Holography= holomorphy vision: analogues of elliptic curves and partonic orbits

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Abstract

Holography= holomorphy principle allows to solve the extremely nonlinear partial differential equations for the space-time surfaces exactly by reducing them to algebraic equations involving an identically vanishing contraction of two holomorphic tensors of different types. In this article, space-time counterparts for elliptic curves and doubly periodic elliptic functions, in particular Weierstrass function, are considered as an application of the method.

Calabi-Yau manifolds are n-complex-dimensional generalizations of elliptic surfaces and recently found to emerge in a quantum field theory based model for energy production in the scattering of blackholes. This motivates the question whether the holography= holomorphy principle could allow the appearance of hyperbolic variant of K3 surface as 2-complex dimensional CY manifolds as analogs of lattice cells for periodic space-time surfaces, perhaps as nonlinear generalizations of 4-dimensional plane waves with the periods serving as counterparts of wave vectors. The space-time surfaces are identified as intersections of 2 6-D surfaces: could they correspond in some cases to hypercomplex variants of 3-complex-dimensional Calabi-Yau manifolds?

Partonic orbits as surfaces at which the Minkowskian signature of the metric transforms from Minkowskian to Euclidian are identified as orbits of partonic 2-surfaces. Holography= holomorphy vision allows to transform $det(g_4) = 0$ condition to a set 1-D Virasoro conditions labelled by points of the partonic 2-surface.

Contents

1 Introduction

Holography = holography principle [L9, L3, L7, L8] leads to an explicit construction of the solutions of field equations by reducing the field equations from extremely nonlinear partial differential equations to algebraic equations. In this article, elliptic curves and functions are considered as an application.

1.1 Holography=holomorphy as the basic principle

Holography=holomorphy principle allows to solve the field equations for the space-time surfaces exactly by reducing them to algebraic equations.

1. Two functions f_1 and f_2 that depend on the generalized complex coordinates of $H = M^4 \times CP_2$ are needed to solve the field equations. These functions depend on the two complex coordinates ξ_1 and ξ_2 of CP_2 and the complex coordinate w of M^4 and the hypercomplex coordinate u for which the coordinate curves are light-like. If the functions are polynomials, denote them $f_1 \equiv P_1$ and $f_2 \equiv P_2$.

   Assume that the Taylor coefficients of these functions are rational or in the expansion of rational numbers, although this is not necessary either.

2. The condition $f_1 = 0$ defines a 6-D surface in H and so does $f_2 = 0$. This is because the condition gives two conditions (both real and imaginary parts for f_i vanish). These 6-D surfaces are interpreted as analogs of the twistor bundles corresponding to M^4 and CP_2. They have fiber which is 2-sphere. This is the physically motivated assumption, which might require an additional condition stating that ξ_1 and ξ_2 are functions of w as analogs of the twistor bundles corresponding to M^4 and CP_2. This would define the map mapping the twistor sphere of the twistor space of M^4 to the twistor sphere of the twistor space of CP_2 or vice versa. The map need not be a bijection but would be single valued.

   The conditions $f_1 = 0$ and $f_2 = 0$ give a 4-D spacetime surface as the intersection of these surfaces, identifiable as the base space of both twistor bundle analogies.

3. The equations obtained in this way are algebraic equations rather than partial differential equations. Solving them numerically is child's play because they are completely local. TGD is solvable both analytically and numerically. The importance of this property cannot be overstated.

4. However, a discretization is needed, which can be number-theoretic and defined by the expansion of rationals. This is however not necessary if one is interested only in geometry and forgets the aspects related to algebraic geometry and number theory.

5. Once these algebraic equations have been solved at the discretization points, a discretization for the spacetime surface has been obtained.

   The task is to assign a spacetime surface to this discretization as a differentiable surface. Standard methods can be found here. A method that produces a surface for which the second partial derivatives exist because they appear in the curvature tensor.

   An analogy is the graph of a function for which the (y, x) pairs are known in a discrete set. One can connect these points, for example, with straight line segments to obtain a continuous curve. Polynomial fit gives rise to a smooth curve.

6. It is good to start with, for example, second-degree polynomials P_1 and P_2 of the generalized complex coordinates of H.

1.2 How could the solution be constructed in practice?

For simplicity, let's assume that $f_1 \equiv P_1$ and $f_2 \equiv P_2$ are polynomials.

1. First, one can solve for instance the equation $P_2(u, w, \xi_1, \xi_2) = 0$ giving for example $\xi_2(u, w, \xi_1)$ as its root. Any complex coordinates w, ξ_1 or ξ_2 is a possible choice and these choices can correspond to different roots as space-time regions and all must be considered to get the full picture. A completely local ordinary algebraic equation is in question so that the situation is infinitely simpler than for second order partial differential equations. This miracle is a consequence of holomorphy.

2. Substitute $\xi_2(u, w, \xi_1)$ in P_1 to obtain the algebraic function $P_1(u, w, \xi_1, \xi_2(u, w, \xi_1)) = Q_1(u, w, \xi_1)$.

3. Solve ξ_1 from the condition $Q_1 = 0$. Now we are dealing with the root of the algebraic function, but the standard numerical solution is still infinitely easier than for partial differential equations.

   After this, the discretization must be completed to get a space-time surface using some method that produces a surface for which the second partial derivatives are continuous.

Very interesting special cases are polynomials with order not larger than 4 since for these the roots can be solved explicitly. I have proposed that P_2 characterizes the cosmological constant as a correspondence between the twistor spheres of M^4 and CP_2 and is characterized by the winding number. In standard cosmology Λ is a constant of Nature but in TGD it is predicted to have a hierarchy of values. The simplest relationship would be $P_2 = \xi_2 - w^n$, n integer. In this case, one can solve $\xi_2(w)$ and substitute it to P_1 to obtain the condition

$$P_1(\xi_1, \xi_2(w), w, u) = 0 \quad . \tag{1.1}$$

If P_1 as a polynomial of ξ_1 has order lower than 5, the roots of ξ_1 can be solved explicitly. Elliptic curves satisfy the condition

$$\xi_1^2 - w^3 + aw + b = 0 \quad . \tag{1.2}$$

The projections of the w-plane are doubly periodic curves and therefore of special interest. For $P_2 = \xi_2 - w^2$ and $P_1 = \xi_1^2 - w\xi_2 + aw + b$, the space-time surface would be a 4-D analog of an elliptic curve. If a and b depend on u, the 3-surface becomes dynamical.

