About honeycombs of hyperbolic 3-space and their relation to the genetic code

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Abstract

$M^8 - H$ duality and the realization of holography in M^8 strongly suggests the importance of tessellations of H^3 (analogous to lattices of E^3) in the TGD based physics. These tessellations form a scale hierarchy and can thus appear in all scales. The hierarchy of effective Planck constants labelling dark matter as phases of ordinary matter indeed predicts quantum coherence in arbitrarily long scales and gravitational quantum coherence corresponds to the largest scales of quantum coherence among basic interactions.

There are 5 Platonic tessellations known as honeycombs: the 4 regular honeycombs correspond to cubic, icosahedral, and 2 dodecahedral honeycombs and a quasiregular icosa-tetrahedral honeycomb having tetrahedra, octahedra and icosahedra as cells. The icosa-tetrahedral honeycomb might define a universal realization of the genetic code as an induced structure so that the genetic code would be much more than a biochemical accident. These 5 Platonic honeycombs could occur also in astrophysical scales as gravitational tessellations. The recent discovery of gravitational hum might have an explanation as gravitational diffraction in this kind of a honeycomb.

In this article the properties of hyperbolic honeycombs are considered in detail and also a detailed view about the realization of DNA double strand in terms of the icosa-tetrahedral honeycomb is considered. The emerging model is surprisingly quantitative. Also a connection with the notion of memetic code and the realization of memetic codons in terms of 21 DNA codons are suggested by the model.

Contents

1 Introduction

$M^8 - H$ duality and the realization of holography in M^8 strongly suggests the importance of tessellations of H^3 (analogous to lattices of E^3) in the TGD based physics. These tessellations form a scale hierarchy and can thus appear in all scales. The hierarchy of effective Planck constants labelling dark matter as phases of ordinary matter indeed predicts quantum coherence in arbitrarily long scales and gravitational quantum coherence corresponds to the largest scales of quantum coherence among basic interactions.

The 4 regular honeycombs correspond to cubic, icosahedral, and 2 dodecahedral tessellations. The quasiregular icosa-tetrahedral honeycomb has tetrahedra, octahedra and icosahedra as cells having triangular faces as cells. These honeycombs serve as candidates for physically interesting tessellations. These 5 honeycombs are unique in that they involve only Platonic solids. I have proposed that the icosa-tetrahedral tessellation might define a universal realization of the genetic code as an induced structure so that the genetic code would be much more than a biochemical accident. The details of this realization are discussed in [L13, L8].

These 5 Platonic tessellations (or honeycombs, I will used these terms interchangeably in the sequel) could occur also in astrophysical scales as gravitational tessellations. The recent discovery of gravitational hum might have an explanation as gravitational diffraction in this kind of a tessellation. The unexpectedly large intensity of hum could be due to the concentration of the radiation intensity in discrete directions and due the fact that in diffraction the amplitude of the scattered field is proportional to the square N^2 of the number N of scatterers rather than N.

Icosa-tetrahedral tessellation relates to the TGD based view of the genetic code. The TGD inspired view of genetic code has evolved during decades.

1. The first model of the genetic code was based on the so-called Combinatorial Hierarchy [K1] [L13] and predicted what I called memetic code realized as sequences of 21 DNA codons. Surprisingly, this model made a comeback as I prepared this article.

2. After several stray paths I ended up from a model of music harmony [L1, L3] [L12, L8] based on Hamiltonian cycles at the icosahedron to a model of genetic code also involving the tetrahedral Hamiltonian cycle.

   The basic observation was that the 12-note scale could correspond to a Hamiltonian cycle of icosahedron such that the steps of the cycle define a quint cycle. 12-note scale is obtained from the quint by octave equivalence. There are 3-types of icosahedral Hamiltonian cycles and each cycle defines 20 3-chords assignable to the triangular faces of the icosahedron and defines a musical harmony.

   One obtains 20+20+20 chords for the 3 different harmonies with symmetry groups Z_6, Z_4 and Z_2. The orbits of these groups define sets of 3-chords. The surprising finding was that if these sets are identified as amino acids, the numbers of the chords are the same as the numbers of DNAs coding for a given amino acid. By adding a tetrahedral Hamiltonian cycle one obtains 64 3-chords. At the level of molecules the music would be "music of light". Since music expresses and generates emotions, the idea that emotions appear already at the molecular level was natural. Different combinations of 3 Hamiltonian cycles with symmetries Z_6, Z_4 and Z_2 would correspond to different moods at bio-molecular level (why just 3?)

   The model made almost correct predictions for the numbers of mRNA codons coding for amino-acids. I have discussed a considerable number of its variants during years and even considered the replacement of icosahedron and tetrahedron with some other geometric object.

   The basic problem was that gluing the tetrahedron and icosahedron together looked ugly and would have allowed only 63 codons. At that time I did not yet realize that an icosahedron and tetrahedron could be parts of a bigger structure.

3. Second model was based on the realization of codons as dark proton triplets assumed to reside at the monopole flux tubes parallel to DNA strands [L3, L8]. Dark proton triplets would neutralize the constant negative charge of -3 units per codon. The model suggested that it might be possible to understand the numbers of DNA, RNA, tRNA and amino acids in terms of entangled states of dark proton triplets representing codons. The model had also problems: in particular, one had to assume an additional binary degree of freedom to get the number DNA and mRNA codons correctly and the proposed identifications of this new degree of freedom did not look quite realistic.

4. Icosa-tetrahedral realization [L13] of the code in terms of icosatetrahedral honeycomb of H^3 was the next step in the evolution of ideas. It was made possible only by the dramatic development of understanding of TGD itself, in particular of its number theoretical aspects related to $M^8 - H$ duality [L9, L10].

   The tessellations of the hyperbolic 3-space H^3 represented as possibly complex mass shell in $M^4_c \subset M^8_c$ and as light-cone proper time = constant hyperboloids in $M^4 \subset M^4 \times CP_2$ are central in the the realization of holography in TGD. Icosa-tetrahedral honeycomb is a completely unique tessellation involving only Platonic solids and all possible platonic solids, tetrahedron, icosahedron, and octahedron are present. Kind of a quantum Platonic holy trinity is in question.

   This led to a proposal of the genetic code in terms of icosa-tetrahedral honeycomb induced to the 3-surface by restriction. This realization could be assignable to the magnetic body of the system involving dark matter in the TGD sense. The realization would be universal and would not be restricted to mere biology. Counterparts of codons and genes can be realized also for higher-dimensional objects, say cell membrane and even brain.

   Icosa-tetrahedral realization led to a proposal that the realizations of the code in terms of dark photon triplets and in terms of dark proton triplets are closely related. I did not however really understand the properties of the icosa-tetrahedral honeycomb when I published the first article about it [L13].

Sequences of N dark cyclotron photon triplets as representations of genes consisting of N dark proton triplets would make possible communications between dark genes by 3N-resonance. Genes would serve as addresses, much like in LISP, and the message would be coded by the modulation of the frequency scale. The details of this picture that were not discussed at that time create problems that are solved by the model based on icosahedral honeycomb.

In this article the properties of hyperbolic honeycombs are considered in detail and also a detailed view about the realization of DNA double strand in terms of the icosa-tetrahedral tessellation is considered. The emerging model is surprisingly quantitative and suggests a lot of new understanding about the dark realization of genetic code. Also a connection with the notion of memetic code [K1] [L4] and the realization of memetic codons in terms of 21 DNA codons are suggested by the model.

I have added to the end of the article a section about the recent progress in the understanding of icosa tetrahedral tessellation (ITT). The improved understanding of the ITT allows us to answer some long standing questions related to the detailed realization of the genetic code. It however turns out that the notion of the super-icosahedron discussed in the original version of this article is not consistent with the improved view. However the 3-D generalization of the vertex figure of the ITT as its inverse image under projection permutes the numbers for the Platonic solids appearing in the super-icosahedron.

2 About honeycombs in hyperbolic 3-space

This section, written in 2023, represents some new understanding related to the tessellations of H^3 known as honeycombs.

2.1 Some preliminaries

Some preliminaries are needed in order to understand Wikipedia articles related to tessellations in general.

1. Schläfli symbol $\{p,r\}$ (`rb.gy/j36tg`) tells that the possibly existing Platonic solid $\{p,r\}$ has r p-polygons as faces meeting at each vertex. For instance, icosahedron $\{3,5\}$ has 5 triangles as faces meeting at each vertex.

   Schläfli symbol generalizes to higher dimensions. The analog of Platonic solid $\{p,r,q\}$ possibly in 4-dimensions and assignable to 3-sphere has q 3-faces which are Platonic solids $\{p,r\}$. This description is purely combinatorial and is recursive. For instance, one can start from 3-D dimensional Platonic solid $\{p,q\}$ with 3-D objects in dimension 4 by replacing p with p,r. One can also project this object to dimension 3. In this manner one obtains a projection of 4-cube (tesseract) $\{4,3,3\}$ for which 3 cubes $\{4,3\}$ meet at each vertex ($2^4 = 16$ of them) and which has 8 3-cubes as faces as a 3-D object.

   In the case of hyperbolic tessellations also strange looking Schläfli symbols $\{(p,q,r,s)\}$ are encountered: icosa-tetrahedral tessellation involving only Platonic solids has symbol $\{(3,3,5,3)\}$. My understanding is that this object corresponds to $\{3,3,5,3\}$ as an analogue of Platonic solid associate with 4-sphere in 5-D Euclidian space and that the fundamental region of this tessellation in H^3 is analogous to a 3-D projection of this object. At a given vertex 3 objects $\{3,3,5\}$ meet. For these objects 5 tetrahedrons meet at a given vertex.

2. Vertex figure is a further central notion. It represents a view of the fundamental region of tessellation from a given vertex. The vertices of the figure are connected to this vertex. It does not represent the entire fundamental region. For instance, for a cube (octahedron) it contains only the 3 (4) nearest vertices. For icosa-tetrahedral tessellation the vertex figure is icosidodecahedron (`rb.gy/3u4pq`). The interpretation of the vertex symbol of the hyperbolic icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`) is a considerable challenge.

3. One cannot avoid Coxeter groups and Coxeter symbols (`rb.gy/48qhg`) in the context of tessellations. They code the structure of the symmetry group of say Platonic solid (tessellation of S^2). This symmetry group is generated by reflections with respect to some set of lines,

usually going through origin. For regular polygons and Platonic solids is its discrete subgroup of rotation group.

The Coxeter group is characterized by the number of reflection hyperplanes H_i and the reflections satisfying $r_i^2 = 1$. The products $r_{ij} = r_i r_j$ define cyclic subgroups of order c_{ij} satisfying $r_{ij}^{c_{ij}} = 1$. Coxeter group is characterized by a diagram in which vertices are labelled by i. The orders of the cyclic subgroups satisfy $c_{ij} \geq 3$. For c_{ij} the generators r_i and r_{ij} commute. For $c_{ij} = 2$ the vertices are not connected, for $c_{ij} = 3$ there is a line and for $c_{ij} > 3$ the number c_{ij} is assigned with the line. For instance, hyperbolic tessellations are characterized by 4 reflection hyperplanes.

For instance, for p-polygon the Coxeter group has 2 generators and the cyclic group has order p. For Platonic solids the Coxter group has 3 generators and the orders of cyclic subgroups are 3, 4, or 5. For icosa-tetrahedral tessellation the order is 4.

2.2 The most interesting honeycombs in hyperbolic 3-space

H^3 allows an infinite number of tessellations. There are 9 types of honeycombs. This makes 76 uniform hyperbolic honeycombs involving only a single polyhedron (`hrb.gy/rs9h5`).

4 of these honeycomes are *regular*, which means that they have identical regular faces (Platonic solids) and the same numbers of faces around vertices. The following list gives the regular uniform honeycombs and their Schläfli symbols $\{p, q, r\}$ telling that each edge has around it regular polygon $\{p, q\}$ for which each vertex is surrounded by q faces with p vertices.

1. H1: 2 regular forms with Schläfli symbol {5,3,4} (dodecahedron) and {4,3,5} (cube).
2. H2: 1 regular form with Schläfli symbol {3,5,3}(icosahedron)
3. H5: 1 regular form with Schläfli symbol {5,3,5} (dodecahedron).

There is a large number of uniform honeycombs involving several cell types. There exists however a "multicellular" honeycomb, which is completely unique in the sense that for it all cells are Platonic solids. This icosa-tetrahedral (or more officially, tetrahedral-icosahedral) honeycomb has tetrahedrons, octahedrons, and icosahedrons as its cells. All faces are triangles. The icosa-tetrahedral honeycomb is of special interest since it might make possible the proposed icosa-tetrahedral realization of the genetic code (`rb.gy/h8xx0`).

From the Wikipedia article about icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`) one learns the following.

1. The Schläfli symbol of icosa-tetrahedral honeycomb is $\{(3, 3, 5, 3)\}$. This combinatorial symbol allows several geometric representations. The inner brackets would refer to the interpretation as an analogue of the Platonic solid assignable to a 4-sphere of Euclidian 5-space. At each vertex 3 objects of type $\{3, 3, 5\}$ would meet. At the vertex of $\{(3, 3, 5)\}$ in turn 5 tetrahedrons meet.

2. Icosa-tetrahedral honeycomb involves tetrahedron $\{(3, 3\}$, octahedron $\{(3, 4)\}$, an icosahedron $\{(3, 5)\}$ as cells. That there are no other honeycombs involving several Platonic solids and only them as cells makes this particular honeycomb especially interesting. Octahedron with Schläfli symbol $\{3, 4\}$ can be also regarded as a rectified tetrahedron havig Schläfli symbol $r\{3, 3\}$.

