p-Adic numbers and TGD

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Abstract

p-Adic physics has become an essential part of quantum TGD and TGD inspired theory of consciousness. The proper interpretation of p-adic physics has turned out to be as the physics of cognition, intention, and imagination. The justification for this identification comes from the inherent nondeterminism of p-adic differential equations making possible to represent quantum jump sequences at spacetime level as p-adic spacetime regions. This representability (not faithfully) of quantum jumps classically explains the self-referentiality of cognition. In this article the basic properties of p-adics as completions of rationals and the possible correspondences between p-adics and reals are discussed. p-Adic length scale hypothesis stating that the primes near prime or prime power powers of two are especially important physically, is considered in biological length scales. Also the basic ideas about p-adic physics as physics of cognition are summarized.

1 Introduction

I ended up with the idea of p-adic physics for about decade ago. I had just a gut feeling that p-adics might be important for TGD and I started to play with them. As a physicist one of my first questions was 'What is the relationship between p-adics and reals?' and I soon ended up the concept of canonical identification mapping p-adics to reals continuously.

1.1 p-Adic mass calculations

I soon realized that so called Mersenne primes $M_n = 2^n - 1$, might be physically especially interesting: the observation was that fundamental mass scales of elementary particle physics seemed to correspond to ratios for the square roots of M_n. This led to the idea that the problem of understanding how elementary mass scales emerge, which is one of the greatest mysteries of elementary particle physics, could reduce to number theory. I indeed ended up to a model of elementary particle masses relying on following ingredients:

a) p-Adic variant of the conformal invariance of superstring models.

b) p-Adic thermodynamics for the superconformal representations.

c) Canonical identification mapping the predictions of p-adic mass squared values to their real counterparts.

d) p-Adic length scale hypothesis stating that p-adic prime p corresponds to a p-adic length scales $L_p = \sqrt{p}l$, $l \simeq 10^4$ Planck lengths and that primes $p \simeq 2^k$, k prime or power or prime are physically especially important.

The predictions of the mass calculations were really fantastic and only very few integer valued parameters were involved.

1.2 Questions

The mass calculations left many things open:

1. What is the origin of the conformal invariance?

Somewhat surprisingly, it turned out that there is whole handful of candidates for conformal symmetries.

a) I soon realized that so called CP_2 type extremals representing elementary particles in TGD indeed allow superconformal invariance. The lightlikeness condition for the lightlike random curve defining the M^4_+ projection of CP_2 type extremal gives standard gauge conditions defining conformal invariance.

b) For some time I identified this conformal invariance with the conformal invariance crucial for the construction of the configuration space geometry: any lightlike 3-surface, in particular the boundary of M^4_+, allows conformal transformations as symmetries: this makes 4-dimensional Minkowski space absolutely unique.

c) The conformal symmetry of CP_2 type extremals generalizes naturally to what might be called quaternion conformal invariance popping up naturally in the number theoretic formulation of TGD. This symmetry is not at all related to the configuration metric. Quaternions emerge indeed naturally when one replaces two-dimensional strings with 4-dimensional spacetime surfaces.

d) A further conformal invariance is related to the elementary particle horizons which are topologically 3- but metrically 2-dimensional surfaces at which the Minkowskian signature of metric of the elementary particle spacetime sheet transforms to Euclidian signature of metric for the CP_2 type extremal glued at it. This conformal invariance is very probably equivalent with the quaternion conformal invariance.

e) Even this is not enough! p-Adic numbers allow algebraic extensions of arbitrary dimension. If one requires that ordinary p-adic numbers allow square root, one ends up with 4-dimensional extensions for $p > 2$ and with 8-dimensional extensions for $p = 2$. This raised the question whether p-adic conformal invariance would be the proper generalization for the conformal invariance of string models. It turned out that this is very probably not the case: the proper generalization is provided by quaternion conformal invariance. p-Adic algebraic extensions play however role in p-adic physics and relate to mathematical cognition.

2. How generally canonical identification applies?

Does it apply also at spacetime level so that one could map real spacetime regions to p-adic and vice versa? What is the interpretation of this map? What about identification between reals and p-adics by common rationals? Is this identification also important?

3. What is the dynamical origin of the p-adic length scale hypothesis?

Here the model for elementary particles led to a possible explanation. CP_2 type extremals correspond to Euclidian spacetime regions and there is 3-dimensional surface at which metric signature transforms to Minkowskian one. At this surface metric is degenerate and this surface is metrically 2-dimensional. The metric two-dimensionality in fact gives rise to conformal invariance which corresponds to quaternion conformal invaraince. This elementary particle horizon is analogous to blackhole horizon and one can generalized Hawking Bekenstein law. In particular, if one requires that the radius of elementary particle horizon corresponds to p-adic length scale L_k one ends up with p-adic length scale hypothesis: the spacetime sheet at which CP_2 type extremal is topologically condensed, corresponds to p-adic length scale L_p, $p \simeq 2^k$, k prime or power of prime.

4. What is the origin and proper interpretation of p-adic physics?
I have considered many alternatives during the decade.

a) The first thing to ask is 'Does p-adic physics prevail below/above some cutoff length scale?'. Many colleagued answered 'Below Planck length scale'. My answer changed rapidly to 'Above some length scale', presumably p-adic length scale. p-Adic topology would be only effective topology making fractal looking structure to look smooth. Real smoothness should hold true below p-adic length scale. It might well be that this hypothesis is true but it has turned out that p-adics are more fundamental than a mere effective description.

b) The next realization was that systems called spin glasses allowing huge ground state degeneracy lead naturally to ultrametric topology. The discrete space of energy minima in fractal energy landscape (valleys inside valleys...) allows ultrametric distance function. Since also p-adic norm is ultrametric, one can consider the possibility that p-adic topology emerges in this manner as effective topology. Indeed the basic variational principle of TGD allows huge vacuum degeneracy. It is quite possible that p-adic topology indeed emerges as effective topology for the space of maxima of Kähler action (absolute minimum of Kähler action). But again: p-adic physics might be something much deeper.

c) The development of TGD inspired theory of consciousness led finally to what I regard the resolution of the problem. The classical nondeterminism of p-adic differential (and field) equations suggests that p-adic physics should be regarded as physics of cognition, imagination, and intention. p-Adic nondeterminism corresponds to the nondeterminism of imagination. p-Adic spacetime regions are topological correlates for thoughts.

1.3 p-Adic physics as physics of cognition

The identification of p-adic physics as physics of cognition led to a trivialization of several technical problems related to the construction of p-adic physics, and more generally, quantum theory in the configuration space consisting of 3-surfaces containing both real and p-adic regions. The basic difficulty in constructing p-adic physics by using real physics as a template was the lack of satisfactory definition of definite integral. It turned out that definite integral is not needed, if one assumes that p-adic configuration space degrees are so called zero modes, which means that each quantum jump involves complete localization in these degrees of freedom. This means that effectively classical degrees of freedom are in question. The world of cognition is indeed completely classical. This by no means trivializes p-adic physics. For instance, p-adic mass calculations make perfect sense since they relate to configuration space spin degrees of freedom, in which p-adic quantum physics is formally similar to real quantum physics.

Although the identification of p-adic physics as physics of cognition seems to be the fundamental one, p-adic physics might emerge as an effective description of fractal structures of real physics in TGD framework suggested strongly by quantum criticality of TGD Universe and by spin glass analogy. The length scale serving as effective p-adic length scales could be also dynamical in this case.

