Space-time surfaces as numbers, Turing and Gödel, and mathematical consciousness

November 30, 2024

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Abstract

In this article a rather detailed view about the realization of Langlands correspondence (LC) is discussed. The geometric and function field versions naturally correspond to each other and the LC itself boils down to the condition that cobordisms for the function pairs (f_1, f_2) defining the space-time surfaces as union of regions defined by their roots are realized as flows in the infinite-D symmetry group permuting space-time regions as roots of a function pair (f_1, f_2) acting in the "world of classical worlds" (WCW) consisting of space of space-time surfaces satisfying holography = holomorphy principle.

Space-time surfaces form an algebra with respect to multiplication and this algebra decomposes to a union of number fields. This suggest a dramatic revision of what computation means physically. The standard view of computation as a construction of arithmetic functions is replaced with a physical picture in which space-times as 4-surfaces have interpretation as almost deterministic computations. Space-time surfaces allow arithmetic operations and also the counterparts of functional composition and iteration are well-defined. This would suggest a dramatic generalization of the computational paradigm and it is interesting to ponder what this might mean.

This also leads to a vision about the geometric correlates of arithmetic and even more general mathematical consciousness based on the vision about space-time surfaces as generalized numbers and providing also a representation of the ordinary complex numbers.

Contents

1 Introduction

The stimulus for this work came the links to Bruno Marchal's posts by Jayaram Bista (see this). The original comments compared the world views behind two Platonisms, the Platonism based on integers or rationals and realized by the Turing machine as a Universal Computer and the quantum Platonism of TGD [L17]. Marchal also talks about Digital Mechanism and claims that it is not necessary to assume a fixed physical universe "out there". Marschal also speaks of mathematical theology and claims that quantum theory and even consciousness reduce to Digital Mechanism.

Later these comments expanded to a vision about the geometric correlates of arithmetic and even more general mathematical consciousness based on the vision about space-time surfaces as generalized numbers and providing also a representation of the ordinary complex numbers.

This also led to a more detailed view about the realization of Langlands correspondence (LC) in which geometric and function field versions naturally correspond to each other and the LC itself boils down to the condition that cobordisms for the function pairs (f_1, f_2) defining the space-time surfaces as their roots are realized as flows in the infinite-D symmetry group permuting space-time regions as roots of a function pair (f_1, f_2) acting in the "world of classical worlds" (WCW) consisting of space of space-time surfaces satisfying holography = holomorphy principle.

That space-time surfaces form an algebra with respect to multiplication and that this algebra decomposes to a union of number fields [L14] means a dramatic revision of what computation means. The standard view of computation as a construction of arithmetic functions is replaced with a physical picture in which space-times as 4-surfaces have interpretation as almost deterministic computations. Space-time surfaces allow arithmetic operations and also the counterparts of functional composition and iteration are well-defined. This would suggest a dramatic generalization of the computational paradigm and it is interesting to ponder what this might mean.

This also leads to a vision about the fundamental geometric correlates of arithmetic and even more general mathematical consciousness based on the vision about space-time surfaces as generalized numbers and providing also a representation of the ordinary complex numbers. The notion of concept, such as a set as a collection of its instances, can be realized at the level of WCW in terms of the locus of the WCW spinor field when space-time surfaces correspond to numbers in generalized sense or to ordinary complex numbers. Second realization analogous to Boolean algebra is in terms of the product of space-time surfaces as elements of the generalized number field. Also the notion of linear space can be realized in this way by realizing the ordering of the elements of the set geometrically. Also the notion of function can be realized.

Of course, my personal view of computation and metamathematics is rather limited: I am just a humble physicist thinking simple thoughts but my sincere hope is that mathematicians would realize how deep the implications of the new physics based number concept has.

2 A more detailed view of the realization of 4-D Langlands correspondence (LC)

Langlands correspondence (LC) [A4, K5, A3, A2] is discussed from the TGD perspective earlier in [K5] [L1, L9, L16]. In [L14] the realization of the analog of Langlands correspondence (LC) based on the holography = holomorphy vision was discussed. Here a more detailed view, in which LC for function fields induced LC for space-time surfaces, will be discussed. The homotopies for function pairs (f,g) (one usually speaks of cobordisms) define the space-time surface and its decomposition in terms of its roots induce flows defining the generalization of the Galois group as allowed permutations of the roots of (f,g).

The natural restriction is that they act as flows induced by the symmetries of WCW, in particular generalized holomorphic transformations. This correspondence maps the group of cobordisms permuting roots of (f,g) defining the analog of the Galois groups to flows in the symmetry group of WCW and maps this group to a subgroup of the symmetry group S of WCW. This implies that the analog of Langlands correspondence would follow as a consequence.

Note that restriction to irreducible polynomials allows a gauge fixing eliminating the huge gauge symmetry due to possibility to multiply with functions, which are non-vanishing inside CD becomes possible. For general analytic functions irreducibility does not have any counterpart but the notions of root and discriminant still make sense. Therefore space-time surface quite generally represents a real number as a product of discriminants assignable to the partonic 2-surfaces if the Taylor coefficients are real.

1. One could define the action for the analog of the Galois group in the function field as cobordisms for functions f_1, f_2 such that they permute the roots as space-time regions representing the roots. Not all permutations are expected to be possible so that the group is smaller than the permutation group in general. This would replace the fundamental group for Riemann surfaces appearing in 2-D Langlands correspondence. This picture could make sense also for partonic 2-surfaces.

2. One can also consider lower-dimensional surfaces of the space-time surfaces. String world sheets could correspond to roots of complex valued functions f_1, f_2, f_3. If f_2 depends on light-coordinate u only, the roots satisfy 2+2+1=5 conditions giving a light-like 3-surface. Light-like partonic orbits could be in question.

   For 4 functions (f_1, f_2, f_3, f_4) such that f_4 is real and therefore depends on a light-like coordinate only, the roots are light-like curves perhaps identifiable as fermion lines as boundaries of the string word sheets. It seems impossible to obtain partonic 2-surfaces as roots for f_i and they should emerge as singularities for which both u and v are constant.

   In all these cases it makes sense to speak of cobordisms. It might be possible talk about cobordisms also partonic 2-surfaces as singularities? The analogy with the Galois group and braid group and realizability as flows for the symmetries of WCW strongly suggests that the cobordism group for light-like 3 surfaces cannot correspond to the infinite fundamental group of the partonic 2-surface. These transformations define an analog of a generalized braid group as a covering of a permutation group.

2.1 What could be the TGD counterpart of the Lie group appearing in LC?

The analog of the Galois/braid group can be identified as the covering of the cobordism group acting on f_1, f_2 and permuting the roots. 4-D variant of the braid group acting on Bohr orbits as a covering group of the cobordism group. The analog of $a^p = \sigma_p a$ would hold true and σ_p is fixed. One would have a projective representation for the covering of the permutation group as an analog of the braid group. The analog of the Frobenius element σ_p would correspond to a Hecke operator in the representation of the group algebra of the WCW symmetry group algebra G acting as symmetries of WCW. Hecke algebra would be sub-algebra representing the analogs of Frobenius elements σ_p or more generally σ_m, m integer. The Hecke algebra could be also non-commutative. The generalized cobordism group/braid group would be represented in the group algebra of G.

