Discretization and Quantum Group Description as Different Aspects of Finite Measurement Resolution

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Abstract

The identification of discretization and quantum group approach as classical and quantal aspects for the description of finite measurement resolution is one of the key ideas of quantum TGD. The article of Etera Livine motivated by loop quantum gravity provides a stimulus leading to a more detailed vision how the two views might relate to each other a group description as its quantum counterpart.

As an outcome the hypothesis can be now formulated more precisely and makes a lot of very general predictions. In particular, the representations of quantum group should be obtained as one decomposes the representations of group with respect to discrete algebraic subgroup. This insight would explain explain and generalize some key observations about quantum group representations (finite number of spins for $SU(2)_q$).

R-matrix defining the action of braid group defines quantum group. A connection with p-adic physics emerges: in p-adic sectors the discretisation is always necessary since only discrete phases (rather than continuous angles) definable as roots of unity and their hyperbolic counterparts exist in the extensions of p-adic numbers. An infinite hierarchy of quantum groups associated with the algebraic extensions of rationals emerges if the interpretation is correct.

1 Introduction

In Thinking Allowed Original there was a link to a very interesting article with title "Designing Curved Blocks of Quantum Space-Time...Or how to build quantum geometry from curved tetrahedra in loop quantum gravity" [A1] telling about the work (http://arxiv.org/abs/1501.00855) of Etera Livine [A3] working at LPENSL in France.

1.1 The idea of the article

The popular article mentions a highly interesting mathematical result relevant for TGD. The idea is to build 3-geometry - not by putting together flat tetrahedra or more general polyhedra along their boundaries - but by using curved hyperbolic tetrahedra or more generally polygons) defined in 3-D hyperbolic space - negative constant curvature space with Lorentz group acting as isometries - cosmic time=constant section of standard cosmology.

As a special case one obtains tesselation of 3-D hyperbolic space H^3. This is somewhat trivial outcome so that one performs a "twisting" - so I understand what is done. Some words about tesselations/lattices/crystals are in order first.

1. In 2-D case you would glue triangles (say) together to get curved surface. For instance, at the surface of sphere you would get finite number of lattice like structures: the five Platonic solids tetrahedron, cube, octahedron, icosahedron, and dodecahedron, which are finite geometries assignable to finite fields corresponding to p=2, 3, and 5 and defining lowest approximaton of p-adic numbers for these primes.

2. In 2-D hyperbolic plane H^2 one obtains hyperbolic tilings (https://en.wikipedia.org/wiki/Uniform_tilings_in_hyperbolic_plane) used by Escher.

3. One can also consider decomposition of hyperbolic 3-space H^3 to a lattice like structure. Essentially a generalization of the ordinary crystallography from flat 3-space E^3 to H^3. There are indications for the quantization of cosmic redshifts completely analogous to the quantization of positions of lattice cells, and my proposal is that they reflect the existing of hyperbolic crystal lattice in which astrophysical objects replace atoms. Macroscopic gravitational quantum coherence due to huge value of gravitational Planck constant could make them possible.

Back to the article and its message. The condition for tetrahedron property stating in flat case that the sum of the 4 normal vectors vanishes generalizes, and is formulated in group SU(2) rather than in group E^3 (Euclidian 3-space). The popular article states that deformation of sum to product of SU(2) elements is equivalent with a condition defining classical q-deformation for gauge group. If this is true, a connection between "quantum quantum mechanics" and hyperbolic manifolds serving as cells of hyperbolic tesselations might exist and would correspond to a transition from flat E^3 to hyperbolic H^3.

1.2 Let loop gravity skeptic talk first

That quantum group could be associated with a transition from Euclidian to hyperbolic 3-geometry looks amazing but it is better to remain skeptic since the work relates to loop quantum gravity and involves specific assumptions and different motivations.

1. For instance, the hyperbolic geometry is motivated by the attempts to quantum geometry producing non-vanishing and negative cosmological constant by introducing it through fundamental quantization rules rather than as a physical prediction and using only algebraic conditions, which allow representation as a tetrahedron of hyperbolic space. This is alarming to me.

2. In loop quantum gravity one tries to quantize discrete geometry. Braids are essential for quantum groups unless one wants to introduce them independently. In loop gravity one considers strings defining 1-D structures and the ordinary of points representing particles at string like entity might be imagined in this framework. I do not know enough loop gravity to decide whether this condition is realized in the framework motivating the article.

