From Principles to Diagrams

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Contents

Abstract

The recent somewhat updated view about the road from general principles to diagrams is discussed. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. Contrary to the original expectations, the twistorial approach is not mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

Second new element is the fusion of twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

1 Introduction

The generalization of twistor diagrams to TGD framework has been very inspiring (and also frightening) mission impossible and allowed to gain deep insights about what TGD diagrams could be mathematically. I of course cannot provide explicit formulas but the general structure for the construction of twistorial amplitudes in $\mathcal{N}=4$ SUSY suggests an analogous construction in TGD thanks to huge symmetries of TGD and unique twistorial properties of $M^4 \times CP_2$. The twistor program in TGD framework has been summarized in [K7].

Contrary to the original expectations, the twistorial approach is not a mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

There are some new results forcing a profound modification of the recent view about TGD but consistent with the general picture. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. The fiber F of Minkowskian twistor space must be identified with sphere S^2 with signature $(-1,-1)$ and would be a variant of the complex space with complex coordinates associated with S^2 and transversal space E^2 in the decomposition $M^4 = M^2 \times E^2$ and one hyper-complex coordinate associated with M^2.

The action principle in 6-D context is also Kähler action, which dimensionally reduces to Kähler action plus cosmological term. This brings in the radii of spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistors space of M^4 and CP_2. For $S(CP_2)$ the radius is of order CP_2 radius R. $R(S^2(M^4))$ could be of the order of Planck length l_P, which would thus become purely classical parameter contrary the expectations. An alternative option is $R(S^2(M^4)) = R$ The radius of S^2 associated with space-time surface is determined by the induced metric and is emergent length scale. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared brings in a further length scale closely related to cosmological constant which is also dynamical and has correct sign to explain accelerated expansion of the Universe. The order of magnitude for L must be radius of the $S^2(X^4)$ and therefore small. This could mean a gigantic cosmological constant. Just as in GRT based cosmology!

This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$ 6-D Kähler action is a sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action and for it the contribution from $S^2(CP_2)$ fiber is assumed to be absent: this could be due to the imbedding of $S^2(X^4)$ reducing to identification $S^2(M^4)$ and is not true generally. Second term in action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$. The independency implies that couplings strengths are independent for them.

The analog for Kähler coupling strength (analogous to critical temperature) associated with $S^2(M^4)$ must be extremely large - so large that one has $\alpha_K(M^4) \times R(M^4)^2 \sim L^2$, L size scale of the recent Universe. This makes possible the small value of cosmological constant assignable to the volume term given by this part of the dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ comes essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. In fact, it turns that one can assumed that the entire 6-D Kähler action contributes if one assumes that the winding numbers (w_1, w_2) for the map $S^2(X^4) \to S^2(M^4) \times S^2(CP_2)$ satisfy $(w_1, w_2) = (n, 0)$ in cosmological scales. The identification of w_1 as $h_{eff}/h = n$ is highly suggestive.

The dimensionally reduced dynamics is a highly non-trivial modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure. Strong constraints come also from the condition that induced spinor structure coming from that for twistor space $T(H)$ is essentially that coming from that of H.

Second new element is the fusion of the twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

In the sequel I will discuss the recent understanding of twistorizalization, which is considerably improved from that in the earlier formulation. I formulate the dimensional reduction of 6-D Kähler action and consider the physical interpretation. There are considerable uncertainties at the level of details I dare believe that basically the situation is understood. After that I proceed to discuss the basic principles behind the recent view about scattering amplitudes as generalized Feynman diagrams.

2 Some mathematical background

First I will try to clarify the mathematical details related to the twistor spaces and how they emerge in the recent context. I do not regard myself as a mathematician in technical sense and I can only hope that the representation based on physical intuition does not contain serious mistakes.

2.1 Imbedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique. Space-times are surfaces in $H = M^4 \times CP_2$. M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept. Strictly speaking, M^4 and its Euclidian variant E^4 allow both twistor space and the twistor space of M^4 is Minkowskian variant $T(M^4) = SU(2,2)/SU(2,1) \times U(1)$ of 6-D twistor space $CP_3 = SU(4)/SU(3) \times U(1)$ of E^4. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

This leads to a proposal - just a proposal - for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4) \times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$. What is nice in this formulation is that one can use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds).

2.2 What does twistor structure in Minkowskian signature really mean?

What twistor structure in Minkowskian signature does really mean geometrically has remained a confusing question for me. The problems associated with the Minkowskian signature of the metric are encountered also in twistor Grassmann approach to scattering amplitudes but are circumvented by performing Wick rotation that is using E^4 or S^4 instead of M^4 and applying algebraic continuation. Also complexification of Minkowksi space for momenta is used. These tricks do not apply now.

Let us try to collect thoughts about what is involved.

1. Instead of M^4 one considers the conformal compactification M^4_c of M^4 identifiable as the boundary of light-cone boundary of 6-D Minkowski space with signature (1,1,-1,-1,-1), whose points differing by scaling are identified. One has a slicing by spheres of signature (-1,-1,-1) and varying radius ρ and these spheres are projectively identified so that one can "fix the gauge" by choosing $\rho = \rho_0$. Since one has light-cone, the contribution $d\rho^2$ to the line element vanishes and one obtains $ds^2 = \rho_0^2 d\phi^2 - \rho_0^2 ds^2(S^3)$. Conformal compactification means that the scale ρ_0 of the metric is not unique. The scaling of the metric of the twistor space ρ_0^2. Conformal invariance of the theory saves from problems.

2. The Euclidian version of the twistor space of M^4 corresponds to the twistor space of S^4 identifiable as $CP_3 = SU(4)/SU(3) \times U(1)$ identifiable in terms of complex 2+2-spinors. The twistor space of M^4_c is $SU(2,2)/SU(2,1) \times U(1)$ (see `https://en.wikipedia.org/wiki/Twistor_theory`) and can be seen as a kind of algebraic continuation of $CP_3 = SU(4)/SU(3) \times U(1)$. This twistor space assignable to S^4 is complex manifold but it is not completely clear to me whether this really guarantees the existence of Kähler structure consistent with the complex structure.

The challenge is to generalize the complex structure of twistor space of E^4 to that for M^4. It turns out that the assumption that twistor space has ordinary complex structure fails. The first guess was that the fiber of twistor space is hyperbolic sphere with metric signature $(1, -1)$ having infinite area so that the 6-D Kähler action would be infinite. This makes no sense. The only alternative, which comes in mind is a hypercomplex generalization of the Kähler structure for M^4 lifted to twistor space, which locally means only adding of S^2 fiber with metric signature $(-1, -1)$.

