Some objections against p-adic thermodynamics and their resolution

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Abstract

There are two basic objections against p-adic thermodynamics. The mass calculations require the presence of states with negative conformal weights giving rise to tachyons. Furthermore, by conformal invariance, L_0 should annihilate physical states so that all states should have vanishing mass squared! In this article a resolution of these objections is discussed. The solution is based on the very definition of thermodynamics and on number theoretic vision predicting quark states with discretized tachyonic mass, which are counterparts for virtual states in QFTs.

Physical states for the entire Universe would be indeed massless but for subsystems such as elementary particles the thermal expectation of the mass squared is non-vanishing. This conforms with the formula of blackhole entropy stating that it is proportional to the mass squared of the blackhole and vanishes for a vanishing mass: this would indeed correspond to a pure state. Higgs mechanism as breaking of gauge symmetry generalizes to apparent breaking of conformal invariance caused by thermodynamic treatment. At M^8 level, the symmetry broken *resp.* unbroken phase of gauge theory has as a TGD counterpart a phase for which the values of mass squared *resp* energy come as roots of a polynomial. In a loose sense, the real valued argument of P serves as a counterpart of the Higgs field.

Contents

1 Introduction

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L1, L2], and classical number fields [K6]. p-Adic mass calculations is a rather successful application of p-adic thermodynamics for the mass squared operator identified as conformal scaling generator L_0. p-Adic thermodynamics can be also understood as a constraint on a real thermodynamics for the mass squared from the condition that it can be also regarded as a p-adic thermodynamics.

The motivation for p-adicization came from p-adic mass calculations [K3, K2].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Boltzmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1+i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [K4] is interpreted as a correlate for cognition. Motivation comes from the observation that piecewise constant functions depending on a finite number of pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

Unlike the Higgs mechanism, p-adic thermodynamics provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry, which guarantees the existence of p-adic Boltzmann weights.

There are two basic objections against p-adic thermodynamics. The mass calculations require the presence of states with negative conformal weights giving rise to tachyons. Furthermore, by conformal invariance L_0 should annihilate physical states so that all states should have vanishing mass squared! In this article a resolution of these objections, based on the very definition of thermodynamics and on number theoretic vision predicting quark states with discretized tachyonic mass, which are counterparts for virtual states in QFTs, is discussed.

Physical states for the entire Universe would be indeed massless but for subsystems such as elementary particles the thermal expectation of the mass squared is non-vanishing. This conforms with the formula of blackhole entropy stating that it is proportional to the mass squared of blackhole and vanishes for vanishing mass: this would indeed correspond to a pure state.

2 Objections and their resolution

The number theoretic picture leads to a deeper understanding of a long standing objection against p-adic thermodynamics [K3] as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra.

If one requires super-Virasoro symmetry and identifies mass squared with a scaling generator L_0, one can argue that only massless states are possible since L_0 must annihilate these states! All states of the theory would be massless, not only those of fundamental particles as in conformally invariant theories to which twistor approach applies! This looks extremely beautiful mathematically but seems to be in conflict with reality already at single particle level!

The resolution of the objection is that *thermodynamics* is indeed in question.

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and describes approximately the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors. The excited states of the entire system indeed are massless [L12].

2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment and the entire mass squared would remain vanishing. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

3. In the p-adic mass calculations [K3, K2], the effective environment for quarks and leptons would in a good approximation consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons). The many-quark state many-quark state associated with the wormhole throat (single quark state for quarks and 3-quark-state for leptons [L5].

4. In M^8 picture [L3, L4], tachyonicity is unavoidable since the real part of the mass squared as a root of a polynomial P can be negative. Also tachyonic real but algebraic mass squared values are possible. At the H level, tachyonicity corresponds to the Euclidean signature of the induced metric for a wormhole contact.

   Tachyonicity is also necessary: otherwise one does not obtain massless states. The super-symplectic states of quarks would entangle with the tachyonic states of the wormhole contacts by Galois confinement.

5. The massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-symplectic representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Galois confinement is satisfied. The tachyonic mass squared is assigned with wormhole contacts with the Euclidean signature of the induced metric, whose throats in turn carry the fermions so that the wormhole contact would form the nearby environment.

   The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state remains massless and massive single particle states receive a compensating negative mass squared from the wormhole contact. Thermal mass squared corresponds to a single particle mass squared and does not take into account the contribution of wormhole contacts except for the ground state.

6. There is a further delicate number theoretic element involved [L7, L9]. The choice of $M^4 \subset M^8$ for the system is not unique. Since M^4 momentum is an M^4 projection of a massless M^8 momentum, it is massless by a suitable choice of $M^4 \subset M^8$. This choice must be made for the environment so that both the state of the environment and the single particle ground state are massless. For the excited states, the choice of M^4 must remain the same, which forces the massivation of the single particle excitations and p-adic massivation.