2 p-Adic numbers

2.1 p-Adic numbers as algebraic extensions of rationals

p-Adic numbers (p is prime: 2,3,5,...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers. p-Adic numbers are representable as power expansion of the prime number p of form:

$$x = \sum_{k\geq k_0} x(k)p^k,\ x(k) = 0, ..., p-1 \ . \tag{1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

2.2 p-Adic norm is ultrametric

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultrametricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \tag{4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \tag{5}$$

This division of the metric space into classes has following properties:

a) Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

b) Distances of points x and y inside single class are smaller than distances between different classes.

c) Classes form a hierarchical tree.

Notice that the concept of the ultrametricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [Parisi]. The emergence of p-adic topology as the topology of the effective spacetime would make ultrametricy property basic feature of physics.

3 The correspondence between p-adics and reals

One can imagine two basic types of correspondences between p-adics and reals: canonical identification and identification via common rationals.

3.1 Canonical identification

There exists a natural continuous map $Id : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k>N} y_k p^{-k} \ , \\ y_k &\in \{0, 1, .., p-1\} \ . \end{aligned} \tag{6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also desimal expansion is not unique $(1 = 0.999...)$ for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite number of pinary digits or single valued and discontinuous and nonsurjective if one makes pinary expansion unique by choosing the one with finite number of pinary digits. The finite number of pinary digits expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

3.2 Identification via common rationals

Besides canonical identification there is also a second natural correspondence between reals and p-adics. This correspondence is induced via common rationals in the sense that one can regard p-adics and reals as different completions of rationals and given rational number can be identified as an element or reals or of any p-adic number field.

a) For instance, if S-matrix is complex rational matrix, one can regard it as either real or p-adic S-matrix. The assumption that the so called CKM matrix describing quark mixings is complex rational, fixes with some empirical inputs the CKM matrix essentially uniquely.

b) If it is assumed that fundamental state space has complex rationals as a coefficient field, it becomes sensible to define tensor factors of Hilbert spaces belonging to different number fields because entanglement is possible with complex rational coefficients.

c) One could also see the basic physics as essentially rational and real and p-adic physics as different manners to see it. The generalized view about imbedding space would be as union of real and p-adic imbedding spaces glued together along common rational points. Spacetime surfaces would in turn decompose into p-adic and real regions glued together along boundaries defined by common rationals.

These examples suggest that this identification is crucial for the construction of quantum TGD is considered.

3.3 Canonical identification as cognitive map?

The question which manner to map the predictions of the p-adic physics to real numbers is the correct one or are both correct in some sense, tested my ability to tolerate mutually conflicting ideas for almost a decade. A more general manner to formulate the problem is as a question how real and p-adic physics relate to each other. This source of frustration disappeared when I realized that real and p-adic physics can be seen in a well-defined sense completions of what might be called algebraic physics and that p-adic physics is physics of cognitive representations.

Both identification maps indeed emerge very naturally if spacetime surfaces can be regarded as 4-surfaces imbedded to a rational imbedding space (in some preferred coordinates) and completed to either real or p-adic surfaces. To which number field completion occurs, depends on in which number field the solution of the field equations converves as power series, and spacetime surface decomposes into regions corresponding to different number fields. p-Adic regions look like fractal dust in a real topology.

In this framework the only sensical interpretation of the p-adic regions is as cognitive representations for the real regions containing matter and obeying the same field equations. The p-adic non-determinism inherent for the p-adic differential equations serves as the physical correlate of imagination. In this framework the identification via common rationals has an interpretation as a self-representation: real spacetime region and p-adic spacetime region are different completions of the common rationals to smooth surfaces obeying field equations. Canonical identification in turn can be interpreted as a map mapping real spacetime regions to p-adic spacetime regions: the p-adic images need not have common rationals with their real pre-images. A good example would be a cognitive representation of the external world inside brain as a p-adic region: sensory input would basically build this correspondence. Both the identification by common rationals and the canonical identification are characterized by a pinary cutoff: in the first case the pinary cutoff results already from the requirement of continuity and in the second case from the requirement that p-adic regions are smooth. This vision solves the conceptual problems but forces to radical modification of the basic dogmas about brain as the only seat of cognition.

4 p-Adic differential and integral calculus

4.1 p-Adic differential calculus

p-Adic differential calculus differs obeys the same formal rules as real differential calculus. p-analytic maps $g : R_p \rightarrow R_p$ satisfy the usual criterion of differentiability and are representable as power series

$$g(x) = \sum_k g_k x^k \ . \tag{9}$$

Also negative powers are in principle allowed. The rules of the p-adic differential calculus are formally identical to those of the ordinary differential calculus and generalize in a trivial manner for the algebraic extensions.

The class of the functions having vanishing p-adic derivatives is larger than in the real case: any function depending on a finite number of positive pinary digits of p-adic number and of arbitrary number of negative pinary digits has a vanishing p-adic derivative. This becomes obvious, when one notices that the p-adic derivative must be calculated by comparing the values of the function at nearby points having the same p-adic norm (here is the crucial difference with respect to real case!). Hence, when the increment of the p-adic coordinate becomes sufficiently small, p-adic constant doesn't detect the variation of x since it depends on finite number of positive p-adic pinary digits only.

p-Adic constants correspond to real functions, which are constant below some length scale $\Delta x = 2^{-n}$. As a consequence p-adic differential equations are non-deterministic: integration constants are arbitrary functions depending on a finite number of the positive p-adic pinary digits. This feature is central as far applications are considered and leads to the interpretation of p-adic physics as physics of cognition which involves imagination in essential manner. The classical non-determinism of the Kähler action, which is the key feature of quantum TGD, corresponds in a natural manner to the non-determinism of language in macroscopic length scales.

4.2 p-Adic integration

The definition of the p-adic integral functions defining integration as inverse of the differentation is straightforward and one obtains just the generalization of the standard calculus. For instance, one has $\int z^n = \frac{z^{n+1}}{(n+1)} + C$ and integral of the Taylor series is obtained by generalizing this. One must however notice that the concept of integration constant generalizes: any function $R_p \rightarrow R_p$ depending on a finite number of pinary digits only, has a vanishing derivative.

Consider next the definite integral. The absence of the well ordering implies that the concept of the integration range (a, b) is not well defined as a purely p-adic concept. A possible resolution of the problem is based on the canonical identification. Consider p-adic numbers a and b. It is natural to define a to be smaller than b if the canonical images of a and b satisfy $a_R < b_R$. One must notice that $a_R = b_R$ does not imply $a = b$, since the inverse of the canonical identification map is two-valued for the real numbers having a finite number of pinary digits. For two p-adic numbers a, b with $a < b$, one can define the integration range (a, b) as the set of the p-adic numbers x satisfying $a \leq x \leq b$ or equivalently $a_R \leq x_R \leq b_R$. For a given value of x_R with a finite number of pinary digits, one has two values of x and x can be made unique by requiring it to have a finite number of pinary digits.

One can define definite integral $\int_a^b f(x)dx$ formally as

$$\int_a^b f(x)dx \quad = \quad F(b) - F(a) \ , \tag{10}$$

where $F(x)$ is integral function obtained by allowing only ordinary integration constants and $b_R > a_R$ holds true. One encounters however a problem, when $a_R = b_R$ and a and b are different. Problem is avoided if the integration limits are assumed to correspond to p-adic numbers with a finite number of pinary digits.

