Can quasi-time crystal be created by a Fibonacci process?: TGD point of view

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Abstract

This article is a commentary of the work of Dumitrescu et al, which is based on a computer simulation of a quantum computer program realizing unitary evolution believed to make sense as a model for that of time quasi-lattice. We do not really understand what makes the time (quasi-)lattices: the needed physics is not understood.

Therefore the key question is how time (quasi-)lattices are possible in TGD Universe: here zero energy ontology (ZEO) provides a mechanism minimizing the dissipation: time reversal occurring in state function reductions gives rise to time reversed dissipation and dissipation in reversed time direction looking like automatic error correction. This relates to the long lifetime of entanglement, easy to achieve in the unitary evolution but more difficult in the dissipative real world.

The popular article at Phys.org talks somewhat misleadingly about 2-D time although the time values in discretization span 2-D algebraic extension of rationals. The effective N-dimensionality in the algebraic sense is a basic prediction of adelic physics, which involves cognitive representations as unique number theoretical discretization of space-time surface relying on the hierarchy of extensions of rationals. In the real physics sense one would have 1-D time but in algebraic sense N-dimensional time.

The claimed dynamical emergence of symmetries making possible symmetry protected short range entanglement for edge states of the ion array is not really understood and is therefore interesting from the TGD viewpoint. Same applies to the notion of topologically preserved long range entanglement: also here the new physics predicted by TGD can help.

The article mentions also the possibility of quantum coherent units of N qubits behaving like single multi-qubit. The notion of dark N-particles emerges naturally from the number theoretical view of TGD. The dark N-particle would be an analog of the color singlet hadron, and the color group would be replaced by the Galois group. The existence of these kinds of states would mean a revolution in quantum computation and there already exists evidence for N-photons.

Contents

1 Introduction

The popular article "Strange phase quantum dimensions" at Phys.org (https://cutt.ly/XL5G2bi) tells about the article "Dynamical topological phase realized in a trapped-ion quantum simulator" published by Dumitrescu et al [B1] (https://cutt.ly/6L5GNjI).

The popular article tells of a new phase of exotic matter created in a quantum computer. This phase has a very long life-time challenging the standard wisdom about physics. This somewhat hypish formulation might create the impression that the new phase was created in the lab: this is not the case.

1.1 What has been done?

The abstract of the original article [B1] gives some idea of what is done.

> Nascent platforms for programmable quantum simulation offer unprecedented access to new regimes of far-from-equilibrium quantum many-body dynamics in (approximately) isolated systems. Here, achieving precise control over quantum many-body entanglement is an essential task for quantum sensing and computation.
>
> Extensive theoretical work suggests that these capabilities can enable dynamical phases and critical phenomena that exhibit topologically-robust methods to create, protect, and manipulate quantum entanglement that self-correct against large classes of errors. However, to date, experimental realizations have been confined to classical (non-entangled) symmetry-breaking orders].
>
> In this work, we demonstrate an emergent dynamical symmetry protected topological phase (EDSPT) , in a quasiperiodically-driven array of ten $^171Yb^+$ hyperfine qubits in Honeywell s System Model H1 trapped-ion quantum processor.
>
> This phase exhibits edge qubits that are dynamically protected from control errors, cross-talk, and stray fields. Crucially, this edge protection relies purely on emergent dynamical symmetries that are absolutely stable to generic coherent perturbations.

This property is special to quasiperiodically driven systems: as we demonstrate, the analogous edge states of a periodically driven qubit-array are vulnerable to symmetry-breaking errors and quickly decohere.

Our work paves the way for implementation of more complex dynamical topological orders [6] that would enable error-resilient techniques to manipulate quantum information.

In the abstract authors tell that they have demonstrated an emergent dynamical symmetry protected topological phase (EDSPT), in a quasi-periodically driven array of ten $^{171}Yb^+$ hyperfine qubits in Honeywell s System Model H1 trapped-ion quantum processor. The temperature is very low since the energy scale for transitions between qubits is that of hyper-fine splittings.

The first challenge is to understand what the acronym EDSPT might mean.

1. One can consult the Wikipedia article (https://cutt.ly/rL5GK7Z) in order to understand the meaning of symmetry protected topological phase (STP). STP is a kind of order in zero temperature quantum state, which has emergent symmetry and a finite energy gap. STP has short range entanglement. protected by a dynamically generated symmetry. Distinct SPT states cannot be transformed to each other without a phase transition but all STP states can be transformed to a trivial product state by a symmetry violating deformation. SPT does not have emergent gauge symmetry nor emergent fractional charge/fractional statistics. The gapless boundary excitations, edge states, are only symmetry (rather than topology) protected.

2. The emergent dynamical symmetry (EDS) means that the symmetry emerges as a dynamical symmetry. It would appear because of the quasiperiodic driving of the system by laser pulses inducing spin rotations which are near 4π in which case the rotations are trivial: one is near criticality.

   These states exhibit edge qubits, which are absolutely stable against generic coherent perturbations unlike their analogs in periodically driven systems (see Fig 1. of https://cutt.ly/6L5GNjI). Why should the aperiodic driving cause this?

1.1.1 Computer simulation of Fibonacci and Floquet processes

The results of computer simulations of time (quasi)-crystals [?] and Floquet time crystal [D1], realized as unitary time evolutions represented by quantum computer programs, are compared. These quantum computer programs leave the real physics behind (quasi-)time crystals open.

1. In the model of time quasi-crystal as a Fibonacci proces, the time values t_n define an analog of growth process. Fibonacci times $t_n = F_n = F_{n-1} + F_n$, which as such are rational, approach asymptotically to $t_n = F_n \simeq \phi^n/\sqrt{5}$, $\phi = (1 + \sqrt{5})/2$ so that one obtains 2 times in algebraic sense defined by the extension of rationals generated by $\sqrt{5}$. The extension of rationals emerges only asymptotically. Note that the time values are asymptotically obtained as scalings from the basic value by ϕ^n. In the simulation only rational values of time coordinate appear. One must of course notice that there is a finite accuracy involved. Also the unitary evolution can be regarded as an analog of the growth process, but at the level of state space.

