Does the notion of Teichmüller element cure the problem of p-adic mass calculations due to the slight failure of Lorentz invariance?

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Abstract

The problem to be discussed in this article relates to the fact that p-adic mass calculations based on p-adic thermodynamics have a problem with Lorentz invariance. The violation of Lorentz invariance is extremely small but if TGD is to be a fundamental theory, it cannot be dismissed. The problem is due to the fact that the canonical identification mapping p-adic valued mass squared to its real counterpart does not map p-adic sums and products to real sums and products.

In the sequel a possible solution of this problem based on the notion of Teichmüller elements defining a representation of finite field G_p in the field of p-adic numbers is discussed.

1 Introduction

The problem to be discussed in this article relates to the fact that p-adic mass calculations based on p-adic thermodynamics have a problem with Lorentz invariance. The violation of Lorentz invariance is extremely small but if TGD is to be a fundamental theory, it cannot be dismissed. The problem is due to the fact that the canonical identification mapping p-adic valued mass squared to its real counterpart does not map p-adic sums and products to real sums and products. It is good to summary first the recent state of the p-adicization program, which in turn led to adelization program.

1.1 The origin of the p-adicization program

Consider first the origin of the p-adicization program.

1. The notion of p-adicization emerged in p-adic thermodynamics for mass squared identifiable as conformal weight allowing to predict masses of elementary particles using the extremely strong constraints coming from the existence of the Boltzman weights forcing a stringy mass squared spectrum. Also p-adic temperature T_p is quantized as $T_p = 1/n$ and has as a real counterpart $T_{p,R} = 1/nlog(p)$. It has turned out that this temperature could correspond to the temperature assigned to the TGD counterpart of quark gluon plasmas and has interpretation as Hagedorn temperature [L17, L15] (see this).

2. Canonical identification $I : x = \sum x_n p^n \to \sum x_n p^{-n}$ from p-adics to reals is the key notion and looks like mixing apples with oranges. The inverse of I obeys the same formula but is two-valued for a finite number of pinary digits. This is not a practical problem since the finite accuracy forces and approximation using finite number of pinary digits on both sides. Canonical identification allows to map p-adic valued mass squared to a real number. The success of p-adic thermodynamics was amazing and encouraged the development to a number theoretic vision of TGD [L13] (see this).

1.1.1 Questions related to p-adicization

When p-adicization is needed/possible?

1. The minimum condition, following from p-adic mass calculations [L6, L13], is that only Lorentz invariants (in p-adic sense) constructed from the momenta of colliding particles are p-adicized and mapped to reals by canonical identification.

   For very large p-adic primes the canonical identification commutes with arithmetics: products and sums are mapped to products and sums. The values of p-adic primes are very large for elementary particles: the electron corresponds to $M_{127} = 2^{127} - 1 \sim 10^{38}$. This implies the convergence of the perturbation series in p-adic thermodynamics is in powers of p and extremely fast.

2. Is it possible to p-adicize space-time surfaces in $H = M^4 \times CP_2$? Holography = holomorphy (H-H) vision [L10, L12, L11, L8] algebraizes the geometric field equations to algebraic equations might make this possible. Riemann geometry is reduced to algebraic geometry, which is universal and the same equations apply for any number field. The theory is number theoretically universal.

3. $M^8 - H$ duality [L16, L14] states that the geometric description of physics in terms of space-time surface in $H = M^4 \times CP_2$ and number theoretic description in terms of surfaces in 8-D momentum space in M^8 identified as octonions, are complementary. $M^8 - H$ duality corresponds to momentum-position duality. M^8 , which is the analog of thr momentum space for particles as 3-surfaces replaced with almost deterministic Bohr orbits.

   At the level of M^8 one has instead of space-time surface, 4-D surfaces representing dispersion relations. The time evolution with respect to light-cone proper time is mapped to a momentum space evolution with respect to mass squared as a dissipation like process involving decays of massive particles to lighter particles: p-adic cooling would be the counterpart for the process [K3, K4].

   At the level of M^8, the field equations can be solved exactly in terms of octonion analytic analytic functions f, in particular polynomials with Taylor coefficients in the extension E of rationals. The local G_2 transformations depending on the generalized complex coordinates of M^8 act dynamical symmetries.

