Summary of Topological Geometrodynamics

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Abstract

Topological Geometrodynamics is a proposal for a unification of fundamental interactions with which I have worked for 42 years. It is based on new view about space-time inspired by the problem of General Relativity Theory with classical conservation laws ("energy problem"). Matter makes the flat Minkowski space M^4 of Special Relativity Theory curved leading to the loss of its symmetries. Poincare invariance implies the conservation laws of energy, momentum, and angular momentum via Noether's theorem so that they are lost in GRT.

If space-time is a 4-surface in space of form $H = M^4 \times S$, S some compact space with very small size, space-time isometries are lifted to those of H and Poincare symmetries are not lost. The geometry of $S = S = CP_2$ codes for the symmetries of standard model.

Topological Geometrodynamics decomposes to two basic threads: physics as geometry and physics as number theory. These complementary approaches related by $M^8 - H$ duality are discussed in this article. TGD is compared with GRT and standard model and the applications of TGD - also those to quantum biology, consciousness and technology - are briefly summarized.

Contents

1 Introduction

Topological Geometrodynamics (TGD) is a 42 year old proposal for a unification of fundamental interactions, which I dare to regard as my life work. TGD is based on new view about space-time inspired by the problem of General Relativity (GRT) with classical conservation laws ("energy problem"). Matter makes the flat Minkowski space M^4 of Special Relativity (SRT) curved leading to the loss of its symmetries. Poincare invariance implies the conservation laws of energy, momentum, and angular momentum via Noether's theorem so that they are lost in GRT.

If space-time is a 4-surface in space of form $H = M^4 \times S$, S some compact space with very small size, space-time isometries are lifted to those of H for general coordinate invariant action depending on Poincare invariant geometric quantities so that Poincare symmetries are not lost and Noether's theorem gives conserved charges. The geometry of $S = S = CP_2$ codes for the symmetries of the standard model: color symmetries correspond to isometry group $SU(3)$ and electroweak symmetries correspond to the holonomies of CP_2 being broken by CP_2 geometry. CP_2 does not allow spinor structure in standard sense [L3] but - as observed already by Hawking and others [A3, A1] - it allows modified spinor structure obtained by coupling spinors to an odd multiple of Kähler gauge potential: for quarks and leptons the coupligs would correspond to $n = 1$ and $n = 3$ but TGD view about color allows to understand leptons as local 3-quark composites so that only quarks are needed as fundamental fermions.

Besides sub-manifold geometry topology becomes important (hence "TGD") since the many-sheeted space-time of TGD is topologically non-trivial in all scales and the physical objects that we see around us correspond directly to space-time sheets topologically condensed on larger space-time sheets and forming a hierarchy.

TGD decomposes to two basic threads: physics as geometry [L5, L4, L19, L51] and physics as number theory [L31, L32, L27] (see **Fig. 1**).

1. In the geometric approach space-time surfaces X^4 correspond to extremals for both volume action and so called Kähler action: this action is predicted by twistor lift of TGD [L21].

2. In number theoretic approach X^4 corresponds to algebraic surface in complexified M^8 obtained as roots for the complexified quaternion-valued "real" or "imaginary" part of a real polynomial with rational coefficients continued to complexified octonion valued polynomial.

These approaches related by $M^8 - H$ duality. The minimal form of duality relies on strong form of holography allowing to deduce $X^4 \subset H$ from the images of 2-D surfaces $X^2 \subset X^4 \subset M^8$ under $M^8 - H$ duality [L43].

Quantum TGD leads to a generalization of the geometrization program of Einstein. Entire quantum theory is geometrized in terms of the notion of the "world of classical worlds" (WCW) consisting of space-time surfaces -preferred extremals (PEs), which are analogs of Bohr orbits. General coordinate invariance implies strong form of holography. The mere existence of WCW Kähler geometry requires maximal isometry group as was shown by Freed to be the case for loop spaces. This leads to the vision that physics is unique from is mere existence. Indeed, the twistor lift of TGD [L49, L50] works only for $H = M^4 \times CP_2$ [A4]. At the number theoretic side the octonionic M^8_c is te unique choice.

Number theoretical vision is also a completely new element and leads to adelic physics [L26] involving both real physics and various p-adic physics identified as correlates of cognition and imagination predicted to be present in all scales. The polynomials defining the $X^4 \subset M^8_c$ give rise to an infinite hierarchy of EQs inducing those of p-adic number fields and of adele. This hierarchy corresponds to an evolutionary hierarchy. The dimension of extension of rationals (EQ) has an interpretation as effective Planck constant $h_{eff} = nh_0$: the prediction is the possibility of quantum coherence in arbitrarily long scales and the values of n define a length scale hierarchy.

TGD has non-trivial applications in all scales.

1. Space-time topology is non-trivial in all scales and besides Einsteinian space-time surfaces with M^4 projection with dimension $D = 4$ also space-time surfaces with $D < 4$ are possible: in particular, so called CP_2 type extremals with $D = 1$ and cosmic strings with $D = 2$.

   The space-time surfaces - also Einsteinian ones - have finite size. In Einsteinian case CP_2 coordinates are many-valued functions of M^4 coordinates, and it is convenient to talk about many-sheeted space-time. Quantum field theory limit (QFT-GRT limit) for Einsteinian space-time surfaces is obtained by replacing the space-time sheets with single slightly metrically deformed region of M^4 and defining gauge potentials and deviation of the metric from M^4 metric as of their deviations for the space-time sheets. Einstein's equations can be regarded as a remnant of Poincare invariance. Many-sheeted space-time is topologically trivial in all scales and the hidden many-sheetedness leads to non-trivial predictions in all scales.

2. For given extension of rationals (EQ) fixed by a polynomial defining defining $X^4 \subset M^8$ there is a unique discretization of space-time surface - cognitive representation - as points common to the real and p-adic variants of X^4 and therefore in EQ. This implies strong correlation between cognition and reality. p-Adic length scale hypothesis emerging from p-adic thermodynamics as a model for particle massivation [L9] and p-adic fractality are very powerful quantitative tools allowing to make non-trivial predictions.

The geometric and number theoretical approaches and $M^8 - H$ duality are briefly discussed in this article. TGD is compared with GRT and standard model (SM) and the applications of TGD - also those to quantum biology and consciousness - are briefly summarized.

For more detailed representations one can consult the books [K2, K3, K4], the article [L26] and articles [L28, L37]. The homepage dedicated to TGD (`http://tgdtheory.fi`) contains online books and articles - also updated versions of published articles.

2 Physics as geometry

One can end up with TGD in two manners (see **Fig. 2**). Either as a solution of energy problem of GRT realizing Einstein's dream about geometrization of classical physicsor as a generalization of hadronic string model or of superstring theory [B7]. In case of hadronic string model the generalization of string to 3-surface would allow to get rid of spontaneous compactification and the landscape catastrophe implied by it.

At fundamental level TGD could be seen as a hybrid of GRT and SRT: the notion of force does not disappear and can be defined as rate for an exchange of conserved quantity which can be Poincare or color charge. This connection with Newtonian limit is more clear than in GRT, where the conservation laws are lost.

2.1 Classical physics as sub-manifold geometry

The new elements are many-sheeted space-time topologically non-trivial in all scales, and topological field quantization implying that physical systems have field identity, field body, in particular magnetic body (MB) central in applications [L2, L1] (see **Fig. 3**).

2.1.1 Induction procedure

One ends up to a geometrization of gravitational field and gauge fields of the standard model as induced fields. Induction means induction of bundle structure is in question. Parallel translation at X^4 is carried out by using spinor connection of H and distances are measured using the metric of H. The components of induced gauge potentials and metric are projections to X^4. Color gauge potentials are identified as projections of Killing vector fields of CP_2 and one can define for them gauge algebra structure. The components of the induced color field are proportional to $H_A J$, where H_A is the Hamiltonian of color isometry and J induced Kähler form. For details see [L3] or the material at my homepage.

