

Does thermodynamics have a representation at the level of space-time geometry?

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Contents

1	Introduction	2
2	Motivations and background	2
2.1	ZEO and the need for space-time correlates for the square root of thermodynamics . . .	3
2.2	Preferred extremals of Kähler action	3
3	Kiehn's topological thermodynamics (TTD)	4
4	Attempt to identify TTD in TGD framework	5
4.1	The role of symplectic transformations	5
4.2	Identification of basic 1-forms of TTD in TGD framework	5
4.3	Instanton current, instanton density, and irreversibility	6
4.4	Also Kähler current and isometry currents are needed	7
4.5	Questions	7

Abstract

R. Kiehn has proposed what he calls Topological Thermodynamics (TTD) as a new formalism of thermodynamics. The basic vision is that thermodynamical equations could be translated to differential geometric statements using the notions of differential forms and Pfaffian system. That TTD differs from TGD by a single letter is not enough to ask whether some relationship between them might exist. Quantum TGD can however in a well-defined sense be regarded as a square root of thermodynamics in zero energy ontology (ZEO) and this leads leads to ask seriously whether TTD might help to understand TGD at deeper level. The thermodynamical interpretation of space-time dynamics would obviously generalize black hole thermodynamics to TGD framework and already earlier some concrete proposals have been made in this direction.

This raises several questions. Could the preferred extremals of Kähler action code for the square root of thermodynamics? Could induced Kähler gauge potential and Kähler form (essentially Maxwell field) have formal thermodynamic interpretation? The vacuum degeneracy of Kähler action implies 4-D spin glass degeneracy and strongly suggests the failure of strict determinism for the dynamics of Kähler action for non-vacuum extremals too. Could thermodynamical irreversibility and preferred arrow of time allow to characterize the notion of preferred extremal more sharply?

It indeed turns out that one can translate Kiehn's notions to TGD framework rather straightforwardly. Kiehn's work 1- form corresponds to induced Kähler gauge potential implying that the vanishing of instanton density for Kähler form becomes a criterion of reversibility and irreversibility is localized on the (4-D) "lines" of generalized Feynman diagrams, which correspond to space-like signature of the induced metric. Heat produced in given generalized Feynman diagram is just the integral of instanton density and the condition that the arrow of geometric time has definite sign classically fixes the sign of produced heat to be positive. In this picture the preferred extremals of Kähler action would allow a trinity of interpretations as non-linear Maxwellian dynamics, thermodynamics, and integrable hydrodynamics.

1 Introduction

R. Kiehn has proposed what he calls Topological Thermodynamics (TTD) [2] as a new formulation of thermodynamics. The basic vision is that thermodynamical equations could be translated to differential geometric statements using the notions of differential forms and Pfaffian system [1]. That TTD differs from TGD by a single letter is not enough to ask whether some relationship between them might exist. Quantum TGD can however in a well-defined sense be regarded as a square root of thermodynamics in zero energy ontology (ZEO) and this leads leads to ask seriously whether TTD might help to understand TGD at deeper level. The thermodynamical interpretation of space-time dynamics would obviously generalize black hole thermodynamics to TGD framework and already earlier some concrete proposals have been made in this direction.

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It indeed turns out that one can translate Kiehn's notions to TGD framework rather straightforwardly.

1. Kiehn's work 1- form corresponds to induced Kähler gauge potential implying that the vanishing of instanton density for Kähler form becomes a criterion of reversibility and irreversibility is localized on the (4-D) "lines" of generalized Feynman diagrams, which correspond to space-like signature of the induced metric. The localization of heat production to generalized Feynman diagrams conforms nicely with the kinetic equations of thermodynamics based on reaction rates deduced from quantum mechanics. It also conforms with Kiehn's vision that dissipation involves topology change.
2. Heat produced in a given generalized Feynman diagram is just the integral of instanton density and the condition that the arrow of geometric time has definite sign classically fixes the sign of produced heat to be positive. In this picture the preferred extremals of Kähler action would allow a trinity of interpretations as non-linear Maxwellian dynamics, thermodynamics, and integrable hydrodynamics.
3. The 4-D spin glass degeneracy of TGD breaking of ergodicity suggests that the notion of global thermal equilibrium is too naive. The hierarchies of Planck constants and of p-adic length scales suggests a hierarchical structure based on CDs withing CDs at imbedding space level and space-time sheets topologically condensed at larger space-time sheets at space-time level. The arrow of geometric time for quantum states could vary for sub- CDs and would have thermodynamical space-time correlates realized in terms of distributions of arrows of geometric time for sub- CDs , sub-sub- CDs , etc...