2. The simulations of quantum Fibonacci process suggest the existence of a symmetry protected phase of ordinary matter such that the symmetry protected edge states at the end of qubit chain have period 3 stable correlations unlike in the bulk inside the chain. In the periodic Floquet process these stable edge state correlations do not emerge. Why? A possible explanation is that the Fibonacci evolution at step $n+1$ is a product of evolutions at levels n and $n-1$ so that there is a kind of repetition involved. Could this repetition prevent the decay of the correlations for the edge states at ends of the ion array?

1.1.2 Description of the Fibonacci process

The authors study what they call Emergent Dynamical Symmetry Protected Topological Phase (EDSPT) in Fibonacci drive, which is essentially a quasiperiodic unitary time evolution. I will talk about the Fibonacci process in the sequel. The Fibonacci process is assumed to create a time quasi-crystal and it does so in absence of dissipation. The Fibonacci process is compared with a Floquet process which is a periodic process supposed to create a time crystal.

1. Unitary time evolution for the Fibonacci process (see Fig 1. of `https://cutt.ly/6L5GNjI`) is realized in terms of two non-commuting unitary evolutions U_x and U_z assignable to 2-gates. These evolutions would start to act at times $t = F_n$, where F_n is Fibonacci number and satisfy the recursion formula $U_{n+1} = U_{n-1}U_n$, $U(0) = U(1) = 1$. This is analogous to $F_{n+1} = F_{n-1} + F_n$, $F(0) = F(1) = 1$ so that unitary evolution can be seen as an analog of the growth process.

   Fibonacci time n is essentially the logarithm of ordinary time for large values of n from $t_n = F_n \simeq \phi^n/\sqrt{5}$. $\phi = (1+\sqrt{5})/2$. $log(t_n) = nlog(\phi)-log(\sqrt{5})$ gives $n = (log(t)+log(\sqrt{5})/log(\phi)$.

   The unitary time evolution has an approximate interpretation as a sequence of evolutions by scalings with respect light-cone proper time a in good approximation identical with linear Minkowski time near origin, which correspond to translations with respect to Fibonacci time n.

2. The 2-gates used in Fibonacci drive represent spin rotations around x- and z-axis by angle $4\pi - \Delta\theta$. The value of $\Delta\theta = .05 \times 4\pi = 2\pi/10$ is rather near 4π. $\Delta\Theta$ happens to be the twist angle in the DNA strand and 1/2 of the angle $2\pi/5$ assignable to the pentagon defining a face of the dodecahedron.

   The spin rotations around z and x-axis generate an infinite subgroup of $SU(2)$. The representations of 2-gates as spin rotations of neighboring 2 spins are $U_{XX_\theta} = exp(-(i/2)\sigma^x \otimes \sigma^x$ and $U_{ZZ_\theta} = exp(-(i/2)\sigma^z \otimes \sigma^z$ (see Fig 1. of `https://cutt.ly/6L5GNjI`). The value of $\Delta\theta = 4\pi - \theta = .05 \times 4\pi = 2\pi/10$ is near 4π.

   The spin rotation represented by U_{XX_θ} corresponds to $cos(\theta)1 \otimes 1 + sin(\theta)\sigma_x \otimes \sigma_x$. Therefore the actions of these unitaries is very near to a rotation by 4π so that the change of the state is small. The spin rotations around z- and x-axis generate an infinite discrete subgroup of $SU(2)$, which at the limit $\theta = 4\pi$ reduces to identity by infinite degeneracy. The interpretation in terms of near criticality makes sense.

3. Also 1-gates representing spin rotations, realized in terms of magnetic fields B_x and B_z having random directions, used to represent perturbations, are involved (see Fig 1. of `https://cutt.ly/6L5GNjI`).

A couple of comments on the findings are in order.

1. Edge state correlation functions are found to have a characteristic 3-periodicity with respect to Fibonacci time, which for light-cone proper time a corresponds to quasiperiodicity coming as powers of 3: could this relate to 3-adicity? In the TGD framework, scalings define fundamental unitary time evolutions and this would be very relevant for the modelling of spin glasses [L17].

2. Quasiperiodicity characterized in terms of Fibonacci numbers is reported to be essential for the generation of stable edge states. The repetitive nature of the time evolutions could be the underlying reason for this.

3. The generation of dynamical symmetries is a part of the proposed paradigm. The systems considered would be characterized by the emergence of an extensive number of local conservation laws, and associated local integrals of motion (LIOMs).

1.1.3 Questions raised by the article

The theoretical background in the model is ordinary condensed matter physics but, as Anderson has said, there is no theory of condensed matter so that the models are phenomenological and to my understanding, also highly speculative.

TGD leads to a new view of condensed matter [L18] involving several new notions. Therefore it would be an instructive exercise to try to transform the notions and proposals of the article to the TGD context. This exercise could also makes it possible to learn about TGD and check the internal consistency of various ideas.

Number theoretic (adelic) physics [L2, L3] is an essential element of TGD, as also $M^8 - H$ duality relating number theoretical and geometric visions of physics.

In TGD, the speculative assumptions of the model could be perhaps reduced to new physics.

1. The zero energy ontology (ZEO) of TGD [K15, K16] [L7, L5, L12, L16, L24] suggests a first principle explanation for the behavior of time (quasi-)crystals. In the article the existence of time crystals is however taken as granted.

2. Edge spin states, possibly identifiable as anyons, could in TGD be replaced with entire irreps of the Galois group Gal or its isotropy group Gal_I leaving a given root invariant, which could be assigned to the edge and boundary states.

3. Multi-spin interactions treating subsets of spins as coherent units would be natural in TGD, which predicts so-called N-particles as quantum coherent units.

4. The automorphism group of quaternions, which defines an analog of the Galois group acting in electron spin degrees of freedom, is a possible candidate for the emergent symmetries responsible for short range entanglement.

1.1.4 What makes possible long range entanglement?

In the TGD framework, long range entanglement could be protected by topology or by Galois symmetry, which might closely relate to the hierarchy of subalgebras of super symplectic algebra (SSA) acting as gauge symmetries [L6].

1. Topological order, which leads to long range entanglement, would be associated with topological quantum computation (TQC) realized in terms of braids represented by monopole flux tube structures [L29, L31].

