What could be the counterpart of T-duality in TGD framework?

M. Pitkänen
Email: matpitka@luukku.com.
http://tgdtheory.com/public_html/.

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1 Introduction

Stephen Crowley sent me a book of Michel Lapidus [2] about zeros of Riemann zeta and also about his own ideas in this respect. The book has been written in avery lucid manner and looks very interesting. The big idea is that the T-duality of string models could correspond to the functional equation for Riemann zeta relating the values of zeta at different sides of the critical line. T-duality [1] is formulated for strings in space $M^d \times S^1$ or its generalization replacing S^1 with higher-dimensional torus and generalized to fractal strings. Duality states that the transformation $R \rightarrow 1/R$ with suitable unit for R defined by string tension is a duality: the physics for these different values of R is same. Intuitively this is due to the fact that the contributions of the string modes representing n-fold winding and those representing vibrations labelled by integer n are transformed to each other in the transformation $R \rightarrow 1/R$.

Lapidus is a mathematician and mathematicians often do not care too much about the physical meaning of the numbers. For a physicist like me it is extremely painful to type the equation $R \rightarrow 1/R$ without explicitly explaining that it should actually read as $R \rightarrow R_0^2/R$, where R_0 is length unit, which must represent fundamental length scale remaining invariant under the duality transformation. Only after this physicist could reluctantly put $R_0 = 1$ but still would feel himself guilty of unforgivable sloppiness. $R_0 = 1$ simplifies the formulas but one must not forget that there are three scales involved rather than only two. The question inspired by this nitpicking is how the physics in the length scales R_1 and R relates to the physics in length scale R. Are dualities - or perhaps holography like relations in question- so that T-duality would follow from these dualities?

2 Could one replace winding number with magnetic charge and T-duality with canonical identification?

How could one generalize T-duality to TGD framework? One should identify the counterpart of the winding number, the three fundamental scales, and say something about the duality transformation itself.

1. In TGD Universe partonic 2-surfaces are the basic object. Partonic 2-surface is not strings and the only reasonable generalization for winding number is as Kähler magnetic charge representing the analog of winding of the partonic 2-surface around magnetically charged 2-sphere of CP_2. Magnetic charge tells how many times partonic 2-surface wraps around the homologically non-trivial geodesic sphere with unit magnetic charge. If the generalization of T-duality holds true, one would expect that the contributions of the oscillations and windings of the partonic two-surface to ground state energy must be transformable to each other by the counterpart of the transformation $R \rightarrow R_0^2/R$ - or something akin to that. Also less concrete and more general interpretations are possible, and below the most plausible interpretation will be considered.

2. The duality $R \rightarrow R_0^2/R = R_1$ gives R_0 as a geometric mean $R_0 = \sqrt{RR_1}$ of the scales R and R_1. What are these three length scales in TGD Universe? The obvious candidate for R is CP_2 size scale. p-Adic mass calculations [3] imply that the primary p-adic length scale $L_{p,1} = \sqrt{p}R$ is of order of Compton length of the elementary particle characterized by the p-adic prime p. The secondary p-adic length scale $L_{p,2} = pR$ in turn defines the size scale of causal diamond (CD) assignable to the magnetic body of the elementary particle characterized by prime p. For instance, for electron this scale corresponds to .1 seconds, a fundamental biological time scale.

   One indeed has $L_{p,1} = \sqrt{L_{p,2}R}$, and CP_2 scale and CD length scale are dual to each other if T-duality holds true. Therefore the duality would relate physics at CP_2 scale - counterpart of Planck length in TGD framework - and in biological scales and would have direct relevance to quantum biology. One has an infinite hierarchy of p-adic length scales and each of them would give rise to one particular instance of the T-duality. Adeles [7] would provide appropriate formulation of T-duality in TGD framework. The corresponding mass scales would be $\hbar/R, \hbar/\sqrt{p}R$ and $\hbar/pR$. The third scale corresponds to a scale, which for electron corresponds to the 10 Hz frequency in the case of photons. The duality would suggest that the physics associated with the frequencies in EEG scale related to the communications from the biological body to magnetic body is dual to the physics in CP_2 scale.

