Holography=holomorphy vision in relation to quantum criticality, hierarchy of Planck constants, and $M^8 - H$ duality

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Abstract

Holography = holomorphy vision generalizes the realization of quantum criticality in terms of conformal invariance. Holography = holomorphy vision provides a general explicit solution to the field equations determining space-time surfaces as minimal surfaces $X^4 \subset H = M^4 \times CP_2$. For the first option the space-time surfaces are roots of two generalized analytic functions P_1, P_2 defined in H . For the second option single analytic generalized analytic function defines X^4 as its root and as the base space of 6-D twistor twistor-surface X^6 in the twistor bundle $T(H) = T(M^4) \times TCP_2)$ identified as a zero section

By holography, the space-time surfaces correspond to not completely deterministic orbits of particles as 3-surfaces and are thus analogous to Bohr orbits. This implies zero energy ontology (ZEO) and to the view of quantum TGD as wave mechanics in the space of these Bohr orbits located inside a causal diamond (CD), which form a causal hierarchy. Also the construction of vertices for particle reactions has evolved dramatically during the last year and one can assign the vertices to partonic 2-surfaces.

$M^8 - H$ duality is a second key principle of TGD. $M^8 - H$ duality can be seen as a number theoretic analog for momentum-position duality and brings in mind Langlands duality. M^8 can be identified as octonions when the number-theoretic Minkowski norm is defined as $Re(o^2)$. The quaternionic normal space $N(y)$ of $y \in Y^4 \subset M^8$ having a 2-D commutative complex sub-space is mapped to a point of CP_2. Y^4 has Euclidean signature with respect to $Re(o^2)$. The points $y \in Y^4$ are lifted by a multiplication with a co-quaternionic unit to points of the quaternionic normal space $N(y)$ and mapped to $M^4 \subset H$ inversion.

This article discusses the relationship of the holography = holomorphy vision with the number theoretic vision predicting a hierarchy $h_{eff} = nh_0$ of effective Planck constants such that n corresponds to the dimension for an extension rationals (or extension F of rationals). How could this hierarchy follow from the recent view of $M^8 - H$ duality? Both realizations of holography = holomorphy vision assume that the polynomials involved have coefficients in an extension F of rationals Partonic 2-surfaces would represent a stronger form of quantum criticality than the generalized holomorphy: one could say islands of algebraic extensions F from the ocean of complex numbers are selected. For the P option, the fermionic lines would be roots of P and dP/dz inducing an extension of F in the twistor sphere. Adelic physics would emerge at quantum criticality and scattering amplitudes would become number-theoretically universal. In particular, the hierarchy of Planck constants and the identification of p-adic primes as ramified primes would emerge as a prediction.

Also a generalization of the theory of analytic functions to the 4-D situation is suggestive. The poles of cuts of analytic functions would correspond to the 2-D partonic surfaces as vertices at which holomorphy fails and 2-D string worlds sheets could correspond to the cuts. This provides a general view of the breaking of the generalized conformal symmetries and their super counterparts as a necessary condition for the non-triviality of the scattering amplitudes.

Contents

1 Introduction

Quantum criticality is realized in terms of conformal invariance in string models and conformal field theories. Holography = holomorphy vision [L15] generalizes the realization of vision to the TGD framework. This view has developed considerably during the last years and provides a general explicit solution to the field equations determining space-time surfaces as minimal surfaces $X^4 \subset H = M^4 \times CP_2$ obeying generalized holomorphy.

By holography, the space-time surfaces correspond to not completely deterministic orbits of particles as 3-surfaces and are thus analogous to Bohr orbits. This leads to zero energy ontology (ZEO) and to the view of TGD as wave mechanics in the space of these Bohr orbits inside causal diamonds (CD), which form a scale hierarchy. The construction of vertices for particle reactions identified as these singular lower-D belonging to the light-like 3-surfaces as interfaces of

Minkowskian and Euclidean space-time regions defining partonic orbits has also evolved dramatically during the last year and one can assign the vertices to the partonic 2-surfaces thanks to the understanding of the modified Dirac equation [L13, L20].

There are two guesses for the vertices. Either as partonic 2-surfaces or as point-like singularities at which the monopole flux tube representing a particle splits to two. The latter interpretation looks more plausible. Fermion lines at light-like partonic orbits would correspond to poles of analytic functions, which are not vertices. At them the minimal surface property would fail. String world sheets having partonic lines as boundaries would be the analogs of cuts as singularities.

M^8-H duality is a second key principle of TGD. It has developed to its recent form [L16] rather slowly during the years via several wrong tracks [L4, L5, L14]. M^8-H duality can be seen as a number theoretic analog for the momentum-position duality and also brings in mind Langlands duality [A2, A1] [L10]. In the recent view, it is essential that M^8 can be identified as octonions when its number-theoretic Minkowski norm is defined as $Re(o^2)$. The quaternionic normal space $N(y)$ of $y \in Y^4 \subset M^8$, having a 2-D commutative complex sub-space, is mapped to a point of CP_2. Y^4 has Euclidean signature with respect to $Re(o^2)$. The points $y \in Y^4$ are lifted by a multiplication with a co-quaternionic unit to points of the quaternionic normal space $N(y)$ and mapped to $M^4 \subset H$ inversion.

This article discusses the relationship of the holography = holomorphy vision developed in [L21, L15] with the number theoretic vision predicting a hierarchy $h_{eff} = nh_0$ of effective Planck constants such that n corresponds to the dimension of an extension rationals. The question is whether and how this hierarchy follows from the recent view of M^8-H duality differing considerably from the earlier view. Also the consistency with the number theoretic vision is discussed.

There are two realizations for the holography = holomorphy vision which need not be mutually exclusive.

1. For the first option, the space-time surfaces are roots of two generalized analytic functions f_1, f_2 of hypercomplex coordinate (having light-like coordinate curves in M^4) and 3 complex coordinates of H. As a special case f_1, f_2 reduces to a pair (P_1, P_2) of polynomials. The minimal surface property holds true for any general coordinate invariant action constructible in terms of the induced geometry and fails only at lower-dimension singularities. This vision has tensions with the number theoretical vision. The roots $P_1 = 0$ and $P_2 = 0$ define 6-surfaces, whose intersection is X^4. A possible interpretation is as analogs of twistor spaces of M^4 and CP_2.

2. There is also a variant of this picture in which the space-time surface is identified as a holomorphic zero section of the twistor surface $X^6 \subset T(M^4) \times T(CP_2)$ defined as a root of single polynomial P. This option makes the nice predictions of the number theoretic vision relying strongly on polynomials with coefficients, which are rational or in an extension F of rationals.

The islands of rationals and their algebraic extensions would be selected by quantum criticality from the ocean of complex numbers. In music, the rational ratios for the frequencies of the Pythagorean scale would represent a similar phenomenon [L1, L7, L8, L12]. In this way quantum criticality defining a hierarchy of number theoretical discretization of WCW would make possible adelic physics and scattering amplitudes would become number-theoretically universal. For the $P = 0$ option, the hierarchy of Planck constants and Galois groups as number theoretical symmetry groups would emerge as predictions. Also p-adic primes essential for p-adic mass calculations would correspond to ramified primes of P. The $P = 0$ option seems to be however consistent with the $(P_1, P_2) = (0, 0)$ option.

The generalization of the theory of analytic functions to the 4-D situation is suggestive.

1. The poles and cuts of analytic functions determine them as an analog of holographic data. Poles could correspond to light-like fermion lines at which the holomorphy fails and the 2-D string worlds sheets connecting partonic orbits of different monopole flux tubes could correspond to the cuts. This failure of analyticity would make non-trivial scattering amplitudes possible, and also lead to the breaking of various conformal symmetries and their super counterpparts and also to the generation of an analog of Higgs vacuum expection [L20].

2. These singularities would define at least a part of the data defining holography allowing to determine the space-time surface (also the 3-surfaces at the ends of space-time surfaces at the boundaries of CD might be needed). This picture brings in mind electrostatics in 2-dimensions where poles and cuts define point charges and line charges as sources. This is nothing but an analog of holography. The light-like fermionic lines at the light-like partonic orbits are the most natural counterparts of poles of analytic function and string world sheets define the analogs of cuts. This would mean a strong analogy with the ordinary complex analysis

3. The singularities would realize a stronger form of quantum criticality than the generalized holomorphy, and there is a rather precise geometric analogy with catastrophe theory predicting a hierarchy of critical surfaces at which some number of roots coincide (cusp catastrophe represents a simple example).

4. The splitting of a closed monopole flux tube representing a particle reaction vertex rise to a fundamental vertex for particle creation involving a creation of a fermion pair which does not seem to correspond to these singularities. The splitting process starts as a touching of the flux tube portions at parallel Minkowskian space-time sheets and generates a wormhole contact, which then splits to two wormhole contacts identifiable as "ends" of final state monopole flux tubes. This involves a splitting of the wormhole throats at a point in which a fermion pair can be created. These two touching points represent genuinely 0-dimensional singularities defining analogs of QFT vertices having no counterpart in 2-D complex analysis. A connection with exotic smooth structures [A5, A6, A4], uniquely distinguishing 4-manifolds, and identifiable as standard smooth structure with defects, proposed to be identifiable in terms of the 2-D singularities, is highly suggestive [L9, L20].

2 About the evolution of the concept of $M^8 - H$ duality

$M^8 - H$ duality [L4, L5, L14] can be seen as a number-theoretic analog of momentum position duality. $M^8 - H$ duality also relates geometric and number theoretic vision of quantum TGD and could serve as a physical realization of Langlands duality [L10]. Note that all single-fermion states are predicted to be massless in the 8-D sense and therefore their 8-D momenta are expected to reside at the 7-D light-cone of M^8.

The development of $M^8 - H$ duality has been a rather tortuous process and has involved several wrong tracks.

