Some fresh ideas about twistorialization of TGD

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Abstract

The reading of the article of Tim Adamo and the recent work of Nima Arkani Hamed and Jaroslav Trnka has inspired a fresh look on twistors and a possible answer to several questions (I have written two chapters about twistors and TGD giving a view about development of ideas).

Both M^4 and CP_2 are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg M^4 as a Cartesian factor. Also CP_2 allows twistor space identifiable as flag manifold $SU(3)/U(1) \times U(1)$ as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in $M^4 \times CP_2$. Both octonionic interpretation of M^8 and triality possible in dimension 8 play a crucial role in the proposed twistorialization of $H = M^4 \times CP_2$. It also turns out that $M^4 \times CP_2$ allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like M^4 projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in M^4 sense. Furthermore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold $CP_3 \times SU(3)/U(1) \times U(1)$.

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via superconformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for $M^4 \times CP_2$ serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of CP_2 twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

1. For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor $1/p^2$ assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.

2. This as also the fact that physical particles are necessarily pairs of wormhole contacts connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused

by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators $G, G^\dagger$ and L bringing in CP_2 scale appear in propagator lines: most importantly, the fact that G and $G^\dagger$ carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has $1/G$ and $1/G^\dagger$ at its ends. Twistorialization applies because all fermion lines are light-like.

3. A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that this happens markedly only in very short length scales comparable to CP_2 scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting CP_2 scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

Perhaps it is not exaggeration to say that the architecture of generalized Feynman diagrams and their connection to twistor approach is now reasonably well-understood. There are of course several problems to be solved. On must feed in p-adic thermodynamics for external particles (here zero energy ontology might be highly relevant). Also the description of elementary particle families in terms of elementary particle functionals in the space of conformal equivalence classes of partonic 2-surface must be achieved.

Contents

1 Introduction

I found from web a thesis by Tim Adamo titled "Twistor actions for gauge theory and gravity" [B4]. The work considers formulation of $N = 4$ SUSY gauge theory directly in twistor space instead of Minkowski space. The author is able to deduce MHV formalism, tree level amplitudes, and planar loop amplitudes from action in twistor space. Also local operators and null polygonal Wilson loops can be expressed twistorially. This approach is applied also to general relativity: one of the challenges is to deduce MHV amplitudes for Einstein gravity. The reading of the article inspired a fresh look on twistors and a possible answer to several questions (I have written two chapters about twistors and TGD [K9, K11] giving a view about development of ideas).

Both M^4 and CP_2 are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg M^4 as a Cartesian factor. Also CP_2 allows twistor space identifiable as flag manifold $SU(3)/U(1) \times U(1)$ as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in $M^4 \times CP_2$. Both octonionic interpretation of M^8 and triality possible in dimension 8 play a crucial role in the proposed twistorialization of $H = M^4 \times CP_2$. It also turns out that $M^4 \times CP_2$ allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like M^4 projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in M^4 sense. Furthermore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold $CP_3 \times SU(3)/U(1) \times U(1)$.

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2 Basic results and problems of twistor approach

The author describes both the basic ideas and results of twistor approach as well as the problems.

2.1 Basic results

There are three deep results of twistor approach besides the impressive results which have emerged after the twistor resolution.

1. Massless fields of arbitrary helicity in 4-D Minkowski space are in 1-1 correspondence with elements of Dolbeault cohomology in the twistor space CP_3. This was already the discovery of Penrose..The connection comes from Penrose transform. The light-like geodesics of M^4 correspond to points of 5-D submanifold of CP_3 analogous to light-cone boundary. The points of M^4 correspond to complex lines (Riemann spheres) of the twistor space CP_3: one can imagine that the point of M^4 corresponds to all light-like geodesics emanating from it and thus to a 2-D surface (sphere) of CP_3. Twistor transform represents the value of a massless field at point of M^4 as a weighted average of its values at sphere of CP_3. This correspondence is formulated between open sets of M^4 and of CP_3. This fits very nicely with the needs of TGD since causal diamonds which can be regarded as open sets of M^4 are the basic objects in zero energy ontology (ZEO).

2. Self-dual instantons of non-Abelian gauge theories for SU(n) gauge group are in one-one correspondence with holomorphic rank-N vector bundles in twistor space satisfying some additional conditions. This generalizes the correspondence of Penrose to the non-Abelian case. Instantons are also usually formulated using classical field theory at four-sphere S^4 having Euclidian signature.

3. Non-linear gravitons having self-dual geometry are in one-one correspondence with spaces obtained as complex deformations of twistor space satisfying certain additional conditions. This is a generalization of Penrose's discovery to the gravitational sector.

Complexification of M^4 emerges unavoidably in twistorial approach and Minkowski space identified as a particular real slice of complexified M^4 corresponds to the 5-D subspace of twistor space in which the quadratic form defined by the SU(2,2) invariant metric of the 8-dimensional space giving twistor space as its projectivization vanishes. The quadratic form has also positive and negative values with its sign defining a projective invariant, and this correspond to complex continuations of M^4 in which positive/negative energy parts of fields approach to zero for large values of imaginary part of M^4 time coordinate.

Interestgingly, this complexification of M^4 is also unavoidable in the number theoretic approach to TGD: what one must do is to replace 4-D Minkowski space with a 4-D slice of 8-D complexified quaternions. What is interesting is that real M^4 appears as a projective invariant consisting of light-like projective vectors of C^4 with metric signature (4,4). Equivalently, the points of M^4 represented as linear combinations of sigma matrices define hermitian matrices.

2.2 Basic problems of twistor approach

The best manner to learn something essential about a new idea is to learn about its problems. Difficulties are often put under the rug but the thesis is however an exception in this respect. It starts directly from the problems of twistor approach. There are two basic challenges.

1. Twistor approach works as such only in the case of Minkowski space. The basic condition for its applicability is that the Weyl tensor is self-dual. For Minkowskian signature this leaves only Minkowski space under consideration. For Euclidian signature the conditions are not quite so restrictive. This looks a fatal restriction if one wants to generalize the result of Penrose to a general space-time geometry. This difficlty is known as "googly" problem.

   According to the thesis MHV construction of tree amplitudes of $\mathcal{N} = 4$ SYM based on topological twistor strings in CP_3 meant a breakthrough and one can indeed understand also have analogs of non-self-dual amplitudes. The problem is however that the gravitational theory assignable to topological twistor strings is conformal gravity, which is generally regarded as non-physical. There have been several attempts to construct the on-shell scattering amplitudes of Einstein's gravity theory as subset of amplitudes of conformal gravity and also thesis considers this problem.

2. The construction of quantum theory based on twistor approach represents second challenge. In this respect the development of twistor approach to $\mathcal{N} = 4$ SYM meant a revolution and one can indeed construct twistorial scattering amplitudes in M^4.

3 TGD inspired solution of the problems of the twistor approach

TGD suggests an alternative solution to the problems of twistor approach. Space-times are 4-D surfaces of $M^4 \times CP_2$ so that one obtains automatically twistor structure at the level of M^4 - that is imbedding space.

It seems natural to interpret twistor structure from the point of view of Zero Energy Ontology (ZEO). The two tips of CD are accompanied by light-cone boundaries and define a pair of 2-spheres in CP_3 since the light-like rays associated with the tips are mapped to points of twistor space. M^4 coordinates for the tips serve as moduli for the space of CDs and can be mapped to pairs of twistor spheres. The points of partonic 2-surfaces at the boundaries of CD reside at light-like geodesics and the conformal invariance with respect to radial coordinate emanating from the tip of CD suggests that the position at light-like geodesic does not matter. Therefore the points of partonic 2-surfaces can be mapped to union of spheres of twistor space.

3.1 Twistor structure for space-time surfaces?

Induction procedure is the core element of sub-manifold gravity. Could one induce the the twistor structure of M^4 to the space-time surface? Would it have any useful function? This idea does not look attractive.

1. Twistor structure assigns to a given point of M^4 a sphere of CP_3 having interpretation as a sphere parametrizing the light-like geodesics emanating from the point. The X^4 counterpart of this assignment would be obtained simply by mapping the M^4 projection of space-time point to a sphere of twistor space in standard manner. This could make sense if the M^4 projection of space-time surface 4-dimensional but not necessary when the M^4 projection is lower-dimensional - say for cosmic strings.

2. Twistor structure assigns to a light-like geodesic of M^4 a point of CP_3. Should one try to generalize this correspondence to the light-like geodesics of space-time surface? Light-like geodesic corresponds to its light-like tangent vectors at x whose direction as imbedding space vector depends now on the point x of the geodesic. The M^4 projection for the tangent vector of light-like geodesics of space-time surface in general time-like vector of M^4 so that one should map time-like M^4 ray to CP_3. Twistor spheres associated with the two points of this geodesic do not intersect so that one cannot define the image point in CP_3 as an intersection of twistor spheres. One could consider the lifts of the light-like geodesics of M^4 to X^4 and map their M^4 projections to the points of CP_3? This looks however somewhat trivial and physically uninteresting.

3.2 Could one assign twistor space to CP_2?

Can one assign a twistor space to CP_2? Could this property of CP_2 make it physically special? The necessary condition is satisfied: the Weyl tensor of CP_2 is self-dual.

3.2.1 CP_2 twistor space as flag manifold

CP_2 indeed allows a twistor structure as one learns from rather technical article about twistor structures (http://www.ams.org/journals/tran/2004-356-03/S0002-9947-03-03157-X/S0002-9947-03-03157-X pdf). The twistor space associated with CP_2 is six-dimensional flag manifold (http://en.wikipedia.org/wiki/Flag_manifold) [A1] $F(1,2,3) = U(3)/\times U(1) \times U(1) \times U(1) = SU(3)/U(1) \times U(1)$ [A2] (http://www.ams.org/journals/tran/2004-356-03/S0002-9947-03-03157-X/S0002-9947-03-03157-X.pdf).

This flag manifold has interpretation as the space of all possible choices of quantization axes for color hyper charge and isospin. Note that the earlier proposal [K11] that the analog of twistor space for CP_2 is CP_3 is wrong.

The twistor space assignable to M^4 can be interpreted as a flag manifold consisting of 2-planes associated with 8-D complexified Minkowski space as is clear from interpretation as projection space CP_3. It might also have an interpretation as the space of the choices of quantization axes. For M^4 light-like vector defines a unique time-like 2-plane M^2 and the direction of the associated 3-vector defines quantization axes of spin whereas the sum of the light-like vector and its dual has only time component and defines preferred time coordinate and thus quantization axes for energy. In fact, the choice of $M^1 \subset M^2 \subset M^4$ defining flag is in crucial role in the number theoretic vision and also in the proposed construction of preferred extremals: the local choice of M^2 would define the plane of unphysical polarizations and as its orthogonal complement the plane of physical polarizations.

Amusingly, the flag manifold $SU(3)/U(1) \times U(1)$ associated with $SU(3)$ made its first appearance in TGD long time ago and in rather unexpected context. The mathematician Barbara Shipman discovered that the the dance of honeybees can be described in terms of this flag manifold [A4] and made the crazy proposal that quark level physics is somehow related to the honeybee dance. TGD indeed predicts scaled variants of also quarks and QCD like physics and in biology the presence of 4 Gaussian Mersenne primes in the length scale range 10 nm- 2.5 μm [K1] suggests that these QCDs might be realized in the new physics of living cell [K2].

