Antipodal duality and TGD

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Abstract

The so called antipodal duality has received considerable attention. The calculations of Dixon et al based on the earlier calculations of Goncharow et al suggests a new kind of duality relating color and electroweak interactions. The calculations lead to an explicit formula for the loop contributions to the 6-gluon scattering amplitude in $\mathcal{N} = 4$ SUSY. The new duality and relates 6-gluon amplitude for the forward scattering to a 3-gluon form factor of stress tensor analogous to a quantum field describing a scalar particle. This amplitude can be identified as a contribution to the scattering amplitude $h + g \to g + g$ at the soft limit when the stress tensor particle scatters in forward direction. The result is somewhat mysterious since in the standard model strong and electroweak interactions are completely separate.

In TGD, there are indeed quite a number of pieces of evidence for this kind of duality but the possibility that only electroweak or color interactions could provide a full description of scattering amplitudes. The number-theoretical view of TGD could however come into rescue.

1 Introduction

I learned of a new particle physics duality from the popular article "Particle Physicists Puzzle Over a New Duality" published in Quanta Magazine (`https://cutt.ly/jZOaDhd`). The article describes the findings of Dixon et al reported in the article "Folding Amplitudes into Form Factors: An Antipodal Duality" [B1] (`https://cutt.ly/EZOsfGl`) This work relies on the calculations of Goncharov et al published in the article "Classical Polylogarithms for Amplitudes and Wilson Loops" [B2] (`https://cutt.ly/sZOsuu6`).

The calculations of Goncharov et al lead to an explicit formula for the loop contributions to the 6-gluon scattering amplitude in $\mathcal{N} = 4$ SUSY. The new duality is called antipodal duality and relates 6-gluon amplitude for the forward scattering to a 3-gluon form factor of stress tensor analogous to a quantum field describing a scalar particle. This amplitude can be identified as a contribution to the scattering amplitude $h + g \to g + g$. The result is somewhat mysterious since in the standard model strong and electroweak interactions are completely separate.

1.1 Findings of Dixon et al

Consider first the findings of Dixon et al [B1].

1. One considers [B2] twistor amplitudes in $\mathcal{N} = \triangle$ SUSY. Only the maximally helicity violating amplitudes (MHV) are considered and one restricts the consideration to planar diagrams (to my best understanding, non-planar diagrams are still poorly understood). The contribution

of the loop corrections is studied and the number of loops is rather high in the computations checking the claimed result.

6-gluon forward scattering amplitude and 3-gluon form factor of stress energy tensor regarded as a quantum field are discussed. Conformal invariance fixes the Lorentz invariants appearing in the 6-gluon forward amplitude and in the 3-gluon form factor of stress tensor to be 3 conformally invariant cross ratios formed from the 3 gluon momenta.

The claimed antipodal duality is found to hold true for each number of loops separately at the limit when one of conformal invariants approaches zero: the interpretation is that momentum exchange between 2 gluons vanishes at this limit. For 6-gluon forward amplitudes, this limit corresponds to in the 3-D space of conformal invariants to the edges of a tetrahedron.

2. $3g \rightarrow 3g$ scattering amplitude is studied at the limit when the scattering is in forward direction. One has effectively 3 gluons but not 3-gluon scattering since there is no momentum conservation constraining the total momentum of 3 gluons except effectively for the forward scattering of the stress tensor.

   As far as total quantum numbers are considered, the stress tensor can give rise to a quantum field behaving like Higgs as far as QCD is considered. The surprising finding is that the so-called antipodal duality applied to the 6-gluon amplitude gives a 3-gluon form factor of the stress tensor, which is scalar having no spin and vanishing color quantum numbers.

3. The antipodal transformation is carried for the 6-gluon amplitude in forward direction so that only 3 gluon momenta are involved. One starts from the 6-gluon amplitude constructed using the standard rules, which require that the amplitude involves only cyclic permutations of the gluons (elements of S_6 of the gluons.

   One considers permutation group $S_3 \subset S_6$ acting in the same way on the first 3 first and 3 remaining gluons, and constructs an S_3 singlet as a sum of the amplitudes obtained by applying S_3 transformations. S_3 operations are not allowed in the twistor diagrammatics since only planar amplitudes are considered usually (the construction of twistor counterparts of non-planar amplitudes is not well-understood).

4. One also constructs the 3-gluon form factor of stress energy tensor by using the twistor rules and considers the so-called soft limit at which the sum of the 3 gluon momenta vanishes so that the effectite particle assignable to the stress tensor scatters in the forward direction. It comes as a surprise that this amplitude is related to the amplitude obtained from the forward 6-gluon amplitude by the antipodal transformation.