The hydrodynamical character of TGD means that field equations reduce to local conservation laws for isometry currents and Kähler gauge current. This requires the extension of Kiehn's formalism to include besides forms and exterior derivative also induced metric, index raising operation transforming 1-forms to vector fields, duality operation transforming k- forms to n-k forms, and divergence which vanishes for conserved currents.

2 Motivations and background

It is good to begin by discussing the motivations for the geometrization of thermodynamics and by introducing the existing mathematical framework identifying space-time surfaces as preferred extremals of Kähler action [2].

2.1 ZEO and the need for space-time correlates for the square root of thermodynamics

Quantum classical correspondence is basic guiding principle of quantum TGD. In ZEO TGD can be regarded as a complex square root of thermodynamics so that the thermodynamics should have correlates at the level of the geometry of space-time.

1. Zero energy states consist of pairs of positive and negative energy states assignable to opposite boundaries of a causal diamond (CD). There is entire hierarchy of CD s characterized by their scale coming as an integer multiple of a basic scale (also their Poincare transforms are allowed).
2. In ZEO zero energy states are automatically time-irreversible in the sense that either end of the causal diamond (CD) corresponds to a state consisting of single particle states with well-defined quantum numbers. In other words, this end of CD carries a prepared state. The other end corresponds to a superposition of states which can have even different particle numbers: this is the case in particle physics experiment typically. State function reduction reduces the second end of CD to a prepared state. This process repeats itself. This suggests that the arrow of time or rather, its geometric counterpart which we experience, alternates. This need not however be the case if quantum classical correspondence holds true.
3. To illustrate what I have in mind consider a path towel, which has been been folded forth and back. Assume that the direction in which folding is carried is time direction. Suppose that the inhabitant of bath towel Universe is like the habitant of the famous Flatland and therefore not able to detect the folding of the towel. If the classical dynamics of towel is time irreversible (time corresponds to the direction in which the folding takes place), the inhabitant sees ever lasting irreversible time evolution with single arrow of geometric time identified as time coordinate for the towel: no changes in the arrow of geometric time. If the inhabitant is able to make measurements about 3-D space the situation he or she might be able to see that his time evolution takes place forth and back with respect to the time coordinate of higher-dimensional imbedding space.
4. One might understand the arrow of time - albeit differently as in normal view about the situation - if classical time evolution for the preferred extremals of Kähler action defines a geometric correlate for quantum irreversibility of zero energy states. There are of course other space-time sheets and other CD s present and it might be possible to detect the alternation of the arrow of geometric time at imbedding space level by making measurements giving information about their geometric arrows of time [1].

By quantum classical correspondence one expects that the geometric arrow of time - irreversibility - for zero energy states should have classical counterparts at the level of the dynamics of preferred extremals of Kähler action. What could be this counterpart? Thermodynamical evolution by quantum jumps does not obey ordinary variational principle that would make it deterministic: Negentropy Maximization Principle (NMP) [6] for statefunction reductions of system is analogous to Second Law for an ensemble of copies of system and actually implies it. Could one mimic irreversibility by single classical evolution defined by a preferred extremal? Note that the dynamics of preferred extremals is not actually strictly deterministic in the ordinary sense of the word: the reason is the enormous vacuum degeneracy implying 4-D spin glass degeneracy [2]. This makes it possible to mimic not only quantum states but also sequences of quantum jumps by piece-wise deterministic evolution.

2.2 Preferred extremals of Kähler action

In Quantum TGD the basic arena of quantum dynamics is "world of classical worlds" (WCW) [8]. Purely classical spinor fields in this infinite-dimensional space define quantum states of the Universe. General Coordinate Invariance (GCI) implies that classical worlds can be regarded as either 3-surfaces or 4-D space-time surfaces analogous to Bohr orbits. Strong form of GCI implies in ZEO strong form of holography in the sense that the points of WCW effectively correspond to collections of partonic 2-surfaces belonging to both ends of causal diamonds (CD s) plus their 4-D tangent space-time data.

Kähler geometry reduces to the notion of Kähler function [5] and by quantum classical correspondence a good guess is that Kähler function corresponds to so called Kähler action for Euclidian

space-time regions. Minkowskian space-time regions give a purely imaginary to Kähler action (square root of metric determinant is imaginary) and this contribution plays the role of Morse function for WCW. Stationary phase approximation implies that in first the approximation the extremals of the Kähler *function* (to be distinguished from preferred extremals of Kähler *action!*) select one particular 3-surface and corresponding classical space-time surface (Bohr orbit) as that defining "classical physics".

