Hierarchy of Planck Constants and Dark Matter Hierarchy

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Abstract

The basic assumption of Quantum TGD (Topological GeometroDynamics) is that space-times are 4-surfaces of the 8-D imbedding space $M^4 \times CP_2$. The recent vision about TGD is briefly summarized with a particular emphasis on the hierarchy of Planck constants proposed to provide a first principle description of dark matter. The hierarchy is realized in terms of the generalization of the imbedding space to a book like structure. The particles at different pages of the book partially labeled by the values of Planck constant are dark relative to each other in the sense that they do not have any local interaction vertices.

1 Introduction

The quantization of Planck constant has been one of the basic themes of TGD since 2005. The starting point was the finding of Nottale [22] that planetary orbits could be interpreted as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = nG M_1 M_2 / v_0$, $v_0/c \simeq 2^{-11}$, $n = 1$ is for the inner planets and $n = 5$ for outer planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.
The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane [10, 11]. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence.

These findings led eventually to a generalization of quantum TGD (chs. Does TGD predict the spectrum of Planck constants and Quantum Hall Effect and the Hierarchy of Planck constants of [4] and ch. Quantum Astrophysics of [3]). Obtained by replacing the 8-dimensional imbedding space \( H = \mathbb{M}_4 \times \mathbb{C}P_2 \) with a more general structure. Let \( CD \) denote the causal diamond \( CD \subset \mathcal{M}^4 \) defined as an intersection of future and past directed light-cones of \( M^4 \). Each \( CD \times \mathbb{C}P_2 \subset H \) is replaced with a book like structure in which products of singular coverings and factor spaces of both \( CD \) resp. \( \mathbb{C}P_2 \) are glued together along \( \mathbb{M}_2 \subset \mathcal{M}_4 \) resp. homologically trivial geodesic sphere \( S^2 \subset \mathbb{C}P_2 \). The coverings and factor spaces are labeled by integers so that the pages of this book like structure are labeled by pairs \((x_a, x_b)\), where \( x_i \) is positive integer for coverings and inverse integer for factor space. Planck constants is by general arguments expressible as \( \hbar = x_a x_b \times \hbar_0 \) and in principle can have all rational values.

In the sequel Quantum TGD in its recent form is described first. After that follow a concise summary about the model of dark matter based on the quantization of Planck constants.

2 The simplest manner to end up with Quantum TGD

One can end up with TGD either as a solution of the energy problem of GRT by replacing space-times with 4-surfaces \( X^4 \) in the space \( H = \mathcal{M}^4 \times S \) so that Poincare transformations act as symmetries of \( H \) rather than \( X^4 \), or as a generalization of string model by replacing strings with 3-D surfaces in \( H \). The requirement that the quantum number spectrum and classical gauge fields determined by the isometries and holonomies of \( H \) are consistent with the standard model leads to a unique choice \( S = \mathbb{C}P_2 \). Family replication phenomenon can be understood in terms of the topology of 2-D space-like sections of 3-D light-like surfaces at which the signature of the induced metric changes (genus-generation correspondence described in ch. Elementary Particle Vacuum Functionals of [6]). Lepton and quark numbers correspond to different chiralities of H-spinors are are separately conserved. Color quantum numbers correspond to color partial waves in \( \mathbb{C}P_2 \) so that both leptons and quarks can have higher color excitations (chs. Massless Particles and Particle Massivation and Recent Status of Leptohadron Hypothesis of [6]).

The simplest mathematical manner to end up with quantum TGD is to start from the local variant of the Clifford algebra of 8-D Minkowski space based on octonionic representation of the gamma matrices obtained as tensor products of octonions and 2-D sigma matrices \( \gamma_0 = 1 \otimes \sigma_1 \), \( \gamma_i = e_i \otimes \sigma_2 \), \( i = 1, \ldots, 7 \), \( e_i^2 = -1 \). The uniqueness of \( M^8 \) and \( \mathcal{M}^4 \times \mathbb{C}P_2 \) as well as the role of hyper-quaternionic space-time surfaces as fundamental dynamical objects indeed follow from rather weak conditions -essentially associativity or co-associativity (hyper-quaternionic plane of complexified quaternions corresponds to the plane spanned by the real unit and units \( i e_i \), where \( i \) is commutative imaginary unit and \( e_i \) are quaternionic imaginary units; the generalization to octonionic case is obvious).

1. The unique feature of \( M^8 \) and any 8-dimensional space with Minkowski signature of metric is that it is possible to have an octonionic representation of the complexified gamma matrices and of spinors (ch. Does the Modified Dirac Equation Define the Fundamental Variational Principle of [2]). This does not require octonionic coordinates for \( M^8 \). The restriction
to a complex quaternionic plane for both gamma matrices and spinors guarantees the
associativity.