The induction of spinor structure allows to avoid the problems related to the definition of spinor structure for general 4-geometry encountered in GRT. For the induced spinor structure induction means projection of gamma matrices to X^4. The definition of gamma matrices is modified when

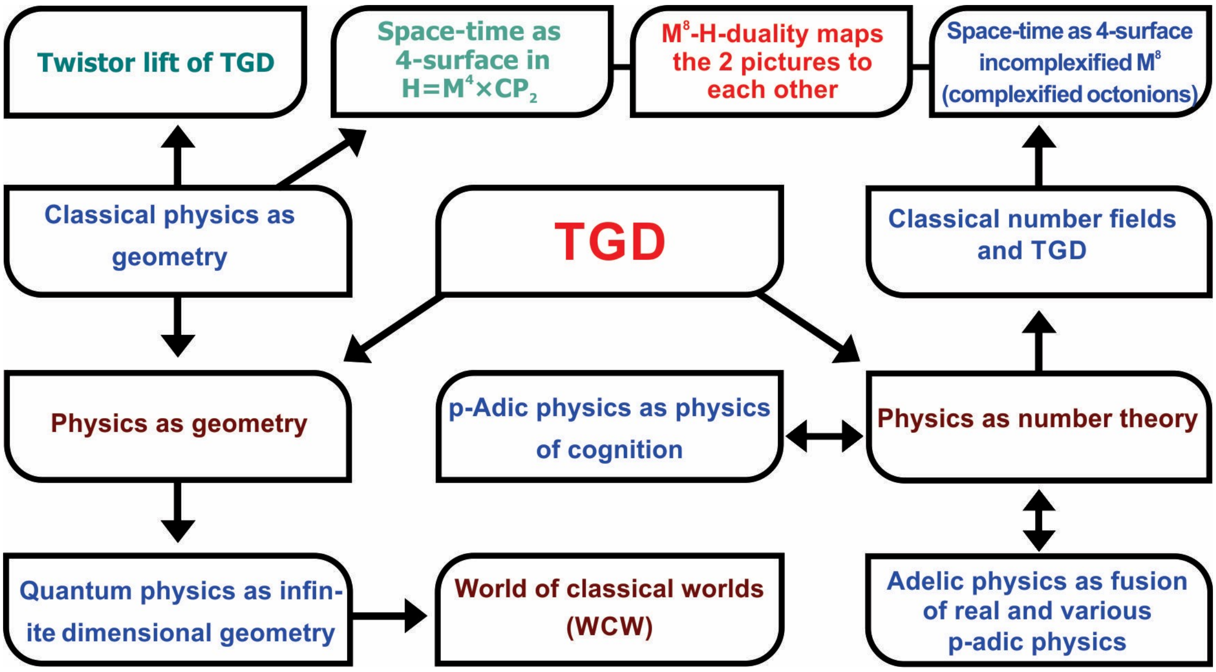

Figure 1: TGD is based on two complementary visions: physcs as geometry and physics as number theory.

classical action defining the space-time dynamics contains besides volume term also Kähler action with the projection of CP_2 Kähler form defining the analog of Maxwell field. Modified gamma matrices are contractions $T^{\alpha k}\gamma_k$ of imbedding space gamma matrices γ_k with canonical momentum currents $T^{\alpha k}$ associated with the action: this is required by the hermiticity of the modified Dirac action and means existence of infinite number of super currents labelled by the modes of the modified Dirac action.

2.1.2 Spacetime is topological complex

Locally the theory is extremely simple: by GCI there are only 4 field-like variables corresponding to a suitable identification of imbedding space coordinates as space-time coordinates. The possibility to choose the coordinates in this manner means enormous simplification since the problems caused by GCI in GRT disappear. It is however obvious that 4 field-like variables does not conform with standard model and GRT. This simplicity is compensated by topological complexity in all scales implied by the many-sheeted space-time. The QFT-GRT limit explained in introduction gives the space-time of gauge theories and GRT.

Geometrically the QFT limit for space-time surfaces having 4-D M^4 projection is obtained by replacing the sheets of many-sheeted space-time with slightly curved region of M^4 and identifying gauge potentials and gravitational field (deviation of the metric from M^4 metric) as superpositions of induced fields at various space-time sheets. Einstein's equations hold true as a remnant of the Poincare invariance.

The presence of space-time regions with M^4 projection of dimension $D < 4$ must be described at QFT limit as particle- or string-like entities. Particle-like entities correspond to CP_2 type extremals

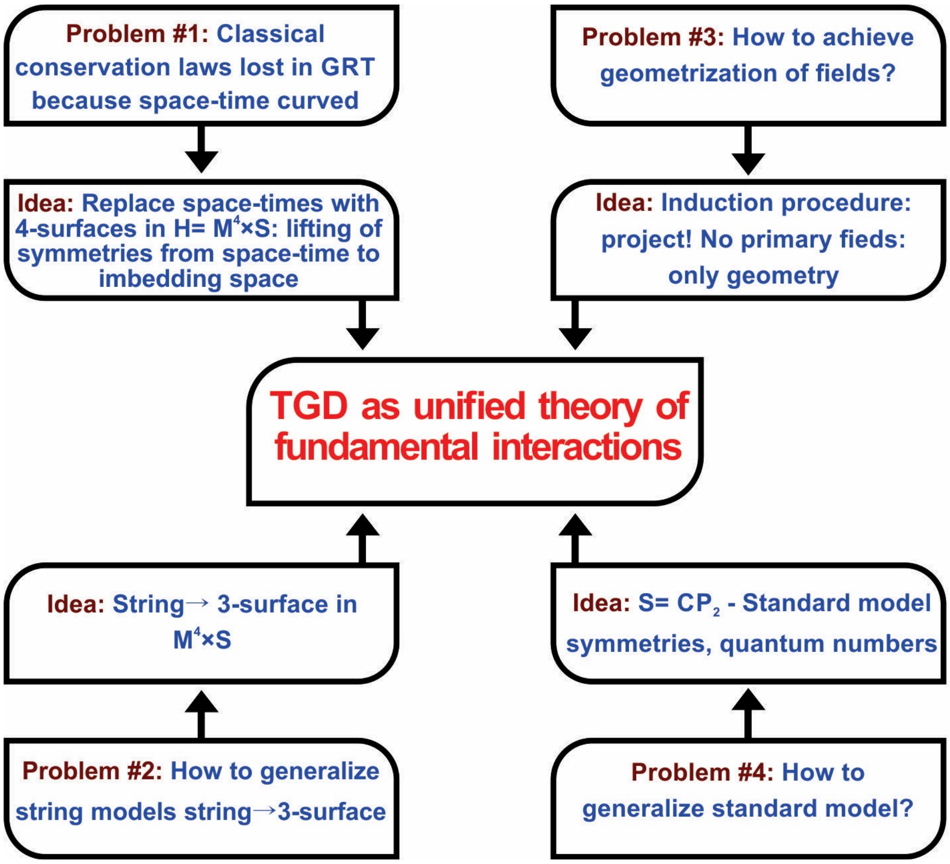

Figure 2: The problems leading to TGD as their solution.

having Euclidian signature of induced metric and light-like M^4 projection. 3-D light-like surfaces serve as boundaries between them and Minkowskian space-time regions: the identification is as partonic orbits carrying fermion number serving as building bricks of elementary particles [L27].

The topology of partonic 2-surface is characterized by its genus (number of handless attached to sphere) and is propose to explain family replication for fermions. Also for bosons 3 families are predicted. The existence of 3 light fermion families is understood in terms of the fact that only 3 lowest genera have global Z_2 as conformal symmetry making possible bound state of 2 handles. For the higher genera handles would behave like particles and mass spectrum would be continuum.

Cosmic strings are fundamental objects of this kind and appear as two different species. Those carrying monopole flux mean deviation from Maxwell's theory. They are unstable against perturbations making their M^4 projection 4-D and transforming them to magnetic flux tubes playing a key role in TGD inspired cosmology.

2.1.3 Twistor lift

One could end up with the twistor lift of TGD from problems of twistor Grassmannian approach originally due to Penrose [B10] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B3, B2, B5, B1, B4].

Twistor lift of TGD [L21, L49, L50] generalizes the ordinary twistor approach [L38, L39] (see **Fig. 4**). The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense.

The basic recipe is simple: replaced fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure

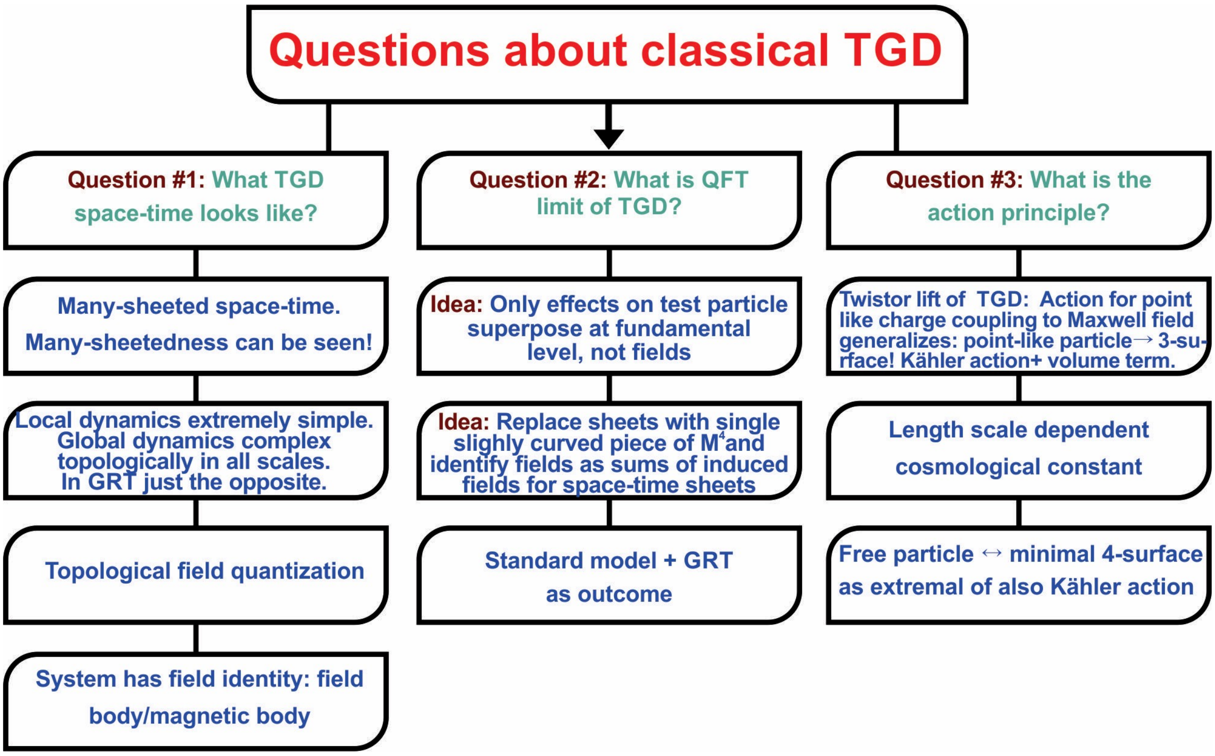

Figure 3: Questions about classical TGD.

requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

The pleasant surprise is that twistor space has Kähler structure only for M^4 and CP_2 [A4]: this had been discovered already when started to develop TGD! Since Kähler structure is necessary for the twistor lift of TGD, TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

2.2 Quantum physics as WCW geometry

2.2.1 WCW as an analog of Wheeler's superspace

Quantum TGD replaces Wheeler's superspace of 3-geometries with the "World of Classical Worlds" (WCW) as the space of 3-surfaces (see **Fig. 5**). The holography forced by general coordinate invariance (GCI) implies their 1-1 correspondence with space-time surfaces identified as preferred extremals (PEs) of the basic variational principle analogous to Bohr orbits. Classical physics becomes an exact part of quantum physics [L5, L4]. Einstein's geometrization of classical physics extends to that of quantum physics.

The geometry of infinite-D WCW (see **Fig. 5**) and physics is highly unique from its mere existence requiring maximal group of isometries: a result proved by Freed for loop spaces [A2]. The group of WCW isometries is identified as the group of symplectic (contact) transformations of $\delta M^3_+ \times CP_2$ having the light-like radial coordinate in the role of complex variable z in conformal field theories

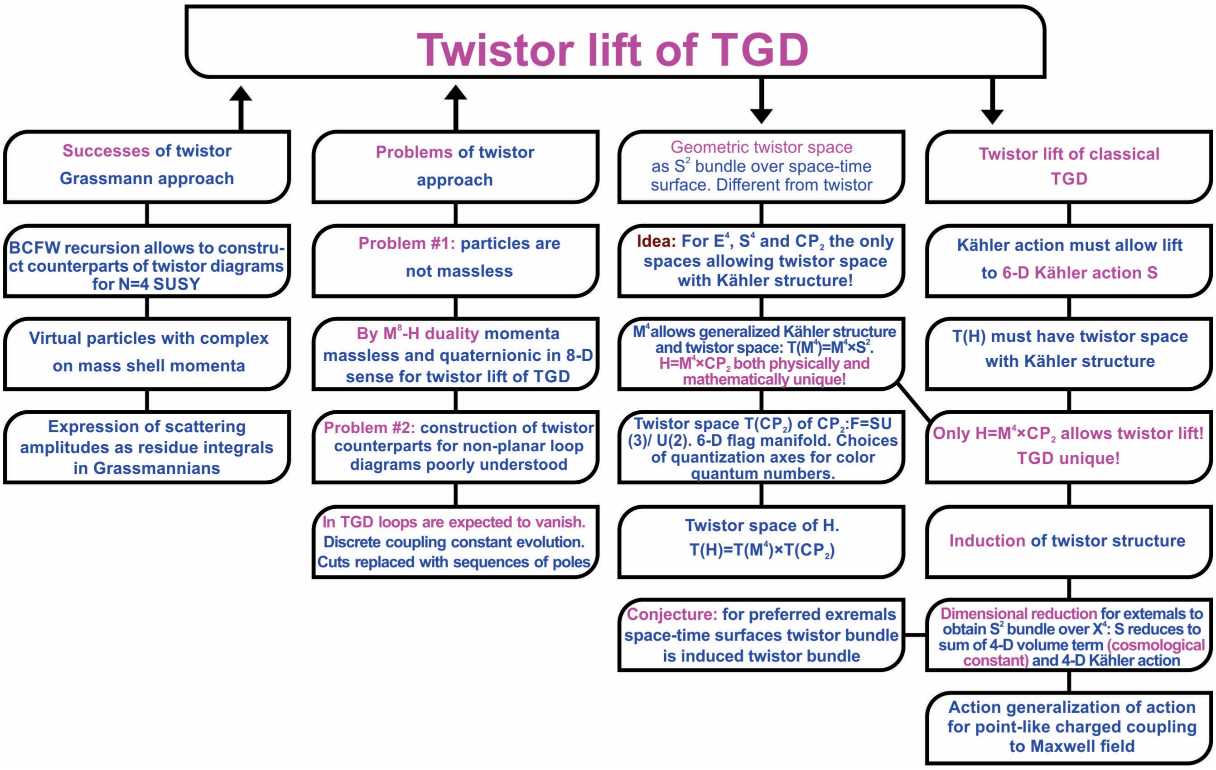

Figure 4: Twistor lift

Remark: The geometric properties of boundary of 4-D light-cone are unique by its metric 2-dimensionality. In particular, the ordinary 2-D conformal symmetries involving local scaling of the radial light-like coordinate give rise to isometries).

The assumption that space-time surfaces as preferred extremals (PEs) are fundamental entities leads to zero energy ontology (ZEO) in which quantum superpositions of pairs (X^3_1, X^3_2) of 3-surfaces at opposite boundaries of causal diamond (CD) and connected by PE represent quantum states [L58]. This leads to a solution of the basic problem of quantum measurement theory due to the conflict between the determinism of field equations and non-determinism of state function reduction (SFR) and quantum measurement theory extends to a theory of consciousness bringing observer a part of the physisal system.

Quantum states are identified as modes of classical WCW spinor fields so that apart from quantum jump the theory is formally classical. WCW spinor structure involves complexified gamma matrices expressible as superpositions of second quantized oscillator operators of the induced spinor fields at space-time so that a geometrization of fermionic statistics is achieved [L19, L51, L55]. The simplest formulation assumes only quark spinors and would predict that lepton are local composites of 3 quarks.

2.2.2 Holography from GCI

Gravitational holography has been one of the dominating themes in recent day theoretical physics. It was originally proposed by Susskind [B9], and formulated by Maldacena as AdS/CFT correspondence [B8]. One application is by Preskill et al to quantum error correcting codes [B6].

By holography implied by GCI the basic variational problem can be seen either as boundary value problem with 3-surfaces at opposite boundaries of CD or as initial value problem caused by

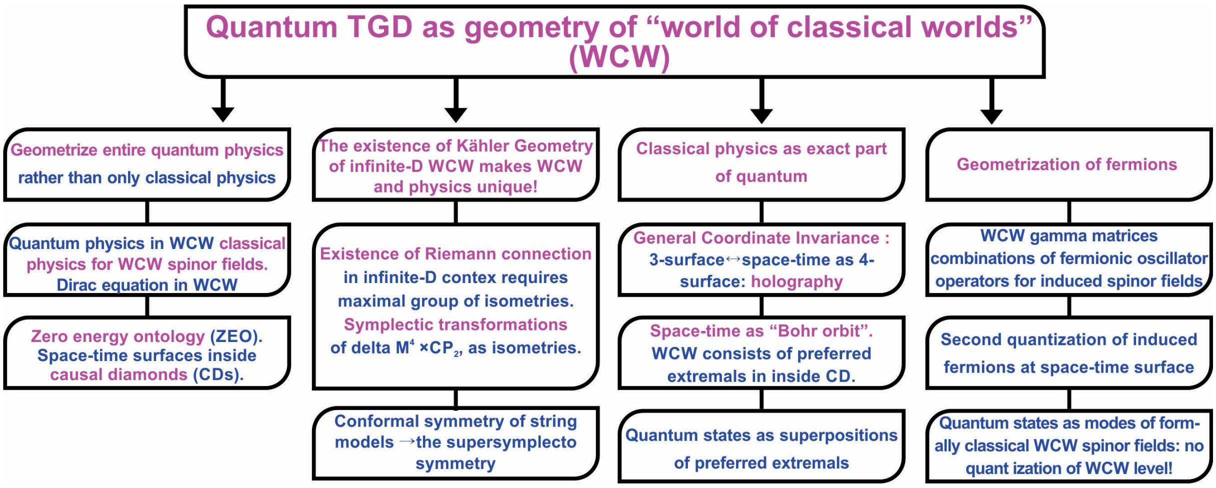

Figure 5: Geometrization of quantum physics in terms of WCW

PE property. Ordinary 3-D holography is thus forced by general coordinate invariance (GCI) and loosely states that the data at 3-surface at either boundary of CD allows to determine $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

Holography poses additional strong conditions on X^4.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SSA_n with radial conformal weights coming as n-multiples of those for the entire algebra SSA and its commutator $[SSA_n, SSSA]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations. An open question is whether this hierarchy corresponds to the hierarchy of EQs with n identifiable as dimension of the extension.