1. The product of $SL(2,C) \times SU(3)$ with electroweak holonomy group is the first candidate but could be too small a group unless the generalized Virasoro and Kac Moody conditions reduce the degrees of freedom dramatically. The reason is that the realization of braindings as flows induced by this group requires that the roots as regions of the space-time surface are related by symmetries. The conformal symmetries of the space-time surfaces acting on the arguments of (f_1, f_2) are much more plausible candidates.
2. Besides conformal symmetries also the isometries proposed to be realizable as symplectic transformations of $\delta M^4 \times CP_2$ and also symplectic transformations of the light-like partonic orbits are good candidates for symmetries and could define the symmetry group S of WCW. The generalized gauge conditions for the corresponding algebras reduced by non-negative integers (half algebra structure) would reduce dramatically the number of degrees of freedom since the number of conformal weights would effectively be reduced to a finite one.

2.2 About the TGD analogs of Frobenius elements?

Langlands correspondence could mean that the homomorphism of the generalized Galois group to the subgroup of S defines "good" representations. The permutations of the roots as spacetime regions should not lead out of the irreducible representation of G. This is guaranteed if the cobordisms correspond to the flows defined by the elements of G.

What could the counterparts of Frobenius elements σ_p be? What is the generalization of the Galois/braid group defined at the function field level? Frobenius element σ_p in number theoretic context for finite fields corresponds to a lift of the Frobenius element $a^p = a$ to the extension so that $a^p = \sigma_p a$. σ_p corresponds to an action of element of the Galois group in the irreducible representation of the Galois of the extension L of the base field K so that it acts trivially on K. In the recent case the counter for σ_p would be a lift of a trivial element of the permutation group to an element of the analog of the braid group acting in the irrep of S. One would have a projective representation or even a more general representation for which phase factors σ_p would be replaced by matrix action.

In the anyonic case the 2-dimensionality is essential. This might be the case also now so that the non-Abelian statistics could be obtained for the partonic orbits. If the exponent of the Kähler function is determined as the product of discriminants assignable to the partonic surfaces, this could be the case. Since the braid strands define holes of 2-D section of partonic orbit and since the cobordism group of the 2-D surface with holes is non-commutative this would be natural.

The step in which the Frobenius element is lifted from a unit to σ_p in the extension of a finite field must correspond to the braided counterpart of the unit element at the level of the symmetry algebra S of WCW. One must consider braided cobordisms of the function algebra.

Since the symmetry group S_n is generated by permutations of two elements it would seem that it is enough to consider braidings for the permutations of neighboring braid strands. From these more complex braidings can be generated. Does this mean that σ_p determines the rest? Presumably this is an oversimplification as the relations for non-spherical Hecke algebras suggests

2.3 About the TGD analogs of Hecke operators?

2.3.1 Spherical Hecke operators

Wikipedia (see) provides information about Hecke operators in the spherical (commutative) case.

According to the Wikipedia article (see this) the so-called spherical Hecke algebras are commutative and therefore represent commuting observables. So called Iwahori-Hecke algebras are in general non-commutative. There are very many kinds of Hecke algebras, inn particular affine Hecke algebra (see this), which is deformation of the the group algebra of the affine Weyl group.

Action of Hecke operators on modular forms in SL(2,Z) is relevant to the number theoretic case in which one considers numbers of roots of third order polynomial equations with two variables and with polynomial coefficients in finite fields. The coefficients a_p of the Fourier series of the modular form codes for the numbers of the solutions in all finite fields except finite number of exceptions.

1. One considers the action of matrices with integer components and with determinant m on modular forms f (not quite functions) defined in the complex upper plane. f is invariant

under modular group $\Gamma = SL(2,Z)$ apart from modular factor: $f(\gamma(z)) = (cz+d)^k f(z)$. k defines the weight of the modular form. Γ corresponds to $m = 1$. Actually factor spaces $\Gamma \setminus M_m$ are considered meaning that the points of M_m differing by a left multiplication by $SL(2,C)$ element have the same action on the argument of f. The presence of the modular factor spoils the full modular invariance.

2. The action of the m:th Hecke operator T_m involves the sum over the translations by elements of $\Gamma \setminus M_m$. The subspace of modular forms with weight k is preserved under the action of Hecke operators T_m. Hecke operators can be regarded as commuting observables. The action of T_m on f creates what is called a normalized cuspidal Hecke eigenform which is an analytic function of $q = exp(i2\pi z)$, which approaches to zero in the upper half plane. The eigenvalues of the Hecke operators are coded by the Fourier coefficients of normalized cuspidal Hecke eigenforms and for prime values of m they code number theoretic information about almost all finite fields.

$SL(2,Z)$ is a subgroup of the Lorentz group. $SL(2,Z+iZ)$ is the Gaussian integer variant of SL(2,Z). Now the action is not restricted to the upper half-plane and the variable $q = exp(i2\pi z)$ diverges in the lower half-plane. Is it possible to generalize the modular functions in H^3 and consider modular groups identified as subgroups of $SL(2,C)$?

Could one consider the 3-D light-cone boundary in which case conformal invariance would not be lost and the variable z could correspond to the complex coordinate of S^2 for which radial light-coordinate is constant. The metric 2-dimensionality implies that conformal transformations accompanied by a local radial scale induce an infinite-D group of isometries and could make the situation effectively 2-D. Could Hamilton-Jacobi structure make possible representations in M^4 sliced by parallel light-cone boundaries?

2.3.2 What could one say about the TGD counterparts of Hecke operators T_p?

What could one say about the TGD counterparts of Hecke operators T_p?

1. The first guess is that T_p should act in the group algebra of S or of its subgroup. The Hecke operators should represent the lifts of the trivial permutations $a^p = a$ to braid group and cannot lead out of the irrep. The commutativity would allow an interpretation as observables. When the particle/braid strand exchange is represented by a non-commutative operation rather than phase factor, one has non-Abelian statistics.

2. Are there physical motivations for the commutativity of the Hecke operators? When they can be non-commutative? Here physical intuition comes to rescue. For anyons this boils down to the question about when anyons are Abelian *resp.* non-Abelian. Non-Abelian anyons require degeneracy of states so that the exchange operation for the strands can lead to a physically different many-particle configuration. The exchange would be represented by a unitary matrix acting in the space of degenerate states.

3. In the TGD framework, this kind of degeneracy is strongly suggested by quantum criticality of the TGD Universe. One prerequisite of criticality is that the Kähler function has the same value for the degenerate configuration. If $exp(K)$ is a power of the product of dicriminants D_i assignable to the partonic 2-surfaces, the value of the product $D = \prod_i D_i$ would be the same. D can be same for a very large number of space-time surfaces.