2. One can also consider a local variant of the octonionic Clifford algebra in $M^8$. This algebra
contains associative subalgebras for which one can assign to each point of $M^8$ a hyper-
quaternionic plane. It is natural to assume that this plane is either a tangent plane of 4-D
manifold defined naturally by the induced gamma matrices $\Gamma_\alpha = \partial_\alpha h^k \Gamma_k$ defining a basis of
tangent space or more generally, by modified gamma matrices $\hat{\Gamma}^\alpha = \partial L/\partial (\partial_\alpha h^k) \Gamma^k$ defined by
a variational principle: these gamma matrices do not define tangent space in general. Kähler
action defines a unique candidate for the variational principle in question. Associativity
condition would automatically select sub-algebras associated with 4-D hyper-quaternionic
space-time surfaces.

3. This vision bears a very concrete connection to quantum TGD. The octonionic formulation of
the modified Dirac equation leads to a highly unique general solution ansatz for the equation
working also for the matrix representation of the Clifford algebra. An open question is
whether the resulting solution as such define also solutions of the modified Dirac equation
for the matrix representation of gammas. Also a possible identification for 8-dimensional
counterparts of twistors as octo-twistors follows: associativity implies that these twistors are
very closely related to the ordinary twistors. In TGD framework octo-twistors provide an
attractive manner to get rid of the difficulties posed by massive particles for the ordinary
twistor formalism (ch. Twisters, $N=4$ Super-conformal Symmetry, and Quantum TGD of
[4]).

4. Associativity implies hyperquaternionic space-time surfaces (also co-associativity is possible
and means associative normal bundle). This leads naturally to the notion of WCW and
local Clifford algebra in this space. Number theoretic arguments imply $M^8 - H$ duality in
the sense that space-time surfaces can be regarded equivalently as surfaces in $M^8$ or $H$ if
the tangent plane of each point of space-time surface contains a preferred plane $M^2 \subset M^4$
identifiable as hyper-complex plane. More general assumption is that $M^2$ depends on point
of space-time but the distribution of these planes integrates to a 2-D surface analogous to
string world sheet. The physical interpretation is as a local decomposition $M^4$ to planes
of physical and non-physical polarizations: this means a number theoretical realization of
gauge invariance. The resulting infinite-dimensional Clifford algebra would differ from von
Neumann algebras in that the Clifford algebra and spinors assignable to the center of mass
degrees of freedom of causal diamond $CD$ would be expressed in terms of octonionic units
although they are associative at space-time surfaces. One can therefore say that quantum
TGD follows by assuming that the tangent space of the imbedding space corresponds to a
classical number field with maximal dimension.

That makes the result so non-trivial is that $M^4 \times CP_2$ codes for the symmetries of standard
model so that unification of the known interactions would reduce to single dynamical principle:
associativity.

3 An overall view about quantum TGD

The great vision is that quantum TGD should provide a geometrization of not only classical but
also of quantum physics in terms of the geometry and spinor structure of the infinite-dimensional
space of 3-surfaces of $H = M^4 \times CP_2$ - the "world of classical worlds" (WCW). Quantum classical
 correspondence has been the basic guideline in the attempts to realize this dream mathematically.
3.1 Basic physical picture

Consider first the identification of WCW. The original vision was that space-like 3-D surfaces of $H = M^4 \times CP_2$ define the points WCW. By general coordinate invariance (GCI) the definition of WCW geometry must assign to any 3-D surface a unique space-time surface. This 4-surface can possess both Minkowskian and Euclidian regions: the latter have interpretation as "lines" of generalized Feynman diagrams. So called $CP_2$ type vacuum extremals having light-like curve as $M^4$ projection (ch. Basic Extremals of Kähler action of [3]) represent a model for the lines of the generalized Feynman diagrams. The boundaries $X_3^l$ between Minkowskian and Euclidian regions are light-like 3-surfaces and the 4-D induced metric is degenerate for them. $X_3^l$ is identified as a carrier of fermionic quantum numbers with arbitrarily high fermion number allowed. Single $CP_2$ type extremal glued to a space-time sheet with Minkowskian signature serves as a space-time correlate for fermions and their super-partners. The pieces of $CP_2$ type vacuum extremals connecting two space-time sheets with Minkowskian signature correspond to a pair of light-like wormhole throats. These are identified bosons and their super-partners.

Zero energy ontology is essential element of quantum TGD and one can interpret it as a formulation allowing to get rid of the non-determinism of Kähler action.

1. The so called Kähler action- Maxwell action associated with the induced $CP_2$ Kähler form in the induced metric- is the unique candidate as far as the dynamics of space-time surfaces is considered. It has a huge vacuum degeneracy corresponding to pure gauge configurations (any 4-surface with $CP_2$ projection which is a Lagrange sub-manifold of $CP_2$, is a vacuum extremal). Vacuum extremals are non-deterministic and the standard form of determinism is expected to fail also for non-vacuum deformations of the vacuum extremals. In order to have determinism in a generalized sense, one must generalize the notion of 3-surface by allowing unions of space-like 3-surfaces with several components with time-like separations. This leads to the zero energy ontology naturally.