2. In the number theoretic vision, the Galois group of extensions of rationals associated with the monic polynomial P determining space-time surface acts as a number theoretic symmetry group.

   Galois confinement [L32, L22, L23, L24] means that physical states are Galois singlets and implies that momentum components are ordinary integers rather than algebraic integers.

   The weaker form of Galois confinement states that states are singlets with respect to the isotropy group of a given root, whose value corresponds to a value of mass squared. Galois confinement defines a universal mechanism for the formation of bound states: for instance, Cooper pairs could be formed in this manner [L18].

   The Galois group is an ideal candidate for a dynamically emerging symmetry, which hower defines a long range order rather than short range order.

3. In the H picture, hierarchies of dynamically emerging symmetries could correspond to hierarchies of supersymplectic symmetry algebras (SSA) [L6] generating groups acting as isometries of the "world of classical worlds" WCW [K5, L20].

   Each level in the hierarchy of subalgebras SSA_n of SSA corresponds to a transformation in which SSA_n acts as a gauge symmetry and its complement acts as genuine isometries of WCW: gauge symmetry breaking in the complement generates a genuine symmetry, which could correspond to Kac-Moody symmetry. By Noether's theorem, the isometries of WCW give rise to local integrals of motion: also super-charges are involved.

   The symmetries would naturally correspond to a long range order. The hierarchies of SSA_n:s, of relative Galois groups and of inclusions of hyperfinite factors [K14, K8] could relate to each other as $M^8 - H$ duality suggests [L28].

1.1.5 What could make possible symmetry protected entanglement?

What about the symmetry protected entanglement?

1. The number theoretic dynamics involves associativity as a dynamical principle. The normal space of 4-surface in M^8 is quaternionic so that the automorphism group $SO(3)$ of quaternions define a natural candidate for a number theoretic symmetry group analogous to the Galois group. These rotations naturally induce rotations of the tangent space of the space-time surface and the idea that the ordinary rotation group of E^3 could be cognitively represented in terms of quaternion automorphisms is natural.

   In the TGD framework, the covering group $SU(2)$ of the quaternionic automorphism group $SO(3)$ plays the role of the Galois group in spin degrees of freedom.

2. The discrete subgroups of quaternionic automorphisms define a natural candidate for the emerging short scale symmetries.

1.1.6 Is there a connection with the physics of DNA?

The work of Dumitrescu et al [B1] also shows intriguing numerical co-incidences suggesting that the TGD analog of the Fibonacci process could have connections with DNA.

1. $\Delta\Theta = 2\pi/10$ happens to be the twist angle between nucleotides in the DNA strand and $1/2$ of the angle $2\pi/5$ assignable to the pentagon appearing in the dodecahedron.

2. One can wonder whether the reported 3-periodicity could relate to the fact that DNA codon consists of 3 nucleotides.

3. The TGD based model for the genetic code [L15] involves the symmetries of icosahedron and tetrahedron. Could the free product of the covering groups of the isometries of icosahedron and (say) tetrahedron (defining finite subgroups of $SU(2)$) having an infinite number of elements emerge in the proposed model as a hidden possibly approximate symmetry group? Very probably, this is not the case.

   Could these two groups correspond to a Galois group associated with a composite polynomial $P_I \cdot P_T$ having the isometry groups of icosahedron and tetrahedron as relative Galois groups? In the model this is probably not the case.

   This kind or organization of finite subgroups to finite groups acting as Galois groups is however an interesting possibility and might relate to the hierarchies of inclusions of hyperfinite factors of type II_1 [K14, **?**] as a counterpart for the hierarchies of inclusions of Galois groups defined by functional composites of polynomials.

1.2 What perspective should I adopt?

The article Dumitrescu et al [B1] tells only about the results of simulations of quantum computer programs rather than reporting an outcome of a real-life experiment.

What perspective should I adopt in order to avoid trying to explain standard physics in terms of new physics? The model of Dumitrescu et al is a simulation based on standard physics inspired speculative assumptions. It does not involve notions like ZEO and number theoretical physics so that it does not make sense to apply the TGD view to explain the findings of the model.

Therefore TGD could be only used to justify the assumptions of the model. The best that I can do is just try to understand what has been done and ask whether the system considered could have analogies in the TGD framework and help to make the TGD inspired view more precise.

2 TGD view about the needed notions of condensed matter physics

TGD relies on two dual visions: geometric and number theoretic views of physics [L20].

1. Geometric approach [K2, K1, K4, K5] is the Einsteinian view about physics as geometry but modified and generalized so that it applies to entire quantum physics. The notion of the "world of classical worlds" (WCW) is fundamental. By its infinite-dimensionality its Kähler geometry is expected to be unique. The existence of the twistor lift of TGD fixes the choice of H uniquely to $H = M^4 \times CP_2$ [L22, L23].

2. Number theoretic approach, adelic physics [L2, L3], is something totally new, and relies on extensions of rationals defining a h_{eff} hierarchy as a hierarchy of effectively dark matters; on p-adic number fields forming an adelic structure [K11]; and on classical number fields [K12] that is reals, complex numbers, quaternions, and octonions.

   One could say that the number theoretic physics provides a discrete cognitive representation of real number based physics relying on geometric notions, in particular on differential geometry.

$M^8 - H$ duality [L9, L10], which is analogous to momentum-position duality relates the geometric and number theoretic visions. Indeed, M^8 is essentially momentum space and the physics at the level of momentum space is algebraic, consider as example only free massless field equations and the free Dirac equation.

The dynamics at the level of M^8 is determined by polynomials P whose roots give mass shells H^3 of $M^4 \subset M^8$ as solutions which provide holographic data for their continuation to a 4-D surface, which obeys number theoretic dynamics stating that the normal space is associative (quaternionic).

This 4-D surface of M^8 is mapped to a space-time surface in $H = M^4 \times CP_2$ by $M^8 - H$ duality. One can say that space-time surfaces are determined by rational polynomials, which could be actually monic polynomials with integer coefficients.