One could perhaps relate the possibility of the p-adic integration constants depending on finite number of pinary digits to the possibility to decompose integration range $[a_R, b_R]$ as $a = x_0 < x_1 < ....x_n = b$ and to select in each subrange $[x_k, x_{k+1}]$ the inverse images of $x_k \leq x \leq x_{k+1}$, with x having finite number of pinary digits in two different manners. These different choices correspond to different integration paths and the value of the integral for different paths could correspond to the different choices of the p-adic integration constant in integral function. The difference between a given integration path and 'standard' path is simply the sum of differences $F(x_k) - F(y_k)$, $(x_k)_R = (y_k)_R$.

This definition has several nice features:

a) The definition generalizes in an obvious manner to the higher dimensional case. The standard connection between integral function and definite integral holds true and in the higher-dimensional case the integral of a total divergence reduces to integral over the boundaries of the integration volume. This property guarantees that p-adic action principle leads to same field equations as its real counterpart. It this in fact this property, which drops other alternatives from the consideration.

b) The basic results of the real integral calculus generalize as such to the p-adic case. For instance, integral is a linear operation and additive as a set function.

One can however represent also skeptic comments against this and also many other definitions of p-adic integral that I have discussed during last ten years.

a) This integral generalizes formally to the finite-dimensional case, but the lack of numerical definition suggests that this integral is not perhaps useful for defining variational principles. On the other hand the p-adic counterparts of the field equations associated with Kähler action make sense even if the Kähler action defined as p-adic valued spacetime integral does not make sense. Even absolute minimization of Kähler action could make sense if it can be defined by the p-adic counterparts of the algebraic conditions stating absolute minimization in the real case.

b) In infinite-dimensional configuration space the existence of p-adic valued integration measure is even more suspectible. Furthermore, if the exponent of Kähler action exists, it does not converge like its real counterpart since it has unit p-adic norm. This suggestes that p-adic degrees of freedom behave like zero modes: each quantum jump involves localization in these degrees of freedom so that it is not necessary to define p-adic configuration space integration.

5 p-Adic length scale hypothesis and biosystems

Primary p-adic length scales correspond to length scales $L_p = \sqrt{p}l$, $l \simeq 10^4$ Planck lengths (essentially equal to CP_2 length scale). One can also define a fractal hierarchy of secondary and higher p-adic length scales $L_{p,n} = p^{n/2}L_p$, $n = 1, 2, ...$ and also these length scales are of importance.

p-Adic length scale hypothesis states that the primes $p \simeq 2^k$, k prime or power of prime, are physically especially interesting. p-Adic length scale hypothesis emerges dynamically from the assumption that the radius of the elementary particle horizon corresponds to any p-adic length scale L_k (or even $L_{k,n}$), and from the generalization of Hawking-Bekenstein law relating to each other the p-adic entropy of elementary particle on one hand, and the area of the elementary particle horizon on the other hand. This idea was discussed very briefly in the introduction and the reader interested in details can consult the chapter "Physical Building Blocks" of [2].

It must be emphasized that p-adic length scales refer to the sizes of *real* spacetime regions. It is yet far from clear how this correspondence emerges but it seems obvious that this must relate directly to the transformation of intention to action: a real region which transform to p-adic regions in quantum jump must have typical size of order L_p.

5.1 Biologically relevant p-adic length scales

The following table lists the p-adic length scales L_p. p near prime power of 2, which might be interesting as far as biosystems are considered. Some overall scaling factor r of order one is present in the definition of the length scale and it is interesting to look whether with a suitable choice of r it is possible to identify p-adic length scales as biologically important length scales. The requirement that $L(151)$ corresponds to the thickness of the cell membrane about 10^{-8} meters gives $r \simeq 1.2$.

k	127	131	137	139	149
$L_p/10^{-10}m$	.025	.1	.8	1.6	50
k	151	157	163	167	169
$L_p/10^{-8}m$	1	8	64	256	512
k	173	179	181	191	193
$L_p/10^{-4}m$	.2	1.6	3.2	100	200
k	197	199	211	223	227
L_p/m	.08	.16	10	640	2560

Table 1. p-Adic length scales $L_p = 2^{k-151}L_{151}$, $p \simeq 2^k$, k prime, possibly relevant to bio physics. The last 3 scales are included in order to show that twin pairs are very frequent in the biologically interesting range of length scales. The length scale $L(151)$ is take to be thickness of cell scale, which is 10^{-8} meters in good approximation.

The study of the table supports the idea that p-adic length scale hypothesis might have explanatory power in biology. What is remarkable is the frequent occurrence of twin length scales related by a factor 2 in the range of biologically interesting p-adic length scales: only 3 of 15 primes in the range do not belong to a twin pair! The fact that these length scales seem to correspond to biologically interesting length scales suggests that twins might be related to replication phenomenon and to the possible 2-adicity in biology: for a given twin pair the smaller length scale would define basic 2-adic length scale. In the following the scales denoted by $\hat{L}(n)$ are related by a factor $r = 1.2$ to the lengthscales $L(n)$ appearing in the table above.

a) $\hat{L}(137) \simeq 7.84E-11\ m$, $\hat{L}(139) \simeq 1.57E-10m$ form a twin pair. This length scales might be associated with atoms and small molecules.

b) The secondary p-adic length scale $\hat{L}(71,2) \simeq .44\ nm$ corresponds to the thickness of the DNA strand which is about .5 nm. Both DNA strand and double helix must correspond to this length scale. The secondary p-adic length scale $\hat{L}(73,2) \simeq 1.77\ nm$ is longer than the thickness of DNA double strand which is roughly 1.1 nm. Whether one could interpret this length scale as that associated with DNA double strand remains an open question. Alpha helix, the basic building block of proteins provides evidence for has radius $1.81\ nm \sim \hat{L}(139)$ and the height of single step in the helix is $.544\ nm$.

c) $\hat{L}(149) \simeq 5.0\ nm$ and $\hat{L}(151) \simeq 10.0\ nm$ form also a twin pair. The thickness of cell membrane of order $10^{-8}\ m \sim \hat{L}(151)$. Cell membrane consists of two separate membranes and the thickness of single membrane therefore corresponds to $\hat{L}(149)$. Microtubules, which are basic structural units of the cytoskeleton, are hollow cylindrical surfaces having thickness $d \sim 11\ nm$, which is not too far from the length scale $\hat{L}(151)$. It has been suggested that microtubules might play key role in the understanding of biosystem as macroscopic quantum system.

c) If neutrinos have masses of order one eV as suggested by recent experiments then the primary condensation level of neutrinos could correspond to $k_Z = 167$ or $k_Z = 13^2 = 169$ and would be the level at which nuclei feed their Z^0 gauge charges. This level is many particle quantum system in p-adic sense and p-adic effects are expected to important at this condensation level. Chirality selection should take place via the breaking of neutrino superconductivity at this level and involve the generation of Z^0 magnetic fields at some level $k < k_Z$, too. $k = 151$ is a good candidate for the level in question.