2.1 The problems with the original forms of $M^8 - H$ duality

The very first form of $M^8 - H$ duality mapped $M^4 \subset H$ to a quaternionic tangent space of $M^4 \subset M^8$: octonions are metrically M^8 with respect to the number theoretic norm $RE(o^2)$. The cold shower [L4, L5] was that the distribution of quaternionic tangent spaces is not in general integrable to a 4-surface Y^4 and it seems that only trivial associative surfaces (M^4) are possible.

This forced the conclusion that the normal spaces must be quaternionic, one could call this co-associativity [L4, L5]. The distribution of normal spaces is indeed known to be associative always. This however forced complexification of M^8 in order to get the metric signature correctly and this led to a multitude of problems.

The form of $M^8 - H$ duality discusses in [L4, L5] assumed the algebraic continuation of polynomials $P(z)$ of complex variable z for which the imaginary unit commutes with octonionic imaginary units to polynomials of complexified octonion.

1. $P(z)$ was assumed to have rational or even integer coefficients smaller than the degree. This led to the proposal that the roots of $P(t)$ correspond to mass squared values m^2 defining mass shells in $M^4 \subset M^8$ as $p_0^2 - \sum p_i^2 = m^2$. This assumption had very nice implications concerning the number theoretic vision. This led to the notion of associative holography. One fixes the 3-surfaces defining the boundary data of the holography to be mass shells $H^3 \subset M^4 \subset M^8$ and continues this data to associative 4-surfaces. The problem was that these holographic data look quite too simple. Deformations in normal direction would seem to be necessary.

2. Perhaps the most serious problem of the approach was that complex roots correspond to complex masses so that one must complexity M^8. This led to rather complicated constructions [L4, L5, L14]. The assumption that the roots are real could solve this problem but looks somewhat ad hoc.

3. The image of $M^4 \subset H$ in M^8 was identified as a co-quaternionic sub-space of complexified octonions M^8_c. The octonionic number theoretic norm is defined without the conjugation with respect to i. There are many choices for the subspace for which the octonionic coordinates are real or purely imaginary and all signatures of the number theoretic norm defined by o^2 are possible. This also led to interpretational problems: to what what particular $M^4 \subset M^8$ does $M^4 \subset H$ correspond to? Does this choice have a physical significance?

2.2 The view of $M^8 - H$ duality in 2024

In [L16], I introduced a dramatic simplification of the earlier version of the $M^8 - H$ duality [L4, L5, L14] allowing to get rid of the complexification of M^8 interpreted as an 8-D momentum space.

The trick is to define the number theoretic norm, not as $o\bar{o}$, but as the real part $RE(o^2)$ of o^2: the real part here means the part of o^2 proportional to the octonionic real unit. This definition applies also to complex numbers and quaternions as subspaces of octonions. This norm is Minkowskian and allows the identification of octonions as M^8 in the metric sense.

In this interpretation, the projections of the points of H to $M^4 \subset H = M^4 \times CP_2$ correspond by $M^8 - H$ duality to the points of $Y^4 \subset M^8 = M^4 \times E^4$ such that the M^4 projection m of the point of X^4 associated with CP_2 point s is mapped by inversion $I : m^k \rightarrow \hbar_{eff} m^k / m_l m^l$ to a point of the quaternionic normal space $N(y(s))$ as its preferred point and naturally projects to the corresponding point of Y^4. Y^4 is Euclidian with respect to the number theoretic norm already described.

Number theoretic holography would be realized by requiring that the mass squared value assignable to the M^4 point of the normal space of point of Y^4 has the same Euclidean mass squared value as the point corresponds to so that Y^4 belongs to the 7-D light-cone of octonions with Minkowskian number-theoretic norm. This correspondence would define an analogue of Wick rotation and pose a constraint on the number theoretic holography. The boundary data given at these Euclidian mass shells S^3 and associativity (quaternionic normal space) would serve as a dynamical principle.

Consider first a simplified view of $M^8 - H$ duality as I understand it now, assuming that the M^4 projection of X^4 is 4-D.

1. The idea is that the points of $X^4 \subset H$ are mapped to the points of $Y^4 \subset M^8$. The M^4 projection of X^4 is mapped to the normal Minkowskian normal space of Euclidian surface E^4. The CP_2 projection associated with a given point of Y^4 parametrizes the normal spaces for it. The assumption that the normal spaces are quaternionic and contain a Minkowskian 2-D commutative space implies that they can be parametrized by CP_2. The normal spaces and 2-D subspaces form integrable distributions and determine Y^4 by number theoretic holography using the integrable distribution of Minkowskian normal spaces as holographic data located at 3-D surfaces. The tangent spaces of Y^4 have an Euclidian number theoretic metric. The points of Y^4 are in 1-1 correspondence with those of CP_2 except at the singularities at which the normal space is not unique.

2. The 4-surface $Y^4 \subset M^8$ is Euclidean. All points of Y^4 have as quaternionic normal space isomorphic to $M^4 \subset M^8$ containing M^2, which is complex and commuting sub-space of octonions. The normal space $N(y)$ of $y \in Y^4$ as M^4 is not unique but this has a geometric interpretation and does not lead to the problem of non-uniquencess of $M^4 \subset M^8$ as the previous view of $M^8 - H$ duality.

   The normal space contains the M^4 point associated with the point of CP_2 as a preferred point having an interpretation as 4-momentum. This point is "active" if there is a fermion at the corresponding point of X^4. One has a geometric representation of the many-fermion state in Y^4 mapped to the points of M^4 projection of X^4 by $M^8 - H$ duality.

3. The new form of $M^8 - H$ duality requires a lift of the point $y \subset Y^4$ to a point of its quaternionic and Minkowskian normal space $N(y)$, which is mapped by inversion to $M^4 \subset H$. Multiplication with an imaginary unit e of Y^4 would perform the lift but can one choose e uniquely?

   Is the uniqueness really necessary? The choice of the basis of quaternionic units for the quaternionic normal space is fixed up to local $U(2)$ rotation. The same is true for the basis of the complement. Could the interpretation be in terms of the M^8 counterpart of the electroweak gauge group? Color symmetries would correspond to the $SU(3) \subset G_2$ of octonionic automorphisms so that the standard model symmetries would be realized number-theoretically. The inverse of the lift would allow to map the points $M^4 \subset H$ to the points of Y^4 and realize the correspondence between points of M^4 and CP_2 for the surfaces representable as graphs of a map $M^4 \rightarrow CP_2$.

4. The spinor modes of fermion in CP_2, can be mapped to the spinor modes in Y^4 for each fermionic momentum. Number theoretical vision suggests that the momentum components are real algebraic integers of the extensions of rationals associated with the space-time surface in question.

This picture applies when the M^4 projection of X^4 is 4-D. If this is not the case the situation is more complex.

1. The ideal CP_2 extremal for which M^4 coordinates are constant, corresponds to a singularity, where the normal space at the point of M^8 is not unique: the normal open spaces span entire CP_2 one has what alebraic geometers call blow up and it occurs often for algebraic surfaces. The vertex of a cone is the basic example: in this case tangent and normals spaces are not unique.

   For deformed CP_2 extremals M^4 projection is a 1-D light-like geodesic line in $M^4 \subset H$ and also in M^8. Along this curve, the normal spaces form give rise to a 3-dimensional surfaces of CP_2.

2. Cosmic strings also correspond to such a singularity: in a 2-D string world sheet $X^2 \subset M^4$, the normal spaces at a given point form a 2-D complex manifold of CP_2.

3. At singularities at which the normal space of Y^4 is not unique, there are additional conditions on the allowed spinor modes since the spinor mode must have the same value for all normal spaces involved. The vanishing of the allowed spinor modes at these points would allow to satisfy these condition.

4. The number-theoretic quantization of M^4 momenta requires that the momentum components are real integers in the algebraic extenssion of the rationals related to the region of X^4 considered. The momentum unit is determined by the size scale of the causal diamond (CD).

What could be the physical interpretation of Y^4?

1. Y^4 can be sliced by the images of r= constant 3-spheres $S^3 \subset CP_2$. Could the time evolution in X^4 with respect to the light-cone proper time of $M^4 \subset H$ have as an analog the evolution of the CP_2 projection with respect to the radial coordinate r of CP_2 defining a slicing of Y^4 by $M^8 - H$ duality? Or can one speak of time evolution below the scale of causal diamond CD, which implies temporal non-locality below its scale.

2. Positive and negative energy states at the half-cones of CD would be mapped to the opposite light-cones in the Minkowskian normal space of Y^4. This brings in mind Wick rotation as the Euclidization trick used in quantum field theories. Could $M^8 - H$ duality define Y^4 as a kind of Euclidization of X^4? If so, one would have both M^4 and CP_2 perspective of the dynamics and also mixed perspectives (cosmic strings). The failure of M^4- and/or CP_2 projection to be 4-D would force mixed perspective.

3 Holography=holomorphy vision

In this section holography=holomorphy vision and its relation to the theory of analytic functions, to the number theoretic vision and to M^8-H duality will be discussed. It must be confessed that during the preparation of this section, the view of the realization of the holography=holomorphy vision simplified dramatically and a considerable portion of the text became obsolete.

3.1 About the geometric description of fermion pair creation vertex in the framework of catastrophe theory

It is good to start from a physical picture. The intuitive idea is that vertices somehow correspond to partonic 2-surfaces X^2 on one hand and to the points of X^2 at which fermion lines turn back in time. Is the fundamental vertex a 2-D parton surface or is it point-like or are both aspects involved? This is the basic question. One must clarify what happens topologically in the creation of a fermion pair. A non-vanishing monopole flux means homological non-triviality of the partonic 2-surfaces.

1. The catastrophe theory picture implies that a given catastrophe involves a hierarchy of singularities such that the number of co-inciding roots and criticality increases. In the case of cusp catastrophe, the tip of the cusp would correspond to the situation in which all 3 roots of the third order catastrophe polynomial coincide.

   There are good reasons to expect that also now this is the generic situation. This would suggest that a point-like vertex initiates a process leading to a creation of a pair of wormhole contacts with opposite monopole fluxes.

