cylinder, one obtains $F/L = \rho v \Gamma$ by Stokes theorem.

If the dynamics of Z^0 fields controls fluid dynamics this picture can be generalized by allowing also Z^0 electric fields. The Z^0 charge densities and Z^0 currents of neutrinos and nuclei cancel each other, they move with the same velocity and one has a neutral Z^0plasma, and HD reduces Z^0 MHD.

For the Z^0 option, the appearance of the density of the fluid in the Magnus force has highly non-trivial implications since it means that *all* nucleons in the liquid flow are effectively dark with large value of h_{eff}, not only those, which reside at magnetic flux tubes. This might well kill this option where as the options in which $A(M^4)$ is involved, survive.

1. At the fundamental level, darkness must reduce to a property of weak bosons propagating along magnetic flux tubes. If magnetic flux tubes are dark also the particles, which touch them are dark. Already earlier it has been concluded that the coupling of ordinary matter to dark gravitational flux tubes by touching makes them effectively dark. For instance, in the case of fountain effect of superfluidity [K24] [L49], this seems to be the only possible interpretation: only superfluid particles touch to dark gravitational flux tubes: it is misleading to say that they are at magnetic flux tubes.

2. Darkness implies that the weak scale is scaled up by $\hbar_{eff}$. What does this mean from the point of view of particle masses? Weak bosons are effectively massless below their dark Compton scale, which for $\hbar_{gr}$ associated with M_E and $\beta_0 = .9$ would be $\Lambda_{gr} \simeq .9$ mm.

   In the standard model framework, this would imply that the Higgs mechanism is realized only in length scales longer than the dark weak scale so that below weak scale quarks would be massless if the Higgs mechanism determines the masses.

   This would not have a considerable effect on the nucleon masses since the contribution of quarks to their masses is only few per cent. In the TGD framework most of the nucleon mass comes from the mass of color magnetic flux tubes. Neutron and proton masses would be identical below the dark weak scale.

3. However, the prediction that electron mass vanishes below say Λ_{gr} looks unrealistic. The situation is saved by the fact that in the TGD framework Higgs mechanism does not determine masses of elementary fermions. Rather, p-adic mass calculations [K8, K2] based on p-adic thermodynamics predict them and weak interactions have nothing to do with the massivation of elementary fermions. Higgs vacuum expectation does not cause massivation but the gradient couplings of Higgs to fermions are naturally proportional to the fermion masses.

4. A further objection against the Z^0 option is following. If ordinary nuclei are dark in hydrodynamical flow, one can wonder what distinguishes between hydrodynamical and super-fluid flows. For instance, why has the fountain effect not been observed? For M^4 and M^4 plus CP_2 options macroscopic quantum coherence is not required but is possible and would explain super-fluid flow and be due to $h_{eff} = h_{gr}$.

2. $A(M^4)$ and $A(M^4) + A(CP_2)$ options

The discussion of the $A(M^4)$ and $A(M^4) + A(CP_2)$ options proceeds along similar lines. Now however large values of h_{eff} would not be necessary and their presence for a super-fluid flow would distinguish it from the ordinary fluid flow.

M^4 contribution to the Kähler charge would replace the Z^0 charge. In this case, nuclei and leptons would screen each other's Kähler charges and in liquid flow their velocities would have opposite directions but magnitudes could be different.

8.5.4 Quantum hydrodynamics is in question

For Lagrangian manifolds associated with non-monopole flux tubes the operators $D_i = p_i - qA_i$ commute and momentum components as eigenvalues determined by $(p_i - qA_i)\psi = 0$ are well-defined so that the interpretation as a classical limit makes sense. The irony is that in this case the value of h_{eff} would be large.

For monopole flux tubes, the Kähler form $J(CP_2)$ is non-trivial. The degeneracies of J determine how many components of v are well-defined.

Besides CP_2 Kähler form also the Kähler form of M^4, strongly suggested by the twistor lift, contributes. The notion of Kähler structure must be modified so that one has a slicing of M^4 by surfaces Y^2 and X^2 such that a given Y^2 with Minkowskian signature intersecting X^2 at point x is orthogonal to X^2 and vice versa.

Y^2 has a hypercomplex structure with an imaginary unit e satisfying $e^2 = 1$ rather than $i^2 = -1$. The square of $J(X^2) + J(Y^2)$ is naturally equal to $g(Y^2) - g(X^2)$. This gives a positive contribution to energy. The Kähler gauge potential contributing to the total Kähler gauge potential is real. The condition would $J^2 = -g$ would force imaginary Kähler gauge potential for Y^2 and make the contribution to energy negative.

Cosmic strings are not realistic models for hydrodynamics but their M^4 deformations could be so since the string tension of the flux tube having interpretation as a length scale dependent cosmological constant depends on the p-adic scale and approaches to zero in long scales. This gives motivation for looking more closely at the situation for cosmic strings $X^4 = X^2 \times S^2 \subset M^4 \times CP_2$. Assume Hamilton-Jacobi structure in M^4 defining an M^4 Kähler form.

1. For a general stationary cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, the covariant derivatives $D_i = p_i - qA_i$ do not commute in Y^2 and X^2 unless X^2 or Y^2 or both are Lagrangian submanifolds. There are 4 basic cases depending on whether X^2 (Y^2) is Lagrangian (L) or non-Lagrangian (n-L). These correspond to pairs (L,L), (n-L,L)(L,n-L),(L,L). In these situations the number of well-defined velocity components is 1+1=2, 2+1=3, 1+2=3, and 2+2.

   For instance, if $X^2 \times Y^2 \subset M^4 \times CP_2$ is a product of Lagrangian 2-surfaces for a given Hamilton-Jacobi structure, the action reduces to a volume term and there is maximum number 4 of well-defined velocity components.

   Only the component D_ϕ along the flow line can be diagonalized for non-Lagrangian $Y^2 \subset CP_2$ and the classical velocity $v_\phi = A_\phi/m$ along the flow line is well-defined. In the n-L situation in $X^2 \subset M^4$ only a single velocity component in $X^2 \subset M^4$ is well-defined and can correspond to a time-like or space-like direction.

   Harmonic oscillator with well-defined energy, momentum component in z-direction and angular momentum L_z would be a good analog for $(n-L, L)$ and $(L, n-L)$ situations. For L, n_L this would correspond to a helical hydrodynamic flow associated with the vortex core with non-vanishing v_z and v_ϕ. About the radial component v_ρ one cannot say anything.

2. The standard MHD picture is that the velocity for a vortex flow is proportional to the magnetic field due to the freezing of the charged particles to the magnetic field lines. This assumption is an idealization since already classically charged particles move along cyclotron orbits along flux lines. This conforms with the above result that the motion in the general case is helical. For cyclotron states this situation corresponds to non-vanishing momentum component p_z and non-vanishing angular momentum component J_z.

For the M^4 deformations of both Lagrangian and cosmic strings to M^4, one expects that the number of well-defined velocity components decreases to the minimal one 1+1=2 corresponding to energy and rotational velocity.

8.6 The TGD view about the flow near boundaries and the generation of turbulence and its decay

The proposal implies a new view about the hydrodynamical flow near boundaries and about the generation of turbulence and its decay.

8.6.1 The flow near boundaries

Consider first a TGD based model for the flow.

1. Outside the cores of vortices and in regions far away from boundaries, dissipation is absent and the flow is gradient flow. The TGD would be in terms of space-time surfaces with vanishing Kähler fields assignable to Lagrangian non-monopole flux tubes. At QFT limit electroweak fields would vanish if the above model is accepted.

2. The absence of dissipation suggests a macroscopic quantum coherence at Lagrangian space-time sheets so that one would have $h_{eff} > h$ at the MB of this region. Superfluid model suggests that the vector potential A is associated with the space-time sheet at which the dark variants of particles with $h_{eff} > h$ reside. Quantization of circulation would be in multiples of $\hbar_{eff} = n\hbar_0$.

   This conforms with the TGD based model for the generation of galactic jets [L48] in which the magnetic fields around galactic blackhole like object are relatively weak but correspond to $h_{eff} = \hbar_{gr} = GMm/v_0$ so that one has quantum coherence in the scale given by gravitational Compton length $\Lambda_{gr} = GM/\beta_0 = r_s/2\beta_0$, $\beta_0 = v_0/c$ which has no dependence on mass m and is in general larger than Scwartschild radius r_s. Λ_{gr} for Earth appears in the TGD based model for superconductivity [L35].

3. What about the monopole flux tube associated with the vortex core? In the model of galactic jets, it would have a considerably smaller value of h_{eff}, perhaps $h_{eff} = h$ [L48]. This assumption would conform with the fact that the flow would be ordinary dissipative flow in this region.

   Remark: One can also consider a fractal hierarchy in which one has at every level a non-dissipative flow apart from vortices. There would be vortices inside vortices inside..., and at the lowest level one would have monopole flux tubes.

4. Near the boundaries one must somehow describe the transversal gradient of the longitudinal velocity field. The natural idea is that small vortices below measurement resolution are present already below the critical value of the Reynolds number R ($R = ud/\nu$) so that the shear would be concentrated in vortex cores.

   Consider two nearby flow lines with slightly different velocities. One can go to a rest system so that the velocities are opposite and replace this pair with a long flattened velocity vortex analogous to a long dipole: A would have as its source B just like B has as its source current j. The vortex core would be now a thin line parallel to the flow. One can replace this structure with a sequence of small vortices just as one can replace a long dipole with a sequence of small dipoles and put them in motion. These vortices could be below the measurement resolution, say having radii in the micron range.

   The flow near boundaries would already contain vortices but they would in general be below the measurement resolution.

8.6.2 The generation of turbulence and its decay

The transition to turbulence would be essentially a self-organization process made possible by energy feed provided by the flow or by some other energy source.

1. In the transition to turbulence, a phase transition increasing h_{eff} for the non-mopole parts and possibly also for the monopole parts of MBs of already existing vortices would take place. It would increase the corresponding parts of flux tubes and make the vortices visible.

   The energy of the flow would not be dissipated but would be used as "metabolic energy" for self-organization. The critical Reynolds number could be due to the condition that circulation is quantized for the vortices as multiples of h_{eff}/m, m the mass of the particle of the flow. Also the formation of bound states of particles by Galois confinement at flux tubes could liberate energy. This would directly relate to the formation of quasiparticles in condensed matter systems.

   Reconnection and braiding would generate complex vortex structures and for high Reynolds numbers the situation would approach chaos.

2. In the hydrodynamic flow in the presence of boundaries the flow would provide the metabolic energy feed whereas in the head-on collision of circular vortices the energy would come from the kinetic energy of the jets. In the burst of a bubble, which scomplex circular vortex ring structures, the metabolic energy would come from the pressure difference between the interior and exterior of the bubble before the creation of the film rupture and from the energy

associated with the string tension. In the case of BECs, laser light can serve as the metabolic energy feed.

3. $h_{eff} > h$ phases at the Lagrangian flux tubes would be generated and this increases the size of the flux tubes. h_{eff} could increase also for the monopole flux tubes implying a larger vortex core. The value of h_{eff} could be however considerably smaller for these flux tubes. Also the reconnection of smaller flux tubes (not plausible with a standard arrow of time) would give rise to larger flux tubes.

Turbulence decays as the metabolic energy feed ceases. How does this take place? The decay of a single linear vortex to parallel vortices has not been observed, which strongly suggests that the dynamics is based on braiding and reconnections leading to the emission of smaller vortices from larger vortices. The eventual outcome would be vortices which are so small that they are below measurement resolution present always near boundaries.

8.6.3 Who is the boss?

Who is the master and who is the slave in the self-organized system?

1. The MB of the entire flow would act as a master controlling the dynamics of the ordinary fluid flow.

2. What about the monopole and non- monopole parts of MB? Who is the master and who is the slave?

   The Lagrangian part of MB as an analog of supra flow could have considerably larger h_{eff}. Could it serve as the master and also control the monopole part of MB?

   However, monopole flux tubes would effectively act as a source of Kähler gauge potential A defining the gradient flow. The dynamics of MB would be essentially topological and involve phenomena like knotting, linking, braiding and reconnection. Could the dynamics of the monopole flux tubes dictate the dynamics of the non-monopole parts just like the moving sources define the non-radiative parts of fields in electrodynamics? Could the monopole part of MB serve as the master for the topological aspects of the flow as the analogy of monopole flux tubes with external field H suggests?

8.6.4 What about the role of time reversals?

What about the role of time reversals? ZEO [K38] [L21] together with the h_{eff} hierarchy predicts that both "small" and "big" (ordinary) SFRs (SSFRs and BSFRs) can occur in all scales.

1. BSFR changes the arrow of time and the outsider with an opposite arrow of time sees BSFR as a classical deterministic evolution leading to the final state of BSFR as the experimental findings of Minev et al suggest [L16]. The proposal is that BSFRs appear in all scales and allow us to understand why the world looks classical despite being genuinely quantal.

2. The generation of turbulence looks like self-organization whereas the decay of the turbulent patterns looks like dissipation. The self-organization aspect is usually explained in terms of non-equilibrium thermodynamics and the necessary energy feed is indeed present. In the TGD picture, the energy feed would make possible an increase of h_{eff} at the MB of the system and since MB controls the system, this would lead to the increase of vortex size and reconnection of microscopic vortices could be involved.

3. One can however ask whether time reversals could play a role in the process and even make spontaneous self-organization without energy feed possible. Could the transition to quantum turbulence in some situations involve a BSFR changing the arrow of time at MB, and lead to maximally self-organized configuration? This would be followed by a second BSFR leading to the decay of the turbulence. In this kind of situation, the self-organization would be essentially decay of large vortices to smaller vortices by reconnections but with a reversed arrow of time occurring after the first BSFR.

Inverse cascade, which is described in [D10], is observed in 2-D hydrodynamic systems with energy feed and looks essentially like the inverse process for the decay of vortices. Large scale vortices and steady states of them are generated. Jupiter and soap films represent examples of systems of this kind. Lars Onsager proposed a model based on statistical mechanics of quantized vortices to explain such behavior. The energy feed would lead to a state with a negative temperature. Nuclear spin systems and condensed matter systems can be forced to states with population reversal by manipulating spins or signs of the interparticle interactions. Authors report the first experimental confirmation of Onsager's model of turbulence in 2-D atomic BEC, in which vortex radius is of ofer micrometer to be compared with 1 Angstrom size in Helium superfluid.

To sum up, although the picture described in this section is is applied to hydrodynamics, it is universal. What is assumed is that current defines integrable flow so that one can assign to it an order parameter defined in terms of space-time geometry. Gradient flow is obtained if the current is conserved and in this case Kähler vacuums provide a model for the complement of vortex cores with a vanishing vorticity. In hydrodynamics and superfluidity the flow corresponds to conserved mass current and in super-conductivity em current but can be something else. The flow of matter would be controlled by the monopole part of MB carrying dark matter and the dynamics would be basically topological as far as turbulence is considered.

Also the vortex core flow is non-dissipative classically if both the CP_2 projection and M^4 projection are at most 2-D. One would have string like objects and dissipation could be understood as a deviation from being a string like object. The very early TGD inspired cosmology [K22, K31, K28, K21] could correspond to this phase.

8.7 Some examples of universality

In the following some applications of the universality of the generation of turbulence are proposed.

8.7.1 The reconnection problem of magnetohydrodynamics

As already mentioned magnetohydrodynamics (MHD) and hydrodynamics (MHD) could have very similar structure. The basic difference could be that in HD Z^0 magnetic fields dominate whereas in MHD magnetic fields dominate. If Weinberg angle vanishes in HD, only $\sum Z^0$ would be non-vanishing, and the difference could relate to Weinberg angle suggesting that in MHD the value of h_{eff} for Lagrangian regions of the vortices is considerably smaller.

Reconnection of magnetic field lines is believed to be the main mechanism for the generation of turbulence in MHD. The problem is that the reconnection rate is systematically predicted to be too low by many orders of magnitudes (`https://cutt.ly/GEq5zDD`). For instance, for solar flares the discrepancy is 13-14 orders of magnitude! One proposed cure is the increase of the local resistivity and therefore the emergence of a new much smaller scale.

The dimensional estimate for the dimensionless reconnection rate R_{SW} in 2-D Sweet-Parker model relies on the observation that in the connection of field lines the frozen charge carrier are transferred from portions of initial flux lines to the portions of re-connected flux lines so that one can speak of incoming and outgoing velocities for charges.

The condition in 2-D case is that the component of electric field normal to the plane of reconnection is conserved: $E_y \sim v_{in}B_{in}v_{out}B_{out}$. E_y defines what is called non-normalized reconnection rate. $v_{out} \simeq v_A = B/\sqrt{\rho}$ follows from the condition that upstream kinetic pressure equals the downstream magnetic pressure. The mass conservation gives $vinL = v_{out}\delta$. The ratio $R_{SP} = v_{in}/v_{out} = B_{out}/B_{in}$ is called normalized or dimensionals reconnection rate. The prediction for the non-normalized reconnection rate is

$$R_{SP} \sim \frac{1}{Re_m^{1/2}} \quad ,$$

where the magnetic Reynolds number is given by $Re_m = v_AL/\eta$. $eta = 1/\sigma_0$ is magnetic diffusivity analogous to viscosity, $v_A = B\sqrt{\rho}$ is the Alfven velocity, and L is the scale of the system. What looks strange to me is that the reconnection rate is dimensionless. Is it impossible to deduce a genuine rate if the reconnection takes place for field lines?

R increases as the effective value of L decreases or the conductivity σ_0 decreases, and it has been proposed that the local increase of resistivity could save the situation but it is difficult to imagine this kind of mechanism in standard MHD.

What is the situation in the TGD framework?

1. The hierarchical structure of many-sheeted space-time brings in an entire hierarchy of scales (dark and p-adic ones). This makes possible the transfer of energy from long to short scales before it is dissipated at short scales. This is the intuitive vision originated by Kolmogorov.

2. The reconnection of magnetic field lines is replaced with that for monopole flux tubes (see A.2 and **Fig. ??**) at the vortex cores. In the simplest model, Lagrangian flux tubes associated with the exteriors of the vortex core would have the generalized Beltrami property and have large h_{eff} - perhaps even $h_{eff} = h_{gr}$ - and be therefore quantum coherent and therefore non-dissipative ($\sigma = \infty$ is the approximation often made in MHD). Lagrangian property would imply vanishing induced Kähler field but non-vanishing em field $\sum \gamma = p \sum 4e^1 \wedge e^2$. Kähler gauge potential would be proportional to velocity field.

3. The monopole flux tubes at vortex cores would have h_{eff} not much larger than h and the vortex core would be therefore dissipative, meaning a large resistivity. The scale L for the entire system appearing in Re_m would be replaced with the size scale of the flux tube, say its length or transversal dimension so that the estimate for the reconnection rate R would increase dramatically if one believes in the naive dimensional analysis based estimate of MHD. Clearly, monopole flux tubes represent symmetry breaking: if the Lagrangian phase has $p = 0$, electroweak symmetry breaking would be in question.

4. The Alfven velocity v_A appearing in R is associated with Alfven waves (`https://cutt.ly/fEq5onl`) plays a key role in the energy transfer in MHD. In the TGD framework, Alfven waves would correspond to two kinds of waves for flux tubes. Either the thickness of the flux tube oscillates but preserves the monopole flux or the shape of flux the tube oscillates but preserves its thickness.

   The estimate $\beta = v/c$ for the phase velocity of the Alfven wave using units with $c = 1$ $\mu_0 = \epsilon_0 = 1$ can be expressed in terms of the relative permittivity $\epsilon_r = \epsilon/\epsilon_0$

   $$\begin{aligned} \beta &= \sqrt{1/\epsilon_r} = \frac{1}{\sqrt{1+\rho/B^2}} = \frac{\beta_A}{\sqrt{1+\beta_A^2}} \ , \\ \beta_A &= \sqrt{\frac{B}{\rho}} \ . \end{aligned} \tag{8.12}$$

   The density ρ could correspond to that at the monopole flux tube or with the space-time regions associated with it.

In the TGD framework it is possible to deduce an estimate for the reconnection rate with a correct dimension.

1. Consider monopole flux tubes that are long and restrict the consideration into plane. The flux tubes intersect this plane at points so that effectively one has point-like particles in 2-D space if one neglects the transversal dimension of the flux tubes. Flux tubes are effectively strings and their orbits are string world sheets.

   The moving flux tubes are bound to intersect sooner or later due to a simple topological fact that the dimension of the string world sheets exceeds the dimension of 3-space by one unit. This means that string world sheets have a discrete set of intersection points in the generic case.

2. The estimate for the rate is obtained from the average velocity v for the flux tube motion and from the average distance L between flux tubes.

$$R_{rec} \sim \frac{1}{\tau_{rec}} = \frac{v}{l} \ . \tag{8.13}$$

The average distance l between flux tubes in plane can be obtained from the density n of the intersections of flux tubes with the plane:

$$l = \frac{1}{n^{1/2}} \quad . \tag{8.14}$$

3. The magnetic flux for monopole flux tubes is conserved and quantized as

$$\Phi_{tube} = \oint_{tube} q_K B_K dS = nm\hbar \quad , \quad \frac{\hbar_{eff}}{\hbar} = m \quad . \tag{8.15}$$

   Note that the cross section of the flux tube is a closed surface!

4. The density of the intersections with the plane with area L^2 the estimate

$$n = \frac{N_{tube}}{L^2} \quad . \tag{8.16}$$

5. The number N_{tube} of flux tubes intersecting the plane can be estimated in terms of total magnetic flux as

$$N_{tube} \sim \frac{\Phi_{tot}}{\langle \Phi_{tube} \rangle} \quad . \tag{8.17}$$

6. This would give for R_{rec} the expression

$$lR_{rec} = \frac{1}{\tau_{rec}} = v \times n^{1/2} = v \times \frac{N_{tube}^{1/2}}{L} \sim v \times \sqrt{\frac{\Phi_{tot}}{\Phi_{tube}}} \frac{1}{L} \quad . \tag{8.18}$$

7. One should estimate the value of v. v corresponds either to the center of mass motion of plasma or to the transverse oscillations of flux tubes which can lead to reconnection if the density of flux tubes is high enough.

   Alfwen waves propagate with the Alfven velocity

$$v = v_A = \frac{B_K}{\sqrt{\rho}} \quad . \tag{8.19}$$

   That there would be no dependence on conductivity would conform with the idea that reconnection is a purely topological process of monopole flux tubes rather than that of plasma.

   An analogous result is expected if v corresponds to the cm velocity of the flux tube.