2. Second conjecture is that PEs are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A4] so that TGD is unique.

2.2.3 Strong form of holography

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated

1. either with light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian,

2. or the space-like 3-surfaces at the ends of CD to determine X^4 as PE (in case that it exists),

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken the strong form of M^8-H duality [L43] mapping $X^4 \subset M^8$ to $X4 \subset H = M^4 \times CP_2$ that it allows to map only certain 2-D sub-manifolds $X^2 \subset X^4 \subset M^8$: SH allows to determine $X^4 \subset H$ from this 2-D data.

2.2.4 Further generalizations

This picture about WCW is not general enough.

1. M^8-H duality [L43] suggests that the notion of WCW applies also M^8 picture. The parameters of polynomials defining $X^4 \subset M^8$ are assumed to be rational. The points of M^8 counterpart of WCW have the rational coefficients of these polynomials as coordinates so that WCW should be discrete in real topology. This should be the case also for H counterpart of WCW. Could one see real and p-adic variants of WCW as completions of this discrete WCW.

2. Adelic physics inspires the question whether p-adic and adelic variants of WCW make sense or is it enough to have number theoretically universal cognitive representations to define unique discretized variants of X^4 and correspondingly discretized WCW.

3. For TGD variant of SUSY [L48, L47] super coordinates for H correspond to hermitian local composites of quark oscillator operators. For super-quarks they correspond to local components with fixed quark number. Leptons can be understand as local composites of quarks - super field components [L55]. SUSY replaces modes of super-field with super-surfaces so that the components of super-field correspond to sets of disjoint 4-surfaces. This is true also for the points of super WCW.

3 Physics as number theory

Number theoretical vision is second thread of TGD. It decomposes to 3 threads corresponding to various p-adic physics [L6] fusing to adelic physics [L27], classical number fields [L7], and infinite primes [L8] (not discussed in the sequel).

3.1 p-Adic and adelic physics and extensions of rationals (EQs)

p-Adic number fields would serve as correlates of cognition and imagination (see **Fig. 6**) . Space-time is replaced with a book like structure having both real and various p-adic space-time sheets as pages. The outcome is adelic physics as fusion of various p-adic physics [L26, L27] (see `http://tinyurl.com/ycbhse5c`). The EQ induces extensions of p-adic numbers fields and of adele giving rise to a hierarchy of physics having interpretation in terms of evolution induced by the increase of the complexity of the EQ.

Adelic physics leads also the hierarchy of Planck constants $h_{eff}/h_0 = n$ with n identified as dimension of EQ labelling phases of ordinary matter behaving like dark mater, and making possible quantum coherence in arbitrarily long time scales essential for understanding living matter.

EQs are characterized by discriminant D assignable to a polynomial giving rise to the extension (for second order polynomials D has expression familiar from school days). Now polynomials with rationals (equivalently integer) valued coefficients are interesting. The primes dividing the discriminant are known as ramified primes and they have a property that for p-adic variant of polynomial degenerate roots appear in $O(p) = 0$ approximation [L45]. The interpretation could be in terms of quantum criticality and physically preferred p-adic primes are identified as ramified primes of extension [L54].

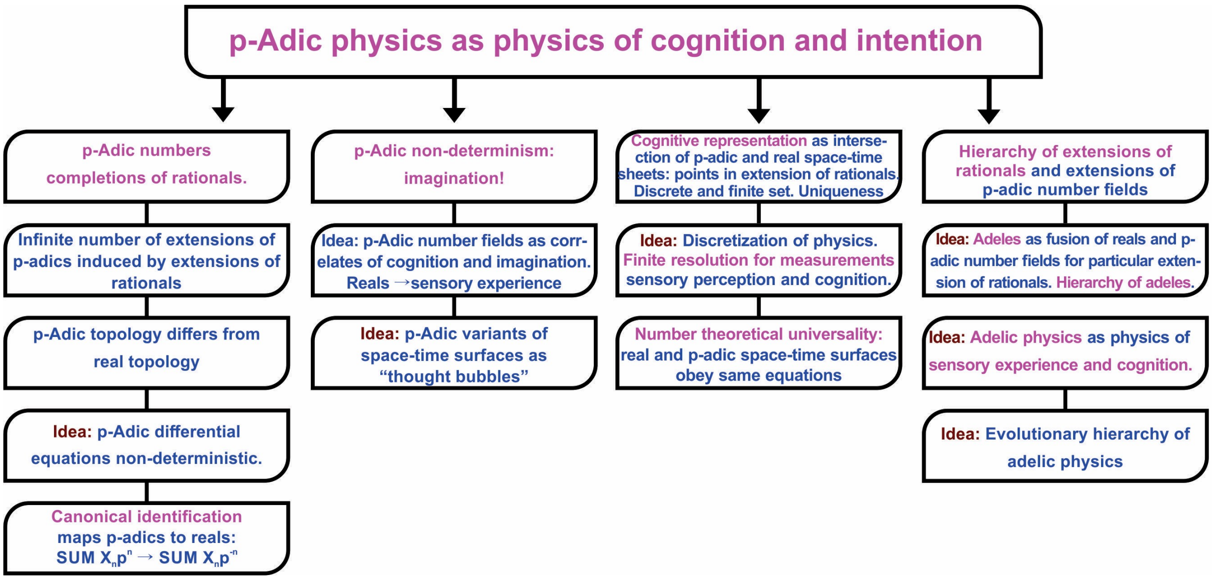

Figure 6: p-Adic physics as physics of cognition and imagination.

Remark: One can also consider polynomials with algebraic coefficients. The notion of Galois group make sense also for real coefficients.

The hierarchy of EQs labelling levels of dark matter hierarchy and of hierarchy of adelic physics follows from M^8-H duality allowing to identify $X^4 \subset M^8$ as a projection of $X^4_c \subset M^8_c$ - identified as complexified octonions O_c - and satisfying algebraic equations associated with a polynomial of degree n.

Real and p-adic physics are strongly correlated and mass calculations represent the most important application of p-adic physics [K1]. Elementary particles seem to correspond to p-adic primes near powers 2^k (there are also indicatons for powers of 3). Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p=2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p=3$ (period tripling as approach to chaos) [I2, I3]. Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2 and the challenge is to understand this [L22].

3.2 Classical number fields

Second aspect of number theoretical vision are classical number fields: reals, complex numbers, quaternions and octonions and their complexifications by a commuting imaginary unit i (see **Fig. 7**).