e) In the previous version of this chapter it was stated that $\hat{L}(167) = 2.73\ \mu m$, $\hat{L}(169 = 13^2) = 5.49\ \mu m$ form a twin pair and correspond to typical length scales associated with cellular structures. Neutrino mass calculations give best predictions for $k = 169$ and this suggests that the generalization of 'k = prime' to 'k = power of prime' should be considered: generalization would allow also $k = 169$ as basic length scale. Also blackhhole elementary particle analogy suggests the generalization of the length scale hypothesis. Furthermore, only $k = 169$ would appear as a new length scale between electron length scale and astrophysical length scales ($k = 3^5, 2^8, 17^2$)! This suggests that the length scales $L(167)$ and $L(169)$ might form effective twin pair. That this could be the case is suggested by the fact that so called epithelial sheets appearing in skin, glands, etc., consisting of two layers of cells play in biosystems same role as cell membranes and are generally regarded as a step of bioevolution analogous to the formation of cell membrane.

f) $\hat{L}(173) = 2.20 \cdot 10^{-5}\ m$ might correspond to a size of some basic cellular structure (A structure consisting of 64 cell layers?). $\hat{L}(179) = 1.75 \cdot 10^{-5}\ m$ and $\hat{L}(181) = 3.52 \cdot 10^{-4}\ m$ form a twin pair. Later it will be found that the pair $k = 179, 181$ might correspond to basic structures associated with cortex.

g) Length scales $\hat{L}(191) = 1.12\ cm$, $\hat{L}(193) = 2.24\ cm$ and $\hat{L}(197) = 9.0\ cm$. $\hat{L}(199) = 18.0\ cm$ are again twins.

5.2 The special role of the p-adic length scale $L(k_Z)$

The length scale $L(k_Z))$ ($L(167) \sim 2.286E-6\ m$) corresponds to typical cell length scale and has very special role in the structure of topological condensate. According to the consideration of condensed matter chapter $k = k_Z$ level is the primary condensation for neutrinos and the first level at which nuclei feed their Z^0 gauge fluxes. $k = k_Z$ p-adic cube is genuine p-adic many particle quantum system. Join along boundaries bonds between $k = k_Z$ blocks provide a manner to construct even larger quantum systems at $k = k_Z$ level. Also $k > k_Z$ levels give rise to quantum systems but with small particle number (particles are now mainly # throats). This suggests that Z^0 fields and neutrinos are in key role in understanding the properties of biosystems. It was already found in the chapter 'TGD and condensed matter' that chirality selection in vivo could be understood in ters of neutrino magnetization in cell length scales.

Topological condensate provides a natural framework for a structured information processing and it would be surprising if biosystems would not have discovered this possibility. One manner to transfer information between different condensate levels is via penetration of gauge fields from lower to higher level. The simplest mechanism is induction, which is especially natural in case of magnetic fields: the # throats on the boundaries of condensate block k_1 rotate in external magnetic field at level $k > k_1$ and the resulting current creates nearly constant magnetic field inside the condensed block. A more exotic mechanism is the direct penetration via formation of # contacts with the quantized magnetic flux: this mechanism was described in more detail in condensed matter chapter.

At levels $k > k_Z$ the direction of the information transfer is mainly from longer to shorter p-adic length scales: Z^0 (or ordinary) magnetic field at level $k > k_Z$ can induce magnetic fields at levels k up to $k = k_Z$. The motion of the nuclear Z^0 # throats in Z^0 magnetic fields at $k = k_Z$ level is bound to create Z^0 magnetic fields inside nuclei so that one obtains direct connection from $k = k_Z$ to $k = k_{nucl} = 113$ level. The motion of Z^0 #

throats in Z^0 fields at $k = k_Z$ level induces also the motion of electromagnetic nuclear # throats at $k = k_{em} = 131$ condensate level and can force such exotic effects as topological evaporation of atom or molecule, when it reaches the boundary of $k = 131$ block. Also the motion of electromagnetic # throats at levels $k < k_Z$ besides inducing magnetic fields at higher condensate levels induces the motion of # Z^0 # throats at level $k = k_Z$. This means that feedback loop $k = k_Z \rightarrow k_{nucl} \rightarrow k_{em} \rightarrow ..... \rightarrow k_Z$ becomes possible. The levels $k = 113$, $k = 131$ and $k = k_Z$ are in fundamental role in this system. The most interesting level is however $k = k_Z$ since only at this level genuine p-adic (very) many particle quantum system is realized.

5.3 Are also Gaussian primes and Eisenstein primes important?

Besides ordinary primes also Gaussian and Eisenstein primes exists and it seems that one define the notion of G-adic and E-adic number fields. This makes these primes very interesting from the point of view of biosystems.

5.3.1 Gaussian primes

Gaussian primes consist of complex integers $e_i \in \{\pm 1, \pm i+\}$, ordinary primes $p \ mod \ 4 = 3$ multiplied by the units e_i to give four different primes, and complex Gaussian primes $r \pm is$ multiplied by the units e_i to give 8 primes with the same modulus squared equal to prime $p \ mod \ 4 = 1$. Every prime $p \ mod \ 4 = 1$ gives rise to 8 nondegenerate Gaussian primes. Pythagorean phases correspond to the phases of the squares of complex Gaussian integers $m + in$ expressible as products of even powers of Gaussian primes $G_p = r + is$:

$$G_p = r + is \ , \quad G\overline{G} = r^2 + s^2 = p \ , \quad p \text{ prime } \& \ p \ mod \ 4 = 1 \ . \tag{11}$$

The general expression of a Pythagorean phase expressible as the phase for a product of even number of Gaussian primes is

$$U = \frac{r^2 - s^2 + i2rs}{r^2 + s^2} \ . \tag{12}$$

By multiplying this expression by a Gaussian prime i, one obtains second type of Pythagorean phase

$$U = \frac{2rs + i(r^2 - s^2)}{r^2 + s^2} \ . \tag{13}$$

5.3.2 Eisenstein primes

Whereas Gaussian primes rely on modulo 4 arithmetics for primes, Eisenstein primes rely on modulo 3 arithmetics. Let $w = exp(i\phi)$, $\phi = \pm 2\pi/3$, denote a nontrivial third root of unity. The number 1-w and its associates obtained by multiplying this number by ± 1 and $\pm i$; the rational primes $p \ mod \ 3 = 2$ and its associates; and the factors $r + sw$ of primes $p \ mod \ 3 = 1$ together with their associates, are Eisenstein primes. One can write Eistenstein prime in the form

$$\begin{aligned} E_p(r,s) &= r-\frac{s}{2}+is(r-\frac{s}{2})\sqrt{3} \ , \\ r^2+s^2-rs &= p \ . \end{aligned} \tag{14}$$

What might be called Eisenstein triangles correspond to the products of powers of the squares of Eisenstein primes and have integer-valued long side. The sides of the orthogonal triangle associated with a square of Eisenstein prime $E_p(r,s)$ have lengths

$$(r^2-rs-\frac{s^2}{2} \ , \ s(r-s)\frac{\sqrt{3}}{2} \ , \ p=r^2+s^2-rs) \ .$$

Eisenstein primes clearly span the ring of the complex numbers having the general form $z=(r+i\sqrt{3}s)/2$, r and s integers. To my very restricted best knowledge, the other algebraic extensions of integers do not allow the notion of prime number.