In TGD inspired theory of consciousness the choice of quantization axis represents a higher level state function reduction and contributes to conscious experience - one can indeed speak about flag manifold qualia. It will be found that the choice of quantization axis is also unavoidable in the conditions stating the light-likeness of 3-surfaces and leading to a generalization of Super Virasoro algebra so that the twistor space of H emerges naturally from basic TGD.

3.2.2 What is the interpretation of the momentum like color quantum numbers?

There is a rather obvious objection against the notion of momentum like quantum numbers in CP_2 degrees of freedom. If the propagator is proportional to $1/(p^2 - Y^2 - I_3^2)$, where Y and I_3 are assigned to quark, a strong breaking of color symmetry results. The following argument demonstrates that this is not the case and also gives an interpretation for the notion of anomalous hyper-charge assignable to CP_2 spinors.

1. Induced spinors do not form color triplets: this is the property of only physical states involving several wormhole throats and the action of super generators and spinor harmonics in cm mass degrees of freedom to which one can assign imbedding space spinor harmonics to be distinguished from second quantizee induced spinors appearing in propagator lines. Color is analogous to rigid body angular momentum and one can speak of color partial waves. The total color quantum numbers are dictated by the cm color quantum numbers plus those associated with the Super Virasoro generators used to create the state [K3] and which also help to correct the wrong correlation between color and electroweak quantum numbers between spinor harmonics.

2. Since CP_2 is projective space the standard complex coordinates are ratios of complex coordinates of C^3: $\{\xi^i = z^i/z_k \ , \ \ i \neq k\}$, where k corresponds to one of the complex coordinates z^i for given coordinate patch (there are three coordinate patches). For instance, for $k = 3$ the coordinates are $(\xi^1, \xi^2)z^1/z^3, z^2/z^3)$. The coordinates z^i triplet representation of $SU(3)$ so that $\{\xi^i, i \neq k\}$ carries anomalous color quantum numbers given by the negatives of the z^k.

3. Also the spinors carry anomalous Y and I_3, which are negative to anomalous color quantum numbers of CP_2 coordinates from the fact that spinors and z^i/z_k form color triplet. These quantum numbers are same for all spinor components inside given CP_2 coordinate patch so that no breaking of color symmetry results in a given patch. The color momentum would appear in the Dirac operator assignable to super Virasoro generators and define most naturally the contribution to region momentum. The "8-momenta" of external lines would be differences of region momenta and their color part would vanish for single fermion states associated with wormhole throat orbits.

3.3 Could one assign twistor space to $M^4 \times CP_2$?

The twistorialization of TGD could be carried by identifying the twistor counterpart of the imbedding space $H = M^4 \times CP_2$. The first guess that comes in mind is that the twistor space is just the product of twistor spaces for M^4 and CP_2. The next thought is that one could identify the counterpart of twistor space in 8-D context as the space of light-like geodesics of H. Since light-like geodesics in CP_2 couple M^4 and CP_2 degrees of freedom and since the M^4 projection of the light-like geodesic is in general time-like, this would allow the treatment of also massive states if the 8-D mass defined as eigenvalue of d'Alembertian vanishes. It however turns that the first thought is consistent with the general TGD based view and that second option yields twistor spaces which are non-compact.

In the following two attempts to identify the twistor space as light-like geodesics is made. I apologize my rudimentary knowledge about the matters involved.

1. If the dimension of the twistor space is same as that for the projective complexifications of M^8 one would dhave $D = 14$. This is also the dimension of projective complexification of octonsions whose importance is suggested by number theoretical considerations. If the twistorialization respects cartesian products then the dimension would be $D = 12$.

2. For M^8 at least the twistor space should have local structure given by $X^8 \times S^6$, where S^6 parametrizes direction vectors in 8-D lightcone. The conformal boundary of the space of light-like geodesics correspond to light-like geodesics of M^4 and this suggests that the conformal boundary of twistor space is $CP_3 \times CP_2$ with dimension $D = 10$.

One can consider several approaches to the identification of the twistor space. One could start from the condition that twistor space describes projective complexification of $M^4 \times CP_2$, from the direct study of light-like geodesics in H, from the definition as flag manifold characterizing the choices of quantization axes for the isometry group of H.

1. The first guess of a category theorist would be that twistorialization commutes with Cartesian products if isometry group decomposes into factors leaving the factors invariant. The naive identification would be as the twelve-dimensional space $CP_3 \times F(1,2,3)$, $F(1,2,3) = SU(3)/U(1) \times U(1)$. The points of H would in turn be mapped to products $S^2 \times S^3 \subset CP_3 \times SU(3)/U(1) \times U(1)$, which are 5-dimensional objects.

   One can criticize this proposal. The points of this space could be interpreted as 2-dimensional objects defined as products of light-like geodesics and geodesic circles of CP_2. They could be also interpreted as space-like geodesics with light-like M^4 projection. Why should space-likegeodesics replace light-like geodesics of H with light-like projection?

   The experience with TGD however suggests that this could be the physical option. p-Adic mass calculations require tachyonic ground states and the action of conformal algebras gives vanishing conformal weight for the physical states. Also massless extremals are characterized by longitudinal space M^2 in which momentum projection is light-like whereas the entire momentum for Fourier components in the expansion of imbedding space coordinates are space-like. This has led to the proposal that it is light-like M^2 projection of momentum that matters. Also the recent vision about generalized Feynman diagrams is that fermions propagate as massless particles in M^4 sense and that massive particles are bound states of massless particles: many-sheeted space-time makes possible to realize this picture. Also the construction of the analog of Super Virasoro algebra for light-like 3-surface leads naturally to the product of twistor spaces as moduli space.

2. The second approach is purely group theoretical and would identify twistor space as the space for the choices of quantization axes for the isometries which form now a product of Poincare group and color group. In the case of Poincare group energy and spin are the observabels and in the case of color group one has isospin and color hypercharge. The twistor space in the case of time-like M^4 projections of 8-momentum is obtained as coset space $P/SO(2) \times SU(3)/U(1) \times U(1) = M^4 \times SO(3,1)/M^1 \times SO(2) \times SU(3)/U(1) \times U(1) = E^3 \times SO(3,1)/SO(2) \times SU(3)/U(1) \times U(1)$. The dimension is the expected $D = 14$. In Euclidian sector one would have $E^4 \times SO(4)/SO(2) \times SO(2) \times SU(3)/U(1) \times U(1)$ having also dimension $D = 14$. The twistor space would not be compact and this is very undesired feature.

   Ordinary twistors define flag manifold for projectively complexified M^4. If this is the case also now one obtains just the naively expected 12-dimensional $CP_3 \times SU(3)/U(1) \times U(1)$ with two spheres replaced with $S^2 \times S^3$. This option corresponds to the "tachyonic" dentification of geodesics of H defining the twistor space as geodesics having light-like M^4 projection and space-like CP_2 projection.

3. One can consider also the space of light-like H-geodesics. Locally the light-like geodesics for which M^4 projection is not space like geodesic can be parametrized by their position defined as intersection with arbitrary time-like hyper-plane $E^3 \subset M^4$. Tangent vector characterizes the geodesic completely since CP_2 geodesics can be characterized by their tangent vector. Hence the situation reduces locally to that in M^8 and light-likeness and projective invariance mean that the sphere S^6 parametrizes the moduli for light-like geodesics at given point of E^3. Hence the parameter space would be at least locally $E^3 \times S^6$. S^6 would be the counterpart of S^2 for ordinary twistors. An important special case are light-like geodesics reducing to light-like geodesics of M^4. These are parametzized by $X^5 \times CP_2$, where X^5 is the space of light-like geodesics in M^4 and defines the analog of light-cone in twistor space CP_3. Therefore the dimension of twistor space must be higher than 10. For M^4 the twistor space has same dimension as projective complexification of M^4.

   One can study the light-like geodesics of H directly. The equation of light-like geodesic of H in terms of curve parameter s can be written as $m^k = v^k s, \phi = \omega s$, $v_k v^k = 1$ for time-like M^4 projection and $v^k v_k = 0$ for light-like M^4 projection. For time-like M^4 projection light-likeness gives $1 - R^2\omega^2 = 0$ fixing the value of ω to $\omega = 1/R$; therefore CP_2 part of the geodesic is characterized by giving unit vector characterizing its direction at arbitrarily chosen point of CP_2 and the modyli sopace space is 3-dimensional S^3. For light-like M^4 projection one obtains $\omega = 0$ so that the CP_2 projection contracts to a point. The hyperbolic space H^3 or Lobatchevski space (mass shell) parametrizing the space of unit four-velocities and S^3 gives the possible directions of velocity at given point of CP_2.

The space of light-like geodesics in H could be therefore regarded as a singular bundle like structure. The interior of the bundle has the space $X^6 = E^3 \times H^3$ of time-like geodesics of M^4 as base and S^3 perhaps identifiable as subspace of flag-manifold $SU(3)/U(1) \times U(1)$ of CP_2 defining CP_2 twistors as fiber. This space couldbe 9-dimensional subspace of $D = 14$ twistor space and consistency with $D = 14$ obtained from previous argument. Boundary consists of light-like geodesics of M^4 - that is 5-D subspace of twistor space CP_3 and fiber reduces to CP_2. The bundle structure seems trivial apart the singular boundary. Again there are good reasons to believe that the twistor space is non-compact which is a highly undesirable feature.

The cautious conclusion is that category theorist is right, and that one must take seriously p-adic mass calculations and generalized Feynman diagrams: the twistor space in question corresponds to space-like geodesics of H with light-like M^4 projection and reduces to the product of twistor spaces of M^4 and CP_2.

I have earlier speculated about twistorial formulation of TGD assuming that the analog of twistor space for $M^4 \times CP_2$ is $CP_3 \times CP_3$ and also noticed the analogy with F-theory [K11]. In the same chapter I have also considered an explicit proposal for the realization of the 10-D counterparts of space-time surfaces as 6-dimensional holomorphic surfaces in $CP_3 \times CP_3$ speculated to be Calabi-Yau manifolds. These speculations can be repeated for $CP_3 \times F(1, 2, 6)$ but with space-time surfaces mapped to 9-D surfaces having interpretation as $S^2 \times S^3$ bundles with space-time surface as a base space. Light-like 3-surfaces would be mapped to 8-D surfaces. Whether they could allow the identification as 4-complex-dimensional Calabi-Yau manifolds with structure group SU(4) as a structure group and Kähler metric with global holonomy contained in SU(4) is a question that mathematician might be able to answer immediately.

3.4 Three approaches to incidence relations

The algebraic realization of incidence relations involves spinors. The 2-dimensional character of the spinors and the possibility to interpret 2×2 Pauli sigma matrices as matrix representation of units of complexified quaternions with additional imaginary unit commuting with quaternionic imaginary units seem to be essential. How could one generalize the incidence relations to 8-D context?

One can consider three approaches to the generalization of the incidence relations defining algebraically the correspondence between bi-spinors and light-like vectors.