At the level of $H = M^4 \times CP_2$ physics is differential geometric and twistor lift implies that the space-time surfaces are determined by field equations and turn out to be minimal surfaces with lower-dimensional singularities: space-time surface is analogous to a soap film with frames [L21]. The volume term in the action corresponds to length scale dependent cosmological constant approaching zero in long scales [L1, L4, L8].

There are many interesting questions to be considered. What are the number theoretic (M^8) and differential geometric (H) descriptions of time (quasi-)crystals? The differential geometric description in H could be as (quasi)-periodic minimal 4-surfaces with singularities [L21]. These space-time surfaces would correspond to a hierarchy of extensions of rationals characterizing the algebraic complexity of these 4-surfaces. Time (quasi-)crystals provide also a test bench for ZEO, which should make their existence possible.

2.1 About the TGD counterparts of key notions

It is instructive to consider the basic notions from the TGD point of view in more detail.

2.1.1 The importance of the energy feed

A small energy feed in terms of laser pulses is necessary to create (quasi-)time chrystals.

1. The article talks about the new phase as a dynamical phase associated with non-equilibrium thermodynamics. Quite generally, the generation of thermodynamic non-equilibrium states requires an energy feed.

   The generation of $h_{eff} = nh_0$ phases of ordinary matter requires energy feed since the energies for states quite generally increase as a function of h_{eff}. The larger the value of h_{eff}, the higher the dimension of extension or rationals, and the larger the algebraic complexity.

   In biology, metabolic energy feed is needed to preserve the distribution of the values of h_{eff} since h_{eff} tends to decrease spontaneously [L26, L27, L25]. If short range order is in question, the values of h_{eff} in TGD picture would be not too large unlike for topological order.

2. The energy feed would be extremely small in the case of time (quasi-)crystals [L18]. This is not easy to understand in the standard physics framework. In TGD, ZEO would change the situation. During the time reversed periods, the system would extract energy from the environment, which would lead to self-organization by time reversed dissipation. This would also provide an automatic error correction procedure requiring no program.

2.1.2 About time (quasi-)crystals

Consider first time (quasi)-crystals.

1. Time (quasi)-crystals are extremely interesting systems. Time (quasi-)crystal manages to behave almost like a perpetuum mobile. The only energy feed consists of periodic or (quasi-)periodic laser pulses.

   The mystery is how the dissipation can be so small. In the zero energy ontology (ZEO) [L7, L24] [K15], which is behind TGD based quantum theory, time (quasi-)crystals could be systems for which (quasi-)periodically occurring "big" state functions reductions (BFSRs) induced by laser pulses would reverse the arrow of time and are followed by second BSFR re-establishing the arrow of time. In living matter various biorhythms, such as sleep-awake rhythm and breathing rhythm, would rely on this kind of sequence.

2. Geometrically, quasicrystal can be understood as a projection of higher-D crystal on lower-D sub-space, say 2-D plane. The higher dimensions are regarded as an auxiliary tool rather than being real. Therefore the claim of the popular article that there are two time dimensions involved is grossly misleading.

3. One way to construct quasicrystals is based on algebraic extensions of rationals, which in TGD framework define a hierarchy of effective Planck constants $h_{eff} = nh_0$, n the dimension of algebraic extension. h_{eff} characterizes phases of ordinary matter which behave in many respects like dark matter.

4. The m roots of an irreducible polynomial define an extension with dimension which is at most $n = m!$. Large values of h_{eff} make possible quantum coherence in long scales and in the TGD framework this would have quite dramatic consequences concerning for instance quantum computations [L29, L31]. Also in biology the implications would be profound.

   As a pedagogical example, which is not directly related to the proposal of the article [**?**], one can consider a discretization of time by starting from a periodic situation $t = nt_0$. One can replace discrete time values in the extension of rationals: say $t_{m,n} = (m + n\sqrt{5})t_0$. One has 2-D lattice in the algebraic sense but 1-D lattice in topological sense. One can also think that one has two 1-D lattices with lattice cell sizes which are in ratio $\sqrt{5}$. Note that this lattice does not directly relate to the lattice considered in the article. A sub-lattice defined by the powers of Golden Mean for which Fibonacci numbers $F(n)$ serve as an approximation, would be in question.

   The roots of the second order polynomial defining Golden mean allow to define this kind of lattice. Technically, this point set could be seen as a projection of 2-D ordinary lattice to a suitably chosen 1-D line (in real sense) so that one obtains algebraic points mentioned. In topological sense, one has a discrete set of points along the real time axis.

5. Quite generally, the algebraic numbers, in particular algebraic integers, of an algebraic extension assignable to the roots of a polynomial with rational coefficients define points of a space, whose dimension is the order of the Galois group in an algebraic sense, not in the sense of real topology.

2.1.3 Fermi ball and Fermi surface

Cognitive representations correspond to points of 4-surface $X^4 \subset M^8$ consisting of momenta for which the components of $M^4 \subset M^8$ momenta are algebraic integers in an extension of rationals defined by a polynomial P defining as its roots mass shells H^3, which in turn define by holography a 4-surface X^4 of M^8 going through them and mapped to H by $M^8 - H$ duality.

At the mass shells H^3 cognitive explosion takes place in the sense that virtual momenta with components, which are algebraic integers, are possible In the interior of X^4, the number of points of cognitive representation is discrete and typically finite. If there is a fundamental fermion at a given point satisfying this condition, one can say that the point of cognitive representation is active.

The momenta inside the Fermi ball-like object [L18] would be active, that is populated by fundamental fermions (quarks in the simplest scenario for which leptons would be 3-quark states). The boundary states would reside at the 2-D boundary of this object and separated by a mass gap in the general case, say in the case of superconductivity. They would be mapped by M^8-H duality to a membrane-like object in H. Also 1-D edges as string connecting membranes populated by fermions would be present.

2.1.4 Edge- and boundary states

The existence of edge and boundary states, the TGD view about boundary layers in hydrodynamical system, and the role of cell membrane in biology motivate the idea that membranes with 2-D E^3 projection and 1-D CP_2 projection are fundamental quantum objects appearing in all scales.

It must be emphasized that the actual presence of membranes as quantum coherent systems is still a speculative idea and basically motivated by hydrodynamical considerations and cell membranes.