GCI implies holography and holography suggests that action reduces to 3-D terms. This is true if one has $j^\mu A_\mu = 0$ in the interior of space-time. If one assumes so called weak form of electric-magnetic duality [4] at the real and effective boundaries of space-time surface (3-D surfaces at the ends of *CDs* and the light-like 3-surfaces at which the signature of induced 4-metric changes so that 4-metric is degenerate), one obtains a reduction of Kähler action to Chern-Simons terms at the boundaries. TGD reduces to almost topological QFT. "Almost" means that the induced metric does not disappear completely from the theory since it appears in the conditions expressing weak form of electric magnetic duality and in the condition $j^\mu A_\mu = 0$ [2].

The strong form of holography implies effective 2-dimensionality and this suggests the reduction of Chern-Simons terms to 2-dimensional areas of string world sheets and possible of partonic 2-surfaces [2]. This would mean almost reduction to string theory like theory with string tension becoming a dynamic quantity.

Under additional rather general conditions the contributions from Minkowskian and Euclidian regions of space-time surface are apart from the value of coefficient identical at light-like 3-surfaces. At space-like 3-surfaces at the ends of space-time surface they need not be identical.

Quantum classical correspondence suggests that space-time surfaces provide a representation for the square root of thermodynamics and therefore also for thermodynamics. In general relativity black hole thermodynamics suggests the same. This idea is not new in TGD framework. For instance, Hawking-Bekenstein formula for blackbody entropy [1] allows a p-adic generalization in terms of area of partonic 2-surfaces [7]. The challenge is to deduce precise form of this correspondence and here Kiehn's topological thermodynamics might help in this task.

3 Kiehn's topological thermodynamics (TTD)

The basic in the work of Kiehn is that thermodynamics allows a topological formulation in terms of differential geometry.

1. Kiehn introduces also the notions of <http://www22.pair.com/csdc/pdf/irevtors.pdf> Pfaffian system [2] [1] and Pfaff dimension as the number of non-vanishing forms in the sequence for given 1-form such as W or Q : $W, dW, W \wedge dW, dW \wedge dW$. Pfaff dimension $D \leq 4$ tells that one can describe W as sum $W = \sum W_k dx^k$ of gradients of D variables. $D = 4$ corresponds to open system, $D = 3$ to a closed system and $W \wedge dW \neq 0$ defines what can be regarded as a chirality. For $D = 2$ chirality vanishes no spontaneous parity breaking.
2. Kiehn's king idea that Pfaffian systems provide a universal description of thermodynamical reversibility. Kiehn introduces heat 1- form Q . System is thermodynamically reversible if Q is integrable. In other words, the condition $Q \wedge dQ = 0$ holds true which implies that one can write $Q = TdS$: Q allows an integrable factor T and is expressible in terms of the gradient of entropy. $Q = TdS$ condition implies that Q correspond to a global flow defined by the coordinate lines of S . This in turn implies that it is possible define phase factors depending on S along the flow line: this relates to macroscopic quantum coherence for macroscopic quantum phases.
3. The first law expressing the work 1-form W as $W = Q - dU = TdS - dU$ for reversible processes. This gives $dW \wedge dW = 0$. The condition $dW \wedge dW \neq 0$ therefore characterizes irreversible processes.
4. Symplectic transformations are natural in Kiehn's framework but not absolutely essential.

Reader is encouraged to get familiar with Kiehn's examples [2] about the description of various simple thermodynamical systems in this conceptual framework. Kiehn has also worked with the differential topology of electrodynamics and discussed concepts like integrable flows known as Beltrami flows. These flows generalized to TGD framework and are in key role in the construction of proposals

for preferred extremals of Kähler action: the basic idea would be that various conserved isometry currents define Beltrami flows so that their flow lines can be associated with coordinate lines [2].

4 Attempt to identify TTD in TGD framework

Let us now try to identify TTD or its complex square root in TGD framework.

4.1 The role of symplectic transformations

Symplectic transformations are important in Kiehn's approach although they are not a necessary ingredient of it and actually impossible to realize in Minkowski space-time.