2. The notion of causal diamond (CD) defined as the intersection of future and past directed light-cones formalizes zero energy ontology. The positions for the tips of causal diamond are labeled by $M^4$ point and a point of future light-cone $M^4_+$ of $M^4$. In absence of any other constraints WCW can be regarded as a union of sub-WCWs associated with CDs labeled by $M^4 \times M^4_+$. $M^4$ would correspond to ordinary Minkowski space $M^4_+$ could allow interpretation in terms of quantum variant of Robertson-Walker cosmology (ch. TGD and Cosmology of [3]). CDs form a fractal hierarchy. p-Adic length scale hypothesis [6] follows if the proper time distance between the tips is quantized as octaves of $CP_2$ time so that $M^4_+$ would reduce to a union of hyperbolic spaces with a quantized value of cosmic time $a = 2^n T(CP_2)$.

3. The challenge is to geometrize the sub-WCW associated with a given CD. It would naturally consist light-like 3-surfaces $X^3 \pm$ with 2-D ends $X^2$ at the light-like boundaries of CD. One could also identify 3-surfaces $X^3$ as pairs of space-like 3-surfaces $X^3 = X^4 \cap \delta CD$. This identification would mean a partial "gauge fixing" made possible by the 4-D general coordinate invariance (GCI). The requirement that these two options are equivalent implies effective 2-dimensionality. In other words, the information about WCW geometry and also about physics must be coded by the partonic 2-surfaces $X^2 = X^3 \cap X^3 + \delta CD$. The already mentioned hyper-quaternionicity condition for Kähler action poses strong conditions on $T(X^4)$ of $X^4$ at points of $X^2$. The time like entanglement coefficients between positive and negative energy states define
$M$-matrix which can be seen as "complex square root" of density matrix expressible as a product of square root of diagonalized density matrix and unitary S-matrix. $M$-matrix is the counterpart for the S-matrix measured in particle physics experiments (chs. *Construction of Quantum Theory: Symmetries* and *Construction of Quantum Theory: $M$-matrix* of [4]). The unitary $U$-matrix between zero energy states assignable to quantum jumps would have role in TGD inspired theory of consciousness [7].

### 3.2 Physics as infinite-D geometry of WCW

The geometrization of quantum physics would mean roughly the geometrization of WCW by introducing Kähler metric and spinor structure. The geometrization program is described in detail in chs. *Construction of Configuration Space Geometry From Symmetry Principles: Part I/Part II* of [2].

1. The geometrization of quantum physics requires among other things the geometrization of Hermitian conjugation. This requires Kähler geometry characterized by Kähler function $K$. $K$ must assign to given 3-surface of $CD$ a unique space-time surface $X^4(X^3)$ and this is achieved if the Kähler function corresponds to a preferred extremal of Kähler action. This space-time surface is the analog of Bohr orbit so that classical physics becomes an exact part of quantum theory in accordance with quantum classical correspondence. This means holography strengthened further by the effective 2-dimensionality. By the failure of the strict non-determinism the holography is achieved only locally and Universe is 3-dimensional and also 4-dimensional in long scales but in discretized sense.

2. Kähler action depends on single parameter- the Kähler coupling strength $g^2_K/4\pi$ analogous to temperature since vacuum functional is identifiable as exponent $\exp(K)$. The condition that $g^2_K/4\pi$ is analogous to critical temperature fixes the theory completely and one can say that TGD Universe is quantum critical.

3. Physical states must correspond to geometric objects. WCW spinor fields satisfy this criterion. WCW Clifford algebra is generated by complexified gamma matrices satisfying anticommutation relations completely analogous to those for fermionic oscillator operators. If these gamma matrices are linear combinations of oscillator operators assignable to second quantized free spinor fields at $X^4$ identified as spinor fields of $H$, a geometrization of fermionic statistics is achieved. The Clifford algebra of WCW is a canonical representative for a von Neumann algebra known as hyper-finite factor (HFF) of type $II_1$ [14].

The so called loop spaces consisting of loops defined by the maps of $S^3$ to some Lie group $G$ are simple analogs of WCW. They can be geometrized and their Kähler metric is unique and possess Kac-Moody group as isometries [12]: the reason is that Riemann connection fails to exist without maximal symmetries. In the recent case there are much stronger reasons to expect the uniqueness of the Kähler geometry -even in the sense that the choice of the imbedding space might be unique and fixed by the preceding number theoretic arguments.

1. What one expects is a union of infinite-dimensional symmetric spaces $G/H$ for suitable groups labeled by an infinite number of zero modes. Kähler metric depends on these parametrically but they do not contribute to the line element. The Kähler form of $CP_2$ induced to $X^4$ characterizes these zero modes and is invariant under symplectic transformations of $CP_2$ and diffeomorphisms of $M^4$. The group of symplectic transformations $\delta M^4 \times CP_2$ characterizes the quantum fluctuating degrees of freedom since it affects the induced metric and therefore also Kähler action. Since induced metric corresponds to the quantum fluctuating degrees of freedom, one can say that quantum dynamics is basically gravitational.
2. Kac-Moody groups act as isometries of loop spaces. In the recent case there are two candidates for the counterparts of these symmetries. The symplectic transformations of $\delta CD \times CP_2$ localized with respect to $X^4$ in an appropriate manner define a generalization of Super Kac-Moody algebra in which the finite-dimensional Lie group $G$ is replaced with the symplectic group of $\delta M^4_+ \times CP_2$. The symplectic algebra has also conformal structure with the role of complex coordinate taken by the radial light-like coordinate of $\delta M^4_+$. There is also super Kac-Moody algebra associated with the isometries of $H$ and respecting light-likeness of $X^3$. In this case light-like coordinate plays the role of complex coordinate. The symmetric spaces in the union should correspond to coset spaces of these infinite-dimensional groups.