Therefore it would seem that p-adicization is possible both at the level of H, M^8 and WCW.

1.1.2 Adelic physics

Adelic physics fuses various p-adic physics by introducing the notion of adele [L1, L2]. In the simplest view, adelic space-time surface can be regarded essentially as a Cartesian product of p-adic space-time surfaces characterized by different p-adic primes and satisfying the same field equations. Multi-p-p-adicity allows also to glue the sectors characterized by different p-adic primes together.

1. One can combine real numbers and p-adic number fields to what is essentially like a Cartesian product: number fields would be like pages of a book intersecting along rationals acting as the back of the book.

   Recently a slightly different view of how to fuse various p-adic physics to an analog of adeles has emerged [L9, L6] [K2]. One can glue two p-adic number fields together along p-adic numbers, which have expansions in terms of integers having both primes as factors. Excluding the expansions which are not in powers of prime, one obtains a structure looking like a Cartesian product of subsets of p-adic number fields, which contain only expansions in powers of the p-adic prime in question.

   The nice feature of this variant is that the transitions changing the value of the p-adic prime of the p-adic space-time surface might become possible. They would be due the presence of the regions in which the expansion of p-adic numbers defining the coordinates are with respect to an integer having both primes as factors. A phase transition changing the p-adic prime could start from a seed at which the pinary expansion has this property which then grows and transforms so that the new p-adic primes becomes dominant.

2. In p-adic mass calculations [L6] p-adic primes assumed to characterize elementary particles, in particular their mass scales. The p-adic prime would correspond to a ramified prime associated with a polynomial characterizing the particle as partonic 2-surface. These transitions might be relevant for the p-adic description of a transition changing the p-adic prime of the particle. The phase transition would be restricted to 2-D singularities of the 3-D light-like partonic orbit associated with the particles and affect the polynomial characterizing the partonic 2-surface and therefore also the spectrum of corresponding ramified primes. A quantum tunnelling between polynomials with different spectrum of ramified primes would be in question and is allowed by the holography=holomorphy vision.

3. Each extension of rational induces extensions of p-adic number fields and extension of the basic adele. Points in the extension of rationals are now common to the pages. The infinite hierarchy of adeles defined by the extensions forms an infinite library, one might say.

4. This leads to an evolutionary hierarchy. The order n of the Galois group as a dimension of extension of rationals is identified as a measure of complexity and of evolutionary level, "IQ". Evolutionary hierarchy is predicted.

5. Also a hierarchy of effective Planck constants interpreted in terms of phases of ordinary matter is predicted. X^4 decomposes to n fundamental regions related by Galois symmetry. Action is n times the action for the fundamental region. Planck constant h is effectively replaced with $h_{eff} = nh_0$, where $h_< h0$ is the minimal value of h_{eff}. Quantum coherence scales are typically proportional to h_{eff}. Quantum coherence in arbitrarily long scales is implied. Dark matter at the magnetic body of the system would serve as controller of ordinary matter in the TGD inspired quantum biology [L18].

   There are reasons to as whether h/h_0 could be the ratio R^2/L_p^2 for CP$_2$ length scale R deduced from p-adic mass calculations and Planck length L_P [L3]. The CP_2 radius R could actually correspond to L_P and the value of R deduced from the p-adic mass calculations would correspond to a dark CP_2 radius $\sqrt{h/h_0}l_P$.

   Also the notions of gravitational Planck constant [K1] [L5, L4], proposed first by Nottale [E1], and electric Planck constant [L7] emerge in the TGD framework. Gravitational (electric) Planck constant would characterize pairs of two masses (charges) and whereas ordinary Planck constant is usually regarded as a universal constant.

1.1.3 Holography = holomorphy vision and functional variants of p-adic number fields

Holography = holomorphy vision [L10, L12, L11] leads to a generalization of the notion of p-adic number field to its functional counterpart emerging naturally at the level of "world of classical worlds" (WCW). This leads to understanding of the origin of p-adic length scale hypothesis.

1. H-H vision relies on the notion of generalized holomorphic structure involving Hamilton-Jacobi structure [L8] in involving complex structure plus complex structure in M^4 plus complex structure in CP_2.