2. What happens that a single closed monopole flux tube splits two two monopole flux tubes? This flux tube has portions at parallel Minkowskian space-time sheets with distance of order CP_2 radius and they carry opposite monopole fluxes. Topologically this process corresponds to a reconnection when the flux tubes are idealized with strings. One must generalize this to that for 3-D objects.

   At the point of touch, the signature of the induced metric must be between Minkowskian and Euclidean signatures so that the induced 4 metric is degenerate. The touching point starts to grow to an Euclidean 3-D region and gives gradually rise to a wormhole contact with a vanishing total monopole flux between the throats. It has opposite wormhole throats with a vanishing total magnetic flux having a 2-D interface with Minkowskian regions. After this both wormhole throats are pinched to two parts with one common point, pinch, between them. Splitting occurs and one has two separate closed flux tubes having the wormhole contacts as their second "end".

3. This process could be also visualized in terms of the 2-D CP_2 projections X^2 and Y^2 of two parallel flux tube portions in M^4, which fuse, evolve to a flux tube portion in CP_2 and split again to 2 wormhole contacts. X^2 and Y^2 touch and fuse to form an intermediate wormhole contact, which evolves to two pieces with opposite monopole fluxes, which are side by side and in CP_2. The two flux tubes separate and develop a pinch, which then splits.

4. The fermion pair would be naturally created at the pinch formed in the final step. For this pinched partonic 2-surface, a global generalized complex structure is not possible since the generalized complex structures at the two parts differ by the conjugation of the hyper hypercomplex coordinate (note that also the conjugation of also CP_2 coordinates might be involved. The holomorphy fails at the pinch at least. The other option is that one just does not have global smooth generalized complex coordinates. The situation would be similar also before the pinch and the holomorphy would fail at the 3-D pieces of the orbits of the throats of the intermediate worm contact.

5. Here one must ask what one means as one speaks of singularity at which the minimal surface property fails. The evolution associated with the splitting of the monopole flux tube involves a development of the opposite light-like wormhole throats as a boundary of a growing 2-D Euclidean region. This evolution defines a 3-dimensional light-like region. Does the trace of

the second fundamental form have a 3-D delta function singularity?: this is in conflict with the guess that the singularities are 2-D [L20]. The final state just before the splitting is a pair of touching wormhole throats. Does this define the 2-D delta function singularity or does the touching point correspond to a point-like delta function singularity analogous to the tip of the cusp catastrophe?

It is good to make a more explicit comparison with the catastrophe theory of Rene Thom [A3].

1. One considers gradient dynamics $dx^i/dt = \partial_i V$ and one considers possible equilibria as extrema of a potential function $V(\{x^i\}, \{a_j\})$. A deep result is that, although the number of the behaviour variables x^i can be large, it is possible to choose a single relevant variable, call it x. Furthermore, in suitable coordinates the potential function can be chosen to be a polynomial. A possible physical identification for the control variables is as 4 space-time coordinates or a subset of them.

2. If the number of control variables a_j is not larger than 4 there are 7 elementary catastrophes and the catastrophes for m behavior variables can be engineered from the catastrophes for $m-1$ behavior variables.

3. The number of sheets of the catastrophe manifold in the space spanned by the behavior variable x and control variables a_i corresponds to the number of roots to the condition $dV/dx = 0$. The catastrophe manifold is a k-dimensional surface for k control variables.

   Different roots correspond to sheets of the catastrophe manifold and meet at lower-dimensional surfaces at which the roots are degenerate. This means criticality in which a dropping to another stable sheet driven by the rapid dynamics can occur. The critical manifolds form a hierarchy and at the tip of the many-sheeted catastrophe region all roots are identical.

3.2 The tension between the holography=holomorphy vision and number-theoretic vision

The relationship between holography=holomorphy vision and number theory vision involving the hierarchy of effective Planck constants and M^8-H duality should be understood better. The updated view M^8-H duality [L16] modifies radically the original vision [L4, L5, L14] based on polynomials $P(o)$ of complexified octonions with rational (or equivalently integer -) coefficients obtained as a continuation of ordinary complex polynomials $P(z)$ of variable $z = x + iy$ with imaginary unit i commuting with the octonionic imaginary units. The earlier vision was consistent with several general ideas such as prediction of the hierarchy of Planck constants and the identification of p-adic primes as ramified primes for these kinds of the polynomials P. It also supported the notion of Galois confinement. These parts of the earlier vision should survive.

The hope is that holography=holomorphy vision allows to get naturally the roots of polynomials and corresponding algebraic extensions and even polynomials with rational or integer coefficients. This would also give ramified primes.

One needs a guiding principle and number theoretical quantum criticality is such a principle. It states that rationals and algebraic numbers correspond to islands in the ocean of complex continuum unstable under perturbations selected by quantum criticality which is the basic principle of TGD. This already implies holography=holomorphy principle but does not fix its details completely. p-Adic primes would characterize elementary particles rather than space-time regions. This suggests that the number theoretic quantum criticality is reduced to single fermion level and allows to identify light-like fermion lines at the light-like orbits of partonic 2-surfaces and assign the ramified primes and $h_{eff} = nh_0$ to them.

3.2.1 Criticism of the $(P_1, P_2) = (0, 0)$ option

There are many tensions to be resolved. The holography=holomorphy vision forces us to reconsider both the earlier view of M^8-H duality and the number theoretical vision. The existing number theoretical vision in turn challenges the detailed realization of the holography=holomorphy vision based on $(P_1, P_2) = (0, 0)$ option.

1. In the earlier version of M^8-H duality [L4, L5, L14] a single polynomial P of single complex variable z with coefficients in the field of rationals (or its extension), continued to a polynomial in a complexification of octonions, defined the holographic data in turn defining the space-time surface.

   Although this approach had shortcomings it also had very nice features. The dimension of the algebraic extension determined by the roots of P defined effective Planck constant and the spectrum of ramified primes of P as factors of its discriminant had interpretation as p-adic primes. The applications of TGD rely on these notions.

2. For the most obvious realization of holography=holomorphy vision a pair (P_1,P_2) of polynomials replaces a single polynomial $P(o)$. Is it possible to reduce the conditions $(P_1,P_2)=(0,0)$ to a single condition $P(z)=0$ for some choice of P and z? In the recent case z would correspond to a complex coordinate at the light-like partonic 2-surface as a slice of the partonic orbit and there are very many choices.

   Putting the lightlike-coordinate u to zero (restriction of fermion line at a light-like partonic orbit) one has polynomials P_i of w, ξ^1, and ξ^2 and one can choose any w, ξ^1, or ξ^2 as dependent variable z. The degree of P_i as a polynomial of z depends on the choice of z. One can find the common 6-D roots of P_i for each choice and they correspond to the intersection of 4-D surfaces $P_1=0$ and $P_2=0$. If the argument w, ξ^1, or ξ^2 is an algebraic number, the roots are algebraic numbers. This leaves a lot of freedom and it is very difficult to figure out the general picture!

   Therefore it is far from clear how to identify fermionic lines represented as points of X^2 such that they are roots of a polynomial with coefficients in some extension of rationals. However, if can identify a unique extension E of rationals, and a unique polynomial $P(z)$ of a highly unique variable z independent of the variables w, ξ^1, and ξ^2, its ramified primes would determine the spectrum of p-adic length scales and $h_{eff}=nh_0$ would corresponds the degree of its Galois group.

3.2.2 Does $P=0$ option for holomorphy = holography option solve the problems?

How could one solve the problems of the $P_1,P_2)$ option?

1. The quantum criticality of TGD suggests that there is a catastrophe theoretic hierarchy of criticalities corresponding to the surfaces $P(z)=0$ giving the space-time surface and as special case partonic 2-surface X^2 as $v=0$ constant section of the partonic orbit X^3. Criticality corresponds to the coincidence of two roots so that one would have $P(z)=0$ and $dP(dz)=0$ at criticality. The roots would give the intersections of the fermionic lines with the partonic 2-surface. The roots of P would define an extension of the coefficient field F of P as an extension of rationals and the ramified primes of P belonging to F.

2. The twistor lift suggests a natural identification of the coordinate z. Twistor lift replaces the space-time surface X^4 with a twistor space X^6 as a S^2 bundle over X^4. X^6 would be determined by the 6-D Kähler action. z could be identified as a complex coordinate of S^2 determined up to holomorphy. Suppose that X^6 is known. A natural identification of X^4 is a section of twistor bundle X^6 can be identifiable as a root of a polynomial $P_{u,t}(z)$, where t can be take to be one of the coordinates (w,ξ^1,ξ^2) (note that $u=constant$ at X^3. The conditions $P=0$ and $dP/dz=0$ at fermionic lines would fix the value of z and if t belongs to F, one obtains an algebraic extension of F.

   The fermion line would be identified sufficiently uniquely if the choice P defining the section is sufficiently unique. In fact, different sections could define different physics. The optimistic expectation is that there is a finite number of sections or at least a finite-dimensional moduli space of sections for a given for a given twistor-surface X^6.

There are 3 obvious choices for the coordinate t corresponding to the set $\{t_1,t_2,t_3\}\equiv\{w,\xi^1,\xi^2\}$. Can one identify the complex coordinate t uniquely or does one obtain 3 kinds of roots also now and what could this mean?

1. If the partonic 2-surface is regarded as a Riemann surface, the natural local coordinate is t_k, and the polynomials $P_k(z,u,t_k)$ are uniquely determined. The choice of t_k is determined apart from holomorphic bijection and Hamilton-Jacobi structure [L21] dictates the choice of w a high degree and in CP_2 Eguchi-Hanson coordinates, favoured by their group theoretical properties, are natural.

2. If all 3 choices of t_k are possible, one obtains 3 kinds of roots. If the roots $z_{k_1,i}$ and $z_{k_2,j}$ coincide, they can correspond to the same point of X^4 but need not do so.

   The partonic 2-surface is many-sheeted with respect to both CP_2 and M^4. Different w-roots would correspond to the multi-sheetedness with respect to CP_2 and different sheets could be assigned with parallel monopole flux tubes going through the w-plane. The physical intuition suggests that, due to the small size of CP_2, the number of roots in CP_2 direction is small CP_2 for a given M^4 point whereas in the direction of M^4 the number of w-roots can be very large giving rise to a large value of $h_{eff}/h_0 = n$.