3. Also the counterpart of coset construction [13] for the representations of Super-Virasoro algebras emerges naturally. The differences of the Super Virasoro generators associated with the super-symplectic and Super Kac-Moody algebras are assumed to annihilate physical states assigned with the partonic 2-surfaces. The four-momenta assignable to these two super-conformal algebras are identical and the interpretation in terms of Equivalence Principle is natural (ch. TGD and GRT of [3]). This gives also a justification for the p-adic thermodynamics for the scaling generator of either Super-Virasoro algebra explaining particle mass as thermal conformal weight analogous to thermal energy (ch. p-Adic Particle Massivation: Elementary Particle Masses of [6]).

4. Super-conformal invariance implies that the light-like directions associated with $\delta M^4_\pm$ and $X^3_l$ are not dynamical locally: this implies the required effective 2-dimensionality.

3.3 The construction of WCW spinor structure and modified Dirac action

WCW gamma matrices should be expressible in terms of the fermionic oscillator operators of second quantized H-spinor fields restricted to space-time surfaces. This construction is described in detail in chs. Configuration Space Spinor Structure and Does the Modified Dirac Equation Define the Fundamental Variational Principle of [2].

1. Dirac action must be generalized, and the first guess is that one just replaces ordinary gamma matrices with the induced gamma matrices $\Gamma_\alpha = \partial_\alpha h^k \Gamma_k$ defined as projections of $H$ gamma matrices to the space-time surface. The Dirac equations for $\Psi$ and $\bar{\Psi}$ are equivalent only if the space-time surface is minimal surface. This implies also the conservation of super currents defined by the local inner products $\bar{\Psi} \Gamma^\alpha \Psi$ of the modes of the spinor fields and second quantized $\Psi$. The exponent of 4-volume for a preferred extremal does not lead to a physically acceptable vacuum functional $\exp(K)$. The difficulty can be circumvented by replacing the induced gamma matrices with the modified gamma matrices $\tilde{\Gamma}^\alpha = (\partial L_K/\partial_\alpha h^k) \Gamma^k$ assignable to the actio density of Kähler action, which by its vacuum degeneracy is a unique choice. Space-time surfaces must be extremals of Kähler action for modified Dirac equation to be internally consistent.

2. Extremal property does not yet guarantee the existence of a fermionic oscillator operator representation of conserved charges. A deformation of the space-time surface induced by a diffeomorphism of $H$ corresponds to a conserved Noether charge only if gives rise to a vanishing second variation of Kähler action. This condition is analogous to the vanishing of the second derivative of a potential function at critical point. The physical interpretation is in terms of quantum criticality. There is however a problem: the second variations associated with the isometries of $H$ vanish identically but the corresponding fermionic charges vanish. The problem how to obtain a quantum representation for Poincare and color quantum numbers remains.
3. The vacuum degeneracy of Kähler action encourages the conjecture that the super-conformal algebra of these deformations acting in zero modes is infinite-dimensional and one has hierarchies of breakings of super-conformal symmetry realized in terms of fractal hierarchies of inclusions of sub-algebras of super-conformal algebra isomorphic to the algebra itself. These hierarchies would replace the symmetry breaking hierarchies of gauge theories and are expected to correspond to inclusion hierarchies of hyper finite factors of type II$_1$. This hierarchy corresponds also to a hierarchy of inclusions in the space of zero modes (induced $CP^1$ Kähler form) such that symmetry breaking brings in new quantum fluctuating degrees of freedom.

4. The 1+3 decomposition of space-time surface arguments around wormhole throat implied by number theoretical allows a dimensional reduction of the modified Dirac equation and one can assign to its 3-D part a spectrum of "energy eigenvalues". The product of these eigenvalues defines a Dirac determinant serving as an excellent candidate for the vacuum functional of the theory. The conjecture is that the Dirac determinant equals to the exponent of Kähler function identified as Kähler action for the preferred extremals. Note that Kähler function is determined only apart from the addition of a real part of a holomorphic function of complex coordinates of WCW depending also on zero modes.

5. Quantum classical correspondence forces to ask how to feed information about various quantum numbers of super conformal representations to the geometry of space-time geometry. These representations define also representations of isometry charges and the problem is how to represent the eigenvalues of these charges in terms of the fermionic oscillator operators. The answer is simple: add a measurement interaction term linear in momentum and color quantum numbers (necessarily in Cartan algebra of SU(3)) to the modified Dirac action. Quantum criticality allows to represent also more general charges in terms of conserved Noether currents implied quantum criticality. This also gives rise to a representation of conserved charges in terms of fermionic oscillator operators. If one accepts the hypothesis that the vacuum functional determined by the Dirac determinant equals to the exponent of Kähler function identified in the proposed manner, the measurement interaction term must induce to the Kähler action an analogous term. A highly attractive condition is that the couplings appearing in the most general measurement interaction term are such that the WCW metric is not affected: in other words, Kähler function is modified by a real part of a holomorphic function. Among other things this condition would fix the value of gravitational constant (ch. *Does the QFT Limit of TGD Have Space-Time Super-symmetry?* of [4]).