What is certain is that the original proposal about the existence of 3-surfaces with boundaries is not favored mathematically. The 3-surfaces are expected to be closed and by their finiteness must define multiple coverings of E^3. This means that the 2-surfaces at which the sheets of the covering meet each other appear effectively as boundaries of 3-surface in E^3. This might be enough.

Note that for magnetic flux tubes the cross section of the flux tube is a closed 2-surface unlike in Maxwellian picture and also now one can ask whether 2-D cylindrical membrane accompanies the flux tube.

M^8-H duality [L9, L10] suggests that these 2-D surfaces of $E^3\subset M^4\subset H$ are accompanied by genuine membrane-like entities.

1. The membranes in H could be seen as H counterparts for the 2-surfaces assignable in M^8 to states separated by a gap from the Fermi surface and therefore analogous to conduction bands. Edge states could be also seen as points at which this 2-surface touches the counterpart of the Fermi surface: at this limit the connecting string would contract to a point.

2. M^8-H duality [L9, L10] requires that the membranes in H are images of corresponding objects in M^8 under M^8-H duality. The pre-images of 3-D singularities of 4-D soap films (such as light-like orbits of partonic 2-surfaces) would correspond to 3-D surfaces in $X^4\subset M^8$ with 1-D commutative normal subspace inside tangent space of X^4 and have well-ordering.

3. The preimages of 2-D singularities (string world sheets and partonic 2-surfaces) would have 2-D commutative normal space inside X^4. 1-D strings would have 3-D normal space in X^4 but this has no special number theoretic property. As curves in H^3 identifiable as string world sheets boundaries they can have commutative normal subspace in H^3.

1. The states assignable to the boundaries of the generalization of Fermi sphere as set of H^3 would define by number theoretic holography a space-time surface, whose $M^4\subset H$ projections is 3-D and CP_2 projection is 1-D. The E^3 projection would be 2-D membranes.

   The CP_2 projection must be dynamical: otherwise the membrane as a M^4 projection cannot be closed [L21]. The projections to H^3 would be 2-D and 1-D strings in the case of edge states.

2. Universe could be filled with membrane-like structures defining the nodes of a network and flux tubes defining connections between the nodes. The boundary between two phases like water and air provides one example.

   I have proposed a quantum model for hydrodynamical turbulence as a generation of vortices in the boundary layer of fluid flow [L19, L34]. The assumption that the phase boundary involves a membrane-like surface with a large value of h_{eff} assigned also to the accompanying monopole magnetic flux tubes [L27, L25] conforms with this view. One can ask whether the boundary states relate to the boundary layer in hydrodynamics as a quantum coherent structure, which is analogous to skin [L19] and predecessor of the nervous system.

   One could assign membranes even with the boundaries of molecules. Edge states might be associated with string- like entities connecting these membranes. All these objects as preferred extremals of the action principle for twistor lift of TGD [L1, L4] would correspond to lower-D singularities of minimal surfaces at which the action principle would be defined by a volume term plus Kähler action.

3. Surface states could be associated with the membrane like structures in turn connected by strings having ends carrying edge states. Number theoretic description predicts that these phases have $h_{eff} = nh_0 > h$, where n is the dimension of the algebraic extension and order of the Galois group of the extension defined by a polynomial P. In many respects, these phases behavelike dark matter. Large value of h_{eff} could theoretically explain the reported long life time [B1] for the TGD analog of the claimed new phase.

2.2 Orders and symmetries

TGD suggests a general view about the realizations of the notions of order and symmetry.

2.2.1 Dynamical emergence of symmetries and their breaking

In [?] the emergence of dynamical symmetries and their breaking is mentioned.

1. The dynamical symmetry breaking cannot correspond to the breaking of Galois symmetry Gal to the isotropy subgroup Gal_I of Galois group leaving invariant a given root of P representing mass squared value. Rather, The breaking of Gal to Gal_I is necessary in order to obtain commutativity with Lorentz and Poincare transformations and this is essential for understanding this symmetry breaking as number theoretical counterpart of Higgs mechanism involving no assumptions about dynamics [L33].

   An interesting question is whether the Galois confined states decouple from thermodynamics of the ordinary matter and have correlation functions stable against thermodynamic perturbations. If Galois confined states indeed correspond to bound states, this is the case at temperature in which bound states are stable.

2. In the M^8 framework hierarchies of dynamically emergent symmetries giving rise to long range entanglement might also correspond to hierarchies of subalgebras SSA_n of super symplectic symmetry algebras (SSA) [L6] associated with the isometries of WCW. The action of the symplectic transformations would be 3-surfaces and therefore "holistic". Therefore also these symmetries should correspond to long range order and SSA_n and Galois groups could relate closely.

   Dynamical symmetry breaking would have an analog at the level of super symplectic isometry algebra (SSA) of WCW [L20] [K2, K1, K5]. TGD predicts the breaking of the full SSA as effective gauge symmetries to SSA_n with conformal weights as n-multiples of those for SSA. Also $[SSA_n, SSA]$ would act as gauge symmetries. The generators with conformal weight smaller than n would act non-trivially as isometries of WCW. The breaking of gauge symmetry generates symmetries acting as isometries. Note that similar breaking is possible also for the Virasoro algebra and Kac-Moody algebras. For SSA the conformal weights are associated with the radial light-like coordinate of the light-cone boundary rather than the complex coordinate.It is not quite clear whether the irreps of these Kac-Moody algebras for reduce to the representations of SSA restricted to the isometry sub-algebra SSA.

3. Besides SSA acting at the level of H at the boundaries of causal diamond (CD), also Kac-Moody algebras associated with the isometries of H and acting at the light-like 3-surfaces assignable to the orbits of partonic 2-surface are involved.

2.2.2 Topological order

At the level of H, the braidings of magnetic flux tubes carrying monopole fluxes (they have no Maxwellian analogs) give rise to topological physics giving rise to long range entanglement: large values of h_{eff} would be essential. As a matter fact, U-shaped flux tubes are the basic entities and could reconnect to form pairs of flux tubes connecting two systems.