5. The duality also involves a simple transformation of the 3 conformal invariants formed from the gluon momenta involved to the 3-gluon form factor of the energy momentum tensor. The antipodal duality holds true at the edges of the 2-D tetrahedron surface defined by the image of the 3-gluon form factor in the space of 3 conformal invariants characterizing the 6-gluon forward amplitude.

   The term antipodal derives from the fact that the 6-gluon amplitude can be expressed as a "word" formed from 6 "letters" and the above described transformation reverses the order of the letters.

6. It is conjectured that this result generalizes to large values of n so that antipodal images of $2n$-gluon scattering amplitude in forward direction could correspond to $n-$-gluon form factor for stress tensor energy and this in turn would be associated with scattering of Higgs and n gluons.

1.2 Questions

Since the stress tensor is a scalar, it is not totally surprising that a term proportional to this amplitude contributes to the scattering amplitude $h + g \rightarrow g + g$, where h denotes Higgs particle. What looks somewhat mysterious is that Higgs is an electro-weakly interacting particle and has no direct color interactions. The description of the scattering in the standard model involves electroweak interactions and involves at least one decay of a gluon to a quark pair in turn interacting with the Higgs.

This inspires several questions.

1. Can one consider more general subgroups $S_m \subset S_{2n}$ and by forming S_m singlets construct amplitudes with a physical interpretation?

2. Can one imagine a deep duality between color and electroweak interactions such that $\mathcal{N} = 4$ SUSY would reflect this duality? Could one even think that the strong and electroweak interactions are in some sense dual?

In TGD color interactions and electroweak interactions are related to the isometries and holonomies of $CP2$ and there indeed exists quite a number of pieces of evidence for this kind of duality. However, the possibility that electroweak or color interactions alone could provide a full description of scattering amplitudes looks unrealistic: both electroweak and color quantum numbers are needed. The number-theoretical view of TGD [L8, L1, L9, L10] could however come into rescue.

2 In what sense could electroweak and color interactions be dual?

Some kind of duality of electroweak and color interactions is suggested by the antipode duality having an interpretation in terms of Hopf algebras (`https://en.wikipedia.org/wiki/Hopf_algebra`): antipode generalizes the notion of inverse for an element of algebra.

TGD contains several mysterious looking and not-well understood features suggesting some kind of duality between electroweak and color interactions. What could make this duality possible in the TGD framework, would be the presence of Galois symmetry, which would allow us to describe electroweak or color particle multiplets number-theoretically using representations of the Galois group.

1. The electric-magnetic duality or Montonen-Olive duality (`https://en.wikipedia.org/wiki/Montonen\OT1\textendashOlive_duality`) is inspired by the homology of CP_2 in TGD [**?**]. The generalization of this duality in gauge theories relates the perturbative description of gauge interactions for gauge group G to a non-perturbative description in terms of magnetic monopoles associated with the dual gauge group G_L. Langlands duality [**?**, **?**] discussed from the TGD perspective in [**?**, **?**] relates the representations of Galois groups and those of Lie groups, and involves Lie group and its Langlands dual. Therefore gauge groups, magnetic monopoles and the corresponding dual gauge group, and number theory seem to be mathematically related, and TGD suggests a physical realization of this view.

2. The dual groups G and G_L should be very similar but electroweak gauge group $U(2)$ and color group $SU(3)$, albeit naturally related as holonomy and isometry groups of CP_2, do not satisfy this condition. Here the Galois group could come into rescue and provide the missing quantum numbers.

3. Depending on the situation, Galois confinement could relate to color confinement or electroweak confinement. In the context of electric-magnetic duality [K4, K1, K5], I have discussed electroweak confinement and as a possible dual description for the electroweak massivation, involving summation of electroweak $SU(2)$ quantum numbers to zero in the scale of monopole flux tubes assignable to elementary particles. The screening of weak isospin would take place by a pair of neutrino and right-handed neutrino in the Compton scale of weak boson or fermion: $h_{eff} > h$ allows longer scales.

4. Also magnetic charge or flux assignable to the flux tubes could make possible a topological description of color hypercharge topologically whereas color isospin could might have description in terms of weak iosospin. I considered this idea already in my thesis. As a matter of fact, already before the discovery of CP_2 around 1980, I proposed that magnetic (homology-) charges 2,-1,-1 for cP_2 could correspond to em charges 2/3,-1/3,-1/3 of quarks and that quark

confinement could be a topological phenomenon. Maybe these almost forgotten ideas might find a place in TGD after all.

Consider now the possible duality between electroweak and color interactions.