1.3 Do Calabi-Yau manifolds as generalizations of elliptic curves appear in TGD?

Calabi-Yau manifolds are n-complex-dimensional generalizations of elliptic surfaces and recently found to emerge in a quantum field theory based model for energy production in the scattering of blackholes [?]. This motivates the question whether the holography= holomorphy principle could allow the appearance of hyperbolic variant of K3 surface as 2-complex dimensional CY manifolds as analogs of lattice cells for periodic space-time surfaces, perhaps as nonlinear generalizations of 4-dimensional plane waves with the periods serving as counterparts of wave vectors. The space-time surfaces are identified as intersections of 2 6-D surfaces: could they correspond in some cases to hypercomplex variants of 3-complex-dimensional Calabi-Yau manifolds?

1.4 Holography= holomorphy vision and a more precise view of partonic orbits

A more precise view about the 3-D light-like trajectories of 2-dimensional parton surfaces is developed on the basis of holograpy= holomorphy hypothesis. Partonic orbits are identified as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian so that the metric determinant vanishes. It turns out that this condition generalizes the Virasoro conditions for 1-dimensional light-like curves to the 3-dimensional light-like partonic orbits. Also an explicit procedure for finding the partonic orbits is discussed.

2 Elliptic curves as an application

One can test whether the numerical method works when the equation giving ξ_1 in terms of w can be solved analytically. For elliptic curves $\xi_1 = \xi_1(w)$, which I have discussed already earlier [L1, L3], this is the case.

2.1 Elliptic curves

The third order polynomial characterizing the elliptic curve (see this) can be be expressed in terms of the root of a third order polynomial $P_3(w)$ as

$$E : \xi_1^2 = 4(w - e_1)(w - e_2)(w - e_3) \ , \tag{2.1}$$

One can choose the complex w in such a manner that the equation contains no term proportional to w^2. This is guaranteed if the condition $e_1 + e_2 + e_3 = 0$ holds true. In this case one obtains the form

$$\begin{aligned} & E : \xi_1^2 = 4w^3 - g_2 w - g_3 \ , \qquad , \\ & g_2 = -4(e_1e_2 + e_2e_3 + e_3e_1) \ , \quad g_3 = 4e_1e_2e_3 \ , \quad e_1 + e_2 + e_3 = 0 \ . \end{aligned} \tag{2.2}$$

2.2 Connection with Weierstrass elliptic functions

There is a connection with Weierstrass elliptic functions, which satisfy the differential equation

$$\wp'(z)^2 = 4\wp(z)^3 - g_2\wp(z) - g_3 \ . \tag{2.3}$$

Clearly, By using z as a complex coordinate instead of w, $\xi_1(w)$ and w for the elliptic curve can be expressed in terms of Weierstrass elliptic function, which is a solution of this differential equation

$$\xi_1(w) = \wp'(z) \ , \quad w(z) = \wp(z) \ . \tag{2.4}$$

Elliptic functions are doubly periodic and using $z = \wp^{-1}(w)$ as a complex coordinate instead of w, this periodicity becomes manifest. The solution possesses a discrete conformal symmetry consisting of a discrete subgroup of 2-D translations and this gives rise to a lattice structure. This conforms with the fact that the elliptic curve, as a compact 2-D surface in the space spanned by coordinates (ξ_1, w) has the topology of a torus and therefore can allow translations as conformal symmetries. This is the case for the elliptic curves considered.

One can represent torus in a complex plane with coordinate z in terms of Weierstrass elliptic function $\wp$ having a double periodicity in z-plane as conformal symmetries. The torus corresponds to the fundamental domain (2-D lattice cell) obtained by identifying the opposite boundaries of the lattice cell. The periods ω_1 and ω_2 define non-orthogonal directions and their ratio $\tau = \omega_1/\omega_2$ is conformal invariant.

One can solve the fundamental periods ω_1 and ω_2 in the following way. Define the auxiliary quantities

$$a_0 = \sqrt{e_1 - e_3}, \qquad b_0 = \sqrt{e_1 - e_2}, \qquad c_0 = \sqrt{e_2 - e_3}, \ . \tag{2.5}$$

The condition $e_1 + e_2 + e_3 = 0$ allows to eliminate e_3 so that one has

$$a_0 = \sqrt{-e_2}, \qquad b_0 = \sqrt{e_1 - e_2}, \qquad c_0 = \sqrt{-e_1}, \ . \tag{2.6}$$

The fundamental periods ω_1 and ω_2 for the elliptic curve can be calculated very rapidly by

$$\omega_1 = \frac{\pi}{\mathrm{M}(a_0, b_0)}, \qquad \omega_2 = \frac{\pi}{\mathrm{M}(c_0, ib_0)} \tag{2.7}$$

Or more explicitly

$$\omega_1 = \frac{\pi}{\mathrm{M}(\sqrt{-e_2}, \sqrt{e_1 - e_2})} \ , \quad \omega_2 = \frac{\pi}{\mathrm{M}(\sqrt{-e_1}, i\sqrt{e_1 - e_2})} \ . \tag{2.8}$$

Here $\mathrm{M}(x, y)$ is defined as arithmo-geometric mean of x and y by a geometric iteration (see this). Assuming $x \geq y \geq 0$ one has

$$a_0 = x, g_0 = y \ , \quad a_{n+1} = (a_n + g_n)/2, g_{n+1} = \sqrt{(a_n g_n)} \ . \tag{2.9}$$

At the limit $n \to \infty$ one has a_{n+1}
$simeqa_n \to a$ and $g_{n+1} \simeq g_n \to g$ and one has $a = (a + g)/2$ and $g = \sqrt{ag}$ implying $a = g$ so that arithmetic and geometric means are identical. Care is required to take the correct sign of square root at each step of iteration (positive in the case considered). The iteration generalizes to the complex case and there probably exist tested programs performing the iteration.

2.3 Elliptic functions and planetary orbits

Weierstrass elliptic functions $\wp$ are periodic in complex plane and this inspires the question of how they rate to the formulas for the elliptic planetary orbits in the gravitational potential $V(r) = k/r$. Choose the mass unit so that the mass is $m = 1$. By spherical symmetry, orbits are planar and angular momentum conservation gives $L = r^2 d\phi/dt$ as a constant of motion. In the radial degree of freedom, energy conservation $E = (dr/dt)^2/2 - k/r + L^2/2r^2$ gives $(dr/dt)^2 = E + k/r - L^2/r^2$. By using $u = 1/r$ as variable, one obtains $(du/dt)^2 = Eu^4 + ku^3 - L^2u^2$ giving $du/dt = \sqrt{Eu^4 + ku^3 - L^2u^2}$, which in turn gives $t = \int du/\sqrt{Eu^4 + ku^3 - L^2u^2}$. This gives the planetary orbit as an elliptic integral. The elliptic integral continued to complex values z of the time coordinate t defines explicitly the inverse of a doubly periodic elliptic function.