These flux tube pairs connecting membrane-like objects as nodes would give rise to dynamical networks and long range entanglement. The braiding of the flux tubes would make TQC possible [L29, L31]. The matter at the flux tubes would be dark in the TGD sense and a large value of h_{eff} would increase the lifetime of flux tube pairs against reconnection.

TGD also predicts non-monopole flux tubes, which are not stable against splitting. An open question is whether they might relate to symmetry protected phases with short range entanglement.

2.2.3 Do Galois groups define a symmetry protected long range order dual to topological order?

In the adelic physics of TGD, Galois symmetries [L14, L13, L11, L24, L28] are central. They could be associated with a symmetry protected order, which would however give rise to long range entanglement and boundary states stable under perturbations respecting Galis symmetries and therefore leaving the Galois group unaffected?

1. The components of $M^4 \subset M^8$ momenta are algebraic integers in an extension of rationals defined by a polynomial P defining as its roots mass shells H^3, which in turne define by number theoretic holography a 4-surface of M^8 going through them. This surface is mapped to space-time surface in H by $M^8 - H$ duality. At the mass shells H^3, a cognitive explosion takes place in the sense that virtual momenta with components, which are algebraic integers, are possible. If there is a fundamental fermion at given points satisfying this condition, one can say that the point of cognitive representation is active.

   The momenta inside the Fermi ball-like object would be active, that is populated by fundamental fermions (quarks in the simplest scenario for which leptons would be 3-quark states). The boundary states would reside at the 2-D boundary of this object and would be mapped to a membrane-like object in H by $M^8 - H$ duality. In super-conductivity also some momenta near the surface would be populated. Also 1-D edges as string connecting membranes populated by fermions would be present.

2. Galois confinement states that physical states are Galois singlets and have total momenta for which components are ordinary integers. Galois confinement would provide a second reason for the stability. One can consider two versions of Galois confinement: with respect to the full Galois group transforming mass shells to each other and with respect to the isotropy group of given root of P leaving the corresponding mass shell invariant.

3. What could the robustness against perturbations correspond in the number theoretic context? The replacement of states with irreps implies robustness in TQC [L29]. Furthermore, the Galois group of the polynomial is not affected by n small enough variations of the coefficients of the polynomial P. In fact, there are good reasons to expect that only monic polynomials (with integer coefficients) are allowed. The change of the order of the polynomial in general affects the Galois group so that might speak of symmetry protected entanglement stable only under perturbations preserving the Galois group.

   However, Galois symmetry is a global property of space-time sheet: should one speak of long scale, global symmetry and assign a phase transition to the change of the symmetry as in the case of long range entanglement. Galois confinement as analog of color confinement, which involves quark entanglement with a long (hadronic) length scale. One speaks also of color-deconfinement phase transition: this would conform with the interpretation in terms of long range entanglement.

4. Cognitive quantum measurements [L11, L24] are identified as SSFR ("small" state function reduction) cascades reducing the irrep of Galois group assignable to the functional composite $P = P_n \circ P_{n-1} \circ ... \circ P_1$ to an unentangled product of irreps of relative Galois groups. This involves a sequence of symmetry breakings involved with phase transitions.

5. Could the entanglement protected by Galois symmetry be equivalent to topologically protected entanglement as the above observations and $M^8 - H$ duality mildly suggest? The following argument suggests that this might be the case.

The dark matter has non-trivial Galois symmetries and resides at the monopole flux tubes. The pairs of U-shaped flux tubes as correlates of long range entanglement are unstable against reconnection.

The value of h_{eff} is expected to correlate with the length of the flux tube and reconnection is proposed to be the basic mechanism, which allows the reactants connected by flux tube pairs to find each other in biocatalysis [L30]. After the reconnection, the reduction of h_{eff} for the U-shaped flux tubes would indeed reduce h_{eff} and induce a reduction of the Galois symmetry.

TGD based view of topological quantum computation (TQC) (see [K7, K6, K13] for the earlier view and [L29, L31] for the recent view) relies on dark matter in number theoretic sense. The best way to generate dark matter would be at quantum criticality: the long range fluctuations would correspond to the presence of phases with large values of h_{eff}. This view differs quite radically from the standard view.

Living matter would be essentially matter at quantum criticality [L25]. Anyons as irreps of the Galois group or its isotropy subgroup would replace particles in TQC. The irreps would define the logical qubits stable against perturbations.

2.2.4 Discrete subgroups of quaternionic automorphisms and short range order

What one can say in the TGD framework about the symmetry protected entanglement unstable against symmetry violating perturbations?

1. The automorphism group $SO(3)$ of quaternions is the analog of the Galois group and is expected to be important in the TGD framework since the normal space of 4-surface in M^8 is associative and therefore quaternionic. Quaternionic automorphisms induce rotations of the tangent space of the space-time surface which suggests that the ordinary rotation group of E^3 could be cognitively represented in terms of quaternion automorphisms. In the TGD framework, the covering group $SU(2)$ of the quaternionic automorphism group $SO(3)$ acts in spin degrees of freedom of fermions.

2. At the level of discrete cognitive representations, the discrete subgroups G of $SU(2)$ acting on spinors at mass shells H^3 and at their $M^8 - H$ images define a natural candidate for the emerging symmetries making possible short range entanglement. The effect of these transformations would be on spinors and therefore local.

2.2.5 A possible connection with McKay correspondence

The hierarchy of finite subgroups G of quaternionic $SU(2)$ could have a direct connection with McKay correspondence, which involves finite subgroups of $SU(2)$ and states that the McKay graphs for their irreps correspond to extended Dynkin diagrams of affine ADE type Lie algebras. The possibility that McKay correspondence indeed assigns to McKay graph representations of affine Lie algebras, is discussed in detail in [L28].

1. For very special choices of generators of the subgroup G of $SU(2)$ finite. By McKay correspondence, these subgroups correspond to ADE type affine algebras. It would be therefore highly interesting to study unitary evolutions based on spin rotations, which generate discrete subgroups of $SU(2)$.

2. At the level of M^8, it would be tempting to identify the edge states as irreps of a subgroup Gal_I of Galois group Gal [L29, L24].