8.7.2 The generation magnetic fields in cosmic length scales

The problem is discussed in the article [D19] of Alexander Schekochihin can be used to summarize basic differences between TGD and standard approach. The problem discussed is the presence of long range magnetic fields in cosmic scales. Maxwellian magnetic fields always require currents to generate them by dynamo effect. In cosmic scales the plasma is however almost collisionless and it is very difficult to understand how magnetic fields could be generated by dynamo mechanism applied in MHD and why they could have such a long range and be preserved. Currents in long ranges are simply missing and if they exist they decay.

The proposal of Schekochihin is that this is possible. The observation is that magnetization M of molecules can be induced already in very weak long range magnetic fields H if such exist.

Assuming the existence of H in cosmic scales, a numerical model providing evidence for the claim is constructed.

What I see as the problem is that such fields H in long scales should not exist if standard cosmology is right! Currents would be random in cosmic scales and long range coherence is lacking.

In the TGD based cosmology the situation is different. Monopole flux tubes carrying magnetic fields analogous to external magnetizing fields H exist already in the primordial cosmology as cosmic strings. Cosmic string world sheets (actually 4-D surfaces) are space-time surfaces with 2-D M^4 projection unstable against thickening of this projection. The thickening of cosmic strings to monopole flux tubes would have produced monopole flux tubes, whose motion induces currents at flux tubes which carry Maxwellian non-monopole magnetic fields analogous to magnetization M requiring the presence of currents. This is a dynamo effect but monopole flux tubes are necessary to generate it by taking the role of H missing from the model of Schekochihin. [This process would have liberated energy transforming to ordinary matter very much like inflaton fields are assumed decay to ordinary matter. The outcome is a solution to the galactic dark matter problem.]

Schekochihin discussed in his lecture (`https://cutt.ly/RW24bTN`) the conjecture that hydrodynamic turbulence in dense plasma could lead to an exponential amplification of magnetic fields (analogous to M) near to the equipartition of energy between kinetic and magnetic degrees of freedom: this equipartition has been observed but is not understood.

In the TGD framework the transfer of energy in plasma turbulence would be due to the generation of vortices, whose cores are accompanied by monopole magnetic flux tubes (H), vortex exteriors can carry ordinary magnetic fields (M) although Kähler gauge field vanishes. They can decay by reconnections to smaller vortices but it would seem that there is lower bound for the vortex size due to the conservation of monopole flux and this would correspond to equipartition of magnetic and kinetic energies in thermal equilibrium [Even nuclei, hadrons and elementary particles would correspond to this kind of flux tubes: flux tubes inside flux tubes inside...].

8.7.3 Bursting bubbles associated with optical cavities in photonic crystals generating jet vortex rings

One can take as an example the bursting bubbles associated with optical cavities in photonic crystals generating jet vortex rings. I am not a specialist so the first challenge is to to understand the above sentence.

1. Photonic crystal (PC) means a periodic structure with a lattice constant, which is half of the wavelength of light in micrometers scale. Photons in this crystal behave like electrons in a lattice. The lattice constant is roughly 10^4 larger than for atomic lattices.

2. Optical cavities (OP) are of size of order 100-1500 nm. Standing waves coupling to plasmons are formed inside the cavity, which leads to amplification of a laser beam. One can speak of a laser without population inversion. The modes inside the cavity are polaritons, which are mixtures of photons and plasmons. They form polariton BEC which can be described by an analog of hydrodynamics.

3. BEC can be regarded as an analog of liquid, it can contain bubbles presumably plasma ions. These bubbles can end up to the boundary of the optical cavity as analogs of soap bubbles and burst. The polariton BEC would form the analog of liquid film bounding a bubble containing plasma.

4. The burst of a bubble would mean generation of a hole at the bubble boundary so that the plasma would burst out. A vortex ring of BEC would be formed around the hole as it is thrown out as a jet. Pressure difference and surface tension for ordinary bubbles would have counterparts. Jet vortex ring would consist of a polariton BEC as an analog of liquid.

If the general vision is correct, an analog of MHD would describe the dynamics of the vortex ring jet. The monopole flux tubes carrying ordinary magnetic fields would define the cores of the BEC vortices.

8.7.4 Generation of vortices in the collision of two circular vortices

It is interesting to see whether the proposed picture allows us to understand a head-one collision of two circular vortices. The article of Chen et al [D16] discusses numerical simulations of the head-on collisions of circular vortex rings of opposite circulations. The article contains illustrations giving a good idea about the time evolution in the collision creating extremely beautiful flow patterns (see **Figs. 17**) and **18**).

In the head-on collision the circular vortex rings with opposite circulations separate from the rest of the fluid, which remains on the collision site, and their radii start to increase. The flux tubes almost reconnect and eventually reconnection inducing splitting to small vortex rings takes place.

In the TGD based model the vortex cores would accompany Kähler magnetic monopole fluxe tubes, which start to increase in size. Liquid flows fuse but flux tubes would stay separate. Eventually they annihilate to smaller monopole flux rings by reconnection. This gives rise to vortex ringlets. **Fig. 17**) illustrates the complexity of the resulting patterns. **Fig. 18**) illustrates a real collision of flux tubes.

The challenge is to see whether the formation of local flux loop extrusions associated with wavy motions of flux tubes preserving topology, and braiding and reconnections of the monopole flux tubes could explain the patterns. Reconnection for a single flux tube can produce a closed flux tube and emission of a closed vortex ringlet. Reconnection between *antiparallel* flux tubes produces two U-shaped flux tubes. Reconnection between *parallel* flux tubes 1 and 2 can produce elementary braiding AC +BD $\rightarrow$ AD +BC. Two reconnections produce a braiding consisting of two subsequent elementary permutations. After thar a reconnection for flux tube 1 (2) can yield a vortex ring around V2 (V1). This is possible also for opposite flux directions if the second flux tube develops a local fold.

The pairs of spikes or "teeth" (Λ vortices) (see sub-figures b) and c) of **Fig. 17**) look strange and it is not obvious how to understand them in the TGD framework. If there is a circular flow around the tooth axis with a non-vanishing circulation and if it corresponds to a monopole flux tube, the monopole flux tube must continue beyond the tip of the tooth. The vortex could disappear because there is no liquid, or could become invisible because the amount of liquid is too small. The members of the tooth pair would be naturally associated with the same flux loop and have opposite circulations and their behaviors should be strongly correlated. This interpretation is supported by the fact that when the Reynolds number is increased, tooth pairs are replaced by vortex loops (sub-figure d) of **Fig. 17**).

8.8 Breaking of the circulation theorem of Kelvin

This section was motivated by the article of Tobias et al [D6] about non-conservation of hydrodynamics circulation for 2-D flows caused by the presence of even weak magnetic fields. The following is just an attempt to interpret the findings described in the article.

8.8.1 Background

It is good to start with the abstract of [D6].

> *In this paper we examine the role of weak magnetic fields in breaking Kelvin's circulation theorem and in vortex breakup in two-dimensional magnetohydrodynamics for the physically important case of a low magnetic Prandtl number (low Pr_m) fluid. We consider three canonical inviscid solutions for the purely hydrodynamical problem, namely a Gaussian vortex, a circular vortex patch and an elliptical vortex patch.*
>
> *We examine how magnetic fields lead to an initial loss of circulation and attempt to derive scaling laws for the loss of circulation as a function of field strength and diffusion as measured by two non-dimensional parameters.*
>
> *We show that for all cases the loss of circulation depends on the integrated effects of the Lorentz force, with the patch cases leading to significantly greater circulation loss. For the case of the elliptical vortex the loss of circulation depends on the total area swept out by the rotating vortex and so this leads to more efficient circulation loss than for a circular vortex.*

For a 2-D incompressible flow, the velocity can be expressed either as a gradient of a scalar function or a rotor of a vector potential in z-direction and thus determined by a scalar function known as stream function. The two scalar functions correspond to real and imaginary parts of an analytic function. The presence of the Lorentz force destroys incompressibility and one loses the conservation of circulation since the velocity field for the vortices is not a gradient anymore. Symmetry breaking as loss of conformal invariance is in question.

The article describes situations in which a stably stratified and hence effectively 2-D flow can lead to a generation of long range correlation and large scale flows. Conservation laws and so called inversion procedure, which I interpret as a generation of large scale vortices from smaller ones than vice versa, is believed to be the reason for this.

Small magnetic field can however inhibit the generation of large scale flows. Magnetic fields can also inhibit shear flow instabilities and lead to a disruption of coherent structures such as vortices. Magnetic fields can also turn the direction of spectral transfer of 2-D turbulence: inverse cascades turn to forward cascades. Magnetic fields seem to be an enemy of the HD turbulence. Why?

8.8.2 TGD view about dissipation and loss of circulation

In the TGD framework, dissipation would mean the reduction of the values of h_{eff} for MBs of vortices: $h_{eff} = nh_0$ as a unit for the quantization of monopole flux is effectively reduced. This could mean several things.

Before continuing one must make clear that one must distinguish between the space-time sheet and the "fundamental region" of the Galois group. There are m sheets corresponding to the "roots" of an irreducible polynomial of order m. The Galois group with $n = h_{eff}/h_0$ elements gives rise to n fundamental regions and their number equals to m for cyclic extensions only. If the Galois group is a permutation group of m objects, its order $m!$ and much larger than the order m of the polynomial.

n is in general not equal to m and corresponds to the order of the Galois group and the order of extension of rationals is expected to decrease. This changes the dimension of algebraic extension of rationals and is expected to lead to both dissipation, the reduction of quantum coherence length scale and of the size of the vortex, and a genuine loss of circulation.

1. Quantum jumps transforming an irreducible polynomial to a reducible polynomial

Irreducible polynomials define connected space-time surfaces formed by m "roots". As the polynomial becomes reducible, say a product of two polynomials, it defines 2 space-time regions with a discrete set of intersection points citebtartGaloisTGD. This is what typically happens in particle reactions and also in SFR so that the processes might relate to each other.

If the WCW quantum state is a superposition of space-time surfaces associated with polynomials of the same degree with rational parameters it can occur that for some parameter values the irreducibility is lost [L33]. An SFR performing localization to these values of parameters would correspond to the decay of the space-time surfaces.

This suggests the following scenario.

1. m as the degree of polynomial is identifiable as the number of space-time sheets and is different from $n = h_{eff}/h_0$. m can correspond to number sheets as a covering of M^4 and also as a covering of CP_2. The latter case corresponds to a bundle of flux tubes and the number of flux tubes can be very large. Both cases can appear simultaneously in which case m is expected to factorize as $m = m(M^4) \times m(CP_2)$.

2. For M^4 coverings, dissipation could correspond to a decay in which the polynomial for critical values of parameters decomposes to a product of polynomials of degrees m_1 and m_2 and vortex decays to vortices with m_1 and m_2 sheets. These structures then leave each other and form separate vortices.

3. In the M^8 picture, in which space-time region corresponds to a "root" of a polynomial, this could mean that the m_2 roots of the polynomial defining the vortex region coincide. The simplest case, perhaps the only realistic situation, corresponds to a co-incidence of $m_2 = 2$ roots so that the polynomial of order m reduces to a product of a second order polynomial and a polynomial of order $m - 2$. The second order polynomial with rational coefficients

would correspond to a single root disjoint from $m-2$ roots. The vortex with $m_2 = 2$ should be small. The interpretation as a reconnection is highly suggestive.

For CP_2 coverings the flux tube bundle decomposes to flux tube bundles consisting of m_1 and m_2 flux tubes.

4. The orders n_1 and n_2 of Galois groups are expected to be smaller than n so that the vortex sizes would be scaled down. Circulation as magnetic flux proportional to $n\hbar_0$ is not expected to be conserved.

2. Cognitive measurement cascade

One can consider the situation also from the point of view of the Galois group with order $n = h_{eff}/h_0$. Dissipation would correspond to the reduction of n.

1. What I call cognitive measurement cascades [L32, L33] occur for extensions of extensions... of rationals Q reprentable as $Q \rightarrow E_1.... \rightarrow E_n$ would mean a stepwise sequence of symmetry breakings in which the representation of Galois group G_n of E_n would first reduce to the product of Galois groups G_n/G_{n-1} for E_n as extension of E_{n-1} and G_{n-1} of E_{n-1} as extension of Q, and the process continues in the similar manner downwards [L33].

2. A given step process would have as a space-time counterpart decay of flux tube to two flux tubes. Various factor groups G_k/G_{k-1} could act in extension of rationals. Only simple Galois groups such as alternating groups A_n would be stable against this process.

   One cannot exclude the possibility that the polynomial decomposes into a product of polynomials and the outcome is two separate space-time surfaces. Also the interpretation in terms of reconnection might make sense.

3. The dimensions n_i of factor groups would be factors of n and one would have $n = \prod n_i$. In the final state the total flux would be equal to $n = \sum n_i$ if the number of flux units is 1 is the initial and final states. Hence the magnetic flux would not be conserved and this could correspond to the non-conservation of circulation. Dissipation would be in question as is clear also from the fact that state function reductions occur. These reductions could be SSFRs.

4. The dissipative period following the generation of turbulence could correspond to this phase and involve genuine loss of information and complexity at the level of a single flux tube. The decay by reconnections could correspond to this process. If BSFR corresponds to an intuitive heureka moment, the sequence of SSFRs would correspond to an analysis period realized quite literally as a decay of vortices.

5. During the generation of turbulence the complexity would increase and time reversal of this process seems to be in question. TGD suggests a genuine time reversal.

8.8.3 A concrete model in terms of flux tubes

Suppose that one takes seriously the model for the flux tubes assigned to the vortices.

1. The Lagrangian non-monopole flux tube associated with the exterior of vortex core would have vanishing Kähler field J. By a generalization of the basic quantization conditions for superfluidity one would have a gradient flow with velocity $v = A/m$, where $A = d\Phi$ is the Kähler gauge potential (note that one does not have genuine gauge invariance). The value of h_{eff} would be large and there would be no dissipation. There would be a macroscopic quantum coherence at the magnetic flux tube in the exterior of the vortex and Beltrami flow or even gradient flow would serve as its space-time correlate.

2. The earlier considerations suggest that electroweak symmetry breaking is absent inside the Lagrangian region in the case of HD vortices and possibly also MHD vortices.

   The reason is that in the Lagrangian region weak bosons or at least Z^0 should behave like a massless boson since the Z^0 field at QFT limit defined as $\sum_{sheets} Z^0$ is non-vanishing and proportional to the sum of $\sum J_{CP_2}$, which is symplectic invariant. The absence of electroweak symmetry breaking below the size scale of the vortex suggests that the Weinberg angle vanishes: $p = sin^2(\theta_W) = 0$. If so, the electromagnetic field is proportional to $J = J_{M^4} + J_{CP_2} = J_{M^4}$ and vanishes if also the M^4 projection of the flux tube is Lagrangian.

3. What about the vortices of MHD? According to [D6], the size of vortices in astrophysical scales is typically considerably larger than that of HD vortices. The same would hold true also for h_{eff}. $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ is suggestive and mass M would be much larger in astrophysical scales: note that gravitational Compton length for particle with mass m is $\Lambda_{gr} = GM/v_0$ [L50, L49].

   Also now $p = 0$ would hold true in the Lagrangian region whereas $p > 0$ would be satisfied inside the vortex core in both cases. In MHD, the classical em field $\sum gamma$ would be non-vanishing both inside and outside the vortex core. This is the case if the M^4 projection of flux tubes is *not* a Lagrangian manifold anymore. Could the distinction between MHD and HD vortices be this?

4. The dissipation for $Re = UD/\nu \leq Re_{cr}$ would occur in HD in smaller scales than in MHD if $\nu/\eta \leq\leq 1$ is true. This suggests that kinematic viscosity ν and magnetic diffusivity $\eta \propto 1/\sigma$ are proportional to h_{eff} in the Lagrangian region.

   ν has dimensions of angular momentum divided by mass so that viscosity has dimensions of angular momentum density. How closely η could relate to the quantity $\hbar_{eff}/m$ serving as a unit of circulation? Could ν and η be proportional to minimal circulation?

5. One should also understand how the generation of the angular momentum of vortices can be consistent with the conservation of angular momentum. Could the angular momenta of dark matter at magnetic flux tube and the angular momentum of the ordinary matter at vortex sum up to zero? The generation of angular momentum of astrophysical objects is an unsolved problem and I have proposed this kind of mechanism as a possible solution to the problem [L36].

8.8.4 What could be the TGD interpretation of inversion

The inversion looks like dissipation meaning a decay of vortices but occurring in a reversed time direction. The most dramatic predictions of TGD based quantum theory is that the arrow of time changes in ordinary state function reductions (SFRs) (I call them "big" SFRs, briefly BSFRs) and that quantum coherence and therefore BSFRs are possible in arbitrary long scales [L21] [K38]. The physics would be apparently classical in long length scales: ZEO BSFRs imply that the physics looks classical for an observer with an arrow of time opposite to the system for which BSFR takes place [L16].

Could the inversion as a generation of larger vortices from smaller vortices, which in the TGD framework should occur in the first stage in the generation of turbulence, be associated with a BSFR in macroscopic scale?

If this interpretation is correct, the introduction of magnetic fields in the hydrodynamic system would induce BSFR and transform inversion back to dissipation. Why should this occur?

Energy feed is needed to increase h_{eff} assignable to vortex MBs. Could it be that electromagnetic and Z^0-magnetic vortices compete over metabolic energy. Could the generation of electromagnetic flux tubes steal the metabolic energy from Z^0-magnetic hydrodynamical flux tubes? If $\nu < \eta$ holds true the formation of magnetic vortices would become possible at smaller length scales and could steal the energy feed.

8.9 Kelvin-Helholtz and Rayleigh-Taylor instabilities

Kelvin-Helmholz instability (K-H) Rayleigh–Taylor instability (R-T) are instabilities of fluid flow.

1. Kelvin-Helmholz instability (K-H) (`https://cutt.ly/TENyKZO`) is caused by shear at boundary of the fluid flow or inside the flow and leads to a generation of vortices. Surface waves in water represent a basic example of K-H. In this case, the perturbation theory fails because the water surface does not anymore allow a description as a graph of a single valued function.

2. Rayleigh–Taylor instability (R-T) (https://cutt.ly/6ENyXzQ) involves two immiscible fluids with different densities. When lighter fluid is pushed against the heavier one, the boundary layer becomes unstable. This pushing can be caused by the gravitational field. This raises the question whether the gravitational Compton length Λ_{gr} could play an essential role in the description of R-T.

   Oil suspended above water in the gravitational field of Earth is one example. The mushroom shaped cloud formed by volcanic eruptions and atmospheric nuclear explosions represents a second example. During the first stage the dynamics is lianer. The second stage of R-T involves a generation of mushroom shaped spikes as heavier fluid forms intrusions inside the lighter one and bubbles as the lighter fluid penetrates inside the heavier fluid. In the third stage, the mushroom shapes interact with each other. Merging of bubbles and spikes to large ones takes place. Also competition takes place as the saturated spikes and bubbles of smaller wavelength are enveloped by larger ones not yet saturated. The dynamics is thus fractal and the process repeats in shorter length scales. The fourth stage corresponds to turbulence and fractality provided that the Reynolds number is large enough.

It has been recently discovered that the fluid equations governing the linear dynamics of the system admit a parity-time symmetry (PT). According to Wikipedia article, K-H occurs when and only when the parity-time symmetry (PT) breaks spontaneously. However, the article about R-T however claims that simultaneous K-H and R-T occur only when PT is spontaneously broken. The intuitive guess is that the failure of PT symmetry must be a general feature for the transition to turbulence. Reynolds number serves as a criterion for the emergence of turbulence caused by K-H.

8.9.1 Complex Hamiltonians with PT symmetry are hermitian

What makes K-H and R-T quantum mechanically so interesting is that the spontaneous breaking of PT symmetry at the level of flow is involved. On the other hand, if PT replaces complex conjugation, complex Hamiltonians can act as Hermitian Hamiltonians.

One can generalize the notion of Hamiltonian (or any Hermitian operator) to that of complex Hamiltonian provided the operator is invariant under PT [B1] (https://cutt.ly/mENpOdq). It turns out that in the TGD framework, one could actually replace PT with CPT transforming the positive and negative energy parts of zero energy states to each other in ZEO. This requires a modification of the inner product so that hermitian conjugation induced by T is replaced with PT involving spatial reflection. The eigenvalues of this operator are real, time evolution is unitary, and states have positive and real norms. A simple example involving addition of term -ix to harmonic oscillator Hamiltonian demonstrates that this is indeed the case.

The addition of the term -ix makes the space complex by the shift $x \rightarrow x - i/2$. This is of special interest in TGD, where one must complexify M^8 and therefore also $M^4 \subset M^8$: there the quark momenta in $X^4 \subset M^8$ correspond to algebraic integers, which can be complex [L46].

1. The restriction to imaginary shifts $x \rightarrow x + iy_0$ of real M^4 coordinates implied by the generalized hermiticity condition allows only imaginary shifts for space-like M^4 coordinates in M^8_c interpreted as momentum space. The reality of the number theoretic norm requires $\sum x \ cdoty_0 = 0$. This selects a 3-D surface of M^4 and reduces M^4 to M^3 for spacelike y_0. This would require an effectively 2-D system.

2. $M^8 - H$ duality would map the momenta to the intersections of geodesic lines with momentum $x + iy_0$ intersecting the opposite boundary of a complexified CD. Quark momenta are algebraic integers in an extension of rationals and can be complex: the real momenta for Galois confined states would belong to M^3.

8.9.2 Spontaneous breaking of PT symmetry in TGD framework

What could PT symmetry and its spontaneous breaking mean in classical TGD having the structure of hydrodynamics (field equations as conservation laws)?

1. Quite generally, CPT symmetry implies PT symmetry in systems in which matter dominates. The theory would be PT invariant and spontaneous PT violation would occur for the solutions

of field equations. Spontaneous violation of PT and even CPT occur in all systems at elementary particle level and large values of h_{eff} could make this possible even in macroscopic scales.

2. If the generalized Beltrami hypothesis is satisfied, the classical dynamics is non-dissipative in each scale. The hypothesis does require PT and C as separate symmetries but in TGD one could loosen this condition by defining the generalized unitary by assuming that hermitian conjugation corresponds to CPT with C realized geometrically as a complex conjugation the level of CP_2.

   C transforms complex structure to its conjugate and changes the sign of the induced Kähler form. This does not seem possible for monopole flux tubes at a given boundary of CD in systems containing only matter. Lagrangian flux tubes do not correspond to complex manifolds and have a vanishing induced Kähler form so that non-trivial action of C could be allowed. The WCW spinor field could be C invariant in this case.