1. To proceed one must make an explicit the definition of twistor space. The 2-D fiber S^2 consists of antisymmetric tensors of X^4 which can be taken to be self-dual or anti-self-dual by taking any antisymmetric form and by adding to its plus/minus its dual. Each tensor of this kind defines a direction - point of S^2. These points can be also regarded as quaternionic imaginary units. One has a natural metric in S^2 defined by the X^4 inner product for antisymmetric tensors: this inner product depends on space-time metric. Kähler action density is example of a norm defined by this inner product in the special case that the antisymmetric tensor is induced Kähler form. Induced Kähler form defines a preferred imaginary unit and is needed to define the imaginary part $\omega(X, Y) = ig(X, -JY)$ of hermitian form $h = h + i\omega$.

2. To define the analog of Kähler structure for M^4, one must start from a decomposition of $M^4 = M^2 \times E^2$ (M^2 is generated by light-like vector and its dual) and E^2 is orthogonal to it. M^2 allows hypercomplex structure, which light-like coordinates $(u = t - z, v = t + z)$ and E^2 complex structure and the metric has form $ds^2 = dudv + dzd\overline{z}$. Hypercomplex numbers can be represented as $h = t + iez$, $i^2 = -1, e^2 = -1$ $i^2 = -1, e^2 = -1$. Hyper-complex numbers do not define number field since for light-like hypercomplex numbers $t + iez$, $t = \pm z$ do not have finite inverse. Hypercomplex numbers allow a generalization of analytic functions used routinely in physics. Kähler form representing hypercomplex imaginary unit would be replaced with eJ. One would consider sub-spaces of complexified quaternions spanned by real unit and units eI_k, $k = 1, 2, 3$ as representation of the tangent space of space-time surfaces in Minkowskian regions. This is familiar already from M^8 duality [K4].

   $M^4 = M^2 \times E^2$ decomposition can depend on point of M^4 (polarization plane and light-like momentum direction depend on point of M^4. The condition that this structure allows global coordinates analogous to $(u, v, z, \overline{z})$ requires that the distributions for M^2 and E^2 are integrable and thus define 2-D surfaces. I have christened this structure Hamilton-Jacobi structure. It emerges naturally in the construction of extremals of Kähler action that I have christened massless extremals (MEs, [K1]) and also in the proposal for the generalization of complex structure to Minkowskian signature [**?**].

   One can define the analog of Kähler form by taking sum of induced Kähler form J and its dual $*J$ defined in terms of permutation tensor. The normalization condition is that this form integrates to the negative of metric $(J \pm *J)^2 = -g$. This condition is possible to satisfy.

3. How to lift the Hamilton Jacobi structure of M^4 to Kähler structure of its twistor space? The basic definition of twistors assumes that their exists a field of time-like directions, and that one considers projections of 4-D antisymmetric tensors to the 3-space orthogonal to the time-like direction at given point. One can say that the projection yields magnetic part of the antisymmetric tensor (say induced Kähler form J) with positive norm with respect to natural metric induced to the twistor fiber from the inner product between two-forms. This unique time direction would be defined the light-like vector defining M^2 and its dual. Therefore the signature of the metric of S^2 would be $(-1, -1)$. In quaternionic picture this direction corresponds to real quaternionic unit.

4. To sum up, the metric of the Minkowskian twistor space has signature $(-1, -1, 1, -1, -1, -1)$. The Minkowskian variant of the twistor space would give 2 complex coordinates and one hyper-complex coordinate. Cosmological term would be finite and the sign of the cosmological term in the dimensionally reduced action would be positive as required. Also metric determinant would be imaginary as required. At this moment I cannot invent any killer objection against this option.

2.3 What does the induction of the twistor structure to space-time surface really mean?

Consider now what the induction of the twistor structure to space-time surface X^4 could mean.

1. The induction procedure for Kähler structure of 12-D twistor space T requires that the induced metric and Kähler form of the base space X^4 of X^6 obtained from T is the same as that obtained by inducing from $H = M^4 \times CP_2$. Since the Kähler structure and metric of T is lift from H this seems obvious. Projection would compensate the lift.

2. This is not yet enough. The Kähler structure and metric of S^2 projected from T must be same as those lifted from X^4. The connection between metric and ω implies that this condition for Kähler form is enough. The antisymmetric Kähler forms in fiber obtained in these two manners co-incide. Since Kähler form has only one component in 2-D case, one obtains single constraint condition giving a commutative diagram stating that the direct projection to S^2 equals with the projection to the base followed by a lift to fiber. The resulting induced Kähler form is not covariantly constant but in fiber S^2 one has $J^2 = -g$.

   As a matter of fact, this condition might be trivially satisfied as a consequence of the bundle structure of twistor space. The Kähler form from $S^2 \times S^2$ can be projected to S^2 associated with X^4 and by bundle projection to a two-form in X^4. The intuitive guess - which might be of course wrong - is that this 2-form must be same as that obtained by projecting the Kähler form of CP_2 to X^4. If so then the bundle structure would be essential but what does it really mean?

3. Intuitively it seems clear that X^6 must decompose locally to a product $X^4 \times S^2$ in some sense. This is true if the metric and Kähler form reduce to direct sums of contributions from the tangent spaces of X^4 and S^2. This guarantees that 6-D Kähler action decomposes to a sum of 4-D Kähler action and Kähler action for S^2.

   This could be however too strong a condition. Dimensional reduction occurs in Kaluza-Klein theories and in this case the metric can have also components between tangent spaces of the fiber and base being interpreted as gauge potentials. This suggests that one should formulate the condition in terms of the matrix $T \leftrightarrow g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}$ defining the norm of the induced Kähler form giving rise to Kähler action. T maps Kähler form $J \leftrightarrow J_{\alpha\beta}$ to a contravariant tensor $J_c \leftrightarrow J^{\alpha\beta}$ and should have the property that $J_c(X^4)$ ($J_c(S^2)$) does not depend on $J(S^2)$ ($J(X^4)$).

   One should take into account also the self-duality of the form defining the imaginary unit. In X^4 the form $S = J \pm *J$ is self-dual/anti-self dual and would define twistorial imaginary unit since its square equals to $-g$ representing the negative of the real unit. This would suggest that 4-D Kähler action is effectively replaced with $(J \pm *J) \wedge (J \pm *J) = J^*J \pm J \wedge J$, where $*J$ is the Hodge dual defined in terms of 4-D permutation tensor ϵ. The second term is topological term (Abelian instanton term) and does not contribute to field equations. This in turn would mean that it is the tensor $T \pm \epsilon$ for which one can demand that $S_c(X^4)$ ($S_c(S^2)$) does not depend on $S(S^2)$ ($S(X^4)$).

4. The preferred quaternionic imaginary unit should be represented as a projection of Kähler form of 12-D twistor space $T(H)$. The preferred imaginary unit defining twistor structure as sum of projections of both $T(CP_2)$ and $T(M^4)$ Kähler forms would guarantee that vacuum extremals like canonically imbedded M^4 for which $T(CP_2)$ Kähler form contributes nothing have well-defined twistor structure. $T(M^4)$ or $T(CP_2)$ are treated completely symmetrically but the maps of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers induce symmetry breaking.