3. In the case of the standard twistor bundle over M^4, S^2 represents the directions of light-like geodesics emanating from a point of M^4. The twistor fibers S^2 at different M^4 points have a common point if there is a light-like geodesic connecting them. This is expected to be a reasonable guess also now.

   Could the w-roots correspond to intersecting twistor spheres for which the points with a different w coordinate are connected by a light-like geodesic of M^4? Since the light-like coordinate u is constant and v is fixed, this is not plausible. This would suggest that the complex coordinate for the S^2 as the cross section of the light cone boundary defines a preferred coordinate identifiable as the coordinate of the twistor sphere.

4. The same question can be posed in the case of CP_2 for which the light-like geodesics are replaced with radial geodesics from the origin of Eguchi-Hanson coordinates directed to the homologically non-trivial geodesic sphere at $r = \infty$. The points pairs at these geodesics would have intersecting twistor spheres. Could the CP_2 points associated with w-roots in the radial direction be located along these radial geodesics of CP_2 so that there could be a large number of w-roots per CP_2 type roots? The complex coordinate of the homologically non-trivial sphere of CP_2 could serve as the analog of the coordinate of the twistor sphere.

One can argue that there is a problem with number theoretic general coordinate invariance (GCI) since the form of the P can change in a generalized holomorphism of H expected to have no physical effect. Is there a unique choice of coordinates allowing to avoid the problem?

1. For the (P_1, P_2) option, the X^4 is identified as an intersection $X^6_1 \cap X^6_2$ of 6-surfaces X^6_i as roots of P_i. Could X^6_i be identified as twistor surfaces as counterparts of the twistor spaces $T(M^4)$ and $T(CP_2)$ with different twistor spheres but the same base space?

   If so,the complex coordinates of the twistor spheres of $T(M^4)$ and $T(CP_2)$ should correspond to the complex coordinates of the twistor sphere of light-like geodesics of the light-cone boundary and of radial geodesics of CP_2 directed from origin to homologically nontrivial sphere of CP_2 "at infinity".

2. The construction of X^6 as an extremal of the 6-D Kähler action for $X^6 \subset T(M^4) \times T(CP_2)$ [K5] identifies the twistor spheres of $T(M^4)$ and $T(CP_2)$. Does this mean X^6_i as twistor bundles are related by the mapping of the space of light-like geodesics of light-cone boundary and/or light-like partonit orbit to the space of radial geodesics of CP_2?

3. This would make the situation highly unique. Holomorphies for a given choice of w *resp.* ξ^1 or ξ^2 would correspond to $SO(3)$ and $U(2)$ acting linearly on the complex coordinate. These groups reduce to $SO(2)$ and $U(1) \times U(1)$ by the choice of the quantization axes. The coordinate w would reduce to the complex coordinate of the twistor sphere of $T(M^4)$ at the light-cone boundary (at least). At partonic orbits the complex coordinate ξ of the geodesic sphere of CP_2 and w would be related by a map characterized by a winding number.

Some comments on the physical interpretation are in order.

1. p-Adic primes are rather large, $M_{127} = 2^{127}-1$ for electrons. I have proposed that one could pose constraints on the size of the polynomial coefficients, say that they are smaller than the degree of the polynomial. In this case it is not clear how to obtain such large ramified primes unless the degree of P is very large. The degree of polynomial increases exponentially in repeated iteration giving rise to an analog for the approach to chaos [L6]. This would increase the dimension of the extension.

2. As found, the polynomial P can be identified as a polynomial of Minkowski-coordinate w, or of CP_2 coordinate ξ^1 or ξ^2. CP_2 is rather small and one expects that in CP_2 directions the number of sheets is rather small so that P as a polynomial of ξ^1 or ξ^2 should have a rather small degree and corresponding h_{eff}/h_0 should be rather small.

   In M^4 there is a lot of room and the degree of P as a polynomial of w can be rather large and therefore also the value of h_{eff}/h_0 for these fermion lines is large and their number can be large. Therefore the corresponding ramified primes and associated p-adic length scales can be rather large in this case. It would seem that the p-adic length scale is naturally assignable to $P(t,u,w)$. If p-adic length scale is assignable to $P(t,\xi^i)$, it should be smaller than CP_2 scale and could correspond to excitations of superconformal and supersymplectic algebras with mass scales which are higher than CP_2 mass scale.

$P=0$ option reduces the pair of polynomials (P_1,P_2) to single polynomial P, allows to interpret the space-time surface X^4 as a section of its twistor space X^6 determined by 6-D Kähler action and to identify fermion lines as surfaces $(P=0, dP/dz=0)$. This view implies the notions of effective Planck constant and ramified primes, and allows to understand number theoretical evolution as the increase of algebraic complexity in two ways: as a collective evolution of the extensions of rationals F appearing as the coefficient field of P and as the evolution at the single particle level for the polynomial P. If the polynomial P can be iterated, a connection with chaos theory [L6] emerges. Also a complexification of catastrophe theory emerges and the space-time surface is analogous to a complexification of the cusp catastrophe.

3.2.3 $(P_1,P_2)=(0,0)$ option and $P=0$ option need not be inconsistent with each other

$P=0$ option starts from the twistor surface X^6 as a known entity and determines X^4 as its section whereas the $(P_1,P_2)=(0,0)$ option represents X^6 as a solution of field equations. Therefore these options need *not* be inconsistent with each other.

1. One can start from a problem. One can argue that the $(P_1,P_2)=(0,0)$ option cannot represent the twistor surface X^6. However, the condition $P_1=0$ or $P_2=0$ defines a 6-D surface X^6_i as a solution. Could these 2 6-surfaces have interpretation as concrete representations for the H projections of two twistor surfaces $X^6_i \in T(M^4)\times T(CP_2)$ having a common base space X^4?

2. How could X^6_i correspond to S^2 bundle over X^4? Could the two disjoint twistor spheres S^2 be counterparts for the twistor spheres of $T(M^4)$ and $T(CP_2)$ having as base-spaces M^4 and CP_2. Could one say that these base spaces are replaced with the projections of X^4 to M^4 and CP_2 to get X^6. Geometrically these two spheres would correspond to the space of light-like rays emanating and to the space of CP_2 radial geodesics of CP_2 emanating from the point $(m,s)\in X^4 \supset H$ and would now be concretely represented as "heavenly spheres". These spheres need not be metric spheres. Note however that the light-cone boundary as the space of light-like geodesics is metrically a 2-sphere.

   In ZEO the non-trivial homology of M^4 twistor sphere could be represented by the effectively missing tips of $cd \subset M^4$, by the axis connecting the tips of the cd, or by the light-like boundaries of cd indeed representing the space of light-rays analogs to the moment of Big Bang and Big Crunch.

3.3 Catastrophe theoretic vision of TGD

The analog catastrophe theoretic vision emerges in the TGD framework. One can consider this vision at the level of the space-time surface and at the level of H and they correspond to the representations of the spacetime surface as a root of two polynomials P_1, P_2 and as a root of single polynomial P.

For the $(P_1, P_2) = (0, 0)$ option, which is mathematically correct but for which the relationship to the basic number theoretical ideas is unclear, one has the following picture.

1. The vanishing of the gradient of V is replaced by the vanishing two generalized complex functions f_1 and f_2, which can be also taken as polynomials P_i with coefficients, which are rational or in an extension F of rationals. The polynomial coefficients of P_i depend on the 4 generalized complex coordinates of H.

2. General coordinate invariance however reduces their number to 2 when one chooses as space-time coordinates a subset of 2 generalized complex coordinates (z^1, z^2) (hypercomplex and complex coordinate in Minkowskian regions and 2 complex coordinates in Euclidean regions). Space-time coordinates become the analogs of complex control variables and the remaining 2 generalized complex H coordinates (w^1, w^2) take the role of complex behavior variables. Instead of $dV/dx = 0$, one has $P_1 = 0$ and $P_2 = 0$.

3. Space-time surface as the counterpart of catastrophe manifold has two generalized complex dimensions. An interesting question is whether it makes sense to regard (P_1, P_2) as a gradient $(partialV/\partial w^1, \partial V/\partial w^2)$ of a complex value potential function with respect to the complex behavior variables. If this is possible, one could have a complexification of the catastrophe theory and the catastrophe could correspond to a complexified cusp with two generalized complex control variables instead of 2 real control variables.

The second option, introduced in this article, is based on a single polynomial with coefficients, which are rational or in an extension F of rationals and elegantly reproduces the earlier basic ideas related to the number theoretic vision.

The catastrophe manifold is replaced with a space-time surface X^4. The condition $dV/dx = 0$ is replaced with the condition $P_{u,t}(z) = 0$ for generalized complex polynomial P, which has as the behavior variable z identified as the complex coordinate of the twistor sphere and the light-like hyper-complex coordinate u and complex variable t are control variables. There are 3 basic ways to choose t as $t \in \{w, \xi^1, \xi^2\}$. P can be interpreted as a complex gradient dV/dz of $V = \int Pdz$ with respect to the behavior variable z so that one has $dV_{u,t}(z)/dz = 0$. The conditions $(P, dP/dz) = 0$ are true for the fermion lines which represent criticality at which two sheets of the catastrophe graph coincide. Fermion lines are analogous to the folds of the cusp catastrophe. Space-time surface as the counterpart of a catastrophe manifold has 2 generalized complex dimensions as also the cusp catastrophe. An interesting question is how unique the function $P(z)$ defining the space-time surface as a section of the twistor bundle X^6 is.

It is interesting to look for the catastrophe theoretic interpretation of the already discussed model for what would happen in the splitting of a closed monopole flux tube to two monopole fluxes leading to a creation of a fermion pair.

1. In the ordinary catastrophe theory, cusp catastrophe might be enough to describe what happens. The abstract cusp catastrophe corresponds to a 2-D surface in 3-space spanned by a behavior variable and two control variables. The upper and lower sheets are stable whereas the intermediate sheet between them is unstable.