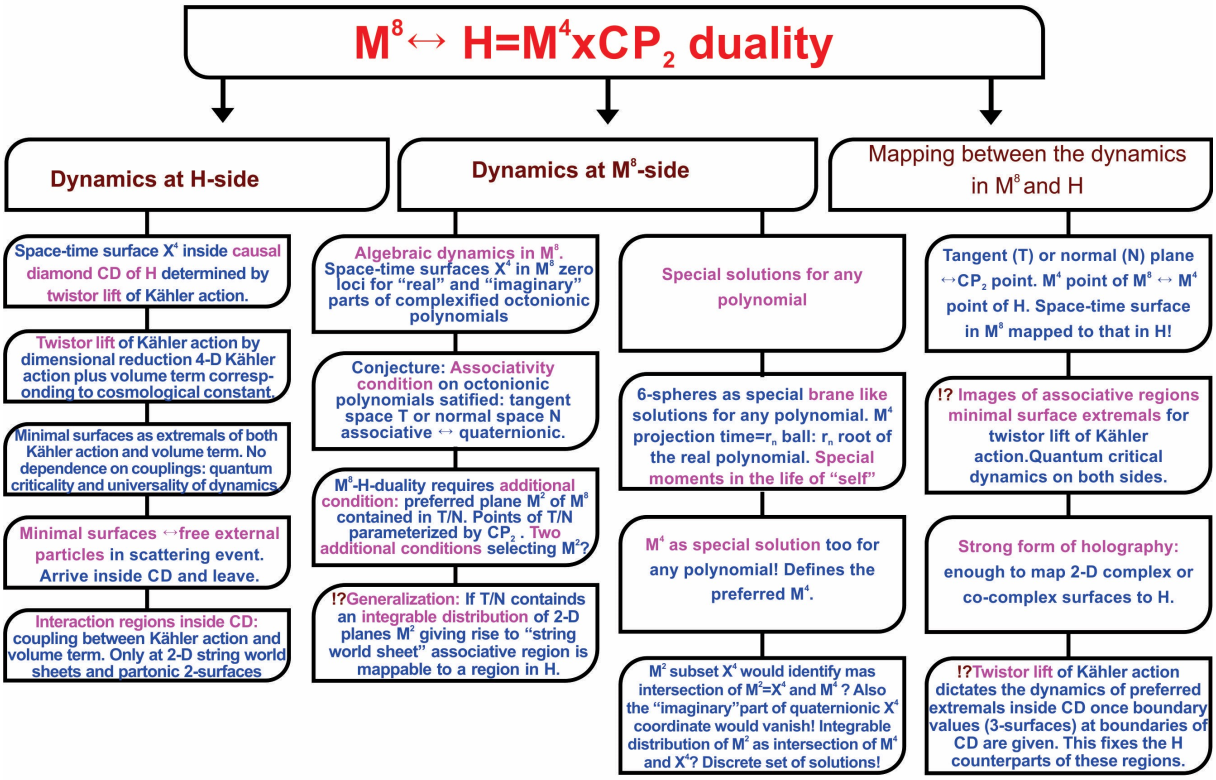

Figure 7: $M^8 - H$ duality

3.2.1 Space-time as 4-surface in $M_c^8 = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8−-projection of $X_c^4 \subset M_c^8 = O_c$. M_c^4 is identified as complexified quaternions H_c [L43, L54]. The dynamics is purely algebraic and therefore local.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X_c^4 as a root for the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.

   The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X_c^4 with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [?] "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

   The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L54]. For ramified primes the p-adic variants of polynomials have degenerate zeros in $O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If the space-time surface corresponds to the projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

Remark: Besides polynomials with rational coefficients also analytic functions with rational coefficients can be considered [L23, L24, L25]. Indeed, one might argue that the existence of periodic phenomena requiring transcendental functions like sine and cosine also requires analytic functions. Space-time surfaces in M^8 would correspond to roots of analytic functions. Number theoretic holography would reduce from a discrete holography to 1-D holography.

There are however serious problems. p-Adic variants of Taylor series do not converge in general. The roots are not algebraic numbers in general and the extensions of rationals obtained are infinite-dimensional. The p-adic analogs of sine and cosine are not periodic and the representation of periodic functions requires extensions of rationals containing roots of unity. After writing the first version of this article, I realized that M^8 is analogous to momentum space and periodic functions are natural only in configuration space, not in momentum space [L52, L53]. Indeed, the induced spinor fields at the level of $H = M^4 \times CP_2$ allow periodic solutions and since all elementary particles are composites of fundamental quarks in TGD, this could be enough.

3.2.2 How to realize $M^8 - H$ duality?

$M^8 - H$ duality (see **Fig. 7**) allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible $M^8 - H$ duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [L51].

Twistor lift allows several variants of this basic duality [L49, L50]. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

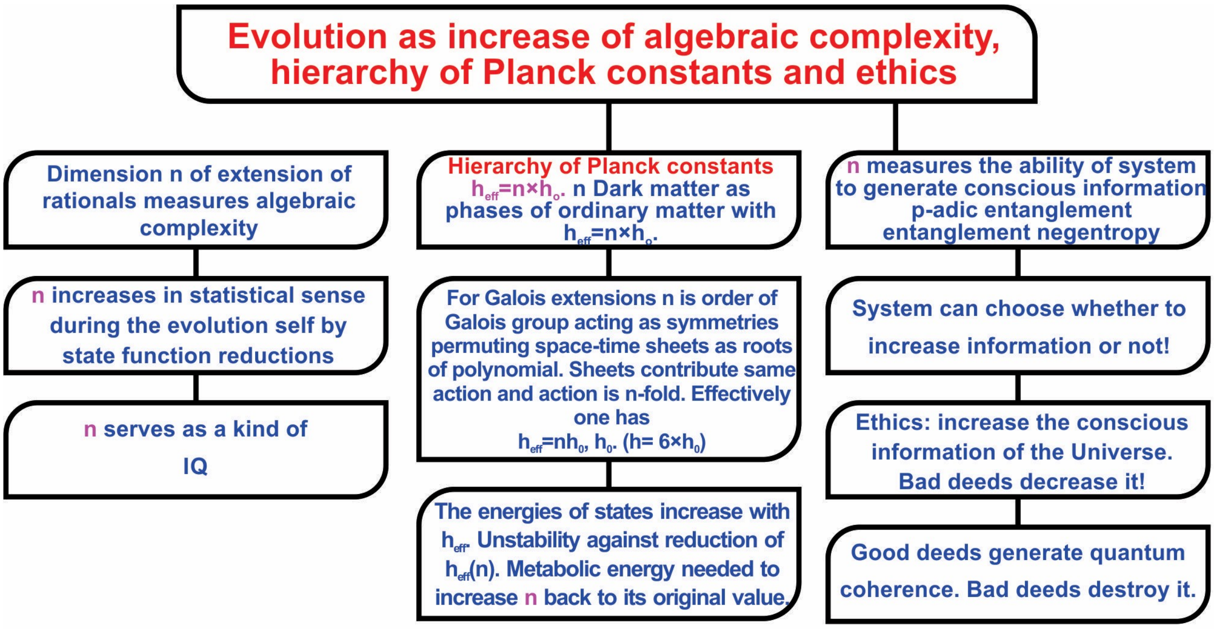

Figure 8: Number theoretic view about evolution

M^8-H duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L43].

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of M^8-H duality and means that tangent - or normal space is associative/quaternionic.

2. Each tangent space of X^4 at x must contain a preferred $M_c^2(x) \subset M_c^4$ such that $M_c^2(x)$ define an integrable distribution and therefore complexified string world sheet in M_c^4. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X_c^2 \subset X_c^4$ in R_c sense. $E_c^2(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space.

One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M_c^2(x)$ and $E_c^2(x)$ define a slicing of X_c^4.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X_c^2 can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M_c^2 would be the simplest solution but also more general complex

sub-manifolds $X^2_c \subset M^4_c$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. SH makes possible $M^8 - H$ duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.

   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X^2_c have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.

3. There is however a delicacy involved: the X^2 might be only metrically 2-D but not topologically. The partonic orbits are 3-D light-like surfaces with metric dimension $D = 2$. The 4-metric degenerates to 2-D metric at them. Therefore their pre-images would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to 2 [L44] . If this happens, SH would not be quite so strong as expected. The study of fermionic variant of $M^8 - H$ correspondence indeed leads to this conclusion.

One can generalize the condition selecting X^2_c so that it selects 1-D surface inside X^2_c. By assuming that R_c-valued "real" or "imaginary" part of complex part of P at this 2-surface vanishes. One obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

3.2.3 What about $M^8 - H$ duality in the fermionic sector?

During the preparation of this article I became aware of the fact that the realization $M^8 - H$ duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about $M^8 - H$ duality also in the bosonic sector and the existing phenomenological picture follows now from $M^8 - H$ duality. There are powerful mathematical guidelines available.

1. Octonionic spinors

By supersymmetry, octonionicity should have also a fermionic counterpart.

1. The interpretation of M^8_c as complexified octonions suggests that one should use complexified octonionic spinors in M^8_c. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L18]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and spinors is replaced with the non-associative octonionic product.

2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L48, L47].

3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have an analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!

   One can define for the octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M^8_c$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal spaces of X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.

4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as a complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about the sigma matrices associated with the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matrices and those of E^4 sigma matrices close to the same $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalized spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2 \otimes 2 = 3 \oplus 1$ so that one must have $1 \oplus 3 \oplus 1 \oplus 3$. The octonionic spinors indeed decompose like $1 + 1 + 3 + \overline{3}$ under $SU(3)$ representing automorphisms of the octonions. $SO(3)$ could be interpreted as $SO(3) \subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

2. Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of the dynamics should be supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for $M^8 - H$ duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corresponds to the imbedding space coordinates.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4 \subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining the space-time surface can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity holds true if also Ψ satisfies associativity/co-associativity condition as proposed above.