3. The vertex figure of icosa-tetrahedral honeycomb (`htrb.gy/3u4pq`), representing the vertices a lines connecting them is icosidodecahedron (`rb.gy/q5w62`), which is a "fusion" of icosahedron and dodecahedron having 30 vertices with 2 pentagons and 2 triangles meeting at each, and 60 identical edges, each separating a triangle from pentagon. From a given vertex VF=60 vertices connected to this vertex by an edge can be seen. In the case of cube, octahedron, and dodecahedron the total number of vertices in the polyhedron is 2(VF+1). It is true also now, one would have 122 vertices in the basic structural unit. The total number of vertices for the disjoint polyhedra is 6+4+12= 22 and since vertices are shared, the number of polyhedra in the basic unit must be rather large.

4. The numbers called "cells by location" could correspond to numbers 30, 20, and 12 for octahedrons, tetrahedrons and icosahedrons respectively inside the fundamental region of the tessellation defining the honeycomb. That the number of icosahedrons is smallest, looks natural. These numbers are quite large. The counts around each vertex are given by (3.3.3.3), (3.3.3), *resp.* (3.3.3.3) for octahedra, tetrahedra, *resp.* icosahedra and tell the numbers of vertices of the faces meeting at a given vertex.

5. What looks intriguing is that the numbers 30, 20, and 12 for octahedrons (O), tetrahedrons (T) and icosahedrons (I) correspond to the numbers of vertices, faces, and edges for I. As if the fundamental region would be obtained by taking an icosahedron and replacing its 30 vertices with O, its 20 faces with T and its 12 edges with I, that is by using the rules *vertex* $\to$ *octahedron*; *edge* $\to$ *I*, *face* $\to$ *T*. These 3-D objects would be fitted together along their triangular faces.

   Do the statements about the geometry and homology of I translate to the statements about the geometry and homology of the fundamental region? This would mean the following replacements:

   (a) "2 faces meet at edge" $\to$ "2 T:s share face with an I".

   (b) "5 faces meet at vertex" $\to$ "5 T:s share face with an O".

   (c) "Edge has 2 vertices as ends" $\to$ "I shares a face with 2 different O:s".

   (d) "Face has 3 vertices $\to$ "T shares a face with 3 different O:s".

   (e) "Face has three edges" $\to$ "T has a common face with 3 I:s".

2.3 An attempt to understand the hyperbolic honeycombs

The following general observations might help to gain some understanding of the honeycombs.

The tessellations of E^3 and H^3 are in many respects analogous to Platonic solids as 2-D objects. The non-compactness implies that there is an infinite number of cells for tessellations. It is important to notice that the radial coordinate r for H^3 corresponds very closely to the hyperbolic angle and its values are quantized for the vertices of tessellation just like the values of spherical coordinates are quantized for Platonic solids. The tessellations for E^3 are scale covariant. For a fixed radius of H^3 characterized by Lorentz invariance cosmic time this is not the case. One can however scale the value of a. What distinguishes between regular tessellations in E^3 and H^3 is that the metric of H^3 is non-flat and has negative curvature. H^3 is homogeneous space meaning that all points are metrically equivalent (this is the counterpart of cosmological principle in cosmology). Since both spaces have rotations as symmetries, this does not affect basic Platonic solids as 2-D structures assignable with 2-sphere if the edges are identified as geodesic lines of S^2. Quite generally, isometries characterize the tessellations, whose fundamental region corresponds to coset space of H^3/Γ by a discrete group of the Lorentz group acting as isometries of H^3. The modifications induced by the replacement $E^3 \to H^3$ relate to the 3-D aspects of the tessellation. This is because the metric is non-flat in the radial direction. The negative curvature implies that the geodesic lines diverge. One can use a counterpart of the standard spherical coordinates and in these coordinates the solid angles assignable to the vertices of Platonic solid are smaller than in E^3. Also the hyperbolic planes H^2 emerging from edges of the tessellation of H^3 diverge in normal direction the angles involved are smaller.

It is useful to start from the description of the Platonic solids. They are characterized combinatorially by integers and geometrically by various kinds of angles. Denote by p the number of vertices/edges of the face and by q the number of faces meeting at vertex.

3. Important constraints come from the topology and combinatorics. Basic equations for the numbers V ,E, and F for the number of vertices, edges and faces are purely topological equations $VE + F = 2$, and the equation $pF = 2E = qV$. Manipulation of these equations gives $1/r + 1/p = 1/2 + 1/E$ implying $1/r + 1/p > 1/2$. Since p and q must be at least 3, the only possibilities for $\{p, q\}$ are $\{3, 3\}$, $\{4, 3\}$, $\{3, 4\}$, $\{5, 3\}$, and $\{3, 5\}$.

2. The angular positions of the vertices at S^2 are basic angle variables. In H^3 hyperbolic angle assignable to the radial coordinate is an additional variable of this kind analogous to the position of the unit cell in the E^3 tessellation. The cosmological interpretation is in terms of redshift.

3. There is the Euclidian angle ϕ associated with the vertex of the face given by π/p. Here there is no difference between E^3 and H^3.

4. The angle deficit δ associated with the faces meeting at a given vertex due to the fact that the faces are not in plane in which case the total angle would be 2π. δ is largest for tetrahedron with 3 faces meeting at vertex and therefore with the sharpest vertex and smallest for icosahedron with 5 triangles meeting at vertex. This notion is essentially 3-dimensional, being defined using radial geodesics, so that the δ is not the same in H^3. In H^3 δ is expected to be larger than in E^3.

5. There is also the dihedral angle θ associated with the faces as planes of E^3 meeting at the edges of the Platonic solid. θ is smallest for a tetrahedron with 4 edges and largest for a dodecahedron with 20 edges so that the dodecahedron is not far from the flat plane and this angle is not far from π. The H^3 counterpart of θ is associated faces identified as hyperbolic planes H^2 and is therefore different.

6. There is also the vertex solid angle Ω associated with each vertex of the Platonic solid $\{p, q\}$ given by $\Omega = q\theta - (q-2)\pi$. For tessellations in E^3 the sum of these angles is 4π. In H^3 its Euclidian counterpart is larger than 4π.

7. The face solid angle is the solid angle associated with the face when seen from the center of the Platonic solid. The sum of the face solid angles is 4π. For Platonic solid with n vertices, one has $\Omega = 4\pi/n$. The divergence of the geodesics of H^3 implies that this angle is smaller in H^3: there is more volume in H^3 than in E^3.

E^3 allows only single regular tessellation having cube as a unit cell. H^3 allows cubic and icosahedral tessellations plus two tessellations having a dodecahedron as a unit cell. Why does E^3 not allow icosahedral and dodecahedral tessellations and how the curvature of H^3 makes them possible? Why is the purely Platonic tetra-icosahedral tessellation possible in H^3?

The first guess is that these tessellations are almost but not quite possible in E^3 by looking at the Euclidian constraints on various angles. In particular, the sum of dihedral angles θ between faces should be 2π in E^3, the sum of the vertex solid angles Ω at the vertex should be 4π. Note that the scaling of the radial coordinate r decreases the dihedral angles θ and solid angles Ω. This flexibility is expected to make possible so many tessellations and honeycombs in H^3. The larger the deviation of the almost allowed tessellation, the larger the size of the fundamental region for fixed a.

Consider now the constraints on the basic parameters of the Platonic solids (`rb.gy/1cuav`) in E^3 while keeping their H^3 counterparts in mind.

1. The values of didedral angle for tetrahedron, cube, octahedron, dodecahedron, and icosahedron are

$$[\theta(T), \theta(C), \theta(O), \theta(D), \theta(I)] \approx [70.3^\circ,\ 90^\circ, 109.47^\circ,\ 116.57^\circ, 138.19^\circ] \ .$$

Note that $r = 5$ tetrahedra meeting at a single edge in E^3 would almost fill the space around the edge. In E^3 $r = 4$ cubes can meet at the edge. In H^3 r should be larger. This is indeed the case for the cubic honeycomb $\{4, 3, 5\}$ having $r = 5$. For $r = 3$ icosahedrons the sum dihedral angles exceeds 2π which conforms with the that $\{3, 5, 3\}$ defines an icosahedral

tessellation in H^3. For the $r = 4$ dodecahedra meeting at the edge the total dihedral angle is larger than 360°: $r = 4$ is therefore a natural candidate in H^3. There are indeed regular dodecahedral honeycombs with Schläfli symbol $\{5, 3, r\}$, $r = 4$ and $r = 5$. Therefore it seems that the intuitive picture is correct.

2. The values of the vertex solid angle Ω for cube, dodecahedron, and icosahedron are given by the formula $\Omega = q\theta - (q-2)\pi$ giving

$$[\Omega(C), \Omega(D), \Omega(I)] \approx [1.57080, 2.96174, 2.63455].$$

The sum of these angles should be 4π for a tessellation in E^3. In E^3 This is true only for 8 cubes per vertex ($\Omega = \pi/2$) so that the cubic honeycomb is the only Platonic honeycomb in E^3. The minimal number of cubes per vertex is 9 in H^3. It is convenient to write the values of the vertex solid angles for D and I as

$$[\Omega(D), \Omega(I)] = [0.108174,\ 0.209651] \times 4\pi \ .$$

The number of D:s *resp.* I:s must be at least 10 *resp.* 5 for dodecahedral *resp.* icosahedral honeycombs in H^3.

3. The basic geometric scales of the Platonic solids are circumradius R, surface area A and volume V. The circumradius is given by $R = (a/2)\tan(\pi/q)\tan(\theta/2)$, where a denotes the edge length. The surface area A of the Platonic solid $\{p, q\}$ equals the area of face multiplied by the number F of faces: $A = (a/2)^2 Fpcot(\pi/p)$. The volume V of the Platonic time is F times the volume of the pyramid whose height is the length a of the face: that is $V = FaA/3$.

Choosing $a/2$ as the length unit, the circumradii R , total face areas A an the volumes V of the Platonic solids are given by

$$[R(T), R(C), R(O), R(D), R(I)] = [\sqrt{3}/2, \sqrt{3}, \sqrt{2}, \sqrt{3}\phi, \sqrt{3-\phi}\phi] \ ,$$

$$[A(T), A(C), A(O), A(D), A(I)] = [4\sqrt{3}, 24, 2\sqrt{3}, 12\sqrt{25+10\sqrt{5}}, 20\sqrt{3}] \ ,$$

and

$$[V(T), V(C), V(O), V(D), V(I)] \approx [\sqrt{8}/3, 8,\ \sqrt{128}/3,\ 20\phi^3/(3-\phi), 20\phi^2/3]$$
$$\approx [.942809, 8, 3.771236, 61.304952, 17.453560] \ .$$

What can one say about icosa-tetrahedral tessellation?

1. Consider first the dihedral angles θ. The values of dihedral angles associated T, O, and I in H^3 are reduced from that in E^3 so that their sum in E^2 scene must be larger than 2π. Therefore at least one of these cells must appear twice in H^3. It could be T but also O can be considered. For $2T + O + I$ and $T + 2O + I$ the sum would be 388.26° *resp.* 427.43° in E^3. $2T + O + I$ *resp.* $T + 2O + I$could correspond to 4 cells ordered cyclically as ITOT resp. IOTO.

2. The values of the vertex solid angle Ω for tetrahedron, octahedron, and icosahedron are given by $[\Omega(T), \Omega(O), \Omega(I)] = [0.043870, 0.108174, 0.209651]4\pi$ If the numbers of T, O and I are $[n(T), n(O), n(I)]$, one must have $[n(T)\Omega(T), +n(O)\Omega(O) + n(I)\Omega(I) > 4\pi$ in H^3.

If the number of the cells for the fundamental domain are really $[N(T), N(O), N(I)] = [30, 20, 12]$, the first guess is that $[n(T), n(O), n(I)] \propto [N(T), N(O), N(I)$ is approximately true. For $[n(T), n(O), n(I)] = [2, 3, 1]n(I)$, one obtains $\Omega = n(T)\Omega(T) + n(O)\Omega(O) + n(I)\Omega(I) = n(I) \times .629 \times 4\pi$. This would suggest $n(I) = 2$ giving $[n(T), n(O), n(I)] = [4, 6, 2]$

3 New results about the relation of the icosa-tetrahedral tessellation to the dark genetic code

How could the icosa-tetrahedral tessellation relate to the proposed dark realizations of the genetic code [L12, L13]?

3.1 About the problems of the earlier view of the dark realizations of the genetic code

Consider first the problems of the earlier views of the realization of the dark genetic codes in terms of dark proton triplets at monopole flux tubes parallel to the ordinary DNA and to the realization in terms of dark photon triplets.

1. The TGD based inspired model of the dark photon genetic code [L1] [L8, L12] assumes that the dark realization of genetic code involves 3 icosahedral Hamiltonian cycles giving rise to 20+20+20 dark DNA codons and the unique tetrahedral Hamiltonian cycle giving the remaining 4 codons.

   The obvious problem of icosa-tetrahedral picture is that one must assume that icosahedron and tetrahedron are disjoint. If they have a common face, the number of faces reduces to 63 and one DNA codon is missing. This raises the question whether icosahedron and tetrahedron could be disjoint pieces of a larger structure.