   For Kähler action $M^4 - CP_2$ symmetry does not make sense. 4-D Kähler action to which 6-D Kähler action dimensionally reduces can depend on CP_2 Kähler form only. I have also considered the possibility of covariantly constant self-dual M^4 term in Kähler action but given it up because of problems with Lorentz invariance. One should couple the gauge potential of M^4 Kähler form to induced spinors. This would mean the existence of vacuum gauge fields coupling to sigma matrices of M^4 so that the gauge grop would be non-compact $SO(3,1)$ leading to a breakdown of unitarity.

There is still one difficulty to be solved.

1. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared, brings in a further length scale. The first guess is that $1/L^2$ is closely

related to cosmological constant, which is also dynamical and $1/L^2$has indeed correct sign to explain accelerated expansion of the Universe. Unfortunately, if $1/L^2$ is of order cosmological constant, the value of the ordinary Kähler coupling strength α_K would be enormous. As a matter of fact, the order of magnitude for L^2 must be equal to the area of $S^2(X^4)$ and in good approximation equal to $L^2 = 4\pi R^2(S^2(M^4))$ and therefore in the same range as Planck length l_P and CP_2 radius R. This would imply a gigantic value of cosmological constant. Just as in GRT based cosmology!

2. This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$, 6-D Kähler action is sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action. For it the contribution from $S^2(CP_2)$ fiber is absent if the imbedding of $S^2(X^4)$ to $S^2(M^4) \times S^2(CP_2)$ reduces to identification with $S^2(M^4)$ so that $S^2(CP_2)$ is effectively absent: this is nottrue generally. Second term in the action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$, which can indeed contribute without breaking of Lorentz symmetry. In fact, one can assume that also the Kähler form of M^4 contributes as will be found.

3. The independency implies that Kähler couplings strengths are independent for them. If one wants that cosmological constant has a reasonable order of magnitude, $L \sim R(S^2(M^4))$ must hold true and the analog $\alpha_K(S^2(M^4))$ of the ordinary Kähler coupling strength (analogous to critical temperature) must be extremely large - so large that one has

$$\alpha_K(M^4) \times 4\pi R(M^4)^2 \sim L^2 \quad ,$$

where L is the size scale of the recent Universe.

This makes possible the small value of cosmological constant assignable to the volume term given by this part of dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ would be essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. One can criticize this identification of 6-D Kähler action as artificial but it seems to be the only option that works. Interestingly also the contribution from M^4 Kähler form can be allowed since it is also extremely small. For canonically imbedded M^4 this contribution vanishes by self-duality of M^4 Kähler form and is extremely small for the vacuum extremals of Kähler action.

4. For general winding numbers of the map $S^2(X^4) \to S^2(M^4) \times S^2(CP_2)$ also $S^2(CP_2)$ Kähler form contributes and cosmological constant is gigantic. It would seem that only the winding numbers $(w_1, w_2) = (n, 0)$ are consistent with the observed value of cosmological constant. Hence it seems that there is no need to pose any additional conditions to the Kähler action if one uses the fact that $T(M^4)$ and $T(CP_2)$ parts are independent!

2.4 A connection with the hierarchy of Planck constants?

A connection with the hierarchy of Planck constants is highly suggestive. Since also a connection with the p-adic length scale hierarchy suggests itself for the hierarchy of p-adic length scales it seems that both length scale hierarchies might find first principle explanation in terms of twistorial lift of Kähler action.

1. Cosmological considerations encourage to think that $R_1 \simeq l_P$ and $R_2 \simeq R$ hold true. One would have in early cosmology $(w_1, w_2) = (1, 0)$ and later $(w_1, w_2) = (0, 1)$ guaranteeing $R_D$grows from l_P to R during cosmological evolution. These situations would correspond the solutions $(w_1 = n, 0)$ and $(0, w_2 = n)$ one has $A = n4\pi R_1^2$ and $A = n \times 4\pi R_2^2$ and both Kähler coupling strengths are scaled down to α_K/n. For $\hbar_{eff}/h = n$ exactly the same thing happens!

   There are further intriguing similarities. $h_{eff}/h = n$ is assumed to correspond *multi-sheeted* (to be distinguished from *many-sheeted*!) covering space structure for space-time surface. Now one has covering space defined by the lift $S^2(X^4) \to S^2(M4) \times S^2(CP_2)$. These lifts define also lifts of space-time surfaces.

Could the hierarchy of Planck constants correspond to the twistorial surfaces for which $S^2(M^4)$ and $S^(CP_2)$ are identified in 1-1 manner? The assumption has been that the n-fold multi-sheeted coverings of space-time surface for $h_{eff}/h = n$ are singular at the ends of space-time surfaces at upper and lower boundaries if causal diamond (CD). Could one consider more precise definition of twistor space in such a manner that CD replaces M^4 and the covering becomes singular at the light-like boundaries of CD - the branches of space-time surface would collapse to single one. What could this collapse mean geometrically? Or should one give up the assumption about singular nature of the covering used to distinguishes many-sheetedness from multi-sheetedness.

2. $w_1 = w_2 = w$ is essentially the first proposal for conditions associated with the lifting of twistor space structure. $w_1 = w_2 = n$ gives $ds^2 = (R_1^2 + R_2^2)(d\theta^2 + w^2 d\phi^2)$ and $A = n \times 4\pi(R_1^2 + R_2^2)$. Also now Kähler coupling strength is scaled down to α/n. Again a connection with the hierarchy of Planck constants suggests itself.

3. One can consider also the option $R_1 = R_2$ option giving $ds^2 = R_1^2(2d\theta^2 + (w_1^2 + w_2^2)d\phi^2$. If the integers w_i define Pythagorean square one has $w_1^2 + w_2^2 = n^2$ and one has $R_1 = R_2$ option that one has $A = n \times 4\pi R^2$. Also now the connection with the hierarchy of Planck constants might make sense.

2.5 Twistorial variant for the imbedding space spinor structure

The induction of the spinor structure of imbedding space is in key role in quantum TGD. The question arises whether one should lift also spinor structure to the level of twistor space. If so one must understand how spinors for $T(M^4)$ and $T(CP_2)$ are defined and how the induced spinor structure is induced.

1. In the case of CP_2 the definition of spinor structure is rather delicate and one must add to the ordinary spinor connection U(1) part, which corresponds physically to the addition of classical U(1) gauge potential and indeed produces correct electroweak couplings to quarks and leptons. It is assumed that the situation does not change in any essential manner: that is the projections of gauge potentials of spinor connection to the space-time surface give those induced from $M^4 \times CP_2$ spinor connection plus possible other parts coming as a projection from the fiber $S^2(M^2) \times S^2(CP_2)$. As a matter of fact, these other parts should vanish if dimensional reduction is what it is meant to be.

2. The key question is whether the complications due to the fact that the geometries of twistor spaces $T(M^4)$ and $T(CP_2)$ are not quite Cartesian products (in the sense that metric could be reduced to a direct sum of metrics for the base and fiber) can be neglected so that one can treat the sphere bundles approximately as Cartesian products $M^4 \times S^2$ and $CP_2 \times S^2$. This will be assumed in the following but should be carefully proven.