2. The stable sheets would correspond to the initial state with a single monopole flux tube and to the final state with two monopole flux tubes. The intermediate wormhole contact would correspond to the unstable sheet of the cusp. The touching of the flux tubes would lead from the initial stable sheet to the unstable sheet. The intermediate wormhole contact would then move along the unstable sheet and end up to a critical point (formation of pinch) and end up to the second stable sheet.

3. In this description, the state of the entire system corresponds to a point of the 3-D space spanned by the 2 control parameters and 1 behavior variable. Therefore it is not possible to identify the control variables of cusp as a subset of space-time coordinates and the behavior variable as an H-coordinate orthogonal to the space-time surface. Rather, the evolution of the system between the initial and final states should correspond to a part of a 4-D catastrophe manifold.

3.4 Holography=holomorphy vision as a realization of quantum criticality and the theory of analytic functions

A generalized complex structure can be seen as a realization of the quantum criticality of the TGD Universe just like 2-D conformal theories can be seen as a realization of criticality. The generalized complex structure of the space-time surface, or Hamilton-Jacobi structure as I call it, combines hypercomplex and complex structures into a 4-D structure [L21].

Hamilton-Jacobi structure involves an integral distribution of the local tangent space-decompositions $M^2(x) \times E^2(x)$ allowing to assign a pair (u, v) of coordinates with light-like coordinate curves to the distribution of $M^2(x)$ and complex coordinates $w, \overline{w}$ to the distribution transversal spaces $E^2(x)$. This structure generalizes to H by introducing complex coordinates (ξ^1, ξ^2) for CP_2.

There are two options, which can be consistent. In the (P_1, P_2) option determining space-time as 4-surface, the spacetime surfaces are identified as roots of two generalized analytic functions $f_1(u, w, \xi^1, \xi^2)$ and $f_2(u, w, \xi^1, \xi^2)$ (polynomials in the special case) defined in H and are minimal surfaces apart from possible 2-D singularities. This option could also allow the identification of the analogs of twistor spaces of M^4 and CP_2 as 6-D roots of P_1 or P_2 having X^4 as intersection and common base space. In the $P = 0$ option space-time surface X^4 is identified as a base space of the twistor-surface X^6 in $T(M^4) \times T(CP_2)$. X^6 could be determined by the (P_1, P_2) option.

This leads to the question, whether the theory of analytic functions could generalize from dimension 2 to dimension 4. I am not a mathematician in a technical sense and know almost nothing about the existing mathematical knowledge. However, I can make guesses by using a physicist's intuition.

1. In the theory of analytic functions, one basic question is whether an analytic function can be constructed from its singularities. This is essentially holography.

   Electrostatics in dimension $D = 2$ serves as an instructive physical example. The electric field can be fixed when the point charges and line charges corresponding to the poles and cuts for the analytic function are known. Only poles are obtained for rational functions. Cuts (such as $z^{1/n}$ and $log(z)$) and essential singularities ($exp(1/z)$) are also possible. Does this picture generalize to the 4-D case?

2. The twistor Grassmannian description of the scattering amplitudes leads to the hypothesis that the amplitudes are coded by the lower-dimensional singularities of the amplitudes. Also this situation corresponds to holography in a very general sense.

3. In TGD, the construction of scattering amplitudes with the help of vertices and the associated light-like parton surfaces leads to the same kind of situation.

These observations motivate the question whether and how the theory of analytic functions in dimension 2 could generalize to dimension 4 in the TGD framework.

1. Does, for example, the representation of an analytic function in terms of its singularities generalize? Suppose that 3-D light-like parton surfaces and associated 2-D partonic 2-surfaces identified as vertices, where the generalized holomorphy breaks down, are known? Can the space-time surface be derived as a generalized holomorphic 4-surface in H, determined by two analytic functions f_1 and f_2 (in the generalized sense) as their roots. Could the partonic 2-surfaces be enough? Or are also string world sheets perhaps identifiable as the counterpart of cuts needed as data. What about the boundary values of the function f_i at the light-like boundaries of causal diamond CD inside which the space-time surfaces are located as analogs of Bohr orbits.

2. Hilbert's principal value theorem (see this) gives an analytic function determined in the complex plane as a sum of contributions corresponding to poles and cuts. Charges and line charges determine the electric field. In poles, the holographic information is determined by residued, and in cuts by the discontinuity of the real-analytical function across the cut.

What can one say about the singularities? Does the singularity correspond to an entire light-like partonic orbit, to a 2-D partonic surface as a section of partonic orbit, to mere light-like fermion line along it, to string world sheet, or to a point-like particle reaction vertex as a defect of a smooth structure as has been assumed hitherto?

1. The number theoretical considerations related to holography=holomorphy vision realized using $P = 0$ option imply that the light-like fermion lines along a light-like partonic orbit are theoretically very special and analogous to the lines of criticality for cusp catastrophe.

   The fermion line is a static structure and one cannot assign to it acceleration in M^4 degrees of freedom as the trace of the second fundamental form defining a generalized acceleration. What could diverge is the CP_2 part of the generalized acceleration having an interpretation as an analog of the Higgs field.

   The light-like coordinate along the light-like fermion line is constant so that metrically it is analogous to a pole of an analytic function. On the other hand, topologically the fermion line is analogous to a cut and it would form part of a boundary of a string world sheet having interpretation as a generalization of cut. The analogy of the fermion line as the critical line of the catastrophe graph of cusp catastrophe suggests that the trace of the second fundamental form in CP_2 degrees of freedom indeed diverges at it. There would be a fold in the CP_2 direction. A possible interpretation is that the fermion line serves as a source of various gauge fields.

2. There is also another kind of singularity: in a creation of a fermion pair in the splitting of closed monopole flux tube, the point at which the fermion and antifermion lines begin would correspond to singularity as an analog for an edge at which fermion turns backwards in time. This singularity is analogous to the vertex of a cusp catastrophe at which 2 folds meet.

   This kind of turning point is analogous to an edge on the path of a Brownian particle. A monopole flux tube would decay to two so that the topological viewpoint of a particle emission would be in question. Also at this singularity one expects the holomorphy and minimal surface property to fail. In fact this could be true inside the entire intermediate wormhole contact formed in the process. The vision about the splitting of monopole flux tubes [L20] leads to the identification of interaction vertices as genuinely point- like singularities at which standard smooth structure has a defect so that an exotic smooth structure is obtained.

3. String world sheets are assumed to connect the 3-D partonic orbits of different monopole flux tubes. The interpretation of string as an analog of a cut is suggestive. For the cuts of the analytic function $z^{1/n}$, the real axis is a seat of discontinuity. By introducing the notion of n-fold covering space of the complex plane one gets rid of the discontinuity. Could the string world sheets correspond to the orbits of stringy singularities such that 2π rotation around the singularities in the generalized complex coordinates of H lead to a point at a different space-time sheet and one obtains n-fold covering. The analogy with line charges suggests that it makes sense to assign dynamics to these objects.

   n-fold covering suggests an n-fold value of the effective Planck constant and the interpretation as a dark phase as n-fold covering of M^4 or subspace M^2. Also n-fold coverings of CP_2 or its geodesic sphere S^2 are possible and would look like copies of M^4 regions inside which the map from CP_2 to M^4 is 1-valued. Now the sheets of the covering would correspond to different copies, for instance flux tubes.

Consider now what happens in singularities from the perspective of field equations.

1. At singularities defined as loci where the minimal surface property fails, the field equations for the *entire* action are valid, but are not separately true for various parts of the action. Generalized holomorphy breaks down. The fermion lines as singularities are completely analogous to the poles of an analytic function in 2-D case and there is analogy with the 2-D electrostatics, where the poles of analytic function correspond to point charges. The fermion lines are boundaries of string world sheets and these are analogous to cuts as line charges. The cut disappears as a singularity when the complex plane is replaced with its covering and the same occurs at the space-time level.

2. The field equations at the singularity give the TGD counterparts of Einstein's equations, analogs of geodesic equations, and also the analogy Newton's F=ma (also in CP_2 degrees of freedom). The generalized 8-D acceleration H^k defined by the trace of the second fundamental form, is localized on the singularities. Singularities can be seen as analogs for the sources of gauge fields. This interpretation could apply to fermion lines analogous to line charges, which correspond to cuts of an analytic function in 2-D electrostatics. At these singularities the CP_2 part of the generalized acceleration H^k could diverge.

   Singularities in a stronger sense can be interpreted as sources for various fermion currents and supercurrents implying their apparent non-conservation at the singularity. This kind of singularity could correspond to a vertex at which a fermion-antifermion pair is created and fermion number conservation is apparently violated since the fermion line ends. Actually only the separate conservation of fermion and antifermion numbers is broken. Here also the M^4 part of H^k would diverge.

One can pose several questions.

1. What are the counterparts of the residues, i.e. holographic data? Does the holographic data correspond to the entire 3-D parton trajectory, on only to the 2-D vertices as partonic surfaces and to the wormhole throats defining the interfaces of Minkowskian and Euclidean space-time sheets, or perhaps only light-like fermionic lines at the light-like orbits of the partonic 2-surfaces?

   Under what conditions are generalized analytic functions $f_1(u, w, \xi^1, \xi^2)$ and $f_2(u, w, \xi^1, \xi^2)$ are determined by this holographic data? It should be possible to determine the Taylor or Laurent coefficients of f_i from the data. Can one imagine an explicit formula? Under what conditions the functions reduce to polynomials with rational (equivalently integer -) coefficients?

   Generalized holomorphy could be seen as a realization of quantum criticality. Physical intuition also suggests that quantum criticality selects extensions of rationals as islands in the sea of real numbers and also makes possible number theoretical physics as adelization of TGD making possible cognitive representations as unique number-theoretical discretizations [L2, L3].

2. Should also the 2-D string world sheets connecting wormhole throats of different monopole flux tubes be included as the physical intuition suggests (these string like entities should not be confused with 4-D cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ and the monopole flux tubes as their thickenings).

