Palmer's IST and TGD

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Abstract

Tony Smith told me about the Invariant Set Theory (IST) of T. N. Palmer involving p-adic numbers and asked how it relates to TGD, where p-adic physics is also central. IS is identified as an invariant set for an iterative dynamics defined by an iteration assignable to quasi-cyclic cosmic evolution. Invariant sets are typically fractals and allow an ultrametric distance function: p-adic distance function is indeed ultrametric. Palmer also assumes that the dynamics is classical and deterministic albeit non-computable so that the challenge is to deduce quantum theory and also quantum measurement theory from this picture. Another challenge is to deduce Einsteinian space-time picture and Einstein's equations as approximate descriptions. In this article IST is briefly summarized and compared with TGD.

1 Introduction

Tony Smith told me about the Invariant Set Theory [B1] of T. N. Palmer (see http://arxiv.org/abs/1605.01051) involving p-adic numbers and asked how it relates to TGD. As a rule this kind of questions are very useful and also now the questions forced to refresh my understanding about the notion of p-adic imbedding space and I realized a possible connection between finite measurement resolution, p-adicization, and hierarchy of inclusions of hyper-finite factors. The work of Palmer involves rather original ideas although our views about physics are radically different.

1. What makes Palmer's work interesting from TGD point of view is that it involves p-adic number fields. p-Adic topology is assumed to provide a natural description for a space U of 3-D Universes. U could be seen as analog of Wheeler's superspace formed by 3-geometries or the "World of Classical Worlds" of TGD. The space I_U is identified as an invariant set (IS) for an iterative dynamics in U assigned to a quasi-cyclic cosmology. IS would be expressible in terms of Cantor sets in the space U. Space-time would correspond to an orbit M_U of 3-space in U.

2. Palmer assumes that the physics is basically classical and deterministic albeit non-computable and that this picture about dynamics could reproduce the predictions of quantum theory. To show that this is the case is a formidable challenge: since one should deduce not only the description in terms of quantum states but also quantum measurement theory and non-deterministic state function reduction with its strange rules.

In TGD Universe classical physics is exact part of quantum physics: the very definition of WCW geometry assigns to 3-surface a 4-surface as analog of Bohr orbit associated with it - the interpretation is in terms of holography. This implies the replacement of path integral with functional integral. There is no attempt to reduce quantum to classical.

3. In Palmer's approach p-adic distance function for the points of invariant set (IS) is introduced and single large p-adic prime is suggested to characterize the topology. This brings strongly in mind models for spin glass energy landscape, which has ultrametric topology (also p-adic topologies are ultrametric). The 3-spaces have metric with Euclidian signature. The challenge is to deduce Einsteinian space-time picture for the orbits M_U in U. The Minkowskian signature of space-time metric is the challenge. Also Einstein's equations should follow from this framework.

   In TGD framework p-adic physics is identified as a physical correlate of cognition and p-adicization of physics is carried at all levels: imbedding space level, space-time level, and WCW level. Single state space characterizes quantum states and can be interpreted as real or p-adic since the coefficient field is assumed to be extension of rationals. All number fields are fused to single structure and one obtains what might be called adelic physics [L1] [K1].

In the following I summarize Palmer's theory in more detail and discuss possible TGD analogies for the notions of Palmer, in particular for the iterative dynamics introduced by Palmer.

2 Palmer's Invariant Set Theory

A rough description of Palmer's IST [B1] (see `http://arxiv.org/abs/1605.01051`) goes as follows.

1. Palmer identifies universe U as 3-space, not space-time. Palmer postulates that the physics of space-time is classical and deterministic and quasi-cyclic. This raises an enormous challenges. One should produce quantum physics from deterministic physics, which is effectively non-deterministic since one cannot computer the prediction. Also Penrose is attracted by this idea.

2. Concerning dynamics, Palmer finds inspiration from the physics of fractals. In this framework one typically considers orbits of particles. The dynamics is given by iteration. For instance, one can fix some $n-1$-dimensional manifold of n-dimensional configuration space (say 2-dimensional surface of 3-space) and follow the fate of particle, which traverses it repeatedly in quasi-cyclic situation. One obtains series of points defining a discrete dynamical system with dynamics defined by iteration. If the dynamics is initial value sensitive, it is not possible to predict next point from the knowledge of previous points. One can consider invariant set for the iteration- I do not know how unique it is. Fractals can be indeed defined as fixed point sets of iteration and it is easy to produce surprisingly realistic looking landscapes by suitably identifying the iterative map.

   The idea is to replace point like particle with entire 3-dimensional Universe! The fixed point set would consist of subset of these 3-D Universe. One can hardly imagine bigger generalization! This makes sense if the dynamics is quasi-cyclic. In relativistic context cyclic time coordinate is necessary.