3. Locally the spinors of the twistorspace $T(H)$ are tensor products of imbedding spinors and those for of $S^2(M^4) \times S^2(CP_2)$ expressible also as tensor products of spinors for $S^2(M^4)$ and $S^2(CP_2)$. Obviously, the number of spinor components increases by factor $2 \times 2 = 4$ unless one poses some additional conditions taking care that one has dimensional reduction without the emergence of any new spin like degrees of freedom for which there is no physical evidence. The only possible manner to achieve this is to pose covariant constancy conditions already at the level of twistor spaces $T(M^4)$ and $T(CP_2)$ leaving only single spin state in these degrees of freedom.

4. In CP_2 covariant constancy is possible for right-handed neutrino so that CP_2 spinor structure can be taken as a model. In the case of CP_2 spinors covariant constancy is possible for right-handed neutrino and is essentially due to the presence of U(1) part in spinor connection forced by the fact that the spinor structure does not exist otherwise. Ordinary S^2spinor connection defined by vielbein exists always. One can however add a coupling to a suitable multiple of Kähler potential satisfying the quantization of magnetic charge (the magnetic flux defined by $U(1)$ connection is multiple of 2π so that its imaginary exponential is unity).

S^2 spinor connections must must have besides ordinary vielbein part determined by S^2 metric also $U(1)$ part defined by Kähler form coupled with correct coupling so that the curvature form annihilates the second spin state for both $S^2(M^4)$ and $S^2(CP_2)$. $U(1)$ part of the spinor curvature is proportional to Kähler form $J \propto sin(theta)d\theta d\phi$ so that this is possible. The vielbein and $U(1)$ parts of the spinor curvature ear proportional Pauli spin matrix $\sigma_z = (1,0;0,-1)/2$ and unit matrix $(1,0;0,1)$ respectively so that the covariant constancy is possible to satisfy and fixes the spin state uniquely.

5. The covariant derivative for the induced spinors is defined by the sum of projections of spinor gauge potentials for $T(M^4)$ and $T(CP_2)$. With above assumptions the contributions gauge potentials from $T(M^4)$ and $T(CP_2)$ separately annihilate single spinor component. As a consequence there are no constraints on the winding numbers w_i, $i = 1,2$ of the maps $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$. Winding number w_i corresponds to the imbedding map $(\Theta_i = \theta, \Phi_i = w_i\phi)$.

6. If the square of the Kähler form in fiber degrees of freedom gives metric to that its square is metric, one obtains just the area of S^2 from the fiber part of action. This is given by the area $A = 4\pi\sqrt{2(w_1^2R_1^2 + w_2^2R_2^2)}$ since the induced metric is given by $ds^2 = (R_1^2 + R_2^2)d\theta^2 + (w_1^2R_1^2 + w_2^2R_2^2)d\phi^2$ for $(\Theta_1 = \theta, \Phi = n_1\phi, \Phi_2 = n_2\phi)$.

3 Surprise: twistorial dynamics does not reduce to a trivial reformulation of the dynamics of Kähler action

I have thought that twistorialization classically means only an alternative formulation of TGD. This is definitely not the case as the explicit study demonstrated. Twistor formulation of TGD is in terms of of 6-D twistor spaces $T(X^4)$ of space-time surfaces $X^4 \subset M^4 \times CP_2$ in 12-dimensional product $T = T(M^4) \times T(CP_2)$ of 6-D twistor spaces of $T(M^4)$ of M^4 and $T(CP_2)$ of CP_2. The induced Kähler form in X^4 defines the quaternionic imaginary unit defining twistor structure: how stupid that I realized it only now! I experienced during single night many other "How stupid I have been" experiences.

Classical dynamics is determined by 6-D variant of Kähler action with coefficient $1/L^2$ having dimensions of inverse length squared. Since twistor space is bundle, a dimensional reduction of 6-D Kähler action to 4-D Kähler action plus a term analogous to cosmological term - space-time volume - takes place so that dynamics reduces to 4-D dynamics also now. Here one must be careful: this happens provided the radius of $S2$ associated with X^4 does not depend on point of X^4. The emergence of cosmological term was however completely unexpected: again "How stupid I have been" experience. The scales of the spheres and the condition that the 6-D action is dimensionless bring in 3 fundamental length scales!

3.1 New scales emerge

The twistorial dynamics gives to several new scales with rather obvious interpretation. The new fundamental constants that emerge are the radii of the spheres associated with $T(M^4)$ and $T(CP_2)$. The radius of the sphere associated with X^4 is not a fundamental constant but determined by the induced metric. By above argument the fiber is sphere for both Euclidian signature and Minkowskian signatures.

1. For CP_2 twistor space the radius of $S^2(CP_2)$ must be apart from numerical constant equal to CP_2 radius R. For $S^2(M^4)$ one an consider two options. The first option is that also now the radius for $S^2(M^4)$ equals to $R(M^4) = R$ so that Planck length would not emerge from fundamental theory classically as assumed hitherto. Second imaginable option is that it does and one has $R(M^4) = l_P$.

2. If the signature of $S^2(M^4)$ is $(-1,-1)$ both Minkowskian and Euclidian regions have $S^2(X^4)$ with the same signature $(-1,-1)$. The radius R_D of $S^2(X^4)$ is dynamically determined.

Recall first how the cosmological constant emerges from TGD framework.

1. The key point is that the 6-D Kähler action contains two terms.

   (a) The first term is essentially the ordinary Kähler action multiplied by the area of $S^2(X^4)$ which is compensated by the length scale, which can be taken to be the area $4\pi R^2(M^4)$ of $S^2(M^4)$. This makes sense for winding numbers $(w_1, w_2) = (1, 0)$ meaning that $S^2(CP_2)$ is effectively absent but $S^2(M^4)$ is present.

   (b) Second term is the analog of Kähler action assignable assignable to the projection of $S^2(M^4)$ Kähler form. The corresponding Kähler coupling strength $\alpha_K(M^4)$ is huge - so huge that one has

$$\alpha_K(M^4) 4\pi R^2(M^4) \equiv L^2 \ , \tag{3.1}$$

   where $1/L^2$ is of the order of cosmological constant and thus of the order of the size of the recent Universe. $\alpha_K(M^4)$ is also analogous to critical temperature and the earlier hypothesis that the values of L correspond to p-adic length scales implies that the values of come as $(M^4) \propto p \simeq 2^k$, p prime, k prime.

   (c) The Kähler form assignable to $M^4$is not assumed to contribute to the action since it does not contribute to spinor connection of M^4. One can of course ask whether it could be present. For canonically imbedded M^4 self-duality implies that this contribution vanishes and for vacuum extremals of ordinary Kähler action this contribution is small.Breaking of Lorentz invariance is however a possible problem. If $\alpha_K(M^4)$ is given by above expression, then this contribution is extremely small.