   If the spontaneous breaking of CPT at the level of space-time surface is possible, it would mean CPT non-invariance of individual space-time surfaces with P and T depending on the CD containing given space-time surfaces. T defined with respect to the center point of CD would permute the 3-surfaces at the opposite boundaries of CD.

   The WCW spinor fields as superpositions of pairs of 3-D quantum states at opposite boundaries of CD are not invariant under this transformation: T and therefore also CPT would permute the 3-D states at the opposite boundaries. Bras would be mapped to kets and vice versa.

   At space-time level CPT violation could make itself visible as the change of the sign of Kähler form of CP_2 or of M^4. CPT violation would occur at the Lagrangian regions of vortices with $h_{eff} > h$ and therefore could take place in long scales.

What does the generalized Beltrami hypothesis imply?

1. The spontaneous violation of PT in ordinary hydrodynamics would correspond in TGD to the breaking of unitary evolution by the occurrence of SSFRs and BSFRs. The sole source of dissipation in ZEO would be reduction of h_{eff}. The reduction of h_{eff} would lead to the reduction of quantum coherence scale and flow of energy to shorter scales. Self-organization as the reverse process in presence of energy feed or induced by time reversal at MB induced by BSFR is also possible and the formation of larger vortices could correspond to this process.

2. PT symmetry would mean absence of dissipation and its spontaneous violation as analog of breaking of unitary time evolution via the occurrence of SFRs.

According to the Wikipedia article, a spontaneous breaking of PT occurs in simultaneous K-H and R-T and possibly already in K-H. What would TGD predict?

1. Consider first the spontaneous violation of PT symmetry classically. The generation of Kähler magnetic fields in vortex cores in the presence of spinning particles would induce T violation. The large value of h_{eff} imply large electroweak violation of P in long (say biological) scales (classical Z^0 fields). The exteriors of vortices carrying Z^0 fields would correspond to regions, where h_{eff} is large, perhaps even equal to $\hbar_{gr}$.

   Do these violations of P and T compensate for each other or is a spontaneous violation of PT possible. Or is the PT violation produced in SFRs?

2. Could the interpretation spontaneous violation of PT in the case of simultaneous K-H and R-T be that the generation of vortices by K-H inside the intrusions (spikes and bubbles) formed by T-H as a flow of energy to shorter scales serves as the counterpart for the dissipation as a counterpart for the breaking of PT.

3. Can K-H alone be enough for the spontaneous violation of PT? This would correspond to reconnection of vortices producing smaller vortices. The boundary of vortex and exterior flow would define the boundary region with shear giving rise to a boundary layer and smaller vortices. This suggests that spontaneous PT violation in the TGD sense characterizes both K-H, R-T and their combination.

Remark: PT symmetry is in a key role in the TGD based model for the role of time reversal at the level of DNA [L53].

9 Are the hydrodynamic quantum analogs much more than analogs?

The hydrodynamic quantum analogs are highly interesting from TGD point of view and Wikipedia article gives a nice summary about them (`https://cutt.ly/xEk5Api`). The quantum-like aspects are associated with a hydrodynamical system consisting of a liquid layer and liquid drop. Liquid surface in a periodic accelerated motion due to shaking: this means energy feed. The fluid bath is just below the criticality for a generation of standing Faraday wave and the bouncind particle indeed generates this kind of wave.

Depending on the values of the parameters, the liquid drop is surfing, bouncing at a fixed position, or "walking" along the surface wave. The surface wave is created by the interaction of particle with the surface. These findings suggest that macrosopic quantum coherence could be involved and quantum phenomena have also classical description. There is energy feed to the systems.

The findings of the group led by Bush and describe in his Youtube lecture (`https://cutt.ly/xEk5Api`) give a nice overall view about the quantum analogs. Bush also suggests a generalization of theory of Vigier involving two pilot waves, which correspond to those associated with wave function and to classical system and theory of Bohm involving single pilot wave assigned to wave function.

The article of Bush et al [D7] describes the findings about the analog of quantum corral. The latter involves electrons inside a circular corral defined by negative ions.

> *Bouncing droplets can self-propel laterally along the surface of a vibrated fluid bath by virtue of a resonant interaction with their own wave field . The resulting walking droplets exhibit features reminiscent of microscopic quantum particles. Here we present the results of an experimental investigation of droplets walking in a circular corral. We demonstrate that a coherent wavelike statistical behavior emerges from the complex underlying dynamics and that the probability distribution is prescribed by the Faraday wave mode of the corral. The statistical behavior of the walking droplets is demonstrated to be analogous to that of electrons in quantum corrals.*

The key questions are following.

1. Could quantum classical correspondence (QCC) be more than an approximation (stationary phase approximation). Note that in TGD QCC is in a well-defined sense exact.

2. Can a macroscopic system can exhibit quantal looking behavior and is there a genuine quantum behavior behind it? In the TGD framework, the hierarchy of effective Planck constants $h_{eff} = nh_0$ labelling phases of ordinary matter located at magnetic body (MB). MB has a hierarchical structure and defines a master slave hierarchy.

   A given level of the hierarchy controls the physics at the lower levels. h_{eff} hierarchy makes quantum coherence possible in arbitrarily long scales at MB and this induces coherence at the level of ordinary matter and makes possible self-organization [L19] The increase of h_{eff} requires however the analogy of metabolic energy feed quite generally.

   There is indeed energy feed to the studied system at frequency of $f = 50$ Hz of the vibrating cylindrical shaker. The standing wave resonance occurs at Faraday frequency $f_F = f/2$. The Faraday frequency has slow time variation with the frequency f and slightly below f_F.

   The system system should be near criticality for the generation of h_{eff} phases. These phases at MB would induce long range correlations of ordinary matter near criticality. The system studied is indeed near criticality for the generation of standing Faraday waves.

3. What could the value of h_{eff} be? The Faraday wave length $\lambda_F = 2\pi\sqrt{2\nu/\mu}$ should be equal to the analog of Compton wavelength $\Lambda_c = \hbar_{eff}/m$, m the mass of the water droplet. λ_F

does not however depend on the mass of the droplet and in the model of the Faraday waves hydrodynamical is determined in the model considered by the properties of the fluid that is friction and kinematic viscosity.

The only possibility is that one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $\hbar_{gr}$ is the gravitational Planck constant introduced by Nottale [E1] [L42] and also appearing in the TGD based model of superconductivity [L35]. This would give $\lambda_F = \lambda gr = GM/v_0 = r_s(M)/2v_0$, where $r_s(M)$ is Schartschild radius. M is naturally the mass of Earth. The minimum value of λ_{gr} corresponds to $v_0/c = 1$ and is $\lambda_{gr} = r_s/2$. Earth's Scwartschild radius is 8.7 mm so that one would have $\lambda_F = 4.35$ mm.

The value of Λ_F for the system studied in the analog of quantum corral by Bush et al is 4.75 mm [D7] and about 10 per cent larger than the minimal value suggesting that $\beta_0 = v_0/c \simeq .92$!

If this single testable prediction is not a nasty coincidence, it would mean an instantaneous breakthrough for the TGD view about quantum gravitation as macroscopic and even astrophysical phenomenon. The only parameter that can be varied in the prediction is β_0. One could measure $\lambda_F = 2\pi\sqrt{2\nu/\mu}$ for different liquids to see whether v_0 codes for the properties of the liquid or whether λ_F is independent of the liquid so that the classical model for Faraday waves could be wrong.

4. The system has a memory in the sense that the induced Faraday wave interpreted as an analog of pilot wave is affected by the bouncing particle and in turns determines particle behavior but not quite completely: an analog of non-deterministic "zitterbewegung" seems to be present for strong enough acccelerations. The observations about the double slit experiment and also about approach to chaotic behavior indeed suggests that the system is not completely deterministic. The findings also suggest that the statistical description of this non-determinism is analogous that in quantum systems.

   In ZEO quantum state as time= constant snapshot is replaced with a space-time surface as preferred extremal (PE) analogous to Bohr orbit. What comes in mind, is that the bouncing corresponds to "small" SFRs (SSFRs). The determinism of PEs is not quite exact that it would serve as correlate for what I call cognitive measurements [L52] as SSFRs. In the TGD inspired theory of consciousness, the loci of non-determism for space-time surfaces as analogs of soap filmds would serve as the seats of mental images quite universally and also represent conscious memories.

5. In this talk Bush interprets the Faraday wave induced by the motion of the droplet along the surface as a kind of pilot wave. In the TGD framework the counterpart of the pilot wave would be the magnetic body (MB) carrying $h_{eff} = nh_0$ phases quantum controlling the behavior of ordinary matter. The magnetic flux tubes assignable to the exteriors of vortex cores are proposed to be present in microscopic scale also below turbulence and to serve as correlates for the vorticity caused by the boundary conditions at the boundary of flowing liquid. Now these boundaries correspond to the boundary between air and liquid bath and air and liquid droplet and could explain how the gravitational magnetic body characterized by $\hbar_{gr}$ enters into the physics of the moving water droplet.

The results discussed in the talk of Bush and the article provide a benchmark test for the general picture provided by TGD and allows to sharpen the TGD view about QCC in quantum hydrodynamics (QHD).

9.1 Summary of the experiments

Consider first a brief layman interpretation of the experiments based on the Youtube lecture of Bush (`https://cutt.ly/xEk5Api`). I must apologize for possible mistakes: I am just a layman as far as hydrodynamics is considered. The description of Faraday waves (`https://cutt.ly/vEk6cSi` should be helpful for the reader.

9.1.1 Faraday waves

One considers a cylinder containing fluid and driven by a shaker, which is a piston, whose position varies in an oscillatory manner with some amplitude A and frequency f. At a certain critical frequency known as Faraday frequency $f_F = f/2$ a standing wave is generated at the surface of the water. This phenomenon is known as Faraday instability.

The parametric oscillator for an infinitesimal fraction of liquid surface is given by

$$\begin{array}{l} \frac{d^2x}{dt^2} + 2\mu\frac{dz}{dt} + \Omega^2(t)z = 0 \ , \\ \Omega^2 = \omega_0^2(1+\alpha(t)) \ . \end{array} \quad (9.1)$$

Here z is the vertical position of the fluid element, μ is the damping rate determined by viscosity of liquid, ω_0 and of shaker is the frequency of oscillation of the fluid element in absence of gravitational acceleration and $\alpha(t)$ is dimensionless oscillating parameter function. One could say that a harmonic oscillator with time dependent frequency $\Omega(t)$ under the damping of a viscous force is in question. Shaking defines the driving force feeding energy to the system.

The time dependent gravitational acceleration $g(t)$ is given in the moving frame of the shaker by the expression

$$g(t) = g + A\omega_0^2 cos(\omega t) = g(1 + \Gamma cos(\omega t)) \ . \quad (9.2)$$

Here A is the amplitude of the vibrations of the and $\omega = 2\pi f$ is its angular frequency of the shaker. From this one has $\Gamma = A\omega_0^2/g$. Note that ω_0 is determined by the properties of the fluid such as density and surface tension. The parameters involved are g, fluid density ρ, surface tension σ, and Faraday frequency $f_F \simeq f_0/2$.

For certain combinations of Γ and ω_0 resonance occurs so that the situation is unstable. The parametric resonance occurs for subharmonics $\omega_0 = \omega \simeq 2/n$. The lowest sub-harmonic corresponds to $n = 2$ and is studied in the recent case. In the recenf case one has $f = 50$ Hz.

The instability occurs in the parameter range

At $\Gamma = \Gamma_F = 4g$, a resonance occurs in the driving frequency range

$$\omega_{min} < \omega < \omega_{max}, \quad \Omega_\pm = 2\omega_0 \pm \sqrt{\Gamma\omega_0^2 - 4\mu^2} \ , \quad (9.3)$$

A standing wave formed at the instability and the damping rate and wave number $k = 2\pi/\lambda_F$ are related by $\mu = 2\nu k^2$, here ν is kinematic viscosity. Faraday wave length $\lambda_F = 2\pi\sqrt{2\nu/\mu}$ depends on the viscosity and friction.

Besides the standing wave, also propagating waves are possible. The instability depends sensitively on parameters like the meniscus at the walls of the vessel and the instability of the fluid layer characteristics and of driving frequency and amplitude can lead to spatiotemporal chaos.

9.1.2 Couder walker

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Floating droplets on vibrating bath were first described by by Jearl Walker in Scientific American in 1978. In 2005, bouncing droplets on vibrain bath were studied by Yves Couder and his lab they discovered most quantum analogs (`https://cutt.ly/TEk5XyN`). John Bush and his lab expanded upon Couder's work and studied the system in more detail.

The droplet can float (also surf), bounce at a single position and walk by bouncing. The essential condition is that no coalescence occurs. The air layer between droplet and water surface is believed to prevent the coalescence. Coalescence does not occur if the droplet does not touch the fluid surface: the air layer prevents this. For a bouncing droplet the frequency of bouncing determined by surface acceleration must be high enough in order to prevent the draining of the air from the layer. Bouncing could be seen as a kind of trampoline effect. Essential is that there is an air layer between droplet and water preventing the direct touching leading to coalescence. Since both the fluid and the droplet move with respect to air, there are surface layers involved and if quantum phenomena are involved, they might relate to the surface layers and the interaction. In the TGD framework, this suggests a possible connection with the proposed model of turbulence. For floating without coalescence, the surface layers should fuse to a single connected structure. The existence of some kind of mattress is suggestive. Mini vortices having interpretation in terms of closed flux tubes is a possible TGD based explanation. Even a surface layer giving rise to a mattress can be imagined. Below a critical acceleration, the droplet makes only a few bounces. Above the criticality the bouncing continues. As the acceleration increases the trajectory transforms from a straight line to chaotic trajectory.

The bouncing of the droplet induces the possibly propagating Faraday wave implying that the system has a memory. For high enough surface acceleration but still below the threshold for the formation of standing waves, the span of the memory increases and chaos is approached.

The initiation of a walking motion requires that the droplet meets the surface in a position in which the surface wave has a large enough slope. The bouncing preserves the particle's momentum component parallel to the surface. If a vertically moving particle meets the surface in a position with non-vanishing slope, it preserves the momentum component parallel to the surface.

After that the particle makes an approximately parabolic orbit and if it meets the surface in a position of a slope of the correct sign, the reflection preserves the parallel momentum component.

From the TGD point of view it is important that there is energy feed and the system is near criticality defined by the Faraday threshold. Also walking is possible in a range of surface accelerations.

Since the bouncing affects the surface and the bouncing indeed creates the Faraday wave. The process is non-Markovian and hereditary since the recent state of the surface is determined by previous bounces. The memory traces about bounces decay exponentially but at critical surface acceleration the memory span becomes formally infinite.

In the lecture of Bush the trajectory equations for both bouncing and surfing are given explicitly. Newton's equation involves beside drag force what is called wave force, which is gravitational force proportional to the gradient of height of the surface which for the walking is asum over exponentially decaying heights changes induced by the previous collisions and for surfing an integral. The collisions serve as instantaneous point sources for a Faraday wave. The collisions approach at the surfing limit continual touching and sum is transformed to an integral. The naive expectation would be that the wave force is present only when the drop (or its air layer) touches the water surface but according to the formula it is present all the time. The equation of motion is however written in an inertial frame of accelerating surface which depends on the position at the surface so that the wave force term is analogous to inertial force like centrifugal force.

9.1.3 Hydrodynamic quantum analogs

There is a long list of hydrodynamics quantum analogs and many of them are discussed in the lecture of Bush (`https://cutt.ly/xEk5Api`). Also the Wikipedia article (`https://cutt.ly/TEk5XyN`) discusses them .

1. Single particle diffraction and interference were studied already by Couder and Fort. According to the Wikipedia article, Bohr and Andersen, and the groups led by Bush and Batelan did not see the interference patterns. Bush however mentions the effects and informs that the experiments have been carried out later and the claimed effects have been found.

   The diffraction was studied in both single slit and double slit experiments. In a single slit experiment a 3 peak structure was observed. This differs from Fraunhofer diffraction appearing in a quantum mechanical situation. If the motion of the droplet is deterministic with the measurement resolution for initial state, this pattern should not be observed and random fluctuations in the experimental conditions should not produce this kind of structure. The diffraction angle also tends to favor quantized values.

   In double slit experiment the diffraction pattern is modified due to the presence of the second slit. This can be understood at a qualitative level since the Faraday wave is affected by the presence of the second slit. Bush suggests that the differences from the quantum case are due to the fact that the droplet is not analogous to a plane wave as in the quantal case. Furthermore, the ratio of λ_F analogous to the de-Broglie wavelength $\lambda_{dB} = \hbar/p$- to the width of the slit is much larger than in quantum scale when it is of order 10^3.

2. Droplets for bound states analogous to molecules. Also walking droplets can form pairs. Bond lengths are quantized in terms of λ_F interpreted as analog of λ_{dB}.

3. The analog of quantum tunnelling through a barrier has been observed. The barrier is now a region of smaller depth. The droplet typically reflects from the barrier but sometimes also transmission takes place. There is an exponential decrease with the width of the barrier.

4. The analog of Zeeman splitting is observed for a bath rotating with a frequency Ω. $2m\Omega$ plays the role of a magnetic field. For a bound state of droplets, the distance between the droplets changes and the sign of the change depends on the relative sign, the direction of the rotation of the bath and of the droplet.

   Bush mentions also the notion of a hydrodynamical analog of spin realized in terms of the direction of the rotational motion of the droplet. The motivation is that there are only two directions of rotational motion and if one has angular momentum, there would be at least 3 different rotational states, with one state without splitting. If the rotational motion of the droplet were preserved at the limit of vanishing Ω, the interpretation as spin would become more convincing.

5. Walkers at circular orbits in a rotating frame are studied. For circular orbits in the rotating frame the centripetal and Coriolis accelerations cancel each other: this gives $\rho v^2/R = 2\rho\Omega v$ (here one must distinguish between rotation frequencies ω and Ω for particle *resp.* bath). This gives $v = 2\Omega R$.

   A quantization of R as in terms of λ_F is observed. According to the lecture, the orbital radii obey the analog of the formula for the cyclotron energies: $R = (n + 1/2)\lambda_F$. Note however that for Landau levels in a constant magnetic field, one has the formula is $R \propto \sqrt{n}$ as one finds from Bohr quantization of rom quantization of the magnetic flux for a constant magnetic field.

   One can however consider the possibility that Ω defines a magnetic field which depends on the distance from the rotation axes. One would actually expect that Ω for a liquid depends on this distance. Assuming that $qB_Z = 2m\Omega f(\rho/\rho_0)$, the quantization of magnetic flux would give $q2m\Omega \int_{\rho_0}^{\rho_n} f(\rho/\rho_0)\rho d\rho 2\pi = n\hbar eff$. This gives $f(\rho) = \rho_0/\rho$ and

$$\rho_n = n\frac{\hbar eff}{m2Q\Omega\rho_0^2} \quad . \tag{9.4}$$

   The vector potential qA_ϕ would be of form

$$qA_\phi = 2m\Omega log(\frac{\rho}{\rho_0}) \quad . \tag{9.5}$$

   In the TGD framework $v \propto A$, where A is Kähler potential is proposed and this would mean that the velocity of the liquid motion varies very slowly and gives rise to a spirals $\phi = (k/\rho)$.

   Given a quantized radius is observed for several values of Ω so that one has plateaus. Could the parameter $\frac{\hbar eff}{m\Omega\rho_0^2}$ be nearly independent of Ω? This would be the case for $\rho_0^2 \propto \hbar_{eff}/m\Omega$. Could this formula be interpreted in terms of Uncertainty Principle?

6. Also unstable rotational orbits with radii smaller than $\lambda_F/2$ are observed. The motions become chaotic for a large acceleration parameter and the jumps between orbits which tend to have a quantized radius of curvature have been observed so that statistical distribution for the radii becomes multimodal.

7. One can also mimic central force by using ferrofluids and magnetic fields to create central force. In this case one obtains analogs of orbitals labelled by integer value average angular momentum and radius R.

   The instantaneous pilot wave approaches the mean wave field at high Me.

8. Quantum corral is a system in which electrons are inside a circular corral formed by negatively charged ions. The hydrodynamical analog of the quantum corral is possible [D7]. In the corral

the depth is smaller. Statistical description gives rise to a similar interference pattern as in the case of quantum corral with λ_{dB} replaced with λ_F. A correlation between position and fluctuation speed bringing in mind the Uncertainty Principle is found.

9. One can also study the analog of scattering. The first example is a walker interacting with a pillar which is represented by a water with smaller depth (surface Schlieren imaging). The surprising find was that the scattering orbit was a logarithmic spiral. It can be produced by an analog of either Coriolis force or Lorentz force $2\pi\gamma_B v \times \Omega$ if $2\pi\gamma_B\Omega$ serves as an analog of the magnetic field. Here Ω is the instantaneous angular velocity with respect to the pillar

   What looks strange is that the walker would create this force associated with the pillar. In the TGD framework the magnetic field would be a Z^0 magnetic field at the MB of the system and would determine the motion of the particle and thus also the value of Ω. In the TGD framework the process would correspond to a phase transition generating dark matter at the Z^0 magnetic flux tubes.

   The second analog is the analog of Friedel oscillations. Instead of a pillar, one studies scattering from a well realized as a region in which the depth of water is larger. In this case long range statistical signature is found to resemble the square of probability density for quantal Friedel oscillations. Also now the λ_F replaces λ_{dB}.

The summary of Bush about the findings is roughly as follows.

1. There are 3 time scales. The fast time scale of bouncing, the intermediate time scale of surfing and the long term time scale for the emergence of statistical behavior. Bush speaks of a mean pilot wave at this limit. There are even indications for the analog of mode superposition. In the TGD framework the counterpart of the pilot wave would be taken by the MB.

2. Resonant interaction between walker/surfer and quasi-chromatic Faraday wave created by droplet is essential.

3. Quantized states emerge from dynamical constraints.

4. Memory effects due the fact that the walker can be said to determine the Faraday wave are basically responsible for the quantum-like behavior.

Bush suggests the following three basic paradigms.

1. Chaotic pilot wave dynamics is proposed. It would involve intermediate switching and multimode quantum-like statistical behaviors.