2.1 H level

At the level of H spinors do not couple classically to gluons and color is not spin-like quantum number.

1. The proposal is that the zero energy states are singlets either with respect to the Galois group or the isotropy group of a given root. Z_3 as a subgroup or possibly normal subgroup of the Galois group would act on the space of fermion momenta for which components are algebraic integers belonging to the extension of rationals defined by P.

2. Color confinement could correspond to Galois confinement. Alternatively, the confinement of color isospin could correspond to Galois confinement whereas the confinement of color hypercharge would have a description in terms of the already mentioned monopole confinement. Both number theoretic and topological color would be invisible.

Could antipodal duality be understood number-theoretically?

1. The antipodal duality produces an S_3 singlet from a twistor amplitude. Could color singlets correspond to Z_3 Galois-singlets and electroweak singlets above Compton scale to Z_2 singlets.

2. Could Z_2 be realized as an exchange of two gluons ordered cyclically in the amplitude? Could one think that S^6 acts as a Galois group or its isotropy group?

   The stress tensor as a Higgs like state is not a doublet. Could one obtain Higgs as a Z_2 doublet by allowing the entire orbit of S_3 but requiring only that Z_3 singlet property holds true?

3. Could all isotropy groups or even all subgroups of S^3 be allowed. Could S_n quite generally have a representation as a Galois group? This picture applies also to $2n$-gluon amplitudes but also more general conditions for Galois singlet property can be imagined.

2.2 M^8 level

The roles of color and electroweak quantum numbers are changed in $M^8 - H$ duality [L4, L5].

1. At the level of M^8, complexified octonionic 2-spinors [L3, L4, L5] decompose to the representations of the subgroup $SU(3) \subset G_2$ of octonionic automorphisms as $1 + \overline{1} + 3 + \overline{3}$. One obtains leptons and quarks with spin but electroweak quantum numbers do not appear as spin-like quantum numbers. This would suggests that one should assume both lepton and quark spinors at the level of H although the idea about leptons as 3-quark composites in CP_2 scale is attractive [L6].

   One can however construct octonionic spinor fields $M^4 \times E^4$ with the spinor partial waves belonging to the representations of $SO(4) = SU(2) \times SU(2)$ decomposing to representation of $U(2)$ with quantum numbers having interpretation as orbital angular momentum like electroweak quantum numbers.

2. At the level of 4-surfaces of M^8, weak isospin doublet could correspond to Galois doublet associated with a Z_2 factor of the Galois group.

2.3 Twistor space level

Also at the level of twistor spaces, the roles of electroweak and color numbers are changed in $M^8 - H$ duality.

1. At the level of H, $M^4 \times CP_2$ is replaced by the product of the twistor spaces $T(M^4)$ and $T(CP_2) = SU(3)/U(1)\times U(1)$. Since spinors are not involved anymore, electroweak quantum numbers disappear. Number theoretic description should apply. Here Galois subgroup Z_2 could help.

   This suggests that $U(2) \subset SU(3)$ must be interpreted in terms of electroweak quantum numbers. There indeed exists a natural embedding of the holonomy group of CP_2 to its isometry group. At the level of space-time, surface color hyper-charge and isopin could correspond to electroweak hyper-charge and isospin. This works if, for given electroweak quantum numbers, the choice of the quantization axes of color quantum numbers depends on the state so that the regions of space-time surface assignable to a fermion depends on its color quantum numbers in H. This would give a correlation between space-time geometry and quantum numbers.

2. At the level of M^8 the twistor space $T(E^4)$ contains information about weak quantum numbers but no information of color quantum numbers since octonionic spinors are given up. Z_6 as a subgroup of the Galois group could help now.

Also the induced twistor structure at the level of space-time surface in H and at the level of 4-surface in M^8 gives strong consistency conditions.

1. The induced twistor structure for the surface $T(X^4) \subset T(H)$ has S^2 bundle structure characterizing twistor space. This structure is obtained by dimensional reduction to $X^6 = X^4 \times S^2$ locally such that S^2 corresponds to the twistor sphere of both $T(M^4)$ and $T(CP_2)$.

2. For cognitive representations as unique number theoretic discretizations of the space-time surface, the twistor spheres S^2 of $T(M^4)$ *resp.* $T(CP_2)$ must correspond to each other. The point of S^2 represents the direction of the quantization axis and the value $\pm 1/2$ of spin *resp.* color isospin or appropriately normalized color hypercharge respectively.

   For quark triplets this kind of correlation can make sense between spin and color hypercharge only and only at the level of the space-time surface. Since the quantization directions of color isospin are not fixed, only the correlation between representations, rather states, is required and makes sense for quarks. This suggests that color isospin at the space-time level must correspond to Galois quantum number.