3. In zero energy ontology hyperbolic geometry emerges in totally different manner. One wants only a discretization of geometry to represent classically finite measurement resolution and Lorentz invariance fixes it at the level of moduli space of CDs. At space-time level discretization would occur for the parameters charactering strings world sheets and partonic 2-surfaces defining "space-time genes" in strong form of holography.

4. One possible reason to worry is that H^3 allows infinite number of different lattice like structures (tesselations) with the analog of lattice cell defining hyperbolic manifold. Thus the decomposition would be highly non-unique and this poses practical problems if one wants to construct 3-geometries using polyhedron like objects as building bricks. The authors mention "twisting": probably this is what would allow to get also other 3-geometries than 3-D hyperbolic space. Could this resolve the non-uniqueness problem? The authors mention "twisting": probably this is what would allow to get also other 3-geometries than 3-D hyperbolic space. Could this resolve the non-uniqueness problem?

   I understand (`http://arxiv.org/pdf/chao-dyn/9502001.pd`) [A2] that hyperbolic tetrahedron can be regarded as a hyperbolic 3-manifold and gives rise to a tesselation of hyperbolic space. Note that in flat case tetrahedral crystal is not possible. In any case, there is an infinite number of this kind of decompositions defined by discrete subgroups G of Lorentz group and completely analogous to the decompositions of flat 3-space to lattice cells: now G replaces the discrete group of translations leaving lattice unaffected. An additional complication in the hyperbolic case is that the topology of the hyperbolic manifold defining lattice cell varies rather than being that of ball as in flat case (all Platonic solids are topologically balls).

1.3 The notion of finite measurement resolution

The notion of finite measurement resolution emerged first in TGD through the realization that von Neumann algebras known as hyper-finite factors of type II_1 (perhaps also of type III_1) emerge naturally in TGD framework. The spinors of "world of classical worlds" (WCW) identifiable in terms of fermionic Fock space provide a canonical realization for them [K1].

The inclusions of hyperfinite factors provide a natural description of finite measurement resolution with included factor defining the sub-algebra, whose action generates states not distinguishable from the original ones. The inclusions are labelled by quantum phases coming as roots of unity and labelling also quantum groups. Hence the idea that quantum groups could allow to describe the quantal aspects of finite measurement resolution whereas discretization would define its classical aspects.

p-Adic sectors of TGD define a correlate for cognition in TGD Universe and cognitive resolution is forced by number theory. Indeed, one cannot define the notion of angle in p-adic context but one can define phases in terms of algebraic extensions of p-adic numbers defined by roots of unity: hence a finite cognitive resolution is unavoidable and might have a correlate also at the level of real physics.

The discrete algebraic extensions of rationals forming a cognitive and evolutionary hierarchy induce extensions of p-adic numbers appearing in corresponding adeles and for them quantum groups should be a necessary ingredient of description. The following arguments support this view and make it more concrete.

2 Quantum groups and discretization as two manners to describe finite measurement resolution in TGD framework

What about quantum groups in TGD framework? I have also proposed that q-deformations could represent finite measurement resolution. There might be a connection between disretizing and quantum groups as different aspects of finite measurement resolution. For instance, quantum group $SU(2)_q$ allows only a finite number of representations (maximum value for angular momentum): this conforms with finite angular resolution implying a discretization in angle variable. At the level of p-adic number fields the discretization of phases $exp(i\phi)$ as roots $U_n = exp(i2\pi/n)$ of unity is unavoidable for number theoretical reasons and makes possible discrete Fourier analysis for algebraic extension.

There exist actually a much stronger hint that discretization and quantum groups related to each other. This hint leads actually to a concrete proposal how discretization is described in terms of quantum group concept.

1. In TGD discretization for space-time surface is not by a discrete set of points but by a complex of 2-D surfaces consisting of strings world sheets and partonic 2-surface. By their 2-dimensionality these 2-surfaces make possible braid statistics. This leads to what I have called "quantum quantum physics" as the permutation group defining the statistics is replaced with braid group defining its infinite covering. Already fermion statistics replaces this group with its double covering. If braids are present there is no need for "quantum quantum". If one forgets the humble braidy origins of the notion begins to talk about quantum groups as independent concept the attribute "quantum quantum" becomes natural. Personally I am skeptic about this approach: it has not yielded anything hitherto.

2. Braiding means that the R-matrix (`https://en.wikipedia.org/wiki/YangBaxter_equation`) characterizing what happens in the permutation of nearby particles is not anymore multiplication by +1 or -1 but more complex operation realized as a gauge group action (no real change to change by gauge invariance). The gauge group action could in electroweak gauge group for instance.