1. The simplest approach assumes that twistor space is Cartesian product of those associated with M^4 and CP_2 separately so that nothing new should emerge besides the quantization of Y_3 and I_3. The incidence relations for Minkowskian and Euclidian situation are discussed in detail later in the section. It might well be that this is all that is needed.

2. Second approach is based on triality for the representations of $SO(1, 7)$ realized for 8-D spaces.

3. Third approach relies on octonionic representations of sigma matrices and replaces $SO(1, 7)$ with the octonionic automorphism group G_2.

The first approach will be discussed in detail at the end of the section.

3.4.1 The approach to incidence relations based on triality

Second approach to incidence relations is based on the notion of triality serving as a special signature of 8-D imbedding space.

1. The triality symmetry making 8-D spaces unique states there are 3 8-D representations of SO(8) or SO(1,7) related by triality. They correspond complexified vector representation and spinor representations together with its conjugate. Could ordinary 8-D gamma matrices define sigma

matrices obtained simply by multiplying them by γ^0 so that one obtains unit matric and analogs of 3-D sigma matrices. Sigma matrices defined in this manner span an algebra which has dimension $d_1 = 2^{D-1}$ corresponding to the even part of 8-D Clifford algebra.

This dimension should be equal to the real dimension of the complex $D \times D$ matrix algebra given by $d_2 = 2 \times D \times D$. For $D = 8$ one one indeed has $d_1 = 128 = d_2$! Hence triality symmetry seems to allow the realization of the incidence relations for 8-vectors and 8-spinors and their conjugates! Could this realize the often conjectured role of triality symmetry as the holy trinity of physics? Note that for the Pauli sigma matrices the situation is different. They correspond to complexified quaternions defining 8-D algebra with dimension $d_1 = 8$, which is same as the dimension d_2 for $D = 2$ assignable to the two 2-spinors.

2. There is however a potential problem. For $D = 4$ the representations of points of complexified M^4 span the entire sigma matrix algebra (complexified quaternions). For $D = 8$ complexified points define 16-D algebra to be contrasted with 128 dimensional algebra spanned by sigma matrices. Can this lead to difficulties?

3. Vector $x^k\sigma_k$ would have geometric interpretation as the tangent vector of the light-like geodesic at some reference point - most naturally defined by the intersection with $X^3 \times CP_2$, where X^3 is 3-D subspace of M^4. X^3 could correspond to time=constant slice E^3. Zero energy ontology would suggests either of the 3-D light-like boundaries of CD: this would give only subspace of full twistor space.

Geometrically the incidence relation would in the 8-D case state that two 6-spheres of 12-D twistor space define as their intersection light-like line of M^8. Here one encounters an unsolved mathematical problem. Generalizing from the ordinary twistors, one might guess that complex structure of 6-sphere could be be crucial for defining complex structure of twistor space. 6-sphere allows almost complex structures induced by octonion structure. These structures are not integrable (do not emerge as a side product of complex manifold structure) and an open problem is whether S^6 admits complex structure (`http://www.math.bme.hu/~etesi/s6-spontan.pdf`) [A3]. From the reference one however learns that S^6 allows twistor structure presumably identified in terms of the space of geodesics.

3.4.2 The approach to incidence relations based on octonionic variant of Clifford algebra

Third approach is purely number theoretical being based on octonions. Only sigma matrices are needed in the definition of twistors and incidence relations. In the case of sigma matrices the replacement of the ordinary sigma matrices with abstract quaternion units makes sense. One could replace bi-spinors with complexified quaternions and identify the two spinors in their matrix representation as the two columns or rows of the matrix.

The octonionic generalization would replace sigma matrices with octonionic units. The non-associativity of octonions however implies that matrix representation does not exist anymore. Only quaternionic subspaces of octonions allow matrix representation and the basic dynamical principle of number theoretic vision is that space-time surfaces are associative in the sense that the tangent space is quaternionic and contains preferred complex subspace. In the purely octonionic context there seems to be no manner to distinguish between vector x and spinor and its conjugate. The distinction becomes possible only in quaternionic subspaces in which 8-D spinors reduces to 4-D spinors and one can use matrix representation to identify vector and and spinor and its conjugate.

In [K9] I have considered also the proposal for the construction of the octonionic gamma matrices (they are not necessary in the twistorial construction). Now octonions alone are not enough since unit matrix does not allow identification as gamma matrix. The proposal constructs gamma matrices as tensor products of σ_3 and octonion units defining octonionic counterpart of the Clifford algebra realized usually in terms of gamma matrices.

Light-likeness condition corresponds to the vanishing of the determinant for the matrix defined by the components of light-like vector. Can one generalize this condition to the octonionic representation? The problem is that matrix representation is lacking and therefore also the notion of determinant is problematic. The vanishing of determinant is equivalent with the existence of vectors annihilated by the matrix. This condition makes sense also now and would say that x as octonion with complexified components produces zero in multiplication with some complexified octonion. This is certainly true for some complexified octonions which are not number field since there exist complexified octonions having no inverse. It is of course easy to construct such octonions and they correspond to light-like 8-vectors having no inverse.

The multiplication of octonionic spinors by octonionic units would appear in the generalization of the incidence relation $\mu^{A\prime} = x^{AA'}\lambda_A$ by replacing spinors and 8-coordinate with complex octonions. This would allow to assign to the tangent vector of light-like geodesic at given point of X^4 a generalized twistor defined by a pair of complexified 8-component octonionic spinors. It is however impossible to make distinction between these three objects unless one restricts to quaternionic spinors and vectors and uses matrix representation for quaternions.

3.5 Are four-fermion vertices of TGD more natural than 3-vertices of SYM?

There are some basic differences between TGD and super Yang-Mills theory (SYM) and it is interesting to compare the two situations from the perspective of both momentum space and twistor space. Here the miminal approach to incidence relations assuming cartesian product $CP_3 \times SU(3)/U(1) \times U(1)$ is starting point but the dimension of spinor space is allowed to be free.

1. In SYM the basic vertex is 3-vertex. Momentum conservation for three massless real momenta requires that the momenta are parallel. This implies that for on mass shell states the vertex is highly singular and this in turn is source of IR divergences. The three twistor pairs would be for real on mass shell states proportional to each other. In twistor formulation one however allows complex light-like momenta and this requires that either λ_i are or $\hat{\lambda}_i$ are collinear. The condition $\lambda_i = \pm(\hat{lambda_i})^*$ implies that twistors are collinear.

2. In TGD framework physical states correspond to collections of wormhole contacts carrying fermion and antifermions at the throats. The simplest states are fermions having fermion number at either throat. For bosons one has fermion and antifermion at opposite throats. External particles are bound states of massless particles. 4-fermion vertex is fundamental one and replaces BFF vertex.

   The basic 4-vertex represents a situation in which there are incoming wormhole contacts which in vertex emit a wormhole contact. For boson exchange incoming fermion and antifermion combine to form the exchanged boson consisting from the fermion and antifermion at opposite throats of the wormhole contact. All fermions are massless in real sense also inside internal lines and only the sum of the massless four-momenta is off mass shell. The momentum of exchanged wormhole contact can be also space-like if energies of fermion and antifermion have opposite signs. The real on mass shell property reduces the number of allow diagrams dramatically and strongly suggests the absence of both UV and IR divergences. Without further conditions ladder diagrams involving arbitrary number of loops representing massess exchanges are possible but simple power counting argument demonstrates that no divergences are generated from these loops.

3. $\mathcal{N} = 4$ SUSY *as such* is not present so that super-twistors might not needed. SUSY is at WCW level replaced with conformal supersymmetry. Right-handed neutrino represents the least broken SUSY and the considerations related to the realization of super-conformal algebra and WCW gamma matrices as fermion number carrying objects suggest that the analogy of $\mathcal{N} = 4$ SUSY with conserved fermion number based on covariantly constant right-handed neutrino spinors emerges from TGD.

Consider now the basic formula for the 3-vertex appearing in gauge theories forgetting the complications due to SUSY.

1. The vertex contains determinants of 2×2 matrices defined by pairs (λ_i, λ_j) and $(\hat{\lambda}_i, \hat{\lambda}_j)$, $i = 1, 2, 3$. $\hat{\lambda}' = -(\lambda^\alpha)^*$ holds true in Minkowskian signature. These determinants define anti-symmetric Lorentz invariant "inner products" based on the 2-dimensional permutation symbol $\epsilon_{\alpha\alpha'}$ defining the Lorentz invariant bilinear for spinors. This form should generalize to the analog of Kähler form.

2. Second essential element is the expression for momentum conservation in terms of the spinors λ and $\hat{\lambda}$. The momentum conservation condition $\sum_k p_k = 0$ combined with the basic identification

$$p^{\alpha\alpha'} = \lambda^{\alpha}\hat{\lambda}^{\alpha'} \qquad (3.1)$$

equivalent with incidence relations gives

$$\sum_{k=1,...,n} \lambda_k^{\alpha}\hat{\lambda}_k^{\alpha'} = 0 \ . \qquad (3.2)$$

The key idea is to interpret λ_k^{α} and $\hat{\lambda}_k^{\alpha'}$ as vectors in n-dimensional space which is Grassmannian $G(2,n)$ since from a given solution to the conditions one obtains a new one by scaling the spinors λ_i and $\hat{\lambda}_j$ by scaling factors, which are inverses of each other. The conditions state that the 2-planes spanned by the λ^{α} and $\hat{\lambda}^{\alpha'}$ as complex 3-vectors are orthogonal. The conservation conditions can be satisfied only for 3-vectors.

Since the expression of momentum conservation as orthogonality conditions is a crucial element in the construction of twistor amplitudes it is good to look in detail what the conditions mean. For future purposes it is convenient to consider N-spinors instead of 2-spinors.

1. The number of these vectors is 2+2 for 2-spinors. For N-component spinors it is $N + N = 2N$. The number of conditions to be satisfied is $2N \times N - N$rather than $2N^2$: the reduction comes from the factor the condition $\hat{\lambda}^{\alpha'} = -(\lambda^{\alpha})^*$ holding for real four-momenta in M^4 case. For complex light-like momenta the number of conditions is $2N^2 = 8$.

2. For $N = 2$ and $n = 3$ with real masses one obtains 6 conditions and 6 independent components so that the conditions allow to solve the constraint uniquely (apart from complex scalings). All momenta are light-like and parallel. For complex masses one has 8 conditions and 12 independent spinor components and conditions imply that either λ_i or $\hat{\lambda}_i$ are parallel so that one has 4 complex spinors . For $n > 3$ the number of conditions is smaller than the total number of spinor components in accordance with the fact that momentum conservation conditions allow continuum of solutions. 3-vertex is the generating vertex in twistor formulation of gauge theories. For $N > 2$ the number conditions is larger than available spinor components and the situation reduces to $N = 2$ for solutions.

3. Euclidian spinors appear in CP_2 degrees of freedom. In $N = 2$ case spinors are complex, "momentum" having anomalous isospin and hyper-charge of CP_2 spinor as components is not light-like, and massless Dirac equation is not satisfied. Hence number of orthogonality conditions is $2 \times N^2 = 8$ whereas the total number of spinor components is $3 \times 2 + 3 \times 2 = 12$ as for complex massless momenta. Orthogonality conditions can be satisfied. For $N > 2$ the real dimension of the sub-paces spanned by spinors is at most 3 and orthogonality condition can be satisfied if N reduces effectively to $N = 2$.