2. Icosahedron and tetrahedron should have a physical realization: what could it be? How the Hamiltonian cycles are realized physically? The cycles are defined only modulo the isometry group I of icosahedron having 60 elements and Z_n $n = 6, 4$ or 2 leaves the cycle and the orbits of this group (amino-acids) invariant. The Hamiltonian cycle has $\#(I/Z_n)$ isometric copies (the numbers of copies are 10, 15, and 32). Does this have a physical significance? How are the 12 frequencies associated with the edges of the cycle realized physically? What is the physical interpretation of octave equivalence: does it have something to do with 2-adicity?

3. In the dark proton realization a given codon would correspond to a selected triangular face of I or T carrying dark protons at the vertices of this face. The original view was that dark 3-proton states would correspond to 64 codons. The problem was that one obtains only 8 states for dark proton triplets from spin and antisymmetrization in spin degrees of freedom would not allow any states unless the spatial wave function is totally antisymmetric and spins are in the same direction.

   In the original proposal also neutrons were assumed so that the codon corresponds to a sequence of 3 nucleons with both spins. 3 nucleons would give rise to 64 states as required. Dark protons can also be effectively neutrons as far as charge is considered. This might be possible if the bonds connecting the dark protons can be both neutral and negatively charged. Weak interactions are as strong as electromagnetic interactions in a given biological scale (such as DNA scale) if the dark Compton length proportional to h_{eff} is larger than this scale and the weak transitions change the dark protons to effective dark neutrons.

   This option leads to a problem with the fact that DNA nucleotides have negative unit charge. One should have protons to neutralize this charge and stabilize DNA. Also variants of the proposal in which there are flux tube connections between dark protons having 2 different neutral states analogous to neutral pion and neutral ρ meson.

   The simplest proposal, which is consistent with the idea that genetic codons correspond to cyclotron transitions of dark proton triplets assignable to the triangular faces of an icosahedron or tetrahedron is as follows. Besides 2 spin states, dark protons can also have 2 states with spin ± 1 corresponding to the analog of rotation in the discrete space defined by the vertices of the triangle. This would give $2^3 \times 2^3 = 64$ states.

The realizations of the genetic code in terms of dark photon triplets and dark proton triplets should correspond to each other. This requires that dark proton triplet realization should naturally correspond to the icosa-tetrahedral realization.

1. The codons identified as dark proton triplets assignable to one of the 20 triangular faces of icosahedron and tetrahedron have in quantum situations a wave function in the discrete space of the faces, which is in general delocalized. Could these wave functions in the set of faces give rise to states in 1-1 correspondence with the icosahedral and tetrahedral codons? There would be 20 wave functions for an icosahedron and 4 wave functions for a tetrahedron. The number of icosahedral states must be tripled to 60 corresponding to the 3 basic types of icosahedral Hamiltonian cycles with symmetries Z_n, $n = 6, 4, 2$.

   The 3 dark protons also have spin degrees of freedom. The dark proton triplet in the ground state(s) would be naturally spontaneously magnetized so that all spins are in the same direction. Also the states in which some dark protons are excited are allowed by Fermi statistics and are needed since these excitations could correspond to the spatial wave functions in face degrees of freedom.

2. Dark photon triplets are needed for communications. The vision is that they correspond to the representation of codons as frequency triplets represented by the realization of icosahedral and tetrahedral Hamiltonian cycles as frequency triplets. The assumption has been that the 3 frequencies of dark 3-photon are associated with the cyclotron (or Larmor transitions if only spin is dynamical) of dark protons of a dark proton triplet.

   Dark photon communications between identical codons would take place by 3-resonance. The de-excitation of the first codon would lead to the excitation of an identical codon: one would have a kind of flip-flop. Also dark genes as sequences of N dark codons could act as a single quantum coherent unit and 3-N resonances between identical dark genes would become possible. The mechanism is very similar to that used in the computer language LISP. The modulation of the frequency scale by modulating the thickness of the monopole flux tubes would make possible coding of the signal and it would be transformed to a sequence of resonance pulses at the receiving end.

   Dark photon triplet states could correspond to wave functions in the space of icosahedral and tetrahedral faces.

3. Cyclotron transitions would be needed in order to generate dark photon triplets. This would require excitations of the dark protons of the spontaneously magnetized ground state(s). If only spin matters, the cyclotron transitions reduce to Larmor transitions. The correspondence with the icosahedral Hamiltonian cycles in terms of dark photon triplets would suggest that these excitations correspond to icosahedral genetic codons as wave functions in the set of faces. The cyclotron transition would provide the energy needed to excite the wave function in the set of faces. 64 transitions would be needed. It is important to notice that cyclotron transitions rather than cyclotron states of dark protons would correspond to codons of icosa-tetrahedral representation represented as wave functions in the set of facs.

   There are however only 8 states per face if only Larmor transitions are allowed. This is much less than the number 20 20+20+4=64 for icosahedral and tetrahedral Hamiltonian cycles. An additional two-valued degree of freedom is needed. The simplest possibility is the assignment to each dark proton an analog of angular momentum eigenstate with spin ± 1 corresponding to a discrete rotation around the triangle. This would give $8 \times 8 = 64$ states per face. Could the excitations of these states correspond to 20+20+20 icosahedral states plus 4 tetrahedral states?

4. Hitherto the considerations have been implicitly classical in that a localization in the set of faces has been assumed. Quantum theory allows us to give up this assumption. Icosahedral realization suggests that dark proton triplet has a icosahedral wave function delocalized to the set of 20 faces with symmetry fixed by the Hamiltonian cycle to Z_n, $n = 6, 4$ or 2, and that the excitation of the dark proton triplet in the face degrees of freedom provides the energy changing the wave function in the set of faces. The same would apply to the tetrahedron with symmetry Z_4 allowing 4 wave functions.

   The orbital and angular momentum degrees of freedom would be coupled. The transition from the ground state for dark proton triplet would excite wave function in the set of faces. This could imply the desired correspondence between the dark proton representations and dark photon realizations of the code.

5. There is a further problem. Spontaneously magnetized states of 3 dark protons would define ground states of codons. The ground state proton triplet cannot have lower energy states and cannot emit dark photon triplets and are therefore "mute" and unable to communicate, presumably necessary for processes like transcription and translation. Note that ground states are however not deaf.

The proposed general view is attractive but the details remain to be understood and problems solved. Here the notion of icosa-tetrahedral tessellation could help. The proposal of [L13] was that the icosa-tetrahedral honeycomb at the light-cone proper time $a = constant$ surfaces identifiable as hyperbolic 3-space H^3 allows to realize the dark genetic code.

The icosa-tetrahedral honeycomb is the unique honeycomb, which involves only Platonic solids. This inspires the question whether genetic code could be universal and realized in all scales by induction, which means that the tessellation of H^3 induces tessellation of 3-surface $X^3 \subset H^3$ by restriction. Also the induction to $H^3(a)$ projection of X^4 makes sense.

The TGD view of holography indeed predicts the special role of hyperbolic 3-spaces. The space-time surfaces in $H = M^4 \times CP_2$ are analogs of Bohr orbits, which go through $H^3(a_n) \subset M^4 \subset H$, where a_n corresponds to a root of the polynomial with integer coefficients determining to a higher degree a given region of the space-time surface by $M^8 - H$ duality [L9, L10].

In the sequel the detailed realization of the genetic code in terms of the icosahedral honeycomb will be discussed with an emphasis on the problems noticed above.

3.2 The realization of the code in terms of icosa-tetrahedral tessellation

The fundamental region of the icosa-tetrahedral tessellation contains 30 octahedrons, 20 tetrahedrons, and 12 icosahedrons and the cautiously proposed interpretation is that the cells meeting at each *edge* of the tessellation have either the cyclic structure TOTI or OTOI, and each vertex involve 3 O:s, 2 T:s and 1 I. Could one interpret this in terms of the dark icosahedral realization of the genetic code?

3.2.1 Ideas related to the detailed realization of the genetic code

The detailed realization of the dark genetic code is far from completely understood and one might hope that icosa-tetrahedral realization could bring in the constraints allowing us to fill in the details. It is useful to proceed by considering basic requirements on the realization of the dark code.

1. There are 3 O:s per single I in vertex if 10 instead of 12 icosahedral cells are included. The reasons for this become clear from the proposed relation between DNA double strand and fundamental cell of icosahedral honeycomb. What could the role of O:S be?

   Imagine that it is possible to arrange the polyhedrons for a given I to cycles as -I-O-T-O-T-O-: here cyclicity is assumed. The two tetrahedrons and I would be disjoint. This would solve the problem due to the common face of T and I (only 63 DNA codons) but give 60+4+4 faces and 68 dark DNA codons. There is however the problem posed by the mute codons. Could the presence of mute DNA codons reduce the number of DNA codons from 68 to 64. This would imply that their transcription allows only 64 dark mRNA codons. Could mute mRNA codons reduce the effective number of mRNA codons to 61 for the standard code (stop codons would be mute)? What about its variants with a smaller number of stop codons?

2. Bioharmony involves 3 icosahedral Hamiltonian cycles. All the combinations of the 3 -cycles with symmetries Z_6, Z_4 and Z_2 predict the same code. These bioharmonies are interpreted as correlates for emotional states appearing already at the basic bio-molecular level. The motivation comes from the fact that the icosa-tetrahedral harmony emerges as a geometric model for the music harmony and music indeed both creates and expresses emotions.

   Could icosahedral honeycomb allow us to understand the realization of these 3 icosahedral Hamiltonian cycles in terms of cyclotron frequency triplets? One must have closed magnetic monopole loops in order to have cyclotron transitions. Could these loops form triangles

of form I-T-O. This would be 6 different triangles and 3 different positions of I for given T. This kind of loop would be assigned with each vertex of the face. Could the magnetic field strengths depend on the loop and for a given T give rise cyclotron frequency triplets characterizing a given icosahedral Hamiltonian cycle.

3. One can criticize the assumption that there is only a single codon per single I and T. I:s could in principle carry several codons. This however gives a restriction that the codons inside given I and T are different and restricts the representative power of the code if it involves more than 2 strands. This restriction is however automatically satisfied for the base-paired codon and anticodon in the DNA double strand!

3.2.2 Dark photon realization of the icosahedral part of the code

Consider first the realization of the icosahedral part of the code in terms of dark photons.

1. The 3 icosahedral Hamiltonian cycles have symmetries. The 20 codons with Z_6 symmetry correspond to 3 6-plets and 1 doublet of Z_6 and for unbroken symmetry the codons inside these multiplets code for the same amino acid. This means 3+1= 4 amino acids. Z_4 symmetry has 5 4-plets and in absence of symmetry breaking this corresponds to 5 amino-acids. Z_2 symmetry as 10 2-plets, and also this symmetry is also almost exact and corresponds to the almost exact symmetry with respect to the third letter of the codon analogous to isospin symmetry.

2. Icosahedral part of the icosa-tetrahedral realization involves 3 icosahedral Hamiltonian cycles characterized by different symmetries. For Z_6 symmetry, there are 6+6+6+2=20 codons codons. These sets of codons can be regarded as orbits of Z_6 and correspond to amino-acids. This if the Z_6 symmetry is not broken. This means 3+1 amino acids in absence of symmetry breaking.

   Z_4 symmetry has 5 4-plets and in absence of symmetry breaking this corresponds to 5 amino-acids coded by 4 codons each. Z_2 symmetry has 10 2-plets and this symmetry is also almost exact. This symmetry corresponds to the almost exact symmetry with respect to the third letter of the codon.

3. Dark photon codons are represented as cyclotron frequency triplets of dark photons created in 3-cyclotron transitions for dark proton triplets involving simultaneous emission of 3 dark photons made possible by quantum coherence. In the case of genes with N codons one has 3N-cyclotron transition and 3N dark proton-state represents a gene as a quantum coherent unit.

3.2.3 Dark proton realization of the icosahedral part of the code

Consider next the dark proton realization of the icosahedral part of the code.

1. The basic problem of the dark proton realization of the code is that there are only 8 dark proton spin states. If one assumes that each dark proton can have spin ± 1 this problem the number of dark proton states is 4 and one obtains 64 states.

   If one allows the states with vanishing spin so that one would have 3 orbital states per dark proton, the number of cyclotron transitions per dark proton is 4. Since lowest energy states are mute and transitions define codons, this could be the correct identification.

2. Icosa-tetrahedral realization should give 20+20+20+4 =64 dark proton triplets assignable to the faces of I and T. Suppose that the cells can be thought of as forming a cycle O-I-O-T-O-T with O and T ends connected. The two T:s have no common faces with O and without additional conditions give rise to 4+4 additional codons giving 68 codons. How can one reduce the number of dark DNA codons to 64?

3. Dark proton codons have a ground state, or possibly several of them, which by definition cannot decay to lower energy states by emission of dark photon cyclotron triplet. Ground state codon is mute since it cannot produce dark photon triplets as 3-chords.

The natural first guess is that the ground states correspond to the 6 combinations 3 icosahedral Hamiltonian cycles and 2 tetrahedral cycles assignable to 2× T. The 3 stop codons are transcribed but not translated so that the interpretation of 3 DNA stop codons as icosahedral ground state dark codons unable to send 3-photon signals is not correct. For mRNA this interpretation could make sense if the mRNA images of DNA stop codons represent ground state codons.

4. Cyclotron excitations of ground state codons are induced by dark photon triplets. Conversely cyclotron de-excitatons generate dark proton triplets except for the ground state codons with minimum total energy. Suppose that there are 6 ground state codons as combinations of 3 dark codon ground states assignable to the 3 icosahedral Hamiltonian cycles and 2 dark proton ground states assignable to tetrahedral cycles of the two T:s. This would give 8 mute states. The total number of dark DNA codons is 60+8=68. Note that the mute states are not deaf: they can receive messages.