3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl} o^k \overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c \rightarrow M^8 \rightarrow M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P \Psi = 0$ gives $m_{kl} P^k \overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases.

$PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X^4_c \rightarrow M^8 \rightarrow M^4$ in terms of Uncertainty Principle.

There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in the M^8 picture?

For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3 \subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

The variation of P at the mass shell of M^8_c (to be precise) could be interpreted in terms of the width of the wave packet associated with a particle. Since the light-like curve at partonic 2-surface for fermion at X^3_L is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K1].

4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L43] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

5. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L43] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

6. The variant Dirac equation would hold true also at 2-surfaces $X^2 \subset X^4$ and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ defines a 2-D variant of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X^1_L. Its H-image would naturally contain the boundary of the string word sheet at X^3_L assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of the string world sheet in H would not carry an induced spinor field.

7. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this is in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

3. The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been a long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of $M^8 - H$-duality for fermions.

1. SH and the expected analogy with the bosonic variant of $M^8 - H$ duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by $M^8 - H$-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.

The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_kP^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in the twistor Grassmann approach!

For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.

   This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L48, L47]. S_{C-S} could emerge as a limit of 4-D action.

   The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^\alpha_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^\alpha_{B,A} = T^\alpha_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.

   Fermionic Noether current would be $J_{F,A} = \overline{\Psi}J^\alpha\Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma}$ defining magnetic flux. The modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what the twistor Grassmannian approach led to [L38, L39].

4 Some applications

The applications of TGD rely on rather few basic notions and ideas applying in all scales - one can loosely say that the length scale reductionism of standard physics is replaced with fractality. Most important of these ideas are p-adic length scale hypothesis, dark matter as phases of ordinary matter with $h_{eff} = nh_0$, and ZEO. Twistor lift of TGD predicts length scale dependent cosmological constant Λ with correct sign approaching zero in long length scales [L49, L50].

4.1 Applications to cosmology and astrophysics

Consider first cosmology.

1. Many-sheeted space-time of TGD implies what might be called many-sheeted Russian doll cosmology. Magnetic flux tubes are the key notion and are obtained as thickening of cosmic string like entities having 2-D M^4 projection. CP_2 projection is in the simplest situation geodesic 2-sphere S^2_I carrying non-vanishing homology charge having interpretation as Kähler magnetic charge or S^2_{II} having induced Kähler form, which vanishes. Note that the cross section of the flux tube is not disk but sphere. S^2_I can be replaced with any complex 2-manifold of CP_2 and also Lagrangian 2-manifolds more general than S^2_{II}. Thus there are two variants of cosmic strings - those carrying monopole flux and and those for which the flux is vanishing.

   Homologically trivial flux tubes are tentatively identified as mediators of gravitational interactions whereas monopole flux tubes are identified as building bricks of various objects in alls scales. Flux tubes would be key players even in biology, chemistry and molecular physics, nuclear physics, and hadron physics.

2. Very early cosmology would rely on cosmic strings with monopole flux. During the analog of inflationary period Einsteinian space-time with space-time surfaces having 4-D M^4 projection would emerge [K4] (for details see the online article at [K5]). Cosmic strings are obviously unstable against thickening to monopole flux tubes. Cosmic flux tubes moving with respect to each other can intersect and reconnect forming gravitationally bound states. This would induce instability so that either cosmic string or both of them develop a tangle and thickens by a phase transitions reducing the length scale dependent cosmological constant Λ liberating energy (magnetic energy is proportional to $1/S$ and volume energy to ΛS) transformed to ordinary particles. This would give rise to the TGD counterpart of inflation leading from cosmic string dominances to radiation dominance and generating ordinary matter.

3. The phase transitions reducing Λ characterizing the cosmic strings occur also later and give rise to periods of accelerated expansion. Even astrophysical objects would suffer this kind of rapid expansions and this would solve the puzzle due to the fact that astrophysical objects do not seem to expand as one might expect although the comove. There is considerable evidence that also Earth radius increased by factor 2 during Cambrian explosion [L16].

TGD provides also a model for the formation of galaxies and stars.

1. The formation of galaxies is one application [L40]. The flux tube picture about galaxies and larger structures is discussed with application to some anomalies strongly suggesting the presence of coherence in scales of even billion light years. This coherence could be induced from quantum coherence in astrophysical scales made possible by the huge values of gravitational Planck constant $h_{gr} = h_{eff} = GMm/v_0$ (proposed originally by Nottale [E2]) assigned to the homologically trivial gravitational flux tubes.

2. The view about the role of new nuclear physics predicted by TGD in the model of solar core [L46] identified as flux tube tangles gives excellent guidelines for the attempts to develop a more detailed understanding about TGD counterparts of blackholes as maximally dense and thus volume filling flux tube spaghettis. One ends up to rather detailed picture making correct predictions about minimum radii of blackholes and neutron stars. The idea about all final states of stars as blackhole like objects in generalized sense emerges.

   The standard blackhole thermodynamics is replaced by two thermodynamics. The first thermodynamics is assignable to the flux tubes as string like entities having Hagedorn temperature T_H as maximal temperature. The second thermodynamics is assignable to gravitational flux tubes characterized by the gravitational Planck constant h_{gr}: Hawking temperature T_B is scaled up by the ratio $\hbar_{gr}/\hbar$ to $T_{B,D}$ and is gigantic as compared to the ordinary Hawking temperature but the intensity of dark Hawking radiation is extremely low.

   The condition $T_H = T_{B,D}$ for thermodynamical equilibrium fixes the velocity parameter $\beta_0 = v_0/c$ appearing in the Nottale formula for $\hbar_{gr}$ and suggests $\beta_0 = 1/h_{eff}$ for the dark nuclei at flux tubes defining star as blackhole like entity in TGD sense. This also predicts the Hagedorn temperature of the counterpart of blackhole in GRT sense to to be hadronic Hagedorn temperature assignable to the flux tube containing dark nuclei as dark nucleon sequences so that there is a remarkable internal consistency. In ZEO quasars and galactic blackholes could be seen as time reversals of each other.

4.2 Particle physics and nuclear physics

As already explained, p-adic mass calculations replace Higgs mechanism with p-adic thermodynamics [L9] so that mass squared is analogous to thermal energy. p-Adic length scale hypothesis allows to understand the particle masses and predicts p-adic length scale hierarchy also outside particle physics.

1. Basic new prediction is the possibility of scaled variants of hadron physics and electroweak physics. The masses of particles of say ordinary hadron physics relate by scaling by a power of $\sqrt{2}$ from those of ordinary hadron physics. There are indications at LHC for bumps with masses of mesons of M_{89} hadron physics [L10]. They would be dark in the sense of having value of h_{eff} differing by factor 512 - the ratio of mass scales of M^{89} hadron physics and ordinary hadron physics characterized by M_{107} - from the standard value and they could be created at quantum criticality in the phase transition believed in QCD framework to correspond de-confinement of quarks so that the M_{89} hadrons would have sizes of ordinary hadrons.

   M^8-H duality strongly suggests that the $SO(4)=SU(2)_L\times SU(2)_R$ as symmetries in the old-fashioned non-perturbative description of hadrons and $SU(3)$ as description in terms of quarks and gluons are related by this duality. This leads for instance to a proposal of deeper formulation of the skyrmion concept applied both in nuclear and hadron physics: skyrmion number as winding number would reduce to winding number n of polynomial of complex number having degree n.

2. Lepto-pion hypothesis was motivated by the observation of pseudo-scalar states in collisions of heavy nuclei near Coulomb threshold ad decaying to electron-positron pairs. These states would be pion like states formed by colored variants of charged leptons [K4] (for details see the online article [K7]).

3. There are as many as 4 Gaussian Mersennes $M_{k,G} = (1+)^k - 1$, $k = 151, 157, 163, 167$ spanning p-adic length scales in the range 10 nm (cell membrane thickness) and 2.5 μ (size of cell nucleus). The idea that these length scales define copies of hadron physics and electroweak physics. These scales could correspond to dark length scales for ordinary hadrons and weak bosons - weak bosons and gluons would behave like massless particles below these scales.

4. Nuclear string model [K4] (for details see the online article [K6]) is an early application of magnetic flux tube concept. “Cold fusion” could be understood in terms of dark nuclei with h_{eff} such that proton Compton length is replaced with that of electron [L20, L29]. This leads to a re-interpretation [L46] of tunnelling phenomenon assumed to take place in nuclear reactions as a phase transition to dark nuclear phase and suggest pre-stellar evolution was preceded by a phase of dark nuclei with ”cold fusion” heating the matter to a temperature at which nuclear reactions start.