What could one say about the counterparts of the Hecke operators in TGD? For Riemann surfaces there is a close correspondence with the homology group and homotopy group of the Riemann surface. In the 4-D case there is a correspondence with the cobordism group acting as flows on pairs (f_1, f_2) and consistent with a flow defined by the symmetry group S of WCW and inducing permutations on their roots as regions of the space-time surface.

Physically the Hecke operators would realize the action of trivial cobordisms of (f_1, f_2) lifted to braid group action in the a covering of the permutation group. The index m as the weight of the modular form should correspond to the degree of the covering and coverings with prime value of m should be special. Non-Abelian statistics allows also nonc-commutative Hecke operators.

The natural first guess is that the Hecke operators T_p are defined as elements of the group algebra of S and are left and right invariant under a discrete subgroup algebra of the symmetry group algebra of WCW. This subgroup should correspond to the homomorphic image of the 4-D analog of the braid group as a covering group of permutations of the roots of (f_1, f_2) represented as homopties of the function field. The guess is that T_p can be expressed as an integral over the elements of the subgroup algebra.

1. In the finite field case, σ_p is an element of Galois group, which acts trivially on the base field K extended to L. This generalizes to extensions of general field K. What is the counterpart of the lift $K \to L$? Permutations are replace with braidings defining n-fold covering of the permutation group. Are prime-fold coverings somehow special? It would be very natural or are Z_p as simple groups of natural subgroups of Z_n. Are powers of permutations $a^p = a$ replaced by multiplication by a phase or are non-commutative braidings possible?

2. Why should the Hecke operators be left and right invariant under a homomorphic image of a discrete subgroup of the braid group? What does the bi-invariance of the Abelian case mean? Is the action of the cobordism group, lifted to braid group action involving the action Frobenius element. The Frobenius elements σ_p should correspond to the Hecke operator T_p in the group algebra S. Does the general formula for a spherical Hecke operator follow from the invariance condition?

3. The lift of an ordinary permutation satisfying $a^p = a$ to an element $a^p = \sigma_p a$, where σ is a Frobenius element of the generalized cobordism group as a braid group. Frobenius element σ_p is a lift of the unit element of the permutation group to an element of the braid group. Unit element is invariant under automorphisms of the permutation group. Also the sigma must have the same property.

   Braid group action must have a counterpart in the group algebra of the symmetry algebra S of WCW. In the Abelian case it must be invariant under left and right action of the image of the subgroup Z_p. Good representations of S are those for which the generalized cobordism group is represented. A semidirect product of braid group with the connected component of S, or rather its Langlands dual, would be in question. Why S or its subgroup is replaced with its Langlands dual is an open question quite generally.

   Automorphic action is a more general option. The representation of the group algebra of a subgroup of S to which the Hecke operator belongs could characterize it. A stronger condition is that the operator as a group algebra element is both left and right invariant and this corresponds to the abelian anyons.

3 TGD view of computationalism and its physical realization

The TGD view of space-time surfaces as numbers [L14] provides the background for the following considerations.

3.1 The replacement of the static universe with a Universe continuously recreating itself

It seems to me that the problems of computationalism emerge from a single ontological assumption: the "system", be it Universe in some sense or God, is fixed. In quantum TGD this is not the case. The Quantum Universe, which could be seen as a counterpart for God, is continually recreating itself and this means the unavoidable increase of algebraic complexity since the dimensions associated with extensions of rationals defining space-time regions unavoidably increase. This in turn implies evolution.

In zero energy ontology (ZEO) "small" state function reductions (SSFRs) [K12], whose sequence generalizes Zeno effect, which has no effect on physical state. SSFRs have and their sequence gives rise to conscious entities, selves. This makes possible memory [L15]: the outcome of SSFR has classical information about the initial state and also about the transition. Therefore the Universe remembers and learns consciously: one can talk about Akashic records.

This dynamical view of the Universe recreating itself and becoming more intelligent by learning about what it was before the previous SSFR is very different from the view of the Universe as a Turing machine or Universal Computer. These notions are static notions (Universe "out there") and computation is based on integers. In the TGD view one obtains an entire hierarchy of computationalisms based on the hierarchy of extensions of rationals. Even transcendental extension can be considered. TGD Universe as a counterpart of the Turing machine is also conscious and has free will.

3.2 A generalization of the number concept

Also the notion of number generalizes from the set N of integers to the set of space-time surfaces, the "World of Classical Worlds" (WCW) [K4, K2, K11, K9, K8, K3]. The TGD view of geometric Langlands duality means that space-time surfaces can be multiplied and summed and form an algebra. This algebra decomposes to a union of number fields with product,division, sum and subtraction. One can identify space-time surfaces forming analogs for hierarchies of algebraic integers, algebraic rationals, etc... So that the mathematics performed by Quantum Platonia is considerably more complex than counting by 5+5 fingers!

These structures are defined by the corresponding structures for function algebras and fields defined in terms of analytic functions of 8 generalized complex coordinates of $H = M^4 \times CP_2$. One of the coordinates is a hypercomplex coordinate with light-like coordinate curves.

1. In TGD space-time surfaces are numbers [L14]. Their dynamics is almost deterministic (at singularities the determinism fails and this forces us to take space-time surfaces instead of 3-surfaces as basic objects). The space-time surface as an almost deterministic time evolution is analogous to a proof of a theorem. The assumptions correspond to the initial state 3-surface and the outcome of the theorem to the final 3-surface. Second interpretation is as analogs of deterministic computer programs. Space-time surface as a proof of a theorem is analogous to its own Gödel number as a generalized number.

2. Cognition always requires a discretization and the space of space-time surfaces ("world of classical worlds", WCW) allows a hierarchy of discretizations. The Taylor coefficients of the two analytic functions defining space-time belong to some extension of rationals forming a hierarchy. Therefore a given space-time surface corresponds to a discrete set of integers/rationals in an extension so that also WCW is discretized. For polynomials and rational functions this set is discrete. This gives a hierarchy. At the level of the space-time surface an analogous discretization in terms of an extension of rationals takes place.

3. Gödel number for a given theorem as almost deterministic time evolution of 3-surface would be parametrized by the Taylor coefficients in a given extension of rationals. Polynomials are simplest analytic functions and irreducible polynomials define polynomial primes having no decomposition to polynomials of a lower degree. They might be seen as counterparts of axioms.

4. One can form analogs of integers as products of polynomials inducing products of space-time surfaces. The space-time surfaces are unions for the space-time surfaces defined by the factors but an important point is that they have a discrete set of intersection points. Fermionic n-point functions defining scattering amplitudes are defined in terms of these intersection points and give a quantum physical realization giving information of the quantum superpositions of space-time surfaces as quantum theorems.

3.3 Could space-time surfaces replaced as integers replace ordinary integers in computationalism?

It is interesting to play with the idea that space-time surfaces as numbers, in particular integers, could define counterparts of integers in ordinary computationalism and metamathematics.

What might be the counterpart for the possibility to represent theorems as integers deduced using logic and for the Gödel numbering for theorems by integers?

