A more precise formulation of zero energy ontology and about the relationship the surfaceology to TGD

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Abstract

This article consists of two parts. In the first part a more precise formulation of zero energy ontology (ZEO) is discussed based on the recent vision of quantum TGD. The development of the mathematical TGD has been a sequence of simplifications and generalizations. Holography = holomorphy vision removes path integral from quantum physics and together with the number theoretic vision might make the bosonic action unnecessary. This means that this vision allows us to solve field equations explicitly and the solution does not depend on the bosonic action.

TGD allows to get rid of primary bosonic fields and fermions are free free fermions at the level of the imbedding space and their localization to space-time surfaces makes them interaction. Pair creation is made possible by the presence of exotic smooth structures possible only in 4-D space-time. This however leads to a problem with the sign of energy. This problem disappears when one realizes that fundamental fermions can have tachyonic momenta and that only the physical states as their bound states, which are Galois singlets, have non-negative mass squared and positive energy.

Nima Arkani Hamed et al have introduced a new approach to QFT generalizing the amplituhedron approach so that it is claimed to work also for theories that are not supersymmetric. This approach, called surfaceology, still starts from the QFT picture but suggests that the large numbers of Feynman amplitudes could be summed to essentially volumes of living in some higher-dimensional spaces. The bold proposal is that the 4-D space-time is not needed at all. On the other hand, surfaceology suggests that the computational algorithms of QFT lead universally to the same result and are analogous to iteration of a dynamics defined in a theory space leading to the same result irrespective of the theory from which one starts from: this is understandable since the renormalization of coupling constants means motion in theory space.

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1 Introduction

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2 Challenging some details of the recent view of TGD

The development of the mathematical TGD has been a sequence of simplifications and generalizations. Holography = holomorphy vision removes path integral from quantum physics and together with the number theoretic vision might make the bosonic action unnecessary. This means that this vision allows us to solve field equations explicitly and the solution does not depend on the bosonic action.

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2.1 Could the classical bosonic action completely disappear from TGD?

Number theoretic vision of TGD and holography = holomorphy principle [L10, L11] forces to challenge the necessity of the classical bosonic action.

1. Any general coordinate action defining the Kähler function K and constructible in terms of the induced geometry gives the same minimal space-time surfaces as extremals and only the boundaries and partonic orbits depend on the action since the boundary conditions stating conservation laws depend on the action. Spinor lift suggests Kähler action for the 6-D twistor surfaces as a unique action principle. But is it necessary?

2. The conjecture $exp(K) \propto D^n$, n an integer, or its generalization to $exp(K) \propto D\overline{D}^n$, where D is a product of discriminants for the polynomials assignable to partonic 2-surfaces define a discrete set of points as their roots, would allow to express vacuum functional completely in terms of number theory. Coupling parameters would be present but evolve in such a way that the condition would hold true.

3. The discriminant D is defined also when the roots assignable to the partonic 2-surfaces are real or even complex numbers. This would conform with the strong form of holography. One could get completely rid of the bosonic action principle. The holomorphy = holography principle would automatically give the non-linear counterpart of massless fields satisfied by the space-time surfaces as minimal surfaces. Could the classical action completely disappear from the theory?

2.2 Could the fermionic interaction vertices be independent of the bosonic action principle

Could the interaction vertices for fermions be independent of the bosonic action principle?

1. The long-held idea is [L7, L12, L5], the vertices appearing in the scattering amplitudes are determined by the modified Dirac equation [L9] determined by the bosonic action associated with the partonic orbits as couplings to the induced gauge potentials. Twistor lift suggests that this action contains volume term and Kähler action.

   But is the modified Dirac action necessary or even physically plausible? The problem is that for a general bosonic action the modified gamma matrices, defined in terms of canonical momentum currents, do not commute to the induced metric unlike the modified Dirac action determined by the mere volume term of the bosonic action. This led to the proposal that this option, consistent also with the fact that, irrespective of the bosonic action, space-time surfaces are minimal surfaces outside singularities at which generalized holomorphy fails, is more plausible.