This integrand gives an elliptic function (see this), which is more general than $\wp$. The integrand $1/\sqrt{(1 - c^2t^2)(1 + E^2t^2)}$ gives Abelian elliptic functions whereas the integrand $1/\sqrt{(1 - t^2)(1 - k^2t^2)}$ gives Jacobi elliptic functions.

The elliptic integral defines the inverse of the Weierstrass elliptic function $\wp$ only for $E = 0$ so that the polynomial under the square root reduces to a third order polynomial. The integrand reduces to $1/u\sqrt{ku - L^2}$. The square root factor vanishes at $u_0 = L^2/k$ which corresponds to the minimal distance r between the two masses and $u = 0$, which corresponds to $r = \infty$. This corresponds to a critical situation in which elliptic orbit transforms to a parabolic orbit. The absence of periodicity at real axis is consistent with the double periodicity of $\wp$ in the complex plane.

One can transform the integrand to a form appearing in $\wp$ by assuming $k = 4$ and by making a linear coordinate change $u \rightarrow v$, $u = v - v_0$, and choosing v_0 in such a way that the v^2 term under the square root vanishes. The required value of v_0 is $v_0 = -L^2/6k$. The parameters g_2 and g_3 in $4v^3 - g_2 v - g_3$ are given by $g_2 = L^4/24$ and $g_3 = -L^6 2^{-5} 3^{-3}$.

One can calculate the inverse of $z = \wp^{-1}(w)$ (complex analog of time coordinate) for a complex argument w (complex analog of the radial coordinate of a planet at the elliptic orbit) by calculating the complex integral

$$\wp^{-1}(w) = \int_{\gamma(w_0 \rightarrow w)} \frac{1}{\sqrt{4w^3 - g_2 w - g_3}} dw \quad .$$

The integration path γ can be chosen in infinitely many ways and a small deformation does not affect the result. The argument of the square root as a polynomial has three roots and the deformation of the integration path in such a way that the deformed curve passes over a root of $4w^3 - g_2 w - g_3$, the integral changes. This gives rise to the infinitely many-valued nature of $\wp^{-1}$(w). For a root with multiplicity 1, the integrand has $1/\sqrt{w - w_0}$ type singularity as the end point of a cut and since the cut means discontinuity, the integral depends on which side of the cut the integration path goes. For a double root there is a pole.

The connection between planetary dynamics and generalized complex surfaces is intriguing and leads to ask whether the connection is more general so that space-time surfaces defined by the conditions $f_1 = 0, f_2 = 0$ represent some dynamical systems, say periodic systems in spherically symmetric potential. These surfaces should allow interpretation as closed surfaces of CP_2 with coordinates ξ_1 and w. These surfaces are characterized some genus and should correspond to a conformal equivalence class characterized by Teichmueller parameters (in the case of torus assignable to the elliptic functions there is only one modular invariant τ defined by the ratio of complex periods). The condition of being closed might require additional constraints. Could closed surfaces as solutions to the conditions $(f_1, f_2) = (0, 0)$ correspond to nonlinear first order differential equations with $\xi_1 = dE/dz$ and $w = E(z)$ defining higher genus analogs of elliptic curves and elliptic functions?

2.4 Calabi-Yau manifolds as generalizations of elliptic curves

I received a like to a very interesting Nature article reporting the work of Driesse et al on calculation of gravitational scattering amplitudes of blackholes in a quantum field theory model. The title of the article [?] (see this) is "Emergence of Calabi Yau manifolds in high-precision black-hole scattering". There is also popular article (see this) describing the findings. The motivation is that blackholes are are elementary particle-like objects characterized byt only mass, spin, and charge.

Let us look first at the abstract of the article.

When two massive objects (black holes, neutron stars or stars) in our universe fly past each other, their gravitational interactions deflect their trajectories. The gravitational waves emitted in the related bound-orbit system-the binary inspiral-are now routinely detected by gravitational-wave observatories. Theoretical physics needs to provide high-precision templates to make use of unprecedented sensitivity and precision of the data from upcoming gravitational-wave observatories. Motivated by this challenge, several analytical and numerical techniques have been developed to approximately solve this gravitational two-body problem. Although numerical relativity is accurate it is too time-consuming to rapidly produce large numbers of gravitational-wave templates. For this, approximate analytical results are also required. Here we report on a new, highest-precision analytical result for the scattering angle, radiated energy and recoil of a black hole or neutron star scattering encounter at the fifth order in Newton's gravitational coupling G, assuming a hierarchy in the two masses. This is achieved by modifying state-of-the-art techniques for the scattering of elementary particles in colliders to this classical physics problem in our universe. Our results show that mathematical functions related to Calabi-Yau (CY) manifolds, 2n-dimensional generalizations of tori, appear in the solution to the radiated energy in these scatterings. We anticipate that our analytical results will allow the development of a new generation of gravitational-wave models, for which the transition to the bound-state problem through analytic continuation and strong-field resummation will need to be performed.

These findings look interesting from the TGD point of view. Calabi-Yau (CY) manifolds have an arbitrary complex dimension n. They generalize the notion of periodic orbit. In 1-D case orbit becomes a complex 2-D manifold, elliptic surface. But complex differential geometry allows a generalization to n-D real periodic orbits and their complex counterparts.

1. Torus is the simplest CY and 2-real-D elliptic doubly periodic surfaces appearing in complex analysis represent the basic example. I have discussed their representations at the level of space-time surfaces in the framework provided by holography= holomorphy vision. Weierstrass surfaces is one example [L13].

   The periods of planetary orbits in Coulomb force expressible in terms of elliptic integrals very probably led to the notion of elliptic Riemann surfaces by making the time variable complex. Elliptic Riemann surfaces are compact but define doubly periodic structures when represented in complex plane? Could the two periods define analogs of momenta?

2. The K3 surface (see this) is a 4-(real)-dimensional CY manifold and a purely algebraic object having a unique topology. It appears in fourth order G^4 in the calculation. K3 surface allows a Kähler metric. It is not clear to me how unique this metric is. The existence of the Kähler metric is important from the TGD point of view since induced metric codes for the Riemannian geometric aspects of TGD.

   Holography= holomorphy vision reduces TGD to algebraic geometry, which can be also regarded as Riemann geometry. Therefore an interesting question is whether a 4-real-D complex K3 surface could be represented in the TGD framework as a complex surface. Does Euclidean signature prevent this or does the K3 surface have a Minkowskian analog obtained by making the second complex coordinate hypercomplex?

   K3 surface can be represented as Fermat quartic surface $x^4 + y^4 + z^4 + t^4 = 0$ in the twistor space CP_3 assigned M^4, or rather, its compact version. Twistor spaces of M^4 and CP_2 appearing as factors of $H = M^4 \times CP_2$ are unique in the sense that they are the only 4-D spaces allowing twistor bundles with a Kähler metric [L14].