   One would obtain only 60 DNA codons, which can be transcribed to mRNA codons if the transcription involving dark photon codons. How could one get 64 as an effective number of DNA codons?

   One can imagine transitions between otherwise mute codons, which generate dark photon triplets coupling to mRNA associated with DNA. Let A, B and C the ground state codons with minimal total dark cyclotron energies in an increasing order for the 3 icosahedral Hamiltonian cycles. If for a given T (two options) the cyclotron transitions are possible only between codons C and B and B and A one obtains 2 DNA-mRNA pairings for both T:s. One would have 60+2+2=64 mRNAs pairing with DNA and effectively 64 DNA codons.

   Note that the transcription produces only 64 dark mRNA codons from 68 dark DNA codons.

   For 64 mRNA codons it could happen that there are no transitions between the 3 icosahedral codons for both choices of T so that there are 6 mute mRNA codons. If there are transitions C → B and B → A, the number of mute icosahedral codons is 4. If there are no transitions between tetrahedral ground state codons, one has effectively 60 mRNA codons since the translation stops due to the absene of dark 3-photon signals to tRNA. If there is a transition between the 2 ground state nRNA codons associated with the two T:s, one obtains 61 effective mRNA codons of the standard realization of the code. The transitions between tetrahedral codons can increase the effective number of mRNA codons.

5. What about tRNA appearing as a pair of amino-acid and single RNA codon. Could the RNA of tRNA and amino-acids correspond to the unique icosahedral honeycomb of H^3 and to icosahedral Hamiltonian cycles so that the number of dark codons in absence of tetrahedral degeneracy would reduce to 32, which is the minimal number of ordinary tRNA codons, which is increased by the non-uniqueness of the ordinary tRNA itself? Note that mute tRNA codons are not deaf: they can receive messages but cannot send them. Obviously, tRNA and amino-acids would correspond to the lowest evolutionary level.

The tentative conclusion would be that in the TGD framework DNA-mRNA transcription is not 1-to-1: information is lost and could say that RNA represents a lower level of evolutionary hierarchy. This would conform with the RNA world vision. The numbers of dark proton DNA and mRNA codons are 68 and 64 respectively. The unavoidable existence of mute codons gives effective DNA codon number 64 as the number of mRNA codons. 3 icosahedral codons can be mute and one obtains 3 stop codons unable to communicate with tRNA. The number of mute codons can also be smaller.

The dark DNA and RNA codons are dynamical and are not fixed to be the same as ordinary codons. This is required only during the communications with ordinary DNA possibly taking place by dark photons transforming to ordinary photons and inducing resonant transitions of ordinary DNA and other basic biomolecules. This strongly suggests that dark DNA and RNA act as kinds of R&D laboratories making it possible to test variants of the genes. Actually their ground states would correspond to 3 icosahedral representations and 2 tetrahedral representations and would correspond to aminoacids via transcription and translation.

Needless to say, this picture is highly speculative and one can probably imagine variants for it. The basic idea is however clear: icosa-tetrahedral tessellation could explain the details of the standard genetic code and its modifications.

3.2.4 Realization of the flux tube structures associated with dark codons

The following represents an attempt to make the above picture more concrete.

1. The selection of 1 O from 3 O:s could mean a selection of an icosahedral Hamiltonian cycle with symmetry group Z_6, Z_4, or Z_2. This gives for icosahedral realization 20+20+20 =60 icosahedral codons. Tetrahedral Hamiltonian cycles associated with the two T:s should give the remaining 4 codons. One can however imagine several ways for how this could occur.

2. The selection of O should correspond to a choice of the icosahedral cycle. What does this mean geometrically? To each dark proton of the codon, one must assign a closed monopole flux tube. The strength of the magnetic field of the flux tube fixes the cyclotron frequency scale for each flux tube. The 20 dark-photon chords defining a given icosahedral bioharmony differ for different choices of O and T. The frequencies are fixed if the Hamiltonian cycle corresponds to a quint cycle such that the frequencies associated with the neighboring vertices of the Hamiltonian cycle differ by a scaling 3/2. This requires that the magnetic field strengths along the cycle differ by scaling 3/2.

3. How to concretely realize the correlation of the bioharmony with the choice of O and T for a given I? Suppose that for a given I, the closed flux tube connects I and the selected O and T. There would be a closed I-O-T flux tube for each vertex of the face defining the codon. This kind of flux tube would define an analog of a string of a musical instrument.

   These closed flux tubes would be hyperbolic analogies of closed circuits formed by Euclidian nearest neighbour lattice bonds. If makes sense to assign to each I a cycle O-I-O-T-O-T, with O and T at ends being connected, the cycle I-O-T would go through the either T, and this implies that tetrahedral codons correspond to the other face of T. One would obtain 64 dark proton codons with 3 mute dark proton codons identifiable as stop codons. In the transcription the signal as a dark photon triplet would not reach the dark RNA codon and the transcription would stop. Could this mean that dark RNA codon attaches first to dark DNA codon and the transcription of DNA to ordinary RNA occurs after that in the usual way.

4. The proposed transitions between ground state codons for icosahedral Hamiltonian cycles modify the cycle geometrically since the O in cycle I-O-T changes. If the transitions for given T are only of C$\rightarrow$ B and $B \rightarrow A$ with energies in increasing order, one can imagine that the O is replaced by a neighboring O in the transition in the O-I-O-T-O-T.

Several questions remain to be answered.

1. The symmetry breaking for the icosahedral codons with Z_n, $m = 6, 4, 2$ should be understood. This symmetry breaking can be assumed to occur at the level of dark mRNA and modify the frequency triplets from those for completely symmetric mRNA codons. The replacement $T \rightarrow U$ might relate to the symmetry breaking.

   UUG, CUG, and the very common AUG appear as start codons. They correspond to symmetry breaking for 6-plet (Z_6) coding for leu and 4-plet (Z_4) coding for ile. All symmetry breakings occur for start codons UUG, CUG, and for codons UAA and UAG and UGA and UGG closely related to stop codons.

2. Can one understand the reduction of the number of mRNA stop codons to 2 or 1 occurring for some variants of the code? In these situations, the stop codon of mRNA can code for an exotic amino acid pyrrolysine and selenocysteine. Could the transition between stop codon of dark mRNA icosahedral Hamiltonian cycle to a stop codon of another Hamiltonian cycle take place such that the dark photon triplet generated couples to tRNA involving the exotic amino acid. Situation would be almost like in the case of DNA where only two ground state codons stop the transcription.

3. What can one say about the strength of the magnetic fields assignable with the monopole flux tubes? Nanometer length scale 1 nm, naturally assignable to the DNA double strand, corresponds from the formula $l_B = 26nm/\sqrt{B/Tesla}$ to 12.2 GHz. What is interesting is that the gravitational Compton frequency for Earth is 67 GHz and defines a lower bound for the gravitational quantum coherence time. If the strengths of the magnetic fields span 7 octaves, the thickness of the flux tube would vary by a factor 10 in the range about .1 nm - 1 nm.

4. Note that the 12-note scale can be realized using powers $(3/2)^k$, $k = 1, ..., 12$, of the fundamental and by using octave equivalence to reduce the note to the basic octave. Since the monopole flux is quantized, the realization of the scale requires variation of flux tube thickness inducing variation of magnetic field strength and therefore of that cyclotron frequency scale.

   There is nothing cherished in the rational quint cycle as the basis of the 12-note scale. For instance, the well-tempered scale actually replaces the Pythagorean scale with an algebraic scale coming in powers of $2^{1/12}$.

3.3 Description of the entire DNA double strand in terms of icosatetrahedral tessellation

The most ambitious model would describe the entire DNA double strand and relate the model bioharmony to the properties of the icosa-tetrahedral tessellation. There are however many questions remaining.

1. Single DNA and RNA strand would correspond to a "half realization" for which the T and I cells would contain only single codon. The splitting of DNA could have a geometric interpretation as an effective replication of the induced tessellation to two tessellations to RNA type tessellations.

2. There are 20 amino-acids and an icosahedron involves 20 faces. Is this a mere accident? Could icosahedral honeycomb describe amino-acid sequences geometrically. tRNA appears as a single unit. tRNA-amino-acid pairing would involve pairing of two icosahedral tessellations as also the pairing of RNA and tRNA in the translation. tRNA would naturally correspond to a single cell of icosahedral tessellation. This would also explain why the number of tRNA molecules is considerably smaller than RNA codons.

3. Does RNA correspond to icosahedral or icosa-tetrahedral tessellation? Tetrahedral Hamiltonian cycles are needed, in particular the dark proton triplets associated with the tetrahedral faces. Therefore icosa-tetrahedral tessellation is the natural option also for RNA.

4. It is thought that DNA and RNA nucleotides float freely in the cellular water and DNA and RNA codons are built from them in replication/transcription. This is probably the case at the biochemical level, whose dynamics is controlled by dark level (I have however considered the possibility that freely floating nucleotides could actually form loosely bound codons).

   At the ark level both replication and transcription would involve replication of the induced icosa-tetrahedral tessellation: a similar process occurs for clay crystals, and is suggested to be a precursor of DNA replication. This process is a holistic quantum process occurring in a single quantum jump. This would explain the incredible accuracy of these processes, which is extremely difficult to understand in the chemical approach.

   The replication would determine the outcome, be it a pair of DNA double strands or of DNA and RNA. After this the chemical processes leading to the formation of chemical codons from nucleotides and their pairing with dark codons of the induced icosa-tetrahedral tessellation would take place.

DNA has a helical structure. Helical tessellations are known to exist (`rb.gy/5ova6`). If icosa-tetrahedral tessellation is induced, the helical structure would most naturally reflect the dynamics of the corresponding space-time surface. This suggests that only a sequence of I:s is selected from the set of 12 I:s in a given fundamental region of the icosa-tetrahedral tessellation.

To see whether this hypothesis can make sense one must use geometrical facts about DNA double helix, which has A-, B-, and Z forms `rb.gy/4kcrm`).

1. B-form is believed to dominate in cells. From the table of the Wikipedia article one learns that for the B-form the rise per base pair (bp) is 3.32 $\AA$, that full turn corresponds to 10.5 bps, and that the pitch of the helix per turn 33.2 $\AA$, which corresponds to 10 bps per turn. The pitch/turn should be equal to $10.5 \times 3.32 = 34.52$ $\AA$. There is obviously a mistake in the table.

2. The solution of the puzzle is that straight DNA in solution has 10.5 bps/turn and 10 bps/turn in solid state (`rb.gy/wqjbh`). If DNA double helix corresponds to solid state then 10 codons correspond to 3 full turns. Therefore my earlier assumption 10 bps/turn in the double helix is correct. 10 codons would correspond 3 full turns and to the length 99.6 $\AA \simeq 10$ nm, which in TGD framework corresponds to the p-adic length scale $L(151)$.

Double DNA strands cannot pair with all 12 I:s associated with the dark DNA. The length $L(151)$ should correspond to 10 I:s taking 80 per cent of the icosahedral volume. Is helical winding enough to achieve this?

1. The total volume of the fundamental region is $V = 20V(T) + 30V(O) + 12V(I) = 341.44$ using $2a$ as length unit. Using the estimate $V_{real} = L(151)^3 = 10^6$ $\AA^3$, one obtains $a = L(151)/2V^{1/3} \simeq 0.07 \times L(151)$. The volume fraction of single icosahedron would be $17.45/V \simeq .05$ and 10 I:s would take 1/2 of the volume.

2. The circumradius of single icosahedron would be $R = \sqrt{3-\phi}\phi a/2 \simeq .1 \times L(151) = 1$ nm. This conforms with the assumption that there are 10 codons per length $L(151)$! The diameter of the B-type DNA strand is 20 $\AA$ is also consistent with the value of the circumradius. Maybe the proposed picture works!

3. Notice that if an icosahedral cell corresponds to 2 tetrahedral cells and 3 tetrahedral cells, then 10 codons is the maximum for the realizable DNA codon.

What can one say about the straight form of DNA?

1. For 10.5 bps/turn for a straight DNA in solution, the smallest portion of strand, which corresponds to integer numbers of turns and of codons is 6 full turns. This corresponds to 63 bps and 21 codons.

2. With an inspiration coming from the notion of Combinatorial Hierarchy [A1, A3] defined in terms of Mersenne primes $M_n = 2^n - 1$ defined by the recursive formula $M(k) = M_{M(k-1)} = 2^{M(k-1)} - 1$, I proposed decades ago that ordinary genetic code could correspond to Mersenne prime $M_7 = 2^7 - 1$ [K1] [L4]. The basic idea is that a system with $2^7 - 1$ states corresponds to a Boolean logic with 7 bits but with one state missing: this state would correspond to empty set in the set theoretic realization or fermionic vacuum state in the realization as a basis for fermionic Fock states. Only 6 full bits can be realized and the number of realizable statements is 64, the number of genetic codons.

3. Memetic code corresponds to the Mersenne prime $M_{127} = M_{M_7} - 1 = 2^{127} - 1$. Now the number of codons would be $2^{126} = 2^{6\times 21}$ and is realizable as sequences of 21 DNA codons! Note that higher Mersenne numbers in the hierarchy were proposed by Hilbert to correspond to Mersenne primes but for obvious reasons this has not been proven.