2. In-line oscillations involve speed fluctuations leading to a correlation between position and speed bringing in mind the Uncertainty Principle. Faraday length $\lambda_F = 2\pi\sqrt{2\nu/\mu}$ analogous to λ_{dB} serves as a statistical signature and the challenge would be to understand its origin. The fact that it seems to be the same at least for a given fluid unlike in wave mechanism where it is proportional to the inverse $1/p$ of momentum, suggests that it is more like Compton wave-length depending on the droplet mass m only. In fact, it could be independent of even m.

   In TGD framework gravitational Compton length $\Lambda_{gr} = \hbar_{gr}/m = GM/v_0$ where M is naturally the mass of Earth and $\beta_0 = v_0/c = 1/2$ seems like a good guess implying that Λ_{gr} is Schartschild radius of Earth about .9 cm might be a possible interpretation. If this is the case, β_0 could code for the variation of ν/μ. Λ_{gr} does not depend on the mass of the droplet, which might serve as a test.

3. Quantized random walks are also a central element. Random walks have steps equal to λ_F, which seems to be a universal feature. Diffusivity is equal to $U\lambda_F = U\hbar_{gr}/m$.

9.2 TGD based view

9.2.1 Quantum classical correspondence

Quantum classical correspondence (QCC) in the TGD framework is much stronger than in standard quantum theory, where it is only approximate (stationary phase approximation). In zero energy ontology (ZEO), the quantum state is a superposition of classical deterministic time evolutions - that is space-time surfaces that are minimal surfaces of very special kind being also extremals of Kähler action: I talk about preferred extremals.

QCC implies many things.

1. Space-time surface is like Bohr orbit, meaning quantization rules. The findings of Bush et al demonstrate these kinds of rules at the level of HD.

2. One can assign to each SFR a superposition of classical evolutions and in good approximation single time evolution, the "average" one. In particular, BSFR with the change of arrow of time, has as a correlate time reversed classical time evolutions which leads from the final state 3-surface to the past. For an observer with a standard arrow of time it looks like deterministic time evolution leading to the final state. This is what Minev et al observed in atomic physics.

3. This implies that SFRs look classical. The world looks classical in all scales although SFRs occur in all scales by h_{eff} hierarchy. Of course, also the basic paradox of quantum measurement theory disappears by ZEO.

9.2.2 h_{eff} hierarchy and gravitational Planck constant

h_{eff} hierarchy realized at the level of magnetic bodies (MBs) acting as controlling agents of lower levels, implies that BSFRs and SSFRs occur in all scales. In particular, hydrodynamics systems should show Bohr quantization and various other quantum effects.

What could these effects be?

1. The interpretation of ordinary quantum measurements relies on classical physics. Without QCC we could not test quantum theory, since everything is based on classical physics at the level of experiment. All the statistical aspects of quantum measurement should have classical correlates.

2. For instance, in double slit experiments you have incoming beam replaced with water droplets in the experiments of Bush et al. 3 peak interference pattern is observed and a possible explanation would be in terms of Bohm's pilot wave. One could even argue that non quantum theory is needed. To me this proposal is obviously wrong.

   The classical interference patterns could be the statistical representation for the outcomes of SFR - actually BSFR at some level of MB hierarchy - which indeed occurs. The only difference between the ordinary double slit experiment and that described by Bush et al is that their experiment the h_{eff} at MB is much larger since the scale is dramatically larger. The first guess is that $h_{eff}/hbar$ is of order 10^{14} (roughly). An educated guess, to be discussed below, is that the scale would correspond to $hbar_{gr} = GMm/v_0$, where M is Earth's mass and $v_0/c = 1/2$. $v_0/c = .92$ turns out be a more realistic guess!

3. Viscosity and magnetic diffusivity could be proportional to $\hbar_{eff}/m$ as proposed inthe TGD based model for the generation of HD and MHD turbulence. Nelson has proposed in his model of stochastic quantum theory that $hbar_eff/m$ plays the role of diffusion constant.

4. Bush represents many examples how water drop experiments provide a statistical representation analogous to interference pattern represented in terms of wave function modulus squared. Double slit experiment is only one example.

What could the value of h_{eff} be?

1. The Faraday wave length

$$\lambda_F = 2\pi\sqrt{2\nu/\mu}$$

   should be equal to either the analog of Compton wavelength $\Lambda_c = \hbar_{eff}/m$, m the mass of the water droplet or to de Broglie wavelength $\lambda_{dB} = \hbar/mv$. λ_F does not depend on the velocity of the droplet so that only Compton wavelength is a possible interpretation.

   The problem is that there is no dependence on the mass of the doplet either. A further problem is that in the hydrodynamical model of the Faraday waves λ_F is determined in the model considered by the properties of the fluid, that is friction and kinematic viscosity. This model could be of course wrong.

2. The only remaining possibility in the TGD framework is that one has

$$\hbar_{eff} = \hbar_{gr} = \frac{GMm}{v_0} \quad , \tag{9.6}$$

   where $\hbar_{gr}$ is the gravitational Planck constant introduced by Nottale [E1] [L42] and also appear in the TGD based model of superconductivity and superfluidity [L35]. This would give

$$\lambda_F = \lambda gr = \frac{GM}{v_0} = \frac{r_s(M)}{2v_0} \quad , \tag{9.7}$$

   where $r_s(M)$ is Schwartschild radius associated with mass M. M is naturally the mass of Earth. The minimum value of λ_{gr} corresponds to $\beta_0 = v_0/c = 1$ and is $\lambda_{gr} = r_s(M)/2$. Earth's Scwartschild radius is 8.7 mm so that one would have $\lambda_F = 4.35$ mm.

   The value of Λ_F for the system studied in the analog of quantum corral by Bush et al is 4.75 mm [D7] and about 10 per cent larger than the minimal value suggesting that $\beta_0 = v_0/c \simeq .92$! The error is about 10 per cent.

3. One can argue that this is a mere coincidence. The usual reductionist thinking is that the ν and μ appear as dissipative parameters in hydrodynamics and their values emerge from quantum theory in atomic scales. One can of course ask, whether the relationship

$$\frac{2\nu}{\mu} = (\frac{GM}{v_0})^2 \quad . \tag{9.8}$$

   could emerge from a microscopic theory. The alternative option suggested by the numerous grave difficulties of the description of the hydrodynamic turbulence is the description of viscosity and friction require new quantum theory predicting quantum coherence in even astrophysical scales?

4. If this correct prediction is not a mere nasty coincidence, it would mean an instantaneous breakthrough for the TGD view about quantum gravitation as a macroscopic and even astrophysical phenomenon. The Equivalence Principle behind $\hbar_{gr}$ would become a cornerstone of models thought to have nothing to do with quantum gravitation.

   The only parameter that can be varied in the prediction is β_0. One could measure λ_F for different liquids to see whether v_0 codes for the properties of the liquid or whether λ_F is independent of the liquid so that the classical model for Faraday waves could be wrong.

   It might be also possible to measure λ_F in Mars for which mass is $.107M_E$ so that λ_{gr} should be by a factor .107 smaller unless v_0 is scaled down by factor .107.

5. It is needless to emphasize how profound implications the inherent connection between dynamics of systems with a size Earth and of size of liquid drop would have. The dependence of the liquid properties determining λ_F on the mass of the planet is totally unexpected and it could be that the classical model is wrong (this is the case in the case of turbulence).

9.2.3 Does the turbulence of air at the surfaces of the droplet and water bath prevent the coalescence?

The mechanism preventing the coalescence of the water droplet with water bath is poorly understood and here macroscopic quantum gravitation could enter the picture.

1. The magnetic flux tubes assignable to the non-dissipating exteriors of vortex cores with gradient flow around the axis of the vortex and assumed to carry Z^0 magnetic fields at their MBs are proposed to be present in microscopic scales also below the criticality for the development of turbulence. They would serve as a microscopic representation for the vorticity caused by the boundary conditions at the boundary of flowing liquid. The cores of Z^0 vortices would be monopole flux tubes and the shear would be concentrated at them.

2. In the recent case the boundaries are between air and liquid bath and air and liquid droplet and the dark matter at the magnetic bodies of air vortices could explain how the gravitational magnetic body characterized by $\hbar_{gr}$ enters into the physics of the moving water droplet.

3. The air layer and perhaps its separation from the surfaces of the liquid drop and liquid bath is analogous to the separation occurring in the generation of turbulence in a liquid flow past a solid body. The TGD based proposal is that in this case the formation of microscopic vortices plays a key role in the separation. The separation of the air layer prevents the touching of the droplet and fluid surface and the coalescence. Circular vortices of air flow analogous to smoke rings would represent the shear due to the radial variation of the vertical velocity component of air flow at the surface of liquid. They would also provide a representation for the separation.

4. The diameter of the circular vortex tube would be naturally $\lambda_F = \lambda_{gr}$. Could the spherical Faraday wave correspond to expanding concentric air vortex rings with radii coming as multiples of Λ_{gr} as a representation of the shear of air. Could they form the mattress preventing direct touch and coalescence?

9.2.4 Should one replace pilot wave with magnetic body?

In his talk Bush interprets the Faraday wave induced by the motion of the droplet along the surface as a kind of pilot wave providing a stastistical description of the system in long time scales resembling the description provided by Schrödinger amplitude. In particular, λ_F appears as a statistical signature in this description.

In the TGD framework the role of the pilot wave would be taken by the magnetic body (MB) of the system carrying $h_{eff} = nh_0$ phases quantum controlling the behavior of ordinary matter. In hydrodynamics magnetic flux tubes assigned with vortices would carry in their cores Z^0 magnetic fields proportion to induced Kähler form whereas the ordinary magnetic field vanishes.The exterior of the core would have vanishing Z^0 magnetic field but Kähler gauge potential would be gradient only in the exterior regions (note that symplectic transformations leaving induced Kähler form invariant are not genuine gauge transformations since they change the induce metric). The simplest model assumes that the Weinberg angle $p = sin^2(\theta_W)$ vanishes in this phase. The interpretation is that below the dark weak scale the electroweak symmetry breaking is absent.

What is encouraging, that the analog of rotation frequency Ω appears in the role of the magnetic field in several quantum-like phenomena discussed by Bush. The prediction is indeed that MB controls the fluid flow and that Ω, that is circulation, is proportional to B_Z whereas velocity is proportional to the vector potential of Kähler form. In one experiment, the instantaneous rotation frequency Ω around a "pillar" causes an analog of Lorentz force. In a second experiment the rotation frequency of the liquid bath gives rise to the analog of Zeeman splitting and analogs of cyclotron orbits.

9.2.5 Classical determinism is not exact

The analog of double slit experiment suggests classical non-determinism. Water drops with the same initial state (modulo measurement resolution) do not behave always in the same manner. If classical non-determinism were exact, this should not be the case.

The work with minimal surfaces [L52] demonstrated that classical non-determinism is probably not quite exact.

1. Space-time surfaces are analogous to soap films spanned by frames (which correspond to initial and final 3-surfaces plus intermediate partonic 2-surfaces) and already for soap films the same frame can allow several soap films. Same occurs now but because of boundary conditions at boundaries of CD (perceptive field) the non-determinism is extremely restricted.

2. There is a finite, discrete non-determinism associated with what I identified as the TGD counterparts of reaction vertices and "very special moments in the life of self". This finite determinism would be the counterpart for quantum non-determinism for space-time surfaces inside a single CD.

3. This non-determinism could have as an adelic counterpart the non-determinism of p-adic differential equations due to the fact that integration constants as functions with a vanishing derivative are not genuine constants as in real case but depend on finite number of the pinary digits. This non-determinism would correspond to cognitive non-determinism having a real counterpart. This non-determinism would correspond to what I call cognitive determinism occurring for the representations of Galois group [L32, L38]

Therefore, if one has a beam of identical droplets in the initial state, they behave differently and one could obtain in the long run a representation for quantum mechanical interference pattern as an analog of the modulus square for a wave function.

9.2.6 Does quantum entanglement have a classical representation?

Can quantum entanglement be represented as a property of the space-time surface?

1. In the TGD framework quantum entanglement has a classical correlate/prerequisite. The flux tube pairs connecting particles as 3-surfaces would serve as prerequisites for the entanglement. This is analog ro ER-EPR correspondence: I actually proposed flux tubes instead of wormholes in GRT sense much before ER-EPR correspondence.

2. The reduction of entanglement in quantum measurement/SFR could correspond to the splitting of a flux tube pair connecting two systems to two U-shaped flux tubes associated with particles or more generally decay of the space-time surface representing systems in measurement interaction to two disjoint space-time surfaces.

   Putting the interaction Hamiltonian on could correspond to the formation of flux tube pair by reconnection and after that the usual description by unitary evolution would be a reasonable model.

3. Entanglement has also a purely classical analog. For instance, superposition of spherical harmonics for a classical field can be seen as an entangled state. At the level of WCW this is also possible.

4. The crucial notion is however the tensor product of Hilbert spaces. I find it impossible to imagine any classical counterpart for it. An entangled two-particle state can have as a classical prerequisite two 3-surfaces connected by flux tube but I am unable to imagine how entanglement could be representable for *single* space-time surface. One must allow quantum superposition of these pairs of this kind of 3-surfaces connected by a flux tube. One has entanglement in WCW degrees of freedom. WCW is needed for entanglement. I see no way to avoid this.

   By the way, in TGD quarks are the only fundamental particles. One does not have fundamental bosons although one can assign to the deformations of 3-surfaces analog of Kac-Moody algebra involving bosonic oscillator operators.

   Quark entanglement due to Fermi statistics is always maximal and cannot be reduced in SFRs: something totally trivial but not realized by most colleagues. Only entanglement at the level of WCW can be reduced. This came as a surprise also to me!

The natural entanglement associated with hierarchies of normal subgroups of Galois groups can be reduced and is reduced in cognitive measurements to which SSFRs can correspond. Cognitive measurement cascades become possible. SSFR is a counterpart for analysis. BSFR is the intuitive eureka moment from the point of view of cognition [L32, L46].

9.2.7 Does Fermi statistics have a classical correlate?

A fermion with momentum p corresponds to a point of $X^4 \subset M^8$ and $M^8 - H$ duality as the TGD counterpart of momentum-position duality maps this point to a point at the 3-surface at either light-like boundary of CD in H depending on sign of the energy. One can put to the same point p several quarks with different spin or electroweak spin. This is not what one would want , that is one fermion per point.

What about twistor lift which provides a geometric description of spin as analog of partial waves in the twistor sphere: twistor space is indeed locally a product of space-time and twistor sphere S^2. Quantization axis of spin means a choice of one direction that is a point of S^2. But both the point and its diametric opposite give the same quantization axis. The interpretation would be that the two choices correspond to two spin directions for fermion. This makes sense for both spin and electroweak spin. The Fermi statistics would mean geometrically that a single point of twistor space can contain only a single fermion.

Twistor lift has a counterpart at the level of M^8 as I realized quite recently. At the level of M^8 twistor lift Fermi statistics would have a classical correlate at the M^8 level and would mean that one cannot put two fermions at the same space-time point. One can say that wave function is involved but there is a localization to a single point representing momentum p and spin.

10 Trying to understand viscosity and critical Reynolds numbers

It is interesting to see whether the critical Reynolds number could be interpreted in terms of criticality for the phase transition generation of dark flux tubes with $h_{eff} > h$ assignable to the flux tubes controlling vortices associated with them. One can also consider the possibility that the angular momenta of dark flux tubes and vortex compensate for each other.

10.1 The notion of viscosity

Kinematic viscosity (`https://cutt.ly/iRuXTsH`) for liquid can be fitted by using the expression

$$\nu(\frac{h}{m},\frac{T}{T_b}) = \frac{h}{m}\times f(\frac{T_b}{T}) \ \ , \quad f(\frac{T_b}{T}) = exp(3.6\frac{T_b}{T}) \ \ . \tag{10.1}$$

(note that one has $c = 1$). T_b is the boiling point for the liquid and m is the average mass of the liquid particle. The expression makes sense between freezing point and boiling point. The model is ad hoc and it is not especially good. The two essential features are proportionality to h suggesting quantum origin the rapid increase with temperature below the boiling point.

From the tables of viscosity (`https://www.engineersedge.com/fluid_flow/kinematic-viscosity-table.htm`), one finds that a natural unit for viscosity is CentiStokes $= 10^{-6}$ m^2/s. CS/c corresponds to a size scale about 3×10^{-15} m/s, which is about $2.3\lambda_p$ where λ_p is proton Compton length. The factor having an exponential temperature dependence brings in mind the inverse of Boltzmann exponent with $3.6T_b$ having a possible interpretation as the energy for a transition of some kind. The formula explains qualitatively the variation of ν by roughly 4 orders of magnitude. This would correspond to a variation of T_b/T by factor 2.9.

10.2 Critical Reynolds numbers

The value Re_{cr} of the critical Reynolds number for the laminar-turbulent transition varies in a wide range (`https://www.ecourses.ou.edu/cgi-bin/ebook.cgi?topic=fl&chap_sec=09.3&page=theory`).

1. For a fully developed pipe flow turbulence with pipe diameter D the value is $Re_{cr} \sim 2300$.
2. For a flow over a flat plate the transition from laminar to turbulent flow occurs at critical distance $D = x_{cr}$ downstream from the leading edge for $Re_{cr} \sim 5 \times 10^5$.

A hydrodynamical model predicts for the laminar flow past plate for the thickness of the liquid layer thickness δ as function of the distances x from the leading edge

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} \quad , \quad Re_x \leq Re_{x_{cr}} \simeq 5 \times 10^5 \quad . \tag{10.2}$$

For a turbulent flow past a flat plate of finite length L, the prediction is

$$\frac{\delta}{x} = \frac{.38}{Re_L^{1/5}} \quad , \quad 5 \times 10^5 \leq Re_L < 10^7 , \tag{10.3}$$

Above $Re_{cr} = 10^7$ only a thin boundary layer forms and the flow develops a thin wake.

10.2.1 Critical Reynolds number as a measure for the ratio of units of angular momentum for the final and initial state

The critical Reynolds number could be essentially a measure for the ratio of the units of dark and ordinary angular momentum. The following estimate suggests that this might make sense.

In the case of laminar flow and using the formulas above, one can estimate the angular momentum associated with a particle of flow as

$$L = m\delta \times U \quad , \quad \delta = 5\sqrt{\frac{\nu}{u}}\sqrt{x} \quad . \tag{10.4}$$

Here x is the length measured from the leading edge of the plate.

At the critical value of $\delta/x = 5/\sqrt{Re_{cr}}$ one cast this equation into the form

$$L = 5m \times Re_{cr}\nu \quad . \tag{10.5}$$

$L = \hbar_{eff}$ would give for the transition

$$Re_{cr} = \frac{\hbar_{eff,f}}{\hbar_{eff,i}} \quad \text{for} \quad \hbar_{eff,i} = 5m\nu \quad . \tag{10.6}$$

The estimate is of course very rough. What is however essential that the identification of kinematic viscosity in terms of h_{eff} could make sense.

Note that one can associated to the vortex a Reynolds number

$$Re_V = \frac{U\delta}{\nu} = \frac{\delta}{D} \times Re_{cr} = 5Re_{cr}^{1/2} \quad . \tag{10.7}$$

10.3 Laminar-turbulent transition as a quantum phase transition?

Could one understand the laminar-turbulent phase transition in terms of quantum hydrodynamics at the level of MB in terms of the formation of $h_{eff} > h$ flux tube structures accompanying the vortices?

1. Suppose that the kinematic viscosity related to angular momentum it makes to speak of "dark" viscosity ν_{dark} and that ν_{dark} decomposes to a product $\hbar_{eff}/m$ with the same temperature dependent factor $f(T/T_b)$ as the ordinary ν. Assume that the critical Reynolds number $Re = UD/\nu$ corresponds to a phase transition $h \to h_{eff}$ making possible a formation of vortices accompanied by pairs of monopole flux tube associated with the core of vortex and non-monopole Lagrangian flux tubes associated with the exterior of the vortex.

2. The transfer of angular momentum from the main flow to vortices is enough to take care of angular momentum conservation. Also the quantized dark angular momentum $m\hbar_{eff}$ at MB could compensate for the angular momentum of the vortex. In this case, the angular momentum of vortices could be considerably larger than one might estimate from the change of the angular momentum of the main flow. This option is of special interest in astrophysical systems.

3. Could the parameter UD at criticality have an interpretation as minimal angular momentum $\hbar_{eff}/m$ for the vortex? Could it be that the angular momentum of the fluid particle of the flow in the region $x \geq x_{cr}$ has angular momentum larger than $\hbar_{eff}$ so that dark magnetic flux tube so that the particle can transform to its dark variat at magnetic flux tube.

The basic question is what the fluid particle is. The intuitive picture is that the increase of viscosity means increase of the fluid particle mass and thus interia. The size of the fluid particle would be caused by the increase of h_{eff} at the MB controlling the ordinary matter in the flow.

If dark matter is formed it could emerge as fluid particles with mass larger than say proton mass which appears as factor h/m in the formula for ν involving also temperature dependent factor increasing at higher temperatures. The increase of the kinematical viscosity ν could mean that the mass m_f of a fluid particle increases with temperature. Suppose that one has $\nu(T/T_b) = \hbar_{eff}(T)/m_f(T)$. If ν would not depend on temperature one would have $\nu(T/T_b) = \nu(1)$, and $\hbar_{eff}(T)$ should be proportional to $m_f(T)$. This is of course not true since the $\nu(T)$ increases with the decreasing temperature. In the range between boiling point and freezing point the change is not however very large.

One should have

$$\frac{f(T/T_b)}{f(1)} = \frac{\hbar_{eff}(T)}{\hbar(T_b)} \times \frac{m(T)}{m_f(T_b)} \geq 1 \ . \tag{10.8}$$

The increase of the viscosity would be indeed due to the formation of larger mass units due to long range correlations induced by MB with larger value of h_{eff}.

10.4 Nottale hypothesis and turbulence

Nottale hypothesis states that it makes sense to talk about gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where M can be the Earth's mass M_E. The gravitational Compton length is given by $\Lambda_{gr} = \hbar_{gr}/m = GMm/v_0 = r_s/2\beta_0$. The "gravitational" kinematic viscosity would be given by $\nu_{gr} = \Lambda_{gr}c$ and independent of the mass m of the fluid particle unless β_0 does not depend on it.

Λ_{gr} does at all on the particle mass. This looks strange. The ratio $m_p/m_e = 1880$ is near to $2^{11} = 2048$ appearing defined by $v_0 = simeq 2^{-11}$ in the Nottale's model for inner planets. This inspires the question whether the proportionality $\beta_0 = m/m_p$ might hold true approximately and realize approximately the expectation that gravitational Compton length is proportional to $1/m$. For instance, could β_0 correspond to the ratio of the p-adic length scales $L(k) \propto 2^{k/2}$ for proton and for the particle with mass m. For the electron one has $k = 127$ and for proton $k = 107$ so that the prediction would be $\beta_0 = 2^{-10}$ and by factor 2 too large.