   The space-time - Palmer calls it M_U, is identified as the orbit of 3-D Universe in some infinite-dimensional space containing, where this dynamics is defined. In Wheeler's view about quantum gravity this space would correspond to space of 3-surfaces and the dynamics would be given by Einstein's equations formulated using Hamiltonian formalism. As Palmer mentions, from the point of view of gravitation the problem is to combine pseudo-Riemannian metric for M_U with the p-adic metric of I_U. Even more serious problem is that this dynamics is not manifestly general coordinate invariant in 4-D space. Neither does this dynamics change the topology of 3-space.

3. Palmer assumes that this Invariant Set - I_U - is the fundamental geometric object. Palmer postulates that the geometry I_U is based Cantor sets having measure zero in U. Cantor set

property implies that the topology of this set cannot be real number based topology but is based on p-adic distance function dhaving powers of prime p as its values. Palmer does not try to define p-adic variant of Riemann geometry involving line element but only postulates distance function between 3-spaces belonging to I_U.

The notion of spin glass landscape identified as minima of free energy is ultrametric and it could allow decomposition sub-landscapes with p-adic distance function. I would guess that all primes are possible.

4. I_U would correspond to an invariant set for an iteration defined by cosmic evolution. The repetition of cosmic evolution means suggests in GRT context cyclic time. Big Bang followed by Big Crunch seems to be the only option consistent with quasi-cyclicity. One must see space-time as orbit in some configuration space and this leads to problems with general coordinate invariance and also special relativity. To get Minkowskian signature one must extend 3-space to space-time and the idea about quasi-cyclic orbit fails.

The formidable task is to reproduce not only the concept of quantum number, the connection with representation of symmetric groups, unitary time development, and also quantum measurement theory. The derivation of a unification of standard model and gravitational interactions from this picture is also a huge challenge.

1. Palmer argues that non-commutativity of momentum and spatial coordinate follows from number theory. I had difficulties to understand the argument. In p-adic framework one can formally define exponential function giving rise to trigonometric and hyperbolic functions: $exp(ix)$ and $exp(x)$ exist if the p-adic norm of x is smaller than one but does not have the physical properties of exponential function such as periodicity and $exp(x)$ fails to converge. Therefore also $cos(x)$ and $sin(x)$ fail to be periodic.

   In order to define the notion of angle with physically acceptable manner (allowing a sensible generalization of exponential and trigonometric functions necessary for Fourier analysis) one must consider phases instead of angles. This conforms with the interference based measurement of angle (actually its cosine and/or sine).

   One must introduce abelian extensions of p-adic numbers by powers $U_{m,n} = (U_n)^m$ of phases $U_n = exp(i2\pi/n)$. For given n only finite number of phases are represented and angles $2\pi/n$ cannot be represented. Continuous angle in the p-adic version of the geometry is replaced with a collection of discrete phases. The most natural manner to do Fourier analysis is to use finite set of phases $U_{m,n}$ as Fourier basis with m identified as angular momentum component or wave vector. Uncertainty Principle in this sense is present already in ordinary Fourier analysis but this requires de-localized wave functions which are not possible in Palmer's purely classical approach.

   Palmer notices that $cos(\phi)$ and ϕ do not make simultaneously sense as p-adic numbers. This is true but I fail to see how this should imply Uncertainty Principle.

2. Palmer claims that complex Hilbert space and Dirac equation and de-Broglie relationship arise from the helical geometry of trajectories near M_U. Palmer also speculates that Bell theorem can be evaded. It is rather difficult to take these claims seriously without precise connection to quantum theory.

3 Points of contact and departure between IST and TGD

There are some points of contact between IST and TGD although the principles and interpretations are very different.

1. The physics of Palmer is classical and deterministic. In TGD classical physics is in a well-defined sense genuine part of quantum theory since the metric of the "world of classical worlds" (WCW) assigns to 3-surface a highly unique preferred extremal of Kähler action as an analog of Bohr orbit. TGD means replacing of path integral with functional integral and quantum theory is accepted basically as such apart from the generalisation by introducing

zero energy ontology (ZEO) and hierarchy of Planck constants. Quantum measurement theory is generalized to a theory of consciousness.

2. p-Adicity is central also in TGD but in different manner. p-Adic physics serves as a correlate for cognition whereas real physics is correlate for sensory experience and matter. Actually both real and p-adic physics for various primes p are fused to adelic physics of sensory and cognitive experience. The fundamental number theoretic structure is a hierarchy of algebraic extensions of rationals defining abeles. Also extensions of rationals by roots of e are allowed: these define finite-D extensions of p-adic numbers. Finiteness of cognitive representations demands finite-D extension of p-adic numbers.