Similar discussion applies for 4-fermion vertex in the case of TGD.

1. Consider first M^4 case ($N = 2$) for $n = 4$-vertex. The momentum conservation conditions imply that fourth momentum is the negative of the sum of the three other and massless. For real momenta the number of conditions on spinors is also now $2 \times N^2 - N = 6$ for $N = 2$. The number of spinor components is now $n \times N = 4 \times N = 8$ so that 2 spinor components characterizing the virtual on mass shell momentum of the second fermion composing the boson remains free in the vertex.

2. In CP_2 degrees of freedom and for $n = 4, N = 2$ the number of orthogonality conditions is $2N^2 = 8$ and the total number of spinor components is $2 \times n \times N = 16$ so that 8 spinor components remain free. The quantization of anomalous hyper-charge and isospin however discretizes the situation as suggested by number theoretic arguments. Also in M^4 degrees of freedom discretisation of four-momenta is suggestive.

3. For $N > 2$ the situation reduces effectively to $N = 2$ for the solutions to the conditions for both Minkowskian and Euclidian signature.

4 Emergence of $M^4 \times CP_2$ twistors at the level of WCW

One could imagine even more dramatic generalization of the notion of twistor, which conforms with the general vision about TGD and twistors. The orbits of partonic 2-surfaces are light-like surfaces and generalize the notion of light-like geodesics. In TGD framework the replacement of point like particle with partonic 2-surface plus 4-D tangent space data suggests strongly that the Yangian algebra defined by finite-dimensional conformal algebra of M^4 generalizes to that defined by the infinite-dimensional conformal algebra associated with all symmetries of WCW.

The twistorialization should give twistorialization of $M^4 \times CP_2$ at point-like limit defined by $CP_2 \times SU(3)/U(1) \times U(1)$. In the following it will be found that this is indeed the case and that twistorialization can be seen as a representation for a choice of quantization axes characterized by appropriate flag manifold.

4.1 Concrete realization for light-like vector fields and generalized Virasoro conditions from light-likeness

The points of WCW correspond to partonic two-surfaces plus 4-D tangent space data. It is attractive to identify the tangent space data in terms of light-like vector fields defined at the partonic 2-surfaces at the ends of light-like 3-surface defining a like of generalized Feynman diagrams so that their would define light-like vector field in the piece of WCW defined by single line of generalized Feynman diagrams. It is also natural to continue these light-like vector fields to light-like vector fields defined at entire light-like 3-surface - call it X^3.

To get some grasp about the situation one can start from a simpler situation, CP_2 type vacuum extremals with 1-D light-like curve as M^4 projection. The light-likeness condition reads as

$$m_{kl}\frac{dm^k}{ds}\frac{dm^l}{ds} = 0 \ , \tag{4.1}$$

One can use the expansion

$$\begin{aligned} m^k &= m_{k,0} + p_0^k s + \sum_{n,i} a_{n,i}\frac{\epsilon_i^k}{\sqrt{n}} s^n \ , \\ \epsilon_i \cdot \epsilon_j &= -P_{ij}^2 \ . \end{aligned} \tag{4.2}$$

Here orthonormalized polarization vectors ϵ_i define 2-D transversal space orthogonal to the longitudinal space $M^2 \subset M^4$ and characterized by the projection operator P^2. M^2 can be fixed by a light-like vector and corresponds to the real section of the twistor space naturally. These conditions are familiar from string (complex coordinate is replaced with s). Here ϵ_i are polarization vectors orthogonal to each other. One obtains the Virasoro conditions

$$L_n = p\cdot p + 2\sum_m a_{n-m}a_m\sqrt{n-k}\sqrt{k} = 0 \tag{4.3}$$

expressing the invariance of light-likeness condition with respect to diffeomorphisms acting on coordinate s. For $n = 0$ one obtains the Virasoro conditions. This can be regarded as restriction of conformal invariance from string world sheets emerging from the modified Dirac equation at their ends at light-like 3-surfaces.

The generalization of these conditions is rather obvious. Instead of functions $m_n^k = \epsilon_n^k s^n$ one considers functions

$$\begin{aligned} m_{n,\alpha}^k &= m^0 + p_0^k s + \sum_{n,i} a_{n,i,\alpha}\epsilon_i^k \frac{s^n}{\sqrt{n}} f_\alpha(x^T) + \sum_{n,i} b_{n,i,\alpha} c_i^k \frac{s^n}{\sqrt{n}} g_\alpha(x^T) \ , \\ s_{n,\alpha}^k &= s_0^k + J_0^k s + c_i^k s^n g_\alpha(x^T) \ , \\ c_i^k \cdot c_j^k &= -\delta_{ij} \ . \end{aligned} \tag{4.4}$$

where s^k denotes CP_2 coordinates. The tangent vecotor J^k characterizes a geodesic line in CP_2 degrees of freedom. There is no reason to restrict the polarization directions in CP_2 degrees of freedom so that the projection operator is flat Eucldian 4-D metric. $\{f_\alpha\}$ is a complete basis of functions of the transversal coordinates for the $s = constant$ slice defined the partonic 2-surface at given position of its orbit. One can assume that the modes are orthogonal in the inner product defined by the imbedding space metric and the integral over partonic 2-surface in measure defined by the $\sqrt{g_2}$ for the 2-D induced metric at the partonic 2-surface

$$\langle f_\alpha, f_\beta \rangle = \delta_{\alpha\beta} \ . \tag{4.5}$$

.

The space of functions f_α is assumed to be closed under product so that they satisfy the multiplication table

$$f_\alpha f_\beta = c^\gamma_{\alpha\beta} f_\gamma \ . \tag{4.6}$$

.

This representation allows to generalize the light-likeness conditions to 3-D form

$$L_{n,\alpha} = p_k p^k + J_k J^k + \sum_{k,\alpha,\beta} \left[2a_{n-k,\alpha} a_{k,\alpha} + 4b_{n-k,\alpha} b_{k,\alpha}\right] \sqrt{n-k}\sqrt{k} = 0 \ . \tag{4.7}$$

These equations define a generalization of Virasoro conditions to 3-D light-like surfaces. The center of mass part now corresponds to conserved color charge vector associated with CP_2 geodesic. One can also write variants of these conditions by performing complexification for functions f_α.

4.2 Is it enough to use twistor space of $M^4 \times CP_2$?

The following argument suggests that Virasoro conditions require naturally the integration over the twistor space for $M^4 \times CP_2$ but that twistorialization in vibrational degrees of freedom is not needed.

The basic problem of Virasoro conditions is that four-momentum in cm degrees of freedom is time-like in the general case. It is very difficult to accept the generalization of the twistor space to $E^3 \times SO(3,1)/SO(2) \times SO(1,1) \times SU(3)/U(1) \times U(1)$ in cm degrees of freedom? The idea about straightforward generalization twistor space to vibrational degrees of freedom seems to lead to grave difficulties. It however seems that a loophole, in fact two of them, exist and is based on the notion of momentum twistors.

1. The key observation is that the selection of M^2 in the Virasoro conditions reduces to a fixing of light-like vector in given M^4 coordinates fixing $M^2 \subset M^4$. This choices defines a twistor in the real section of the twistor space. Could twistors emerge through this kind of condition? In the quantization of the theory which must somehow appear also in TGD framework, the selection of quantization axes must be made and means selection of point of a flag manifold defining the twistor spaces associated with M^4 and CP_2. In quasiclassical picture only the components of the tangent vector in CP_2 degrees of freedom have well-defined isospin and hypercharge so that J_k would be a linear combination of I_3 and Y. Standard complex coordinates transforming linearly at their origin under U(2) indeed have this property.

   Could the integration over twistor space mean in WCW context an integration over the possible choices of the quantization axes necessary in order to preserve isometries as symmetries? Four-momenta of external lines itself could be assumed to be massless as conformal invariance strongly suggests.

2. Consider now the problem. Virasoro conditions require that M^4 momentum is massive. This is not consistent with twistorialization. Momentum twistors for which external light-like momenta characterizing external lines are differences $p_i = x_i - x_{i-1}$ of the "region momenta" x_i assigned with the twistor lines [B5] (`http://arxiv.org/pdf/1008.3110v1.pdf`) might solve

the problem. In the recent case region momenta x_i would correspond to those appearing in Virasoro conditions and light-like momenta of outgoing lines would correspond to their differences. Similar identification would apply to color iso-spin and hyper-charge. For SYM massless real momenta in the condition $p_i = x_i - x_{i-1}$ implies that all three momenta are parallel, which is a catastrophic result. In the TGD based twistor approach region momenta can be however real and massless : this would give rise to dual conformal invariance leading to Yangian symmetries. In this picture Super Virasoro conditions would separate completely from twistorialization and apply in overall cm degrees of freedos: this is indeed what has been assumed hitherto.

It is easy to see that that region momenta can be real and light-like in TGD framework. A generalization of the condition $p_i = x_i - x_{i-1}$ from 3-vertex to 4-fermion vertex is needed (4-particle vertex requires super-symmetrization but this is not essential for the argument). 4-fermion vertex involves interaction between 2-fermions via Euclidian wormhole contact (this will be discussed later) inducing their scattering. For massless external fermion second internal line is a wormhole contact carrying massless fermion and anti-fermion at its opposite throats. The region momentum associated with this line can be defined as sum of the light-like region momenta associated with the throats. If the external particle is boson like carrying - in general non-parallel - light-like momenta at its throats, then p_i is sum of their light-like momenta.

Concerning the identification of region momenta, one could consider also another option inspired by the vision that also the fermions propagating in the internal lines are massless.

1. For this option also region momenta are light-like in accordance with the idea about twistor diagrams as null polygons and the idea about light-light on mass shell propagation also on internal lines. One can consider two options for the fermionic propagator.
   (a) In twistor description the inverse of the full massless Dirac propagator would appear in the line in twistor formalism and this would leave only non-physical helicities making the lines virtual: the interpretation would be as a residue of $1/p^2$ pole.
   (b) The M^2 projection of the light-like momentum associated with the corresponding internal line would be time-like. In CP_2 degrees of freedom J^k could be replaced by its projection to the plane spanned by isospin and hypercharge. The values of the sum of transverse E^2 momentum squared and in cm and vibrational degrees of freedom would be identical.

   Indeed, one possible option considered already earlier is that M^4 momentum is always light-like and only its longitudinal M^2 part is precisely defined for quantum states (as for partons inside hadron). The original argument was that if only the M^2 part of momentum appears in the propagators, one can have on mass shell massless particles without diverging propagators: in twistorial approach one gets rid of the ordinary propagators in the case gauge fields. The integration over different choices of M^2 associated with the internal line and having interpretation as integration over light-like virtual momenta would guarantee overall Lorentz invariance. This would allow also the use of the M^2 part of four-momentum - an option cautiously considered for generalized Feynman diagrams - without losing isometries as symmetries.
2. The fermion propagator could also contain CP_2 contribution. Since only Cartan algebra charges can be measured simultaneously, J^k would correspond to a superposition of color hypercharge and isospin generators. The flag manifold $SU(3)/U(1)\times U(1)$ would characterize possible choices of quantization axes for CP_2. Also in the case of CP_2 only the "polarization directions" orthogonal to the plane defined by I_3 and Y could be allowed and it might be possible to speak about CP_2 polarization perhaps related to Higgs field. The dimension of $M^4 \times CP_2$ in vibrational degrees of freedom would effectively reduce to 4. Number theoretically this could correspond to the choice of quaternionic subspace of the octonionic tangent space.