Interestingly, for a neutrino mass about .1 eV this hypothesis would give $\Lambda_{gr} \sim 200$ AU which is the length of the heliospheric magnetotail at the side of the downwind.

10.4.1 Encouraging observations

There are several encouraging observations.

1. Λ_{gr} for Earth appears both the TGD based model of superconductivity [L35] and in the model for the hydromic quantum analogues as a correct prediction for the Faraday wavelength.

   Λ_{gr} could relate to the length or radius of the vortex. Since the MB of the system is responsible for the generation of coherence as induced quantum coherence, the simplest interpretation would be in terms of the length of the dark magnetic flux associated with the vortex.

2. The Nottale model for the inner planetary orbits assumes $\beta_0 \simeq 2^{-11}$. From Schwartscild radius of Sun one has for the gravitational Compton length of Sun $\Lambda_{gr}(Sun) = GM_{Sun}/v_0 =$ 6 Mm to be compared with the radius $R_E = 6.4$ Mm of Earth. This would suggest a dark graviton BEC in the scale of Earth and a deep connection between the gravitational physics of Earth and Sun.

In MHD, magnetotail is analogous to a wake of hydrodynamic flow past a body. The length of the magnetotail is about $D_R \simeq 10^3 R_E$: "10^3" suggests that $\beta_0 = 2^{-11}$ appearing in $\hbar_{gr}$ for the inner planets is involved.

If the parameter UD appearing in Re corresponds $\nu_{gr} = \hbar_{gr} c/m\Lambda_{gr} c$, one has $D = \Lambda_{gr} c/U$. The velocity U of the solar wind varies in the range 300-800 km/s, that is $U/c \in \{2 - 5.4) \times \beta_0$, where $\beta_0 = 2^{-11}$, which is perhaps not an accident. $U = 4 \times 10^2$ km/s serves as a nominal value. $\Lambda_{gr}(Sun) = 6 \times 10^3$ km for $\beta_0 = 2^{-11}$ and for the nominal value of U gives the estimate $D = 700 R_E$ to be compared with $D_R \sim 10^3 R_E$.

One can look at the situation also in the case of solar magnetotail. The solar magnetotail has length about $D_R = 200$ AU ($AU \simeq 2.3 \times 10^4 R_E$) at the downside of the flow. The center of the Milky Way could contain the source of the galactic wind defining the mass M appearing in $\hbar_{gr}$. One can imagine two options.

1. The mass appearing in $\hbar_{gr}$ for the galaxy could be the total mass M_{MW} of the Milky Way. The estimates forM_{MW} vary in the range $M_{MW}/M_{Sun} \in [10^{11} - 10^{12}]$ and are based on the halo model of dark matter. Dark energy and matter are estimated to contribute about 95 per cent to the mass of the Universe.

   In the TGD framework, the flat velocity spectrum for stars rotating around galaxies is explained in terms of dark cosmic strings predicting automatically flat velocity spectrum. Since the the galactic wind from the galactic jet emerging from the galactic blackhole-like entity should not affect to the gravitational field associated with solar magnetotail, the estimate for the visible mass of the galaxy reduces by a factor $\sim .05$ to $M(M_{MW}/MSun = 5.0 \times 10^{10}$ giving for the Schwarschild radius the estimate $r_S(MW) = 15 \times 10^{10}$ km. The estimate for the ratio $r_{S,MW}/D_R$ is $r_{S,MW}/D_R = 5.0$.

   The estimate of the ratio D_{pr}/D_R for the predicted value D_{pr} of D is

   $$\frac{D_{pr}}{D_R} = \frac{r_{S,MW}}{D_R} \times \frac{c}{U\beta_0} \simeq 5 \times \frac{c}{U\beta_0} \ .$$

   By $c/U\beta_0 \geq 1$, $D_{pr}/D_R \geq 5$ is true even for ($\beta_0 = 1, U = c$). Of course, the idea that the galactic wind would blow with the speed of light, does not seem plausible.

2. The galactic wind could correspond to a galactic jet emerging from the blackhole-like entity in the center of the Milky Way having mass about $4 \times 10^6 M_{Sun}$. In this case, r_S is reduced by a factor 10^{-6} and one obtains $D_{pr}/D_R = (r_{S,MW}/D_R) \times (c/U\beta_0) \simeq 10^{-6} \times (c/U\beta_0)$. If $\beta_0 = 2^{-11} \simeq m_e/m_p$ appears as a universal parameter then a good guess is ($\beta_0 = 2^{-11}, U/c = \beta_0$). For this guess, the ratio equals unity.

Remark: The mass of the Moon is $.012 M_E$. For $\beta_0 = 1$, this would correspond to $\Lambda_{gr} \simeq 10^{-4}$ m, which could be some biological length scale.

10.4.2 Does the transition to turbulence correspond to a large change of h_{eff}?

The simplest option is that the variation of β_0 explains the temperature variation of kinematic viscosity in terms of a slow variation of $\hbar_{eff} \leq\leq \hbar_{gr}$. h_{gr} is not a plausible candidate for understanding the kinematic viscosity but can be replaced with its electromagnetic analog $\hbar_{em}$ or Z^0

analog $\hbar_Z$. Z^0 option is attractive in hydrodynamics whereas the electromagnetic analog might have a role in MHD.

In the transition to turbulence, a dramatic change of h_{eff} seems to take place.

1. Are both $\hbar_{eff}$ and the mass m_f of the fluid particle scaled up by Re_{cr} so that ν would remain invariant? In the case of $\hbar_{gr}$ this would be naturally the case.

2. Is only $\hbar_{eff} = \hbar_{gr}$ scaled up by Re_{cr} so that $\nu = \hbar_{eff}/m_f$ would be scaled up by Re_{cr}? If one accepts the notion of gravitational quantum coherence, one can consider the change of M and Earth mass M_E and solar mass M_{Sun} appear as natural candidates.

10.4.3 Could critical Reynolds numbers be understood in terms of the Nottale's hypothesis and its generalization?

One can try to understand the two critical Reynolds numbers in terms of $\Lambda_{gr} = GM/v_0$. Assuming Nottale's formula and the proposed connection between $\hbar_{eff}$ and ν, the ratio M/β_0 would change at criticality by factor Re_{cr}. The masses of Earth and Sun are natural candidates to consider. The critical quantum numbers depend on the geometry of the flow but this could be explained by the change of β_0.

1. For the pipe flow, one has $Re_{cr} = 2300$. Perhaps it is not a mere accident that $Re_{cr} = 2300$ is not too far from $1/\beta_0 = 2^{11} = 2048 \sim m_p/m_e$ associated with the inner planets of the Solar system. If the initial state corresponds to $\beta_0 \simeq .92$ for the initial state as suggested by the model for the quantum-like aspects of hydrodynamics, one has $Re_{cr} = 2226$ and the error .5 per cent.

   Could the transition to a turbulent pipe flow correspond for the final state to $\beta_0 \simeq 1 \rightarrow 2^{-11}$ for $\Lambda_{gr} = GM_{Sun}/\beta_0 \simeq 3.2km$, $\beta_0 \simeq .9$ so that one would have $\Lambda_{gr} \simeq R_E$ after the transition. The Earth's crust has thickness between 5-70 km: could this variation relate to the variation of β_0 in range $(.64, .045)$?

2. Consider next the flow past a planar object with $Re_{cr} = 5 \times 10^5$. The ratio of the masses of Sun and Earth is $M_{Sun}/M_E \simeq 3 \times 10^5$, which is not far from $Re_{cr} \sim 5 \times 10^5$. Could $Re_{cr} \sim 5 \times 10^5$ correspond to a phase transition $M_E \rightarrow M_{Sun}$ and $\beta_0 \simeq 1 \rightarrow 3\beta_0/5$?

M_E, M_{Sun}, and the values $\beta_0 \simeq 1$ and $\beta_0 \simeq 2^{-11} \simeq m_e/m_p$ could appear in the model for the transition to turbulence. The dependence of the Re_{cr} for the pipe flow on the mass of the planet is a rather dramatic prediction, and could kill the proposal.

The natural assumption is that the gravitational flux tubes have length Λ_{gr} so that the phase transition would mean emergence of longer flux tubes corresponding to the gravitational Compton length hierarchy $\Lambda_{gr} \in \{.1\ \ m, 3.2\ \ km, 644\ \ Mm\}$.

Needless to emphasize, these proposals are only a light-hearted thought game taking seriously the notion of macroscopic quantum gravitational coherence.

10.5 Trying to understand kinematic viscosity

The model for the hydrodynamical quantum analogs leads to a proposal, which is completely crazy from the reductionistic point of view and looks like a return to astrology. The motivation is that the Faraday wave length λ_F appearing as analog of Compton length equals the gravitational Compton length associated with the gravitational Compton length associated with the gravitational Planck constant proposed by Nottale [E1].

10.5.1 Kinematic viscosity cannot be described in terms of $\hbar_{gr}$ for the masses of Earth and Sun

The first thing to notice is that ν_{gr} is several orders of magnitude larger than kinematic viscosity.

One obtains for ν_{gr}

$$\nu_{gr} = \Lambda_{gr} c = \frac{GMc}{\beta_0} = \frac{r_s c}{2\beta_0} \quad . \tag{10.9}$$

This corresponds for Earth's mass $M = M_E$ and $\beta_0 = 1$ to the Schwarshild radius 0.87 cm of Earth. This scale is by a factor roughly 10^{13} times longer than the Compton length of proton assignable to CentiStoke 10^{-6} m^2/s as a unit of kinematic viscosity.

Therefore the value of ν_{gr} is however very large as compared to the values of ν for ordinary liquids and the reduction of β_0 would make the value of ν_{gr} even larger. Therefore ordinary viscosity cannot correspond to $\hbar_{gr}$ for any astrophysical mass.

One can of course ask, whether h_{eff} could correspond to $\hbar_{gr}$ but for a smaller non-astrophysical - say mass M of some geological unit or of a unit assignable to atmosphere. Note that the variationof β_0 could allow us to understand the dependence on temperature.

10.5.2 Could one understand kinematic viscosity in terms of masses of geological or atmosperic objects?

Could one understand kinematic viscosity in terms of masses of geological or atmosperic objects?

1. As already noticed, the decrease of the velocity parameter β_0 with temperature perhaps related to the decrease of thermal velocity could be enough to explain the temperature dependence of ν. This raises the question whether the basic scale for ν could be set by some natural astrophysical or geological mass.

2. For M_E, one has $\Lambda_{gr} \simeq 10^6$ m^2/s whereas centiStoke as natural unit of η is 10^{-6} m^2/s and defines lower bound for it (the range of variation is 4 orders of magnitude) so that the mass M for ν should be smaller than M_E by 8-12 orders of magnitude. Therefore only geological objects appearing as basic building bricks of Earth's crust can be considered. Note that also β_0 appears as a parameter.

   The size scale L of an object of density $\rho_{ave} \sim 5.5 \times 10^3$ kg/m^3 with mass $M \sim 3 \times 10^{-13} M_E$ corresponding to $\nu = 1$ centiStoke would be about $L = 640$ m. There is no obvious identification.

   Could the object in question correspond to an atmospheric basic unit? The density of air is 1.2 kg/m^3 so that the size scale of the object would be 32 km. Note that the eye of the hurricane has a radius 16-32 km.

The basic objection is that the value of the kinematic viscosity would depend on local physics at Earth and this seems highly implausible.

It seems that one must distinguish between classical hydrodynamics assignable with ordinary matter with $h_{eff} = h$ and quantum hydrodynamics assignable to dark matter with $h_{eff} > h$. In particular, one must distinguish between quantum gravitational aspects of hydrodynamics assignable to $\hbar_{gr}$ involving mass of Earth or Sun.

10.6 Also the notions of $\hbar_{em}$ and $\hbar_Z$ make sense

It is of course not necessary to assume $\hbar_{eff} = \hbar_{gr}$. One can also consider the electromagnetic and weak variants of $\hbar_{gr}$.

1. For hydrodynamics, dark Z^0 interaction looks natural and one would have $\hbar_Z/\hbar = Q_Z 4\pi\alpha_Z/\beta_0$, where $Q_Z = N$ is the total Z^0 charge the number of elementary particles with Z^0 charge giving rise to the particle. Z^0 would be effectively massless below dark weak length scale.

   In the electromagnetic case, one has $\hbar_{em}/\hbar = Q_{em} 4\pi\alpha/\beta_0$. A highly interesting possibility suggested by the model of the vortices is that electroweak symmetry breaking is absent for the Lagrangian MB controlling the region of vortex exterior to the core (one would have $p = sin^2(\theta_W) = 0$). This raises the question whether electromagnetic and Z^0 situations are equivalent in this case.

2. These formulas make sense only for $\hbar_{em}/\bar{>}1$ and $\hbar_{em}/\bar{>}1$ and this gives the following criterion for darkness

$$\begin{array}{l} Q_Z \geq \frac{\beta_0}{4\pi\alpha_Z} \ , \\ Q_{em} \geq \frac{\beta_0}{4\pi\alpha_{em}} \ , \end{array} \tag{10.10}$$

In the electromagnetic case and for $\beta_0 = 1$, the transition would take place for completely ionized atoms with charge $Q_{em} > 10$. Natrium with $Z = 11$ would be the first dark completely ionized atom (ionization energy for the ground state electron is about 1.645 keV). Dark proton sequences at flux tubes consisting of dark proton triplets realizing genetic code would be the basic example from TGD inspired quantum biology [L55, L39, L38].

The interpretation would be that when a perturbation series fails to converge, a phase transition takes place. The new coupling strength is $Q_{em}e^2/\hbar_{em}$ resp. $Q_Z g_Z^2/\hbar_Z$ and is equal to $v_0/4\pi$ so that the perturbative expansion is universal and has the same coupling strength for all interactions. This conforms with the assumption that all classical fields are induced from the geometry of the embedding space.

Also now one can define dark Compton lengths for electromagnetic and Z^0 ions as

$$\Lambda_{em} = \frac{\hbar_{em}}{m} \quad , \tag{10.11}$$

$$\Lambda_Z = \frac{\hbar_Z}{m} \quad , \tag{10.12}$$

$$\tag{10.13}$$

where m is the mass of the em or Z^0 charged particle at the flux tube. One can of course ask whether the notion of Z^0 makes sense.

The em charged particles at flux tubes could be protons or biologically important dark ions as proposed in the TGD based model for quantum biology. There would be $N = Q_{em}$ dark protons associated with or at the flux tube so that their density is $\Lambda_{eff}/\lambda_c(m)$. Similar interpretation applies in the Z^0 case.

If em and weak interactions are dark at gravitational flux tubes, the weak scale is scaled up by $h_{gr}/\hbar$ to $10^{-4}\ m, 3\ m, 6\ km\}$ corresponding to various dark gravitational Compton lengths. Therefore one could regard neutrons and protons as having weak Z^0 charge since the weak charge of neutrons is 50 times larger than that of protons. Dark neutrinos would be responsible for the screening of weak charge of dark nuclei.

10.6.1 Could one understand hydrodynamical viscosity and magnetohydrodynamical diffusivity in terms of $\hbar_{em}$ and $\hbar_Z$?

The variation of hydrodynamical kinematic viscosity could have explanation in terms of the variation of β_0. The basic units with Z^0 charge $Q_Z = Nq_z$ could correspond to vortex like entities. For gases one would have $\hbar_{em} = \hbar$ and dark matter would have no role in the dynamics.

For MHD plama the the picture would be similar and one can consider vortex like units and plasmoids as basic units with charge Q_{em}. TGD counterparts would be magnetic and Z^0 magnetic flux tubes: here one can consider the the core of the vortex as a monopole flux tube or its exterior and Lagrangian flux tube.

Consider the kinematic viscosities.

1. Consider first the Z^0 case. The Z^0 charges of proton and neutrino are by a factor about 1/50 smaller than those of proton and electron. Stability requires that the weak charges of neutrinos and dark nuclear neutrons sum up to zero. It is convenient to talk about the length scale

$$L_\nu = \frac{\nu}{c} = \frac{\hbar_Z}{m} = \frac{4\pi\alpha_Z q_z^2 N}{\beta_0}\frac{\hbar}{m} \quad . \tag{10.14}$$

Here N is the number of dark Z^0 charge carriers at magnetic flux tube.

The already described formula for the kinematic viscosity reads as $L_\nu = \lambda_c(n) \times f(T/T_b)$. This suggest the identification as $\hbar_{eff}/\hbar = f(T/T_b)/f(1)$, $f(1) = exp(3.6) \simeq 16.3$. At boiling point and above it one would have $\hbar_{eff}/\hbar = 1$.

This would give for $m = m_n$ the following formula

$$N = \frac{1}{4\pi\alpha_Z q_Z^2} \times \beta_0 \frac{f(T/T_b)}{f(1)} \quad . \tag{10.15}$$

N would be the total number of neutrons with Z^0 charge q_Z within volume defined by the scale L_ν. If one has $\beta_0 = k\frac{f(T/T_b)}{f(1)}$, N is constant and L_ν scales like $1/\beta_0$. Could N correspond to the number of neutrons for a dark atomic nucleus with $\hbar_{eff}/\hbar = \frac{4\pi\alpha_Z q_z^2 N}{\beta_0}$? The decrease of β_0 would increase quantum correlation length and viscosity.

2. Electromagnetic case can be treated in similar way.

10.6.2 Could dark quantum coherence scales for dark gravitation, dark Z^0 and dark em interaction be identical?

Could dark em, and Z^0, and gravitational quantum coherence scales be identical in some situation? Could this condition make possible astrophysical quantum coherence and symmetry restoration of electroweak interactions at the level of MB?

The general conditions for the equality of the quantum coherence scales are as follows.

$$\begin{aligned}
\Lambda_{em} &= \Lambda_Z = \lambda_{gr} \quad , \\
\Lambda_{em} = \frac{\hbar_{em}}{m_e} &= \frac{e^2 q^2 N_p}{\beta_0} L_{c,e} \sim \frac{10^{-2} N_p}{\beta_0} \times 2 \times 10^{-12} \ \ m \quad , \\
\Lambda_Z = \frac{\hbar_{em}}{m_\nu} &= \frac{e^2 q^2 N_n}{\beta_0} L_{c,\nu} \sim \frac{10^{-2} N_n}{\beta_0} \times 10^{-6} \ \ m \sim \Lambda_{gr} \quad , \\
\Lambda_{gr} &= \frac{GM}{\beta_0} \quad , \\
(M, \beta_0) &\in \{(M_E, \beta \simeq 1), (M_{Sun}, \beta_0 \simeq 1), (M_{Sun}, \beta_0 \simeq 2^{-11})\} \quad .
\end{aligned} \tag{10.16}$$

This gives the conditions

$$\begin{aligned}
N_p &= \frac{\beta_0}{e^2 q^2} \frac{\Lambda_{gr}}{L_{c,e}} \quad , \\
N_n &= \frac{\beta_0}{g_Z^2 q_Z^2} \frac{\Lambda_{gr}}{L_{c,\nu}} \quad .
\end{aligned} \tag{10.17}$$

This gives for $\Lambda_{gr} \in \{1.0 \ \ cm, 3.2 \ \ km, 6.4 \ \ Mm\}$.

$$\begin{aligned}
N_p &= \frac{4\pi\beta_0}{\alpha e^2} \frac{\Lambda_{gr}}{L_{c,e}} \in \frac{4\pi\beta_0}{\alpha e^2} \times \{.5 \times 10^{10}, 1.6 \times 10^{15}, \\
N_n = \frac{4\pi\beta_0}{\alpha_Z q_Z^2} \frac{\Lambda_{gr}}{L_{c,\nu}} &\in \frac{4\pi\beta_0}{\alpha_Z q_Z^2} \times \{2.5 \times 10^{16}, 8.0 \times 10^{21}, 1.8 \times 10^{25}\} \quad .
\end{aligned}$$

The only natural interpretation is that these scales correspond to flux tube lengths. Assume that one has $\beta_0 = 1$.

1. For $\beta_0 = 1$, the density of protons would be in all three cases about 5×10^{12} per meter: of order 2 protons per electron Compton length. This is of the same order of magnitude as deduced for the density of dark protons in magnetic flux in the model of "cold fusion". For $\beta_0 = 2^{-11}$, where would be roughly one proton per 10^{-8} m, this is the p-adic length scale $L(151)$ and thickness of neuronal membrane.

2. For the Z^0 case with $\beta_0 = 1$ the density of neutrons would be roughly 2.5×10^{15} per meter for all options so that there would be one neutron per length 4×10^{-16} m. The Compton length of the neutron is $\lambda_n = 3.8 \times 10^{-16}$ m so that there would be roughly 1 neutron per neutron Compton length. This suggests that nuclear flux tubes are in question for $\beta_0 = 1$.

   If one assumes that $\beta_0 = 2^{-11} \simeq m_p/m_e$, the density would be roughly 1 neutron per electron Compton length. The TGD based proposal for solar cores is that they correspond to this kind of nuclear flux tubes.

Both dark neutrinos and neutrons and dark electrons and protons would neutralize each other. This suggests a connection with Pollack effect [I1] in which part protons of water molecules form sequences at dark flux tubes in the presence of a metabolic energy feed. Every fourth water molecule would give one proton which would be transformed to dark proton. Pollack effect is the cornerstone of the TGD inspired model of quantum biology [L55]. In the recent case, the flow would provide the energy needed to transform the protons to dark protons.

10.6.3 Gravitational de-Broglie wavelength and hydrodynamic length scale hierarchies

It is possible to define quantum gravitational de-Broglie wavelength as

$$\Lambda_{gr,dB} = \frac{\hbar_{gr}}{mv} = \frac{GM}{v_0 v} = \frac{r_s c^2}{2\beta_0 \beta} \ . \tag{10.19}$$

The length scale $UD = \Lambda_{gr}/m$ gives $D = \Lambda_{gr,dB}$ for U proposed to correspond to the length of magnetopause as an analog of wake in MHD.

In TGD, the p-adic length scale hierarchies $L_p \propto p^{1/2}$ assignable to $p \simeq 2^k$ for some integers k, play a central role [K15]. $p = 2$ defines length scale hierarchy in powers of $\sqrt{2}$ giving as a sub-hierarchy in powers of 2, which could correspond to a hierarchy of period doublings in approach to chaos.