1. In TGD space-time surfaces are numbers. Their dynamics is almost deterministic (at singularities the determinism fails and this forces us to take space-time surfaces instead of 3-surfaces as basic objects). The space-time surface as an almost deterministic time evolution is analogous to a proof of a theorem. The assumptions correspond to the initial state 3-surface and the outcome of the theorem to the final 3-surface. The second interpretation is as analogs of deterministic computer programs. Space-time surface as a proof of a theorem is analogous to its own Gödel number as a generalized number.

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3.4 Adeles and Gödel numbering

Adeles in TGD sense [L2, L3, L12] inspire another interesting development generalizing the Gödelian view of metamathematics.

1. p-Adic number fields are labelled by primes and finite fields induced by their extensions. One can organize the p-adic number fields to adele and the same applies to their extensions so that one has an infinite hierarchy of algebraic extensions of the rational adele. TGD brings something new to this picture.

2. Two p-adic number fields for which elements are power series in powers of p_1 *resp.* p_2 with coefficients smaller than p_1 *resp.* p_2, have common elements for which expansions are in powers of integers $n(k_1, k_2) = p_1^{k_1} \times p_2^{k_2}$, $k_1 > 0, k_2 > 0$ [L12, L16]. This generalizes to the intersection of $p_1, p_2, ...p_n$. One can decompose adeles for a union of p-adic number fields which are glued together along these kinds of subsets. This decomposition is general in the description of interactions between p-adic sectors of adeles. Interactions are localized to these intersections.

3. Mathematical cognition would be based on p-adic numbers. Could one think that ordinary integers should be replaced with the adelic integers for which the p_i:th factor would consist of p-adic integers of type p_i. These integers are not well-ordered so that the one cannot well-order theorems/programs/etc... as in Gödel numbering. The number of p-adic integers is much larger than natural numbers since the pinery expansion can contain an infinite number of terms and one can map p-adic integers to real numbers by what I call canonical identification. Besides this one has fusion of various p-adic number fields.

An interesting question is how this changes the Gödelian views about metamathematics. It is interesting to play with the idea that space-time surfaces as numbers, in particular generalized integers, could define counterparts of integers in ordinary computationalism and metamathematics.

3.5 Numbering of theorems by space-time surfaces?

What might be the counterpart for the possibility to represent theorems as integers deduced using logic and for the Gödel numbering for theorems by integers?

1. In TGD space-time surfaces are numbers. Their dynamics is almost deterministic (at singularities the determinism fails and this forces us to take 4-D space-time surfaces instead of 3-surfaces as basic objects). The space-time surface as an almost deterministic time evolution is analogous to a proof of a theorem. The assumptions correspond to the initial state 3-surface and the outcome of the theorem to the final 3-surface. The second interpretation is as an analog of a deterministic computer program. The third interpretation as a biological function. Space-time surface as a proof of a theorem is analogous to its own Gödel number, but now as a generalized number. One can define the notions of prime , integer , rational and transcendental for the space-time surfaces.

   The counterparts of primes, determined by pairs of irreducible polynomials, could be seen as axioms. The product operation for space-time surfaces generates unions of space-time surfaces with a discrete set of intersection points, which appear as arguments of fermionic n-point functions allowing to define fermionic scattering amplitudes. Also other arithmetic operations are possible.

2. Also functional composition, essential in computationalism, is possible. One can take any pair $(h_1(z), h_2(z))$ of analytic functions of a complex coordinate z and form a functional composites $h_1 \circ f_1$ or $h_2 \circ f_2$. One can also iterate with respect to h_1 or h_2. As a special case on can take $h_1 = h_2 = h$. This would make it possible to realize recursion, essential in computationalism. The iteration increases the degree of the polynomial and therefore also the number of roots with an exponential rate so that the complexity of the space-time surface increases. The iteration leads also to fractals. An interesting question is how this could relate to biological evolution.

   Also self-referentiality becomes possible: one could identify z as one of the genuinely complex H coordinates and perform parameter dependent iteration for z as argument of f_1 using $z \to f_1(z, ...)$ with parameters defined by other H coordinates.

3. One can consider even more general maps $(f_1, f_2) \to G(f_1, f_2) = (g_1(f_1, f_2), g_2(f_1, f_2))$ and iterate also them. The special case $(g_1, g_2) = (g_1(f_1), g_2(f_2))$ gives iterations of functions g_i of a single complex variable appearing in the construction of Mandelbrot and Julia fractals.

Extensions of rationals, Galois groups, and ramified primes assignable to polynomials of a single complex variable are central in the number theoretic vision. It is not however completely clear how they should emerge from the holography= holomorphy vision.

1. If the functions $g_i \equiv P_i$ are polynomials, which vanish at the origin $(0, 0)$ (this is not a necessary condition), the surfaces $(f_1, f_2) = (0, 0)$ are roots of $(P_1(f_1, f_2), P_2(f_1, f_2)) = (0, 0)$. Besides these roots, there are roots for which (f_1, f_2) does not vanish. One can solve the roots $f_2 = h(f_1)$ from $g_2(f_1, f_2) = 0$ and substitute to $P_1(f_1, f_2) = 0$ to get $P_1(f_1, h(f_1)) \equiv P_1 \circ H(f_1)) = 0$. The values of $H(f_1)$ are roots of P_1 and are algebraic numbers if the coefficients of P_1 are in an extension of rationals. One can assign to the roots discriminant, ramified primes, and Galois group. This is just what the phenomenological number theoretical picture requires.

2. In the earliest approach to $M^8 - H$ duality summarized in [L6, L7, L13] polynomials P of a single complex coordinate played a key role. Although this approach was a failure, it added to the number theoretic vision Galois groups and ramified primes as prime factors of the discriminant D, identified as p-adic primes in p-adic mass calculations. Note that in the general case the ramified primes are primes of algebraic extensions of rationals: the simplest case corresponds to Gaussian primes and Gaussian Mersenne primes indeed appear in the applications of TGD [K6, K7].

   The problem was to assign a Galois group and ramified primes to the space-time surfaces as 4-D roots of $(f_1, f_2) = (0, 0)$. One can indeed define the counterpart of the Galois group defined as analytic flows permuting various 4-D roots of $(f_1, f_2) = (0, 0)$ [L14].

Since the roots are 4-D surfaces, it is far from clear whether there exists a definition of discriminant as an analog for the product of root differences. Also it is unclear what the notion of ramified prime could mean. However, the ordinary Galois group plays a key role in the number theoretic vision: can one identify it? An possible identification of the ordinary Galois group and ramified primes would by the assignment to maps defined by $(f_1, f_2) \rightarrow (P_1(f_1, f_2), P_2(f_1, f_2))$ would be in terms of $(P_1(f_1, f_2), P_2(f_1, f_2)) = (0, 0)$ giving the roots $P_1(f_1, h(f_1)) = 0$ as values of $h(f_1)$. The roots belong to extension rationals even when f_i are arbitrary analytic functions of H coordinates but correspond geometrically to 4-surfaces.