2. Fermion pair creation (and emission of bosons as Galois singlet bound states of fermions and antifermions is possible only for 4-D space-time surfaces. The existence of exotic smooth structures in dimension $D = 4$ [L2] makes possible pair creation vertices [L12, L5]. A given exotic smooth structure corresponds to the unique standard ordinary smooth structure with defects and vertices would correspond to defects at which the fermion line turns backwards in time. The defects would be associated with partonic 2-surfaces at which the generalized holomorphy of the function pair (f_1, f_2) with respect to generalized complex coordinates of H (one of them is hypercomplex coordinate) fails, perhaps only at the defect.

3. There is an objection against this proposal. The creation of fermion pairs with opposite sign of single fermionic energy suggests that a given light-like boundary of CD can contain fermions with both signs of energy. This does not conform with the assumption that the sign of the *single* particle energy is fixed and opposite for the opposite boundaries of CD. Should one only require that the total energy has a fixed sign at a given boundary of the CD?

   Could one only require that the sign of the energy is fixed only for physical states formed as many-fermions states and identified as Galois singlets and that the physical states can also contain negative energy tachyonic fermions or antifermions. Could this make sense mathematically?

2.3 Extension of the fermionic state space to include tachyonic fundamental fermions as analogs of virtual fermions

I recently received from Paul Kirsch a link to an interesting article about the possibility to describing tachyons in a mathematically consistent way [B6] (see this). The basic problem is that for tachyons the number of positive energy particles is not well-defined since Lorentz transformation can change positive energy tachyons to negative energy tachyons and vice versa. The proposed solution of the problem is the doubling of the Hilbert space which includes both incoming and outgoing states. To me this looks like a mathematically sensible idea and might make sense also physically.

Surprisingly, this proposal has a rather concrete connection with zero energy ontology (ZEO).

1. In the simplest formulation of ZEO, the fermionic vacua at the passive *resp.* active boundaries of CD correspond to the fermionic vacua annihilated by annihilation operators *resp.* creation operators as their hermitian conjugates. In the standard QFT only the second vacuum is accepted and this allows only a single arrow of geometric time.

2. ZEO allows both arrows and a given zero energy state is a state pair for which the fermionic state at the passive boundary of CD remains fixed during the sequence of small state function reductions (SSFRs) and corresponding time evolution which lead to the increase of CD in a statistical sense. The state at the active boundary changes and this corresponds to the subjective time evolution of a conscious entity, self. SSFRs are the TGD counterparts of repeated measurements for observables which commute with the observables whose eigenstates the states at the passive boundary are.

3. The doubled state space is highly analogous to the space of fermionic states in ZEO involving positive and negative energy physical particles at the opposite boundaries of CD. If one also allows single fermion tachyonic states then one could have fermions with wrong sign of energy at a given boundary of CD. If bosons correspond to fermion-antifermion pairs such that either fermion or antifermion is tachyonic, one obtains boson emission and physical bosons can have correct sign of mass squared. In the vertex identified as a defect of the standard spinor structure, either fermion or antifermion would be tachyonic. Since several vertices involving the change of the sign of the fermion or antifermion momentum are possible, outgoing physical fermions and antifermions with a correct sign or energy can be produced. Recall that both the physical leptons and quarks involve fermion-antifermion pairs in the recent picture based on closed monopole flux tubes associated with a pair of Minkowskian space-time sheets.

4. Tachyonic single fundamental fermion states (quarks or leptons) are natural in the number theoretic vision of TGD. The components of the fermionic momenta for a given extension of rationals are algebraic integers and mass squared for them can be tachyonic. These states are analogs of virtual fermions of the standard QFT which also can have tachyonic momenta. Physical states are assumed to be Galois singlets so that the total momentum for a bound state of fermions and antifermions has integer valued components and mass squared is integer. The condition that mass squared energy have a fixed sign for the physical states at a given boundary of the CD is natural and has been made.

3 Surfaceology, twistors, and TGD

The inspiration coming from the work of Nima Arkani-Hamed and colleagues concerning the twistor Grassmannian approach [B3, B7, B4, B2, B8, B1, B5] provided a strong boost for the development of TGD. I started from the problems of the twistor approach and ended up with a geometrization of the twistor space in terms of sub-manifold geometry with twistor space represented as a 6-surface. Also the twistor space of CP_2 played a key role.