6. Measurement interaction term allows also a generalization of space-time super-symmetries. Measurement interaction term suggests a unique form for the anti-commutation relations of fermionic oscillator operators and gives rise to $\mathcal{N} = \infty$ SUSY algebra in the general case so that any mode of induced spinor field corresponds to a super-symmetry in the sense of spectrum generating algebra. Super-symmetry is broken at the level of masses from the beginning. This forces a generalization of chiral and vector super-fields and leads to a detailed proposal for the QFT limit of Quantum TGD. Very probably this construction can be used at the level of basic quantum TGD.

### 3.4 HFFs and TGD

The Clifford algebra of WCW at a given point (3-surface) and fermionic oscillator operator algebra define canonical representations for HFF of type II$_1$. One can consider also the local Clifford algebra defined by Clifford algebra valued functions in WCW associated with a given $CD$. This algebra is naturally HFF of type II$_\infty$ expressible as a tensor product of HFF of type II$_1$ and a factor of type $I_\infty$ associated with wave mechanical systems such as hydrogen atom. If one extends the local Clifford algebra element to union of $CD$s with the lower tips labeled by $M^4$, one expects HFF of
type $III_1$ encountered also in quantum field theories in $M^4$. The resulting vision is described in detail in ch. Was von Neumann Right After All of [4] and in the following only the basic ideas are summarized.

1. The inclusions of $\mathcal{N} \subset \mathcal{M}$ of HFFs are important in quantum TGD. Quantum groups appear in topological QFTs based on braids and braids emerge also in TGD naturally. Non-commutative physics relates closely to these inclusions and the quantum Clifford algebra could be interpreted in terms of quantum coset spaces $\mathcal{M}/\mathcal{N}$ interpreted as $\mathcal{N}$ modules. These correspond to finite-D matrix Clifford algebra with $\mathcal{N}$-valued matrix elements and possess a fractal dimension defined by the index of inclusion, which is quantized and expressible in terms of quantum phase $q = \exp(i2\pi/n)$, $n = 3, 4, \ldots$. Fractal dimension measures the correlations between matrix elements of finite-D Clifford algebra caused by the non-commutativity.

2. In TGD framework the inclusions have interpretation in terms of a finite measurement resolution rather than some exotic physics emerging in Planck length scale (ch. Construction of Quantum Theory: Symmetries of [4]. The action of the included algebra $\mathcal{N}$ on zero energy states creates states not distinguishable from the original one in the measurement resolution used. $\mathcal{N}$ takes the role of complex numbers and one can speak about $\mathcal{N}$ rays as representations of physical states. The effect of element of $\mathcal{N}$ on zero energy state should not affect scattering probabilities defined by $\mathcal{M}$-matrix in the finite measurement resolution defined by $\mathcal{N}$. Tracing over $\mathcal{N}$ is the simplest manner to achieve this and works for HFFs of type $II_1$ and $I_\infty$. The generalization of Connes tensor product in which tensor factors correspond to positive and negative energy parts of zero energy states, defines this kind of $\mathcal{M}$-matrix and in this case matrix elements are proportional to projectors to $\mathcal{N}$. For HFFs of type $III_1$ traces are infinite and one must used more general description analogous to the calculation of thermodynamical expectation value for the integration over $\mathcal{N}$ degrees of freedom in order to obtain well-defined reaction rates.

3. Discretization provides the space-time correlate for the finite measurement resolution. Partonic 2-surface $X^2$ is replaced by the set of points at which fermions reside. At the level of SUSY algebra this means a cutoff of $\mathcal{N}$ to a finite value. The orbits of these points define braids with some maximum strand number, and a connection with topological QFTs emerges allowing to understand why the quantum groups assignable to the inclusions appear also in braid theory. WCW reduces effectively to a Cartesian power of $H$ in the discretization.

4. In the theory of factors of type III, the notion of state obtained as a generalization of thermal state (or density matrix) plays a key role. In zero energy ontology thermal states are replaced by their ”complex square roots” and define the M-matrix. An interesting question is how strong conditions the mere HFF of type $III_1$ property poses on $\mathcal{M}$-matrix and S-matrix. Path integral notions does not allow a rigorous mathematical formulation. In TGD framework only the complex square root of the density matrix is needed and corresponds to the exponent of Kähler function defining WCW integration as a mathematically well-defined operation free of infinities. Finite measurement resolution would correspond in the general case to integration over $\mathcal{N}$ degrees of freedom with weighting defined by square root of density matrix. An interesting question is whether a Universal $\mathcal{M}$-matrix with ideal measurement resolution exists in some sense and whether it is possible to obtain $\mathcal{M}$-matrix with a given finite measurement resolution from this by ”integrating” over an appropriate sub-algebra $\mathcal{N}$.