4. Could 6 full turns of straight DNA define a memetic codon? During the transcription and replication, DNA double strand opens and becomes straight. Could memetic code be established during the transcription and replication periods? A further intriguing observation is that the cell membrane involves proteins consisting of 21 amino-acids.

3.4 Some questions

Many questions remain to be answered.

1. Hamiltonian cycles are fixed only modulo the 60-element isometry group I of icosahedron. Subgroups Z_n, $n = 6, 4$ or 2 as invariance groups of their orbits defining amino-acids coded by DNA codons assigned to them. Therefore the space I/Z_n corresponds to the space of orbits of Hamiltonian cycles having 10, 15, *resp.* 32 elements for $n = 6, 4$, *resp.* 2 . Suppose that the Hamiltonian cycles for various icosahedrons of the fundamental region proposed to be associated with the sequence of 10 DNA codons differ by a non-trivial isometry assignable to I/Z_n. Does this have physical implications or is it mere gauge degeneracy?

2. The wave functions defining quantal variants of the genetic codons can be assumed to be products of wave functions for the position of the face and 3-proton states assignable to a given face should form an orthonormal set. The face wave functions associated with tetrahedra are trivially orthogonal with those of second tetrahedron and icosahedron. For a fixed choice of the icosahedral or tetrahedral Hamiltonian cycle orthogonality can be realized for the wave functions associated with the position of the face.

   If the icosahedral face wave functions correspond to different Hamiltonian cycles then orthogonality of protonic states for a given face can guarantee the orthogonality. This is possible if the number of protonic states is larger than the number of icosahedral wave functions. This requires 20+20+20+20 protonic states so that four protonic 4 states are left if their number is 64.

3. Why Hamiltonian cycle and quint cycle? Without Hamiltonian cycles the number of frequencies defining 3-chords would be 30 and is reduced to 12 for Hamiltonian cycles. Hamiltonian cycles assigned to the genetic code define an additional symmetry as shifts along the cycle, which are represented as 3/2 scalings modulo octave equivalence. The quint cycle defines the 12 frequencies for a given magnetic field strength and the chords of different cycles consist of different combinations of frequencies.

   What does the Hamiltonian cycle as a 1-D closed path correspond physically?

The proposal that the fundamental region of the icosa-tetrahedral honeycomb could have interpretation as a kind of super-icosahedron raises several interesting questions.

1. Assume that the sequence of 10 DNA (2 codons missing) to the super-icosahedron having icosahedrons as 12 super-edges, tetrahedrons as 20 super-faces, and 30 octahedrons as super-vertices. Combinatorial equivalence suggests that one cdefine icosahedral Hamiltonian cycles as sequences of 12 icosahedrons serving as superedges. Could one define higher level icosahedral genetic codes in terms of icosahedral Hamiltonian cycles. The orthogonality of the face wave functions for the different Hamiltonian cycles would require the assignment of the analogs of dark proton triplets to the super-faces.

2. What could the notion of a super-Hamiltonian cycle as a sequence of 12 dark DNAs mean? The proposed interpretation is that the collection of tetrahedral and 3 icosahedral Hamilton's cycles defines a correlate of a mood, emotional state. It is difficult to say whether the mood is the same for all cells of the entire organism, for the genome of a single cell, for the genes, for the sequences of 10 DNAs, or for codons.

   Super-Hamiltonian cycle associated with the super-icosahedron would have as its edges icosahedrons with the associated 12 dark DNA codon. If the 12 icosahedrons can correspond to different Hamiltonian cycles, one would have a correlate for a sequence of moods. Hamiltonian cycle property allows only 60 sequences of this kind. Without this restriction one would have N^{12} mood sequences, where N is the number of Hamiltonian cycles.

3. One can of course ask whether super-octahedron and super-tetrahedron could make sense and whether they could combine to form a super-icosa-tetrahedron. Does one have any tessellation for which fundamental region would correspond to super-tetrahedron with tetrahedron as

interior, 4 octahedrons as 4 super-vertices and 4 icosahedrons as super-edges. There is no mention of this kind of tessellation but it is known that hyperbolic tessellations constructible using the standard methods do exist.

One could even ask whether there could exist a fractal hierarchy of these super-structures constructible from the super-Platonic solids of the previous level and whether it could be realized as a hierarchy associated with dark DNA. This would mean a hierarchy of increasingly refined emotions emerging as the length of genes and DNA increases.

4 Further progress in the understanding of the icosa tetrahedral realization of the genetic code

TGD leads to two models of the genetic code. The first model emerges from a model of music harmony based on the combination icosahedral and tetrahedral geometries [L1] [L12]. The second model relies on the representation of the genetic codons as entangled triplets of dark protons at the monopoles flux tubes defining the dark variant of DNA accompanying the ordinary DNA [L8].

It took quite a long time to understand why both icosahedra and tetrahedra are needed and how the two models are related. The solution of the puzzle came from a universal model of the genetic code based on a completely unique tessellation of 3-D hyperbolic space H^3 realized as the light-cone proper time constant hyperboloid of the Minkowski space. This icosa tetrahedral tessellation (ITT) (known also as tetrahedral-icosahedral tessellation) makes sense in all scales and I have proposed its realization at the level of DNA in [**?**]essellationH3. The model involves several intuitive elements and the best way to proceed is to try to improve the existing understanding and to identify the possible weaknesses of the model.

I am not a professional hyperbolic crystallographer and my view of ITT (see this) relies on guesswork guided by physical and biological intuition based on what I call icosa tetrahedral model for the genetic code. In this article I represent some results based on using standard results from Platonic solids to deduce the numbers of tetrahedrons, octahedrons and icosahedrons emanating from a given vertex of the tessellation. The study of the vertex figure of ITT leads to a rather plausible guess for a manner to obtain ITT as a "blow-up" of icosahedral tessellation (IT).

The improved understanding of the icosa tetrahedral tessellation allows to answer some long standing questions related to the detailed realization of the genetic code. It however turns out that the notion of the super-icosahedron discussed in [L17] is not consistent with the improved view. However the 3-D generalization of the vertex figure of the ITT as its inverse image under projection permutes the numbers for the Platonic solids appearing in the super-icosahedron.

This section provides an answer to the question how many icosahedrons, octahedrons and tetrahedrons meet at the vertex of ITT: the answer comes by studying the vertex figure of ITT: these numbers are 12, 30, and 20. The study of the vertex figure of ITT suggests that the ITT can be constructed as a "blow-up" of the icosahedral tessellation (IT) by replacing icosahedral vertices with tetrahedra and dodecahedral vertices by pentagons and adding between icosahedral tetrahedra and dodecahedra octahedra as analogs of edges. Icosahedral and dodecahedral bioharmonies correspond to 12-note *resp.* assignable to Western *resp.* Eastern music. One can ask whether octahedral 4-codons should also be allowed.

The picture provided by RID is consistent with the earlier notion of "super-icosahedron". The model of the genetic code generalizes: besides the icosahedral Hamilton cycles (HCs) and codons for the three icosahedral codes and the tetrahedral HC and corresponding codons, also a unique dodecahedral HC and associated 5-codons plus pentahedral HC and codons are in principle possible. The fundamental region deduced from RID corresponds to a sequence of 10 or 12 DNA codons as proposed already earlier on the basis "super-icosahedron model".

The model allows us to understand the symmetry breaking of genetic codons. In particular, tetrahedral codons correspond to 3 stop codons and the codon coding for trp. A given codon corresponds either to I/T or D/pentahedron. The fundamental region represents a sequence of 10 or 12 DNAs so that all codons of the Hamiltonian cycle are used and the HC corresponds to a section of DNA. Fundamental region represents both DNA strands.

4.1 More precise views about some aspects of the icosa tetrahedral realization of the genetic code

The improved understanding of the icosa tetrahedral tessellation allows to answer some long standing questions related to the detailed realization of the genetic code.

4.2 What does the fundamental domain of ITT look like?

One basic question is how many tetrahedra (T), icosahedra (I) and octahedra (O) emerge from a given vertex of ITT.

4.2.1 The first guess was wrong as always

The wrong guess was that one can answer this question just from the knowledge of the solid angles associated with vertices of these Platonic solids. The solid angles are naturally defined as ratios of spherical areas to the radial distance squared and at the limit of very small hyperbolic radial distance approaching Euclidean distance, the total solid angle at this limit is 4π as in the Euclidean case.

However, the metric in the radial direction is non-Euclidean for the negatively curved hyperbolic 3-space H^3 so that the edges from the vertex diverge whereas in Euclidean spherical geometry they converge. Note that H^3 has 3-D rotation group as isometries so that the notion of Platonic solid applies also in H^3.

The lines emanating from the vertex are shared by neighboring T, O, and I emanating from the vertex. Two neighboring lines are associated with a triangular face shared by two Platonic solids involved.

The basic condition for the numbers $n(i)$ of the Platonic solids involved is $\sum_{i\in\{T,O,I\}} n_i\ \Omega_i$ $=4\pi$. Consider first the *Euclidean* case. One can find the general formulas for the solid angles from Wikipedia (see this).

1. Platonic solids are classified by 2 integers $\{q,p\}$ stating that q $p-$polygons meet at a given vertex. In the recent case one has only 3-polygons, that is triangles, for all Platonic solids involved. One has

$$[q(T),q(O),q(I)] = [3,4,5] \ , \quad [p(T),p(O),p(I)] = [3,3,3] \ .$$

2. Dihedral angle angle is the interior angle between the faces of the Platonic solid and satisfies the general formula

$$\theta(q,p) = 2asin(\frac{cos(\pi/q)}{sin(\pi/p)}) \ .$$

In the *Euclidean* case the solid angle at the vertex is given as

$$\Omega(q,p) = q\theta(q,p) - (q-2)\pi \ .$$

3. Suppose that all vertices are identical as the fact that there is only a single vertex figure. The vertex of vertex figure, call it V, involves $n(T) \equiv n(3)$ tetrahedrons, $n(O) \equiv n(4)$ octahedrons and $n(I) \equiv n(5)$ icosahedrons. The sum of the solid angles equals to 4π, which gives

$$\sum_{q\in\{3,4,5\}} n(q)[q\theta(q,3) - (q-2)\pi] = 4\pi \ .$$

This gives

$$\sum_{q\in\{3,4,5\}} n(q)\ qasin(\frac{cos(\pi/q)}{sin(\pi/p)}) - (n(3)+2n(4)+3n(5))\pi = 4\pi \ .$$

4. In the Euclidean case, one can guess the solution to the condition by starting from the icosa tetrahedral model [L8, L12] for the genetic code, which is a fusion of 3 icosahedral codes associated with Hamiltonian cycles (HCs) with symmetry groups Z_6, Z_4, Z_2 and of a single tetrahedral code defined by the unique tetrahedral HC. In the proposed model based on ITTs [L13, L17], the octahedron is passive and does not contribute to the code. A reasonable guess based on this model is $n(I) = 3$ and $n(T) = 1$.

   The normalized vertex solid angles are

$$\frac{[\Omega(3), \Omega(4), \Omega(5)]}{4\pi} = [0.043871, 0.1082, 0.2097] \ .$$

   The consistency condition is

$$\frac{n(T)\Omega(T) + n(O)\Omega(O) + n(I)\Omega(I)}{4\pi} = 1 \ .$$

   This leaves only the guess $[n(T), n(O), n(I)] = [1, 3, 3]$ under consideration giving for the sum the value 0.9974 in the accuracy used partially determined by the approximation $\pi \simeq 3.1415926535897$.

4.2.2 The vertex figure for ITT contains the needed information

The vertex figure V codes the information about the fundamental domain as one can easily see in the case of say cube. Consider now the vertex figure V for ITT.

1. The vertex figure is obtained by cutting a 3-sphere around a vertex is a 2-D object to which the vertices as edges and faces of the fundamental region of the solid are projected. For ITT, the vertex figure is rhombicosidodecahedron (RID) (see this). This is an Archimedean solid, one of thirteen convex isogonal nonprismatic solids constructed of two or more types of regular polygon faces.

   RID has *20 disjoint triangular* faces (as also I has), *30 square* faces, which share their corners with other square faces, *12 disjoint pentagonal* faces (as D has), 60 vertices, and 120 edges. The numbers of faces are much larger than in the Euclidean case.

   The 20 triangular faces correspond naturally to intersections of 20 T:s with the sphere, the 30 square faces to the intersections with 30 octahedra, and 12 pentagons to the intersections with 12 icosahedra. From V one finds squares and common edges with triangles and pentagons.

2. The illustrations of RID (see this) gives a 2-D analog for what it means that the tessellation has different 3-D Platonic solids as building bricks. Interestingly, the faces of RID are faces of the duals of the Platonic solids T, O and and I. In RID the triangles are disjoint and share sides with squares.

   Since RID and ITT are combinatorially closely related, this suggests that the disjoint triangles of RID correspond disjoint T:s of ITT and the squares of RID having sharing only corners correspond to O:s of ITT sharing only edges whereas D:s would correspond to I:s.

Could an analog for the construction of RID allow to deform the hyperbolic icosahedral tessellation (IT) to ITT?

1. The construction would rely on the correspondence *triangle - self dual T, pentagon - I as dual of D, square - O as dual of cube*. One could generalize the correspondence to *triangle→ T , pentagon → I, and square → O.*

   The recipe would be as follows. Start from the hyperbolic icosahedral tessellation (IT) $\{(3, 5, 3\}$ with 3 I:s $\{5, 3\}$ meeting at each edge. One could blow-up the icosahedral vertices to T:s and glue to the faces of a given T 4 O:s. O:s would also share faces with other T:s and I:s but not with O:s if there is a combinatorial analogy with RID.