Hence one can consider the possibility that the action is just the sum of full 6-D Kähler actions assignable to $T(M^4)$ and $T(CP_2)$ but with different values of α_K if one has $(w_1, w_2) = (n, 0)$. Also other $w_2 \neq 0$ is possible but corresponds to gigantic cosmological constant.

Given the parameter L^2 as it is defined above, one can deduce an expression for cosmological constant Λ and show that it is positive.

(a) 6-D Kähler action has dimensions of length squared and one must scale it by a dimensional constant: call it $1/L^2$. L is a fundamental scale and in dimensional reduction it gives rise to cosmological constant. Cosmological constant Λ is defined in terms of vacuum energy density as $\Lambda = 8\pi G\rho_{vac}$ can have two interpretations. Λ can correspond to a modification of Einstein-Hilbert action or - as now - to an additional term in the action for matter. In the latter case positive Λ means negative pressure explaining the observed accelerating expansion. It is actually easy to deduce the sign of Λ.

$1/L^2$ multiplies both Kähler action - $F^{ij}F_{ij}$ ($\propto E^2 - B^2$ in Minkowskian signature). The energy density is positive. For Kähler action the sign of the multiplier must be positive so that $1/L^2$ is positive. The volume term is fiber space part of action having same form as Kähler action. It gives a positive contribution to the energy density and negative contribution to the pressure.

In $\Lambda = 8\pi G\rho_{vac}$ one would have $\rho_{vac} = \pi/L^2 R_D^2$ as integral of the $-F^{ij}F_{ij}$ over S^2 given the π/R_D^2 (no guarantee about correctness of numerical constants). This gives $\Lambda = 8\pi^2 G/L^2 R_D^2$. Λ is positive and the sign is same as as required by accelerated cosmic expansion. Note that super string models predict wrong sign for Λ. Λ is also dynamical since it depends on R_D, which is dynamical. One has $1/L^2 = k\Lambda$, $k = 8\pi^2 G/R_D^2$ apart from numerical factors.

The value of L of deduced from Euclidian and Minkowskian regions in this formal manner need not be same. Since the GRT limit of TGD describes space-time sheets with Minkowskian signature, the formula seems to be applicable only in Minkowskian regions. Again one can argue that one cannot exclude Euclidian space-time sheets of even macroscopic size and blackholes and even ordinary concept matter would represent this kind of structures.

(b) L is not size scale of any fundamental geometric object. This suggests that L is analogous to α_K and has value spectrum dictated by p-adic length scale hypothesis. In fact, one can introduce the ratio of $\epsilon = R^2/L^2$ as a dimensionless parameter analogous to coupling strength what it indeed is in field equations. If so, L could have different values in Minkowskian and Euclidian regions.

(c) I have earlier proposed that $R_U \equiv 1/\sqrt{1/\Lambda}$ is essentially the p-adic length scale $L_p \propto \sqrt{p} = 2^{k/2}$, $p \simeq 2^k$, $k$prime, characterizing the cosmology at given time and satisfies $R_U \propto a$ meaning that vacuum energy density is piecewise constant but on the average decreases as $1/a^2$, a cosmic time defined by light-cone proper time. A more natural hypothesis is that L satisfies this condition and in turn implies similar behavior or R_U. p-Adic length scales would be the critical values of L so that also p-adic length scale hypothesis would emerge from quantum critical dynamics! This conforms with the hypothesis about the value spectrum of α_K labelled in the same manner [L1].

(d) At GRT limit the magnetic energy of the flux tubes gives rise to an average contribution to energy momentum tensor, which effectively corresponds to negative pressure for which the expansion of the Universe accelerates. It would seem that both contributions could explain accelerating expansion. If the dynamics for Kähler action and volume term are coupled, one would expect same orders of magnitude for negative pressure and energy density - kind of equipartition of energy.

Consider first the basic scales emerging also from GRT picture. $R_U \sim \sqrt{1/\Lambda} \sim 10^{26}$ m = 10 Gly is not far from the recent size of the Universe defined as $c \times t \sim 13.8$ Gly. The derived size scale $L_1 \equiv (R_U \times l_P)^{1/2}$ is of the order of $L_1 = .5 \times 10^{-4}$ meters, the size of neuron. Perhaps this is not an accident. To make life of the reader easier I have collected the basic numbers to the following table.

$$\begin{array}{lll} m(CP_2) \simeq 5.7 \times 10^{14}\ \ GeV \ , & m_P = 2.435 \times 10^{18}\ \ GeV \ , & \frac{R(CP_2)}{l_P} \simeq 4.1 \times 10^3 \ , \\ R_U = 10\ \ Gy \ , & t = 13.8\ \ Gy \ , & L_1 = \sqrt{l_P R_U} = .5 \times 10^{-4}\ \ m \ . \end{array} \tag{3.2}$$

Let us consider now some quantitative estimates. $R(X^4)$ depends on homotopy equivalence classes of the maps from $S^2(X^4) \to S^2(M^4)$ and $S^2(X^4) \to S^2(CP_2)$ - that is winding numbers w_i, $i = 0, 2$ for these maps. The simplest situations correspond to the winding numbers $(w_1, w_2) = (1, 0)$ and $(w_1, w_2) = (0, 1)$. For $(w_1, w_2) = (1, 0)$ M^4 contribution to the metric of $S^2(X^4)$ dominates and one has $R(X^4) \simeq R(M^4)$. For $R(M^4) = l_P$ so Planck length would define a fundamental length and Planck mass and Newton's constant would be quantal parameters. For $(w_1, w_2) = (0, 1)$ the radius of sphere would satisfy $R_D \simeq R$ (CP_2 size): now also Planck length would be quantal parameter.

Consider next additional scales emerging from TGD picture.

(a) One has $L = (2^{3/2}\pi l_P/R_D) \times R_U$. In Minkowskian regions with $R_D = l_P$ this would give $L = 8.9 \times R_U$: there is no obvious interpretation for this number in recent cosmology. For $(R_D = R)$ one obtains the estimate $L = 29$ Mly. The size scale of large voids varies from about 36 Mly to 450 Mly (see `https://en.wikipedia.org/wiki/Void_(astronomy)`).

(b) Consider next the derived size scale $L_2 = (L \times l_P)^{1/2} = \sqrt{L/R_U} \times L_1 = \sqrt{2^{3/2}\pi l_P/R_D} \times L_1$. For $R_D = l_P$ one has $L_2 \simeq 3L_1$. For $R_D = R$ making sense in Euclidian regions, this is of the order of size of neutrino Compton length: 3 μm, the size of cellular nucleus and rather near to the p-adic length scale $L(167) = 2.6$ m, corresponds to the largest miracle Gaussian Mersennes associated with $k = 151, 157, 163, 167$ defining length scales in the range between cell membrane thickness and the size of cellular nucleus. Perhaps these are co-incidences are not accidental. Biology is something so fundamental that fundamental length scale of biology should appear in the fundamental physics.