5.3.3 G-adic and E-adic number fields

It seems possible to generalize the notion of p-adicity so that could speak about G-adic and E-adic number fields. The properties of the Gaussian and Einsenstein primes indeed strongly suggest a generalization for the notion of p-adic numbers to include what might be called G-adic or E-adic numbers.

a) Consider for definiteness Gaussian primes. The basic point is that the decomposition into a product of prime factors is unique. For a given Gaussian prime one could consider the representation of the algebraic extension involved (complex integers in case of Gaussian primes) as a ring formed by the formal power series

$$G=\sum_n z_n G_p^n \ . \tag{15}$$

Here z_n is Gaussian integer with norm smaller than $|G_p|$, which equals to p for $p \ mod \ 4=3$ and $\sqrt{p}$ for $p \ mod \ 4=1$.

b) If any Gaussian integer z has a unique expansion in powers of G_p such that coefficients have norm squared smaller than p, modulo G arithmetics makes sense and one can construct the inverse of G and number field results. For $p \ mod \ 4=1$ the extension of the p-adic numbers by introducing $\sqrt{-1}$ as a unit is not possible since $\sqrt{-1}$ exists as a p-adic number: the proposed structure might perhaps provide the counterpart of the p-adic complex numbers in case $p \ mod \ 4=1$. Thus the question is whether one could regard Gaussian p-adic numbers as a natural complexification of p-adics for $p \ mod \ 4=1$, perhaps some kind of square root of R_p, and if they indeed form a number field, do they reduce to some known algebraic extension of R_p?

c) In case of Eisenstein numbers one can identify the coefficients z_n in the formal power series $E=\sum z_n E_p^n$ as Eisenstein numbers having modulus square smaller than p associated with E_p and similar argument works also in this case.

d) What is interesting from the physics point of view is that for $p \ mod \ 4=1$ the points G_p^n and E_p^n are on the logarithmic spiral $z_n=p^{n/2}exp(in\phi_0/2)$, where ϕ is the Pythagorean (Eisenstein) phase associated with G_p^2 (E_p^2). The logarithmic spiral can be written also as $\rho=exp(nlog(p)\phi/\phi_0)$. This reminds strongly of the logarithmic spirals, which are fractal structures frequently encountered in self-organizing systems: perhaps G- and E-adics might provide the mathematics for the modelling of these structures.

d) p-Adic length scale hypothesis should hold true also for Gaussian primes, in particular, Gaussian Mersennes of form $(1 \pm i)^k - 1$ should be especially interesting from TGD point of view.

i) The integers k associated with the lowest Gaussian Mersennes are following: $2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 11$ $k = 113$ corresponds to the p-adic length scale associated with the atomic nucleus and muon. Thus all known charged leptons, rather than only e and τ, as well as nuclear physics length scale, correspond to Mersenne primes in the generalized sense.

ii) The primes $k = 151, 157, 163, 167$ define perhaps the most fundamental biological length scales: $k = 151$ corresponds to the thickness of the cell membrane of about ten nanometers and $k = 167$ to cell size about 2.56 μm. This strongly suggests that cellular organisms have evolved to their present form through four basic stages.

iii) $k = 239, 241, 283, 353, 367, 379, 457$ associated with the next Gaussian Mersennes define astronomical length scales. $k = 239$ and $k = 241$ correspond to the p-adic time scales .55 ms and 1.1 ms: basic time scales associated with nerve pulse transmission are in question. $k = 283$ corresponds to the time scale of 38.6 min. An interesting question is whether this period could define a fundamental biological rhytm. The length scale $L(353)$ corresponds to about 2.6×10^6 light years, rouhgly the size scale of galaxies. The length scale $L(367) \simeq \times 3.3 \times 10^8$ light years is of same order of magnitude as the size scale of the large voids containing galaxies on their boundaries (note the analogy with cells). $T(379) \simeq 2.1 \times 10^{10}$ years corresponds to the lower bound for the order of the age of the Universe. $T(457) \sim 10^{22}$ years defines a completely superastronomical time and length scale.

e) Eisenstein integers form a hexagonal lattice equivalent with the root lattice of the color group $SU(3)$. Microtubular surface defines a hexagonal lattice on the surface of a cylinder which suggests an interpretation in terms of E-adicity. Also the patterns of neural activity form often hexagonal lattices.

5.3.4 Do Gaussian Mersennes define 'miracle frequencies' in living matter?

Ordinary and Gaussian Mersenne primes are of special importance in elementary particle length scales. All charged leptons, atomic nuclei, hadrons and intermediate gauge bosons correspond to ordinary or Gaussian Mersennes. The number theoretical, and there are reasons to assume that also biological, miracle is that there are four subsequent Gaussian Mersennes in the biologically most interesting length scale range. The values of k for these length scales L_p, $p \simeq 2^k$, k prime, are $k = 151, 157, 163, 167$ and correspond to the p-adic length scales 10 nm, 80 nm, 640 nm, and 2560 nm. The next p-adic length scale is also very special and corresponds to $k = 13^2 = 169$ which is not prime but a power of prime and very rare as such. It is quite possible that neutrinos could metastably topologically condense at these length scales so that one would have four metastable neutrino physics besides the stable one corresponding to $k = 169$ (this on basis of the data about neutrino mass squared differences, see the chapter "p-Adic Mass Calculations: Elementary Particle Masses" of [2]).

The photon energies corresponding to these length scales are $E(151) = 124.0$ eV (UV), $E(157) = 15.5$ eV (UV), $E(163) = 1.9375$ eV (red light) and $E(167) = .4844$ eV (near infrared). The energy corresponding to $k = 169$ is $E(169) = .2422$ eV. One must notice that there is few per cent uncertainty related to an overall scaling of length scales and energies. These energies indeed seem to correspond to biologically important photon energies.

a) $E(163) = 1.9375$ eV corresponds to wavelength of 640 nm which is with .6 per cent accuracy equal to the wave length 644 nm of the photon absorbed in photosynthesis associated with chlorophyll b). For chlorophyll a) the wavelength is 680 nm and deviation is 6 per cent. This suggests that photosynthesis leads to a generation of positive energy ME representing the stored energy and having length of near to $L(167)$.

b) From the yield of 48 kJ/mole of energy in ADP-to-ATP transformation, .4976 eV corresponds to the energy liberated when ATP decays to phosphor atom and ADP and is few per cent higher than $E(167) = .4844$ eV. In the spirit of the topological self-referentiality, one might play with the thought that also the stored energy, rather than only binding energy, is represented topologically. If so, this energy might be simply stored as positive energy ME carrying this energy disappearing when ATP gives up its energy. It is unclear whether the vibration energy quantum .52 eV of water hydrogen bond could relate to $E(167)$.

In the chapter "Biosystems as superconductors" a model for ATP as a universal 'energy currency' is developed. The model is based on the hypothesis that $E(167)$ MEs, rather than theoretically and empirically questionable high energy phosphate bonds, serve as the energy currency. This leads also to a model for the coherent locomotion relying on the assumption that the hydrogen ion current accompanying the phosphorylation of ADP molecules to ATP molecules is generated by the leakage of the protonic supra currents from the flux tubes of Earth's magnetic field to the atomic spacetime sheets. The macroscopic quantum coherence of the protonic supra currents allows to understand the coherency of the locomotion, which is miracle in the framework of the standard biochemistry.

c) $k = 169$ corresponds to energy $E(169) = .2422$ eV and belongs to the region of hydrogen bond energies, which depend on which kind of molecules hydrogen bond connects with each other. The range of weak hydrogen bond energies is $.13 - .3$ eV and in the near infrared. Also strong hydrogen bonds with energies extending up to 1.6 eV are possible but the hydrogen bonds associated with the biological molecules such as those connecting the DNA nucleotides are weak. The maximum binding energy for water hydrogen bond equals to $E(169)$ with one per cent accuracy. Notice however that hydrogen bond energy depends on its environment: typically the energy of the first bond in DNA is largest which gives rise to what might be called zipper effect. Negative energy MEs with this frequency should be very important and allow better understanding of the collective properties of water.