3. The earlier proposal is that the ordinary Galois group can be assigned to the partonic 2-surfaces so that points of the partonic 2-surface as roots of a polynomial give rise to the Galois group and ramified primes. The most elegant way to realize this is to introduce 4 polynomials (P_1, P_2, P_3, P_4). The roots of (P_1, P_2, P_3) allow to solve the 3 complex coordinates as a function of the hypercomplex coordinate u. This surface can be identified as a string world sheet.

   The additional condition $P_4(u) = 0$ gives roots which are algebraic numbers if the coefficients of P_4 are in an extension of rationals. Note that only real roots are allowed.

   The interpretation of the roots for u would be as singularities of the space-time surface located at the partonic 2-surfaces where the minimal surface property fails and the trace of the second fundamental form diverges. These points would correspond to vertices for the creation of a fermion pair and would represent defects of the standard smooth structure giving rise to an exotic smooth structure [L17, L8, L18].

Cognition always requires a discretization.

1. The space of space-time surfaces ("world of classical worlds", WCW) allows a hierarchy of discretizations. The Taylor coefficients of the two analytic functions f_1, f_2 defining space-time belong to some extension E of rationals forming a hierarchy. Therefore a given space-time surface corresponds to a discrete set of integers/rationals in an extension of rationals so that also WCW is discretized for given E. For polynomials and rational functions this set is discrete. This gives a hierarchy. At the level of the space-time surface an analogous discretization in terms of E takes place.

2. Gödel number for a given theorem as almost deterministic time evolution of 3-surface would be parametrized by the Taylor coefficients in a given extension of rationals. Polynomials are simplest analytic functions and irreducible polynomials define polynomial primes having no decomposition to polynomials of a lower degree. Polynomial primes might be seen as counterparts of axioms. General analytic functions are analogous to transcendentals.

3. One can form analogs of integers as products of polynomials inducing products of space-time surfaces as their roots. The space-time surfaces are unions for the space-time surfaces defined by the factors but an important point is that they have a discrete set of intersection points. Fermionic n-point functions defining scattering amplitudes are defined in terms of these intersection points and give a quantum physical realization giving information of the quantum superpositions of space-time surfaces as quantum theorems.

4 A more detailed view of the arithmetics of space-time surfaces

The idea that the Universe could be performing arithmetics with space-time surfaces as classical worlds is fascinating. What could the physical meaning of the product and sum be and could they correspond to real physical interactions to which one can assign scattering amplitudes?

4.1 Sum and product for the space-time surfaces

In the case of the sum, the basic restriction is the condition that the space-time surfaces appearing as summands allow a common Hamilton-Jacobi structure [L10] in M^4 degrees of freedom in turn inducing it for the space-time surfaces. The summed space-time surfaces must have a common hypercomplex coordinate with light-like coordinate curves and a common complex coordinate. For the product this is not required.

1. One can form analogs of integers as products of polynomials inducing products of space-time surfaces as their roots. The product is defined as the root of $(f_1, g) * (f_2, g) = (f_1 f_2, g))$. The space-time surface defined by the product is the union of the space-time surfaces defined by the factors but an important point is that they have a discrete set of intersection points. In this case there are no restrictions on Hamilton-Jacobi structures.

   One can argue that the product represents a mere free two-particle state in topological and geometric sense. On the other hand, fermionic n-point functions defining scattering amplitudes are defined in terms of these intersection points and could give a quantum physical realization giving information of the quantum superpositions of space-time surfaces as quantum theorems. This would raise dimensions $D = 4$ and $D = 8$ in a completely unique role.

2. Could the sum of space-time surfaces $(f_1, g) = (0, 0)$ and $(f_2, g) = (0, 0)$ defined as a root of $(f_1, g)+(f_2, g) = (f_1+f_2, g))$ define a topologically and geometrically non-trivial interaction? If the functions f_1 and f_2 have interiors of causal diamonds CD_1 and CD_2 with different tips as supports (does the complex analyticity allow this?) and CD_1 and CD_2 are located within a larger CD then both f_1 and f_2 are nonvanishing only in the intersection $CD_1 \cap CD_2$.

   Generalized complex analyticity requires a Hamilton-Jacobi structure [L10] inside CD. It must have a common hypercomplex coordinate and complex M^4 coordinate inside CD and therefore inside $CD_1 \cap CD_2$ and also inside CD_1 and CD_2? Suppose that this condition can be satisfied.

   Outside $CD_1 \cap CD_2$ either f_1 and f_2 is identically vanishing and one has $f_1 = 0$ and $f_2 = 0$ as disjoint roots representing incoming particles in topological sense. In the intersection $CD_1 \cap CD_2$ $f_1 + f_2 = 0$ represents a root having interpretation as interaction. f_i "interfere" in this region and this interference is consistent with relativistic causality.

   One could also assign to the sum a tensor product in fermionic degrees of freedom and define n-point functions and restrict their arguments to the self-intersection points of the intersection region $CD_1 \cap CD_2$. One could also say that the sum represents $z = x + y$ in such a way that both summands and sum are realized geometrically.

At this moment it is unclear whether both product and sum or only product or some could be assigned with topological particle interactions. From the number theoretic point of perspective one would expect that both are involved.

4.2 Could the Hilbert space of pairs (f_1, f_2) have an inner product defined by the intersection of corresponding space-time surfaces?

The pairs (f_1, f_2) can be formally regarded as elements of a complex Hilbert space. There is however a huge gauge invariance: the multiplication of f_i by analytic functions, which are non-vanishing inside the CD, does not affect the space-time surface. The localization of the scalar multiplication means a huge reduction in the number of degrees of freedom. Note that the multiplication with a scalar does not change the spacetime surface but this does not destroy the field property. Since $f_1 = constant = c$ does not correspond to any space-time surface (this would require $c = 0$) the multiplication with a constant does not correspond to a multiplication with a space-time surface.

The local complex scalings are local variants of complex scalings of Hilbert space vectors which do not affect the state: one cannot however replace Hilbert space by a projective space and the same applies now. Could space-time surfaces define a classical representation for the analogs of local wave functions forming a local counterpart of a Hilbert space?

How could one realize the Hilbert space inner product?

1. Could one consider a sensible inner product for the pairs (f_1, g) having CD as a dynamic locus (SSFRs) [L11]. The only realistic option consistent with the local scaling property seems to be that the locus of the integral defining the inner product must be the intersection of the space-time surfaces defined by (f_i, g_i). By their dimension, the space-time surfaces have in the generic case a discrete set of intersection points so that the inner product is non-trivial. What suggests itself is that the inner product is determined by the intersection form of the space-time surfaces, most naturally its trace. The norm would in turn correspond to self-intersection form. Does this give rise to a positive definite inner product?

2. The situation would be the same as in the fermionic degrees of freedom where also intersection points would appear as arguments of n-point functions. That 4-D surfaces are in question conforms with the idea of generalized complex and symplectic structures reducing the number of degrees of freedom from 8 to 4.

4.3 More details about the arithmetics of space-times surfaces

Some basic facts about the arithmetics of the space-times surfaces deserve a separate discussion.