What can one conclude?

1. Since the choice of quantization axis is same for all modes and forces them to a space orthogonal to that defined by quantization axes, one can say that all modes are characterized by the twistor space for $M^4 \times CP_2$ and there is no need to consider infinite-dimensional generalization of the twistor space only $M^4 \times CP_2$ twistors would be needed and would have interpretation as the integration over the choices of quantization axes is natural part of quantum TGD.

2. The use of ordinary massless Dirac operator is very attractive option since it gives the inverse of massless Dirac operator as effective propagator in twistor formalism and requires that only non-physical helicities propagate. Massless on mass shell propagation is possible only for fermions as fundamental particles. If one wants also CP_2 contribution to the propagator then restriction to $I_3 - Y$ plane might be necessary. This option does not look too promising.

3. From the TGD point of view twistor approach to gauge theory in M^4 would not describe not much more than the physics related to the choice of quantization axes in M^4. The physics described by gauge theories is indeed in good approximation to that assignable to cm degrees of freedom. The remaining part of the physics in TGD Universe - maybe the most interesting part of it involving WCW integration - would be described in terms of infinite-dimensional super-conformal algebras.

4.3 Super counterparts of Virasoro conditions

Although super-conformal algebras have been applied successfully in p-adic mass calculations, many aspects related to super Virasoro conditions remain still unclear. p-Adic mass calculations require only that there are 5 super-conformal tensor factors and leaves a lot of room for imagination.

1. There are two super conformal algebras. The first one is the super-symplectic algebra assignable to the space-like 3-surface and acts at the level of imbedding spaceand is induced by Hamiltonians of $\delta M^4_\pm \times CP_2$. Second algebra is Super Kac-Moody algebra acting on light-like 3-surfaces as deformations respecting their light-likeness and is also assignable to partonic 2-surfaces and their 4-D tangent space. Do these algebras combine to single algebra or do they define separate Super Virasoro conditions? p-Adic mass calculations assume that the direct sum is in question and can be localized to partonic 2-surfaces by strong form of holography. This makes the application of p-adic thermodynamics [K3] sensical .

2. Do the Super Virasoro conditions apply only in over all cm degrees of freedom so that spinors are imbedding space spinors. They would thus apply at the level of the entire 3-surfaces assigned to external elementary particles and containing at least two wormhole contacts. In this case the resulting massive states would be bound states of massless fermions with non-parallel light-like momenta and the resulting massivation could be consistent with conformal invariance.

This is roughly the recent picture about the situation. One can however consider also alternatives.

1. Could the Super Virasoro conditions apply to invididual partonic 2-surfaces or even at the lines of generalized Feynman diagrams but in this case involve only the longitudinal part of massless M^4 momentum?

2. Could Super-Virasoro conditions be satisfied at partonic 2-surfaces defining vertices in the sense that the sum of incoming super Virasoro generators annihilate the vertex identified. In cm degrees of freedom this condition would be satisfied in cm degrees of freedom momentum conservation holds true. In vibrational degrees of freedom the condition is non-trivial but in principle can be satisfied. The fermionic oscillator operators at incoming legs are related linearly to each other and the problem is to solve this relationship. In the case of N-S generators the same applies. For Virasoro generators the conditions are satisfied if the Virasoro algebras of lines annihilate the state associated with them separately.

These options do look too plausible and would make the situation un-necessarily complex.

4.3.1 How the cm parts of WCW gamma matrices could carry fermion number?

Super counterparts of Virasoro conditions must be satisfied for the entire 3-surface or less probably for the light-like lines of generalized Feynman diagram. These conditions look problematic, and I have considered earlier several solutions to the problem with a partial motivation coming from p-adic thermodynamics.

The problem is following.

1. In Ramond representation super generators are labeled by integers and string models suggest that super generator G_0 and its hermitian conjugate have ordinary Dirac operator as its cm term and vibrational part has fermion number ± 1. This does not conform with the non-hermiticity of G_0 and looks non-sensical and it seems difficult to satisfy the super Virasoro conditions in non-trivial manner.

2. There exist a mechanism providing the cm part of G_0 with fermion number? Right-handed neutrino is exceptional: it is de-localized into entire X^4 as opposed to other spinor components localized to string world sheets and has covariantly constant zero modes with vanishing momentum. These modes seem to provide the only possible option that one can imagine. The fermion number carrying gamma matrices in cm degrees of freedom of H would be defined as $\Gamma^\alpha = \gamma^\alpha \Psi_\nu$ and $\Gamma^{\alpha\dagger} = \overline{\Psi}_{\nu_R}\gamma^\alpha$, where Ψ_{ν_R} represents covariantly constant right-handed neutrino. The anticommutator gives imbedding space metric as required. Right-handed neutrino would have a key role in the mathematical structure of the theory.

3. For Neveu-Scwartz representation WCW gamma matrices and super generators are labeled by half odd integers and in this case all generators would have fermion number ± 1. The squares of super generators give rise to Virasoro generators L_n and L_0 should be essentially the mass squared operator as $G_{1/2}G_{-1/2} + hc.$. This operator should give the d'Alembertian in $M^4 \times CP_2$ or its longitudinal part. This is quite possible but it seems that Ramond option is the physical one.

The two spin states of covariantly constant right handed neutrino and its antiparticle could provide a fermion number conserving TGD analog of $\mathcal{N} = 4$ SUSY since the four oscillator operators for Ψ_{ν_R} would define the analogs of the four theta parameters.

What is the nature of the possible space-time supersymmetry generated by the right-handed neutrino? Do different super-partners have different mass as seems clear if different super-partners can be distinguished by their interactions. If they have different masses do they obey same mass formula but with different p-adic prime defining the mass scale? This problem is discussed the article [?] and in the chapter [K6].

4.3.2 About the SUSY generated by covariantly constant right-handed neutrinos

The interpretation of covariantly constant right-handed neutrinos (ν_R in what follows) in $M^4 \times CP_2$ has been a continual head-ache. Should they be included to the spectrum or not. If not, then one has no fear/hope about space-time SUSY of any kind and has only conformal SUSY. First some general obsrevations.

1. In TGD framework right-handed neutrinos differ from other electroweak charge states of fermions in that the solutions of the modified Dirac equation for them are delocalized at entire 4-D space-time sheets whereas for other electroweak charge states the spinors are localized at string world sheets [K12].

2. Since right-handed neutrinos are in question so that right-handed neutrino are in 1-1 correspondence with complex 2-component Weyl spinors, which are eigenstates of γ_5 with eigenvalue say $+1$ (I never remember whether $+1$ corresponds to right or left handed spinors in standard conventions).

3. The basic question is whether the fermion number associated with covariantly constant right-handed neutrinos is conserved or conserved only modulo 2. The fact that the right-handed neutrino spinors and their conjugates belong to unitarily equivalent pseudoreal representations of SO(1,3) (by definition unitarily equivalent with its complex conjugate) suggests that generalized Majorana property is true in the sense that the fermion number is conserved only modulo 2. Since ν_R decouples from other fermion states, it seems that lepton number is conserved.

4. The conservation of the number of right-handed neutrinos in vertices could cause some rather obvious mathematical troubles if the right-handed neutrino oscillator algebras assignable to different incoming fermions are identified at the vertex. This is also suggested by the fact that right-handed neutrinos are delocalized.

5. Since the ν_R:s are covariantly constant complex conjugation should not affect physics. Therefore the corresponding oscillator operators would not be only hermitian conjugates but hermitian apart from unitary transformation (pseudo-reality). This would imply generalized Majorana property.

6. A further problem would be to understand how these SUSY candidates are broken. Different p-adic mass scale for particles and super-partners is the obvious and rather elegant solution to the problem but why the addition of right-handed neutrino should increase the p-adic mass scale beyond TeV range?

If the ν_R:s are included, the pseudor3al analog of $\mathcal{N}=1$ SUSY assumed in the minimal extensions of standard model or the analog of $\mathcal{N}=2$ or $\mathcal{N}=4$ SUSY $\mathcal{N}=2$ or even $\mathcal{N}=4$ SUSY is expected so that SUSY type theory might describe the situation. The following is an attempt to understand what might happen. The earlier attempt was made in [K6].

1. Covariantly constant right-handed neutrinos as limiting cases of massless modes

For the first option covariantly constant right-handed neutrinos are obtained as limiting case for the solutions of massless Dirac equation. One obtains 2 complex spinors satisfying Dirac equation $n^k\gamma_k\Psi = 0$ for some momentum direction n^k defining quantization axis for spin. Second helicity is unphysical: one has therefore one helicity for neutrino and one for antineutrino.

1. If the oscillator operators for ν_R and its conjugate are hermitian conjugates, which anticommute to zero (limit of anticommutations for massless modes) one obtains the analog of $\mathcal{N}=2$ SUSY.

2. If the oscillator operators are hermitian or pseudohermitian, one has pseudoreal analog of $\mathcal{N}=1$ SUSY. Since ν_R decouples from other fermion states, lepton number and baryon number are conserved.

Note that in TGD based twistor approach four-fermion vertex is the fundamental vertex and fermions propagate as massless fermions with non-physical helicity in internal lines. This would suggest that if right-handed neutrinos are zero momentum limits, they propagate but give in the residue integral over energy twistor line contribution proportional to $p^k\gamma_k$, which is non-vanishing for non-physical helicity in general but vanishes at the limit $p^k \rightarrow 0$. Covariantly constant right-handed neutrinos would therefore decouple from the dynamics (natural in continuum approach since they would represent just single point in momentum space). This option is not too attractive.

2. Covariantly constant right-handed neutrinos as limiting cases of massless modes

For the second option covariantly constant neutrinos have vanishing four-momentum and both helicities are allowed so that the number of helicities is 2 for both neutrino and antineutrino.

1. The analog of $\mathcal{N}=4$ SUSY is obtained if oscillator operators are not hermitian apart from unitary transformation (pseudo reality) since there are 2+2 oscillator operators.

2. If hermiticity is assumed as pseudoreality suggests, $\mathcal{N}=2$ SUSY with right-handed neutrino conserved only modulo two in vertices obtained.

3. In this case covariantly constant right-handed neutrinos would not propagate and would naturally generate SUSY multiplets.

3. Could twistor approach provide additional insights?

Concerning the quantization of ν_R:s, it seems that the situation reduces to the oscillator algebra for complex M^4 spinors since CP_2 part of the H-spinor is spinor is fixed. Could twistor approach provide additional insights?

As discussed, M^4 and CP_2 parts of H-twistors can be treated separately and only M^4 part is now interesting. Usually one assigns to massless four-momentum a twistor pair $(\lambda^a, \hat{\lambda}^{a'})$ such that one has $p^{aa'} = \lambda^a\hat{\lambda}^{a'}$. Dirac equation gives $\lambda^a = \pm(\hat{\lambda}^{a'})^*$, where $\pm$ corresponds to positive and negative frequency spinors.