   In TGD, CP_3 generalizes to its hypercomplex variant with one complex coordinate made hyperbolic and corresponds to $SU(3,1)/SU(3) \times U(1)$ [L14]. This generalization allows to identify the base space of the twistor bundle as M^4, rather than its compactified version. The hyperbolic counterpart of the quartic Fermat surface might serve as a one particular space-time surface in holography= holomorphy vision [L4, L7, L11, L14].

   In TGD, a generalized complex manifold is obtained from a complex manifold by making one complex coordinate hypercomplex. $H = M^4 \times CP_2$ and space-time surfaces X^4 in H are generalized complex manifolds. Suppose that the double periodicity of the 2-dimensional case generalizes so that the hyperbolic variant of K3 surface could correspond to a lattice cell of a 4-D periodic structure. Could one assign the hyperbolic counterpart of K3 surface a 4-D variant of a plane wave? This would conform with the view that gravitational waves are involved with the scattering of blackholes. Could kind of representation generalize to all kinds of plane waves and could K3 be one of the simplest examples?

3. The 3-complex-dimensional CYs were not mentioned in the article. They appear in the spontaneous compactification of the string models. Now the topology is not unique and the famous number 10^{500} was introduced as a rough estimate for their number. This turned out to be an untestable and fatal production.

   What a 3-real-dimensional periodic "orbit" and its complex generalization could mean? By holography= holomorphy vision [L4, L7, L11, L14], space-time surfaces are representable as intersections of 2 3-D generalized complex manifolds X^6 and Y^6 in H and could be seen as analogs of twistor spaces for M^4 and CP_2. The twistor space CP_3 is a CY manifold. Also the $SU(3)/U(1) \times U(1)$ as the twistor space of CP_2 is a Kähler manifold [A2]: this makes TGD unique.

   Could it happen that X^6 or Y^6 is a generalized CY manifold with one hypercomplex coordinate? 6-D real periodicity would require double periodicity also in hyperbolic coordinate, which looks unrealistic since by hyper-complex analyticity only the second real hyperbolic coordinate of the pair (u, v) appears as argument in the function pair $(f_1, f_2) : H \to C^2$ defining the space-time surface as its root. It would seem that only one hypercomplex coordinate can allow the periodicity? 3-D (2-D) generalized complex homology would be non-trivial for these 6-surfaces (space-time surfaces).

3 Holography= holomorphy vision and a more precise view of partonic orbits

3-D light-like partonic orbits are a central piece of the TGD view of elementary particles. In the sequel a more precise identification of these surfaces is considered.

It is however useful to start by clarifying some basic aspects of dynamics in the TGD Universe.

1. There are two kinds of degrees of freedom in TGD: geometric, i.e. degrees of freedom of the space-time surface, and fermionic. All elementary particles are made up of fermions and antifermions: bosons emerge. There are no bosonic primary quantum fields.

2. The basic result from the solution of the Dirac equation for H spinor fields, assuming that M^4 has a non-trivial Kähler structure [L12], is that the mass scale of colored partial waves of fermions is given by CP_2 mass scale and there are no free massless gluons or quarks. However, massless color singlets for which the difference in the numbers of quarks and antiquarks is a multiple of three, are possible. This gives baryons and mesons. p-Adic thermodynamics gives small thermal masses for the massless modes states appearing as ground states for the generalization of super-conformal representations.

   Here comes a crucial difference between QCD and TGD. In lattice QCD there would be no $g-2$ anomaly whereas the approach based on the information given by physical hadrons imply the anomaly (see this). In TGD, color singlets, in particular hadrons, are indeed the fundamental objects. The anomaly would be real and the new physics implied by TGD predicts it [L12]. For example, copies of hadron physics at larger mass scales are predicted. Also the color singlets formed from higher color partial waves of quarks and leptons give rise to an infinite number of new hadrons and also leptons: I have called them lepto hadrons and there is evidence for them [K3]. This could not be farther from the notion of the desert assumed in GUTs. It will be exciting to see whether QCD or TGD is right.

3. The arguments of the n-point functions of the second quantized free fermion fields of H (scattering amplitudes) are points of the spacetime surface so that the dynamics of the spacetime surface affects the scattering amplitudes. Effectively, the spacetime surface defines the classical background in terms of the induced fields: induced metric, spinor connection, etc... Free fermion field do not allow pair creation in ordinary QFTs. The possibillity of exotic smooth structures for 4-D space-times comes in rescue here [L10] [?] The exotic smooth structure can be seen as the ordinary smooth structure with defects. Defects define analogs of vertices for the creation of fermion pair interpreted as turning of a fermion line in time direction. Since bosons correspond bound states of fermions and antifermions rather than primary quantum fields, all interaction vertices reduce to this vertex.

A particle can be seen in two ways:

1. Particle as a 3-surface and its Bohr orbit as a four-surface X^4.

2. Particle as a fermion and its orbit, the fermion line, is a light-like curve, maybe even a light-like geodesic line in $M^4 \times CP_2$ or M^4.

The spacetime surface X^4 has a rich anatomy and this leads to a more detailed view of what particles are.

1. X^4 has internal structure and the 3-D partonic orbits define light-like surfaces X^3 at which the Minkoski signature of the surface becomes Euclidean so that the metric determinant vanishes.

2. A fermion line would be an intersection of 2-D string world sheet and a 3-D light-like partonic orbit. The proposal is that string world sheet can be obtained as the intersection of two spacetime surfaces X^4 and Y^4 if they have the same Hamilton-Jacobi structure at the level of H [L4], i.e. allow the same generalized complex H coordinates u, w, ξ_1, ξ_2 and their conjugates ($\overline{u} = v$). The corrected view of generalized analyticity however forces to challenges this assumption although it is physically very attractive.

   One can ask whether the mutual interactions of particles as space-time surfaces occur only when they have the same Hamilton-Jacobi (H-J) structure. If so, the interactions can be described in terms of their intersections consisting of string world sheets and fermion lines at their boundaries. If so, a strong analogy with string models would emerge. The second option is that the intersections are discrete. Also now fermionic n-point functions are well-defined.

   Also the self-interactions could be described by considering infinitesimal deformation of the space-time surface preserving H-J structure and finding the string world sheets in this case.

3. In TGD, the genus of the parton surface is an important topological quantum number [K2]. The genera $g = 0, 1, 2$ corresponds to the observed fermion generations. $g = 2$ allows a bound state for the 2 handles of the sphere that are like particles. This is because $g \leq 2$ allows global conformal symmetry. In the $g \geq 2$ topology, g handles are like particles in a multiparticle state, and the mass spectrum of the states is continuous, unlike for elementary particles.