3. What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color [K3].

The induced twistor structure for 4-surfaces in M^8 does not correspond to dimensional reduction but one expects an analogous correlation between spin and electroweak quantum numbers induced by the mapping of the twistor spheres S^2 to each other.

1. This correlation spin H-spinors correspond to tensor products of spin and electroweak doublets and all elementary particles are constructed from these.

2. Something seems to be however missing: also M^4 spinors should have a $U(1)$ charge to make the picture completely symmetric. The spinor lift strongly suggests that also M^4 has the analog of Kähler structure [L7] and this would give rise to $U(1)$ charge for M^4 spinors [L2] [K1]. This coupling could give rise to small CP breaking effects at the level of fundamental spinors [L7].

The experimental picture about strong and electroweak interactions suggests that the description of standard model interactions as either color interactions or electroweak interactions combined with a number theoretic/topological description of the missing quantum numbers is enough.

1. In hadron physics, only electroweak quantum numbers are visible. Color could be described using number-theory and topology and also these descriptions might be dual. In the QCD picture at high energies only color quantum numbers are visible and electroweak quantum numbers could be described number-theoretically. For a given particle, electroweak confinement would work above its Compton scale of weak scale.

2. In the old fashioned hadron physics conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) relate hadron physics and electroweak physics in a manner which is not fully understood since also quark confinement is still poorly understood. PCAC reflects the massivation of hadrons and can be also seen as caused by the massivation of quarks and leptons and makes successful predictions. In the TGD framework PCAC is applied to the model of so-called lepto-hadrons [K6].

   One can say that hadronic description uses $SO(4) = SU(2)_L \times U(2)_R$ or rather, $U_{ew}(2)$ as a symmetry group whereas QCD uses $SU(3)$ in accordance with the duality between color and electroweak interactions. This conforms with the $M^8 - H$ duality.

3. What about CP_2 type extremals (wormhole contacts), which have Euclidean metric. Could electroweak spin be described as the spin of an octo-spinor and could M^4 spin be described number-theoretically.

What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color.

2.4 Antipodal duality and TGD: two years later

Avi Shrikumar asked about the antipodal duality (see this), which has been discovered in QCD but whose origin is not well-understood.

Antipodal duality implies connections between strong and electroweak interactions, which look mysterious since in the standard model these interactions are apparently independent. This kind of connections were discovered long before QCD and expressed in terms of the conserved vector current hypothesis (CVC) and partially conserved axial current PCAC hypothesis for the current algebra.

I looked at the antipodal duality as I learned [K2] of it but did not find any obvious explanation in TGD at that time. After that I however managed to develop a rather detailed understanding of how the scattering amplitudes emerge in the TGD framework. The basic ideas about the construction of vertices [L11, L12] are very helpful in the sequel.

1. In TGD, classical gravitational fields, color fields, electroweak fields are very closely related, being expressed in terms of CP_2 coordinates and their gradients, which define the basic field like variables when space-time surface 4-D M^4 projection. TGD predicts that also M^4 possesses Kähler structure and gives rise to the electroweak U(1) gauge field. It might give an additional contribution to the electroweak U(1) field or define an independent U(1) field.

   There is also a Higgs emission vertex and the CP_2 part for the trace of the second fundamental fundamental form behaves like the Higgs group theoretically. This trace can be regarded as a generalized acceleration and satisfies the analog of Newton' s equation and Einstein's equations. M^4 part as generalized M^4 acceeration would naturally define graviton emission vertex and CP_2 part Higgs emission vertex.

   This picture is bound to imply very strong connections between strong and weak interactions and also gravitation.

2. The construction of the vertices led to the outcome that all gauge theory vertices reduce to the electroweak vertices. Only the emission vertex corresponding to Kähler gauge potential and photon are vectorial and can contribute to gluon emission vertices so that strong interactions might involve only the Kähler gauge potentials of CP_2 and M^4 (something new).

3. The vertices involving gluons can involve only electroweak parity conserving vertices since color is not a spin-like quantum number in TGD but corresponds to partial waves in CP_2. This implies very strong connections between electroweak vertices and vertices involving gluon emission. One might perhaps say that one starts the U(1) electroweak vertex and its M^4 counterpart and assigns to the final state particles as a center of mass motion in CP_2.

If this view is correct, then the standard model would reflect the underlying much deeper connection between electroweak, color and gravitational interactions implied by the geometrization of the standard model fields and gravitational fields. This connection would remain hidden in the gauge theory approximation.

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