This led to rather dramatic results. Most importantly, the twistor lift of TGD is possible only for H=$M^4 \times CP_2$ since only M^4 and CP_2 allow twistor space with Kähler structure [A1]: TGD is unique. The most recent result [L8] is that one can formulate the twistor-lift in terms of 6-surfaces of H (rather than 6-surfaces in the product of the twistor spaces of M^4 and CP_2). These twistor surfaces represent twistor spaces of M^4 and CP_2 or rather their generalizations, their intersection would define the space-time surface. Therefore one can formulate the twistor lift without the the 12-D product of twistor spaces of M^4 and CP_2.

During last years I have not followed the work of Nima and others since our ways went in very different directions: Nima was ready to give up space-time altogether and I wanted to replace it with 4-surfaces. I was also very worried about giving up space-time since twistor is basically a notion related to a flat 4-D Minkowski space.

However, in Quanta Magazine there there was recently a popular article telling about the recent work of Nima Arkani Hamed and his collaborators (see this). The title of the article was "Physicists Reveal a Quantum Geometry That Exists Outside of Space and Time". The article discusses the notions of amplituhedron and associahedron [L1] which together with the twistor Grassmann approach led to considerable insights about theories with $\mathcal{N} = 4$ supersymmetry. These theories are however rather limited and do not describe physical reality. In the fall of 2022, a Princeton University graduate student named Carolina Figueiredo realized that three types of particles lead to very similar scattering amplitudes. Some kind of universality seems to be involved. This leads to developments which allow to generalize the approach based on $\mathcal{N} = 4$ SUSY.

This approach, called surfaceology, still starts from the QFT picture, which has profound problems. On the other hand, it suggests that the calculational algorithms of QFT lead universally to the same result and are analogous to iteration of a dynamics defined in a theory space leading to the same result irrespective of the theory from which one starts from: this is understandable since the renormalization of coupling constants means motion in theory space.

3.1 Surfaceology and TGD

How does the surfaceology relate to TGD?

1. What one wants are the amplitudes, not all possible ways to end up them. The basic obstacle here is the belief in path integral approach. In TGD, general coordinate invariance forces holography forcing to give up path integral as something completely unnecessary.

2. Surfaceology and brings strongly in mind TGD. I have talked for almost 47 years about space-time as surfaces without any attention from colleagues (unless one regards the crackpot label and the loss of all support as such). Now I can congratulate myself: the battle that has lasted 47 years has ended in a victory. TGD is a more or less mature theory.

   It did not take many years to realize that space-times must be 4-surfaces in H=$M^4\times$CP$_2$, which is forced by both the standard model symmetries including Poincare invariance and by the mathematical existence of the theory. Point-like particles are replaced with 3-surfaces or rather the 4-D analogs of their Bohr orbits which are almost deterministic. These 4-surfaces contain 3-D light-like partonic orbits containing fermion lines. Space-time surfaces can in turn be seen as analogs of Feynman graphs with lines thickened to orbits of particles as 3-surfaces as analogs of Bohr orbits.

3. In holography=holomorphy vision space-time surfaces are minimal surfaces realized as roots of function pairs (f_1, f_2) of 4 generalized complex coordinates of H (the hypercomplex coordinate has light-like coordinate curves) [L8]. The roots of f_1 and f_2 are 6-D surfaces analogous to twistor spaces of M^4 and CP_2 and their intersection gives the space-time surface. The condition $f_2 = 0$ defines a map between the twistor spheres of M^4 and CP_2 and identifies the twistor spheres of M^4 and CP_2 [K1]. f_2 defines a slowly varying background whereas f_1 determines the fast dynamics. Outside the 3-D light-like partonic orbits appearing as singularities and carrying fermionic lines, these surfaces are extremals of any general coordinate invariant action constructible in terms of the induced geometry. In accordance with quantum criticality, the dynamics is therefore universal.

   Holography=holomorphy [L10, L11] vision generalizes ordinary holomorphy, which is the prerequisite of twistorialization. Now light-like 4-D momenta are replaced with 8-momenta which means that the generalized twistorialization applies also to particles massive in 4-D sense.

This strongly resembles what the popular article talks about surfaceology: the lines of Feynman diagrams are thickened to surfaces and lines are drawn to the surfaces which are however not space-time surfaces. Also Nima Arkani-Hamed admits that it would be important to have the notion of space-time.