   Could Gal_I provide a cognitive representation for a discrete subgroup $G \subset SU(2)$ of $SU(2)$ as covering quaternionic acting on electron spin? I have proposed this kind of cognitive representation as a possible deeper level explanation of Galois correspondence [L28]. *Gal* would act as the Weyl group of the extended ADE Dynkin diagram defined by the McKay graph.

   This proposal involves also a quantum analog of number theory in which ordinary sum and multiplication are replaced by tensor product $\otimes$ and direct sum $\oplus$ for the irreps of G

appearing in the McKay graph. This quantum number theory could define a kind of cognitive representation of number theory.

3. If McKay correspondence is realized in the TGD sense [L28], edge and boundary states could correspond at the level of H to the representations of Kac-Moody algebra satisfying the Kac-Moody and Virasoro gauge conditions for the conformal weights smaller than the integer n characterizing SSA_n. Kac-Moody representations for the isometries and holonomies of H are indeed assigned with the 3-D light-like orbits of partonic 2-surfaces in p-adic mass calculations.

   The TGD inspired explanation of McKay correspondence proposed in [L28] suggests that the finite subgroups of the rotation group and affine Lie-algebras assignable to the hierarchy at the level of H relate to each other. The Galois group would coincide with the Weyl group of an affine ADE Lie algebra acting as a dynamical symmetry. McKay correspondence would reflect $M^8 - H$ duality and would be the physics counterpart [K9, K10] of Langlands correspondence [A3, A2].

4. If Galois groups give rise to stable long range entanglement, one can ask whether the cognitive representations of $G \subset SU(2)$ could absolutely stabilize the corresponding symmetry protected entanglement and whether this could take place for boundary and edge states for which the energy degeneracy could correspond to representation of Gal_I identifiable as representation of G.

These considerations raised a question that I have not considered earlier.

1. The proposal is that the order n of the Galois group defines an effective Planck constant $h_{eff}/h_0 = n$. Can one assign to the order of the discrete subgroup of quaternionic $SU(2)$ an effective Planck constant in a similar way? This could make sense for finite discrete subgroups but not for infinite ones.

2. This question is highly interesting in the case of the Fibonacci process since the 2-gates induced rotations on incoming qubits which are near to a rotation of 4π, which means near-criticality. The 2-gates act as identical rotations of neighboring spins around x- or z-axis. These rotations must generate an infinite discrete subgroup of $SU(2)$. One cannot assign a finite value of h_{eff} to the order of this group.

   Can one conclude that it does not make sense to assign h_{eff} to discrete subgroups of $SU(2)$? Or could the effective Planck constant for the free product of the finite subgroups corresponds to the product of effective Planck constants as the orders of these groups? Or could the subgroups form hierarchies of relative Galois groups assignable to extensions generated by functional composites of polynomials?

2.3 Quantum coherent multispin states

The article [?] also mentions multispin interactions. These can be realized at the level of models but it is far from clear whether standard physics allows to realize them. In the TGD framework, Galois confinement gives rise to N-particles as bound states of N virtual particles behaving like coherent quantum units [L15, L32].

For instance, dark N-photon states behaving like a single quantum coherent unit, which can be emitted and absorbed, are predicted. The emission process makes it possible to change qubits for an entire block of qubits simultaneously rather than doing this qubit by qubit. The analogy with color confinement is obvious. This would make possible methods to create and control many-body quantum entanglement.

Rather remarkably, there is empirical evidence for N-photon states with this property [D2, D3] discussed from the TGD viewpoint in [L18].

2.4 Fibonacci process and $M^8 - H$ duality

To sum up, TGD suggests the following overall view about analogs of the systems studied in [?].

1. At the level of M^8, number theoretical physics suggests Galois symmetries and/or discrete subgroups G of $SU(2)$ as candidates for emerging symmetries making possible symmetry protection. Galois confinement would be an essential element. The difference between Galois and subgroups G is that Galois symmetries act trivially on Galois singlets unlike G.

2. At the level of H, the braidings of magnetic flux tubes, which carry monopole fluxes (they have no Maxwellian analogs) would give rise to topological order making possible long range entanglement. Topological order could correspond to large values of h_{eff} and symmetry protected order to rather small values of h_{eff}. Large value of h_{eff} makes possible long range entanglement. Topological order is not necessary unless it is implied by the $M^8 - H$ duality. This might be the case: dark matter is assumed to reside in monopole flux tubes.

3. Galois group characterizes an entire space-time region number theoretically and is stable against small deformations of the polynomial defining the space-time surface. Both Galois symmetry and topology are therefore holistic notions. This suggests that topological order and Galois symmetries provide dual prerequisites for the long range entanglement although they need not be dual descriptions.

   Indeed, $M^8 - H$ duality is a continuous map so that both topological and number theoretic aspects should be present in both number theoretic and differential geometric descriptions. Strings, string world sheets, and membrane like objects appearing as singularities of minimal surface in H would have number theoretical counterparts in M^8 as real, complex, quaternionic, etc.. surfaces.

The finite subgroups G of $SU(2)$ could be responsible for the symmetry protected order.

1. If one cannot assign h_{eff} to G, one cannot assign a long range order to it. G does not make itself visible in h_{eff} although it can be represented by the Galois group and makes itself visible in the structure of Galois singlets.

2. The article also talks of a hierarchy of emerging dynamical symmetries. Emergence has natural interpretation in terms of extensions of rationals and hierarchies could correspond to sequences of composites $P_n \circ P_{n-1} \circ ... \circ P_1$ of polynomials but how the emergence could be assigned with the discrete subgroups of $SU(2)$?

The TGD view McKay correspondence [L28] is that Galois groups can provide cognitive representations for the dynamical symmetries defined by finite discrete subgroups of $SU(2)$ and also of $SU(n)$.

1. The Galois group of extension would coincide with the Weyl group for an extended ADE Lie algebra and acting on the irreps of isotropy subgroup of Galois group forming which correspond to the roots of a polynomial whose roots carry irreps of the isotropy group and whose tensor products with the canonical representation define the McKay diagram.

2. The vertices of the McKay diagram are invariant under the isotropy group, which leaves the McKay diagram invariant. These Galois=Weyl extensions would be very special. The finite subgroups of quaternionic $SU(2)$ would be in a physically special role.

