The notion of four-momentum in TGD

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Abstract

One manner to see TGD is as a solution of the energy problem of General Relativity in terms of sub-manifold gravity. The translations act now as translations of 8-D imbedding space $M^4 \times CP_2$ rather than in space-time itself and four-momentum can be identified as Noether charge. The detailed realization of this vision however involves several conceptual delicacies. What does Equivalence Principle mean in this framework: equivalence of gravitational and inertial momenta or just Einstein's equations or their generalization? What is the precise definition of inertial and gravitational four-momenta? What does quantum classical correspondence mean and could Equivalence Principle reduce to it? p-Adic mass calculations and the notion of generalised conformal invariance provide strong constraints on the attempts to answer these questions. This article provides the most recent view about the most plausible looking answer to these questions. Twistor Grassmann approach relies on the Yangian variant of 4-D conformal symmetry generalizing in TGD framework to Yangian variants of huge conformal symmetry algebras due to the effective 2-dimensionality of light-like 3-surfaces. This suggest also a generalization of four-momentum bringing in multilocal contributions analogous to interaction energy and also this is discussed in some detail.

1 Introduction

The starting point of TGD was the energy problem of General Relativity [?]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [?, ?]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of zero energy ontology (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [?] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [?, ?]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [?]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

2 Problems

2.1 Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [?], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of zero energy ontology (ZEO) [K1, ?], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [?]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [?].

2.2 Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

2.3 What Equivalence Principle (EP) means in TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations. What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [?]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

   Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of Equivalence Principle. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. I must admit that the notion of coset representation creates uneasy feeling in my stomach. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.

2. Does EP reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D spacetime and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [**?**] have motivated theuse of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

1. For the coset option the situation is unclear. Even the definition of coset representation is problematic. If Super Kac-Moody generators vanish at partonic 2-surfaces one would have just direct sum Super-Virasoro algebras and coset representations would reduce to that for symplectic group containing only single tensor factor. If Super-Kac Moody generators do not vanish at partonic 2-surfaces. one must extend the symplectic generators by making them local with respect to partonic 2-surface in order to get a closed algebra. The imbedding of Kac-Moody to Sympl might be well-defined since isometries form subgroup of symplectic transformations. But is it possible to speak about a direct sum of Super Virasoro algebras in this case? It seems that only the inclusion electroweak part to symplectic part represented in terms of fermionic currents allows this and would bring in two tensor factors so that one would have 3 tensor factors. If one counts fermionic tensor factors assignable to 2-D transversal part of Kac-Moody algebra one would have 5 tensor factors. This seems however tricky.

2. For the $P_{class,I} = P_{quant,gr}$ option the number of tensor factors is naturally five. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!). Therefore this option seems to be slightly favored. Note that Kac-Moody generators are local isometries localized with respect to the coordinates of light-like 3-surface and vanish at partonic 2-surface so that one can speak about direct sum of algebras.

Clearly, both experimental input and Occam's razor seem to favor the option reducing Equivalence Principle to Quantum Classical Correspondence.

For this option however the GRT inspired interpretation of Equivalence Principle at space-time level remains to be understood. Is it needed at all? The condition that the energy momentum tensor of Kähler action has a vanishing divergence leads in General Relativity to Einstein equations with cosmological term. In TGD framework preferred extremals satisfying the analogs of Einstein's equations with several cosmological constant like parameters can be considered.

Should one give up this idea, which indeed might be wrong? Could the divergence of of energy momentum tensor vanish only asymptotically as was the original proposal? Or should one try to generalize the interpretation? QCC states that quantum physics has classical correlate at space-time level and implies EP. Could also quantum classical correspondence itself have a correlate at space-time level. If so, space-time surface would able to represent abstractions as statements about statements about.... as the many-sheeted structure and the vision about TGD physics as analog of Turing machine able to mimic any other Turing machine suggest.g machine suggests.

2.4 TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see fig. `http://www.tgdtheory.fi/appfigures/fieldsuperpose.jpg` or fig. 11 in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

   This description applies also to gauge fields: classical electro-weak gauge potentials of standard model correspond to superpositions of induced gauge potentials. Classical color gauge potentials correspond to superpositions of projections of CP_2 Killing vector fields defining induce color gauge potentials.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

   The currents assigned with topological inhomogenities defined by topologically condensed matter serve as sources of in Einstein-YM equations.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

2.5 How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

2.5.1 Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n --> T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

2.5.2 The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only timelike superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred etxremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that timelike translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

3 Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [?]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [?], where also references to the work of pioneers can be found.

3.1 Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [?] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [?] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N}=4$ SUSY). One of the conditions conditions is that the tensor product $R\otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n=0$ and $n=1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n=1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h=0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n>0$ are $n+1$-local in the sense that they involve $n+1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

3.2 How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N}=4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [?] and Virasoro algebras [?] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

2. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD\times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. A generalization of the Equivalence Principle is in question. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.3 Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

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