2.1 All physical states are massless!

These arguments strongly suggest that pure states, in particular the state of the entire Universe, are massless. Mass would reflect the statistical description of entanglement using the density matrix. The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural. This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K5] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and $M^8 - H$ duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L10, L11].

2.2 Relationship to Higgs mechanism

Polynomials P have two kinds of solutions depending on whether their roots determine either mass or energy shells. For the energy option a space-time region corresponds by $M^8 - H$ duality to a solution spectrum in which the roots correspond to energies rather than mass squared values and light-cone proper time is replaced with linear Minkoski time [L3, L4]. The physical interpretation of the energy shell option has remained unclear.

The energy shell option gives rise to a p-adic variant of the ordinary thermodynamics and requires integer quantization of energy. This option is natural for massless states since scalings leave the mass shell invariant in this case. Scaling invariance and conformal invariance are not violated.

One can wonder what the role of these massless virtual quark states in TQC could be. A good guess is that the two options correspond to phases with broken *resp.* unbroken conformal symmetry. In gauge theories to phases with broken and unbroken gauge symmetries. The breaking of gauge symmetry indeed induces breaking of conformal symmetry and its breaking is more fundamental.

1. Particle massivation corresponds in gauge theories to symmetry breaking caused by the generation of the Higgs vacuum expectation value. Gauge symmetry breaking induces a breaking of conformal symmetry and particle massivation. In the TGD framework, the generation of entanglement between members of state pairs such that members having opposite values of mass squared determined as roots of polynomial P in the most general case, leads to a breaking of conformal symmetry for each tensor factor and the description in terms of p-adic thermodynamics gives thermal mass squared.

2. What about the situation when energy, instead of mass squared, comes as a root of P. Also now one can construct physical states from massless virtual quarks with energies coming as algebraic integers. Total energies would be ordinary integers. This gives massless entangled states, if the rational integer parts of 4-momenta are parallel. This brings in mind a standard twistor approach with parallel light-like momenta for on-mass shell states. Now however the virtual states can have transversal momentum components which are algebraic numbers (possibly complex) but sum up to zero.

   Quantum entangled states would be superpositions over state pairs with parallel massless momenta. Massless extremals (topological light rays) are natural classical space-time correlates for them. This phase would correspond to the phase with unbroken conformal symmetry.

3. One can also assign a symmetry breaking to the thermodynamic massivation. For the energy option, the entire Galois group appears as symmetry of the mass shell whereas for the mass squared option only the isotropy group does so. Therefore there is a symmetry breaking of the full Galois symmetry to the symmetry defined by the isotropy group. In a loose sense, the real valued argument of P serves as a counterpart of the Higgs field.

   If the symmetry breaking in the model of electroweak interaction corresponds to this kind of symmetry breaking, the isotropy group, which presumably involves also a discrete subgroup of quaternionic automorphisms as an analog of the Galois group. Quaternionic group could act as a discrete subgroup of $SU(2) \subset SU(2)_L \times U(1)$. The hierarchy of discrete subgroups associated with the hierarchy of Jones inclusions assigned with measurement resolution suggests itself. It has the isometry groups of Platonic solids as the groups with genuinely 3-D action. $U(1)$ factor could correspond to Z_n as the isotropy group of the Galois group. In the QCD picture about strong interactions there is no gauge symmetry breaking so that a description based on the energy option is natural. Hadronic picture would correspond to mass squared option and symmetry breaking to the isotropy group of the root.

To sum up, in the maximally symmetric scenario, conformal symmetry breaking would be only apparent, and due to the necessity to restrict to non-tachyonic subsystems using p-adic thermodynamics. Gauge symmetry breaking would be replaced with the replacement of the Galois group with the isotropy group of the root representing mass squared value. The argument of the polynomial defining space-time region would be the analog of the Higgs field.

3 Some further comments about the notion of mass

In the sequel some further comments related to the notion of mass are represented.

3.1 $M^8 - H$ duality and tachyonic momenta

Tachyonic momenta are mapped to space-like geodesics in H or possibly to the geodesics of X^4 [L3, L4, L8]. This description could allow to describe pair creation as turning of fermion backwards in time [L11]. Tachyonic momenta correspond to points of de Sitter space and are therefore outside CD and would go outside the space-time surface, which is inside CD. Could one avoid this?

1. Since the points of the twistor spaces $T(M^4)$ and $T(CP_2)$ are in 1-1 correspondence, one can use either $T(M^4)$ or $T(CP_2)$ so that the projection to M^4 or CP_2 would serve as the base space of $T(X^4)$. One could use CP_2 coordinates or M^4 coordinates as space-time coordinates if the dimension of the projection is 4 to either of these spaces. In the generic case, both dimensions are 4 but one must be very cautious with genericity arguments which fail at the level of M^8.

2. There are exceptional situations in which genericity fails at the level of H. String-like objects of the form $X^2 \times Y^2 \subset M^4 \subset CP_2$ is one example of this. In this case, X^6 would not define 1-1 correspondence between $T(M^4)$ or $T(CP_2)$.