1. The first - presumably non-physical - option would correspond to limiting case and the twistors λ and $\hat{\lambda}$ would both approach zero at the $p^k \to 0$ limit, which again would suggest that covariantly constant right-handed neutrinos decouple completely from dynamics.

2. For the second option one could assume that either λ or $\hat{\lambda}$ vanishes. In this manner one obtains 2 spinors λ_i, $i = 1,2$ and their complex conjugates $\hat{\lambda}_i$ as representatives for the super-generators and could assign the oscillator algebra to these. Obviously twistors would give something genuinely new in this case. The maximal option would give 2 anti-commuting creation operators and their hermitian conjugates and the non-vanishing anti-commutators would be proportional to $\delta_{a,b}\lambda_i^a(\lambda^b)_j^*$ and $\delta_{a,b}\hat{\lambda}_i^{a'}(\hat{\lambda}^{b'})_j^*$. If the oscillator operators are hermitian conjugates of each other and (pseudo-)hermitian, the anticommutators vanish.

An interesting challenge is to deduce the generalization of conformally invariant part of four-fermion vertices in terms of twistors associated with the four-fermions and also the SUSY extension of this vertex.

4.3.3 Are fermionic propagators defined at the space-time level, imbedding space level, or WCW level?

There are also questions related to the fermionic propagators. Does the propagation of fermions occur at space-time level, imbedding space level, or WCW level?

1. Space-time level the propagator would defined by the modified Dirac operator. This description seems to correspond to ultramicroscopic level integrated out in twistorial description.

2. At imbedding space level allowing twistorial description the lines of generalized Feynman diagram would be massless in the usual sense and involve only the fermionic propagators defined by the twistorial "8-momenta" defining region momenta in twistor approach.This allows two options.

   (a) Only the projection to M^2 and preferred $I_3 - Y$ plane of the momenta would be contained by the propagator. The integration over twistor space would be necessary to guarantee Lorentz invariance.

   (b) M^4 helicity for internal lines would be "wrong" so that M^4 Dirac operator would not annihilate it. For ordinary Feynman diagrams the propagator would be $p^k\gamma_k/p^2$ and would diverge but for twistor diagrams only its inverse $p^k\gamma_k$ would appear and would be well-defined. This option looks attractive from twistor point of view.

3. If WCW level determines the sermonic propagator as in string models, bosonic propagator would naturally correspond to $1/L_0$. The generalization of the fermionic propagator could be defined as G/L_0, where the super generator G contains the analog of ordinary Dirac operator as cm part. The square of G would give L_0 allowing to define the generalization of bosonic propagator. The inverse of the fermionic propagator would carry fermion number.

   This is good enough reason for excluding WCW level propagator and for assuming that the fermion propagators defined at imbedding space level appear in the generalized Feynman diagrams and Super Virasoro algebra are applied only in particle states as done in p-adic mass calculations.

The conclusion is that the original picture about fermion propagation is the only possible one. If one requires that ordinary Feynman diagrams make sense then only the M^2 part of 4-momentum can appear in the propagator. If one assumes that only twistor formalism is needed then propagator is replaced with its inverse in fermionic lines and if polarization is "wrong" the outcome is non-vanishing. This situation has interpretation in terms of homology theory. One could also the interpret the situation in terms of residue calculus picking up $p^k\gamma_k$ as the residue of the pole of $1/(p^2 + i\epsilon)$.

4.4 What could 4-fermion twistor amplitudes look like?

What can one conclude about 4-fermion twistor amplitudes on basis of $\mathcal{N}=4$ amplitudes? Instead of 3-vertices as in SYM, one has 4-fermion vertices as fundamental vertices and the challenge is to guess their general form. The basis idea is that $\mathcal{N}=4$ SYM amplitudes could give as special case the n-fermion amplitudes and their supersymmetric generalizations.

4.4.1 A attempt to understand the physical picture

One must try to identify the physical picture first.

1. Elementary particles consist of pairs of wormhole contacts connecting two space-time sheets. The throats are connected by magnetic fluxes running in opposite directions so that a closed monopole flux loop is in question. One can assign to the ordinary fermions open string world sheets whose boundary belong to the light-like 3-surfaces assignable to these two wormhole contacts. The question is whether one can restrict the consideration to single wormhole contact or should one describe the situation as dynamics of the open string world sheets so that basic unit would involve two wormhole contacts possibly both carrying fermion number at their throats.

   Elementary particles are bound states of massless fermions assignable to wormhole throats. Virtual fermions are massless on mass shell particles with unphysical helicity. Propagator for wormhole contact as bound state - or rather entire elementary particle would be from p-adic thermodynamics expressible in terms of Virasoro scaling generator as $1/L_0$ in the case of boson. Super-symmetrization suggests that one should replace L_0 by G_0 in the wormhole contact but this leads to problems if G_0 carries fermion number. This might be a good enough motivation for the twistorial description of the dynamics reducing it to fermion propagator along the light-like orbit of wormhole throat. Super Virasoro algebra would emerged only for the bound states of massless fermions.

2. Suppose that the construction of four-fermion vertices reduces to the level of single wormhole contact. 4-fermion vertex involves wormhole contact giving rise to something analogous to a boson exchange along wormhole contact. This kind of exchange might allow interpretation in terms of Euclidian correlation function assigned to a deformation of CP_2 type vacuum extremal with Euclidian signature.

   A good guess for the interaction terms between fermions at opposite wormhole contacts is as current-current interaction $j^\alpha(x)j_\alpha(y)$, where x and y parametrize points of opposite throats. The current is defined in terms of induced gamma matrices as $\overline{\Psi}\Gamma^\alpha\Psi$ and one functionally integrates over the deformations of the wormhole contact assumed to correspond in vacuum configuration to CP_2 type vacuum extremal metrically equivalent with CP_2 itself. One can expand the induced gamma matrix as a sum of CP_2 gamma matrix and contribution from M^4 deformation $\Gamma_\alpha = \Gamma_\alpha^{CP_2} + \partial_\alpha m^k \gamma_k$. The transversal part of M^4 coordinates orthogonal to $M^2 \subset M^4$ defines the dynamical part of m^k so that one obtains strong analogy with string models and gauge theories.

3. The deformation Δm^k can be expanded in terms of CP_2 complex coordinates so that the modes have well defined color hyper-charge and isospin. There are two options to be considered.

   (a) One could use CP_2 spherical harmonics defined as eigenstates of CP_2 scalar Laplacian D^2. The scale of eigenvalues would be $1/R^2$, where R is CP_2 radius of order 10^4 Planck lengths. The spherical harmonics are in general not holomorphic in CP_2 complex coordinates ξ_i, $i=1,2$. The use of CP_2 spherical harmonics is however not necessary since wormhole throats mean that wormhole contact involves only a part of CP_2 is involved.

   (b) Conformal invariance suggests the use of holomorphic functions $\xi_1^m\xi_2^n$ as analogs of z^n in the expansion. This would also be the Euclidian analog for the appearance of massless spinors in internal lines. Holomorphic functions are annihilated by the ordinary scalar Laplacian. For conformal Laplacian they correspond to the same eigenvalue given by the constant curvature scalar R of CP_2. This might have interpretation as a spontaneous breaking of conformal invariance.

The holomorphic basis z^n reduces to phase factors $exp(in\phi)$ at unit circle and can be orthogonalized. Holomorphic harmonics reduce to phase factors $exp(im\phi_1)exp(in\phi_2)$ and torus defined by putting the moduli of ξ_i constant and can thus be orthogonalized. Inner product for the harmonics is however defined at partonic 2-surface. Since partonic 2-surfaces represent Kähler magnetic monopoles they have 2-dimensional CP_2 projection. The phases $exp(im\phi_i)$ could be functionally independent and a reduction of inner product to integral over circle and reduction of phase factors to powers $exp(in\phi)$ could take place and give rise to the analog of ordinary conformal invariance at partonic 2-surface. This does not mean that separate conservation of I_3 and Y is broken for propagator.

(c) Holomorphic harmonics are very attractive but the problem is that it is annihilated by the ordinary Laplacian. Besides ordinary Laplacian one can however consider conformal Laplacian [?] (`http://en.wikipedia.org/wiki/Laplace_operators_in_differential_geometry#Conformal_Laplacian`) defined as

$$D_c^2 = -6D^2 + R \ , \tag{4.8}$$

and relating the curvature scalars of two conformally scaled metrics. The overall scale factor and also its sign is just a convention. This Laplacian has the same eigenvalue for all conformal harmonics. The interpretation would be in terms of a breaking of conformal invariance due to CP_2 geometry: this could also relate closely to the necessity to assume tachyonic ground state in the p-adic mass calculations [K3].

The breaking of conformal invariance is necessary in order to avoid infrared divergences. The replacement of M^4 massless propagators with massive CP_2 bosonic propagators in 4-fermion vertices brings in the needed breaking of conformal invariance. Conformal invariance is however retained at the level of M^4 fermion propagators and external lines identified as bound states of massless states.

4.4.2 How to identify the bosonic correlation function inside wormhole contacts?

The next challenge is to identify the correlation function for the deformation δm^k inside wormhole contacts.

Conformal invariance suggests the identification of the analog of propagator as a correlation function fixed by conformal invariance for a system defined by the wormhole contact. The correlation function should depend on the differences $\xi_i = \xi_{i,1} - \xi_{i,2}$ of the complex CP_2 coordinates at the points $\xi_{i,1)}$ and $\xi_{i,2}$ of the opposite throats and transforms in a simple manner under scalings of ξ_i. The simplest expectation is that the correlation function is power r^{-n}, where $r = \sqrt{|\xi_1|^2 + |\xi_2|^2}$ is U(2) invariant coordinate distance. The correlation function can be expanded as products of conformal harmonics or ordinary harmonics of CP_2 assignable to $\xi_{i,1}$ and $\xi_{i,2}$ and one expects that the values of Y and I_3 vanish for the terms in the expansions: this just states that Y and I_3 are conserved in the propagation.

Second approach relies on the idea about propagator as the inverse of some kind of Laplacian. The approach is not in conflict with the general conformal approach since the Laplacian could occur in the action defining the conformal field theory. One should try to identify a Laplacian defining the propagator for δm^k inside Euclidian regions.

1. The propagator defined by the ordinary Laplacian D^2 has infinite value for all conformal harmonics appearing in the correlation function. This cannot be the case.

2. If the propagator is defined by the conformal Laplacian D_c^2 of CP_2 multiplied by some numerical factor it gives fro a given model besides color quantum numbers conserving delta function a constant factor nR^2 playing the same role as weak coupling strength in the four-fermion theory of weak interactions. Propagator in CP_2 degrees of freedom would give a constant contribution if the total color quantum numbers for vanish for wormhole throat so that one would have four-fermion vertex.

   This option does not look physically attractive. If four-fermion vertex involves always wormhole contact carrying fermion and antifermion at its throats, the interpretation as effective boson exchange is possible and one can assume that the vertex contains instead of L^2 a factor proportional to $1/p^2$. It will be shows later that this description leads to gauge theory like picture. A further possibility is that L^2 is replaced by p-adic length scale square L_p^2 associated with p^2. This would discretize coupling constant evolution.