Sol-gel transition involves the generation of hydrogen bonds and thus $k = 169$ MEs might be involved with this transition. Hence it would be interesting to look for the effects of coherent light with this frequency on water and to the sol-gel phase transition and its reversal. Also irradiation of DNA by photons with energy $E(167)$ might yield interesting effects. It deserves to be noticed that $E(169)$ which correspond rather nearly to the hydrogen bond energy is liberated together with the corresponding momentum when only the second member of $k = 167$ ME pair liberates its energy.

d) What about the miracle frequencies in ultraviolet? A not very plausible possibility is that these frequencies are associated with atomic transitions. They could also correspond to energies associated with structures with corresponding lengths. For $k = 151$ MEs parallel to lipids of cell membrane are a possible candidate and it is known that the charging of the mitochondrial energy batteries occurs at its membrane. $k = 163$ and $k = 167$ seem to be related to metabolism and one can wonder whether the same could hold true for all the miracle length scales.

i) Perhaps the simplest possibility is that MEs with length of $L(167)$ are in question but em field corresponds to $n = 2^8 = 256$ harmonic serving as a topological correlate for a Bose-Einstein condensate of 256 photons with energy $E(167)$. During the discharging of the mitochondrial energy battery the value of n would gradually decrease. During the charging process, which perhaps involves p-adic-to-real phase transition representing the transformation of intention to action, the reversal of this process would occur. It is also possible that the notion of momentum battery makes sense. During discharging coherent momentum would be given to the biomolecules involved. This could make possible coherent locomotion at the cellular level.

ii) If one assumes that also 124.0 eV and 15.5 eV correspond to minimum length MEs representing energy packets, one ends up to an alternative idea about how metabolism might work. 124 eV bunches of energy with length equal the cell membrane thickness could be first divided to 8 bunches of 15.5 eV at the membrane of the mitochondrias, then these bunches could be divided to .19 eV bunches and finally .48 eV bunches would result. This would be like wares coming to a market store in big packets containing smaller packets containing... Now however every subpacket of the energy packet have larger size that the packet by uncertainty principle. The problem here is that rather complex topological processes are needed to liberate the energy in this case.

To sum up, the study of the effect of the 'miracle frequencies' in living matter might be very revealing concerning the understanding of the biocontrol and demonstrate unexpected effects.

6 p-Adic physics as physics of cognition

6.1 p-Adic non-determinism and cognition

p-Adic non-determinism follows from the fact that functions with vanishing derivatives are piecewise constant functions in the p-adic context. More precisely, p-adic pseudo constants depend on the pinary cutoff of their arguments and replace integration constants in p-adic differential equations. In case of field equations this means roughly that the initial data are replaced with initial data given for a discrete set of time values chosen in such a manner that unique solution of field equations results. Solution can be fixed also in a discrete subset of rational points of the imbedding space. Presumably the uniqueness requirement implies some unique pinary cutoff.

Thus the spacetime surfaces representing solutions of p-adic field equations are analogous to spacetime surfaces consisting of pieces of solutions of the real field equations. Thus p-adic reality is much like thedream reality consisting of rational fragments glued together in illogical manner or pieces of child's drawing of body containing body parts in more or less chaotic order.

The obvious interpretation for the solutions of the p-adic field equations is as a geometric correlate of imagination. Plans, intentions, expectations, dreams, and cognition in general are expected to have p-adic cognitive spacetime sheets as their geometric correlates. A deep principle seems to be involved: incompleteness is characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

The basic quiding principle in the construction of quantum TGD is the correspondence between quantum and classical physics, the former being realized at the level of infinite-dimensional configuration space and the latter being realized at spacetime level. This correspondence can be generalized even to the level of quantum jump sequences defining selves in the sense that quantum jump sequences characterizing contents of consciousness of self can (by p-adic nondeterminism) be represented cognitively as p-adic spacetime regions. Real spacetime regions behave also non-deterministically by the nondeterminism of Kähler action and this nondeterminism can be identified as the nondeterminism of the sensory (symbolic) representations provided by language.

6.2 p-Adic–real phase transitions and matter-mind interaction

If one accepts the idea that real and p-adic spacetime regions are correlates for matter and cognitive mind, one encounters the question how matter and mind interact. A good candidate for this interaction is the phase transition leading to a transformation of the real spacetime regions to p-adic ones and vice versa. These transformations can take place in quantum jumps. p-Adic-to-real phase transition would have interpretation as a transformation of thought into a sensory experience (dream or hallucination) or to an action. The reverse phase transition might relate to the transformation of the sensory experience to cognition. Sensory experiences could be also transformed to cognition by initial values realized as common rational points of a real spacetime sheet representing sensory input and a p-adic spacetime sheet representing the cognitive output. In this case the cognitive mental image is unique only in case that p-adic pseudo constants are ordinary constants.

6.3 p-Adic teleportation

Massless extremals (MEs) are an excellent candidate for a hierarchy of life forms representing MEs. MEs represent classical gauge fields propagating with a light velocity such that the shape of the wave form is preserved. They allow the coding of any pulse shape to the shape of the classical gauge field and are thus optimal for representing information classically. This is why MEs are in a key role in TGD based theory of consciousness including the model of EEG (see the chapter "Quantum model of EEG and nerve pulse"). For the p-adic MEs conservation laws allow reflection in a spatial or temporal direction, and one can consider gluing of pieces of ME to get zigzag curves with reflections in spatial or time direction. In the reflection to the direction of geometric past a time reversed copy of the cognitive representation is formed, in the next reflection a copy of the original is formed, etc... This mechanism makes possible both the meme replication and their transmission with a superluminal velocity. Even transfer of memes to the geometric past becomes possible. Time reversed cognition is the basic prediction very much analogous to the prediction of the antimatter in Dirac's theory of electron. Obviously p-adic teleportation and real–p-adic phase transition provide also general mechanisms for a large variety of paranormal effects.

6.4 Cognitive degeneracy and the survival of the fittest

The construction of quantum TGD suggests strongly that p-adic spacetime regions do not contribute to configuration space geometry and thus to physics in the conventional sense. This has nontrivial implications. First of all, all spacetime surfaces which differ only cognitively are physically equivalent and one can speak about cognitive degeneracy. This means that physical system with a large cognitive degeneracy is analogous to a system with a large state degeneracy. If the final states of quantum jumps have roughly the same probabilities, this means that quantum jumps lead with highest probability to those states for which cognitive degenerary is highest. The mere ability to imagine would mean winning in the fight for survival. The possible weak point of this hypothesis is the assumption that probabalities for various final states are roughly the same. Indeed, if the cognitive degeneracy is roughly equal to the negative exponent of the Kähler function, as proposed earlier (see the chapter "Information theoretic aspects of TGD inspired theory of consciousness"), the two exponents cancel in the total probability for quantum jump to given physical state, and the argument fails.