   Real space-time surface represents "reality"classically. p-Adic space-time surface represents imagined reality. p-Adicity corresponds to reality only locally. The reason is p-adic non-determinism: integration constants are only piecewise constants for p-adic differential equations. For instance, in strong holography (SH) one necessarily cannot continue collections of string world sheets and partonic 2-surfaces (just "two-surfaces"in the sequel) to a preferred 4-D extremal. p-Adically this is possible by pseudo-constants. One can imagine something but it cannot be necessary realized!

   Imagination mimics reality locally and p-adic space-time surface satisfied p-adic variants of local field equations for Kähler action. They indeed make sense. Also the infinite number of vanishing conditions for Noether changes of symplectic symmetry algebra at the ends of CD characterizing strong holography make sense also p-adically as algebraic conditions. Hence everything generalizes from real to p-adic although the integral of Kähler action density as such does not make sense p-adically.

   One can speak about real and p-adic variants of space-time surfaces obtained by strong form of holography as preferred extremals of Kähler action. Algebraic continuation of 2-D string worlds sheets and partonic 2-surfaces to 4-surface is in question and supersymplectic symmetries are in fundamental role.

3. p-Adicity allows to talk about cognition and define a genuine notion of information obviously highly relevant to cognition. For instance, p-adic norm allows to define an analog of Shannon entropy, which can be negative and in this case characterizes information assignable to entanglement, which must be algebraic. The entanglement, which in real sense is always entropic can define p-adic entanglement which is negentropic and Negentropy Maximization Principle (NMP) guarantees that negentropic entanglement gradually increases in the Universe. This implies evolution: universe learns about itself. Consistency requires that Hilbert spaces have as coefficients extensions of rationals defining finite dimensional extensions of p-adics.

3.1 M_U is analogous 4-D space-time surface in $M^4 \times CP_2$

In understood that M_U would have metric with Euclidian signature. In TGD Universe space-time surface consists of regions with Euclidian and Minkowskian signature in induced metric and Euclidian regions correspond to generalized Feynman graphs.

The identification as orbit does not allow topology changes for M_U, which in TGD are absolutely essential. I think that this is fatal disease of Parker's Newtonian picture. In TGD space-time surfaces have non-trivial topologies and even more general topology than manifold topology. Generalized Feynman graphs are singular as 4-manifolds just like Feynman graphs are singular as 1-manifolds at vertices. Euclidian regions of space-time surface define the "lines"of Feynman graphs. At vertices these "lines" meet.

3.2 The space U is analogous to WCW in TGD

4-D General Coordinate invariance also at the level of U is highly desirable. This is the weak point of Palmer's approach. In Wheeler's superspace approach one would consider the space of 3-metrics but this leads to the loss of time and one ends up with many other difficulties- such as realization of fermionic dynamics.

The set U is analogous to the world of classical worlds (WCW) in TGD. To make this comprehensible I must explain what WCW is. This requires the choice of ontology. There are two options to choose from: positive energy ontology (PEO) of standard physics and zero energy ontology (ZEO) of TGD.

1. PEO: The oldest and roughest formulation WCW is the space of 3-surfaces in $M^4 \times CP_2$ to which the definition of Kähler metric assigns a unique space-time surface for 4-D general coordinate transformations to act on (holography from general coordinate invariance). This did not work.

2. ZEO: In ZEO the roughest articulation for WCW is as the space of 3-D surfaces consisting of pairs of 3-surfaces at the opposite boundaries of causal diamond $CD \times CP_2$, where CD is intersection of future and past directed light-cones. These pairs represent geometrically events connected by a history represented by preferred extremal of Kähler action.

   WCW decomposes to sectors assignable to CDs with different size scales assumed to be discrete for number theoretical reasons: the distance between tips of CD is multiple of CP_2 time.

3.3 Could I_U correspond to a sub-space of WCW in TGD?

The space U is analogous to the "world of classisal worlds" of TGD consisting of 3-surfaces related by holography to space-time surfaces. SH reduces this set to the set of string world sheets and partonic 2-surfaces (just "2-surfaces" in the sequel)

Should require the existence of analog of I_U in TGD? Does it somehow help in the formulation of the dynamics? The strong form of holography (SH) following from General Coordinate Invariance states that 2-surfaces determine space-time surfaces as preferred extremals of Kähler action. This description is extremely economic and it is far from clear whether one should introduce I_U at all.

By SH he notion of U and therefore also I_U should reduce to a set of preferred 2-surfaces and I_U would provide even more economical description. On the other hand, p-adic variants fo space-time surfaces realizing finite measurement resolution might be quite enough.

Suppose that one forgets above skeptic arguments and takes the notion of I_U seriously. Could some subspace of WCW define a fixed set of iteration identified as analog of cosmic evolution? The obvious objection is that now the iteration cannot classical but quantal.