1. Symplectic symmetries of WCW induced by symplectic symmetries of CP_2 and light-like boundary of CD are important also in TGD framework [3] and define the isometries of WCW. As a matter of fact, symplectic group parameterizes the quantum fluctuating degrees of freedom and zero modes defining classical variables are symplectic invariants. One cannot assign to entire space-time surfaces symplectic structure although this is possible for partonic 2-surfaces.
2. The symplectic transformations of CP_2 act on the Kähler gauge potential as $U(1)$ gauge transformations formally but modify the shape of the space-time surface. These symplectic transformations are symmetries of Kähler action only in the vacuum sector which as such does not belong to WCW whereas small deformations of vacua belong. Therefore genuine gauge symmetries are not in question. One can of course formally assign to Kähler gauge potential a separate $U(1)$ gauge invariance.
3. Vacuum extremals with at most 2-D CP_2 projection (Lagrangian sub-manifold) form an infinite-dimensional space. Both M^4 diffeomorphisms and symplectic transformations of CP_2 produce new vacuum extremals, whose small deformations are expected to correspond to preferred extremals. This gives rise to 4-D spin glass degeneracy [7] to be distinguished from 4-D gauge degeneracy.

4.2 Identification of basic 1-forms of TTD in TGD framework

Consider next the identification of the basic variables which are forms of various degrees in TTD.

1. Kähler gauge potential is analogous to work 1-form W . In classical electrodynamics vector potential indeed has this interpretation. $dW \wedge dW$ is replaced with $J \wedge J$ defining instanton density ($E_K \cdot B_K$ in physicist's notation) for Kähler form and its non-vanishing - or equivalently 4-dimensionality of CP_2 projection of space-time surface - would be the signature of irreversibility. $dJ = 0$ holds true only locally and one can have magnetic monopoles since CP_2 has non-trivial homology. Therefore the non-trivial topology of CP_2 implying that the counterpart of W is not globally defined, brings in non-trivial new element to Kiehn's theory.
2. Chirality $C - S = A \wedge J$ is essentially Chern-Simons 3-form and in ordinary QFT non-vanishing of $C - S$ if present in action - means parity breaking in ordinary quantum field theories. Now one must be very cautious since parity is a symmetry of the imbedding space rather than that of space-time sheet.
3. Pfaff dimension equals to the dimension of CP_2 projection and has been used to classify existing preferred extremals. I have called the extremals with 4-D CP_2 projection chaotic and so called CP_2 vacuum extremals with 4-D CP_2 projection correspond to such extremals. Massless extremals or topological light rays correspond to $D = 2$ as do also cosmic strings. In Euclidian regions preferred extremals with $D = 4$ are possible but not in Minkowskian regions if one accepts effective 3-dimensionality. Here one must keep mind open.

Irreversibility identified as a non-vanishing of the instanton density $J \wedge J$ has a purely geometrical and topological description in TGD Universe if one accepts effective 3-dimensionality.

1. The effective 3-dimensionality for space-time sheets (holography implied by general coordinate invariance) implies that Kähler action reduces to Chern-Simons terms so that the Pfaff dimension is at most $D = 3$ for Minkowskian regions of space-time surface so that they are thermodynamically reversible.
2. For Euclidian regions (say deformations of CP_2 type vacuum extremals) representing orbits of elementary particles and lines of generalized Feynman diagrams $D = 4$ is possible. Therefore Euclidian space-like regions of space-time would be solely responsible for the irreversibility. This is quite strong conclusion but conforms with the standard quantum view about thermodynamics according to which various particle reaction rates deduced from quantum theory appear in kinetic equations giving rise to irreversible dynamics at the level of ensembles. The presence of Morse function coming from Minkowskian regions is natural since square root of thermodynamics is in question. Morse function is analogous to the action in QFTs whereas Kähler function is analogous to Hamiltonian in thermodynamics. Also this conforms with the square root of TTD interpretation.

4.3 Instanton current, instanton density, and irreversibility

Classical TGD has the structure of hydrodynamics in the sense that field equations are conservation laws for isometry currents and Kähler current. These are vector fields although induced metric allows to transform them to forms. This aspect should be visible also in thermodynamic interpretation and forces to add to the Kiehn's formulation involving only forms and exterior derivative also induced metric transforming 1-forms to vector fields, the duality mapping 4-k forms and k-forms to each other, and divergence operation.

It was already found that irreversibility and dissipation corresponds locally to a non-vanishing instanton density $J \wedge J$. This form can be regarded as exterior derivative of Chern-Simons 3-form or equivalently as divergence of instanton current.