   Two analytic function pairs $f = (f_1, f_2)$, in particular, polynomials P_1 and P_2 (or analytic functions) define space-time surfaces as their roots. The extermely non-linear field equations reduce to local algebraic equations and the solutions are minimal surfaces. This happens irrespective of the action principle as long as the action is general coordinate invariant and expressible in terms of induced geometry. Only the singularities at which the minimal surface propert fails can distinguish between different classical actions. This results means universality of the dynamics and suggests that the values of action exponential are expressible in terms of number theoretic invariants.

   Twistor lift favors the sum of Kähler action and volume terms emerging from 6-D Kähler action in the product space of twistor spaces of M^4 and CP_2 by dimensional reducting giving rise to a generalization of twistor space having space-time surface as base space and sphere CP_1 as a fiber.

2. If one assumes that the Taylor coefficients are in some extension E of rationals, the reduction of field equations to algebraic equations implies that it is possible to define the p-adic variants of the space-time surfaces and WCW could have p-adic variants. The same applies at the level of M^8.

3. WCW would decompose to sectors characterized by the extensions E. The natural topology is therefore ultrametric and WCW would be like a spin glass landscape decompositing to regons characterized by p-adic numbers fields and their extensions and having functional fields as their generalizations.

   The same is true at the level of M^8 [L16, L14]: in this case the pair of analytic functions is replaced with a single octonioni analytic function with Taylor coefficients in E. The local G_2 transformations depending on the generalized complex coordinates of M^8 are dynamical symmetries as also the octonion analytic maps g defining hierarchies with respect to functional composition. Also now prime polynomials are well-defined.

4. The notion of p-adic number field generalizes. p-Adic analogs of function fields can be identified in terms of solutions of field equations. The functional composition of polynomial primes g=(P_1, P_2) with (f_1, f_2) defining the space-time surface defines what might be called hierarchies. In particular, the iterates of composition of (P_1, P_2) can act on (f_1, f_2) producing 4-D analogs of Mandelbrot fractals and Julia sets. One can identify primes polynomial pairs (P_1, P_2) as those having no further functional decomposition. In one variable case, relevant for M^8 , the polynomials with prime degree are polynomial primes. Also in H one can consider the hierarchies of iterates of $(P_1, 1)$ and $(1, P_2)$ acting on (f_1, f_2) . In this case prime degree gives prime polynomial.

5. Functional p-adic number fields represented as space-time surfaces can be assigned with prime polynomial pairs (P_1, P_2) and in special cases with $(P_1, 1)$ or $(1, P_2)$ having degree p emerge naturally. There is category morphism to ordinary p-adic number fields. All polynomials with the same prime degree could be physically equivalent, which means a huge symmetry. p-Adic length scale hypothesis emerges naturally from this picture and polynomials with degree 2 and 3 are physically very special since in this case polynomial equations for iterates can be solved explicitly. This gives rise to the p-adic length scale hypothesis stating that primes p near q^k are of special interest for $q = 2$ and $q = 3$.

1.2 The problem

p-Adic mass calculations involve canonical identification $I : x = \sum_n x_n p^n \sum x_n p^{-n}$ mapping the p-adic values of mass squared to real numbers. The momenta p_i at the p-adic side are mapped to real momenta $I(p_i)$ at the real side. Lorenz invariance requires $I(p_i \cdot p_j) = I(p_i) \cdot I(p_j)$. The predictions for mass squared values should be Lorentz invariant. The problem is that without additional assumptions the canonical identification I does not commute with arithmetics operations.

Sums are mapped to sums and products to products only at the limit of large p-adic primes p and mass squared values, which correspond to $x_n \leq\leq p$. The p-adic primes are indeed large: for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$. In this approximation, the Lorentz invariant inner products $p_i \cdot p_j$ for the momenta at the p-adic side are indeed mapped to the inner products of the real images: $I(p_i \cdot p_j) = I(p_i) \cdot I(p_j)$. This is however not generally true.

The question following.

1. Should this slight failure of Lorentz invariance be accepted as being due to the approximate nature of the p-adic physics or could it be possible to modify the canonical identification? It should be also noticed that in zero energy ontology [K5], the finite size of the causal diamond (CD) reduces Lorent symmetries so that they apply only to Lorenz group acting on either vertex of the CD.