1. One candidate for a preferred subset would be set of pairs 3-surfaces with members at opposite boundaries of CD, which are maxima of Kähler function. This would be analogous to the set of free energy minima in spin glass energy landscape allowing ultrametric distance function. The invariant set is given p-adic metric. Spin glass energy landscapes are provided by this ultrametric topology by kind of minimax principle. The distance between two valleys is the highest mountain between them. This space could decompose to subsets with p-adic topologies.

   Tis idea is very nice and I believe that it makes sense but is not enough. Later however adelic physics emerged and p-adic physics for various primes p and real numbers where unified to adelic physics describing matter and cognition.

2. Or could TGD counterpart of I_U be the subset of 3-surfaces at the boundary of CD remaining fixed during given life cycle of self. The parts of zero energy states at this boundary would have locus in this set. The size of CD increases in the sequence of re-incarnations and both boundaries drift farther away. To talk about invariant set one should take a limit at which the number of life cycles becomes large. Of course, it is not clear whether the invariant set exists since NMP implies endless evolution.

3. The most attractive identification of $I_U$is as a subset of 2-surfaces (string world sheets and partonic 2-surfaces determining space-time surface if SH is true.

The requirement of simplicity does not favor the idea of iteration and certainly it is not a natural tool for the formulation of TGD since TGD is quantum theory per se.

3.4 TGD counterpart of iteration

Palmer identifies the basic step of iteration as entire cosmic evolution. In TGD framework one could identify its analog as a single lifetime of self which is predicted to re-incarnate: selves also for a fractal hierarchy of sub-Universes one might say. Now iteration would be quantal operation governed by NMP.

1. In zero energy ontology (ZEO) based generalization of quantum measurement theory defining at the same time theory of conscious entities, one ends up with this kind of iterative structure. Zero energy states are pairs of states at opposite boundaries of causal diamonds (CDs, which for pedagogical purposes can be identified essentially as intersections of future and past directed light-cones). The members of pairs have opposite quantum numbers and are analogous to initial and final states in ordinary ontology - events. Event becomes the primary ontological entity, state is not needed.

2. State function reductions occur at either boundary of CD and repeated state function reductions leave the other boundary and members of state pairs at it invariant. This is Zeno effect. The second boundary changes so that the distance betwen tips of CD increases: this gives rise to experienced time flow. Also the members of state pairs at the changing boundary change.

3. The sequence of reductions at fixed boundary defines self as conscious entity - as generalized Zeno effect one might say. The first reduction to opposite boundary of CD is forced by Negentropy Maximization Principle (NMP) to eventually occur and means that self dies.

   This implies re-incarnation of self at opposite boundary and its geometric time defined by the increase of distance between tips of CD flows in opposite direction since now it is the opposite boundary of CD drifts farther away reduction by reduction. The size of CD thus steadily increases. The first reduction to the opposite boundary correspond to the quantum measurements in the usual sense. The possibility of two time direction is new and seems to be realized in living matter as already Fantappie discovered. Examples are phase conjugate laser beams and spontaneous self assembly. By the fractality of TGD this makes sense in all scales in that of the entire Universe.

   By strong form of holography string world sheets and partonic 2-surfaces - briefly 2-surfaces - are the objects carrying information about construction of quantum states. The repeated iteration of life cycles plus projection to the boundary of same light-cone would mean that this process leads asymptotically to asymptotic 2-surfaces obtained by iteration. What comes in mind is analogy with Mandelbrot fractals: these of course differ in the sense that the fractal is subset of 2-surfaces. Now it would be 2-surface. One can consider also closer analogies but again this takes too far.

4. In TGD single step of iteration would be quantal and correspond to the state function reduction sequence at fixed boundary of CD leaving it and the states at it invariant. Since dissipation is due to the state function reductions and dissipative dynamics leads to asymptotic self-organization patterns, one might hope that quantum iteration does this also in ZEO.

   One could consider the situation at the limit of very long sequences of life cycles as CD becomes very large. One might hope that quantum iteration leads to fixed point quantum state or at least that the 3-surface involved would become analogous to fixed point of classical dynamics based on iteration. One must however remember than quantum non-determinism could make this impossible.

5. WCW metric in principle allows to calculate distance between 3-surfaces. This metric is Euclidian Kähler metric and the restriction of this metric to the counterpart of I_U might be ultrametric although it is difficult to see why the distance would be proportional to powers of p.

6. The p-adic variants of 3-surfaces suggest an amazingly simple manner to define the p-adic distance function in the set of 3-surfaces in accordance with finite measurement resolution.

One consider the discrete point sets assignable to two 3-surfaces and defines the distance as minimum distance between points of these sets. One would have difference of numbers in a finite-dimensional extension of rationals and the p-adic norm of this number could be well defined.

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