   Note that one cannot exclude alternative variants of T-duality. In particular, Planck scale and CP_2 length scale as candidates R_1 and R could be considered.

3. What is the interpretation of these three length scales? CP_2 length scale corresponds naturally to the size scale of wormhole contacts. They are Euclidian regions of space-time surface and represent lines of generalized Feynman graphs. Both general arguments and the construction of elementary bosons forces [4] to assign to these regions braid strands playing a role of Euclidian strings. Parallel translation along the strands is essential in the construction of fermionic bilinears as invariant under general coordinate transformations and gauge transformations [4]. The ends of these strands carry fermion and anti-fermion numbers. The counterpart of string tension involved appearing in stringy mass formula implied by super-conformal invariance is indeed determined by R and p-adic thermodynamics [3] leads to a detailed and successful calculations for elementary particle masses using only p-adic thermodynamics, super-conformal invariance, and p-adic length scale hypothesis as basic assumptions.

4. The wormhole throats carrying fermion number are Kähler magnetic monopoles and the wormhole must be accompanied by a second wormhole throat carrying opposite magnetic charge and also a neutrino pair neutralizing the weak isospin so that weak massivation takes place. The end of the flux tube containing the neutrino pair is virtually non-existent at low energies. The length scale for this string must correspond to Compton length for elementary particle given essentially by primary p-adic length scale $L_{p,1}$. The more restrictive assumption that this length scale corresponds to the Compton length of weak bosons looks un-necessarily restrictive and looks also un-natural.

5. The excitations with mass scale $\hbar/pR$ would correspond to excitations assignable to entire CD, maybe assignable to the flux tubes of the magnetic bodies of elementary particles defining also string like objects but in macroscopic scales. For electron the scale is of order of the circumference of Earth. This dynamics would naturally correspond to the dynamics in Minkowskian space-time regions. The dynamics at intermediate length scale would be intermediate between the Euclidian and Minkowskian dynamics and reduce to that for light-like orbits of partonic 2-surfaces with metric intermediate between Minkowskian and Euclidian.

6. A natural interpretation for T-duality in this sense is in terms of strong form of holography. The interior dynamics at length scale R *resp.* pR assigned to Euclidian *resp.* Minkowskian regions of space-time surface corresponds by holography to the dynamics of light-like orbits of partonic 2-surfaces identified as wormhole throats. Therefore the dynamics in Euclidian and Minkowskian regions are dual to each other. Therefore T-duality in TGD sense would follow from the possibility of having both Euclidian and Minkowskian holography. Strong form of holography in turn reduces to strong form of General Coordinate Invariance, which has turned out to be extremely powerful principle in TGD framework.

3 Is the physics of life dual to the physics in CP_2 scale?

The duality of life with elementary particle physics at CP_2 length scale - the TGD counterpart of Planck scale - looks rather far-fetched idea. There is however already earlier support for this idea.

1. p-Adic physics is physics of cognition, and one can say that living systems are in the algebraic intersection of real and p-adic worlds: the intersection of cognition and matter. Canonical identification maps p-adic physics to real physics. This map takes p-adic integers which are small in p-adic sense to larger integers in real sense and thus maps long real scales to short real scales. Clearly this map is jhighly analogous to the T-duality. p-Adic length scales are indeed explicitly related with the above identification of the T-duality so that canonical identification might be involved with T-duality.

   If this interpretation is correct, cognitive p-adic representations in long real length scales would give representations for the physics in short length scales. EEG range of frequencies allowing communication to the magnetic bodies is absolutely essential for brain function. CDs would correspond to the real physics scale associated with the cognitive representations. These cognitive representations are indeed exactly what our science is building so that T-duality would make also scientist as a part of the big vision!