2. The inspection of the RID shows that T:s and I:s do not have common faces and meet only at V. O:s share faces with I:s and T:s. Besides the vertex figure there are also T:s,I:s, and O:s emerging from the origin. They have triangular 3 ,4, 5 triangular faces respectively and they contribute to the genetic code. Second important point is that RID contains only one half of the vertex figure. The natural interpretation would be that these halves correspond to DNA strands. This however requires that the fundamental domain is realized at the magnetic body.

3. The maximally symmetric solution to the condition $\sum_{i\in\{T,O,I\}} n_i\ \Omega_i = 4\pi$ would be

$$\Omega_i = 4\pi \frac{}{(n(T)+n(O)+n(I))=\frac{4\pi}{62}}.$$

4.3 Some progress in the detailed realization of the genetic code

ITT emerged as a mathematization of the icosa tetrahedral realization of the genetic code and it is interesting to see whether the new results allow us to gain some understanding about the issues related to the detailed realizations.

In the original vision [L1], it was unclear whether there are 3 different I:s or only a single I realizing one of the 3 HCs at time. Also the relationship between T and I was unclear. The proposal was that there is a single I and T that shares a common face with it. The idea about a common face was however somewhat fuzzy and I have discussed several ways to understand the details of the genetic code, in particular those assignable to stop codons.

Also the selection of a single active triangle as a representation of the codon was adhoc. The natural idea is that the Hamiltonian cycle selects all codons so that the fundamental region represents a portion of DNA: in fact 10 codons.

4.3.1 The revised view of the genetic code

The number 12 of pentagons is the number of the faces of D, the number of squares is the number of O:s and the number of edges of I, and the number 20 T:s is the number of faces of I. This conforms with the idea of blow-up in which vertices of I are replaced with T:s.

The structure of RID suggests a rather dramatic revision of the view about how the genetic code is realized at the level of ITT assignable to the magnetic body of DNA double strand. The interpretation that O:s serve in the role of edges is attractive and suggests that there are no codons associated with them. The role of the possible O codons as edges means that they are determined by I and D codons and that the Hamiltonian cycle for the squares is not a useful concept. Furthermore, there is no analog of edge between O codons which intersect at their corners.

This leads to the following picture.

1. Codons are associated with RID, that is with both the intersections of T:s, O:s, and I:s with the S^2 and also with the triangles emanating from the vertex V to RID. One can interpret the 20 triangles and 12 pentagons and 30 squares as potential genetic codons.

2. The notion of Hamiltonian cycle generalizes for the blow-ups and the edges of the cycle connects the blow-up vertices: 20 triangles for the blow-up of D and 12 pentagons for the blow-up of D. There are also tetrahedral with 1+3 vertices and codons with 1+5 vertices. For I:s having triangles as vertices there is a larger number of Hamiltonian cycles. For D:s having pentagons as vertices there is only one Hamiltonian cycle. Hamiltonian cycles represent a piece of DNA strand.

   Octave Equivalence implies that the frequency scaling in transition between two neighboring vertices for $2^{1/V}$, where the number of vertices is $V = 12$ *resp.* $V = 20$ for the I *resp.* D, type Hamilatonian cycle D type Hamiltonian cycle is $2^{1/20}$. This corresponds to the micro

scales used in Eastern music. For the tetrahedral cycle it is $2^{1/4}$: this corresponds to the chord $C, E, G\sharp$. For its analog for D, the scaling is $2^{1/6}$.

3. RID and its mirror image needed to obtain the fundamental domain represent the 20 DNA icosahedral codons or 12 dodecahedral codons.

In [L17], I proposed a heuristic model for the 10-codon piece of DNA sequence a candidate for the fundamental region of IIT. The idea was that it corresponds to what I called super-icosahedron (SI) having icosahedrons as 12 super-edges, tetrahedrons as 20 super-faces, and 30 octahedrons as super-vertices. What is worrying is that 2 DNAs would be missing so that there would be 10 Is.

The guess was essentially correct: the RID has 20 regular disjoint triangular faces (as also I has), 30 square faces, which share their corners with other square faces, 12 regular disjoint pentagonal faces (as D has) plus 60 vertices, and 120 edges. The triplet (20 triangles, 30 squares, 12 pentagons) contains the same numbers as appear in SI. The correct identification of SI could indeed be as the fundamental domain of ITT if one glues to RIDs together (consider cube as an simple example). ITT could be seen as a 3-D combinatorial lift of RID obtained by the inverse of the projection to the sphere defining the vertex figure (triangle $\rightarrow$ T, square $\rightarrow$ O, pentagon $\rightarrow$ I): this is supported by the view what vertex figure means. Could the sequence of 12 DNAs correspond to (20 T:s, 30 O:s, 12 I:s) as the 3-D inverse image of the RID?

4.3.2 Solutions of some problems provided by the new view

The model of genetic code has suffered from some cronic problems.

1. If each vertex of IT is replaced with T in the blow-up identified as ITT, a single icosahedral triangle of IT would be replaced with 3 T:s. A natural identification is in terms of a genetic codon with 3 letters, one T per letter.

2. The icosahedral realization leads the following problem. The icosahedral HC with Z_6 symmetry corresponds to 3 Z_6 orbits with 6 triangles and one orbit with 2 triangles (6+6+6+2=20). This corresponds to 5 amino acids (AAs) identified as orbits of Z_6. The HC with Z_4 symmetry contains 5 orbits with 4 triangles ($5 \times 4 = 20$) and gives 5 AAs. The HC with Z_2 symmetry gives 10 orbits and therefore 10 AAs. One has 19 AAs altogether. One AA is missing.

3. Tetrahedral cycle involves 4 triangles and Z_3 symmetry is natural. The triangle opposite to V would give a codon coding for single AAs and the remaining triangles related by Z_3 symmetry corresponding to 3 ordinary DNA codons. The frequencies of these dark codons differ only by order and this suggests that they code for a single AA (vertex of T and the vertices opposite to it). The triplet could correspond to stop codons coding for no physical AA. The singlet codon could correspond to the missing AA, most naturally trip. Another less plausible option corresponds to (ile,ile,ile,met). One cannot interpret this multiplet as a symmetry broken quadruplet since there are 5 quartets. The interpretation as a symmetry breaking of (met,met) (ile, met) however works.

4. There are also problems related to the chemical realization of the dark code. There are several slightly different chemical realizations of the code, which are not complete and violate the symmetries, which are exact for the dark realization. Also the number of stop codons van vary.

4.3.3 Representation of codons as 3-chords

There are also questions related to the HC and its realization and also the realization of the codons as cyclotron frequency triplets.

1. Icosahedral HC corresponds to a sequence of 12 vertices to which one can assign T:s. The basic idea of bioharmony is that one assigns to each vertex a note of 12-note scale and the

notes associated with the triangular faces define the 20 chords of the harmony for a given T as dark counterparts of DNA codons.

The 12 edges connecting the faces in the simplest model corresponds to a scaling of frequency by 3/2 or by $2^{7/12}$ corresponding to Pythagorean and well-tempered scales (HC as quint cycle module octave equivalence). Various HCs correspond to different realizations of the genetic code in terms of 3-chords realized as cyclotron frequency triplets assignable to the triangular faces of I and interpreted as different harmonies as representations of moods: this aspect is absent in the standard view.

2. How the cyclotron frequencies are assigned with the vertices of I. One could consider the situation also from the point of view of IIT as a blow-up of IT. Each vertex of I has T as a blow-up. One should assign a cyclotron frequency with this particular vertex of T.

3. Could one assign the frequency triplets with the 3 T:s associated with the blow-ups of the triangular face of I? A given cyclotron frequency is most naturally associated with the vertex which it shares with an active I. This seems necessary since the letters of the codon defined by the 3 T:s should be independent. Octave equivalence allows only one frequency triplet unless the order of frequencies $C, E, G\sharp$ matters. At the level of DNA it matters but at the level of AAs it does not. The T triplets could correspond to different DNA codons coding for the same AA.

4. The situation is very similar for the start codon and stop codons. The start codon of the gene coding for the met is very special. Could one assign the start codon to the triangle opposite to the active vertex of T, so that it would effectively reduce to a doublet and the three codons coding for ile to the other faces of T?

What DNA codons do the T codons correspond to? One can consider two options for which T codons would code for DNA triplet and singlet and there would be no symmetry at the level of T.

1. At the level of the ordinary DNA, T codons could correspond to 3 stop codons and a codon coding trp or to 3 ile codons and met. There would be symmetry breaking for the I quartet giving rise to (ile,ile,ile, met) but this is not possible since there are 5 AAs coded by 4 DNAs. Therefore this option fails.

2. For the second option dark T codons correspond to DNA codons coding for (ile,ile,ile,met). One doublet would code for (stop,stp) and a second doublet would break Z_2 symmetry and code for (stop,trp). This option might also allow us to understand the small deviations from the standard genetic code for which stop codons occasionally code for a real AA.

5 About the genetic code and icosa tetrahedral tessellation of hyperbolic 3-space

The TGD based model for the genetic code [L17] relies on icosa tetrahedral hyperbolic tessellation (ITT) realized in the hyperbolic 3-space H^3 representable as a light-cone proper time constant hyperboloid of light-cone of M^4 or as a mass shell in momentum space.

1. The general idea that genetic codons as 6-bit units of ordinary "bitty" intelligence are accompanied by emotional intelligence represented in terms bio-harmonies serving as correlates for emotions. Music indeed expresses and creates emotions [L1] [L6, L12, L13, L17]. This view has far reaching implications. In particular, it means that emotions are present already at the biomolecular level. In the TGD Universe, life is universal and can appear in very many scales. This would be true also for the genetic code realized in terms of the icosa tetrahedral tessellation of H^3 which can appear in arbitrary scales.

2. This interpretation of the genetic code belongs to the category of the intuitive "must-be-true" hypothesis of TGD, whose status has remained unclear. One reason for this is that I am not a specialist in the field of hyperbolic tessellations. Once again I realized that my understanding is far from perfect and decided to clarify my thoughts once again.

3. ITT involves tetrahedra (T), octahedra (O) and icosahedra (I). Genetic code would correspond to a fusion of 3 properly chosen icosahedral Hamiltonian cycles representing 12-note scale (there are many options) and one tetrahedral Hamilton cycle, which is unique. I have an intuitive geometric interpretation for this 3-1 structure: 3 I:s share 3 faces of T. This leaves one free face of T serving as an additional codon. This gives 20+20+20+1=64-3 codons and the missing 3 codons could correspond to stop codons. Also O:s are involved and the intuitive idea is that O is passive in the sense that it represents a void in the sense that the vertices, edges and faces of the octahedron can be regarded as those of octahedron or I. How to make this idea more concrete?

There has been a dramatic evolution in the basic understanding of TGD during 2024-2025 and it is time to update the views of ITT and also to summarize the recent overall TGD based view about quantum biology.

5.1 A more precise view of ITT

The ITT in the hyperbolic 3-space H^3 (honeycomb) is completely unique because it includes as cells all Platonic solids, tetrahedron (T), octahedron (O) and icosahedron (I) for which the faces are equilateral triangles. One can characterize the tessellation by giving the numbers of 3-cells meeting at vertices, edges and faces.

Consider first the vertices.

1. The vertex figure of the ITT (see this) represents what an observed at a given vertex sees as intersection of a vertex-centered ball with the ITT. For instane, for cube, vertex figure is square for C(ube) and O, pentangon for I and triangle for T and D(odecahedron).

   For ITT vertex figure corresponds to an Archimedean solid known as icosadodecahedron (ID), which can be regarded as a hybrid of I and D. The 12 pentagons at the vertices of I as vertex figures of 12 I:s and the 20 triangles as vertex figures of 20 T:s correspond to vertices of D. ID has 20 triangular faces and 12 pentagonal faces, totaling 32 faces, with 30 identical vertices, at which two triangles and two pentagons touch, and 60 edges separating a triangle from a pentagon.

2. O is passive in the sense that only 20 I and 12 T but not O meet at the given vertex, hence the attribute "icosatetrahedral". One can say that O represents a void. Octahedron is a lower-dimensional example of this phenomenon: the square defining the vertex figure of O does not define a face appearing at the vertex. Only 4 triangles meet at a given vertex. This brings in mind giant voids of cosmology having galaxies at their boundaries. I have proposed that the tessellations of H^3 realized as cosmological time $a = constant$ hyperboloids in the light-cone of M^4 could explain the observed quantization phenomena for the redshifts [L20]. Could these large voids have something to do with the O:s of ITT?

3. How could one understand the 3-1 correspondence for I:s and T:s? A given T of the vertex figure is surrounded by 3 I:s. This suggests the T+3I defines a unit giving a realization of the genetic code, 4 units of this kind would meet at a given vertex.

   The proposed interpretation of T+ 3I as a unit conformas with the proposed view of the genetic code. I:s have 20 triangular faces and since the I:s have no common faces, this motivates the proposal that the 3I give rise to 20+20+20 icosahedral codons. The I:s would realize a Hamiltonian cycle with a symmetry group which is Z_6, Z_4 or Z_2. Z_2 would act as reflections or rotations. Z_6 cycle is unique, there are 2 Z_2 cycles and a large number of Z_2 cycles.