   Also the homological charge of the partonic 2-surface, identifiable as Kähler magnetic charge of the space-time surface is an important topological quantum number.

This article was born as an attempt to develop a more precise view about the 3-D light-like trajectories of 2-dimensional parton surfaces on the basis of holograpy= holomorphy hypothesis (H-H).

1. CP_2 type extremals [K1] have a 1-D light-like curve as M^4 projection. Their complex deformations satisfy H-H. The projection corresponds to a coordinate curve of the hypercomplex coordinate u or v. The generalization of the $(f_1, f_2) = (0, 0)$ hypothesis to them turned out to be impossible: CP_2 projection turned out to be 3-dimensional and failed to satisfy H-H.

   The assumption that the space-time surface X^4 is invariant under generalized conjugation taking u to v and vice versa, implies that X^4 must have two sheets with $v = v_0$ or $u = u_0$ permuted by the generalized conjugation. They meet at the 3-surface X^3 $u = v$. This implies $u_0 = v_0$ implying that two M^4 coordinates u, v are constant and one has 3-D surface of CP_2 invariant under complex conjugation of complex H coordinates. At this surface, the light-like curves for the two sheets meet at an edge, which has an interpretation in terms of an exotic smooth structure in turn having interpretation in terms of a vertex for a creation of a fermion pair.

2. Partonic orbits can be identified as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian so that the metric determinant vanishes and the induced 4-metric degenerates to an effectively 2-D metric.

3. The light-like u or v coordinate lines can have edges at the partonic orbits. This has led to a proposal for how exotic smooth structures necessary for defining fermion pair creation vertices emerge via partonic orbits as defects of the standard smooth structure [L10, L2]. Fermion pair as a fermion returning backwards in time would correspond to the edge of u (or v) coordinate line. These conditions generalize the Virasoro conditions for 1-dimensional light-like curves to the 3-dimensional light-like partonic orbits.

4. The light-likeness of the coordinate lines generalizes the Virasoro conditions for 1-dimensional light-like curves to the 3-dimensional light-like partonic orbits and one obtains a set of 1-D Virasoro conditions parametrized by the points of the partonic 2-surface. In fact, the 1-D Virasoro conditions emerged first for CP_2 type extremals [K1] and led to the realization that the generalization of conformal invariance in some sense must be a fundamental symmetry of TGD: the discovery of holography= holomorphy principle finally led to a detailed understanding of this symmetry [L6]. Also an explicit procedure for finding the partonic orbits is discussed.

3.1 The identification of the partonic orbits

It took a considerable time to realize that holography= holomorphy vision has delicate technical problems and the recent view was found by trial and error.

3.1.1 Definition of hypercomplex conjugation

What does one mean with the generalization of the complex conjugation when applied to the argument of f? Could it correspond a) to $(u, w, \xi_1, \xi_2) \rightarrow (u, \overline{w}, \overline{\xi_1}, \overline{\xi_2})$ so that there is no hypercomplex conjugation or b) to $(v, w, \xi_1, \xi_2) \rightarrow (u, \overline{w}, \overline{\xi_1}, \overline{\xi_2})$ so that there is hypercomplex conjugation.

1. For option a), the roots of f and $\overline{f}$ represent the same surface. For the roots of f the contribution of complex coordinates to g_{uv} and g_{vw} is vanishing but the components $g_{u\overline{w}}$ and there is only the contribution of M^4 metric to g_{uv}. Partonic orbits are not possible.

2. For option b), the roots of the conjugate $\overline{f}$ do not coincide with the roots of f unless symmetries exist. If the space-time surface is invariant under the generalized conjugation (in analogy with complex plane), it must be a union of the u-type and v-type regions defines the space-time surface. Hypercomplex conjugation would be a non-local symmetry transforming to each other two parts of the space-time surface. The 3-surface $u = v$ would be a 3-dimensional surface along which the two space-time regions would be glued together.

Consider the option b) in more detail.

1. How to identify the u- and v-type regions? In the model for elementary particles, Euclidian regions as deformations of CP_2 extremals connect two Minkowskian space-time sheets, which are extremely near to each other having a distance of order CP_2 radius. Could the two Minkowskian space-time sheets correspond to u- and v-type regions and could generalized complex conjugation $(u, w, \xi_1, \xi_2) \leftrightarrow (v, \overline{w}, \overline{\xi_1}, \overline{\xi_2})$ transform then to each other.

2. Could the 3-surface X^3 at which the sheets intersect so that both u and v coordinates associated with the sheets are identical, define the 2-surface X^2 along which the sheets are glued together? Could this surface be identifiable as the light-like partonic orbit.

   The wormhole contact identified in this way has 3-D CP_2 projection and does not correspond to the CP_2 type extremal. It is not clear whether this is a problem or not.

3. Presumably, there would be discontinuity associated with the derivatives of the embedding space coordinates at X^3, where the u- and v-type time evolutions at the two sheets would be glued together.

4. Could X^3 be interpreted in terms of an exotic smooth structure [A3, A4, A1] allowing an interpretation as the standard smooth structure with defects? Could the u- lines transform to v-lines at X^3 and give rise to edges violating the standard smoothness.

   Also the partonic orbits could define analogous defects since the u- *resp.* v-lines could have an edge. The identification of fermion lines as these kinds of lines allow the interpretation of defects as vertices for the creation of fermion-antifermion pair as turning of fermion line backwards in time [L10, L2]?

3.1.2 Technical problems of the holography= holomorphy vision

Consider first the technical problems related to the finding of the roots of (f_1, f_2) appearing in the Euclidean space-time regions. Note that this is only an ansatz, which is less general than H-H and need not work for wormhole contacts as deformations of CP_2 type extremals [K1].

1. The first problem is that in Minkowskian regions defining the parallel space-time sheets one has two kinds of solutions for which hypercomplex coordinate u *resp.* its conjugate v appears in f_i *resp.* its conjugate. These should correspond to a single solution and the only way is to consider their union. The two regions in question have a natural identification as Minkowskian space-time sheets connected by a wormhole contact with an Euclidean signature of the induced metric.

At the surface, where the two sheets are glued, f_i must be invariant under conjugation, which for real coefficients of f_i requires $u = v$ and reality of various complex coordinates or at least that the surface in question is invariant under complex conjugation.

2. In Euclidean regions, the realization of H-H, using $(f_1, f_2) = (0, 0)$ ansatz assuming that either hypercomplex coordinate u or v is a dynamical variable, leads to a problem. Either u or v is a complex analytic function f of CP_2 coordinates and its reality implies $Im(f) = 0$ so that CP_2 projection is 3-dimensional, which means the failure of the holomorphy with respect to the CP_2 coordinates. For a moment I thought that Wick rotation might help but this was not the case.