   Could one use partial projections to M^2 and S^2 in this case? Could $T(X^4)$ be divided locally into a Cartesian product of 3-D M^4 part projecting to $M^2 \subset M^4$ and of 3-D CP_2 part projected to $Y^2 \subset CP_2$.

3. One can also consider the possibility of defining the twistor space $T(M^2 \times S^2)$. Its fiber at a given point would consist of light-like geodesics of $M^2 \times S^2$. The fiber consists of direction vectors of light-like geodesics. S^2 projection would correspond to a geodesic circle $S^1 \subset S^2$ going through a given point of S^2 and its points are parametrized by azimuthal angle Φ. Hyperbolic tangent $tanh(\eta)$ with range $[-1, 1]$ would characterize the direction of a time like geodesic in M^2. At the limit of $\eta \rightarrow \pm\infty$ the S^2 contribution to the S^2 tangent vector to length squared of the tangent vector vanishes so that all angles in the range $(0, 2\pi)$ correspond to the same point. Therefore the fiber space has a topology of S^2.

   There are also other special situations such as $M^1 \times S^3$, $M^3 \times S^1$ for which one must introduce specific twistor space and which can be treated in the same way.

During the writing of this article I realized that the twistor space of H defined geometrically as a bundle, which has as H as base space and fiber as the space of light-like geodesic starting from a given point of H need not be equal to $T(M^4) \times T(CP_2)$, where $T(CP_2)$ is identified as $SU(3)/U(1) \times U(1)$ characterizing the choices of color quantization axes.

1. The definition of $T(CP_2)$ as the space of light-like geodesics from a given point of CP_2 is not possible. One could also define the fiber space of $T(CP_2)$ geometrically as the space of geodesics emating from origin at $r = 0$ in the Eguchi-Hanson coordinates [K1] and connecting it to the homologically non-trivial geodesic sphere S^2_G $r = \infty$. This relation is symmetric.

   In fact, all geodesics from $r = 0$ end up to S^2. This is due to the compactness and symmetries of CP_2. In the same way, the geodesics from the North Pole of S^2 end up to the South Pole. If only the endpoint of the geodesic of CP_2 matters, one can always regard it as a point S^2_G.

   The two homologically non-trivial geodesic spheres associated with distinct points of CP_2 always intersect at a single point, which means that their twistor fibers contain a common geodesic line of this kind. Also the twistor spheres of $T(M^4)$ associated with distinct points of M^4 with a light-like distance intersect at a common point identifiable as a light-like geodesic connecting them.

2. Geometrically, a light-like geodesic of H is defined by a 3-D momentum vector in M^4 and 3-D color momentum along CP_2 geodesic. The scale of the 8-D tangent vector does not matter and the 8-D light-likeness condition holds true. This leaves 4 parameters so that $T(H)$ identified in this way is 12-dimensional.

   The M^4 momenta correspond to a mass shell H^3. Only the momentum direction matters so that also in the M^4 sector the fiber reduces to S^2 . If this argument is correct, the space of light-like geodesics at point of H has the topology of $S^2 \times S^2$ and $T(H)$ would reduce to $T(M^4) \times T(CP_2)$ as indeed looks natural.

3.2 Conformal confinement at the level of H

The proposal of [L15], inspired by p-adic thermodynamics, is that all states are massless in the sense that the sum of mass squared values vanishes. Conformal weight, as essentially mass squared value, is naturally additive and conformal confinement as a realization of conformal invariance would mean that the sum of mass squared values vanishes. Since complex mass squared values with a negative real part are allowed as roots of polynomials, the condition is highly non-trivial.

$M^8 - H$ duality [L3, L4] would make it natural to assign tachyonic masses with CP_2 type extremals and with the Euclidean regions of the space-time surface. Time-like masses would be assigned with time-like space-time regions. In [L13] it was found that, contrary to the beliefs held hitherto, it is possible to satisfy boundary conditions for the action action consisting of the Kähler action, volume term and Chern-Simons term, at boundaries (genuine or between Minkowskian and Euclidean space-time regions) if they are light-like surfaces satisfying also $detg_4 = 0$. Masslessness, at least in the classical sense, would be naturally associated with light-like boundaries (genuine or between Minkowskian and Euclidean regions).

3.3 About the analogs of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

3.3.1 Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L14] define a unique discretization of 4-surface in M^4 and, by $M^8 - H$ duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L14, ?] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L6]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3) \setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

   The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

   Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and $M^8 - H$ duality in terms of associativity [L3, L4]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

3.3.2 De Sitter manifolds as tachyonic analogs of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ (`https://cutt.ly/XVsdLwY`).

3.3.3 Do complexified hyperbolic manifolds as analogs of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T = 1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H^3_c with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogs of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogs of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H^3_c would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H^3_c is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$ contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H^3_c could be such in a complexified sense.

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