3.5 Number theoretic vision

Number theoretic vision involves three threads: p-adic physics, classical number fields , and the notion of infinite prime. These threads are discussed in the chapters of the first part of [5].
p-Adic physics became an integral part of quantum TGD after the success of p-adic mass calculations based on p-adic thermodynamics, superconformal invariance, and p-adic length scale hypothesis (chs. Massless Particles and Particle Massivation and p-Adic Mass Calculations: Elementary Particle Masses of [6]) and has made possible surprisingly detailed quantitative predictions.

1. p-Adic length scale hypothesis states that primes \( p \approx 2^k \), \( k = 1, 2, \ldots \) are of special importance physically. The quantization of proper time distance between the tips of \( CD \) having interpretation in terms of quantization of cosmic time, implies p-adic length scale hypothesis.

2. The challenge is to fuse real physics and various p-adic physics to a single coherent whole. The fact that all these number fields are obtained as completions of rationals, inspires the generalization of number concept obtained by gluing together reals and various p-adic number fields along rational points and common algebras to form a book like structure with number fields representing "pages" of the book (ch. TGD as Generalized Number Theory: p-Adicization Program of [5]).

3. This idea generalizes to the level of imbedding space. It would be nice if also complex numbers as a maximal completion of rationals would emerge. As a matter fact, the space \( M^4 \times M^4_+ \) labeling the tip pairs of \( CDs \) without the quantization condition allows interpretation in terms of complexification of reals and of \( M^4_+ \). Space-time surfaces at different pages -realities and p-adicities- meet only along rational points and common algebras.

4. The notion of completion can be abstracted. At the level of WCW 3-surfaces common to assigned to two different number fields would have representation in terms of functions making sense in both number fields. Polynomials and rational functions with coefficients which are rationals or in some algebraic extension would be in question.

5. Number theoretic Universality states that for common points of WCW M-matrices must coincide and must therefore be expressible in terms of rational numbers and allowed algebraic numbers and is expected to give strong constraints on M-matrix. One can of course consider also infinite-dimensional extensions containing transcendental numbers such as \( \pi \) and ordinary QFT suggests that they are unavoidable.

6. TGD inspired theory of consciousness suggests an interpretation of the p-adic space-time sheets as correlates for cognition and intentionality. Zero energy ontology allows a leakage between sectors of WCW corresponding to different number fields and the interpretation could be in terms of transformation of intention to action. In this framework evolution could involve the evolution of algebraic complexity as a migration to algebraically increasingly complex sectors of WCW with higher dimensions of algebraic extensions of p-adic numbers.

Classical number fields (ch. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper-Counterparts of [5]) form an important part of quantum TGD, and I have already described how one ends up to quantum TGD from the idea that associativity defines the basic dynamical principle both at classical and quantum level. One important implication is hyper-quaternionicity stating that the modified gamma matrices defined by Kähler action span a quaternionic sub-space of octonionic space in the octonionic representation of 8-D gamma matrices. An unproven implication is that hyper-quaternionic space-time surface must be preferred extremals of Kähler action and thus also quantum critical.

What makes infinite primes (ch. TGD as a Generalized Number Theory: Infinite Primes of [5]) so interesting is that their construction is structurally equivalent with a repeated second quantization of a super-symmetric arithmetic quantum field theory. At lowest level finite primes label the states. At \( N \)-th level the states are labeled by infinite primes at this particular level and
at the next level the many particle states of the previous level define the elementary particles of
the new level. The structure of many-sheeted space-time with each new sheet representing a new
level has a similar hierarchical structure.

4 Hierarchy of Planck constants and the generalization of
the notion of imbedding space

In the following the recent view about the structure of imbedding space forced by the quantization
of Planck constant is summarized. The question is whether it might be possible in some sense
to replace $H$ or its Cartesian factors by their necessarily singular multiple coverings and factor
spaces. One can consider two options: either $M^4$ or the causal diamond $CD$. The latter one is
the more plausible option from the point of view of WCW geometry.

4.1 The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter
as a hierarchy of phases of matter with non-standard value of Planck constants was much faster
than the evolution of mathematical ideas and quite a number of applications have been developed
during last five years.

1. The starting point was the proposal of Nottale [22] that the orbits of inner planets correspond
to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant
$\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal (chs. TGD and Astro-physics
and Quantum Astro-physics of [3]) was that ordinary matter condenses around dark matter which
is a phase of matter characterized by a non-standard value of Planck constant whose value is
gigantic for the space-time sheets mediating gravitational interaction. The interpretation of
these space-time sheets could be as magnetic flux quanta or as massless extremals (ch. Basic
Extremals of Kähler action of [3]) assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have an enormous value of
Compton length meaning that the density of matter at these space-time sheets must be very
slowly varying. The string tension of string like objects implies effective negative pressure
characterizing dark energy so that the interpretation in terms of dark energy might make
sense (ch. TGD and Cosmology of $\mathcal{K}$). TGD predicted a one-parameter family of Robertson-
Walker cosmologies with critical or over-critical mass density and the ”pressure” associated
with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the standard
view about space-time. Particles with different Planck constant must belong to ”different
worlds” in the sense local interactions of particles with different values of $\hbar$ are not possible.
This inspires the idea about the book like structure of the imbedding space obtained by
gluing almost copies of $H$ together along common ”back” and partially labeled by different
values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at dif-
ferent pages of the book like structure cannot appear in the same vertex of the generalized
Feynman diagram. The phase transitions in which partonic 2-surface $X^2$ during its travel
along $X^3_t$ leaks to another page of book are however possible and change Planck constant.
Particle (say photon -) exchanges of this kind allow particles at different pages to interact.
The interactions are strongly constrained by charge fractionization and are essentially phase
transitions involving many particles. Classical interactions are also possible. It might be that
we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [23] (ch. The Notion of Wave-Genome and DNA as Topological Quantum Computer of [9]).