3. One can consider also a third - perhaps artificial option - motivated for Dirac spinors by the need to generalize Dirac operator to contain only I_3 and Y. Holomorphic partial waves are also eigenstates of a modified Laplacian D_C^2 defined in terms of Cartan algebra as

$$D_C^2 \equiv \frac{aY^2 + bI_3^2}{R^2} \quad , \tag{4.9}$$

where a and b suitable numerical constants and R denotes the CP_2 radius defined in terms of the length $2\pi R$ of CP_2 geodesic circle. The value of a/b is fixed from the condition $Tr(Y^2) = Tr(I_3^2)$ and spectra of Y and I_3 given by $(2/3, -1/3, -1/3)$ and $(0, 1/2, -1/2)$ for triplet representation. This gives $a/b = 9/20$ so that one has

$$D_C^2 = (\frac{9}{20}Y^2 + I_3^2) \times \frac{a}{R^2} \quad . \tag{4.10}$$

In the fermionic case this kind of representation is well motivated since fermionic Dirac operator would be $Y^k e_k^A \gamma_A + I_3^k e_k^A \gamma_A$, where the vierbein projections $Y^k e_k^A$ $Y^k e_k^A$ and $I_3^k e_k^A$ of Killing vectors represent the conserved quantities along geodesic circles and by semiclassical quantization argument should correspond to the quantized values of Y and I_3 as vectors in Lie algebra of $SU(3)$ and thus tangent vectors in the tangent space of CP_2 at the point of geodesic circle along which these quantities are conserved. In the case of S^2 one would have Killing vector field L_z at equator.

Two general remarks are in order.

1. That a theory containing only fermions as fundamental elementary particles would have four-fermion vertex with dimensional coupling as a basic vertex at twistor level, would not be surprising. As a matter of fact, Heisenberg suggested for long time ago a unified theory based on use of only spinors and this kind of interaction vertex. A little book about this theory actually inspired me to consider seriously the fascinating challenge of unification.

2. A common problem of all these options seems to be that the 4-fermion coupling strength is of order R^2 - about 10^8 times gravitational coupling strength and quite too weak if one wants to understand gauge interactions. It turns out however that color partial waves for the deformations of space-time surface propagating in loops can increase R^2 to the square $L_p^2 = pR^2$ of p-adic length scale. For D_C^2 assumed to serve as a propagator of an effective action of a conformal field theory one can argue that large renormalization effects from loops increase R^2 to something of order pR^2.

4.4.3 Do color quantum numbers propagate and are they conserved in vertices?

The basic questions are whether one can speak about conservation of color quantum numbers in vertices and their propagation along the internal lines and the closed magnetic flux loops assigned with the elementary particles having size given by p-adic length scale and having wormhole contacts at its ends. p-Adic mass calculations predict that in principle all color partial waves are possible in cm degreees of freedom: this is a description at the level of imbedding space and its natural counterpart at space-time level would be conformal harmonics for induced spinor fields and allowance of all of them in generalized Feynman diagrams.

1. The analog of massless propagation in Euclidian degrees of freedom would correspond naturally to the conservation of Y and I_3 along propagator line and conservation of Y and I_3 at vertices. The sum of fermionic and bosonic color quantum numbers assignable to the color partial waves woul be conserved. For external fermions the color quantum numbers are fixed but fermions in internal lines could move also in color excited states.

2. One can argue that the correlation function for the M^4 coordinates for points at the ends of fermionic line do not correlate as functions of CP_2 coordinates since the distance between partonic 2-surface is much longer than CP_2 scale but do so as functions of the string world sheet coordinates as stringy description strongly suggests and that stringy correlation function satisfying conformal invariance gives this correlation. One can however counter argue that for hadrons the color correlations are different in hadronic length scale. This in turn suggests that the correlations are non-trivial for both the wormhole magnetic flux tubes assignable to elementary particles and perhaps also for the internal fermion lines.

3. I_3 and Y assignable to the exchanged boson should have interpretation as an exchange of quantum numbers between the fermions at upper and lower throat or change of color quantum numbers in the scattering of fermion. The problem is that induced spinors have constant anomalous Y and I_3 in given coordinate patch of CP_2 so that the exchange of these quantum numbers would vanish if upper and lower coordinate patches are identical. Should one expand also the induced spinor fields in Euclidian regions using the harmonics or their holomorphic variants as suggested by conformal invariance?

   The color of the induced spinor fields as analog of orbital angular momentum would realized as color of the holomorphic function basis in Euclidian regions. If the fermions in the internal lines cannot carry anomalous color, the sum over exchanges trivializes to include only a constant conformal harmonic. The allowance of color partial waves would conform with the idea that all color partial waves are allowed for quarks and leptons at imbedding space level but define very massive bound states of massless fermions.

4. The fermion vertex would be a sum over the exchanges defined by spherical harmonics or - more probably - by their holomorphic analogs. For both the spherical and conformal harmonic option the 4-fermion coupling strength would be of order R^2, where R is CP_2 length. The coupling would be extremely weak - about 10^8 times the gravitational coupling strength G if the coupling is of order one. This is definitely a severe problem: one would want something like L_p^2, where p is p-adic prime assignable to the elementary particle involved.

   This problem provides a motivation for why a non-trivial color should propagate in internal lines. This could amplify the coupling strength of order R^2 to something of order $L_p^2 = pR^2$. In terms of Feynman diagrams the simplest color loops are associated with the closed magnetic flux tubes connecting two elementary wormhole contacts of elementary particle and having length scale given by p-adic length scale L_p. Recall that $\nu_L\overline{\nu}_R$ pair or its conjugate neutralizes the weak isospin of the elementary fermion. The loop diagrams representing exchange of neutrino and the fermion associated with the two different wormhole contacts and thus consisting of fermion lines assignable to "long" strings and boson lines assignable to "short strings" at wormhole contacts represent first radiative correction to 4-fermion diagram. They would give sum over color exchanges consistent with the conservation of color quantum numbers at vertices. This sum, which in 4-D QFT gives rise to divergence, could increase the value of four-fermion coupling to something of order $L_p^2 = kpR^2$ and induce a large scaling factor of D_C^2.

5. Why known elementary fermions correspond to color singlets and triplets? p-Adic mass calculations provide one explanation for this: colored excitations are simply too massive. There is however evidence that leptons possess color octet excitations giving rise to light mesonlike states. Could the explanation relate to the observation that color singlet and triplet partial waves are special in the sense that they are apart from the factor $1/\sqrt{1+r^2}$, $r^2 = \sum \xi_i\overline{\xi}_i$ for color triplet holomorphic functions?

4.4.4 Why twistorialization in CP_2 degrees of freedom?

A couple of comments about twistorialization in CP_2 degrees of freedom are in order.

(a) Both M^4 and CP_2 twistors could be present for the holomorphic option. M^4 twistors would characterize fermionic momenta and CP_2 twistors to the quantum numbers assignable to deformations of CP_2 type vacuum extremals. CP_2 twistors would be discretized since I_3 and Y have discrete spectrum and it is not at all clear whether twistorialization is useful now. There is excellent motivation for the integration over the flag-manifold defining the choices of color quantization axes. The point is that the choice of conformal basis with well-defined Y and I_3 breaks overall color symmetry $SU(3)$ to $U(2)$ and an integration over all possible choices restores it.

(b) Four-fermion vertex has a singularity corresponding to the situation in which p_1, p_2 and $p_1 + p_2$ assignable to emitted virtual wormhole throat are collinear and thus all light-like. The amplitude must develop a pole as $p_3 + p_3 = p_1 + p_2$ becomes massless. These wormhole contacts would behave like virtual boson consisting of almost collinear pair of fermion and anti-fermion at wormhole throats.

4.4.5 Reduction of scattering amplitudes to subset of $\mathcal{N} = 4$ scattering amplitudes

$\mathcal{N} = 4$ SUSY provides quantitative guidelines concerning the actual construction of the scattering amplitudes.

1. For single wormhole contact carrying one fermion, one obtains two $\mathcal{N} = 2$ SUSY multiplets from fermions by adding to ordinary one-fermion state right-handed neutrino, its conjugate with opposite spin, or their pair. The net spin projections would be $0, 1/2, 1$ with degeneracies (1,2,1) for fermion helicity 1/2 and $(0, -1/2, -1)$ with same degeneracies for fermion helicity -1/2. These $\mathcal{N} = 2$ multiplets can be imbedded to the $\mathcal{N} = 4$ multiplet containing 2^4 states with spins $(1, 1/2, 0, -1/2, -1)$ and degeneracies given by $(1, 4, 6, 4, 1)$. The amplitudes in $\mathcal{N} = 2$ case could be special cases of $\mathcal{N} = 4$ amplitudes in the same manner as they amplitudes of gauge theories are special cases of those of super-gauge theories. The only difference would be that propagator factors $1/p^2$ appearing in twistorial construction would be replaced by propagators in CP_2 degrees of freedom.

2. In twistor Grassmannian approach to planar SYM one obtains general formulas for n-particle scattering amplitudes with k positive (or negative helicities) in terms of residue integrals in Grassmann manifold $G(n, k)$. 4-particle scattering amplitudes of TGD, that is 4-fermion scattering amplitudes and their super counterparts would be obtained by restricting to $\mathcal{N} = 2$ sub-multiplets of full $\mathcal{N} = 4$ SYM. The only non-vanishing amplitudes correspond for $n = 4$ to $k = 2 = n - 2$ so that they can be regarded as either holomorphic or anti-holomorphic in twistor variables, an apparent paradox understandable in terms of additional symmetry as explained and noticed by Witten. Four-particle scattering amplitude would be obtained by replacing in Feynman graph description the four-momentum in propagator with CP_2 momentum defined by I_3 and Y for the particle like entity exchanged between fermions at opposite wormhole throats. Analogous replacement should work for twistorial diagrams.

3. In fact, single fermion per wormhole throat implying 4-fermion amplitudes as building blocks of more general amplitudes is only a special case although it is expected to provide excellent approximation in the case of ordinary elementary particles. Twistorial approach could allow the treatment of also $n > 4$-fermion case using subset of twistorial n-particle amplitudes with Euclidian propagator. One cannot assign right-handed neutrino to each fermion separately but only to the elementary particle 3-surface so that the degeneration of states due to SUSY is reduced dramatically. This means strong restrictions on allowed combinations of vertices.

Some words of critism is in order.

1. Should one use CP_2 twistors everywhere in the 3-vertices so that only fermionic propagators would remain as remnants of M^4? This does not look plausible. Should one use include to 3-vertices both M^4 and CP_2 type twistorial terms? Do CP_2 twistorial terms trivialize as a consequence of quantization of Y and I_3?

2. Nothing has been said about modified Dirac operator. The assumption has been that it disappears in the functional integration and the outcome is twistor formalism. The above argument however implies functional integration over the deformations of CP_2 type vacuum extremals.

5 Could twistorialization make sense in vibrational degrees of freedom of WCW?

An obvious question is whether the notion of twistor makes sense in vibrational degrees of freedom of WCW?

1. Could one map light-like 3-surfaces to the points of an infinite-dimensional analog of twistor space generalizing or perhaps even defining WCW and its analytic continuation analogous to that of M^4? Could one map partonic 2-surfaces to higher-dimensional spheres of this generalized twistor-space. Note that 4-D tangent space data would distinguish between different light-like 3-surfaces associated with the same partonic 2-surfaces.