The formulas and predictions for different options are summarized by the following table.

Option	$L=\frac{2^{3/2}\pi l_P}{R_D}\times R_U$	$L_2=\sqrt{Ll_P}=\sqrt{\frac{2^{3/2}\pi l_P}{R_D}}\times L_1$
$R_D=R$,	$29\ Mly$,	$\simeq 3\ \mu m$,
$R_D=l_P$,	$8.9R_U$,	$\simeq 3L_1=1.5\times 10^{-4}\ m$,

(3.3)

In the case of M^4 the radius of S^2 cannot be fixed it remains unclear whether Planck length scale is fundamental constant or whether it emerges.

3.2 Estimate for the cosmic evolution of R_D

One can actually get estimate for the evolution of R_D as function of cosmic time if one accepts Friedman cosmology as an approximation of TGD cosmology.

(a) Assume critical mass density so that one has

$$\rho_{cr}=\frac{3H^2}{8\pi G}\ .$$

(b) Assume that the contribution of cosmological constant term to the mass mass density dominates. This gives $\rho\simeq\rho_{vac}=\Lambda/8\pi G$. From $\rho_{cr}=\rho_{vac}$ one obtains

$$\Lambda=3H^2\ .$$

(c) From Friedman equations one has $H^2=((da/dt)/a)^2$, where a corresponds to light-cone proper time and t to cosmic time defined as proper time along geodesic lines of space-time surface approximated as Friedmann cosmology. One has

$$\Lambda=\frac{3}{g_{aa}a^2}$$

in Robertson-Walker cosmology with $ds^2=g_{aa}da^2-a^2d\sigma_3^2$.

(d) Combining this equations with the TGD based equation

$$\Lambda=\frac{8\pi^2G}{L^2R_D^2}$$

one obtains

$$\frac{8\pi^2G}{L^2R_D^2}=\frac{3}{g_{aa}a^2}\ . \tag{3.4}$$

(e) Assume that quantum criticality applies so that L has spectrum given by p-adic length scale hypothesis so that one discrete p-adic length scale evolution for the values of L. There are two options to consider depending on whether p-adic length scales are assigned with light-cone proper time a or with cosmic time t

$$T=a\ \ (Option\ \ I)\ \ ,\ \ \ T=t\ \ (Option\ \ II) \tag{3.5}$$

Both options give the same general formula for the p-adic evolution of $L(k)$ but with different interpretation of $T(k)$.

$$\frac{L(k)}{L_{now}}=\frac{T(k)}{T_{now}}\ \ ,\ \ \ T(k)=L(k)=2^{(k-151)/2}\times L(151)\ \ ,\ \ \ L(151)\simeq 10\ \ nm\ \ . \tag{3.6}$$

Here $T(k)$ is assumed to correspond to primary p-adic length scale. An alternative - less plausible - option is that $T(k)$ corresponds to secondary p-adic length scale $L_2(k) = 2^{k/2}L(k)$ so that $T(k)$ would correspond to the size scale of causal diamond. In any case one has $L \propto L(k)$. One has a discretized version of smooth evolution

$$L(a) = L_{now} \times \frac{T}{T_{now}} \quad . \tag{3.7}$$

(f) Feeding into this to Eq. 3.4 one obtains an expression for $R_D(a)$

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{L(a)} \times g_{aa}^{1/2} \quad . \tag{3.8}$$

Unless the dependences on cosmic time compensate each other, R_D is dynamical and becomes very small at very early times since g_{aa} becomes very small. $R(M^4) = l_P$ however poses a lower boundary since either of the maps $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$ must be homotopically non-trivial. For $R(M^4) = l_P$ one would obtain $R_D/l_P = 1$ at this limit giving also lower bound for g_{aa}. For $T = t$ option $a/L(a)$ becomes large and g_{aa} small.

As a matter of fact, in very early cosmic string dominated cosmology g_{aa} would be extremely small constant [K2]. In late cosmology $g_{aa} \rightarrow 1$ holds true and one obtains at this limit

$$\frac{R_D(now)}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a_{now}}{L_{now}} \times l_P \simeq 4.4\frac{a_{now}}{L_{now}} \quad . \tag{3.9}$$

(g) For $T = t$ option R_D/l_P remains constant during both matter dominated cosmology, radiation dominated cosmology, and string dominated cosmology since one has $a \propto t^n$ with $n = 1/2$ during radiation dominated era, $n = 2/3$ during matter dominated era, and $n = 1$ during string dominated era [K2]. This gives

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{t}\sqrt{g_{aa}}\frac{t(end)}{L(end)} = (\frac{8}{3})^{1/2}\frac{\pi}{n}\frac{t(end)}{L(end)} \quad .$$

Here "end" refers the end of the string or radiation dominated period or to the recent time in the case of matter dominated era. The value of n would have evolved as $R_D/l_P \propto (1/n)(t_{end}/L_{end})$, $n \in \{1, 3/2, 2\}$. During radiation dominated cosmology $R_D \propto a^{1/2}$ holds true. The value of R_D would be very nearly equal to $R(M^4)$ and $R(M^4)$ would be of the same order of magnitude as Planck length. In matter dominated cosmology would would have $R_D \simeq 2.2(t(now)/L(now)) \times l_P$.

(h) For $R_D(now) = l_P$ one would have

$$\frac{L_{now}}{a_{now}} = (\frac{8}{3})^{1/2}\pi \simeq 4.4 \quad .$$

In matter dominated cosmology $g_{aa} = 1$ gives $t_{now} = (2/3) \times a_{now}$ so that predictions differ only by this factor for options I and II. The winding number for the map $S^2(X^4) \rightarrow S^2(CP_2)$ must clearly vanish since otherwise the radius would be of order R.

(i) For $R_D(now) = R$ one would obtain

$$\frac{a_{now}}{L_{now}} = (\frac{8}{3})^{1/2} \times \frac{R}{l_P} \simeq 2.1 \times 10^4 \quad .$$

One has $L_{now} = 10^6$ ly: this is roughly the average distance scale between galaxies. The size of Milky Way is in the range $1 - 1.8 \times 10^5$ ly and of an order of magnitude smaller.

(j) An interesting possibility is that $R_D(a)$ evolves from $R_D \sim R(M^4) \sim l_P$ to $R_D \sim R$. This could happen if the winding number pair $(w_1, w_2) = (1, 0)$ transforms to $(w_1, w_2) = (0, 1)$ during transition to from radiation (string) dominance to matter (radiation) dominance. R_D/l_P radiation dominated cosmology would be related by a factor

$$\frac{R_D(rad)}{R_D(mat} = (3/4)\frac{t(rad,end)}{L(rad,end)} \times \frac{L(now)}{t(now)}$$

to that in matter dominated cosmology. Similar factor would relate the values of R_D/l_P in string dominated and radiation dominated cosmologies. The condition $R_D(rad)/R_D(mat) = l_P/R$ expressing the transformation of winding numbers would give

$$\frac{L(now)}{L(rad,end)} = \frac{4}{3}\frac{l_P}{R}\frac{t(now)}{t(rad,end)} .$$

One has $t(now)/t(rad, end) \simeq .5\times 10^6$ and $l_P/R = 2.5\times 10^{-4}$ giving $L(now)/L(rad, end) \simeq 125$, which happens to be near fine structure constant.