5. The 10 year old anomaly of solar nuclear physics [E1, E3] means that the nuclear abundances inside Sun deduced from solar seismology are higher than those deduced at the surface of the Sun and from meteorites. TGD based model assumes that tunnelling effect in nuclear reaction involves formation of dark nuclei in the proposed sense as intermediate states so that dark fusion would be essential element of nuclear physics. The explanation would be that nuclei can be in both ordinary and dark states inside Sun and dark nuclei are visible via seismology. The high temperature excites the dark nuclei: the increase of h_{eff} indeed requires energy and this explains why also ”cold fusion” requires energy feed.

4.3 Living matter, biochemistry, and consciousness

The model for living matter relies heavily on the notions of MB carrying $h_{eff} > h$ phases behaving like dark matter and ZEO.

4.3.1 ZEO based quantum measurement theory extends to a theory of consciousness

ZEO based quantum measurement theory [L58] leads to a quantum theory of consciousness (see **Fig. 9**)by lifting the observer from an outsider to part of physical system. In particular, the theory predicts that the arrow of time changes in "big" (ordinary) state function reductions (BSFRs) as opposed to "small" SFRs (SSFRs) as the counterparts of weak measurements (see **Fig. 10**).

This suggests that self-organization in all scales reduces to dissipation with reversed arrow of time. The energies of states increase with h_{eff} and h_{eff} tends to be reduced spontaneously. This means that energy feed is needed to preserved the distribution for h_{eff}: in biology this corresponds to metabolic energy feed. The energy feed necessary for self-organization would reduce to dissipation of self-organizing system in reversed time direction. Dark matter at MB of the system would serve as a master controlling the ordinary matter serving in the role of slave. Note that there would be master-slave hierarchy of MBs ordered by h_{eff}.

This would happen at magnetic and have dramatic implications. Time reversed dissipation looks like energy feed from the environment to system. Self-organization involves always energy feed and generation of structures rather than their disappearance in apparent conflict with second law. Self-organization would correspond to dissipation in reversed time direction implied by generalized second law. No specific mechanisms would be required and only metabolic energy storages- systems able to receive the energy dissipated in reversed time direction - are enough. Obviously this provides a totally new vision about energy technology.

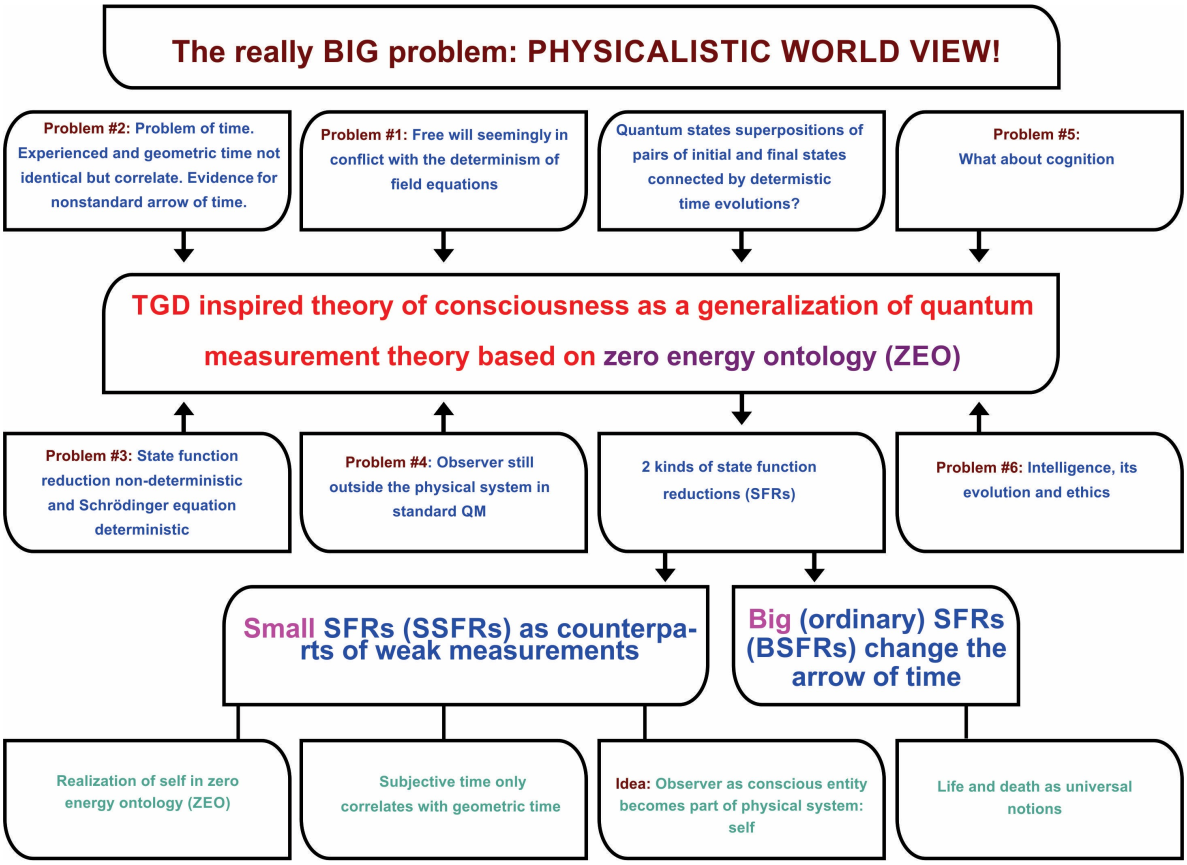

Figure 9: Consciousness theory from quantum measurement theory

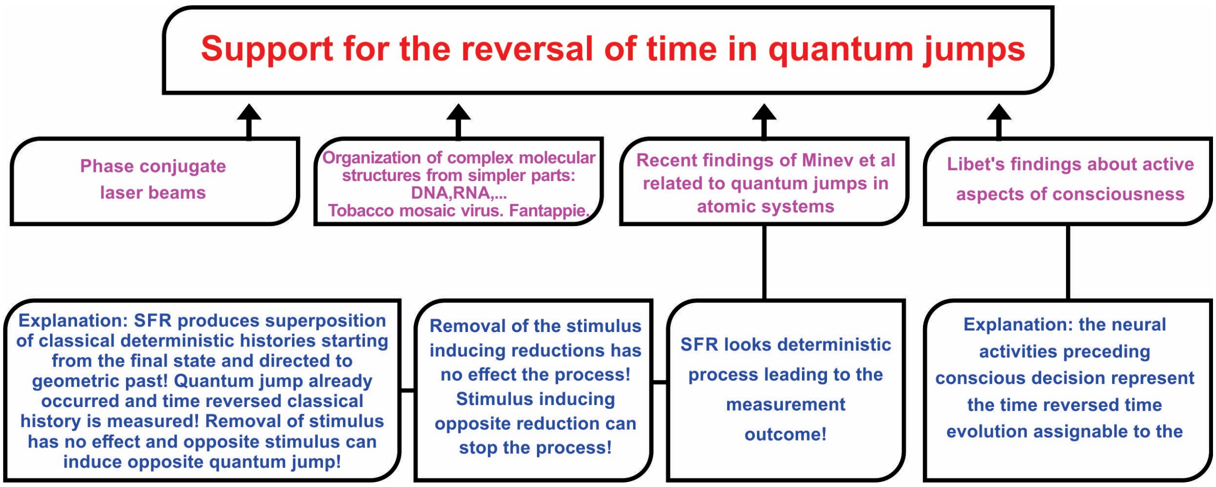

Figure 10: Time reversal occurs in BSFR

4.3.2 The notion of magnetic body

Magnetic body (MB) would carrying dark matter would serve as the boss controlling ordinary matter at flux tubes.

1. MB has as building bricks magnetic flux quanta. Typically flux tubes and flux sheets. It consists of two kinds of flux quanta. Flux can be vanishing, which corresponds to Maxwellian case. The flux can be also non-vanishing and quantized and corresponds to monopole flux. In monopole case magnetic field requires no current to create it. This option is not possible in Maxwellian world. These flux tubes play a key role in TGD Universe in all scales.

2. Also Earth's magnetic field with nominal value $B_E = .5$ Gauss would have these two parts. Monopole part corresponds to the "endogenous" magnetic field $B_{end} = .2$ Gauss explaining strange effects of ELF em radiation to the physiology and behavior of vertebrates [J1]. The presence of this part identifiable as monopole flux explains why Earth has magnetic field: this field should have decayed long time ago in Maxwellian world since it requires currents to generate it and they disappear. Magnetic fields of permanent magnets could have a monopole part consisting of flux quanta. Electromagnets would not have it.

3. MB would carry dark matter as $h_{eff} = n \times h_0$ phases and act as a "boss" controlling ordinary matter [L42]. Communication to and control of biological body (ordinary matter) would be based on dark photons, which can transform to ordinary photons and vice versa. Molecular transitions would be one form of control.