   They would also give rise to inclusion hierarchies of hyperfinite factors of type II_1 such that each step would involve the Weyl group of some extended diagram as the Galois group. These hierarchies could correspond to hierarchies of composites of polynomials with Galois=Weyl property.

3 Appendix: Isometries and holonomies of WCW as counterparts of exact and broken gauge symmetries

The detailed interpretation of various candidates for the symmetries of WCW [L6] has remained somewhat obscure. At the level of H, isometries are exact symmetries and analogous to unbroken gauge symmetries assignable to color interactions. Holonomies do not give rise to Noether charges and are analogous to broken gauge symmetries assignable to electroweak interactions. This observation can serve as a principle in attempts to understand WCW symmetries.

The division to isometries and holonomies is expected to take place at the level of WCW and this decomposition would naturally correspond to exact and broken gauge symmetries.

3.1 Isometries of WCW

The identification of the isometries of WCW is still on shaky ground.

1. In the H picture, the conjecture has been that symplectic transformations of δM^4_+ act as isometries. The hierarchies of dynamically emerging symmetries could relate to the hierarchies of sub-algebras (SSA$_n$) of super symplectic algebra SSA [L6] acting as isometries of the "world of classical worlds" (WCW) [K5] [L20].

   Each level in the hierarchy of subalgebras SSA$_n$ of SSA corresponds to a transformation in which SSA_n acts as a gauge symmetry and its complement acts as genuine isometries of WCW: gauge symmetry breaking in the complement generates a genuine symmetry, which could correspond to Kac-Moody symmetry. By Noether's theorem, the isometries of WCW would give rise to local integrals of motion: also super-charges are involved. These charges are well-defined but they need not be conserved so that the interpretation as dynamically emerging symmetries must be considered.

   The symmetries would naturally correspond to a long range order. The hierarchies of SSA$_n$:s, of relative Galois groups and of inclusions of hyperfinite factors [K14, K8] could relate to each other as $M^8 - H$ duality suggests [L28].

   What can one say about the algebras SSA_n and the corresponding affine analogs KM_n (for affine algebras the generalized Cartan matrix is a product of a diagonal matrix with integer entries with a symmetric matrix). If n is prime, one can regard these algebras as local algebras in a finite field $G(p)$. Also extensions $G(p, n)$ of $G(p)$ induced by extensions of rationals can be considered. KM algebras in finite fields define what are called the incomplete Kac-Moody groups. Some of their aspects are discussed in the article "Abstract simplicity of complete Kac-Moody groups over finite fields" [A1]. It is shown that for $p > 3$, affine groups are abstractly simple, that is, have no proper non-trivial closed subgroups. Complete KM groups are obtained as completions of incomplete KM groups and are totally disconnected: this suggests that they define p-adic analogs of Kac-Moody groups. Complete KM groups are known to be simple.

2. There are also different kinds of isometries. Consider first the light-cone boundary $\delta M^4_+ \times CP_2$ as an example of a light-like 3-surface. The isometries of CP_2 are symmetries. ΔM^4_+ is metrically equivalent with sphere S^2. Conformal transformations of S^2, which are made local with light-like coordinate r of δM^4_+, induce a conformal scaling of the metric of S^2 depending on r. It is possible to compensate for this scaling by a local radial scaling of r depending on S^2 coordinates such that the transformation acts as an isometry of δM^4_+.

   These isometries of ΔM^4_+ form an infinite-D group. The transformations of this group differ from those of the symplectic group in that the symplectic group of δM^4_+ is replaced with the isometries of δM^4_+ consisting of r-local conformal transformations of S^2 involving S^2-local radial scaling. There are no localizat of CP_2 isometries. This yields an analog of KM algebra.

   This group induces local spinor rotations defining a realization of KM algebra. Also super-KM algebra defined in terms of conserved super-charges associated with the modified Dirac action is possible. These isometries would be Noether symmetries just like those defined by SSA.

3. What about light-like partonic orbits analogous to $\delta M^4_+ \times CP_2$. Can one assign with them Kac-Moody type algebras acting as isometries?

   The infinite-D group of isometries of the light-cone boundary could generalize. If they leave the partonic 2-surfaces at the ends of the orbit X^3_L, they could be seen as 3-D general coordinate transformations acting as internal isometries of the partonic 3-surface, which cannot be regarded as isometries of a fixed subspace of H. These isometries do not affect the partonic 3-surface as a whole and cannot induce isometries of WCW.

However, if X^3_L is connected by string world sheets to other partonic orbits, these transformations affect the string world sheets and there is a real physical effect, and one has genuine isometries. Same is true if these transformations do not leave the partonic 2-surfaces at the ends of X^3_L invariant.

3.2 Holonomies of WCW

What about holonomies at the level of WCW? The holonomies of H acting on spinors induces a holonomy at the level of WCW: WCW spinors identified as Fock states created by oscillator operators of the second quantized H spinors. This would give a generalized KM-type algebra decomposing to sub-algebras corresponding to spin and electroweak quantum numbers. This algebra would have 3 tensor-factors. p-Adic mass calculations imply that the optimal number of tensor factors in conformal algebra is 5 [K3]. 2 tensor factors are needed.

1. SSA would give 2 tensor factors corresponding to δM^4_+ (effectively S^2) and CP_2. This gives 5 tensor factors which is the optimal number of tensor factors in p-adic mass calculations [K3]. SSA Noether charges are well-defined but not conserved. Could SSA only define a hierarchy of dynamical symmetries. Note however that for isometries of H conservation holds true.

2. Also the isometries of δM^4 and of light-like orbits of partonic 2-surfaces give the needed 2 tensor factors. Also this alternative would give inclusion hierarchies of KM sub-algebras with conformal weights coming as multiples of the full algebra. The corresponding Noether charges are well-defined but can one speak of conservation only in the partonic case? One can even argue that the isometries of $\delta M^4_+ \times CP_2$ define a more plausible candidate for inducing WCW isometries than the symplectic transformations. p-Adic mass calculations conform with this option.

To sum up, WCW symmetries would have a nice geometric interpretation as isometries and holonomies. The details of the interpretation are however still unclear and one must leave the status of SSA open.

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