   The orbits of the symmetry group would correspond to amino-acids. Z_6 would give rise to 3 6-element orbits and 1 2-element orbit. Z_4 would give rise to 5 4-element orbits and Z_2 to 10 2-element orbits. This explains almost exactly the numbers of DNA codons coding for a given amino-acid. The 3 I:s share 3 common faces with T, which leaves one free face for T to which one can assign a tetrahedral genetic codon. The 3 missing tetrahedral faces would correspond to stop codons.

4. Interesting questions concern the interpretation of the cycles. The Hamilton cycle connects the nearest neighbor vertices of the Platonic solid. Does the cycle correspond to a closed monopole flux tube? What does it mean that one face (at least) for a given 3I+T unit is active and represents a codon: does it have protons at its vertices as the alternative realization of the genetic code in terms of the states of 3-proton triplets suggests [L6]? Can the 3I+T units of ITT contain different Hamiltonian cycles so that emotions could be local. Does DNA strand correspond to a linear structure as a substructure of ITT. Is the induction of ITT to 1-D, 2-D and even 3-D structures representing genetic code possible? Could for instance, cell membrane and microtubules represent 2-D realization of the genetic code. Could the brain and even the biological body represent a 4-D realization. Could these realizations be time dependent as the failure of strict non-determinism of the classical dynamics dictated by holography = holomorphy vision suggests: if so, even 4-D realization would be possible.

Also the numbers of cells meeting edges and faces characterize ITT.

1. Edge transitivity means that all edges are symmetry related just as the vertices are. At a given edge I, I, O, and T meet in a cyclic order IIOT.

2. 2 3-cells cells meet at a given face. Only I and T can share faces. and the shared faces correspond to O faces. O does not appear at the vertices, being realized as a "ghost" cell being analogous to the square appearing in O and having no physical realization as a face.

   If one assumes that all I-T interfaces also involve an O interface then the number of 20 O:s surrounded by 3 I:s implies that presence of $4 \times 20 = 80$ O:s assignable to a single vertex. Since each O has 8 faces, there would be 10 O:s per vertex. The numbers of (T,I,O) per vertex would be (20,12,10).

To sum up, the conjecture that the genetic code is realized in terms of ITT is now at a rather firm basis. During the last few years several ideas of TGD have reached a rather strong status as the understanding of the basic mathematical ideas of TGD has increased and TGD is now a mature mathematical theory and can be applied in all scales.

5.2 How the genetic code is realized at the level of the magnetic body of DNA double strand?

Suppose that the proposed view of the ITT realized at the level of the magnetic body (MB) of DNA is correct that dark genetic codons as induction of ITT from the MB of DNA have as a chemical counterpart of DNA or RNA double strand. How the more precise view of ITT affects the earlier model discussed in [L17].

First a couple of facts.

1. The numbers of (T,I,O) per vertex should be (20,12,10) if the T-I interface always involves O. Therefore also DNA codons correspond to faces of O:s and DNA sequences can be identified as a sequence of faces of O:s.

2. 10 DNA codons define the shortests DNA sequence for which the twist is a full multiple of 2π. One should have a sequence of triangles representing genetic codons and each codon should correspond to a face of I and to a 3-chord of a fixed Hamiltonian cycle defining a bioharmony.

This raises the following questions.

1. Does the sequence of 10 O:s correspond to a single ITT vertex and does DNA correspond to a sequence of ITT vertices such that each vertex corresponds to an O and associated 20 T:s and 12 I:s?

2. Do the two DNA strands correspond to separate dark strands or does a single dark strand correspond to both of them as the fact that the DNA strands are conjugates of each other as the latest proposal assumes. Assume this. Single O has 3+3 faces and has two disjoint triangular faces. Could these two faces correspond to DNA codon and its conjugate?

3. This sequence of 10 O:s corresponds to a sequence of 12 I:s. 2 I:s would be "empty" and would not correspond to dark proton triplet: what does this mean? Does this mean that all vertices of the I and T carry ordinary protons and the activation of the codon transforms the ordinary protons of the face to dark proton triplet. I have considered a possible interpretation of this. In the state in which DNA is opened (transcription) the 2 codons would become active and correspond to dark proton triplets.

4. What distinguishes between I and T type active codons? When the dark proton triplet is of T type and when it is of I type? Could the presence of the Hamilton cycle, the assignment of 3-chords to the faces, and resonance interaction allow us to understand this? Does the 3-chord assigned to the face determine whether the dark proton triplet belongs to the T or I type Hamiltonian cycle? Is there some symmetry breaking mechanism selecting from the T type conds the one while the remaining ones act as stop codons. Could the presence of I or T type Hamiltonian cycle in given I or T determine whether it can define an active codon and whether an associated ordinary proton triplet can be transformed to a dark one?

   The cyclotron frequencies assignable to T type codons are different from those assignable to I type codons if the frequency ratio for two subsequent vertices of the cycle is 3/2 for the Hamilton cycle at I in the Pythagorean model.

   Note that the basic problem of the Pythagorean model of harmony (known already by Pythagoras) is that the full Hamiltonian cycle, involving 12 frequency scalings by factor 3/2, does not give quite precisely a full multiple of octaves. One must allow irrational frequency scaling of $2^{1/12}$ on a well-tempered 12-note scale to get rid of the problem. This might relate to the symmetry breaking.

   For a tetrahedron with 4 vertices the frequency ratio should be also such that the cycle spans a multiple of octaves. This is not possible for rational scalings. In any case, I and T options are not consistent and this suggests that the 3-chords select between I and O options. The chords dictated by the character of the Hamilton's cycle select whether the face is of type I or O. The presence of the Hamiltonian cycle would be necessary for the transformation of the ordinary proton triplets to dark proton triplets and only the I or T type cycle can be realized.

   In the standard realization of the code there are 3 stop codons, which are transcribed to mRNA but are not translated to amino-acids. There are 4 codons of type T. There should be a symmetry breaking in the sense that 3 of them are not translated. This could be due to the failure of 3-chord resonance conditions so that there would be no tRNAs with the required resonance frequency triplest. Only a single tetrahedral codon would be translated for the standard realization of the code. This model also allows deviations from the standard realization of the code.

5.3 Pollack effect and ATP→ ADP+P_i transformation

The molecules XP, where X ∈ {A,T,C,G} denote DNA nucleotides, are basic building blocks of DNA. The molecules XP are stable unlike the more complex molecules. The molecules ATP, ADP and GTP, GDP involve 2 or 3 phosphate ions. The latter molecules are essential for the metabolism and appear as carriers of metabolic energy assigned in the TGD view to the dark protons at the magnetic body associated with the molecule. What distinguishes them from the mononucleotides appearing in DNA and RNA?

We talked with Ville-Einari Saari (a member of our Zoom group) about whether it might be possible to build stable negentropic systems with a large Planck constant h_{eff}. Without any stabilizing mechanism, large h_{eff} systems are unstable against the decrease in h_{eff} because their energies increase with h_{eff}, so as free systems they require a continuous energy input and only flow equilibrium is possible. This is the case in the case of XDP and XTP and this makes for ADP and GTP to transfer metabolic energy.

In water, the Pollack effect is a fundamental process and produces dark protons that transform into ordinary ones in an attosecond time scale. This expectation comes from the observation of

exotic phases of water with effective stoichiometry $H_{1.5}O$ having attosecond life time. The explanation is that a phase transition in which every fourth proton becomes a dark proton at monopole flux tubes takes place under external energy feed. The negatively charged exclusion zone (EZ) created in the Pollack effect by radiation is an example of this effect. The essential prerequisite for the Pollack effect is external energy feed and TGD has led to various generalizations of the Pollack effect. In particular formation of biomolecules generates binding energy and this could stabilize dark phase [L14, L18, L21] and cold plasmas are excellent candidates for the carriers of stable dark phases.

An illustrative example is provided by transformation of chemical energy to a usable energy as a transition ATP$\rightarrow$ ADP $+$P$_i$, where P$_i$ is inorganic phosphorus. This process occurs spontaneously. The reverse process requires metabolic energy input and mitochondria are specialized to produce ATP from ADP. The process ADP$\rightarrow$ ATP$\rightarrow$ ... can be seen as a kind of a karmic cycle.

1. The phosphorus P appearing in ATP and ADP ions is organic. It is not clear what this really means and biologists argue about a mysterious high energy phosphate bond which would carry the metabolic energy to the final uses as ATP transforms back to ADP $+$P$_i$. In the TGD framework, the interpretation is that ATP and also ADP involves a dark proton at the MB that neutralizes the negatively charged system and is generated by the generalization of the Pollack effect in the formation of ATP or ADP.

2. The conversion of the chemical energy into a usable form occurs in the mitochondria in a biochemical machine that resembles a rotating turbine of a power plant. 3 ATP are produced in one revolution of the turbine from three ADP. This would strongly suggest that a precursor of dark genetic codon as dark proton triplet is involved.

   Google informs that the lifespan of the ATP varies enormously: when the environment needs energy, its lifespan is shortened. In vivo it varies from a few seconds to about 100 seconds whereas in vitro ATP can be almost stable.

What about DNA and RNA?

1. DNA and RNA have a stable negative charge (as Google informs): there is a negative charge of 3 units per codon. A natural guess is that it corresponds to the exclusion zone (EZ) of the Pollack effect. This suggests that there must be a stable positive charge in the form of dark proton triplets at the magnetic body associated with the DNA and the proposal is that these triplets define dark codons. What stabilizes the negative charge of DNA and therefore also the dark protons and makes the negentropic state stable.

2. Bound states are formed between phosphates and DNA nucleotides. If their chemical binding energy is so high that the total binding energy, which is reduced by the energy of the dark proton, remains positive, the state is stable. I have suggested earlier [L18] that the formation of biomolecules as bound states can stabilize the dark protons, so the creation of biomolecules would also produce negentropy at the magnetic body. In fact, the formation of biomolecules as bound states during the biological evolution would have generated the dark protons at the monopole flux tubes of their magnetic bodies.

   To sum up, negentropic states can be stabilized in this way and do not require a constant input of metabolic energy to maintain dark h_{eff} in the sense of flow equilibrium. DNA and RNA would be completely exceptional bio-molecules in this respect and would fully deserve the name information molecule.

5.4 How large h_{eff} states are stabilized?

The quantum critical state is unstable by definition because the $h_{eff} \geq h$ states are more energetic than the $h_{eff} = h$ states and spontaneously decay into these.

One way to avoid this would be for the $h_{eff} \geq h$ molecule to form a bound state, for example with a molecule or a larger structure. The electric field of the larger charged structure and that in turn a state where h_{eff} would be stabilized. However, I do not understand the details of the mechanism. How to build a state in which $h_{eff} \geq h$ dark protons are possible in the minimum energy state. Is this possible if only the electromagnetic interaction is involved?

This is a fundamental question. So let's start from a clean table.

1. In the case of DNA and cell membranes, h_{eff} stabilization is related to the presence of electric fields, but do they produce the stabilization or are they a consequence of it?

   A $h_{eff} \geq h$ state and a state bound with another state are created so that the $h_{eff} \geq h$ state stabilizes because the dissociation is no longer energetically favorable. It should be noted that due to their large negative charge DNA and the cell membrane are biologically completely unique. Charge separation does also occur at the level of the brain and the whole body and its sign correlates with the level of consciousness: the sign of the voltage changes during sleep. The Earth itself also has an electric field, which suggests that the biosphere is conscious.

2. In the case of DNA, the bound state would be between phosphate and deoxyribose. Would the large $h_{eff} = h_{em}$ somehow be made possible by the longitudinal and radial electric fields of DNA or is it a consequence of a stabilization mechanism? Maintaining the electric field requires energy, so metabolic energy input is still necessary but at the level of classical fields. But do electric fields maintain dark protons at the monopole flux tubes or vice versa?

   The problem: In the case of DNA, the repulsive energy of the negative charges of the phosphates destabilizes the state. In addition, there is repulsion between the dark protons in the flux tubes. Charge separation, where the dark protons and the phosphate ions are far apart, requires energy because the neutral ground state is of minimum energy.

   The solution of the problem: Some interaction energy must compensate for the increase in interaction energy. Could strong interactions of the dark protons in the flux tubes, proposed to form dark nuclei with a scale down nuclear binding energy, be involved? The strong interaction would stabilize the repulsive energy of the negative charge of the phosphates, the same would happen for the dark protons. Long range electric field would be a consequence, not the cause.

   (a) The TGD-based model of cold fusion [L2, L5, L11, **?**] indeed assumes that the dark protons in the magnetic flux tubes form an analogy of the atomic nucleus and the scaled binding energy of the nucleus would produce the binding energy. Strong interactions in the TGD sense would play a key role in biology and also in electrolysis. This would be new and revolutionary.

   (b) Of course, one could try to cope with just electromagnetic interactions.

   i) The negative electrostatic energy would be between the dark protons and the negative charge of the phosphates. One would expect this energy to be small, but is it for flux tubes?

   ii) What about the role of water? It can become positively charged (and for example $Mg^+ + ionsdo), which can produce a Coulomb bound state. Mg^+ + ions are naturally present in monopole flux$

iii) The binding energy is related to the bound state between negatively charged phosphates and riboses. The problem is that ribose molecules are not permanently positively charged. This doesn't seem promising.

(c) In the case of the cell membrane, the electric field associated with the membrane potential should accompany large values of h_{eff}. A decrease in the field strength below a critical value would lead to a decrease in the value of h_{em}, perhaps down to $h_{eff} = h$ because h_{em} is proportional to the field value and quantized as an integer. The scale of quantum coherence would be reduced and a nerve impulse would be generated.