This raises interesting questions.

1. Could this kind of hierarchy correspond to a hierarchy $\beta_{0,n} = p^{-n}\beta_{0,1}$ giving a period doubling hierarchy for $p = 2$? The velocity hierarchy and the associated length scale hierarchy would respect UP. Could the vortex lengths or radii for vortex hierarchies in hydrodynamic turbulence be described in this manner?

2. Could $\beta_0(Sun) \simeq 2^{-11}$ correspond to $\beta_{0,11}$ level for $\beta_{0,1} \simeq 1$. As found, $\beta_{0,1} \simeq .92$ predicts correctly the Faraday wavelength for hydrodynamic quantum analogs. For $\beta_{0,1} = .94$, $R_E = \Lambda_{gr,Sun}$ holds true exactly. For $\beta_{0,1} = .89$ $Re_{cr} = 2300$ for critical Reynolds number in pipe flow is predicted.

The original motivation for the Nottale hypothesis comes from the Bohr orbit model of the planetary system. This model involves an ad hoc feature. For the outer planets of the solar system, one must assume $\beta_0(out) = \beta_0(in)/5$. $p = 5 = 2^2 + 1$ is prime but not near a power of 2. Another interpretation is that β_0 is not changed but the principal quantum numbers come as $n = 2, 4, 5$ for the inner planets and as $n = 5k$ for $k = 2, ..., 6$, for the outer planets. Earth could be interpreted as an inner or outer planet.

This would suggest a secondary hierarchy in powers of 5 and $\beta_{0,1} = \beta_0(Sun)$.

Could 2-adic fractality conform with this rule? Could the rule be that the allowed 2-adic length scales proportional to $2^{k/2}$ must be as near as possible to the radius of an elliptic Bohr orbit for the principal quantum number n satisfying the Bohr conditions. For an elliptic orbit, the radius of the orbit could be defined as the geometric mean $\sqrt{ab}$. This condition also predicts the ellipticity of the orbit.

The fractal orbits with radii $r \propto 2^k$, $k = 3, 4$ have radii proportional to $2^3 = 8, 2^4 = 16$ and fit rather satisfactorily with the circular Bohr orbits with $n = 3, 4$ and radii proportional to $3^2 = 9, 4^2 = 16$ (Mercury and Venus). Earth and the outer planets would correspond to $2^{k+4+1/2}$ $k = 0, 1, ..6$ with $2^{4+1/2} \simeq 22.6$ as an approximation of $n^2 = 25$ for $n = 5$ orbit (Earth) in the

Bohr model: the 2-adic length scale is 10 per cent smaller than the prediction of Bohr model for a circular orbit.

Since the inner and outer planets seem to be separate systems, one can consider the possibility that $\beta_{0,1}(out)$ for solar-planet gravitational flux tubes satisfies $\beta_{0,1}(out) = 1.1\beta_{0,1}(in)$. This requires $\beta_{0,1} \leq .9$. The values $k/2 \in \{2,3,4,4+1/2,5,6\}$ would provide a reasonable fit for the outer planets and would correspond to $n \in \{1,2,3,4,5,6\}$.

Remark: The Bohr orbits are assumed to correspond to magnetic flux tubes carrying dark matter delocalized along the orbit. The wave function for the dark matter BEC along the orbit could be a restriction of the 3-D hydrogen wave function at the orbit. For a circular orbit the angular dependence would be trivial in accordance with the interpretation that the angular momentum vanishes for these orbits in a quantum sense. Ordinary matter would be localized at the orbit and perform classical motion.

11 Testing of the vision

Eventually the basic concepts of TGD applied to condensed matter physics should be tested. The following lists some challenges.

11.1 Observation of dark matter

The observation of dark matter as $h_{eff} = nh_0$ phases in condensed matter systems is one basic goal (`allb`qcritdark1,qcritdark2,qcritdark3). Macroscopic quantum phases, emergence of additional degrees of freedom, and the effective increase of the dimension of the momentum space from 3 $3k(n,K)$, where k is a numerical factor determined by the number K of particles forming the Galois confined states and by the dimension n of the extensionof rationals, are possible. Also photon scattering via the formation of polaritons could allow us to "see" the structure of dark matter at the level of MB as an interference pattern. The analog of X-ray diffraction would be in question.

11.2 Topological physics at space-time and imbedding space levels

The basic new physics element is the topological physics in the TGD sense based on non-trivial space-time topology at the fundamental level.

Some examples are in order.

1. Magnetic flux tubes are always closed, which means non-trivial first homotopy making possible the topological variant of the geometric phase.

2. Flux tube braidings would be a basic concept of topological hydrodynamics. Reconnections as changes of braid topology would be central and bring in 2-braids and knots of 2-D flux sheets in 4-D space-times (also intersections at discrete points replace links of 1-braids).

3. In TGD inspired biology systems have U-shaped flux tubes as tentacles with which they generate connections to the environment by reconnecting in which two U-shaped flux tubes of different systems such as molecules form a pair of flux tubes.

   For instance, friction could be due to the formation of flux tube pairs. Static friction would be generated and the de-reconnection of flux tube pairs would require energy.

Also topological defects due to the embedding space topology are possible. The monopole flux reflects the non-trivial topology of CP_2. Skyrmions result from the constraint that the ball $B^3 \subset M^4$ is mapped to the sphere S^3 of $E^4 \subset M^8$ or equivalently of CP_2.

11.3 The new view about gauge fields

1. The new view about gauge fields and also electromagnetic fields relies on flux tubes. Flux tubes appear as two types: monopole flux tubes and non-monopole ones. Monopole flux tubes require no current to preserve the magnetic field.

This would explain magnetic fields in cosmic scales, why Earth's magnetic field has not disappeared [L7], and also the huge magnetic fields of magnetars and neutron stars. Could the fields H, M, and B of Maxwell's theory correspond to monopole fields, non-monopole fields induced by the motion of their flux quanta, and to their sum $B = M + H$.

2. The twistor lift of TGD [L13, L15] predicts that also M^4 should have Kähler structure defined by a self-dual constant Kähler form for which the electric part would be imaginary. This implies a global CP breaking in M^4 that could induce a matter-antimatter asymmetry. 3 quarks would prefer to form baryons and antiquarks to form leptons as 3 antiquarks composites in primordial Universe and after the annihilation the remaining baryons would represent matter and leptons antimatter. This is possible only by the TGD view about color [L22, L34].

   The mechanism of CP (T) violation could be essentially the same as in the topological insulators destroying the boundary conductivity by T violation. In the condensed matter case the magnetic field would receive $U(1)$ contributions from both CP_2 and M^4 degrees of freedom. The magnetic interaction energy with spin would have opposite signs for opposite spin directions and lead to CP and T violation. For cosmic strings and flux tubes the M^4 magnetic part would be small, which would explain the smallness of the CP violation. Since M^4 Kähler form contributes also to the $U(1)$ part of em and Z^0 fields, it could have small effects also at the level of condensed matter if M^4 projection of the flux tube is 4-D.

3. Wormhole contacts identified as pieces of deformed CP_2 type extremals serve as basic building bricks of elementary particles. The wormhole throats are identified as partonic surfaces and their orbits are light-like curves performing zitterbewegung. One can assign to them a Kac-Moody type algebra with non-negative conformal weights. This algebra is very much like gauge algebra but not quite. For instance, there is a hierarchy of representations for which only the generators with conformal weight larger than some maximal conformal weight h_{max} annihilate the physical states. Could these analogs of gauge algebras assignable to $M^2 \times CP_2$ isometries allow a realization of synthetic gauge groups acting also in M^4 spin degrees of freedom [D8] (`https://cutt.ly/4Wy39B5`?

11.4 Number theoretical physics

Number theoretical physics brings in new elements and involves in an essential manner $M^8 - H$ duality.

1. The hierarchy of effective Planck constants and p-adic physics as physics of cognition involving p-adic length scale physics means a completely new element of quantum theory central for understanding of various supra phases.

2. Galois confinement is a central notion. Quantum states would be Galois singlets above the quantum coherence scale defined by h_{eff} and become unconfined states below this scale. The situation is highly reminiscent of color confinement. At M^8 level, the assumption that momenta are algebraic integers for the extension of rationals considered implies that confined states have total momenta, which are ordinary integers and that the rational integer parts momenta of K composite particles are identical. This implies a reduction of translational degrees of freedom so that the density dn/dE of confined states increases and among other things leads to a reduction of Fermi energy.

   Galois confinement could serve as a universal mechanism for the formation of bound states: this includes atoms and molecules, atomic nuclei, and hadrons. Color confinement can be one particular example of this if Galois group is represented as a subgroup of color group: Z_3 is the obvious guess but also more general discrete subgroups of $SU(3)$ are possible. Also the discrete subgroups of the rotation group $SO(3)$ and its covering group $SU(2)$ could be representable as Galois groups and appear in the ADE hierarchy for inclusions of HFFs. $M^8 - H$ duality would give a very concrete ideas about the momentum space and space-time geometries of the bound states. Momenta in M^8 would form a representation of Galois group mapped to H by $M^8 - H$ duality.

3. The number theoretical phase transitions changing the polynomial that determines $X^4 \subset M^8$ and therefore the extension of rationals and the Galois group as symmetry group would be a new element. Discrete degrees of freedom would appear or disappear. The scaling of the number of states within the Fermi ball could be one signature. Extensions could also give rise to quasicrystals.

   The change of the fidelity described as the metric of the parametrized space of quantum states would take place. Fidelity would be coded by the Kähler metric of WCW and geometric phase by the Kähler form of WCW. This is because, the WCW Kähler metric induces the metric of quantum states depending on the parameters coding for the X^4 as a point of WCW.

4. Negentropy Maximization Principle (NMP) and adelic physics provide a new view about quantum measurements and about second law. In particular, a vision about how the information about measurement is stored in the space-time geometry modified in the measurement, emerges.

11.5 ZEO and new view about quantum measurement theory and thermodynamics

ZEO allows "big" state function reductions (BSFRs) in long scales. If time reversal indeed occurs, it induces a long lasting effective time reversal at the level of ordinary matter (genuine time reversals at this level last a very short time). Dissipation would effectively occur with an opposite arrow of time and lead to the formation of self-organization patterns [L19]. The findings of Minev et al discussed in [L16] support the new view about quantum theory.

The most dramatic implications would be to biology. In particular, homeostasis could be understood as self-organized quantum critical (SOQC) [L41]. Condensed matter systems in the presence of energy feed playing the role of metabolic energy feed could exhibit primitive aspects of living systems.

Note that at the QFT limit most of the information about the TGD based new physics is lost since both space-time topology and number theoretic structure is lost so that QFT is not able to test the relevant effects. However, it might be possible to make this hidden level visible.

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A Appendix

A.1 Comparison of TGD with other theories

Table 1 compares GRT and TGD and **Table 2** compares standard model and TGD.

A.2 Glossary and figures

The following glossary explains some basic concepts of TGD and TGD inspired biology.

- ***Space-time as surface***. Space-times can be regarded as 4-D surfaces in an 8-D space $M^4 \times CP_2$ obtained from empty Minkowski space (M^4) by adding four small dimensions (CP_2). The study of field equations characterizing space-time surfaces as "orbits" of 3-surfaces (3-D generalization of strings) forces the conclusion that the topology of space-time is non-trivial in all length scales.

- ***Geometrization of classical fields***. Both weak, electromagnetic, gluonic, and gravitational fields are known once the space-time surface in H as a solution of field equations is known.

  Many-sheeted space-time (see **Fig. 4**) consists of space-time sheets with various length scales with smaller sheets being glued to larger ones by ***wormhole contacts*** (see **Fig. 5**) identified as the building bricks of elementary particles. The sizes of wormhole contacts vary but are at least of CP_2 size (about 10^4 Planck lengths) and thus extremely small.

  Many-sheeted space-time replaces reductionism with ***fractality*** . The existence of scaled variants of physics of strong and weak interactions in various length scales is implied, and biology is especially interesting in this respect.

	GRT	TGD
Scope of geometrization	classical gravitation	all interactions and quantum theory
Spacetime		
Geometry	abstract 4-geometry	sub-manifold geometry
Topology	trivial in long length scales	many-sheeted space-time
Signature	Minkowskian everywhere	also Euclidian
Fields		
classical	primary dynamical variables	induced from the geometry of H
Quantum fields	primary dynamical variables	modes of WCW spinor fields
Particles	point-like	3-surfaces
Symmetries		
Poincare symmetry	lost	Exact
GCI	true	true - leads to SH and ZEO
	Problem in the identication of coordinates	$H = M^4 \times CP_2$ provides preferred coordinates
Super-symmetry	super-gravitation	super variant of H: super-surfaces
Dynamics		
Equivalence Principle	true	true
Newton's laws and		
notion of force	lost	generalized
Einstein's equations	from GCI and EP	remnant of Poincare invariance at QFT limit of TGD
Bosonic action	EYM action	Kähler action + volume term
Cosmological constant	suggested by dark energy	length scale dependent coefficient of volume term
Fermionic action	Dirac action	Modified Dirac action for induced spinors
Newton's constant	given	predicted
Quantization	fails	Quantum states as modes of WCW spinor field

Table 1: Differences and similarities between GRT and TGD

	SM	TGD
Symmetries		
Origin	from empiria	reduction to CP_2 geometry
Color symmetry	gauge symmetry	isometries of CP_2
Color	analogous to spin	analogous to angular momentum
Ew symmetry	gauge symmery	holonomies of CP_2
Symmetry breaking	Higgs mechanism	CP_2 geometry
Spectrum		
Elementary particles	fundamental	consist of fundamental fermions
Bosons	gauge bosons, Higgs	gauge bosons, Higgs, pseudo-scalar
Fundamental fermions	quarks and leptons	quarks: leptons as local 3-quark composites
Dynamics		
Degrees of freedom	gauge fields, Higgs, and fermions	3-D surface geometry and spinors
Classical fields	gauge fields, Higgs	induced spinor connection
	SU(3) Killing vectors of CP_2	
Quantal degrees of freedom	gauge bosons,Higgs,	quantized induced spinor fields
Massivation	Higgs mechanism	p-adic thermodynamics with superconformal symmetry

Table 2: Differences and similarities between standard model and TGD

- ***Topological field quantization (TFQ)*** . TFQ replaces classical fields with space-time quanta. For instance, magnetic fields decompose into space-time surfaces of finite size representing flux tubes or -sheets. Field configurations are like Bohr orbits carrying "archetypal" classical field patterns. Radiation fields correspond to topological light rays or massless extremals (MEs), magnetic fields to magnetic flux quanta (flux tubes and sheets) having as primordial representatives "cosmic strings", electric fields correspond to electric flux quanta (e.g. cell membrane), and fundamental particles to CP_2 type vacuum extremals.

- ***Field body*** (FB) and ***magnetic body*** (MB). Any physical system has field identity - FB or MB - in the sense that a given topological field quantum corresponds to a particular source (or several of them - e.g. in the case of the flux tube connecting two systems).

  Maxwellian electrodynamics cannot have this kind of identification since the fields created by different sources superpose. Superposition is replaced with a set theoretic union: only the *effects* of the fields assignable to different sources on test particle superpose. This makes it possible to define the QFT limit of TGD.

- ***p-Adic physics*** [K15] as a physics of cognition and intention and the fusion of p-adic physics with real number based physics are new elements.

- ***Adelic physics*** [L11, L14] is a fusion of real physics of sensory experience and various p-adic physics of cognition.

- ***p-Adic length scale hypothesis*** states that preferred p-adic length scales correspond to primes p near powers of two: $p \simeq 2^k$, k positive integer.

- A ***Dark matter hierarchy*** realized in terms of a hierarchy of values of effective Planck constant $h_{eff} = nh_0$ as integers using $h_0 = h/6$ as a unit. Large value of h_{eff} makes possible macroscopic quantum coherence which is crucial in living matter.

- ***MB as an intentional agent using biological body (BB) as a sensory receptor and motor instrument*** . The personal MB associated with the living body - as opposed to larger MBs assignable with collective levels of consciousness - has a hierarchical onion-like layered structure and several MBs can use the same BB making possible remote mental interactions such as hypnosis [L5].

- ***Cosmic strings Magnetic flux tubes*** belong to the basic extremals of practically any general coordinate invariant action principle. Cosmic strings are surfaces of form $X^2 \times Y^2 \subset M^4 \times CP_2$. X^2 is analogous to string world sheet. Cosmic strings come in two varieties and both seem to have a deep role in TGD.

  Y^2 is either a complex or Lagrangian 2-manifold of CP_2. Complex 2-manifold carries monopole flux. For Lagrangian sub-manifold the Kähler form and magnetic flux and Kähler action vanishes. Both types of cosmic strings are are simultaneous extremals of both Kähler action and volume action: this holds true quite generally for preferred extremals.

  Cosmic strings are unstable against perturbations thickening the 2-D M^4 projection to 3-D or 4-D: this gives rise to monopole (see **Fig. ??**) and non-monopole magnetic flux tubes. Using $M^2 \times Y^2$ coordinates, the thickening corresponds to the deformation for which $E^2 \subset M^4$ coordinates are not constant anymore but depend on Y^2 coordinates.

- ***Magnetic flux tubes and sheets*** serve as "body parts" of MB (analogous to body parts of BB), and one can speak about magnetic motor actions. Besides concrete motion of flux quanta/tubes analogous to ordinary motor activity, basic motor actions include the contraction of magnetic flux tubes by a phase transition possibly reducing Planck constant, and the change in thickness of the magnetic flux tube, thus changing the value of the magnetic field, and in turn the cyclotron frequency. Transversal oscillatory motions of flux tubes and oscillatory variations of the thickness of the flux tubes serve as counterparts for Alfwen waves.

  Reconnections of the U-shaped flux tubes allow two MBs to get in contact based on a pair of flux tubes connecting the systems and temporal variations of magnetic fields inducing motor actions of MBs favor the formation of reconnections.

  In hydrodynamics and magnetohydrodynamics reconnections would be essential for the generation of turbulence by the generation of vortices having monopole flux tube at core and Lagrangian flux tube as its exterior.

  Flux tube connections at the molecular level bring a new element to biochemistry making it possible to understand bio-catalysis. Flux tube connections serve as a space-time correlates for attention in the TGD inspired theory of consciousness.

- ***Cyclotron Bose-Einstein condensates (BECs)*** of various charged particles can accompany MBs. Cyclotron energy $E_c = hZeB/m$ is much below thermal energy at physiological temperatures for magnetic fields possible in living matter. In the transition $h \rightarrow h_{eff}$ E_c is scaled up by a fractor $h_{eff}/h = n$. For sufficiently high value of h_{eff} cyclotron energy is above thermal energy $E = h_{eff}\ ZeB/m$. Cyclotron Bose-Einstein condensates at MBs of basic biomolecules and of cell membrane proteins - play a key role in TGD based biology.

- ***Josephson junctions*** exist between two superconductors. In TGD framework, ***generalized Josephson junctions*** accompany membrane proteins such as ion channels and pumps. A voltage between the two super-conductors implies a ***Josephson current***. For a constant voltage the current is oscillating with the ***Josephson frequency*** . The Josephson current emits ***Josephson radiation***. The energies come as multiples of ***Josephson energy***.

  In TGD generalized Josephson radiation consisting of dark photons makes communication of sensory input to MB possible. The signal is coded to the modulation of Josephson frequency depending on the membrane voltage. The cyclotron BEC at MB receives the radiation producing a sequence of resonance peaks.

- ***Negentropy Maximization Principle*** (NMP). NMP [K9] [L41] is the variational principle of consciousness and generalizes SL. NMP states that the negentropy gain in SFR is non-negative and maximal. NMP implies SL for ordinary matter.

- ***Negentropic entanglement*** (NE). NE is possible in adelic physics and NMP does not allow its reduction. NMP implies a connection between NE, the dark matter hierarchy, p-adic physics, and quantum criticality. NE is a prerequisite for an experience defining abstraction as a rule having as instances the state pairs appearing in the entangled state.

- ***Zero energy ontology (ZEO)*** In ZEO physical states are pairs of positive and negative energy parts having opposite net quantum numbers and identifiable as counterparts of initial and final states of a physical event in the ordinary ontology. Positive and negative energy parts of the zero energy state are at the opposite boundaries of a ***causal diamond*** (CD, see **Fig. 12**)) defined as a double-pyramid-like intersection of future and past directed light-cones of Minkowski space.

  CD defines the "spot-light of consciousness": the contents of conscious experience associated with a given CD is determined by the space-time sheets in the imbedding space region spanned by CD.

- ***SFR*** is an acronym for state function reduction. The measurement interaction is universal and defined by the entanglement of the subsystem considered with the external world [L21] [K38]. What is measured is the density matrix characterizing entanglement and the outcome is an eigenstate of the density matrix with eigenvalue giving the probability of this particular outcome. SFR can in principle occur for any pair of systems.

  SFR in ZEO solves the basic problem of quantum measurement theory since the zero energy state as a superposition of classical deterministic time evolutions (preferred extremals) is replaced with a new one. Individual time evolutions are not made non-deterministic.

  One must however notice that the reduction of entanglement between fermions (quarks in TGD) is not possible since Fermi- and als Bose statistics predicts a maximal entanglement. Entanglement reduction must occur in WCW degrees of freedom and they are present because point-like particles are replaced with 3-surfaces. They can correspond to the number theoretical degrees of freedom assignable to the Galois group - actually its decomposition in terms of its normal subgroups - and to topological degrees of freedom.

- ***SSFR*** is an acronym for "small" SFR as the TGD counterpart of weak measurement of quantum optics and resembles classical measurement since the change of the state is small [L21] [K38]. SSFR is preceded by the TGD counterpart of unitary time evolution replacing the state associated with CD with a quantum superposition of CDs and zero energy states associated with them. SSFR performs a localization of CD and corresponds to time measurement with time identifiable as the temporal distance between the tips of CD. CD is scaled up in size - at least in statistical sense and this gives rise to the arrow of time.

  The unitary process and SSFR represent also the counterpart for Zeno effect in the sense that the passive boundary of CD as also CD is only scaled up but is not shifted. The states remain unchanged apart from the addition of new fermions contained by the added part of the passive boundary. One can say that the size of the CD as analogous to the perceptive field means that more and more of the zero energy state at the passive boundary becomes visible. The active boundary is however both scaled and shifted in SSFR and states at it change. This gives rise to the experience of time flow and SSFRs as moments of subjective time correspond to geometric time as a distance between the tips of CD. The analog of unitary time evolution corresponds to "time" evolution induced by the exponential of the scaling generator L_0. Time translation is thus replaced by scaling. This is the case also in p-adic thermodynamics. The idea of time evolution by scalings has emerged also in condensed matter physics.