(k) For the twistorial lifts of space-time surfaces for which cosmological constant has a reasonable value , the winding numbers are equal to $(w_1, w_2) = (n, 0)$ so that $R_D = \sqrt{n}R(S^2(M^4))$ holds true in good approximation. This conforms with the observed constancy of R_D during various cosmological eras, and would suggest that the ratio $\frac{t(end)}{L(end)}$ characterizing these periods is same for all periods. This determines the evolution for the values of $\alpha_K(M^4)$.

$R(M^4) \sim l_P$ seems rather plausible option so that Planck length would be fundamental classical length scale emerging naturally in twistor approach. Cosmological constant would be coupling constant like parameter with a spectrum of critical values given by p-adic length scales.

3.3 What about extremals of the dimensionally reduced 6-D Kähler action?

It seems that the basic wisdom about extremals of Kähler action remains unaffected and the motivations for WCW are not lost. What is new is that the removal of vacuum degeneracy is forced by twistorial action.

(a) All extremals, which are minimal surfaces remain extremals. In fact, all the known extremals except vacuum extremals. For minimal surfaces the dynamics of the volume term and 4-D Kähler action separate and field equations for them are separately satisfied. The vacuum degeneracy motivating the introduction of WCW is preserved. The induced Kähler form vanishes for vacuum extremals and the imaginary unit of twistor space is ill-defined. Hence vacuum extremals cannot belong to WCW. This correspond to the vanishing of WCW metric for vacuum extremals.

(b) For non-minimal surfaces Kähler coupling strength does not disappear from the field equations and appears as a genuine coupling very much like in classical field theories. Minimal surface equations are a generalization of wave equation and Kähler action would define analogs of source terms. Field equations would state that the total isometry currents are conserved. It is not clear whether other than minimal surfaces are possible, I have even conjectured that all preferred extremals are always minimal surfaces having the property that being holomorphic they are almost universal extremals for general coordinate invariant actions.

(c) Thermodynamical analogy might help in the attempts to interpret. Quantum TGD in zero energy ontology (ZEO) corresponds formally to a complex square root of thermodynamics. Kähler action can be identified as a complexified analog of free energy. Complexification follows both from the fact that $\sqrt{g}$ is real/imaginary in Euclidian/Minkowskian space-time regions. Complex values are also implied by the proposed identification of the values of Kähler coupling strength in terms of zeros and pole of Riemann zeta in turn identifiable as poles of the so called fermionic zeta defining number theoretic partition function for fermions [K4] [L1, L2]. The thermodynamical for Kähler action with volume term is Gibbs free energy $G = F - TS = E - TS + PV$ playing key role in chemistry.

(d) The boundary conditions at the ends of space-time surfaces at boundaries of CD generalize appropriately and symmetries of WCW remain as such. At light-like boundaries between Minkowskian and Euclidian regions boundary conditions must be generalized. In Minkowkian regions volume can be very large but only the Euclidian regions contribute to Kähler function so that vacuum functional can be non-vanishing for arbitrarily large space-time surfaces since exponent of Minkowskian Kähler action is a phase factor.

(e) One can worry about almost topological QFT property. Although Kähler action from Minkowskian regions at least would reduce to Chern-Simons terms with rather general assumptions about preferred extremals, the extremely small cosmological term does not. Could one say that cosmological constant term is responsible for "almost"?

It is interesting that the volume of manifold serves in algebraic geometry as topological invariant for hyperbolic manifolds, which look locally like hyperbolic spaces $H_n = SO(n,1)/SO(n)$ [A1] [K3]. See also the article "Volumes of hyperbolic manifolds and mixed Tate motives" (see `http://arxiv.org/abs/alg-geom/9601021`). Now one would have $n = 4$. It is probably too much to hope that space-time surfaces would be hyperbolic manifolds. In any case, by the extreme uniqueness of the preferred extremal property expressed by strong form of holography the volume of space-time surface could also now serve as topological invariant in some sense as I have earlier proposed. What is intriguing is that AdS_n appearing in AdS/CFT correspondence is Lorentzian analogue H_n.

To sum up, I have the feeling that the final formulation of TGD has now emerged and it is clear that TGD is indeed a quantum theory of gravitation allowing to understand standard model symmetries. The existence of twistorial formulation is all that is needed to fix the theory completely. It makes possible gravitation and predicts standard model symmetries. This cannot be said about any competitor of TGD.

4 Basic principles behind construction of amplitudes

Basic principles of the construction summarized in this section could be seen as axioms trying to abstract the essentials. The explicit construction of amplitudes is too heavy challenge at this stage and at least for me.

4.1 Imbedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique.

(a) As already explained, M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept as one might guess from the fact that the projection of Kähler form naturally defines the preferred quaternionic imaginary unit defining the twistor structure for space-time surface. Both M^4 and its Euclidian variant E^4 allow twistor space and the twistor space of M^4 is Minkowskian variant $T(M^4) = SU(2,2)/SU(2,1) \times U(1)$ of 6-D twistor space $CP_3 = SU(4)/SU(3) \times U(1)$ of E^4. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

(b) This leads to a proposal for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4) \times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$. What is nice in this formulation is that one can use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds). It has been already described how this approach leads to a profound understanding of the relationship between TGD and GRT. Planck length emerges whereas fundamental constant as also cosmological constant emerges dynamically from the length scale parameter appearing in 6-D Kähler action. One can say, that twistor extension is absolutely essential for really understanding the gravitational interactions although the modification of Kähler action is extremely small due to the huge value of length scale defined by cosmological constant.

(c) Masslessness (masslessness in complex sense for virtual particles in twistorialization) is essential condition for twistorialization. In TGD massless is masslessness in 8-D sense for the representations of superconformal algebras. This suggests that 8-D variant of twistors makes sense. 8-dimensionality indeed allows octonionic structure in the tangent space of imbedding space. One can also define octonionic gamma matrices and this allows a possible generalization of 4-D twistors to 8-D ones using generalization of sigma matrices representing quaternionic units to to octonionic sigma "matrices" essential for the notion of twistors. These octonion units do not of course allow matrix representation unless one restricts to units in some quaternionic subspace of octonions. Space-time surfaces would be associative and thus have quaternionic tangent space at each point satisfying some additional conditions.