4.3.1 Intersections of the space-time surfaces obtained in product operations

The intersections of the space-time surfaces obtained in the products of space-times surfaces are highly interesting since they determine fermionic scattering amplitudes in the many-partic states. There are several cases to be considered. In particular, one can consider the function fields and the more general function algebra.

1. Consider first the product of space-time surfaces f_1, g and f_2, g is induced by the product $(f_1, g) * f(f_2, g) = (f_1 f_2, g)$. One obtains union of the space-time surfaces $(f_i, g$. If f_1 and f_2 correspond to mutually consistent Hamilton-Jacobi (H-J) structures, 3 complex conditions $(f_1, f_2, g) = (0, 0, 0)$ are satisfied in the intersection, which is therefore a 2-D hypercomplex generalized holomorphic string world sheet. That string world sheet appearing as an intersection of the space-time surfaces suggests that the string model is a genuine part of TGD as has been assumed.

   If the H-J structures are not consistent, the complex conditions depend on light-like coordinate u and its dual v and one cannot speak of a generalized holomorphic string world sheet. The arguments of the fermionic n-point functions appearing in the scattering amplitudes would be restricted on these string world sheets.

2. One can also consider self-intersections emerging in the power $(f, g) * (f, g) = (f^2, g)$. In this case one must consider the limit in which $f_2 \to f_1 = f$. The self-intersection corresponds to a string world sheet. The higher powers of f would have multiple copies of self-intersections on top of each other. What suggests itself is local bound states consisting of several fermions associated with string world sheets such that fermions are on top of each other at different sheets. Fermi statistics realized at the level of H allows only a finite number of them corresponding to spin and electroweak quantum numbers (color degrees of freedom are realized as color partial waves in CP_2) and they form an analog of super multiplet. I have already earlier proposed this kind of interpretation in the situation when space-time sheets intersect at a discrete point. Fermi statistics would restrict the powers of f to that corresponding to the H spinor components.

3. Also the more general product $(f_1, g_1) * (f_2, g_2) = (f_1 f_2, f_2 g_2)$, replacing the function field with function algebra, makes sense. If the H-J structures are not consistent, the product gives the union of surfaces (f_1, g_1) and (f_2, g_2) as holomorphic surfaces but with different H-J structures. The intersection of these surfaces involves 8 complex conditions parametrized by light-coordinate u and its dual v so that a discrete set of intersection points is possible and makes sense. The arguments of fermionic n-point functions are restricted to this discrete set and one obtains a situation resembling quantum field theories.

4. String world sheets could also be represented as the roots of complex valued functions f_1, f_2, f_3. If f_2 depends on light-coordinate u only, the roots satisfy 2+2+1=5 conditions so that the root is a 3-D surface. Light-like partonic orbits could be in question.

   For 4 functions (f_1, f_2, f_3, f_4) such that f_4 is real and depends on a light-like coordinate only, one obtains light-like curves perhaps identifiable as fermion lines and boundaries of the string word sheets. If f_4 is complex, there are no solutions. Note that it seems impossible to obtain partonic 2-surfaces using conditions for f_i since this requires fixing of the both light-like coordinates u and v as functions of the remaining coordinates or putting them to constant. They should emerge as singularities.

5. If the pairs (f_1, g_1) and (f_2, g_2) correspond to different H-J structure, the space-time surface corresponding to the product $(f_1 f_2, g_1 g_2)$ consists only of the roots of (f_1, g_1) and (f_2, g_2) and their intersection defined by 4 complex conditions in the generic case. The presence of two coordinates u and v however makes possible a discrete intersection. For the same H-J structure the intersection of (f_1, g_1) and (f_2, g_2) is empty.

   If the intersection points of 4-surfaces belong to the intersection of 3-D holographic data, the intersection points could belong to partonic orbits. Could a strong form of holography allow the reduction to 2-D case? Could the space-time surface be determined by 2-D data at partonic 2-surfaces and by the intersection with the light-like passive boundary of CD? This point set could define punctures permuted by the flows defined by the symmetries of WCW. This group could be seen as a rather direct analog of the Galois group.

   For $g_1 = g_2 = g$, the product $(f_1 f_2, g)$ consists of the roots of (f_1, g) and (f_2, g) and the intersection is 2-D string world sheet. The string world sheet would play a key role in the definition of the fermionic scattering amplitudes in this case.

4.3.2 How to treat CP_2 type extremals?

CP_2 type extremals and their deformations have Euclidean induced metric. I have not considered their treatment in function algebra/field context. The basic property of the simplest CP_2 type extremals is that the contribution of M^4 to the induced metric vanishes so that one has just the Kähler metric and form of CP_2. CP_2 type extremals have light-like curves as M^4 projections. Also piecewise constant light-like geodesics of M^4 are suggested by generic action principles. One can also consider light-like curves or geodesics of $H = M^4 \times CP_2$ and the geometric view of the Higgs mechanism suggests this kind of projection.

For CP_2 type extremals, M^4 coordinates are functions of a single coordinate s of CP_2 and this seems to be a violation of holomorphy: one complex coordinate splits to two real coordinates. On the other hand, the function algebra of H involves generalized holomorphic functions. Is it possible to overcome this problem?

The function algebra/field approach should treat them in such a way that the Euclidean character of the induced metric is respected. This might be possible by a judicious choice of the functions (f, g). Nothing prevents from considering also functions f which have the form $f = f_1(w, \xi^1, xi^2) - f_2(u)$ where $f_2(u)$ is real (one can multiply f_2 complex constant. This means u-dependent constant shift for the complex coordinates for the complex value space of f_1 and preserves its complex structure.

The condition $g = 0$ eliminates one complex coordinate and leaves $w = w(\xi^1, \xi^2)$. $f = 0$ gives $s \equiv Re(f_1(w(\xi^1, \xi^2), \xi^1, xi^2)) = f_2(u)$ and $t \equiv Im(f_1(w(\xi^1, \xi^2), \xi^1, xi^2) = 0$. One has two real CP_2 coordinates (s, t) and the second one is expressible in terms of the light-like coordinate u. This happens also in the case of CP_2 type extremals.

5 How mathematical consciousness could be realized at the fundamental level

5.1 Could ordinary arithmetic operations be realized consciously in terms of arithmetic operations for the space-time surfaces?

Could arithmetic operations be realized at the fundamental level. We have learned in the basic school algorithms for the basic arithmetics as stable associations and the basic arithmetics does not involve conscious thought except in the beginning when we learn the rules by concrete examples. This is very similar to what large language models do.

However, idiot savants [J1, J2] can decompose numbers into prime factors without any idea about the concept of prime numbers and certainly do not do this consciously by an algorithm or by logical deduction. Could this process occur spontaneously at a fundamental level and for some reason idiot savants could be able to do this consciously, perhaps because they are not able to do this using usual cognitive tools. I have considered the TGD inspired model for this [K10, K1]. The basic idea of various models is the same. The decomposition of a number to its factors is a spontaneous quantum process observed by the idiot savant.