2. The model for dark nucleons as three quark states led to one of the greatest surprises of my professional life [5, 2]. Under rather general conditions the three quark states for nucleon are in one-one correspondence with the DNA, RNA, tRNA codons, and aminoacids for vertebrate genetic code and there is natural physical correspondence between DNA triplets and aminoacids. This suggests that genetic code is realized at the level of hadrons and that living matter is a kind of emulation for it, or that living matter is representation for matter at hadron level. This leads to rather far reaching speculations about biological evolution - not as random process - but a process analogous R&D applied in industry [2]. New genes would be continually tested at the level of dark matter and the modifications of genome could be carried out if there is a transcription process transforming dark DNA to ordinary DNA.

3. The secondary p-adic mass scale of electron corresponds to the 10 Hz frequency, which defines a fundamental biorhythm. Also to current quark masses, which are actually not so well-known but are in MeV range, one can assign biologically interesting time scales in millisecond range. This suggests that all elementary particles induce physics in macroscopic time scales via their CD:s containing their magnetic bodies.

The unavoidable and completely crazy looking question raised by T-duality is whether there is intelligent life in the Euclidian realm below the CP_2 length scale - inside the lines of generalized Feynman graphs. This kind of possibility cannot be avoided if one takes holography absolutely seriously. In purely mathematical sense TGD suggests even stronger form of holography based on the notion of infinite primes [6]. In this holography the number theoretic anatomy of given space-time point is infinitely complex and evolves. The notion of quantum mathematics replacing numbers by Hilbert spaces representing ordinary arithmetics in terms of direct sum and tensor product suggest the same [7]. Space-time point would be in this picture its own infinitely complex Universe - the Platonia.

4 Could one get expression for Kähler coupling strength from restricted form of modular invariance?

The contributions to the exponent of the vacuum functional, which is proportional to Kähler action for preferred extremal, are real *resp.* imaginary in Euclidian *resp.* Minkowskian regions. Under rather general assumptions (weak form of electric-magnetic duality defining boundary conditions at wormhole throats plus additional intuitively plausible assumption) these contributions are proportional to the same Chern-Simons term but with possibly different constant of proportionality [1].

These terms sum up to a Chern-Simons term with a coefficient analogous to the complex inverse gauge coupling

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_K^2} \quad .$$

The real part would correspond to Kähler function coming from Euclidian regions defining the lines of generalized Feynman diagrams and imaginary part to Minkowskian regions. There are could arguments suggesting that With the conventions that I have used $\theta/2\pi$ is counterpart for $1/\alpha_K$ and there are good arguments that it corresponds to finte structure constant in electron length scale. Furthermore, T-duality would suggest that τ is proportional to $1+i$ so that one would have

$$\frac{\theta}{2\pi} = \frac{4\pi}{g_K^2} \quad .$$

This condition would fit nicely with the fact that Chern-Simons contributions from Minkowskian and Euclidian regions are identical. If this equation holds true the modular transformations must reduce to those leaving this relationship invariant and can only permute the complex and real parts and thus leave τ invariant. One could also interpret this value of τ as physically especially interesting representation and assign to all values of τ related by modular transformation an isotropy group leaving it fixed. All other physically equivalent values would be obtained as SL(2,Z) orbit of this value.

The counterpart of T-duality should somehow relate dynamics in Minkowskian and Euclidian regions and this raises the question whether it corresponds to $\tau \rightarrow i\tau$ and is represented by some duality transformation

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad ,$$

where $(a, b; c, d)$ defines a unimodular matrix $(ad - bc = 1)$ with integer elements, that is in $SL(2, Z)$. The electric-magnetic duality $\tau \rightarrow -1/\tau$ [1] and the shift $\tau \rightarrow \tau + 1$ are the generators of this group. It is not however quite clear whether they can be regarded as gauge symmetries in TGD framework. If they are gauge symmetries, then the critical values of Kähler coupling strength defined as fixed points of coupling constant evolution must form an orbit of $SL(2, Z)$. It could be also that modular symmetry is broken to a subgroup of $SL(2, Z)$ and this subgroup leaves τ invariant in the case of minimal symmetry.