4. Dark photons with large h_{eff} serve as as communication and control tools. Josephson frequencies would be involved with the communication of sensory data to MB and cy-

clotron frequencies with control by MB. Dark photons are assumed to transform to bio-photons [L15, L14] with energies covering visible and UV associated with the transitions of bio-molecules. The control by MB which layers having size even larger than that of Earth means that remote mental interactions are routine in living matter. EEG would be a particular example of these communications: without MB it is difficult to understand why brain would use such large amounts of energy to send signals to outer space.

5. The experiments of Blackman and others led originally to the notion of h_{eff} hierarchy. The large effects of radiation at ELF frequencies could be understood iin terms of cyclotron transitions in $B_{end} = .2$ Gauss if the value of h in $E = hf$ is replaced with h_{eff}, which would be rather large and possibly assignable to gravitational flux tubes with $\hbar_{eff} = \hbar_{gr} = GMm/v_0$.

   MB would control BB by cyclotron radiation - possibly via genome accompanied by dark genome at flux tubes parallel to the DNA strands. Cyclotron Bose-Einstein condensates of bosonic ions, Cooper pairs of fermionic ions, and Cooper pairs of protons and electrons would appear in living matter and $h_{eff} = h_{gr}$ hypothesis predicts universal energy spectrum in the range of bio-photon energies.

   Cell membrane could act as generalized Josephson junction generating dark Jophson radiation with energies given by the sum for ordinary Josephson energy and of the difference of cyclotron energies for flux tubes at the two sides of the membrane. The variation of the membrane potential would induce variation of the Josephson frequency and code the sensory information at cell membrane to a dark photon signal sent to MB.

6. In ZEO field body and MB correspond to 4-D rather than 3-D field patterns. Quantum states are replaced by quantum counterparts of behaviors and biological functions. The basic mechanism used by MB would be generation of conscious holograms by using dark photon reference beams from MB and their reading. In ZEO also the time reversals of these processes are possible and make possible to understand memory as communications with geometric past. Sensory perception and memory recall would be time reversals of each other and correspond to sequences of SSRs. Motor action would correspond to BSRs.

4.3.3 Life is not mere chemistry

The dogma about biology as mere bio-chemistry is given up in TGD framework.

1. Bio-catalysis remains a mystery in bio-chemical approach. MB carrying dark matter could provide the needed mechanisms.

   According to TGD view about catalysis, the U-shaped flux tubes associated with the MBs of reactants reconnect to a pair of flux tubes connecting the molecules [L36]. This happens if there is cyclotron resonance for dark cyclotron radiation assignable to massless extremals (MEs) associated with U-shaped flux tubes. This requires that the flux tubes have same strength of magnetic field and therefore same thickness by flux quantization. The same value of h_{eff} guarantees resonance. The next step is the shortening of the flux tubes by a reduction of h_{eff} and liberating energy kicking the reactants over the potential wall making the process extremely slow otherwise.

2. Also valence bonds and hydrogen bonds could correspond to magnetic flux tubes characterized by $h_{eff} = h_{em} = n \times h_0$, where n is now rather small number ($h = 6h_0$). This leads to a model for valence bond energies of atom with n increasing as one moves to right along the row of the periodic table providing insights to the biological roles of various molecules in biology [L30]. For instance, the molecules involving atoms towards right end of the periodic table would be natural carriers of metabolic energy whereas at the left end of row would be naturally involved with biocontrol via cyclotron frequencies.

3. The physics of water is full of anomalies [I5]. TGD suggests an explanation [L33] in terms of flux tubes assignable to hydrogen bonds [L33, L35]. These flux tubes could correspond also to values of $h_{eff} > h$ so that these flux tube could be long and give rise to long range quantal correlations. Water could be seen as a manyphase system. The MBs assignable to

water molecule clusters could mimick the cyclotron frequency spectrum of invader molecules and make possible water memory and primitive immune system based on reconnections of U-shaped flux tubes of water cluster and invader molecule [L56]. In this framework water would represent a primitive life form.

In Pollack effect [I4] exclusion zones (EZs) are induced at the boundary between gel phase and water by energy feed such as IR radiation. The negative charge of EZs is explained as a formation of flux tubes carrying dark protons having interpretation as dark nuclei. A simple model for linear dark proton triplets predicts their states to be in 1-1 correspondence with DNA, RNA, tRNA, and amino-acids and the numbers of codons coding for given amino-acid are predicted to be same as for vertebrate genetic code [L34, L41]. The model thus predicts deep connections between nuclear physics, condensed matter physics, chemistry, and biology usually thought to be rather disjoin disciplines.

EZs are able remove impurities from interior in conflict with second law. TGD based explanation of the mystery is change of the arrow of time induced by TGD counterpart of ordinary state function reduction in ZEO) [L58]: self-organization would be dissipation with reversed arrow of time at at the magnetic body (MB) of system acting as master and forcing time reversed evolution at the level of ordinary bio-matter serving as a slave.

DNA has one negative charge per nucleotide, microtubules are negatively charged, also cell is negatively charged, ATP carries 3 units of negative charge. This together with ZEO suggests that Pollack effect plays a key role in bio-control and macroscopic SFRs play a key role in living matter.

4.4 About possible technological applications

Dark nuclei with small binding energy created from protons by analog of Pollack effect and spontaneously decaying to ordinary nuclei liberating almost the ordinary nuclear binding energy could make possible "cold fusion" based new energy technology and also allow to artificial produce various elements [L57].

ZEO allows to imagine several applications.

1. A new energy technology in which system gets energy by effectively sending negative energy to a system able to receive it. Also quantum communications involved time reversal able to overcome the barrier due to the finite light-velocity can be imagined. Biology could actually use both these mechanisms.

2. Artificial life able to evolve spontaneously relying ZEO based view about self-organization would be one application. There is evidence that biological systems are quantum critical [I1]. Quantum criticality is universal feature of TGD Universe. Quantum critical systems [L17] would be accompanied by large h_{eff} phases at the level of MB, ordinary criticality plus suitable energy feed would be the prerequisites.

3. Magnetic flux tubes acting as braids make possible topological quantum computation using dark matter and flux tubes [L12, L13, L11]. Large values of h_{eff} could solve the basic problem of standard quantum computation due to too small value of h leading to a rapid loss of quantum coherence. ZEO suggests a new view about quantum computation like activity utilizing time reversal to perform computation going forth and back in time implying evolution of the computing system.

5 Comparison of TGD with other theories

Table 1 compares GRT and TGD and **Table 2** compares standard model and TGD.

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	GRT	**TGD**
Scope of geometrization	classical gravitation	all interactions and quantum theory
Spacetime		
Geometry	abstract 4-geometry	sub-manifold geometry
Topology	trivial in long length scales	many-sheeted space-time
Signature	Minkowskian everywhere	also Euclidian
Fields		
classical	primary dynamical variables	induced from the geometry of H
Quantum fields	primary dynamical variables	modes of WCW spinor fields
Particles	point-like	3-surfaces
Symmetries		
Poincare symmetry	lost	Exact
GCI	true	true - leads to SH and ZEO
	Problem in the identication of coordinates	$H = M^4 \times CP_2$ provides preferred coordinates
Super-symmetry	super-gravitation	super variant of H: super-surfaces
Dynamics		
Equivalence Principle	true	true
Newton's laws and		
notion of force	lost	generalized
Einstein's equations	from GCI and EP	remnant of Poincare invariance at QFT limit of TGD
Bosonic action	EYM action	Kähler action + volume term
Cosmological constant	suggested by dark energy	length scale dependent coefficient of volume term
Fermionic action	Dirac action	Modified Dirac action for induced spinors
Newton's constant	given	predicted
Quantization	fails	Quantum states as modes of WCW spinor field

Table 1: Differences and similarities between GRT and TGD

	SM	TGD
Symmetries		
Origin	from empiria	reduction to CP_2 geometry
Color symmetry	gauge symmetry	isometries of CP_2
Color	analogous to spin	analogous to angular momentum
Ew symmetry	gauge symmery	holonomies of CP_2
Symmetry breaking	Higgs mechanism	CP_2 geometry
Spectrum		
Elementary particles	fundamental	consist of fundamental fermions
Bosons	gauge bosons, Higgs	gauge bosons, Higgs, pseudo-scalar
Fundamental fermions	quarks and leptons	quarks: leptons as local 3-quark composites
Dynamics		
Degrees of freedom	gauge fields, Higgs, and fermions	3-D surface geometry and spinors
Classical fields	gauge fields, Higgs	induced spinor connection
	SU(3) Killing vectors of CP_2	
Quantal degrees of freedom	gauge bosons,Higgs,	quantized induced spinor fields
Massivation	Higgs mechanism	p-adic thermodynamics with superconformal symmetry

Table 2: Differences and similarities between standard model and TGD

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