3. This forces to give up $(f_1, f_2) = (0, 0)$ ansatz and assume only H-H. The original vision was that the Euclidean region as a wormhole contact corresponds to a deformation of a canonically embedded CP_2 so that it has a light-like coordinate curve of u or v as M^4 projection. These space-time surfaces are holomorphic so that field equations are satisfied.

   The gluing condition implies constancy condition $v = v_0$ *resp.* $u = u_0$ and v *resp.* u is replaced with a real CP_2 coordinate $s(u)$ *resp.* $s(v)$. M^4 complex coordinate w can be a function of CP_2 coordinates.

4. The gluing condition for the two sheets requires $u_0 = v_0$ which for $u = m^0 + m^3$ and $v = m^0 - m^3$ gives $m^0 = 2u_0$ and $m^3 = 0$. At the points of this 3-surface) there is an edge at which the coordinate curves for u and v meet: the interpretation could be in terms of an exotic smooth structure [A3, A4, A1] as standard smooth structure with a defect to which fermion pair creation or fermion scattering vertex can be assigned. The two sheets are glued together along a 3-surface X^3 with 3-D CP_2 projection invariant under complex conjugation. The CP_2 projection X^3 must contain a homologically non-trivial 2-surface since the wormhole contact must carry a monopole flux between the space-time sheets.

This tentative picture would relate several key ideas of TGD: H-H involving hypercomplex numbers, the notion of light-like partonic orbit, the idea that exotic smooth structures make possible non-trivial scattering theory in 4 dimensional space-time. One can compare this picture with the intuitive phenomenological picture.

3.1.3 The 3-D light-like orbits of partonic 2-surfaces

The trajectories of partonic 2-surfaces are singularities at which the Euclidean induced 4-geometry transforms into Minkowskian. The light-like dimension implies $\sqrt{|det(g_4)|} = 0$. The challenge is to derive the partonic orbits from this.

1. H-J structure defines Kähler structure $M^4 \subset H$ inducing that of X^4 and is independent of holography= holomorphy hypothesis. The induced Kähler structure of X^4 is defined by the projection of the sum of M^4 and CP_2 Kähler forms and need not be the same as that of M^4. If the proposal holds true, these structures differ only at the partonic orbits. The generalized complex coordinates of X^4 (hypercomplex coordinate u (or v) and complex coordinate w) are a subset of the generalized complex coordinates of H, which also include 2 complex coordinates of CP_2.

   The induced Kähler structure of X^4, which is more or less equivalent with Hamilton-Jacobi structure, defines a slicing of X^4 by light-like 3-surfaces with one light-like curves, which can be taken to correspond to the hypercomplex coordinate u, which is constant along the lines $u = u_0$. Also its dual slicing, assignable to the v-surface is well-defined.

   The 4-metric is hermitian and is a tensor of type (1,1) having only 4 independent components. The only non-vanishing component of the induced 3-metric g^3 at X^3 defined by the projection of the 4-metric is $g_{w\overline{w}}$ so that the slice is metrically 2-dimensional. Light-cone boundary provides a simple example of this.

2. The space-time surface X^4 is defined by the conditions $(f_1, f_2) = (0, 0)$, where f_1 and f_2 are analytic functions $H = M^4 \times CP_2 \rightarrow C^2$ depending only on the hypercomplex coordinate u with light-like coordinate curves and complex coordinates w, ξ_1 and ξ_2 of H but not on the coordinate v as hypercomplex conjugate u and the conjugates $\overline{w}, \overline{\xi_1}, \overline{\xi_2}$. The surfaces are same.

   As a special case, f_i are polynomials or rational functions. Additional restrictions can be posed on the coefficients of the polynomials. The conditions $(f_1, f_2) = (0, 0)$ have been studied in some cases [L13].

3. $\sqrt{det(g_4)} = 0$ gives an additional condition and gives a 3-D light-like partonic orbit X^3.

3.1.4 $det(g_4) = 0$ condition as a generalization of Virasoro conditions

The $g^4_{uv} = 0$ condition has an interpretation as a generalization of the Virasoro conditions of string models to the 4-D context.

1. If the situation were 2-dimensional instead of 4-D, the $det(g_4) = 0$ condition would give a light-like curve and the light-likeness would give rise to the Virasoro conditions. This was actually one of the first observations as I discovered CP_2 extremals, whose M^4 projection is a light-like curve for the Kähler action [K1]. For the action defined by the sum of Kähler action and volume term the light-like curves are replaced with light-like geodesics of M^4 and possibly of H. The conditions as such are not Virasoro conditions. It is the derivative of the conditions with respect to the curve parameter, which gives the Virasoro conditions. By taking Fourier transform one obtains the standard form of the Virasoro conditions.

   The Virasoro conditions can fail at discrete points and these singularities have an interpretation as vertices and also as points at which the generalized holomorphy fails. The poles and zeros of the ordinary analytic function are analogs for this.

2. By holomorphy= holography vision alone implies that the space-time surface is sliced by light-like curves. These curves satisfy Virasoro conditions so that one has a generalization of Virasoro conditions to a bundle of conditions parameterized by points of a 3-D section of the space-time surface. Space-time surface itself does not define a light-like orbit of the 3-surface.

3. For the 4-D generalization, the light-like curve is replaced by a 3-D light-like parton trajectory identifiable as a 2-D bundle of light-like curves so that 1-D Virasoro conditions are true for each curve. The analogs of Virasoro conditions are indeed very natural also now because 2-D conformal invariance is generalized to 4-dimensional one. The Virasoro conditions have one integer, the conformal weight. Now the Fourier transform with respect to the coordinates of X^4, say u and w gives conditions labelled by two integers having interpretation as conformal weights.

   This suggests that conditions can be seen as analogs of Virasoro conditions. Their generalization gives rise to analogs of the corresponding gauge conditions for the Kac-Moody algebra, just like in the string model. A lot of physics would be involved.

4. A new element brought by TGD is that algebras would have non-negative conformal weights meaning that an entire fractal hierarchy of isomorphic algebras is predicted such that sub-algebra and its commutator with the entire algebra annihilate the physical states [L6]. This makes possible a hierarchy of gauge symmetry breakings in which a subspace of the entire algebra transforms from a gauge algebra to a dynamical algebra.

3.2 How to find the partonic orbits?

In the sequel, the partonic orbit refers to the light-like boundary at which the signature of the induced metric changes from Minkowskian to Euclidian. In the Minkowskian region $(f_1, f_2) = (0, 0)$ ansatz works and,depending on which sheet one considers, the passive coordinate v or u becomes constant at the boundary.