The naive Maxwellian assumption would be that a nerve impulse is generated when the voltage is too high: there would be a di-electric breakdown, just as is supposed to happen in a Tesla coil. The fact that exactly the opposite happens is a central mystery of biology. A decrease in h_{em} would explain the mystery. One can pose an interesting and somewhat nosy question: has it really been tested that breakdown is the correct mechanism in Tesla coils?

Also now the strong interactions with monopole flux tubes would stabilize the state.

(d) The negative charge on the surface of the Earth's electric field and the protons and ions in the gravitational flux tubes and electric flux tubes and their strong interaction would stabilize the biosphere as a conscious system.

5.5 Does the presence of ITT at the MB reveal itself in the structure of DNA the surrounding water

Does the presence of ITT at the MB of DNA reveal itself in the structure of DNA and the surrounding water. How does the presence of O:s, T:s and I:s at the MB reflect itself in the properties of chemical DNA and possibly of water? Could the structure of water around DNa reflect the projection of hyperbolic tessellation at 3-D Euclidean space E^3.

Do the octahedrons of the field body have any counterpart in the nearby environment of DNA.

1. Here Google tells that the water around DNA indeed involves octahedral structures besides tetrahedral structures which generally present (see this). They occur in the form of hexahydrated metal cations (see this), such as $[Mg(H_2O)_6]^{2+}$ with positive charge of 2 units. Mg^{+2} ions are bosons and could form Bose-Einstein condensate-like states. The 6 water molecules reside at the 6 vertices of O and its two opposite disjoint faces could correspond to two dark codons generated by Pollack effect from water molecules.

2. These octahedral complexes are commonly found in the major groove or the phosphate backbone region of the DNA, where they are thought to shield the negative charges and stabilize the overall structure. This assumption is natural also in the TGD based view. Only 15 percent of Mg^{+2} ions is estimated to touch phosphate oxygens directly. They would form a kind of cloud, which conforms with the idea that they serve as stabilizers. That they accompany the vertices of the octahedron conforms with the idea that the vertices involve negative charges created as protons are transformed to dark protons.

3. Mg^{+2} ions screen 88-89 percent of the negative DNA charge. If one can assign this kind of octahedron with a net charge of +2 units with each genetic codon, one unit of negative charge remains unscreened for both strands. Fraction 2/3 of total charge would be screened. This is considerably less than 88-89 percent so that not all Mg^{+2} ions would be associated with the vertices of the octahedra.

Could one understand the correspondence between ITT and DNA double strand more concretely? The natural guess is that the vertex figure of ITT relates to the structure of DNA double strand.

1. Could the pentagon associated with the deoxyribose (or ribose in the case of RNA) serve as a counterpart for the pentagon appearing in the vertex figure of ITT? The vertex figure has 12 pentagons, which could correspond to 12 DNA codons defining a cycle in the sense that the total twist angle of the double helix is $3 \times 2\pi$ in the open configuration of the DNA double strand.

   For a non-open double strand 10 DNA codons define a full cycle. One could say that there are 2 missing DNA codons and 2 empty IIT pentagons without dark protons triplets defining a gap separating the dark codons. If the corresponding $[Mg(H_2O)_6]^{2+}$:s, whose opposite triangles would represent DNA codon and its conjugate, are present at all, they should not give rise to dark protons. Mg^{+2} ions giving rise to Bose-Einstein condensate could give rise to quantum coherence at the level of ordinary DNA and make possible the simultaneous generation of 2 dark proton triplets by Pollack effect.

2. Could also Mg^{+2} ions be dark? The findings of Blackman [**?**] can be explained in terms of bosonic Ca^{++} ions which have cyclotron frequency 15 Hz in the endogenous magnetic field $B_{end} \simeq .2$ Gauss consisting of gravitational monopole flux tubes. They are dark in the sense that they have a very large gravitational Planck constant $\hbar_{eff} = \hbar_{gr} \sim 10^{15}$ [**?**] implying that the cyclotron photons can have energies in the range of visible photons. Mg^{+2} has cyclotron frequency 12.5 Hz for $B_{end} \simeq .2$ Gauss. The crucial assumption is that besides protons, also other metallic ions can be dark in the sense of having large h_{eff}. This suggests that also Mg^{+2} associated with a single codon as a face of ITT is dark in the sense it resides at the MB. The interpretation could be that its wave function is delocalized at the gravitational flux tube of the Earth's surface. When Mg^{+2} is observed its wave function would localize to the surface of Earth, meaning "dropping" from the gravitational flux tube. The effects of electromagnetic radiation with this frequency on DNA could be tested.

In fact, all metal ions M form $[[M(H_2O)_6]^{2+}]^n$:s complexes (see this). The number of water molecules involved is known as the solvation number and is 6 for the third and fourth period of the periodic table containing Mg and Ca. The bosonic Mg and Ca ions are also involved with microtubules and cell membrane (see this). This gives support for the proposed 2-D realization of the genetic code in terms of dark proton triplets.

3. The ordinary codon should correspond to the dark codon as a triangle at the MB with dark protons at its vertices. At the level of DNA there is no triangle. Could the 1-D quasiperiodic lattice formed by the DNA codons correspond to periodic boundary conditions at the MB so that the linear codon as a unit cell of the lattice has a triangle as a counterpart at the level of ITT? 3 chemically identical pentagons associated with the codon should correspond to a single pentagon at ITT. A single $[Mg(H_2O)_6]^{2+}$ octahedron associated with the major groove should correspond to a single O of ITT? Whether there is indeed only a single O per pair of codon and its conjugate could be perhaps tested. One could argue that symmetry requires that both strands involve $[Mg(H_2O)_6]^{2+}$ octahedron. However, only the other strand is active. This could mean that only its codons contain the $[Mg(H_2O)_6]^{2+}$ octahedron.

4. What about the tetrahedral structures, which also characterize water, around DNA? Here Google informs that in the hydration shell of DNA tetrahedral ordering is present and is essential for the stability of DNA. The presence of tetrahedral ordering could reflect the presence of ITT at the magnetic body associated with DNA and also a region of water environment. There is an enhanced tetrahedral ordering in the DNA minor grooves (see https://pubs.acs.org/doi/10.1021/jp907513wthis). The DNA molecule imprints its helical structure to the tetrahedral structure of water. The TGD interpretation is that the faces of tetrahedra also correspond to the faces of the $Mg(H_2O)_6]^{2+}$ octahedron. This could be the analog for the I-T faces of ITT identifiable also as octahedral faces? An interesting question is whether the ribose pentagon could somehow correspond to a vertex figure of icosahedron also at the level of DNA.

5.6 Hen-egg questions related to the genetic code

Biology involves a long list of hen-egg questions [L15, L7]. What came first: metabolism, basic information molecules, bio-catalysis, or genetic code? Which biomolecules emerged first: RNA, DNA, or amino acids? TGD provides tentative general answers to these questions in terms of the dark genetic code, whose realization in terms of ITT was present from the beginning. It is instructive to consider these questions in the framework provided by the recent views about the realization of the genetic code in terms of ITT about the emergence of dark matter via the generalization of the Pollack effect. One can also try to develop an overall view.

Consider first the emregence of the basic structures.

1. The dark variants of DNA, RNA, tRNA, amino acids were present from the beginning and realized in terms of dark proton triplets assigned with ITTs at MBs. Stable dark realizations of the DNA, RNA and dark protons at MB were stabilized by the formation of corresponding biomolecules as bound states with the binding energy of the state compensating for the larger energy of the dark proton [L18]. Hence one cannot say which came first.

2. The lifetimes of the basic biomolecules serve as guidelines in the attempts to build an overall view about whether the dark protons at the magnetic body of a biomolecule are relevant for its functioning.

   (a) DNA is extremely long-lived: 521 years in bone. Also the negative charge associated with its phosphates is stable. The TGD based conclusion is that the dark protons at the magnetic body of DNA are stable. There is however a metabolic cost also in this case. The classical long range electric along DNA are a crucial aspect of DNA and make possible large values of h_{em} assignable to the DNA. Also the nuclear membrane potentials are crucial for the survival of the DNA nucleotide. Metabolic energy feed is needed to preserve the charge separations generating the classical electric fields.

(b) Also the negative charge of RNA is stable but the lifetimes of RNA molecules vary in a wide range. mRNA has a lifetime from minutes to ours and the average lifetime of 2-20 mins. The lifetime can however be much longer, even days and can persist an organism's lifetime. Special RNAs such as tRNA, rRNA, circular RNAs and nuclear RNAs are very stable and long-lived.

The finite life-time of RNA could be due to the instability of the -OH bond associated with the ribose making possible the transition to the -OH $\rightarrow O^-$ + dark proton at its magnetic body. This would be essential for the ability of RNA to act as a catalyst and could explain the varying lifetime. The stable negative charge of RNA serves as a signature for the presence of dark protons. The dark protons triplets would make possible the communications of RNA with dark DNA and dark tRNA by 3N-resonance.

(c) Amino acids (see this) do not possess a stable negative charge, which suggests that they do not have dark protons at their magnetic body stably. However, Google AI tells that, a C=O bond in a protein can be temporarily converted into a gem-diol structure ($C(OH)_2$) intermediate in an enzyme's active site during catalytic action. This process is a form of nucleophilic addition of water across the carbonyl double bond, which is often a key step in reactions such as the hydrolysis of peptide bonds (catalyzed by peptidases/proteases) or other reactions involving carbonyl-containing substrates. In the TGD framework this could mean that during the enzyme catalysis a proton from C-OH is transferred to the magnetic body of the protein and drops back later. ATP could quite generally provide the needed metabolic energy to achieve this.

The emergence of communications and control was a crucial step in evolution.

(a) Cyclotron frequency triplets as chords assignable to the ITT made possible resonant communications between field bodies by 3N-resonance involving both frequency and energy resonance. The communications between levels involving different values of h_{eff} (and different length scales) involved only energy resonance and very probably 3N-resonance was replaced by the ordinary resonance. This led to an automatic generation of communication and control networks between field bodies characterized by varying values of h_{eff} and biological bodies. Dark cyclotron radiation and frequency modulated dark Josephson radiation inducing a sequence of pulses at the receiver's end are basic mechanisms suggested by TGD [L19].

(b) Large h_{eff} stability possible for DNA and RNA led to a generation of intelligence based on algebraic complexity and to a control by MB. This led to an evolutionary explosion. The electric and gravitational field bodies assignable to the Earth and the Sun were in essential roles [L18].

The emergence of replication was a crucial step. At the chemical level replication reduces to the replication of DNA. A doubling of the DNA strand must occur. In the bio-chemistry approach replication is something which is just accepted.

(a) In the TGD framework, the analog of the replication problem is encountered already at the level of particle physics. Fermion fields are free fields in $H = M^4 \times CP_2$ as also the induced spinor fields at the space-time surfaces defined by them: how is fermion pair creation possible at all? The solution is simple and possible only in 4-D space-time: fermion makes a V-turn in time direction generalized [L23]. The vertex of V corresponds to a 3-D edge of the space-time surface [L22, L16, L24] at which the standard smooth structure has a defect [A4, A5, A2]. The magnetic body assignable to the dark DNA as a 3-surface would make a V-turn and induce DNA replication by transcription of the dark DNA to ordinary DNA.

(b) What was the first replicator and when did it emerge? This classical question becomes obsolete in the proposed framework. The replication could be a general property of space-time surfaces and therefore of the 3-surfaces associated with the dark DNA molecules realizing ITT at the magnetic body of DNA. There are many interesting questions to be pondered. For instance, how to relate the usual view about the role of various catalysts involved with the replication and what is the role of "big" state

function reductions (BSFRs) changing the arrow of time in the process. Could the BSFR have a V-turn as a classical counterpart?

What bio-catalysis is and how did it emerge?

(a) In biocatalysis the reactants must find each other in a dense molecular crowd. How can they recognize each other's presence? In the simplest picture the U-shaped monopole flux tubes emerging from the reactants reconnect to form flux tube pairs connecting them. The shortening of the flux tube pair would force the reactants together and could be induced by a reduction of h_{eff} shortening the flux tube lengths.

(b) The potential wall preventing the bio-chemical reaction must be overcome. The shortening of the monopole flux tubes could liberate metabolic energy while the reduction of h_{eff} could help to overcome the potential wall. The attachment of a biocatalyst carrying large h_{eff} protons to the reacting system could also provide energy allowing it to overcome the potential wall.

(c) How are biocatalysts generated? In general, biocatalysts are unstable. The instability can be inherent or their degradation can be programmed for metabolic reasons since they are needed only when used. If bio-catalysts provide energy to overcome potential walls, they must carry dark protons and their generation requires metabolic energy feed, which also raises the algebraic complexity, "IQ" of the catalysts so that it can take the role of a midwife. ATP is a universal way to provide metabolic energy and dark protons in a standardized way. An alternative option is creation of chemical binding energy making it possible to generate dark protons with large h_{eff}.

(d) The dark proton of the catalyst should transform to an ordinary one in the reaction and liberate the energy needed to overcome the potential wall. Catalysts could be either inherently h_{eff} unstable or the instability could be induced in the reaction and induce the decay of the catalyst. Often the catalyst indeed decays after the reaction. Catalysts often have ATPs attached to them and ATP–¿ADP is a basic aspect of catalysis.

Note that in the translation of mRNA to proteins mRNA serves as a template and degrades after the translation. This could be due to the catalysis of the translation requiring the reduction of h_{eff} inducing a chemical instability. The instability could relate to the -OH sidegroup of the ribose.

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