An alternative, and perhaps a more realistic, manner to see the situation is that a physical system with high cognitive degeneracy has large repertoare of transformations of cognitive spacetime sheets to real spacetime sheets and is thus highly adaptive and survives for this reason.

6.5 How to test p-adic physics?

The obvious question is how to test p-adic physics empirically. Since the times of Newton physicists have modelled the world using solely real numbers. Hence a careful reconsideration of the relationship between physical theories and experimental physics is needed before one can answer this question.

The basic heuristic guideline is that thinking is essentially p-adic sensory experiencing. Hence the reduction of the Cartesian theories–experimental science dichotomy to p-adic–real dichotomy seems natural. Just as experimental science is an extension of the everyday real sensory experience, theories would represent an extension of the everyday p-adic sensory experience (common sense thinking). Thus the basic test is how well p-adic physics based theories describe cognition.

Also indirect experimental testing is possible. The p-adic models for physical systems are models for cognitive models of real physics. The successes of these highly predictive models of models support the paradigm of p-adic cognition. A representative example is provided by the elementary particle mass calculations based on p-adic thermodynamics and characteristically involving only very few integer valued parameters and exponentially sensitive dependence of the masses on single integer parameter. Therefore either the success of the model is extremely unprobable statistical miracle or the model indeed describes physical reality (see the fourth part of "TGD and p-Adic Numbers" [2]. A second example is the consistency of the very strong predictions of p-adic length scale hypothesis with empirical facts. It is also evident that, in TGD framework at least, the physics of matter must be based on real numbers. For instance, p-adic non-determinism would mean that basic conservation laws would not hold true expect in piecewice manner: at the level of imagination this cannot be allowed but not in the laboratory.

p-Adic–real phase transitions transform thought to action and sensory input to thought.

The theories of brain functioning should at the fundamental level involve modelling of these transitions and the success or failure of these models serves as a further test for p-adic physics as physics of cognition.

7 Problems related to the notion of self, interaction between matter and mind, and construction of quantum TGD

In the sequel I want to discuss a triangle in which the three vertices correspond to problems related to a) the notion of self, b) the quantum description of matter-mind (cognition-matter) interaction, and c) the generalization of configuration space metric and spinor structure and the construction of configuration space spinor fields needed when three-surfaces are assumed to consist of both real and p-adic regions.

What makes these problems very closely related is the hypothesis that p-adic physics is the physics of cognition whereas real physics is the physics of matter.

7.1 Entanglement as quantum description of cognition-matter interaction?

Cognition and matter certainly interact. The open question is how this occurs.

a) At purely geometric level the adjacent real and p-adic regions have a common boundary, which consists of common rational points. Together with the field equations this implies that the physics in the p-adic region is correlated with that of an adjacent real region so that p-adic region forms a cognitive representation of the real region classically.

b) The quantum jump in which p-adic region is transformed to a real region has interpretation as a transformation of an intention to action.

c) Generation of quantum entanglement is the fundamental interaction at quantum level if one forgets everything related to the geometry. The question is whether also quantum entanglement between p-adic and real degrees of freedom is possible and makes possible generation of correlations between the real and p-adic systems. If p-adic-real quantum entanglement is not possible, it seems that there can be no quantum correlation between contents of cognition and real world. One can argue that even ordinary quantum measurement involves a step correlating the reading of the measurement apparatus with the cognitive mental image localizable in brain. Most naturally the entanglement between the spacetime regions belonging to different number fields would be reduced in the state function reduction step of the quantum jump followed by the state preparation phase.

Thus one must ask whether one can generalize quantum theory in such a manner that this entanglement is possible. It seems that the vision about quantum physics based on rational numbers at fundamental Fock space level might allow to understand the nature of the needed generalization. The idea is very simple: rational numbers are common to both reals and all p-adic number fields, and if entanglement coefficients between orthonormal state basis in different number fields are rational (or algebraic) numbers, inner products are rational (algebraic) number-valued.

7.2 Problems related to the notion of self

The notions of self and quantum jump have developed slowly. In the beginning quantum jump was regarded as something totally irreducible without any further structure. Gradually the notion of quantum jump differentiated to a structure containing the unitary time development characterized by S-matrix; the counterpart of state function reduction implying generalization of the standard quantum measurement theory; and the part identifiable as a state preparation process. Only quite recently I realized that only bound state entanglement is stable against state preparation and that this state of affairs has very deep implications.

The notion of self emerged from the need to understand the historical dimension of self identity and the autonomous nature of conscious systems. Self as a system able to remain unentangled in subsequent quantum jumps allows to formalize mathematically these aspects of having self. Also this notion has evolved and a beautiful connection with statistical physics and the theory of qualia was one outcome of this development. In the real context it seems that selves are not possible at all since even the slightest perturbation generates quantum entanglement.

The simplest hypothesis is that spacetime surface decomposes to real and p-adic regions, and the regions belonging to different number fields do not quantum entangle at all. This hypothesis has intuitive appeal but, as it seems, does not allow the generation of quantum correlation between cognitive states and states of matter. One can however weaken this hypothesis by assuming that the unitary operator U can generate entanglement between p-adic and real spacetime regions but that state function reduction reduces this entanglement so that self always corresponds to a well defined number field.

For the states normalized to unity (common to both p-adics and reals) rational (algebraic number) entanglement between different number fields indeed seems to be possible. The reason is that inner products containing the physics are rational (algebraic) for these states and it is s matter of taste to which number field one assumes them to belong. Although these entangled states itself would not belong to any definite number field, their inner products would make sense.

7.3 Generalizing the construction of the configuration space geometry

A problematics analogous to that related with the entanglement between real and p-adic number fields is encountered also in the construction of the configuration space geometry. The original construction was performed in the real context. What is needed are Kähler geometry and spinor structure for the configuration space of three-surfaces, and a construction of the configuration space spinor fields. What (as I believe) solves these immense architectural challenges are the equally immense symmetries of the configuration space (superconformal and supercanonical symmetries). What I hope that everything of physical interest reduces to the level of algebra (rational or algebraic numbers) and that topology (be it real or p-adic) disappears totally at the level of the matrix elements of the metric and of S-matrix.

7.3.1 Generalizing the construction for configuration space metric

It is not enough to generalize this construction to the p-adic context or infinite-p p-adic context. 3-surfaces contain both real and p-adic regions and should be able to perform the construction for this kind of objects.

a) Very naively, one could start from the Riemannian construction of the line element which tells the length squared between infinitesimally close points at each point of the Riemann manifold. The notion of line element involves the notion of nearness and one obviously cannot do without topology here. The line element makes formally sense sense for real and p-adic contexts but not for the situation in which 3-surface contains both real and p-adic regions: it does not make sense to sum real and p-adic line-elements together. One can however construct a collection of real and p-adic line-elements coming from various regions of the 3-surface.

b) The notion of line-element is not actually needed in the quantum theory. Only the matrix elements of the configuration space metric matter and one could consider the possibility that configuration space metric is a collection of these matrix elements for real and p-adic regions with the deformations of 3-surface selected so that they vanish only in real or p-adic regions with fixed value of p. If the rational boundaries between the regions of the 3-surface belonging to different number fields correspond to zero modes (and behave effectively clasically), there is no need to construct the metric associated with these modes.

c) With these assumptions metric tensor would reduce to a direct sum of tensors belonging to different number fields. One cannot exclude the possibility that the values of the matrix elements are rational or complex rational (or algebraic) so that everything would algebraize and topology would disappear at the level of matrix elements completely.

d) The explicit construction of the matrix elements of the metric in the real context involves canonical symmetries, and thus also configuration space Hamiltonians, whose definition involves integrals over 3-surfaces. Definite integrals are problematic in the p-adic context, as is clear from the fact that in-numerable number of definitions of definite integral have been proposed. One might however hope that one could reduce the construction in the real case to that for the representations of superconformal and canonical symmetries, and analytically continue the construction from the real context to the p-adic contexts by *defining* the matrix elements of the metric to be what the symmetry respecting analytical continuation gives.