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase (ch. Quantum Hall Effect and the Hierarchy of Planck Constants of [4]). If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [22] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius $r_s$ of order scaled up Planck length $l_{Pl} = \sqrt{\hbar \kappa G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology [10, 11]. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and aminoacids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings (ch. Nuclear String Model of [6]).

4.2 The most general option for the generalized imbedding space

Simple physical arguments pose constraints on the choice of the most general form of the imbedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for $M^4$, $CD$, $CP_2$, or $H$. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where $S^2$ is geodesic sphere of $CP_2$, $M^4 = M^4 \setminus M^2$ and $CP_2 = CP_2 \setminus S^2$ have fundamental group $Z$ since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. $CP_2$ allows two geodesic spheres which left invariant by $U(2 \text{ resp. } SO(3))$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is unacceptable for non-vacuum extremals so that only homologically trivial geodesic sphere $S^2$ would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of $M^4$ are not possible. Therefore only the singular coverings and factor spaces of $CP_2$ over the homologically trivial geodesic sphere $S^2$ would be possible. This however looks a non-physical outcome.
The question how the observed Planck constant relates to the integers is far from trivial and I have considered several options. The basic idea is that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to the tilings for factor space or vice versa).

3. In general case there are four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C - C$, $C - F$, $F - C$, and $F - F$, where $C$ ($F$) signifies for covering (factor space) and first (second) letter signifies for $CD$ ($CP_2$) and correspond to the spaces $(CD \times G_a) \times (CP_2 \times G_b)$, $(CD \times G_a) \times CP_2 / G_b$, $CD / G_a \times (CP_2 \times G_b)$, and $CD / G_a \times CP_2 / G_b$.

4. The groups $G_i$ correspond to cyclic groups $Z_n$. One can also consider an extension by replacing $M^2$ and $S^2$ with its orbit under more general group $G$ (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2$ or $S^2$. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

### 4.3 How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers $n_a$ and $n_b$ defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of $CD$ (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2 / 4\pi \hbar$ on the other hand.

1. One can assign to Planck constant to both $CD$ and $CP_2$ by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $h(CD)$ and $h(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by $G_i$. This requires $r(X) = h(X) h_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $h^2(X)$, $X = M^4$, $CP_2$ corresponds to the scaling of the covariant metric tensor $g_{ij}$ and performs an over-all scaling of $H$-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\bar{h}^2(CP_2)$, one obtains the scaling of $M^4$ covariant metric by $r^2 \equiv \bar{h}^2 / \hbar_0^2 = h^2(M^4) / h^2(CP_2)$ whereas $CP_2$ metric is not scaled at all.

3. The condition that $h$ scales as $n_a$ is guaranteed if one has $h(CD) = n_a \hbar_0$. This does not fix the dependence of $h(CP_2)$ on $n_b$ and one could have $h(CP_2) = n_b \hbar_0$ or $h(CP_2) = \hbar_0 / n_b$. The intuitive picture is that $n_b$-fold covering gives in good approximation rise to $n_a n_b$ sheets and multiplies YM action by $n_a n_b$ which is equivalent with the $h = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $h(CP_2) = \hbar_0 / n_b$ and $h = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.
This gives the following formulas \( r \equiv h/h_0 = r(M^4)/r(CP_2) \) in various cases.

\[
\begin{array}{cccccc}
C - C & F - C & C - F & F - F \\
\ell & n_a n_b & \frac{n_a}{n_b} & \frac{n_b}{n_a} & \frac{1}{n_a n_b}
\end{array}
\]

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products \( n_F = 2^k \prod_s F_s \), where \( F_s = 2^{2^s} + 1 \) are distinct Fermat primes, are favored. The reason would be that quantum phase \( q = \exp(i\pi/n) \) is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to \( s = 0, 1, 2, 3, 4 \) so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of \( n_F \) of fundamental p-adic length scale. \( n_F = 2^{11} \) corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, \( CP_2 \) radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of \( 2^{11} \) seem to be especially favored as values of \( n_a \) in living matter (ch. TGD and EEG of [8]).

### 4.4 A simple model for fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [17] at the level of basic quantum TGD as integer QHE for non-standard value of Planck constant (ch. Quantum Hall Effect and the Hierarchy of Planck Constants of [4]).