The TGD view is crystallized in Geometric Langlands correspondence [L8] is realized naturally in TGD and implying correspondence between geometric and number theoretic views of TGD.

1. Space-time surfaces form an algebra decomposing to number fields so that one can multiply, divide, sum and subtract them. By holography= holomorphy vision, space-time surfaces are holomorphic minimal surfaces with singularities to which the holographic data defining scattering amplitudes can be assigned.

2. What is marvellous is that the minimal surfaces emerge irrespective of the classical action as long as it is general coordinate invariant and constructed in terms of induced geometry: action makes itself visible only at the partonic orbits and vacuum functional. This corresponds to the mysterious looking finding of Figueiredo.

   There is however a unique action and it corresponds to Kähler action for 6-D generalization of twistor space as surface in the product of twistor spaces of M^4 and CP_2. These twistor spaces of M^4 and CP_2 must allow Kähler structure and this is only possible for them. TGD is completely unique. Also number theoretic vision as dual of geometric vision implies uniqueness. A further source of uniqueness is that non-trivial fermionic scattering amplitudes exist only for 4-D space-time surfaces and 8-D embedding space.

3. Scattering amplitudes reduce at fermionic level to n-point functions of free field theory expressible using fermionic propagators for free leptonic and quark-like spinor fields in H with arguments restricted to the discrete set of self-intersections of the space-time surfaces and in more general case to intersections of several space-time surfaces. This works only for 4-D space-time surfaces and 8-dimensional H. Also pair creation is possible and is made possible by the existence of exotic smooth structures [L11, L12], which are ordinary smooth structures with defects identifiable as the intersection points. Therefore there is a direct correspondence with 4-D homology and intersection form. One can say that TGD in its recent form provides an exact construction recipe for the scattering amplitudes.

4. There is no special need to construct scattering amplitudes in terms of twistors as proposed in [L3, L4] although this is possible since the classical realization of twistorialization is enough and only fermions with spin 1/2 and isospin 1/2 are present as fundamental particles. Since all particles are bound states of fundamental fermions propagating along fermion lines associated with the partonic orbits, all amplitudes involve only propagators for free fermions of H. The analog of twistor diagrams correspond to diagrams, whose vertices correspond to the intersections and self-intersections for space-time surfaces.

3.2 Could quantum field theories be universal

The findings of Nima Arkani Hamed and his collaborators, in particular Carolina Figueiredo, suggest a universality for the scattering amplitudes predicted quantum field theories. Is it possible to understand this universality mathematically and what could its physical meaning be?

The background for these considerations comes from TGD, where holography = holomorphy principle and $M^8 - H$ duality relating geometric and number theoretic visions fixing the theory to a high degree.

1. Space-time surfaces are holomorphic surfaces in $H = M^4 \times CP_2$ and therefore minimal surfaces satisfying nonlinear analogs of massless field equations and representing generalizations of light-like geodesics. Therefore generalized conformal invariance seems to be central and also the Hamilton-Jacobi structures [L6] realizing this conformal invariance in M^4 in terms of a pair formed by complex and hypercomplex coordinate, which has light-like coordinate curves.

2. Quantum criticality means that minima as attractors and maxima as repulsors are replaced with saddle points having both stable and unstable directions. A particle at a saddle point tends to fall in unstable directions and end up to a second saddle point, which is attractive with respect to the degrees of freedom considered.

   Zero energy ontology (ZEO) predicts that the arrow of time is changed in "big" state function reductions (BSFRs). BSFRs make it possible to stay near the saddle point. This is proposed to be a key element of homeostasis. Particles can end up to a second saddle point by this kind of quantum transition.

3. Quantum criticality has conformal invariance as a correlate. This implies long range correlations and vanishing of dimensional parameters for degrees of freedom considered. This is the case in QFTs, which describe massless fields.

   Could one think that the S-matrix of a massless QFT actually serves as a model for transition between two quantum critical states located near saddle points in future and past infinity? The particle states at these temporal infinities would correspond to incoming and outgoing states and the S-matrix would be indeed non-trivial. Note that masslessness means that mass squared as the analog of harmonic oscillator coupling vanishes so that one has quantum criticality.

What can one say of the massless theories as models for the quantum transitions between two quantum critical states?