4.2 Strong form of holography

Strong form of holography (SH) following from general coordinate invariance (GCI) for space-times as surfaces states that the data assignable to string world sheets and partonic 2-surfaces allows to code for scattering amplitudes. The boundaries of string world sheets at the space-like 3-surfaces defining the ends of space-time surfaces at boundaries of causal diamonds (CDs) and the fermionic lines along light-like orbits of partonic 2-surfaces representing lines of generalized Feynman diagrams become the basic elements in the generalization of twistor diagrams (I will not use the attribute "Feynman" in precise sense, one could replace it with "twistor" or even drop away). One can assign fermionic lines massless in 8-D sense to flux tubes, which can also be braided. One obtains a fractal hierarchy of braids with strands, which are braids themselves. At the lowest level one has braids for which fermionic lines are braided. This fractal hierarchy is unavoidable and means generalization of the ordinary Feynman diagram. I have considered some implications of this hierarchy in [K6].

4.3 The existence of WCW demands maximal symmetries

Quantum TGD reduces to the construction of Kähler geometry of infinite-D "world of classical worlds" (WCW), of associated spinor structure, and of modes of WCW spinor fields which are purely classical entities and quantum jump remains the only genuinely quantal element of quantum TGD. Quantization without quantization, would Wheeler say.

By its infinite-dimensionality, the mere mathematical existence of the Kähler geometry of WCW requires maximal isometries. Physics is completely fixed by the mere condition that its mathematical description exists. Super-symplectic and other symmetries of "world of classical worlds" (WCW) are in decisive role. These symmetry algebras have conformal structure and generalize and extend the conformal symmetries of string models (Kac-Moody algebras in particular). These symmetries give also rise to the hierarchy of Planck constants. The super-symplectic symmetries extend to a Yangian algebra, whose generators are polylocal in the sense that they involve products of generators associated with different partonic surfaces. These symmetries leave scattering amplitudes invariant. This is an immensely powerful constraint, which remains to be understood.

4.4 Quantum criticality

Quantum criticality (QC) of TGD Universe is a further principle. QC implies that Kähler coupling strength is mathematically analogous to critical temperature and has a discrete spectrum. Coupling constant evolution is replaced with a discrete evolution as function of p-adic length scale: sequence of jumps from criticality to a more refined criticality or vice versa (in spin glass energy landscape you at bottom of well containing smaller wells and you go to the bottom of smaller well). This implies that either all radiative corrections (loops) sum up to zero (QFT limit) or that diagrams containing loops correspond to the same scattering amplitude as tree diagrams so that loops can eliminated by transforming them to arbitrary small ones and snipping away moving the end points of internal lines along the lines of diagram (fundamental description).

Quantum criticality at the level of super-conformal symmetries leads to the hierarchy of Planck constants $h_{eff} = n \times h$ labelling a hierarchy of sub-algebras of super-symplectic and other conformal algebras isomorphic to the full algebra. Physical interpretation is in terms of dark matter hierarchy. One has conformal symmetry breaking without conformal symmetry breaking as Wheeler would put it.

4.5 Physics as generalized number theory, number theoretical universality

Physics as generalized number theory vision has important implications. Adelic physics is one of them. Adelic physics implied by number theoretic universality (NTU) requires that physics in real and various p-adic numbers fields and their extensions can be obtained from the physics in their intersection corresponding to an extension of rationals. This is also enormously powerful condition and the success of p-adic length scale hypothesis and p-adic mass calculations can be understood in the adelic context.

In TGD inspired theory of consciousness various p-adic physics serve as correlates of cognition and p-adic space-time sheets can be seen as cognitive representations, "thought bubbles". NTU is closely related to SH. String world sheets and partonic 2-surfaces with parameters (WCW coordinates) characterizing them in the intersection of rationals can be continued to space-time surfaces by preferred extremal property but not always. In p-adic context the fact that p-adic integration constants depend on finite number of pinary digits makes the continuation easy but in real context this need not be possible always. It is always possible to imagine something but not always actualize it!

4.6 Scattering diagrams as computations

Quantum criticality as possibility to eliminate loops has a number theoretic interpretation. Generalized Feynman diagram can be interpreted as a representation of a computation connecting given set X of algebraic objects to second set Y of them (initial and final states in scattering) (trivial example: $X = \{3, 4\} \to 3 \times 4 = 12 \to 2 \times 6 \to \{2, 6\} = Y$. The 3-vertices $(a \times b = c)$ and their time-reversals represent algebraic product and co-product.

There is a huge symmetry: all diagrams representing computation connecting given X and Y must produce the same amplitude and there must exist minimal computation. The task of finding this computation is like finding the simplest representation for the formula X=Y and the noble purpose of math teachers is that we should learn to find it during our school days. This generalizes the duality symmetry of old fashioned string models: one can transform any diagram to a tree diagram without loops. This corresponds to quantum criticality in TGD: coupling constants do not evolve. The evolution is actually there but discrete and corresponds to infinite number critical values for Kahler coupling strength analogous to temperature.

4.7 Reduction of diagrams with loops to braided tree-diagrams

(a) In TGD pointlike particles are replaced with 3-surfaces and by SH by partonic 2-surfaces. The important implication of 3-dimensionality is braiding. The fermionic lines inside light-like orbits of partonic 2-surfaces can be knotted and linked - that is braided (this is dynamical braiding analogous to dance). Also the fermionic strings connecting partonic 2-surfaces at space-like 3-surfaces at boundaries of causal diamonds (CDs) are braided (space-like braiding).

Therefore ordinary Feynman diagrams are not enough and one must allow braiding for tree diagrams. One can also imagine of starting from braids and allowing 3-vertices for their strands (product and co-product above). It is difficult to imagine what this braiding could mean. It is better to imagine braid and allow the strands to fuse and split (annihilation and pair creation vertices).

(b) This braiding gives rise in the planar projection representation of braids to a generalization of non-planar Feynman diagrams. Non-planar diagrams are the basic unsolved problem of twistor approach and have prevented its development to a full theory allowing to construct exact expressions for the full scattering amplitudes (I remember however that Nima Arkani-Hamed et al have conjectured that non-planar amplitudes could be constructed by some procedure: they notice the role of permutation group and talk also about braidings (describable using covering groups of permutation groups)). In TGD framework the non-planar Feynman diagrams correspond to non-trivial braids for which the projection of braid to plane has crossing lines, say a and b, and one must decide whether the line a goes over b or vice versa.

(c) An interesting open question is whether one must sum over all braidings or whether one can choose only single braiding. Choice of single braiding might be possible and reflect the failure of string determinism for Kähler action and it would be favored by TGD as almost topological quantum field theory (TQFT) vision in which Kähler action for preferred extremal is topological invariant.

4.8 Scattering amplitudes as generalized braid invariants

The last big idea is the reduction of quantum TGD to generalized knot/braid theory (I have talked also about TGD as almost TQFT). The scattering amplitude can be identified as a generalized braid invariant and could be constructed by the generalization of the recursive procedure transforming in a step-by-step manner given braided tree diagram to a non-braided tree diagram: essentially what Alexander the Great did for Gordian knot but tying the pieces together after cutting. At each step one must express amplitude as superposition of amplitudes associated with the different outcomes of splitting followed by reconnection. This procedure transforms braided tree diagram to a non-braided tree diagrams and the outcome is the scattering amplitude!

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