7.3.2 Configuration space spinors

One must also construct spinor structure. Also this construction relies crucially on superconformal and supercanonical symmetries. Spinors at a given point of the configuration space correspond to the Fock space spanned by fermionic oscillator operators and again one might hope that supersymmetries would allow algebraization of the whole procedure. Configuration space spinor fields depend on the point of the configuration space and here the hopes are based on the construction of an orthonormal basis, whose elements are normalized to unity with respect to an inner product involving the integral over the configuration space. p-Adic configuration space integral poses deep technical problems but again analytical continuation from the real context using supersymmetries might save the situation.

7.3.3 Generalizing the notion of configuration space spinor field

Assume that all these difficulties can be overcome using supersymmetry based analytical continuation and that everything is algebraic at the S-matrix level. How to generalize of the notion of configuration space spinor field?

a) For a moment restrict the consideration to the space of 3-surfaces with fixed decomposition to real and p-adic regions such that the boundaries between regions belonging to different number fields consist of fixed sets of rational numbers. The whole configuration space can be regarded as a union over all these sectors. If the rational boundaries can be regarded as zero modes (classical degrees of freedom in which localization occurs in each quantum jump), there is no need to integrate over these boundaries in the inner product for configuration space spinors fields. Assume that this is the case.

b) Assume that one has found an orthonormal basis for the spaces of real and p-adic regions of the 3-surfaces with a fixed rational boundary.

c) Armed with these assumptions (plus many other about which I am not yet conscious of) one can construct formal products of the configuration space spinor fields belonging to different regions. One can also construct formal sums of the products with rational or algebraic entanglement coefficients. Although these expressions do not belong to any definite number field, their inner products are complex rational numbers and this is all that is needed for doing physics.

This construction, if it really works, would mean that it is possible to construct a quantum theory which is able to describe also the interaction between cognitive representations and matter as well as cognitive representations characterized by different values of p-adic prime p.

7.3.4 Is the trivial solution the only one?

It is quite possible that the trivial solution to these challenges is the only possible one. The line element vanishes in all p-adic sectors and p-adic spacetime regions correspond to zero modes in which a complete localization occurs in each quantum jump so that there is no need to define p-adic configuration space integral. The p-adic counterparts of the field equations defined by the Kähler action are satisfied. This option saves from the trouble of defining the four-dimensional integral defining the Kähler action. One also avoids the challenge of giving p-adic meaning to the exponent of the Kähler action defining the vacuum functional.

Since configuration space metric does not pose any conditions on the fermionic oscillators operators, they could but need not anticommute to zero.

1. Do fermionic oscillator operators anticommute to zero...

If fermionic oscillator operators anticommute to zero, fermionic oscillator algebra reduces to Grassmann numbers and thus also fermionic degrees of freedom represent classical zero modes. The fact that classical theory is an essential part of all quantum theories could be interpreted as reflecting the fact that cognition obeys purely classical physics.

2. ... or do they generate quaternion conformal algebra?

On physical grounds it seems that fermionic oscillator operators cannot generate a mere Grassmann algebra. p-Adic mass calculations are based on p-adic thermodynamics which assumed p-adic conformal invariance and p-adic Super Kac Moody algebra based on fermionic oscillator operators. Certainly fermionic oscillator operators cannot anticommute to zero in this case and in cognitive degrees of freedom one would have quaternion conformal fermionic field theory in a fixed p-adic background spacetime. The non-anticommutativity of fermions is consistent with the vanishing of the p-adic configuration space metric since quaternion conformal algebra is not related to the configuration space metric.

One might hope that this theory allows to define various symmetry generators purely representation theoretically without any reference to the problematic integrals over p-adic 3-surface. This seems to be the case. For Kac Moody representations momentum and mass squared operators do not correspond to conserved charges defined by integrals. Quaternion conformal invariance implies effective two-dimensionality in the sense that the quantization of the theory reduces to that in a commutive sub-manifold of the quaternionic spacetime. At each point of a commutative submanifold quaternionic tangent space reduces to some subspace of quaternions isomorphic to the field of complex numbers. The integrals defining conserved charges are integrals over circles in the real context and integration can be done by residy calculus without any resort to a numerical definition of the integral. The residy calculus would provide a natural p-adic generalization for these integrals. Clearly this option could solve all technical problems related to the construction of the configuration space spinors and configuration space spinor fields and be consistent with p-adic mass calculations.

From the point of view of cognition the situation would look like follows (assuming the option allowing quaternion conformal representations).

a) Quaternion conformal fermionic algebra has interpretation as infinite-dimensional Boolean algebra and as a physical correlate of logical thinking.

b) Cognitive representations would be completely classical in spacetime degrees of freedom and induced by the common boundaries of the real and p-adic regions.

c) The unitary time development operator U could generate rational or algebraic entanglement between different number fields in both bosonic and fermionic degrees of freedom. Thus the mapping of the real to cognitive by quantum entanglement would occur in the same manner as in quantum measurement. Thus even the trivial solution gives all what can dream of and explains why the world of cognitive experience is classical.

This picture suggests that our deepest physical theories do not only reflect the structure of the physical world but also the structure and limitations of our cognitive consciousness.

a) Fermionic quaternion conformal symmetries are realized as p-adic cognitive representations unlike the supercanonical symmetries and superconformal symmetries of lightlike manifolds of 4-dimensional Minkowski space related to the configuration space metric. Only the standard model symmetries (Poincare, color, and electroweak super Kac Moody symmetries) are realized at the level of cognitive representations in terms of effectively two-dimensional quaternion conformal invariance. One can say that cognition represents spacetime as effectively two-dimensional and makes it to look like the world of superstring models.

b) The cognitive non-representability of supercanonical and conformal symmetries of lightlike manifolds of 4-dimensional Minkowski space could provide answer to several intriguing questions. Why these symmetries, which are realized at the level of sensory qualia but not at the level of cognition, have not been invented although the massless extremals possessing supercanonical symmetries seem to be everywhere and have fantastic explanatory power in the physics of living matter? Why the conformal symmetries of the lightlike manifolds of 4-dimensional Minkowski space, which is extremely natural and obvious mathematically, have still not found their way to the mathematical physics literature although they could have been invented already at the times of Einstein and quantum gravitational holography almost forces to discover these symmetries? Could the effective two-dimensionality of cognition and heavy left-brainy character of the recent day theoretical physics explain the otherwise mysterious looking success of super string models despite the fact that the world of super strings is so far from the experiental reality: did the cognitive representation of the world mask the world behind it?

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