1. Are these theories free theories in the sense that both dimensional and dimensionless coupling parameters associated with the critical degrees of freedom vanish at quantum criticality. If the TGD inspired proposal is correct, it might be possible to have a non-trivial and universal S-matrix connecting two saddle points even if the theories are free.

2. A weaker condition would be that dimensionless coupling parameters approach fixed points at quantum criticality. This option looks more realistic but can it be realized in the QFT framework?

QFTs can be solved by an iteration of type $DX_{n+1} = f(X_n)$ and it is interesting to see what this allows to say about these two options.

1. In the classical gauge theory situation, X_{n+1} would correspond to an $n+1$:th iterate for a massless boson or spinor field whereas D would correspond to the free d'Alembertian for bosons and free Dirac operator for fermions. $f(X_n)$ would define the source term. For bosons it would be proportional to a fermionic or bosonic gauge current multiplied by coupling constant. For a spinor field it would correspond to the coupling of the spinor field to gauge potential or scalar field multiplied by a dimensional coupling constant.

2. Convergence requires that $f(X_n)$ approaches zero. This is not possible if the coupling parameters remain nonvanishing or the currents become non-vanishing in physical states. This could occur for gauge currents and gauge boson couplings of fermions in low enough resolution and would correspond to confinement.

3. In the quantum situation, bosonic and fermionic fields are operators. Radiative corrections bring in local divergences and their elimination leads to renormalization theory. Each step in the iteration requires the renormalization of the coupling parameters and this also requires empirical input. $f(X_n)$ approaches zero if the renormalized coupling parameters approach zero. This could be interpreted in terms of the length scale dependence of the coupling parameters.

4. Many things could go wrong in the iteration. Already, the iteration of polynomials of a complex variable need not converge to a fixed point but can approach a limit cycle and even chaos. In more general situations, the system can approach a strange attractor. In the case of QFT, the situation is much more complex and this kind of catastrophe could take place. One might hope that the renormalization of coupling parameters and possible approach to zero could save the situation.

It is interesting to compare the situation to TGD? First some general observations are in order.

1. Coupling constants are absorbed in the definition of induced gauge potentials and there is no sense in decomposing the classical field equations to free and interaction terms. At the QFT limit the situation of course changes.

2. There are no primary boson fields since bosons are identified as bound states of fermions and antifermions and fermion fields are induced from the free second quantized spinor fields of H to the space-time surfaces. Therefore the iterative procedure is not needed in TGD.

3. CP_2 size defines the only dimensional parameter and has geometric meaning unlike the dimensional couplings of QFTs and string tension of superstring models. Planck length scale and various p-adic length scales would be proportional to CP_2 size. These parameters can be made dimensionless using CP_2 size as a geometric length unit.

The counterpart of the coupling constant evolution emerges at the QFT limit of TGD.

1. Coupling constant evolution is determined by number theory and is discrete. Different fixed points as quantum critical points correspond to extensions of rationals and p-adic length scales associated with ramified primes in the approximation when polynomials with coefficients in an extension of rationals determine space-time surfaces as their roots.

2. The values of the dimensionless coupling parameters appearing in the action determining geometrically the space-time surface (Kähler coupling strength and cosmological constant) are fixed by the conditions that the exponential of the action, which depends n coupling parameters, equals to its number theoretic counterparts determined by number theoretic considerations alone as a product of discriminants associated with the partonic 2-surfaces [L8, ?]. These couplings determine the other gauge couplings since all induced gauge fields are expressible in terms of H coordinates and their gradients.

3. Any general coordinate invariant action constructible in terms of the induced geometry satisfies the general holomorphic ansats giving minimal surfaces as solutions. The form of the classical action can affect the partonic surfaces only via boundary conditions, which in turn affects the values of the discriminants. Could the partonic 2-surfaces adapt in such a way that the discriminant does not depend on the form of the classical action? The modified Dirac action containing couplings to the induced gauge potentials and metric would determine the fermioni scattering amplitudes.

4. In TGD the induction of metric, spinor connection and second quantized spinor fields of H solves the problems of QFT approach due to the condition that coupling parameters should approach zero at the limit of an infinite number of iterations. Minimal surfaces geometrizes gauge dynamics. Space-time surfaces satisfying holography = holomorphy condition correspond to quantum critical situations and the iteration leading from one critical point to another one is replaced with quantum transition.

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