1. The first first thing to notice that division is the time reversal of multiplication: one has co-algebra structure. ZEO [L5, L4] [K12] allows both operations and co-operations and the decomposition of an integer to factors would correspond to a product with a reversed arrow of time. Could pairs of BSFR involving temporary time reversal be involved and be easier for idiot savants than for people with ordinary cognitive abilities? Could the arrow of time in ordinary cognition be highly stable and make these feats impossible? Could the time reversal for the formation of the product of space-time surfaces as generalized numbers make ordinary conscious arithmetics possible?

2. M^8-H duality and geometric Langlands correspondence [L14] suggest that the exponent of the Kähler function $ex(K)$ for the region of the space-time surface represented by the polynomial with integer coefficients is some power D^m of the discriminant D of a polynomial, which has integer coefficients. D decomposes to a product of powers of ramified primes p_i, which are p-adically special. For a product $(P_1,g)*(P_2,g) = (P_1P_2,g)$ of space-time surfaces, the exponent of Kähler function is product of those for factors and thus product of powers of D_m for f_1 and f_2. A polynomial must be involved and I have considered the possibility that a particular discriminant D could correspond to a partonic 2-surface determining polynomial assignable to the singularity of the space-time surfaces as a minimal surface [L14].

3. One can say that for polynomials (P_1,P_2) with integer coefficients, the space-time surface represents an ordinary integer identifiable as D with $exp(K) \propto D^m$. For a topological single particle state, P is irreducible but can be unstable against a splitting to 2 surfaces unless the D is prime. If $exp(K)$ is conserved in the decay process, the splitting can produce a pair of space-time surfaces such that one has $D = D_1D_2$. This would represent physically the factorization of an integer to two factors, co-multiplication as the reversal of the multiplication operation. ZEO allows both.

   The preservation of the exponent of the Kähler function in the splitting reflects quantum criticality meaning that the initial and final states are superpositions of space-time surfaces with the same value of $exp(K)$. The thermodynamic analog is a microcanonical ensemble is a closed system in a thermodynamic equilibrium involving only states of the same energy.

4. This consideration generalizes trivially to the case of the sum. The product for the discriminants corresponds to the sum for their logarithms. If the system is able to physically represent the logarithm of the discriminant and also experience this representation consciously, then the product of space-time surfaces corresponds to the product of discriminants and to sum of their logarithms.

   The natural base for the logarithm is defined by some ramified prime p appearing in the discriminant. The measurement corresponding to the measurement of the exponent k of p^k would be scaling pd/dp corresponding to the scaling generator of conformal algebra extended to a 4-D algebra in the TGD framework.

If discriminant involves only a single ramified prime, the p-adic logarithm is uniquely defined. Just as in the case of co-product, the space-time surface representing integer $k = k_1 + k_2$ represented by an irreducible polynomial (f, g) splits to two space-time surfaces (f_1, g) and (f_2, g) representing representing integers k_1 and k_2.

5.2 Could mathematical consciousness have a realization in terms of quantum dynamics for WCW spinor fields

The dream of a mathematician interested in physics and consciousness is to understand what mathematical consciousness is and how it is realized at the fundamental level. What is the fundamental representation of numbers? What could be the representation of conscious arithmetics? Can one have quantum physical representations for higher level entities such as functions and even calculus? Could it be possible to represent at the level of conscious experience notions like Hilbert space and linears operators. This would make possible also a conscious representation of notions like Lie algebra.

One can start from the following observations.

1. LC for function fields induces in TGD the 4-D version of the geometric LC naturally since the pairs f_1, f_2 correspond to space-time surfaces in H. Every function pair corresponds to a space-time surface. This construction works also in 2-D case. One can also consider 2-D complex surfaces in 4-D complex space or 2-surfaces as roots of f_1, f_2, f_3.

2. The proposal is that one can assign to each space-time surface a product of discriminants D_i assignable to partonic 2-surfaces and D_i defines a number which is integers for polynomials with rational coefficients. This discriminant is defined as a product of root differences and makes sense for all functions, not only polynomials as also the notion of root. The notion of ramified prime makes sense as long as one has polynomials or rational functions with coefficients in an algebraic extension of rationals. One has representation of numbers as space-time surfaces.

   Space-time surfaces also represent numbers in a much more abstract sense. Keeping $f_2 = g$ fixed one has a function field defined by functions f_1 and its elements are represented by space-time surfaces and one can sum and multiply space-time surfaces. The use of irreducible polynomials allows a gauge fixing. The product of space-time surfaces in this sense induces a product of discriminants and there of ordinary complex numbers. For the sum of space-time surfaces this is not the case.

The following considerations are just the first speculations concerning the space-time and WCW correlates of mathematical consciousness.

1. Mathematics is an abstraction process creating concepts. Classically, a concept is represented as a set theoretic union of its instances. Quantum superposition is a natural realization of the quantum concept. Superpositions of space-time surfaces would give rise to WCW spinor fields as zero energy states in ZEO having interpretation as concept: the part of WCW in which these fields are non-vanishing would correspond to the classical concept. Quantum concept involves a large number of different perspectives and is much richer than its classical counterpart.

2. Logical deduction is a central aspect of mathematics. The almost deterministic time evolution of space-time surfaces is the natural counterpart of logical deduction. The seats of non-determinism could correspond to steps in which Boolean route branches. The slight non-determinism of the holography could correspond to different ways to arrive at the same conclusion.

3. At quantum level, the dynamics of the SSFRs would correspond to a logical deduction. In the fermionic sector, Fock basis realizes Boolean algebra and the fermionic dynamics could express at quantum level what the implication $A \rightarrow B$ means. WCW spinor field could be seen as a not totally deterministic logical deduction leading from premises to conclusions.

4. What could be the quantum counterpart for the notion of function, say for a polynomial? The key idea is to replace ordinary product and sum with the corresponding operations for the space time surfaces represented in turn as operations for functions (f, g) representing the numbers appearing in the expression $f_X = \sum_n a_n X^n$. The coefficient a_n is replaced by a function pair (f_{a_n}, g) representing the number a_n as a product of discriminants. X is represented with the function pair (f_X, g) and X^n is represented in terms of function (f_X^n, g) and one forms the pair $(F, G) = (f_{a_n} f_X^n, g)$. This process is carried out for all values of n to give $(\sum f_{a_n} f_X^n, g)$.

   The roots of (F, g) define the space-time surface representing the value of F for a particular choice of argument X. The function itself is represented as a quantum superposition of its values for various arguments X that is as WCW spinor field having the locus in the set of values of X represented as sub-manifold of WCW.

   It seems that the same recipe for functions of several arguments X_i? It would seem that the same recipe works also now: one just replaces what Taylor series for several variables with its counterpart.

5.2.1 The notion of set

One should also represent the notion of set.

1. Sets of space-time surfaces in WCW defining the locus of WCW spinor fields has been already considered. This representation is analogous to a single particle wave function.

2. A "many-particle" representation would be as a many-particle state with a particle at each element of the set. The product of space-time surfaces is the natural candidate for the formation of many-particle states. The product would thus give rise to a union of space-time surfaces and allow to define sets consisting of complex numbers. This would involve the tensor product of the fermionic state spaces associated with the elements of the set. This gives rise to the tensor product of fermionic states spaces and one obtains also fermionic representation of the Boolean algebra defined in the intersection of the space-time surfaces.