   What is so nice that something very closely resembling the action of quantum variant of gauge group (say electroweak gauge group) emerges. If the discretization is by the orbit of discrete subgroup H of $SL(2,C)$ defining hyperbolic manifold $SL(2,C)/H$ as the analog of lattice cell, the action of the discrete subgroup H is leaves "lattice cell" invariant but could induce gauge action on state. R-matrix defining quantum group representation would define the action of braiding as a discrete group element in H. Yang-Baxter equations would give a constraint on the representation.

This description looks especially natural in the p-adic sectors of TGD. Discretization of both ordinary and hyperbolic angles is unavoidable in p-adic sectors since only the phases, which are roots of unity exist (p-adically angle is a non-existing notion): there is always a cutoff involved: only phases $U_m = exp(i2\pi/m)$, $m < r$ exist and r should be a factor of the integer defining the value of Planck constant $h_{eff}/h = n$ defining the dimension of the algebraic extension of rational numbers used [**?**] In the same manner hyperbolic "phases" defined by roots $e^{1/mp}$ of e (the very deep number theoretical fact is that e is algebraic number (p:th root) p-adically since e^p is ordinary p-adic number, a quantum group theoretically extremely important fact perhaps not noticed by the community!). The test for this conjecture is easy: check whether the reduction of representations of groups yields direct sums of representations of corresponding quantum groups.

3. In TGD framework H^3 is identified as light-cone proper time=constant surface, which is 3-D hyperboloid in 4-D Minkowski space (necessary in zero energy ontology). Under some additional conditions a discrete subgroup G of SL(2,C) defines the tesselation of H^3 representing finite measurement resolution. Tesselation consists of a discrete set of cosets gSL(2,C). The right action of the discrete subgroup of SL(2,C) on cosets would define the analog of gauge action and appear in the definition of R-matrix.

   The original belief was that discretization would have continuous representation and powerful quantum analog of Lie algebra would become available. It is not however clear whether this is really possible or whether this is needed since the R-matrix would be defined by a map of braid group to the subgroup of Lorentz group or gauge group. The parameters defining the q-deformation are determined by the algebraic extension and it is quite possible that there are more than one parameters.

4. The relation to integrable quantum field theories in M^2 [**?**, **?**] (`http://www.sns.ias.edu/~malda/Zamolodchikov.pdf`) is interesting. Particles are characterized by Lorentz boosts in SO(1,1) defining their 2-momenta besides discrete quantum numbers. The scattering reduces to a permutation of quantum numbers plus phase shifts. By 2-particle irreducibility defining the integrability the scattering matrix reduces to 2-particle S-matrix depending on the boost parameters of particles, and clearly generalizes the R-matrix as a physical permutation of particles having no momentum. Could this generalize to 4-D context? Could one speak of the analog of this 2-particle S-matrix as having discrete Lorentz boosts h_i in sub-group H as arguments and representable as element $h(h_1, h_2)$ of H: is the ad hoc guess $h = h_1 h_2^{-1}$ trivial?

5. The popular article says that one has $q > 1$ in loop gravity. As found, in TGD quantum deformation has at least two parameters are needed in the case of SL(2,C). The first corresponds to the n:th root of unity ($U_n = exp(i2\pi/n)$) and second one to $n \times p$:th root of e^p. One could do without quantum group but it would provide an elegant representation of discrete coset spaces. It could be also powerful tool as one considers algebraic extensions of rationals and the extensions of p-adic numbers induced by them.

   One can even consider a concrete prediction for the unit of quantized cosmic redhifts if astrophysical objects form tesselations of H^3 in cosmic scales. The basic unit appearing in the exponent defining the Lorentz boost would depend on the algebraic extension involved and of p-adic prime defining effective p-adicity and would be $e^\eta = e^{k/np}$, $0 \leq k < np$. The hyperbolic "phase" relates by by the standard formula to the redshift: $1+z = e^\eta = e^{k/np}$. The relationship to to the cosmic recession velocity $\beta = v/c$ is obtained from $exp(\eta) = \gamma\times(1+\beta) = \sqrt{(1+\beta)/(1-\beta)}$, $\gamma = 1/\sqrt{1-\beta^2}$: $\beta = (exp(2\eta)-1)/(exp(2\eta)+1) = (exp(2k/np)+1)/(exp(2k/np)-1) \simeq k/np$. The recession velocity v is approximately quantized in multiples of $v_0 = c/np$.

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