1. The dual of C-S 3-form given by $*(A \wedge J)$ defines what I have called instanton current. This current is not conserved in general and the interpretation as a heat current would be natural. The exterior derivative of C-S gives instanton density $J \wedge J$. Equivalently, the divergence of instanton current gives the dual of $J \wedge J$ and the integral of instanton density gives the analog of instanton number analogous to the heat generated in a given space-time volume. Note that in Minkowskian regions one can multiply instanton current with a function of CP_2 coordinates without losing closedness property so that infinite number similar conserved currents is possible. The heat 3-form is expressible in terms of Chern-Simons 3-form and for preferred extremals it would be proportional to the weight sum of Kähler actions from Minkowskian and Euclidian regions (coefficients are purely imaginary and real in these two regions). Instead of single real quantity one would have complex quantity characterizing irreversibility. Complexity would conform with the idea that quantum TGD is complex square root of thermodynamics.
2. The integral of heat 3-form over effective boundaries associated with a given space-time region define the net heat flow from that region. Only the regions defining the lines of generalized Feynman diagrams give rise to non-vanishing heat fluxes. Second law states that one has $\Delta Q \geq 0$. Generalized second law means at the level of quantum classical correspondence would mean that depending on the arrow of geometric time for zero energy state ΔQ is defined as difference between upper and lower or lower and upper boundaries of CD . This condition applied to CD and sub- CD :s would generalize the conditions familiar from hydrodynamics (stating for instance that for shock waves the branch of bifurcation for which the entropy increases is selected). Note that the field equations of TGD are hydrodynamical in the sense that they express conservation of various isometry currents. The naive picture about irreversibility is that classical dynamics generates CP_2 type vacuum extremals so that the number of outgoing lines of generalized Feynman diagram is higher than that of incoming ones. Therefore that the number of space-like 3-surfaces giving rise to Chern-Simons contribution is larger at the end of CD corresponding to the final (negative energy) state.
3. A more precise characterization of the irreversible states involves several non-trivial questions.

- (a) By the failure of strict classical determinism the condition that for a given CD the number outgoing lines is not smaller than incoming lines need not provide a unique manner to fix the preferred extremal when partonic 2-surfaces at the ends are fixed. Could the arrow of geometric time depend on sub- CD as the model for living matter suggests (recall also phase conjugate light rays)?

In ordinary quantum mechanical approach to kinetic equations also the reactions, which decrease entropy are allowed but their weight is smaller in thermal equilibrium. Could this fact be described as a probability distribution for the arrow of time associated for the sub- CD s, sub-sub- CD s, etc... ? Space-time correlates for quantal thermodynamics would be probability distributions for space-time sheets and hierarchy of sub- CD s.

- (b) 4-D spin glass degeneracy suggests breaking of ergodic hypothesis: could this mean that one does not have thermodynamical equilibrium but very large number of spin glass states caused by the frustration for which induced Kähler form provides a representation? Could these states correspond to a varying arrow of geometric time for sub- CD s? Or could different deformed vacuum extremals correspond to different space-time sheets in thermal equilibrium with different thermal parameters.

4.4 Also Kähler current and isometry currents are needed

The conservation Kähler current and of isometry currents imply the hydrodynamical character of TGD.

1. The conserved Kähler current j_K is defined as 3-form $j_K = *(d * J)$, where $d * J$ is closed 3-form and defines the counterpart of $d * dW$. Field equations for preferred extremals require $*j_K \wedge A = 0$ satisfied if one Kähler current is proportional to instanton current: $*j_K \propto A \wedge J$. As a consequence Kähler action reduces to 3-dimensional Chern-Simons terms (classical holography) and Minkowskian space-time regions have at most 3-D CP_2 projection (Pfaff dimension $D \leq 3$) so that one has $J \wedge J = 0$ and reversibility. This condition holds true for preferred extremals representing macroscopically the propagation of massless quanta but not Euclidian regions representing quanta themselves and identifiable as basic building bricks of wormhole contacts between Minkowskian space-time sheets.
2. A more general proposal is that all conserved currents transformed to 1-forms using the induced metric (classical gravitation comes into play!) are integrable: in other words, one has $j \wedge dj = 0$ for both isometry currents and Kähler current. This would mean that they are analogous to heat 1-forms in the reversible case and therefore have a representation analogous to $Q = TdS$, $W = PdV$, μdN and the coordinate along flowline defines the analog of S , V , or N (note however that dS, dV, dN would more naturally correspond to 3-forms than 1-forms, see below) A stronger form corresponds to the analog of hydrodynamics for one particle species: all one-forms are proportional (by scalar function) to single 1-form which is $A \wedge J$ (all quantum number flows are parallel to each other).