One must solve the induced metric for a given solution $f = (f_1, f_2) = (0, 0)$ in Minkowskian region and find what happens to it at the boundary. This means moving from mere algebraic geometry to differential geometry because the induced metric depends on the partial derivatives of the imbedding coordinates. The complexity of the task depends on how strong assumptions one makes.

3.2.1 Two alternative identifications of partonic orbits

One can consider two alternative identifications of partonic orbits.

1. One could start from a completely general solution in Minkowskian region and consider only the $det(g_4) = 0$ condition without any additional assumptions such as the Hamilton-Jacobi structure.

2. If one assumes holography= holomorphy principle, 3-surfaces with $g^4_{uv} = 0$ implying $det(g_4) = 0$ condition, are good candidates candidates for partonic orbits, which must be metrically 2-dimensional. Since the signature transforms to Euclidian, the induced metric must receive a CP_2 contribution, which implis the conditions $det(g_4) = 0$ and $g^4_{uv} = 0$ implying metric 2-dimensionality.

Simple physical considerations help to understand what the partonic orbits look like. The simplest surface to consider is deformed M^4 for which CP_2 projection is a geodesic line: $\Phi = \omega t$. The induced metric is $g_{tt} = 1 - R^2\omega^2, g_{ij} = -\delta_{ij}$, where R is CP_2 length scale. For $R^2\omega^2 = 1$, the time-like direction becomes light-like. Something analogous happens also in the general case. The rapid time variation of the $\xi_i(w, u)$ and $\xi_i(\overline{w}, v)$ is what can change the sign of $det(g_4)$. Some partial derivatives $\partial_u \xi_i(u, 2)$ and $\partial \overline{\xi_i}(v, \overline{w})$ must have order of magnitude $1/R$. Therefore the numerical calculation must start from a situation in which these time derivatives are large.

To find the partonic orbits defined in the way already discussed, it is useful to find a region of space-time surface whether the gradients of CP_2 coordinates as functions of u coordinate are of order $1/R$ so that g^4_{uv} can be near zero.

3.2.2 $det(g_4) = 0$ condition as a possible definition of the parton orbit

This section gives some idea about how concrete calculations might proceed. The condition $det(g_4) = 0$ is a natural guess for the precise definition of the partonic orbit as light-like 3-surfaces at which 4-metric degenerates to 2-dimensional metric.

The condition $det(g_4) = 0$ is a natural guess for the precise definition of the partonic orbit as light-like 3-surfaces at which 4-metric degenerates to 2-dimensional metric. Consider in more detail the $det(g_4 = 0)$ option for partonic surfaces using H-J coordinates but without assuming H-H vision. The following also describes how to calculate the induced metric.

For X^4 Kähler form is obtained by inducing the sum of Kähler forms of M^4 and CP_2 and is in general different from that M^4. The H-J coordinates are however the same. If the coordinates of X^4 are not H-J coordinates one must $det(g_4) = 0$ condition without hermiticity conditions on the induced metric. This requires an additional computational effort.

For H-J coordinates for X^4, the $det(g_4) = 0$ is equivalent with the $g^4_{uv} = 0$ condition and the situation simplifies dramatically and one must find the 3-surfaces with $g^4_{uv} = 0$.

1. The general form of the induced metric is

$$g_{\alpha\beta} = h_{kl}\partial_\alpha h^k \partial_\beta h^l \quad . \tag{3.1}$$

   For H-J coordinates, α and β refer to $u, v, w, \overline{w}$ and k and l refer to $u, v, w, \overline{w}, \xi_1, \xi_2$. The metric of H in these coordinates can be written easily. From this, we one can calculate the induced metric.

2. For the generalized complex coordinates, not necessarily consistent with the H-J structure, the rows of the induced metric g can be written as a matrix in the general case in the form

$$\begin{matrix} (g_{uu} & g^4_{uv} & g_{uw} & g_{u\overline{w}}) \\ (g_{vu} & g_{vv} & g_{vw} & g_{v\overline{w}}) \\ (g_{wu} & g_{wv} & g_{ww} & g_{w\overline{w}}) \\ (g_{\overline{w}u} & g_{\overline{w}v} & g_{\overline{w}w} & g_{\overline{w}\overline{w}}) \end{matrix} \quad (3.2)$$

All components of the metric are in general non-vanishing.

3. Holomorphy, implying that the embedding space metric and induced metric are tensors of type (1,1), implies the vanishing of a large fraction of elements of g^4. This gives

$$\begin{pmatrix} 0 & g^4_{uv} & 0 & g_{u\overline{w}} \\ g_{vu} & 0 & g_{vw} & 0 \\ 0 & g_{wv} & 0 & g_{w\overline{w}} \\ g_{\overline{w}u} & 0 & g_{\overline{w}w} & 0 \end{pmatrix} \quad (3.3)$$

The symmetry $g_{\alpha\beta} = g_{\beta\alpha}$ leaves only 4 independent matrix elements. $g^4_{uv}, g_{u\overline{w}}, g_{vw}, g_{w\overline{w}}$. The determinant of this metric vanishes if a partonic orbit is in question.

This is the expression of the induced metric in the Euclidean regions. Partonic orbit corresponds to the interfaces at which $g_{uv} = 0$ is true. The field equations ($f_1 = 0, f_2 = 0$) in the Euclidean region must be solved using Wick rotation.

4. In the Minkowskian regions, where u (or v) serves as a parameter, g^4_{uv} reduces to its Minkowskian contribution and components g_{vw} and $g_{u\overline{w}}$ vanish. Partonic orbits are not possible in these regions. The induced 3-metric g_3 at light-like u coordinate lines in Minkowskian regions reduces to

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_{w\overline{w}} \\ 0 & g_{\overline{w}w} & 0 \end{pmatrix} \quad (3.4)$$

The situation is metrically 2-dimensional. Also g_4 is metrically 2-dimensional if the metric changes from Minkowskian to Euclidean so that g^4_{uv} vanishes.

5. If one has $f_2 = \xi_2 - w$ and $f_1(\xi_1, w, h)$ is a polynomial of degree $n < 5$ with respect to w, analytic expressions for $\xi_i(h, w)$ are possible and the analytic calculation of the partial derivatives can be considered. Otherwise, we have to use numerical methods. One could hope that a symbolic program for calculating partial derivatives could be found .

6. If the reduction of the condition $det(g_4) = 0$ to the condition $g^4_{uv} = 0$ indeed takes place, the key variable is

$$g^4_{uv} = \partial_u h^k \partial_v h^l = g^0_{uv} + s_{k\overline{l}} \partial_u s^k \partial_v \overline{s}^l \ . \quad (3.5)$$

Here g^0_{uv} denotes the M^4 contribution to the induced metric. For $det(g_4) = 0$ the M^4 and CP_2 contributions cancel each other and one has

$$g^0_{uv} = -s_{k\overline{l}} \partial s^k \overline{s}^l \ . \quad (3.6)$$

A generalization of a light-like geodesic of H to a bundle of light-like curves parameterized by the points of the partonic 2-surface is in question.