   Are the genuinely 2-D string world sheets analogous to hypercomplex cuts? At them, the hypercomplex analytic function, which depends on the coordinate of a light-like curve of M^4, would be discontinuous. The 4-surface would be either discontinuous along the stringy curve or space-time surface could correspond to a covering of a region of M^4 so that a rotation of 2π around the stringy curve leads to another sheet and rotation of $n2\pi$ rotation is needed in to return to the original point. I have assigned a fractional spin to this kind of situation. In CP_2 degrees of freedom the same phenomenon is possible and fractional charges could be assigned with them.

3. What about the 3-surfaces at the light-like boundaries of the CD defining the "ends" of X^4. Should these 3-surfaces be given as data or could they be fixed from holographic data provided by the fermion lines and string world sheets?

4. Is it possible to have a general formula for the functions f_1 and f_2 (or for P) determining the space-time surface X^4 analogous to the formula of Hilbert in terms of the holographic data? This formula would be a classical analog for the determination of the scattering amplitudes from their lower-dimensional momentum space singularities in the twistor Grassmannian approach.

4 Beltrami flows, intebrable flows and holography = holomorphy hypothesis

Beltrami flows [?, ?, ?, ?, ?] (see this) appear in several contexts. Google AI informs that Beltrami flow is a force-free flow field at 3-sphere. The simplest Hopf fibration (see this) is from 3-sphere to 2-sphere and fibers correspond to circles and there are numerous generations of Hopf fibration: the fibration $S^5 \to CP_2$ is of special interest in TGD.

4.1 Some background

Some background about Beltrami flows is in order.

1. For the Beltrami flow (see this) velocity field satisfies $curl(v) = \Lambda v$ so that $curl(v)$ is parallel to v. In fluid dynamics Beltrami flow corresponds to a flow for which vorticity $\omega = \nabla \times v$ and velocity v are parallel. $\omega \times v = 0$ gives $\omega = \nabla \times v = \alpha(x,t)v$. Beltrami flows in S^3 satisfy this condition and are exact solutions to Euler equations.

2. In magnetohydrodynamics one can replace velocity field with magnetic field B and of the current j satisfies $j = \nabla \times B = \alpha B$ implying the vanishing of the Lorentz force $j \times B$. The current flows along field lines and in TGD the flow of particles along monopole flux tubes is the counterpart for this flow. These Beltrami flows involve the linking and knotting of magnetic field lines. Similar situation prevails in hydrodynamics.

Jenny Lorraine Nielsen has proposed that the Hopf fibration $S^1 \to S^9 \to CP_4$ could provide a theory of everything (see this) and that Beltrami flows (see this) associated with this kind of fibrations play a key role in physics. The scalar Λ, which depends on position, appearing in the definition of Beltrami flow has dimensions of $1/length$. Mass has dimension of $\hbar/length$ so that $1/\lambda$ should be identified as an analog of Compton length. These flows are topologically very interesting and involve linking and knotting of the flow lines.

The claim of Jenny Nielsen is that it is possible to understand particle massivation in terms of Beltrami flows. Higgs expectation defining the mass spectrum in the standard model is identified as $hbar\Lambda$ for the eigenvalue Λ of the lowest eigenmode of Beltrami flow. It would seem that Λ is assumed to be constant: this is not necessary. It must be possible to relate Λ to the radius of S^3 and one chooses it suitably to get Higgs vacuum expectation. To get masses of fermions one must put them in by hand as couplings of fermions to Higgs so that one does not really predict fermion masses: to my best understanding, the situation remains the same as in the standard model. TGD leads to a predictive model for the masses of elementary fermion [K4, K3] [L11] allowing also to predict hadron masses [L23].

4.2 Motivations for considering Beltrami flows and integrable flows from the TGD point of view

The field equations of TGD reduce to conservation laws for isometry charges so that TGD is analogous to hydrodynamics. Beltrami flows, generalized to 4-D situation, are indeed a basic aspect of TGD [K2, K1] and it is interesting to try to relate them to the new vision of TGD, in particular the holography = holomorphy principle [L17, L22, L27, L21, L18].

Only irrotational Beltrami flows are possible in the plane. The simplest integrable planar flows, having interpretation as irrotational and incompressible Beltrami flows, reduce to gradient flows except at singularities.

These flows do not correspond to the flows defined by complex analytic maps or symplectic maps of plane, whose generalizations to higher dimensions play a key role in TGD. These flows, definable for the generalized complex structure of $H = M^4 \times CP_2$ and $X^4 \subset H$ accompanied by Kähler structure and symplectic structure, allow global coordinates along their time and have a natural interpretation as hydrodynamical flows in the induced Kähler field.

The complex flows give rise to maps, which have as singularities are 2-D partonic surfaces and string world sheets. In the view of scattering amplitudes, vertices correspond to partonic 2-surfaces at which the lines of the analogs of Feynman diagrams meet each other [L28] whereas string world sheets as intersections of two 4-surfaces with common Hamilton-Jacobi structure [L21] characterize interactions as contact interactions.

4.3 Beltrami flows and integrable flows in TGD

The generalization of Beltrami flows to 4-D context is one of the key ideas of TGD [K2, K1] but I have not discussed them explicitly in the recent framework based on holography = holomorphy vision (H-H)) [L17, L22].

1. The motivation is that TGD is formally hydrodynamics in the sense that field equations express local conservation of isometry charges of $M^4 \times CP_2$. There is actually infinite-dimensional algebra of conserved charges. The proposal is that in TGD, the Beltrami flows generalize genuinely 4-dimensional flows and correspond to classical field configuration for which the 4-D Lorentz force involving electric components vanishes.

2. The definition of the Beltrami flow is however different since one cannot regard the magnetic field as a vector field in 4 dimensions. For field equations Kähler current typically vanishes but can be also light-like. The counterpart of Beltrami flow states that Kähler current is proportional to the corresponding axial current:

$$j^\mu = D_\nu J^{\mu\nu} = \alpha \times \epsilon^{\mu\nu\alpha\beta} A_\nu J_{\alpha\beta}$$

.

The divergence of j^μ vanishes and this must be true also for the instanton current.

This is the case if the CP_2 projection of the space-time surface is at most 3-dimensional. If it is 4-D the parameter α must vanish since the divergence of the axial current gives instanton density $\epsilon^{\mu\nu\alpha\beta} J_{\mu\nu} J_{\alpha\beta}$, which is non-vanishing for CP_2 and by self-duality proportional to $J^{\mu\nu} J_{\mu\nu}$. Hence the only option is $\alpha = 0$ for $D = 4$.

3. If these Beltrami flows are integrable, they can give a physical realization of some, perhaps all, space-time coordinates as coordinated varying along the flow lines of some isometry current. The time component of 4-force has interpretation as dissipation power and also vanishes. These non-dissipative configurations play a key role in TGD and are natural when space-time surfaces are identified as quantum coherence regions.

4. The key idea sharpening dramatically the notion of Beltrami flow supported by H-H vision is that complex analytic maps $f : z \to f(z)$ allow us to construct integrable flows. What matters physically would be singularities: poles and zeros. Without them these maps would be just general coordinate transformations.

In TGD, this generalizes to 4 dimensions by the introduction of generalized complex structure in $H = M^4 \times CP_2$. The presence of hypercomplex coordinates in M^4 motivates the term "generalized". In 4-D context, poles and zeros as singularities of a flow correspond to string world sheets and partonic 2-surfaces. The second key idea is that fermions at the flow lines serve as markers and provide information about the flow. In the cognitive sector they realize Boolean logic.

Complex structure is often accompanied by Kähler structure. Its generalization to the Hamilton-Jacobi structure [L21] of H and M^4 involves hypercomplex structure. Kähler structure involves symplectic structure and the symplectic symmetries of H induce isometries of the "world of classical worlds" (WCW) [L15] as also the generalized holomorphic transformations of H.

Symplectic *resp.* holomorphic transformations preserve areas *resp.* angles, which in 2-D case are canonically conjugate variables so that these transformations should be very closely related. Symplectic flows are not gradient flows but one can assign to their flow lines a global coordinate the Hamilton canonically conjugate to the Hamilton of the flow. Also complex analytic flows allow this.

4.3.1 Flows in the complex plane

Flows in the plane are not usually regarded as interesting Beltrami flows since in this case the condition $\nabla \times v = \alpha v$ cannot be satisfied for integrable flows as gradient flows unless the vorticity and a position dependent eigenvalue α vanish. There are however other ways to satisfy the integrability. Complex analytic maps define define integrable flows in more general sense.

One can start from flows in plane, in particular integrable flows.

1. Integrability means that the flow lines of the flow give rise to globally defined coordinate lines which fill the space smoothly. Intuition suggests that without integrability and the existence of a global coordinate along flow lines, the flow would be more like a random motion analogous to the motion of gas particles. Integrability would bring in smoothness and the flow looks like a fluid flow.

2. Integrability in strongest form requires that the velocity v for the flow line is a gradient $v = grad(\phi)$ of the global coordinate in question. This implies $\nabla \times v = 0$ and $\alpha = 0$. This condition is very strong and implies irrotationality so that a rotational flow is only possible in a global sense. There are however milder ways to guarantee the integrability.

3. Note that exotic smooth structures [A5, A6, A4] possible in TGD [L19, L9, L26] would correspond to flows for which smoothness fails at singularities to make possible fermionic interactions, although fermions are free in TGD. But this is possible only for 4-D space-time.

1. Flows of plane defined by complex analytic maps

The flows defined by complex analytic maps define integrable flows.

1. In the case of complex plane, analyticity conditions for a map $f : z \rightarrow= (u, v)$ give Cauchy-Riemann conditions $\partial_x v^x = \partial_y v^y$ and $\partial_y v^x = -\partial_x v^y$ expressing complex analyticity. Neither $\nabla \times v$ nor $\nabla \cdot v$ vanishes. One has neither gradient flow or incompressible flow.