1. $\tau \rightarrow i\tau$ would permute Euclidian and Minkowskian regions with each other and is therefore a candidate for the T-duality. This condition cannot be satisfied in generic case but one can ask whether for some special choices of τ these transformations could generate a non-trivial sub-group of modular transformations. This subgroup

   To see whether this is the case let us write explicitly the condition $\tau \rightarrow i\tau$:

   $$\frac{a\tau + b}{c\tau + d} = \frac{\theta}{2\pi} + i\frac{1}{\alpha_K} \quad , \quad \alpha_K = \frac{g_K^2}{4\pi} .$$

   The condition allows to solve τ as

   $$\tau = \frac{a - id}{2c}\left[1 + \epsilon_1\sqrt{1 + \frac{4ibc}{(d - ia)^2}})\right] \quad , \quad \epsilon_1 = \pm 1 \quad .$$

2. For

$$d = \epsilon a \ , \ \epsilon = \pm 1$$

implying $a^2 - bc = 1$, the solution simplifies since the argument of square root is real. One has

$$\tau = \frac{a}{2c}(1-\epsilon i)\left[1+\epsilon_1\sqrt{1-\epsilon\frac{\sqrt{a^2-1}}{a}}\right] \ .$$

The imaginary and real parts of τ are identical: this might allow an interpretation in terms of the fact that Chern-Simons terms from two regions are identical (normal derivatives are however discontinuous at wormhole throat). Certainly this is a rather strong prediction.

3. Does this mean that SL(2,C) is broken down to the 4-element isotropy group generated by this transformation? If so, a the condition just deduced could allow to deduce additional constraints on the value of Kähler coupling strength, which is in principle fixed by the criticality condition to have only finite number of values? By the earlier arguments - related to p-adic mass calculations and the heuristic formula for the gravitational constant - the value of Kähler coupling strength is in a good approximation equal to fine structure constant at electron length scale:

$$\alpha_K = \frac{g_K^2}{4\pi} \simeq \alpha \ , \ \frac{1}{\alpha} \simeq 137.035999084 \ .$$

4. One obtains the following estimate for $a/2c$ from the estimate for α_K by considering the imaginary part of τ:

$$\frac{a}{2c}\left[1+\epsilon_1\sqrt{1-\epsilon\frac{\sqrt{a^2-1}}{a}}\right] = \frac{1}{\alpha_K} \ .$$

At the limit $a \rightarrow \infty$ one has

$$\frac{a}{2c}\left[1+\epsilon_1\sqrt{1-\epsilon}\right] = \frac{1}{\alpha_K} \ .$$

The simplest option at this limit corresponds to $\epsilon = 1$ giving

$$\frac{a}{2c} \simeq 137.035999084 \ .$$

Note that $a/2c = 137$ is not allowed by determinant condition so that the deviation of α_K from $1/137$ is predicted. One must have $a > 137 \times 2c \geq 2 \times 137$. This implies

$$1+\epsilon_1\sqrt{1-\frac{\sqrt{a^2-1}}{a}} = 1 + .0026\epsilon_1 \ .$$

By expanding the square root in first order to Taylor series one obtains the condition

$$\frac{a}{2c}\left(1+\epsilon_1\frac{1}{2^{3/2}\times 137c}\right) \simeq \frac{1}{\alpha_K} \ .$$

For large enough values of a and c it is possible to have arbitrary good approximation to fine structure constant. Note that the integers a and c cannot have common factors since this together with determinant condition $a^2 - bc = 1$ would lead to contradiction.

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