3.3 Considerations inspired by LLM summaries of TGD articles

Tuomas Sorakivi prepared LLM summaries about some articles related to TGD, in particular the article [L13] in which the relation of the holography = holomorphy vision to elliptic surfaces and the notion of partonic orbits are considered.

The discussions and the LLM summaries inspired considerations related to the general view about the definition of the partonic orbits involving the conditions $\sqrt{g_4} = 0$ assuming generalized holomorphy and to the details related to the model for the pairs of space-time sheets connected by wormhole contacts.

3.3.1 What conjugation means for generalized complex coordinates

Generalized complex structure involves hypercomplex coordinates and this involves non-trivial delicacies related to the counterpart of generalized complex conjugation.

1. The expression for g_{uv} involves conjugation of CP_2 cooordinates ξ^k. It is important to note that conjugation means that means

$$\overline{\xi}^k(u,w) \rightarrow \overline{\xi}^k(v,w) \ .$$

   This is because v is the hypercomplex conjugate of u. In the conditions $f_i = 0$, only the hypercomplex and therefore real u coordinate occurs in the functions $f_i(u, w, \xi^1, \xi^2)$, $i = 1, 2$.

2. Relevant question concerns the interpretation of the fact that the hypercomplex conjugation $u \rightarrow v$ is involved? The presented model for a pair of spacetime sheets is that, for example, the upper sheet has an active coordinate u and the lower one has v.

   Conjugation would take from the "upper" spacetime sheet to the "lower" one if both are involved. This would indicate that the sheets are the relations of generalized complex conjugation. This is not a necessary assumption, but it is possible and I have suggested it.

3. This formal interpretation seems strange, but in ordinary complex conjugation it is like this. $x + iy$, $y \geq 0$ corresponds to the upper half plane and $x - iy$, $y \geq 0$ to the lower half plane. Conjugation takes from the upper half plane to the lower one. On the real axis $y = 0$ the planes meet.

   So two 4-D Minkowski spacetime sheets would be generalizations of the half planes. The real axis would be the Euclidean 3-D CP_2 inside the extremal: it is not the same as the parto orbit: the language model had mixed them up. In the used H-J coordinates, $u = t - z = v = t + z$, that is $z = 0$, would hold. This 3-surface in the direction of time would correspond to the world line of a particle at rest in M^4.

3.3.2 Connection with particle massivation and ideas of Connes

The fact that this 3-surface inside the CP_2 type extremal is like a particle at rest necessarily means that there are 2 space-time sheets and they are connected by a wormhole contact. Massification has necessarily occurred.

1. If only one space-time sheet is involved, it is a half-plane equivalent of one of the two. Is this possible? Could the light-like 3-D orbit of the parton surface be a track edge in the Minkowski region? Is such a solution possible or are wormhole contacts and a pair of space-time sheets necessarily needed. In any case, the fermion lines would be on partonic 2-surfaces, so a partonic surface is needed.

2. Interestingly, a top French mathematician Connes ended up proposing that the Higgs mechanism in non-commutative geometry would correspond to the Minkowski space doubling in the same way. Also in TGD framework the massivation would occur in the same way!

   I have been in Schrödinger's cat-like state regarding this question: it would seem that the boundary conditions do not allow boundaries at all. On the other hand, I have also considered the possibility allowing light-like boundaries.

3. The fact that only the coordinate u or v appears in the generalized analytic functions f_1 and f_2 means that an analogy is made between the wave motion at the speed of light $t-z$ or $t=z$ and the coordinate on which the wave depends. In the string model, the terms left mover and right mover are used.

   The situation in which both space-time sheets are involved would correspond in the string model to the fact that a wave coming along one space-time sheet is reflected back on this three-surface of CP_2 type extremal and returns along the other space-time sheet.

   If a single sheet with light-like boundaries are possible, it would correspond to massless particles. Either a left-mover or a right-mover, but not both. On the other hand, p-adic thermodynamics predicts that photons and gravitons also have a small mass.

3.3.3 Testing whether the conditions $g_{uv} = 0$ allow solutions

Tuomas had, using the language model, come up with a proposal to investigate whether there are analytical solutions to the condition $g_{uv} = 0$ on a partonic surface. If there are, then we can be satisfied. On the other hand, it could happen that there are none. I thought about it at night and found out that such solutions really do exist. The task is to find such a simple situation that numerical calculations are not needed.

1. I already made a simplifying assumption earlier that f_2 is of the form $f_2 = \xi^2 - w^n$. There would be no u-dependence at all. $f_2 = 0$ would give $\xi_2 = w^n$. There would be no need to find the roots either.

   A more general solution would be $f_2 = P_2(\xi^2, w)$ without u-dependence. Now the roots of the polynomial must be solved. This does not change the situation.

2. We could make a similar assumption for f_1, but assume u-dependence.

   $$f_1 = f_1(\xi_1, w, u) = \xi_1 - g(w, u) \ .$$

   .

   We can simplify it even further by assuming

   $$g(w, u) = uh(w) \ .$$

   So we can solve ξ_1 as

   $$\xi_1 = uh(w) \ .$$

3. Now we have everything we need to solve the condition $g_{uv} = 0$.

   (a) The CP_2 metric $s_{x^i\overline{\xi}^j}$ is known. Here we must remember that conjugation means $u \to v$!

   (b) The vanishing condition $g_{uv} = 0$ gives

   $$s_{k\overline{l}} \partial_u \xi^k \partial_v \overline{\xi}^l = -1 \ .$$

   (c) The non-vanishing partial derivatives are

   $$\partial_u \xi^1 = h(w) \partial_v \overline{\xi}^1 = h(\overline{w}) \ .$$

   This gives

   $$s_{1\overline{1}} h(w) h(\overline{w}) = -1 \ .$$

(d) The component of the CP_2 metric $s_{1\overline{1}} \leq 0$ appears in the formula (the CP_2 metric is Euclidean) and is known and is proportional to $1/(1+r^2)$ [L5],

$$r^2 = \xi^1\overline{\xi}^1 + \xi^2\overline{\xi}^2$$

and depends on the uv via

$$\xi^1\overline{\xi}^1 = uvg(w)g(\overline{w}) \ .$$

The equation can be solved for the uv function in terms of a function $k(w,\overline{w}$ deducible from the condition:
$uv = k(w,\overline{w})$.
In the (u,v) plane, this is a hyperbola for the given values of w. So there are solutions. We can breathe a sigh of relief.

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