   This notion of set gives naturally rise to the notion of cardinal numbers as the number of elements of the set expected to be realized also at the level of the sensory experience whereas the number assignable to the space-time surfaces as a product of discriminants corresponds to the algebraic view of number.

5.2.2 The notion of linear space

Linear space differs from a set in that it consists of ordered numbers representing coordinates whereas for the set the ordering does not matter. Besides this linear superposition is possible. Could one use the definition of the set but order the function pairs (f_i, g) in their product $(\prod_i f_i, g)$?

1. Linear superposition so that in the most general case the coordinates are complex. Number theoretical discretization using an extension or rationals and even finite fields are possible. Subsets are replaced by sub-spaces. Note a more general product $(\prod_i f_i, \prod_i g_i)$ makes sense in the function algebra context.

   What comes to mind is that the ordering could be induced by the ordering of CDs associated with f_i with respect to the temporal positions of the tips of their passive boundaries. The arguments of (f_i, g) inside corresponding CDs could differ by Poincare translation, say time translation. For instance, hypercomplex coordinates u with light-like coordinate curve, could differ by a time translation. The WCW spinor field having this set of space-time surfaces as a locus would correspond to a linear space as a quantum concept. This ordering gives rise to the notion of number as ordinal.

2. What could the counterpart for a linear map? The space of linear maps $U \to V$ is linear space and corresponds to $m \times n$ matrices and to the Cartesian product $V_1 \times V_2$. This space can be realized using the proposed. Linear map A corresponds to the point $A_{mn}|u_m\rangle u_m\langle v_n|$

when U and V have basis $\{u_m\}$ and $\{v_n\}$. This would make possible conscious realization of Lie-algebras and various structures central in quantum theory.

3. What about differential calculus? What could be the realization of derivatives at the space-time level? In the case of a function F of a single variable, one should associate to a function F for a given value of argument its derivative with that value of argument and represent this combination as a space-time surface. This should be abstracted to the WCW spinor field representing the function and its derivative. The association of the function and its derivative could correspond to the product $(f_F * f_d F/dX, g)$ defining a space-time surface. Also now ordering of the factors is required and could be given by CD ordering. This idea generalizes to the case of functions of several variables and also to the case when the output consists of several functions of several variables.

   This did not say anything about how the derivative is estimated at the fundamental level. Derivative is a linear operation and could be represented in the way discussed above. It is not quite clear this operation could be reduced to an operation at the level of function algebra. Numerically one should consider small deformations of the argument X of the function and they should not lead out of the locus WCW spinor field. The standard formula would give the estimate for the derivative.

5.2.3 Tegmark and TGD

Max Tegmark has proposed that all that exists at the fundamental level is mathematics. Tegmark could be called a Platonist. What exists at the fundamental level would be Platonia, the world of mathematical objects. In some way, Tegmark wants to add consciousness to Platonia (see this) and of course faces the same problem as the materialists.

There is no hint of what qualia, the contents of consciousness, could be attached to mathematical objects. What would happen when $2^{1/2}$ gets depressed or falls in love with $3^{1/2}$. What happens when $5^{1/2}$ suffers jealousy towards roots of higher order polynomials because they are more complex algebraic numbers. What emotions could the roots of a polynomial feel and what sensory qualia could they experience? What about more complex structures like sets and Hilbert spaces?

Something is needed and it is a geometric representation of numbers and a quantum jump and its representation: it brings in awareness and free will.

1. A representation of numbers as physical quantum objects is needed. Frenkel wondered in his marvellous AfterMath podcast (see this) how the numbers are physically represented. Frenkel emphasized that numbers cannot not presented in spacetime. TGD offers a solution to the puzzle: numbers are represented as spacetime surfaces in $H = M^4 \times CP_2$. Holography=holomorphy vision [L16, L17] makes this representation possible. The details of the emerging view are discussed in the article describing the TGD view of Langlands correspondence [L14].

   (a) The spacetime surface can correspond to numbers in a functional algebra with product $(f_1, f_2) * (g_1, g_2) = (f_1 f_2, g_1 g_2)$ or, more restrictively, as elements of a function field with product $(f_1, g) * (g_1, g) = (f_1 f_2, g)$ with g fixed so that one obtains a family of function fields.

   (b) Space-time surfaces also correspond to complex numbers: $M^8 - H$ duality [L13]. The discriminant determined by the product of the differences roots of the function for 2-D parton surfaces determines the discriminant, which is a complex number in the general case and defined also for general analytic function. In school days we encountered the discriminant while solving roots of a second order polynomial.

   Space-time surfaces form an entire evolutionary hierarchy (or rather hierarchies) depending on how algebraically complex they are. Also p-adic number fields and their extensions as correlates of cognition emerge naturally through the hierarchy of algebraic extensions of rationals.

Now we have the numbers represented as space-time surfaces and one can ask how abstractions so central for mathematical thinking emerge. For instance, how could sets and Hilbert spaces and operators in Hilbert spaces could?

1. Since we are in the quantum world, we start to build wave functions in WCW, the Platonia. More specifically we construct WCW spinor field, Ψ in space and therefore also in the space the complex numbers represented as spacetime surfaces. We get quantum Platonia. The WCW spinors correspond to many-fermion Fock states in quantum field theory for a given 4-surafce and Ψ assigns to the space-time surfaced such a spinor. The spinor fields of WCW are induced from the second quantized free spinor fields of H and the 4-dimensionality of space-time surfaces and associated exotic smooth structures make possible fermion-pair creation but only in 4-D space-time.

2. WCW spinor field Ψ can be restricted, for example, to the set of positive and odd numbers, i.e. corresponding spacetime surfaces. The subset of WCW in which Ψ is non-vanishing, defines a subset of numbers and the concept in the classical sense as the set of its instances. For example, one obtains the concept of an odd number as a set of space-time surfaces representing odd integers. Ψ can be also restricted to the roots of polynomials of a certain degree (corresponding space-time surfaces): this gives the notion of the root of a polynomial of a given degree. Quantum concept is not a mere set but an infinite number of different WCW spinor fields that give different perspectives on the concept.

3. There is also a second way to define the notion of set: Boolean algebraic definition is possible using function algebra consisting of pairs (f_1, f_2) of some function field obtained by keeping $f_2 == g$ fixed and defining the product as $(f_1, g) * (f_2, g) = (f_1 f_2, g)$. The product in the function field induces the product of space-time surfaces and this product is just the union of space-time surfaces. A given set of ordinary complex numbers represented in terms of discriminants defined by the roots of analytic functions defined at partonic 2-surfaces corresponds to the product of the spacetime surfaces corresponding to the numbers in the sense of functional algebra.

   What is of fundamental importance is that these space-time surfaces in general have a discrete set of intersection points so that there is an interaction in fermionic degrees of freedom and one obtains n-point scattering amplitudes. Fermionic Fock states restricted to the intersection indeed define naturally a Boolean algebra.

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