2. Or could one consider something more elegant and ask under what additional conditions Lorentz invariance is respected in the sense that inner products for momenta on the p-dic side are mapped to inner products of momenta on the real side.

In the sequel a possible solution of this problem based on the notion of Teichmüller elements defining a representation of finite field G_p in the field of p-adic numbers is discussed.

2 Does the Lorentz invariance for p-adic mass calculations require the p-adic mass squared values to be Teichmüller elements?

The so called Teichmüller elements of the p-adic number field could allow to realize exact Lorentz invariance.

1. Teichmüller elements $T(x)$ associated with the elements of a p-adic number field satisfy $x^p = x$, and define therefore a finite field G_p, which is not the same as that given by p-adic integers modulo p. Teichmüller element $T(x)$ is the same for all p-adic numbers congruent modulo p and involves an infinite series in powers of p.

   The map $x \to T(x)$ respects arithmetics. Teichmüller elements of for the product and sum of two p-adic integers are products and sums of their Teichmüller elements: $T(x_1 + x_2) = T(x_1) + T(x_2)$ and $T(x_1 x_2) = T(x_1)T(x_2)$.

2. If the thermal mass squared is Teichmüller element, it is possible to have Lorentz invariance in the sense that the p-adic mass squared $m_p^2 = p^k p_k$ defined in terms of p-adic momenta p_k is mapped to $m_R^2 = I(m_p^2)$ satisfying $I(m_p^2) = I(p^k)I(p_k)$. Also the inner product $p_1 \cdot p_2$ of p-adic momenta mapped to $I(p_1 \cdot p_2) = I(p_1) \cdot I(p_2)$ if the momenta are Teichmüller elements.

3. Should the mass squared value coming as a series in powers of p mapped to Teichmüller element or should it be equal to Teichmüller element?

   (a) If the mass squared value is mapped to the Teichmüller element, the lowest order contribution to mass squared from p-adic thermodynamics fixes the mass squared completely. Therefore the Teichmüller element does not differ much from the p-adic mass squared predicted by p-adic thermodynamics. For the large p-adic primes assignable to elementary particles this is true.

(b) The radical option is that p-adic thermodynamics and momentum spectrum is such that it predicts that thermal mass squared values are Teichmüller elements. This would fix the p-adic thermodynamics apart from the choice of p-adic number field or its extension. Mass squared spectrum would be universal and determined by number theory. Note that the p-adic mass calculations predict that mass squared is of order $O(p)$: this is however not a problem since one can consider the m^2/p.

This would have rather dramatic physical implications.

1. If the allowed p-adic momenta are Teichmüller elements and therefore elements of G_p then also the mass squared values are Teichmm̈uller elements. This would mean theoretical momentum quantization. This would imply Teichmüller property also for the thermal mass squared since p-adic thermodynamics in the approximation that very higher powers of p give a negligible contribution give a finite sum over Teichmm̈uller elements. Number theory would predict both momentum and mass spectra and also thermal mass squared spectrum.

   What does it mean that the product of Teichmüller elements is Teichmüller element? The product xy can be written as $\sum_k (xy)_k p^k$, $(xy)_k = \sum_l x_{k-l} y_l$. For Teichmüller elements $(xy)_k$ has no overflow digits. This is true also for $I(xy)$ so that $I(xy) = I(x)I(y)$. Similar argument applies to the sum.

2. The number of possible mass squared values in p-adic thermodynamics would be equal to the p-adic prime p and the mass squared values would be determined purely number theoretically as Teichmüller representatives defining the elements of finite field G_p. The p-adic temperature [L13], which is quantized as $1/T_p = n$, can have only p values $0, 1, ...p-1$ and $1/T_p = 0$ corresponds to high temperature limit for which p-adic Boltzman weights are equal to 1 and the p-adic mass squared is proportional to $m^2 = \sum g(m)m/\sum(g(m))$, where $g(m)$ is the degeneracy of the state with conformal weight $h = m$. $T_p = 1/(p-1)$ corresponds to the low temperature limit for which Boltzman weights approach rapidly zero.

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