2. One can also consider velocity fields $j = (j^x, j^y)$ satisfying the Cauchy-Riemann conditions. The exponentiation of v defines a flow as the analytic map $z \rightarrow f(z) = u + iv$ of the complex plane which in the case of the plane is of the same form as the generator of the flow. The flow lines can be identified as coordinate lines of the new coordinates u and v and defined by the conditions $Im(f) = v = constant$ and $Re(f) = u = constant$ so that the flow is integrable.

3. In the case of a complex plane, both the holomorphic vector fields j and maps f can however have poles and zeros as singularities and it is important to make a clear distinction between these two interpretations. Zeros of the map $f = (u, v)$ correspond to point-like vortex cores and poles to point-like sources and sinks at which the analyticity fails. If f is interpreted as an electric or magnetic field, poles correspond to charges as sources of the electric field and vortices to point currents as sources of the magnetic field.

4. One can also allow cuts. They appear if a complex analytic map is many-valued, such as fractional power and it is made discontinuous by taking only a single branch. Second option is to allow a covering in which case the complex plane becomes many-sheeted. In TGD, this picture is generalized to a 4-D situation.

2. Symplectic flows in plane

One can consider also symplectic flows in plane E^2 endowed with Kähler form $J_{xy} = -J_{yx} = 1$, which is negative of the tensor squared of the metric $g_{ij} = \delta_{ij}$ of E^2. Symplectic flows preserve the signed area defined by the symplectic form which in complex coordinates corresponds to the Kähler form which in complex coordinates defines a geometric representation of the imaginary unit.

The flows defining infinitesimal generators of the symplectic transformations are in the general case of the form $j^k = J^{kl}\partial_l H$, where index raising is by the metric. In the case of plane E^2 the explicit expression is $(j^x, j^y) = (\partial_y H, -\partial_x H)$, where H is the Hamiltonian of the flow, which defines conserved "energy" constant along flow lines. The vanishing of the divergence $D_k j^k$ means the preservation of the area.

Symplectic flow is not a gradient flow but it allows a global coordinate varying along the flow lines. This follows from the existence of the canonical conjugate H^c of H, whose Poisson bracket with H equals to on: $\{H^c, H\} = \partial_k H^c J^{kl} \partial_l H = 1$. The equation for H^c along the flow lines of H is $dH^c/dt = \{H^c, H\} = 1$ and is solve by $H^c = t$ so that H_c defines the gradient flow giving rise to a global coordinate. The plane decomposes to a union of flow lines as $H = E$ surfaces.

4.3.2 The 2-dimensional flows related to the simplest Hopf fibration $S^3 \to S^2$

Consider first the Hopf fibration $S^3 \to S^2$. The simplest visualization of the fibration is in terms of inverse images of the circles S^1 of S^2 in S^3 under bundle projection. The fibers associated with the points S^2 correspond to linked, non-intersecting circles in S^3. The twist or linkage is characterized by an integer known as Chern number. That the inverse images are smooth 2-surfaces, is highly non-trivial and is due to the fact that the flow in S^2 is integrable. Any integrable flow allows similar smooth lift.

For visualization purposes, one can represent S^2 as E^2 and S^3 as E^3. For instance, the inverse images of the circles $S^1 \subset S^2$ with a constant latitude θ, identified as flow lines, define a slicing of $E^3 \setminus Z$, where Z is z-axis, by tori $S^1 \subset S^1$ the origin of E^3 and projecting to a circle with center point at the origin of E^2. Poles of S^2 correspond to tori which degenerate to a single point, the origin E^3. The inverse images of closed flow lines in S^3 are tori for any integrable flow.

The flows of S^3 consistent with the Hopf fibration are unions of toric flows at the tori $S^1 \times S^1$ characterized by 2 winding numbers (n_1, n_2) project to circles $S^1 \subset S^2$. Note that the flow in S^2 is not geodesic flow. The flows of charged particles along closed cosmic strings with homologically trivial $S^2 \subset CP_2$ as cross section and define analogs of these flows.

Besides Betrami flows $\nabla \times v = \alpha v$ in S^3 also other flows S^2 loosely related to Hopf fibrations and its generalization are interesting in the TGD framework. Since S^2 has complex and Kähler structures, the integrable flows of S^2 should be reducible to analytic maps $f : z \to f(z)$ of S^2 to itself. From the TGD point of view, especially interesting flows flow are magnetohydrodynamics geodesic flows of CP_1 (and CP_2) coupled to its Kähler form as $U(1)$ field for which S^3 (S^5) define the fiber of U^1 bundle.

1. At the fermionic the presence of the S^1 as fiber of S^3 brings in a coupling of S^2 spinors to a covariantly constant Kähler form of S^2, which corresponds to a $U(1)$ symmetry assignable to S^1. In the case of S^2, the coupling is not necessary but in the case of CP_2 the Hopf fibration $S^5 \to CP_2$ allows Spin$_c$ structure and leads to the standard model couplings and symmetries in TGD.

2. S^2 with Kähler structure can be visualized for the standard embedding $S^2 \to E^3$ as a covariantly constant magnetic field B orthogonal to S^2. Another way to describe B is as a covariantly constant antisymmetric 2-tensor in S^2.

3. At the hydrodynamical level, one can consider hydrodynamics in which geodesic free motion couples to the magnetic field defined by the Kähler form via Lorentz force. The magnetic force causes a twisting so that the motion is not anymore along a big circle. The flow lines tend to turn towards the North Pole or South Pole and approach/or leave the poles from South or North. Chiral symmetry is clearly violated.

   For the lift of this flow to S^3 flow lines define a union of non-intersecting linked circles S^1 as fibers of $S^3 \to S^2$ giving rise to tori in the case of closed flow lines. If the S^2 flow is integrable, it is possible to label the fiber circles by a time coordinate, so that they are expected to combine to form a smooth 2-D manifold. Vortex singularities must correspond to single fiber S^1, possibly contracted to a point.

4. The basic question is whether a given flow is integrable rather than like a random motion of gas molecules for which flow lines can intersect and do not form a smooth filling of the space. Complex and Kähler structures make sense also for S^2. The conclusion is that analytic maps $z \to f(z)$ of a complex coordinate of S^2 define an integrable flow. The real and imaginary parts of $f(z)$ define the velocity field v. Also symplectic flows define flows global coordinate along the flow lines so that the flow lines allow a lift to tori in S^3.

5. There are two kinds of singularities at which the analyticity fails: zeros correspond to vortices and poles to sources and sinks. Everywhere else the flow is locally incompressible and irrotational so that both the divergence and rotor of the velocity field vanish. If the flow has no singularities it can be regarded as a mere coordinate change. Singularities contain the physics. It would seem that only integrable flows allow a lift to flows in S^3.

4.3.3 Hopf fibration $S^5 \to CP_2$

In TGD, the projection $S^5 \to CP_2$ is the crucial Hopf fibration since it makes it possible to provide CP_2 with a respectable spinor structure. The Kähler coupling gives rise to the standard model couplings and symmetries and $H = M^4 \times CP_2$ is physically unique: weak interactions are color interactions in CP_2 spin degrees of freedom (charge and weak isospin). What is essential is the coupling of the Kähler gauge potential to spinors. This in turn leads to a Dirac equation in $H = M^4 \times CP_2$ and the induced Dirac equation at the space-time surface X^4.

1. At the hydrodynamical level one has a geodesic flow coupled to the self-dual Kähler form of CP_2 . One has Euclidian analogs of constant electric and magnetic fields, which are of the same magnitude. They would be orthogonal in E^4 but in CP_2 their inner product gives constant instanton density. In this case the inverse images of the flow lines not linked.

2. Also now complex analytic maps $f : CP_2 \to CP_2$ define integrable flows with singularities guaranteeing that the inverse images of flow lines in S^5 are 2-D smooth manifolds. There are two complex coordinates and one can have poles with respect to both of them. Both poles and zeros are replaced with 2-D surfaces and also the analogs of cuts appearing if many-valued maps f are allowed.

3. Also symplectic maps define flows global coordinate along the flow lines so that the flow lines allow a lift to tori in S^5.

4.3.4 CP_2 type extremals

At the next level one can consider CP_2 type extremals, which are deformations of the canonical embedding of CP_2 as an Euclidean 4-surface of $H = M^4 \times CP_2$ for which M^4 coordinates are constant. They can be said to define basic building bricks of particles in TGD. The CP_2 type extremal has locally the same induced metric and Kähler structure as CP_2 but its M^4 projection is a light-like curve, light-like geodesic in the simplest situation. It also ends, that is holes realized as 3-surfaces.

1. The above situation for which time is time parameter as 5:th coordinate is replaced with M^4 time coordinate u varying along the light-like curve. Also now the complex analytic functions $f : CP_2 \to CP_2$ define integrable flows. Time coordinate labels 3-D sections of the flow.

2. Now these flows would carry real physics. Induced Dirac equation effectively reduces to 1-D Dirac equation for fermion lines identified and holomorphy solves it, very much like in string models.

   The physical interpretation is very concrete. The addition of fermions to fermion lines serves as an addition of a marker making the flow visible. Fermions as markers allow to get information about the underlying geometric flow making itself visible via the time evolution of the many-fermion state.

   In TGD, fermions also realize Boolean logic at quantum level and the time evolutions between fermionic states can be seen as logical implication $A \to B$. Spinor structure as square root of metric structure fuses logic and geometry to a larger structure.

4.3.5 Flows at space-time surfaces $X^4 \subset H$

In holography = holomorphy vision space-time surfaces are roots for a pair $f = (f_1, f_2) : H \to C^2$ of two generalized analytic functions f_i of one real hypercomplex coordinate u of M^4, and the remaining 3 complex coordinates of H. Let us denote one of the complex coordinates by w, which can be either an M^4 or CP_2 coordinate.

1. The roots give space-time surfaces as minimal surfaces solving the field equations for any classical action as long as it is general coordinate invariant and constructible in terms of induced geometry. The extremely nonlinear field equations reduce to local algebraic conditions and Riemannian geometry to algebraic geometry.