- ***BSFR*** is an acronym for "big" SFR, which is the TGD counterpart of ordinary state function reduction with the standard probabilistic rules [L21] [K38]. What is new is that the arrow of time changes since the roles of passive and active boundaries change and CD starts to increase in an opposite time direction.

  This has profound thermodynamic implications. Second law must be generalized and the time corresponds to dissipation with a reversed arrow of time looking like self-organization for an observed with opposite arrow of time [L19]. The interpretation of BSFR is as analog of biological death and the time reversed period is analogous to re-incarnation but with non-standard arrow of time. The findings of Minev et al [L16] give support for BSFR at atomic level. Together with h_{eff} hierarchy BSFR predicts that the world looks classical in all scales for an observer with the opposite arrow of time.

A.3 Figures

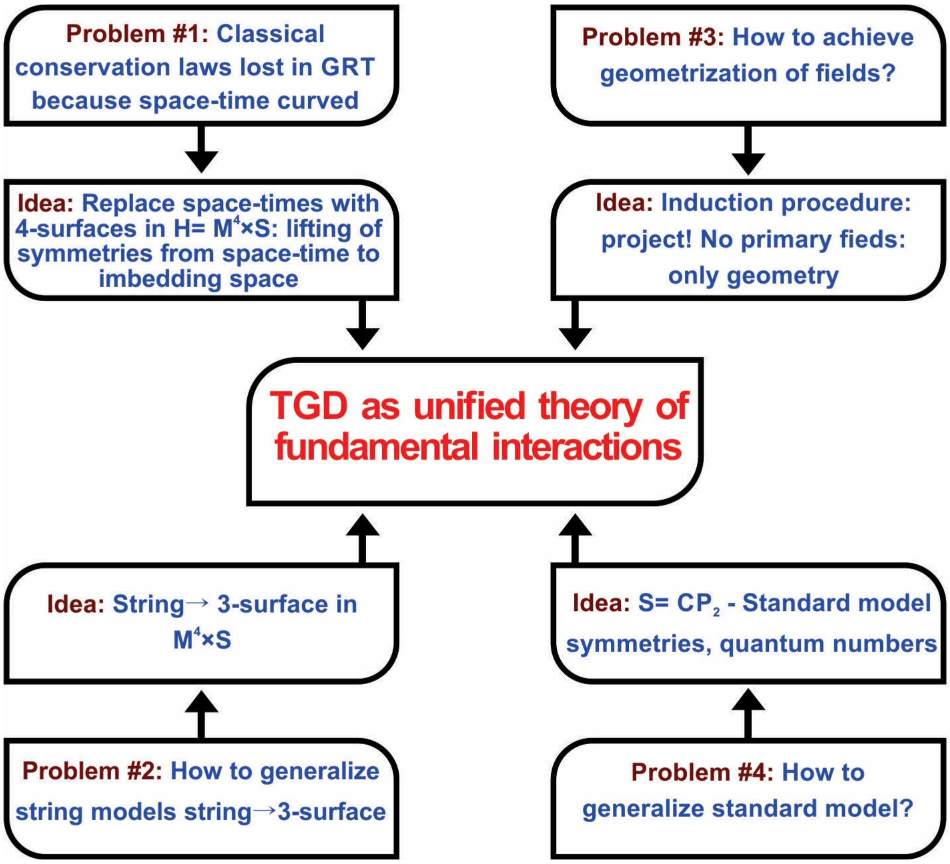

Figure 1: The problems leading to TGD as their solution.

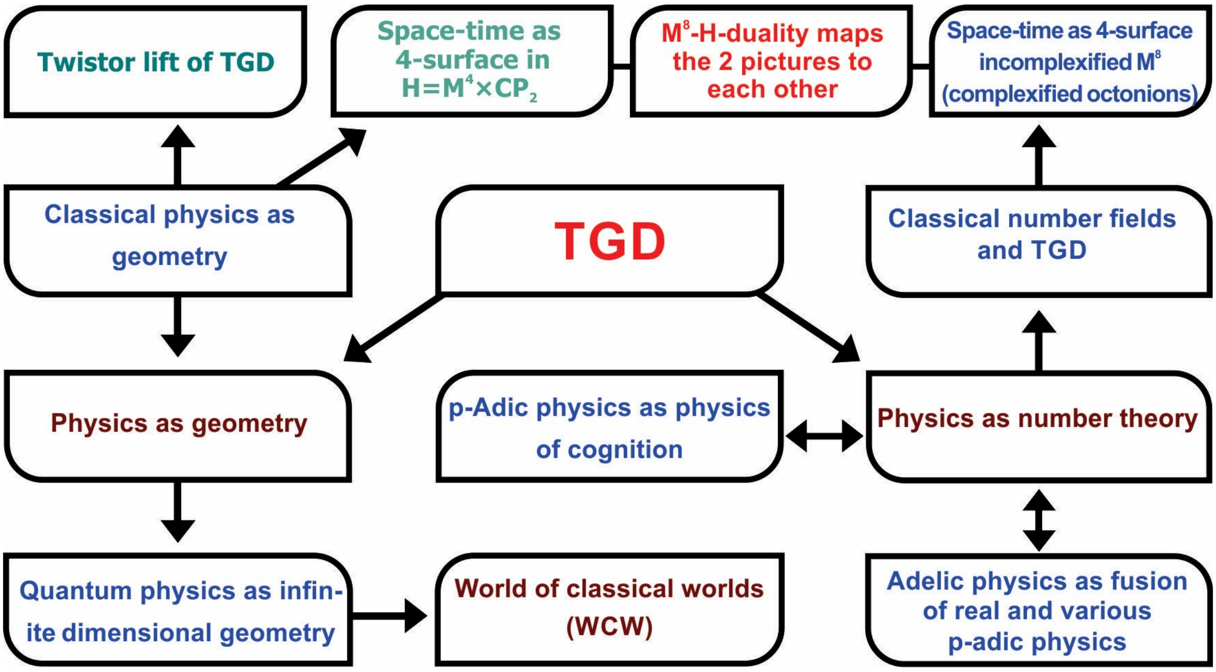

Figure 2: TGD is based on two complementary visions: physics as geometry and physics as number theory.

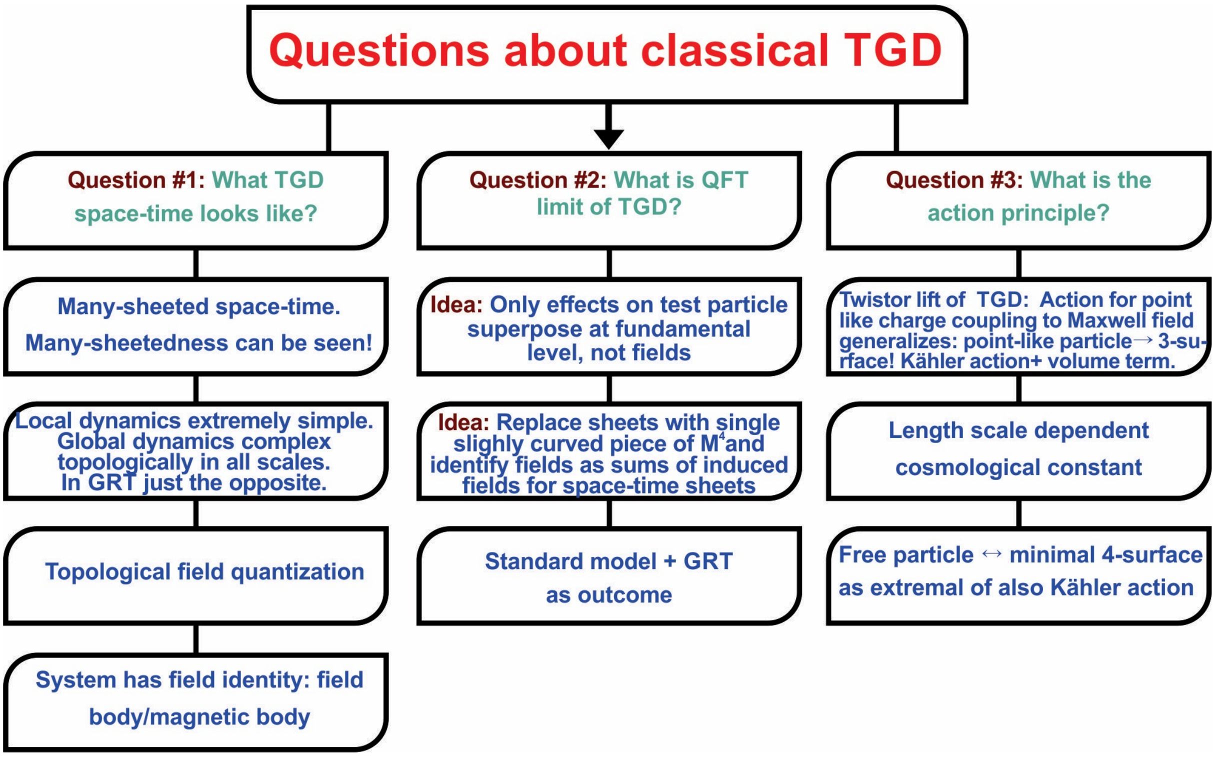

Figure 3: Questions about classical TGD.

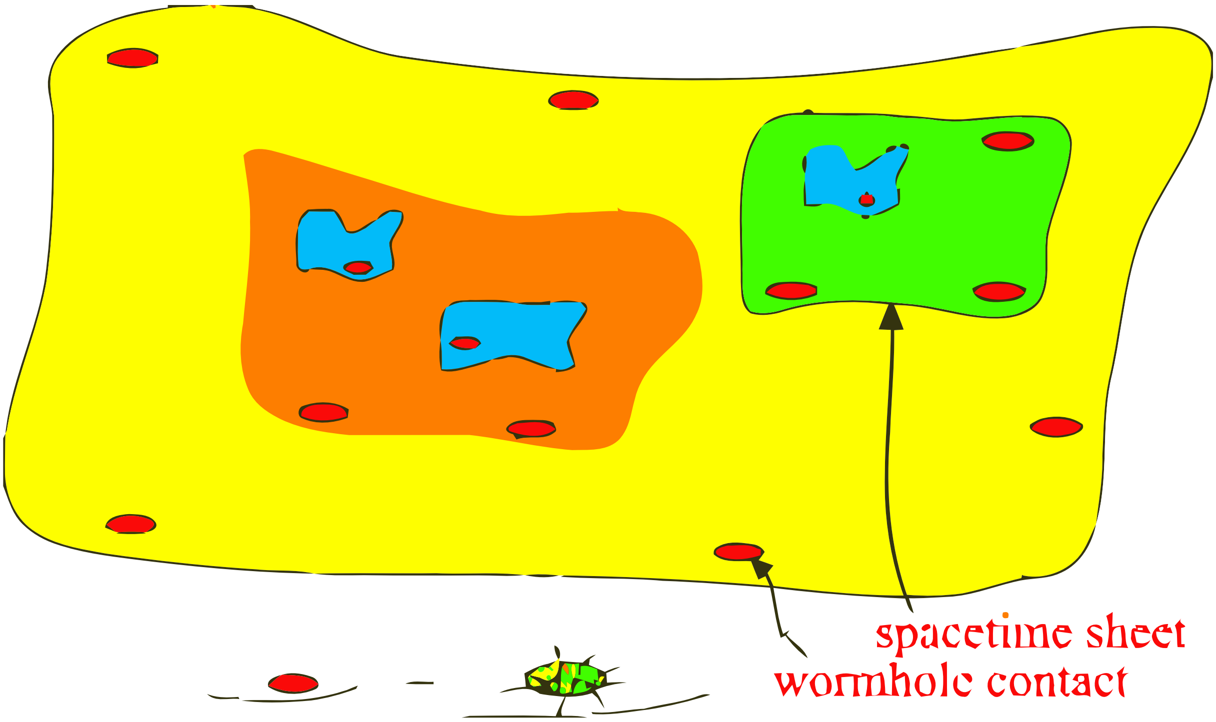

Figure 4: Many-sheeted space-time.

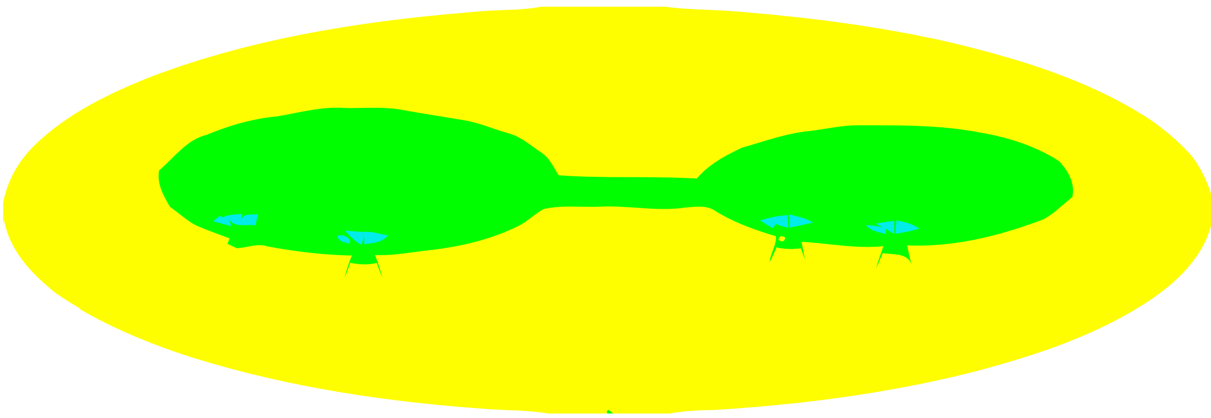

Figure 5: Wormhole contacts.