The formula for the quantized Hall conductance is given by

\[
\sigma = \nu \times \frac{e^2}{h},
\]

\[
\nu = \frac{n}{m}.
\]

Series of fractions in \( \nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9..., 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7... \) with odd denominator have been observed as are also \( \nu = 1/2 \) and \( \nu = 5/2 \) states with even denominator [17].

The model of Laughlin [16] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [18]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

Before proposing the TGD based model of FQHE as IQHE with non-standard value of Planck constant, it is good to represent a simple explanation of IQHE effect. Choose the coordinates of the current currying slab so that \( x \) varies in the direction of Hall current and \( y \) in the direction of the main current. For IQHE the value of Hall conductivity is given by \( \sigma = j_y/E_x = n_e ev/\nu B = n_e e/B = Ne^2/h BS = Ne^2/mh \), were \( m \) characterizes the value of magnetized flux and \( N \) is the total number of electrons in the current. In the Landau gauge \( A_y = x B \) one can assume that energy eigenstates are momentum eigenstates in the direction of current and harmonic oscillator Gaussians in \( x \)-direction in which Hall current runs. This gives

\[
\Psi \propto \exp(iky) H_n(x + kl^2) \exp\left(-\frac{(x + kl^2)^2}{2l^2}\right), \quad l^2 = \frac{h}{eB}.
\]

Only the states for which the oscillator Gaussian differs considerably from zero inside slab are important so that the momentum eigenvalues are in good approximation in the range \( 0 \leq k \leq \)
\[ k_{\text{max}} = \frac{L_x}{l^2}. \] Using \( N = \left( \frac{L_y}{2\pi} \right) \int_0^{k_{\text{max}}} dk \) one obtains that the total number of momentum eigenstates associated with the given value of \( n \) is
\[ N = eBdL_xL_y/\hbar = n. \]
If \( \nu \) Landau states are filled, the value of \( \sigma \) is \( \sigma = \nu e^2/\hbar. \)

The interpretation of FQHE as IQHE with non standard value of Planck constant could explain also the fractionization of charge, spin, and electron number. There are \( 2 \times 2 = 4 \) combinations of covering and factor spaces of \( CP_2 \) and three of them can lead to the increase or at least fractionization of the Planck constant required by FQHE.

1. The prediction for the filling fraction in FQHE would be
\[
\nu = \nu_0 \frac{k_0}{h}, \quad \nu_0 = 1, 2, \ldots .
\] \[ (3) \]
\( \nu_0 \) denotes the number of filled Landau levels.

2. Let us denote the options as C-C, C-F, F-C, F-F, where the first (second) letter tells whether a singular covering or factor space of \( CD \) (\( CP_2 \)) is in question. The observed filling fractions are consistent with options C-C, C-F, and F-C for which \( CD \) or \( CP_2 \) or both correspond to a singular covering space. The values of \( \nu \) in various cases are given by the following table.

<table>
<thead>
<tr>
<th>Option</th>
<th>C-C</th>
<th>C-F</th>
<th>F-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu )</td>
<td>( \nu_0 \frac{k_0}{n_a} )</td>
<td>( \nu_0 k_0 )</td>
<td>( \nu_0 k_0 )</td>
</tr>
</tbody>
</table>

\[ (4) \]

There is a complete symmetry under the exchange of \( CD \) and \( CP_2 \) as far as values of \( \nu \) are considered.

3. All three options are consistent with observations. Charge fractionization allows only the options \( C - C \) and \( F - C \). If one believes the general arguments stating that also spin is fractionized in FQHE then only the option \( C - C \), for which charge and spin units are equal to \( 1/n_b \) and \( 1/n_a \) respectively, remains. For \( C - C \) option one must allow \( \nu_0 > 1 \).

4. Both \( \nu = 1/2 \) and \( \nu = 5/2 \) state has been observed \[17, 20\]. The fractionized charge is believed to be \( e/4 \) in the latter case \[21, 19\]. This requires \( n_b = 4 \) allowing only \( (C, C) \) and \( (F, C) \) options. \( n_i \geq 3 \) holds true if coverings and factor spaces are correlates for Jones inclusions and this gives additional constraint. The minimal values of \( (\nu_0, n_a, n_b) \) are \((2, 1, 4)\) for \( \nu = 1/2 \) and \((10, 1, 4)\) for \( \nu = 5/2 \) for both \( C - C \) and \( F - C \) option. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field \[18\]. \( n_b = 2 \) would be inconsistent with the observed fractionization of electric charge for \( \nu = 5/2 \) and with the vision inspired by Jones inclusions implying \( n_i \geq 3 \).

5. A possible problematic aspect of the TGD based model is the experimental absence of even values of \( m \) except \( m = 2 \) (Laughlin’s model predicts only odd values of \( m \)). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) both \( n_a \) and \( n_b \) must be odd. This would require that \( m = 2 \) case differs in some manner from the remaining cases.
References


[23] P. P. Gariaev, G. G. Tertishni, A. V. Tovmash (2007), Experimental investigation in vitro of holographic mapping and holographic transposition of DNA in conjunction with the information pool encircling DNA, New Medical Tehcnologies, #9, pp. 42-53. The article is in Russian but Peter Gariaev kindly provided a translation of the article to English.