4.5 Questions

There are several questions to be answered.

1. In Darboux coordinates in which one has $A = P_1 dQ^1 + P_2 dQ^2$. The identification of A as counterpart for $W = PdV - \mu dN$ comes first in mind. For thermodynamical equilibria one would have $TdS = dU + W$ translating to $TdS = dU + A$ so that Q for reversible processes would be apart from $U(1)$ gauge transformation equal to the Kähler gauge potential. Symplectic transformations of CP_2 generate $U(1)$ gauge transformations and dU might have interpretation in terms of energy flow induced by this kind of transformation. Recall however that symplectic transformations are not symmetries of space-time surfaces but only of the WCW metric and act on partonic 2-surfaces and their tangent space data as such.
2. Does the conserved Kähler current j_K have any thermodynamical interpretation? Clearly the counterparts of conserved (and also non-conserved quantities) in Kiehn's formulation would be 3-forms with vanishing curl $d(*j_K) = 0$ in conserved case. Therefore it seems impossible to

reduce them to 1-forms unless one introduces divergence besides exterior derivative as a basic differential operation.

The hypothesis that the flow lines of these 1-forms associated with j_K vector field are integrable implies that they are gradients apart from the presence of integrating factor. Reduction to a gradient ($j = dU$) means that U satisfies massless d'Alembert equation $d * dU = 0$. Note that local polarization and light-like momentum are gradients of scalar functions which satisfy massless d'Alembert equation for the Mikowskian space-time regions representing propagating of massless quanta.

3. In genuinely 3-dimensional context S, V, N are integrals of 3-forms over 3-surfaces for some current defining 3-form. This is in conflict with Kiehn's description where they are 0-forms. One can imagine three cures and first two ones look
 - (a) The integrability of the flows allows to see them as superposition of independent 1-dimensional flows. This picture would make it natural to regard the TGD counterparts of S, V, N as 0-forms rather than 2-forms. This would also allow to deduce $J \wedge J = 0$ as a reversibility condition using Kiehn's argument.
 - (b) Unless one requires integrable flows, one must consider the replacement of $Q = TdS$ resp. $W = PdV$ resp. μdN $Q = TdS$ resp. $W = PdV$ resp. μdN where $W, Q, dS, dV, \text{ and } dN$ with 3-forms. So that S, V, N would be 2-forms and the 3-integrals of dS, dV, dN over 3-surfaces would reduce to integrals over partonic 2-surfaces, which is of course highly non-trivial but physically natural implication of the effective 2-dimensionality. First law should now read as $*W = T*dS - *dU$ and would give $d*W = dT \wedge *dS + Td*dS + d*dU$. If S and U as 2-forms satisfy massless d'Alembert equation, one obtains $d*W = dT \wedge *dS$ giving $d*W \wedge d*W = 0$ as the reversibility condition. If one replaces $W \leftrightarrow A$ correspondence with $*W \leftrightarrow A$ correspondence, one obtains the vanishing of instanton density as a condition for reversibility. For the preferred extremals having interpretation as massless modes the massless d'Alembert equations are satisfied and it might that this option makes sense and be equivalent with the first option.
 - (c) In accordance with the idea that finite measurement resolution is realized at the level of modified Dirac equation, its solutions at lightlike 3-surfaces reduces to solutions restricted to lines connecting partonic 2-surfaces. Could one regard $W, Q, dS, dV, \text{ and } dN$ as singular one-forms restricted to these lines? The vanishing of instanton density would be obtained as a condition for reversibility only at the braid strands, and one could keep the original view of Kiehn. Note however that the instanton density could be non-vanishing elsewhere unless one develops a separate argument for its vanishing. For instance, the condition that isometries of imbedding space say translations produce braid ends points for which instanton density also vanishes for the reversible situation might be enough.

To sum up, it seems that TTD allows to develop considerable insights about how classical space-time surfaces could code for classical thermodynamics. An essential ingredient seems to be the reduction of the hydrodynamical flows for isometry currents to what might be called perfect flows decomposing to 1-dimensional flows with conservation laws holding true for individual flow lines. An interesting challenge is to find expressions for total heat in terms of temperature and entropy. Blackhole-elementary particle analogy suggest the reduction as well as effective 2-dimensionality suggest the reduction of the integrals of Chern-Simons terms defining total heat flux to two 2-D volume integrals over string world sheets and/or partonic 2-surfaces and this would be quite near to Hawking-Bekenstein formula.

Books related to TGD

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