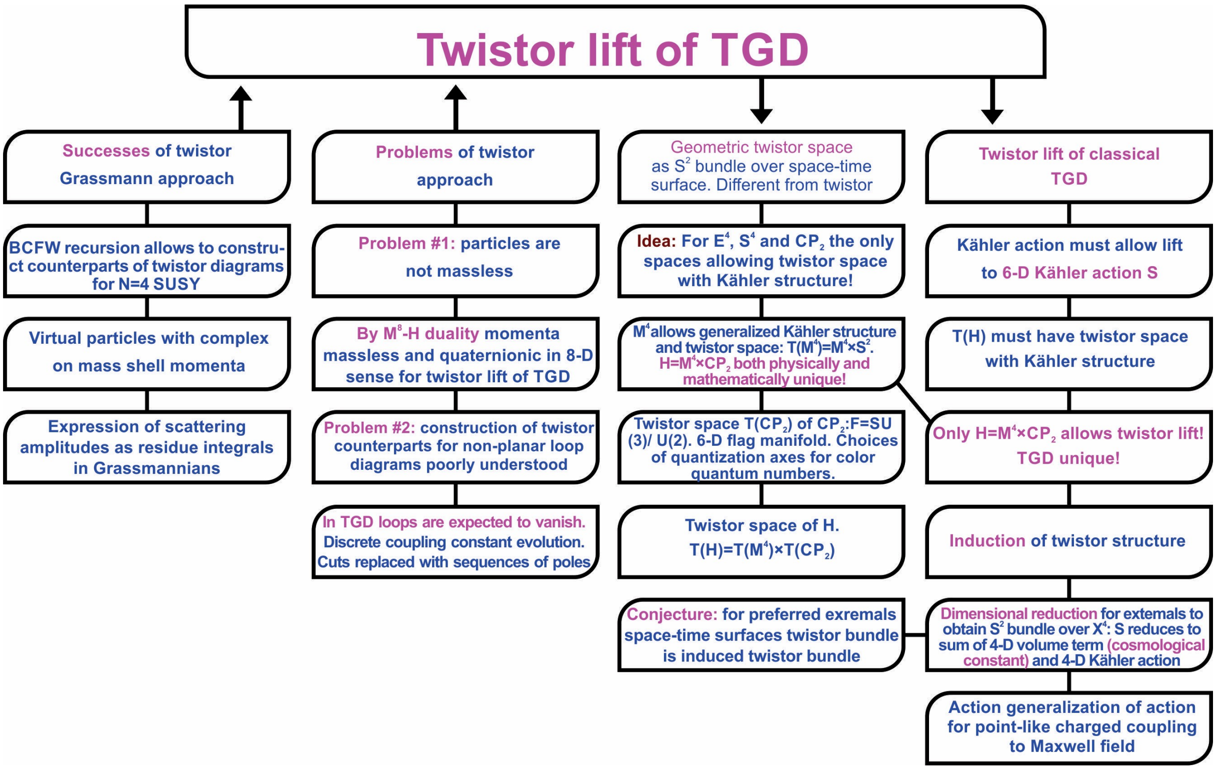

Figure 6: Twistor lift

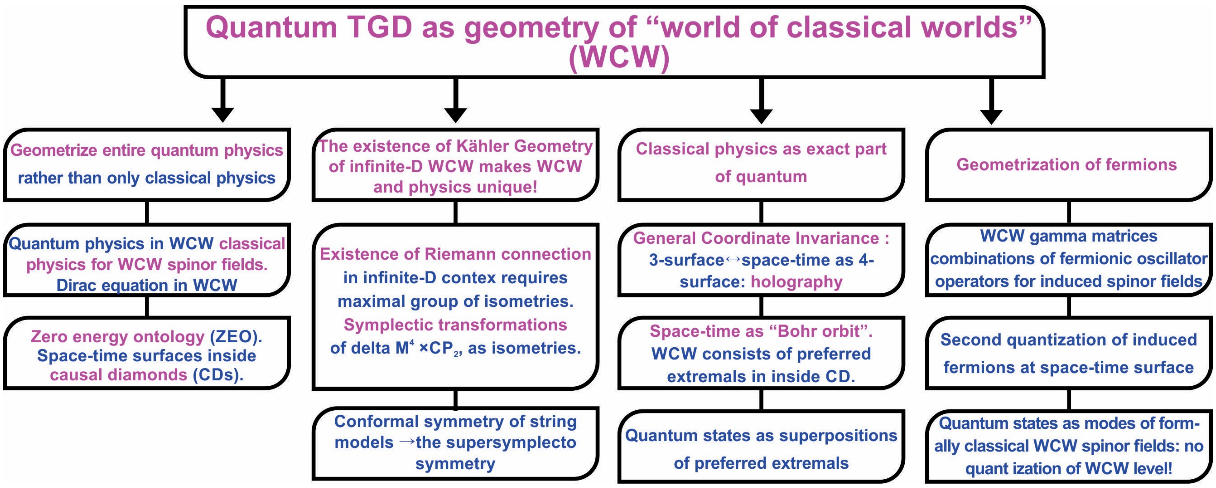

Figure 7: Geometrization of quantum physics in terms of WCW

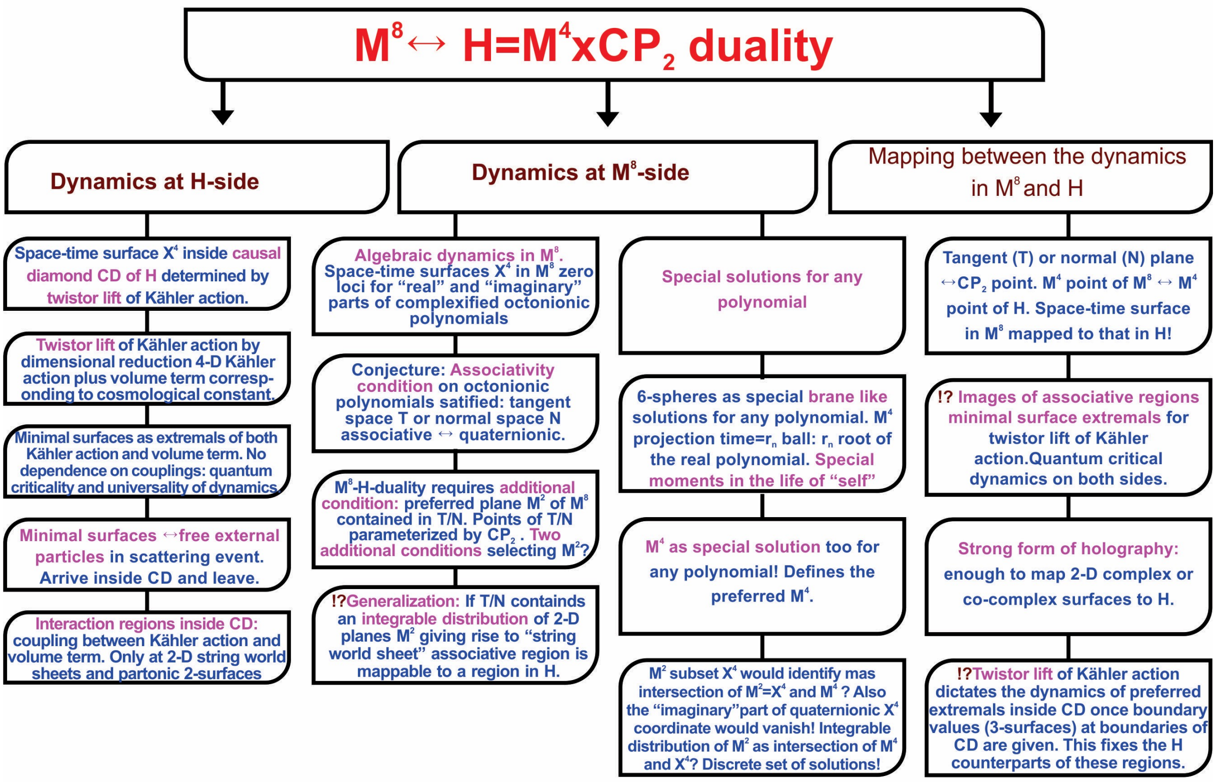

Figure 8: $M^8 - H$ duality

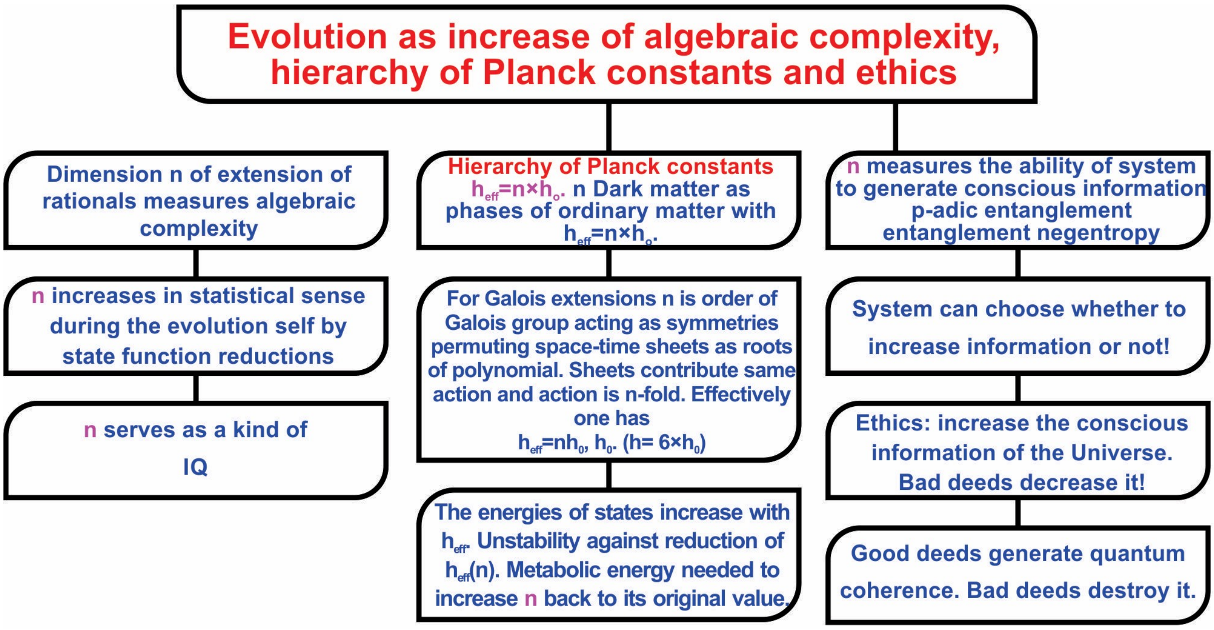

Figure 9: Number theoretic view of evolution

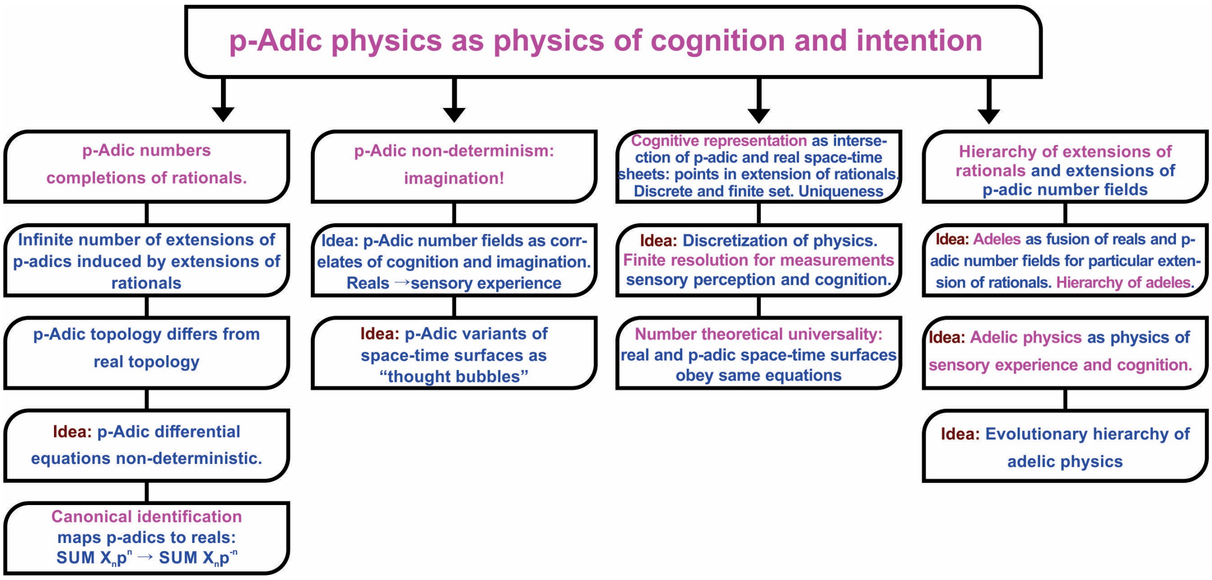

Figure 10: p-Adic physics as physics of cognition and imagination.

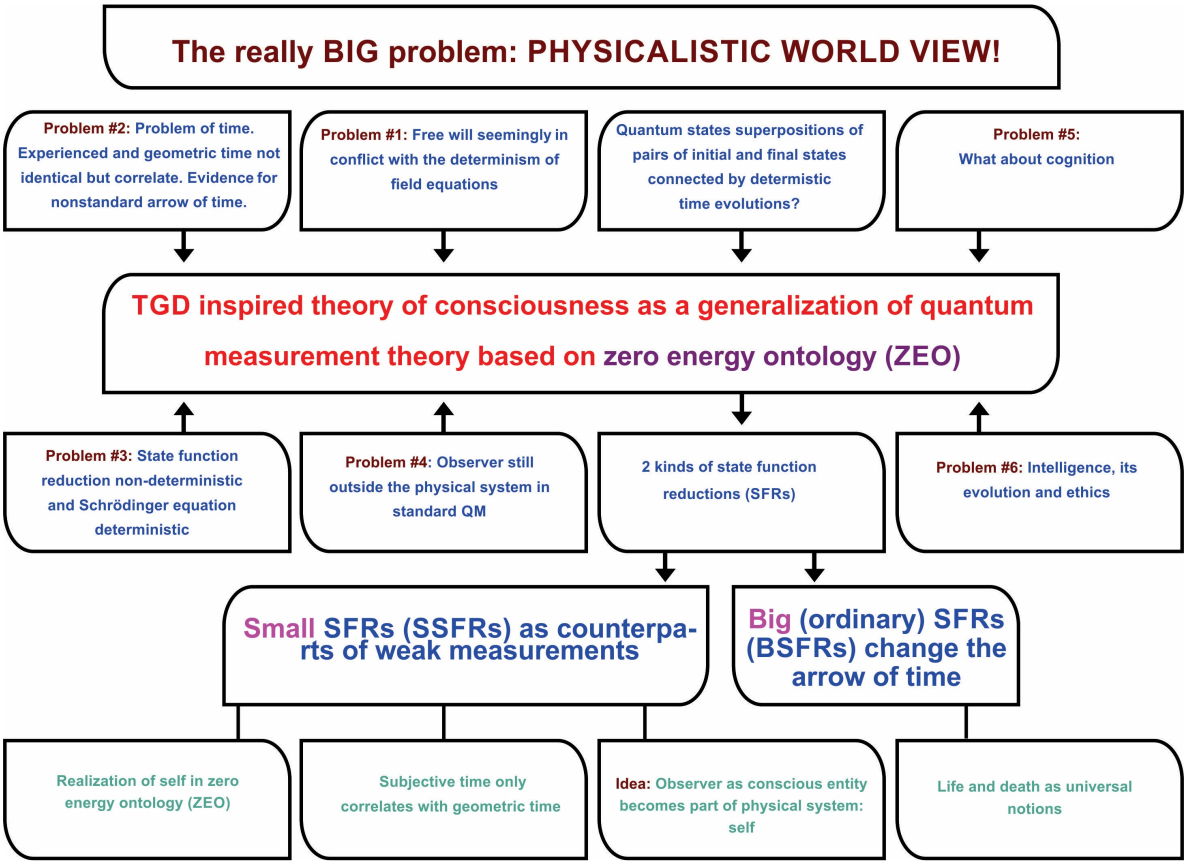

Figure 11: Consciousness theory from quantum measurement theory

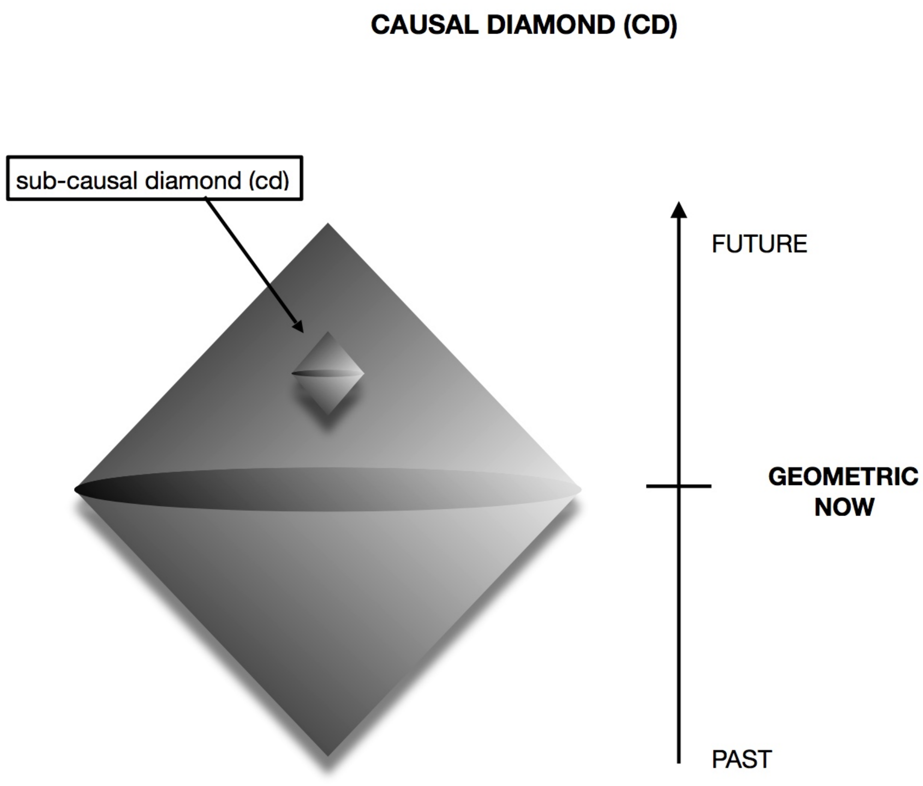

Figure 12: Causal diamond

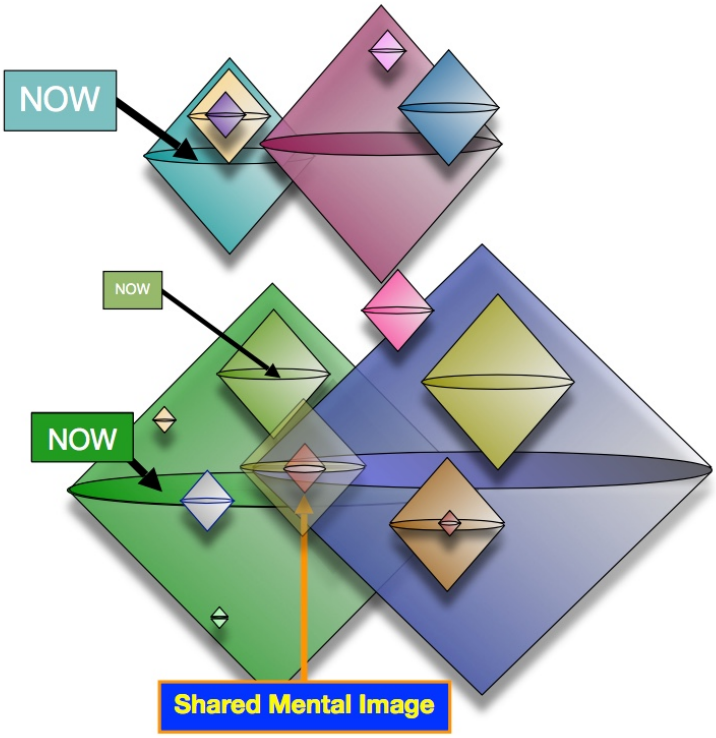

Figure 13: CDs define a fractal "conscious atlas"

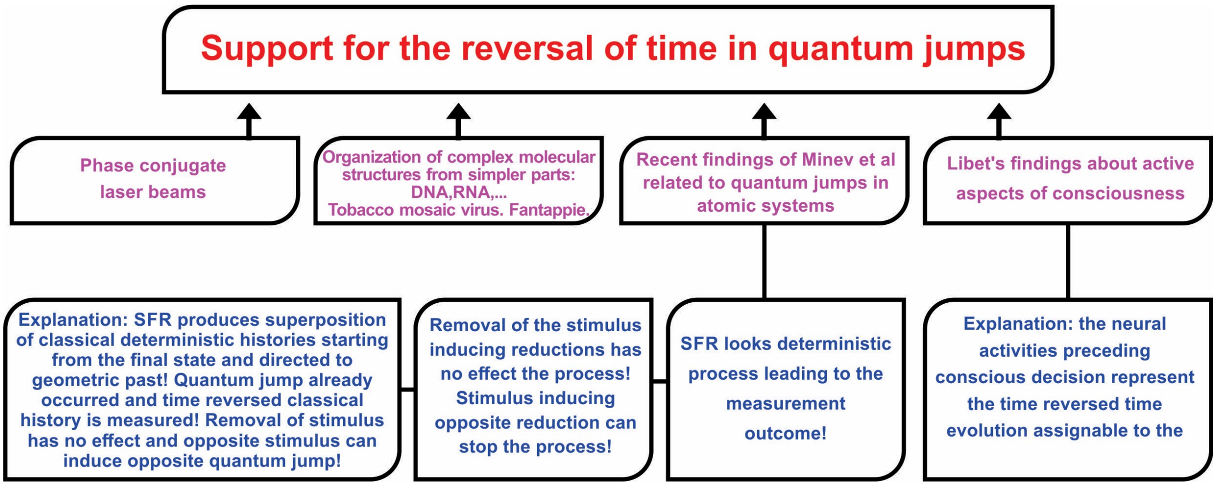

Figure 14: Time reversal occurs in BSFR

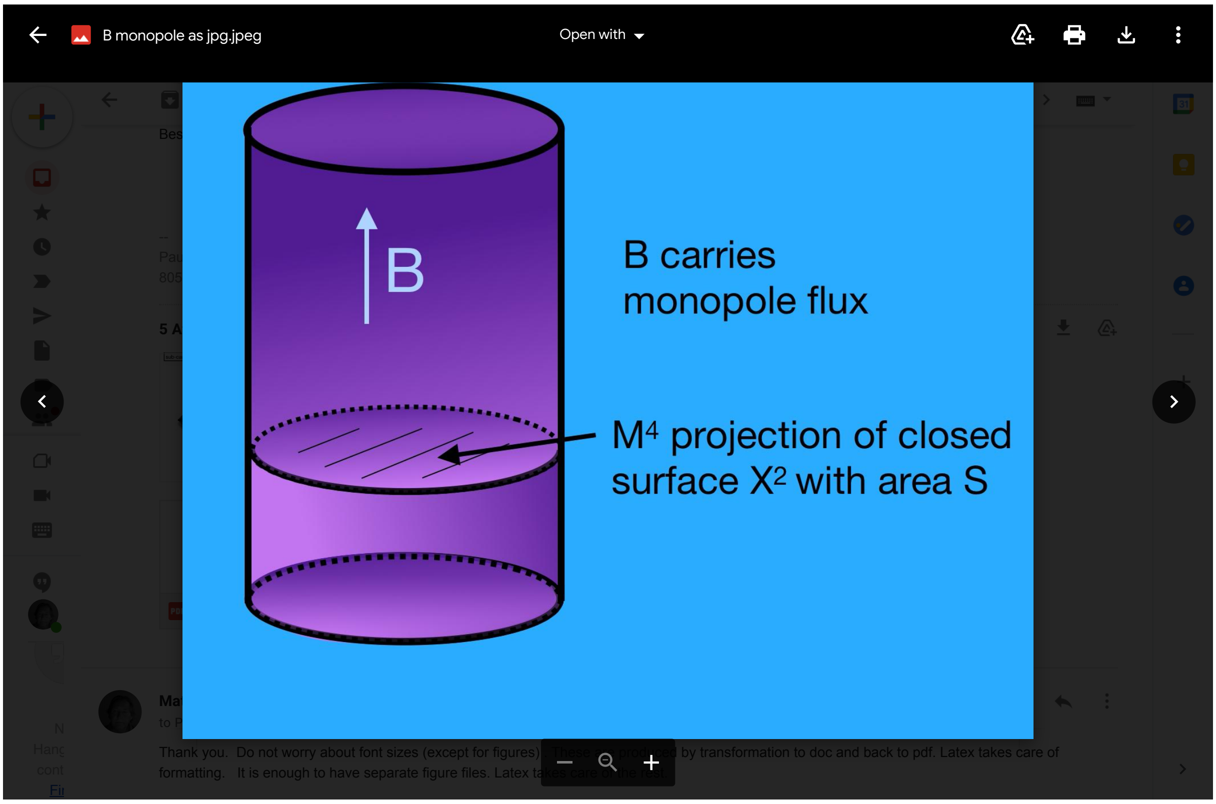

Figure 15: The M^4 projection of a closed surface X^2 with area S defining the cross section for monopole flux tube. Flux quantization $e\oint B\cdot dS = eBS = kh$ at single sheet of n-sheeted flux tube gives for cyclotron frequency $f_c = ZeB/2\pi m = khZ/2\pi mS$. The variation of S implies frequency modulation.

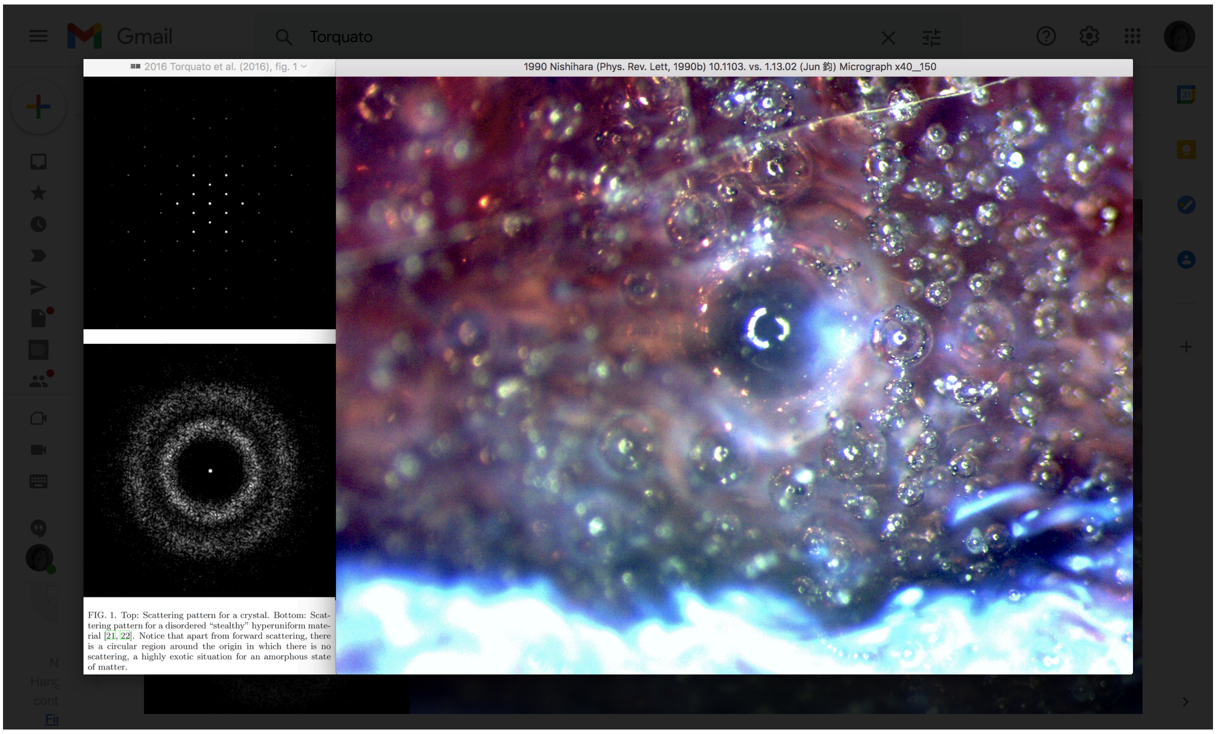

Figure 16: The scattering from a hyperuniform amorphous material shows no scattering in small angles apart from the forward peak (`https://cutt.ly/ZWyLgjk`). This is very untypical in amorphous matter and might reflect the diffraction pattern of dark photons at the magnetic body of the system.

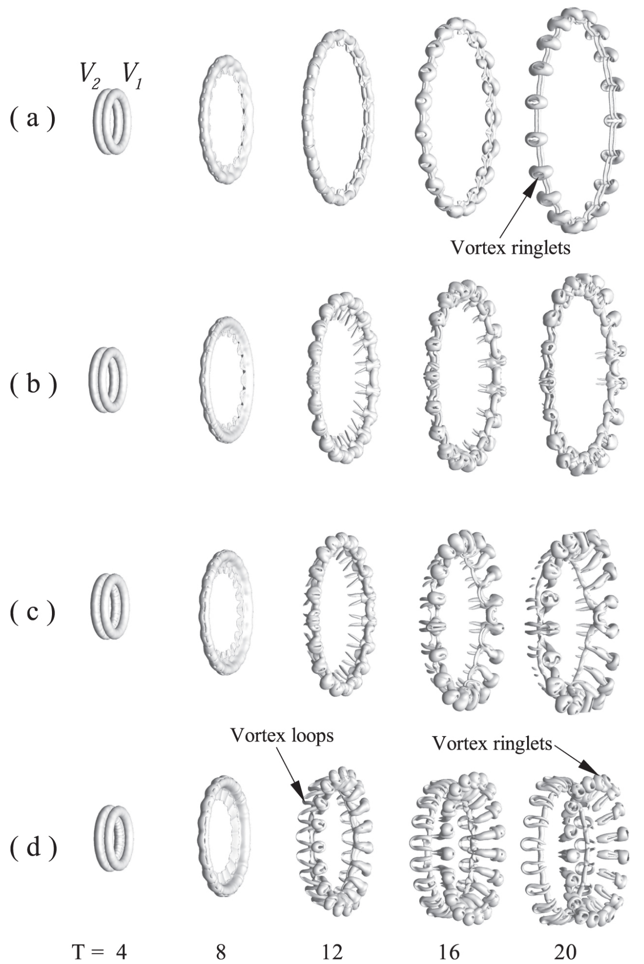

Figure 17: Illustrations of flow patterns resulting in a numerical simulation of a head-on collision of vortex rings with opposite circulations.

Figure 18: Illustrations of flow patterns resulting in a real head-on collision of vortex rings with opposite circulations.