2. X^4 shares one hypercomplex coordinate and one complex coordinate with H and both X^4 and H have generalized complex structure. X^4 has hypercomplex coordinate u ($u = t - z$ of M^2 in the simplest situation) and complex coordinate w (coordinate of complex plane E^2 in the simplest situation). This defines the Hamilton-Jacobi structure of X^4.

3. Complex analytic maps of X^4 are of the form by $(u \to f(u), w \to g(u, w))$. Integrable flows are induced by these maps. If there are no singularities they correspond to general coordinate transformations. The map by f having singularities generates a new Hamilton-Jacobi structure.

4. Poles and zeros in the w-plane correspond to 2-D string world sheets. The counterparts of zeros and poles for hypercomplex plane, parameterized by a discrete set of values of the real hypercomplex coordinate u correspond to singular partonic 2-surfaces with complex coordinate w at the light-like orbit of a partonic 2-surface.

   These singular partonic 2-surfaces can be identified as TGD counterparts analogs of vertices at which fermionic lines can change their direction. At these surfaces the trace H of the second fundamental form vanishing everywhere else by minimal surface property has a delta function like singular. Its CP_2 part has an interpretation as analog of Higgs vacuum expectation value. The claim of Jenny Nielsen is analogous to this result. In TGD also the M^4 part of H is non-vanishing and corresponds to a local acceleration concentrated at the singularity. An analog of Brownian motion is in question.

   One could very loosely say that the parameter α for Beltrami flow vanishes everywhere except at singularities where it has interpretation as value of the analog of Higgs expectation as the trace of the second fundamental form.

   String world sheets in turn mediate interactions since they connect to each other the light-like orbits of partonic 2-surfaces. This view conforms with the basic physical picture of TGD.

A summary of the situation would look like follows.

1. It would seem that in TGD the flows in CP_1 and CP_2 are more important than flows in S^3 and S^5 but that the integrable flows allow a lift of the flow lines to smooth manifolds of the total space. The spheres provide the needed Kähler form guaranteeing the twisting of the flow and making in the case of S^2 possible arbitrarily complex flow topologies as knotting, braiding, and linking. Also 2-knots are possible in 4-D context.

2. The flows with a coupling to the induced Kähler form have a clear physical interpretation and the fermion lines central in the TGD based view of scattering amplitudes could correspond to the flow lines. The flows without singularities define general coordinate transformations. What about the Kähler flows expected to have singularities? Could they have some physical interpretation?

   String world sheets are identifiable as intersections of two space-time surfaces with the same H-J structure, this applies also to self-intersections. Partonic 2-surfaces in turn are counterparts of vertices at which the TGD counterparts of Feynman lines meet [L28]. These singularities play a key role in the construction of scattering amplitudes in the TGD framework. Also the singularities of the complex flows in the presence of Kähler force have this kind of singularities as counterparts vortices and sinks and sources. Could the flow singularities correspond to self intersections and partonic 2-surfaces?

3. Could the analytic maps with singularities defined by Kähler flow allow to define Hamilton-Jacobi structure in geometric terms using the information about its singularities as self-intersections.

4. The realization that fermion lines very concretely serve as markers of a hydrodynamic flow.

5 How to create monopole flux loops?

Yesterday evening we had a very fruitful discussion with Marko Manninen about wormhole contacts and monopole flux loops. The question was following: *How do wormhole contact pairs, which correspond to elementary particles, arise?*. We pondered this a lot, but the final outcome remained vague and bothered me and I continued to ponder the problem during the night. Finally, I realized that the problem was structurally the same as another problem: how is the creation of fermion pairs possible in TGD even though spinor fields in $H = M^4 \times CP_2$ are free fields? The solution was also the same and possible only for 4-D spacetime surfaces.

5.1 Some basic facts

I will summarize the basic facts first and then the results.

1. A wormhole contact is a Euclidean spacetime region that connects two Minkowski spacetime sheets. They appear as two different types.

   (a) One that arises when the sheets touch each other. This region is not stable because there is no net magnetic flux flowing from one sheet to the other.

   (b) One with a monopole flux. One can talk about a very short piece of flux tube. Since the flux is conserved, it cannot arise when the sheets touch. On the other hand, the conservation of flux stabilizes the tube so that it cannot be split. It must be however made clear that their emergence is not in conflict with the fundamental conservation laws.

   How could monopole wormhole contacts arise? This is the basic question we pondered. During late-night reflections it became clear that the attempts that first came to my mind did not work.

2. Some more facts are in order.

   (a) The boundary conditions of the field equations of TGD do not allow open flux tubes, i.e. cylinders with ends from which flux would escape into vacuum. The flux losses at the ends would compensate each other, but this is not enough. Local conservation is required and is not possible. Flux tubes must be closed loops in which monopole flux flows.

   (b) This implies that wormhole contacts carrying monopole flux must emerge in pairs. They can be visualized as two wormhole contacts, i.e. magnetic flux flows from throat A_1 to throat B_1, from there to throat B_2 on the lower sheet and from there to throat A_2 and further to throat A_1 on the upper sheet. This makes a closed loop. At least massive elementary particles would correspond to such. Point-like fermions would inhabit the throats. What single wormhole contact looks like is familiar from visualization of them in general relativity (see this). Now however the size is extremely small since the contact has the size scale of CP_2 about 10^4 Planck lengths.

   (c) Magnetic flux corresponds to Kähler magnetic charge. It must be conserved as it flows along the tube. The total magnetic charge would be zero if open flux tubes were allowed. Here the hydrodynamic analogy to incompressible flow, which is actually mathematically very accurate, helps.

   (d) But what happens when one has several closed flux tubes carrying magnetic flux and possibly fusing and decaying? Is the sum of the total fluxes conserved for closed flux tubes in this kind of reaction? This would generalize the conservation of flux inside a single tube. The flux for a closed flux tube would be analogous to an electric charge but should not be confused with it.

If conservation holds true, one could conclude that if a monopole flux tube breaks up into two flux tubes with fluxes n_1 and n_2, then the flux remains $n = n_1 + n_2$?

A reasonable half-guess is that flux can be considered as some kind of conserved charge. This makes sense if the sign of the flux can be operationally defined. If a flux tube has some kind of chirality (DNA helix is an example of this) or parity, then it determines the sign of the flux regardless of the position of the flux tube.

Different chiralities could also be represented by flux tubes that are complex conjugates of each other with respect to the complex conjugation of CP_2. This would correspond to charge-conjugation. Holography = holomorphy hypothesis predicts this kind of geometric charge conjugation affecting the space-time surface. A given space-time sheet would contain only matter or antimatter and this could explain the mysterious matter-antimatter symmetry. Matter and antimatter would live at different space-time sheets.

(e) In any case, even the creation of a single closed monopole flux tube is impossible. So the simplest possibility is that they are created in pairs so that the fluxes are opposite. How would this happen?

5.2 Connection with the similar problem related to the creation of fermion pairs

Here a connection to a similar problem related to the fermion pair creation emerges.

1. In TGD, the spinor fields in $H = M^4 \times CP_2$ are free [L25, L24]. This is the only option. If a quartic term coupling were added to the action, the result would be a non-renormalizable theory and thus a catastrophe.

   The problem is that in TGD gauge bosons do not correspond to primary quantum fields but emerge as bound states of fermions and antifermions. Is the creation of fermion pairs from a vacuum or from a classical induced field then possible at all? In QED, it is possible in the quantized electromagnetic field but also when the field is classical and this is in fact the approximation made in applications like atomic and molecular physics.

2. What could the pair creation correspond geometrically? An intuitive picture of what happens has been clear since Dirac's time. The fermion line turns back in time when a pair is created. This picture generalizes to TGD. The V-shaped fermion line has an edge with an infinite acceleration. This is not possible in physics respecting smoothness.

3. Now comes the connection with zero energy ontology. In TGD, the particle is geometrically replaced by a "Bohr orbit" of a 3-surface and the infinite acceleration at the edge of the V corresponds to the failure of the holomorphy at a singular 3-surface. These singularities are the fundamental prediction and correspond to the mild non-determinism of the holography = holomorphy principle. Field equations are not violated; these singular 3-surfaces at which the minimal surface property fails are analogous to the poles or zeros of analytic functions. These singular surfaces can be identified as generalized interaction vertices.

   At the edge of the V, the minimal surface property fails and the trace of the second fundamental form, which is the generalization of the acceleration form 1-D to a 4-D object, is infinite. Its CP_2 part transforms under symmetries like the Higgs. The Higgs would be non-zero and only at the vertices and have an infinite value there. The M^4 part corresponds to the ordinary acceleration and the particle's earthly pilgrimage would be geometrically analogous to 8-D Brownian motion such that the vertices represent special moments in its elementary particle life since in TGD inspired theory of consciousness they correspond to conscious choices even at this level.

4. A couple of years ago I realized that the edge singularity of a fermion line could correspond to the phenomenon known as "exotic smooth structure" [?]ossible in TGD [?] It is an exotic smooth structure, which reduces to the standard smooth structure except for defects that

are 3-surfaces: they are "edges" and correspond geometrically to particle vertices in TGD. Differentiability is broken.

The exotic smooth structures only occur in dimension $D = 4$. Pair creation and non-trivial fermionic dynamics are only possible in 4-D spacetime! A really bad problem of general relativity turns into a triumph in TGD.

5.3 Do also monopole flux tubes turn back in time?

Now we are very close to solving the original problem. So: how do monopole flux loops arise?

1. The emergence of a pair formed by closed monopole flux tubes corresponds to a situation when a closed monopole flux tube coming from the geometric future turns back to the geometric future! More general reversals can occur and the emission of virtual particles at vertices corresponds to this.

   Therefore, a particle pair in geometric sense can be created in the classical induced field in a geometric sense. What happens to fermion lines also happens to 3-surfaces.

2. This view generalizes: a closed monopole flux tube can break up, thus creating, for example, two monopole flux tubes and the total flux is conserved. It should be however made clear that the proposed mechanism does not throw the textbooks of physics to a paper basket but only generalizes and makes them more precise by providing concrete geometric description of elementary particles as extended geometric objects.

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