2. The geometric co-incidence relations for light-like geodesics of M^4 as intersections of twistorial spheres should generalize to the condition that two partonic 2-surfaces at the opposite ends of CD are connected by a light-like 3-surface.

The conservative conclusion from previous considerations is that twistor description applies only in cm degrees of freedom and has very natural interpretation as a manner to achieve Lorentz and color invariance. Hence the twistorialization in vibrational degrees of freedom does not look like an attractive idea. This idea however has however some very attractive features and therefore deserved a more detailed debunking.

5.1 Algebraic incidence relations in the infinite-D context reduce to effectively 4-D case

The generalization of algebraic incidence relations to infinite-dimensional context looks like a highly non-trivial if not inpossible.

It is good to start with motivating observations.

1. One could replace light-like vector of M^4 or H with light-like tangent vector X at point of WCW. Could one generalize the spinor pair (λ, μ) associated with a light-like M^4 geodesic to a pair of spinors of WCW identifiable as fermionic Fock states assignable to positive/negative energy parts of zero energy states associated with the future and past boundaries of WCW or rather with the ends of the light-like 3-surface at boundaries of CD? The formulas $d_1 = 2^{D-1}$ and $d_2 = 2D \times D$ are not encouraging and the only reasonable option seems to be that the spinorial dimension must correspond to the dimension of the space generated by creation operator type gamma matrices which is indeed as WCW dimension.

2. If the spinor pair represents positive and negative energy parts of a zero energy state, does the co-incidence relation have interpretation as a quantum classical correspondence mapping zero energy states consisting of fermions to light-like momenta in WCW and therefore (tangents of) light-like geodesics of WCW? This kind of correspondence between space-time surfaces and quantum states would be just what the physical interpretation of TGD requires. Infinite-D momenta would correspond to pairs of initial and final states defining physical events in positive energy ontology. A weaker correspondence is that single fermion states generated by WCW gamma matrices are in 1-1 correspondence with the tangent space algebra represented as Kac-Moody generators and in this case the situation seems much promising since bosonic representations of Kac-Moody algebra can act in the same manner as a representation in terms of fermionic bilinears. This would be the counterpart of incidence relation now.

3. What could be the interpretation of the infinite-D hermitian operator $X^{AA'}\sigma_A$, which should relate positive and negative energy parts of the Fock state to each other? Could the algebra of these vectors span the infinite-D algebra of WCW and could isometry generators and WCW gamma matrices (or sigma matrices) span together a super-conformal algebra? This would be analog for the finite-dimensional super-conformal algebra associated with ordinary twistors. X defines a light-like tangent vector: could the interpretation be in terms of infinite-dimensional momentum vector for which light-likeness condition generalizes ordinary light-likeness condition allowing massivation in M^4 just as p-adic mass calculations suggest?

5.2 In what sense the numbers of spinorial and bosonic degrees of freedom could be same?

The detailed consideration of spinors reveals what looks like a grave difficulty: 2-dimensional considerations suggests that the number of spinorial degrees of freedom of WCW should be same as the dimension of WCW. N-dimensional spinor space has however dimension, which is exponentially larger than the dimension WCW. Stating it in slightly different manner: the space of complexified WCW gamma matrices expressible in terms of fermionic oscillator operators is exponentially smaller than the space of fermionic Fock states generated by them. As such this need not spoil hope about algebraic incidence relations but would spoil the nice super-symmetry between bosonic and fermionic dimensions. Could the situation be saved by considering only single fermion states or by ZEO or could a generalization of octonionic sigma matrices help?

The condition that single fermion states are on 1-1 correspondence with bosonic states, which correspond to tangent vectors that is Kac-Moody type algebra, makes sense. The representation of tangent space momentum vector identified as Kac-Moody generator as fermionic bilinear and the condition that it annihilates physical state would be the counterpart for the representation of momentum as bilinear in spinors appearing in twistor. The analog of incidence relation would express the action of Kac-Moody generator on fermion state or its commutator action on super generator.

The attempt to generalize momentum conservation conditions essential for the twistor formalism however fails. The generators of the Cartan algebra of Kac-Moody algebra commute but central extension spoils the situation and one can talk only about the cm parts of Cartan algebra Kac-Moody generators as conserved quantities.

5.3 Could twistor amplitudes allow a generalization in vibrational degrees of freedom?

The original idea was that twistorialization could make sense in vibrational degrees of freedom. It soon became clear that this is not needed since twistorialization in cm degrees of freedom is all the is needed. Therefore the answer to the question of the title is "No".

5.3.1 Twistorialization in minimal sense is possible

It has been already found that twistorialization in $M^4 \times CP_2$ emerges naturally from the integration over selections of quantization axes for Super Virasoro algebra. The amplitudes have the general Grassmannian form and the additional structures comes from vertices determined by super conformal invariance and from integration over WCW.

One can of course ask whether twistorialization could make sense in more general sense so that the integration over WCW 4-D tangent space degrees of freedom could be carried out by introducing twistor like entities in vibrational degrees of freedom: essentially this would mean representation of bosonic Kac-Moody algebra in terms of fermionic bilinears and this kind of representations indeed exist: the condition implying these representations would be that the sums of fermionic and bosonic Kac-Moody generators annihilate the vertices. One might say that small deformation of partonic 2-surface corresponds to generation of fermion pairs and has therefore physically observable.

5.3.2 Twistorialization in strong sense in vibrational degrees of freedom fails

The obvious question is whether twistorial amplitudes could allow a generalization obtained by replacing 2-spinors with N-spinors with N even approaching infinity. Skeptic could argue that the treatment of CP_2 degrees of freedom in terms of momenta is wrong: for quantum states one must use color quantum numbers: color isospin, hypercharge and the value of the Casimir operator. As a matter fact, the number of these parameters is three and happens to be the same as the number of components of unit vector characterizing the direction of CP_2 geodesic for which all color generators define conserved charges classically.

It its quite possible that the twistor approach does not make sense for color quantum numbers. It could however make sense for WCW degrees of freedom and co-incidence relations would allow to assign to tangent vector characterizing light-like 3-surfaces as orbit of parton in terms of positive and negative energy states at its ends. Quantum classical correspondence would be realized and even this would be a wonderful result concerning the interpretation of the theory, especially quantum measurement theory.

Therefore it is interesting to find whether twistor amplitudes allow a formal generalization at least. The essential elements is the reduction of the construction of amplitudes to that for on mass shell vertices with on mass shell property generalized to allow complex light-like momenta. From vertices one can build more general amplitudes by using simple basic operations and ends up with a recursion formula for the n-particle loop amplitudes in terms of Grassmannian. The especially interesting feature from TGD point of view is that the integrals are residue integrals and make sense also p-adically since for algebraic extension of p-adic numbers $2\pi = N \times sin(2\pi/N)$ gives the definition of p-adic 2π: here N corresponds to the largest root of unity involved with the extension. Hence twistorial construction could provide a universal solution to the p-adicization problem.

The algebraic incidence relations were already earlier discussed by allowing also the option $N > 2$ (N is power of two). It was found that the incidence relations can be satisfied but that the solutions reduce essentially to those for $N = 2$. Since this point is important one can look in more detail what happens for $N > 2$-spinors (N is power of 2 in finite-D case)?

1. For general amplitude the number of conditions to be satisfied - the dimension of the Grassmannian $G(k, n)$ - depends only on the number n of the particles and the number k of positive helicity external particles. For 3-vertex and $k = 2$ with complex light-like momenta at most $n = 3$ spinors λ^α *resp.* $\hat{\lambda}^{\alpha'}$ are linearly independent so that their number reduces effectively to $n_{eff} \leq 3$. For $N = 2$ and $n_{eff} = 3$ both $\lambda\alpha$ and $\hat{\lambda}^{\alpha'}$ span the entire 3-D complex space and no solutions are obtained without posing additional conditions on the spinors. Already for $N = 2$ either λ_i or $\hat{\lambda}_i$ are linearly independent. If this holds also now for - say - λ_i and $\hat{\lambda}^{\alpha'}$ span only 2-plane both, one obtains a solution. In other words, solutions given by 2-spinors give rise to solutions given by N-spinors reducing to 2-spinors effectively. Very probably there are no other solutions. Without these conditions one obtains $2 \times n_{eff} \times 3 - 3 = 15$ conditions and the effective number of spinor components is only $2 \times 3 \times 1 = 12 < 15$.

2. The reduction implies that in M^4 vibrational degrees of freedom some 4-D sub-space of tangent space of WCW is always selected and vibrational momenta in vertex belong to this plane. Momentum conservation however allows different 4-D sub-spaces in different vertices: the 4-D spaces at vertices connected by line must intersect along 1-D space at least. Hence the physics in vibrational degrees of freedom would reduce to 4-D only at vertices. An interesting question is whether this might be true for the dynamics of Kähler action at vertices or - if momentum conservation indeed holds true - in the sense that the light-like 3-surface corresponds to a motion of partonic 2-surface in 4-D subspace of single particle WCW. Same applies in CP_2 vibrational degrees of freedom.

3. Similar considerations apply in the case of 4-vertex since the number of conditions depends on N^2 and requires the effective reduction of N to $N = 2$.

These strange conditions on the dynamics reducing it to effectively four-dimensional one encourage to conclude that twistorial approach in vibrational degrees of freedom produces only problems. In $M^4 \times CP_2$ degrees it should work with minor modifications.

6 Scattering amplitudes in positive Grassmannian: TGD perspective

The quite recent but not yet published proposal of Hamed and his former student Trnka has gained a lot of attention. There is a popular article in Quanta Magazine about their work at `https://www.simonsfoundation.org/quanta/20130917-a-jewel-at-the-heart-of-quantum-physics/`. There is a video talk by Jaroslav Trnka about positive Grassmannian (the topic is actually touched at the end of the talk but it gives an excellent view about the situation) at `http://www.maths.dur.ac.uk/events/Meetings/LMS/2013/PNTPP13/talks/0438trnka.pdf` [B2] and a video talk of Nima Arkani-Hamed at `http://susy2013.ictp.it/video/05_Friday/2013_08_30_Arkani-Hamed_4-3.html#` [?]. One can also find the slides of Trnka at `http://www.staff.science.uu.nl/~tonge105/`

igst13/Trnka.pdf [B2]. For beginners like me the article of Henriette Envang and Yu-tin Huang serves as an enjoyable concretization of the general ideas [B6].

The basic claim is that the Grassmannian amplitudes reduce to volumes of positive Grassmannians determined by external particle data and realized as polytopes in Grassmannians such that their facets correspond to logarithmic singularities of a volume form in one-one correspondence with the singularities of the scattering amplitude. Furthermore, t the factorization of the scattering amplitude at singularities corresponds to the singularities at facets. Scattering amplitudes would characterize therefore purely geometric objects. The crucial Yangian symmetry would correspond to diffeomorphisms preserving the positivity property. Unitarity and locality would be implied by the volume interpretation. Nima concludes that unitarity and locality, gauge symmetries, space-time, and even quantum mechanics emerge. One can however quite well argue that its the positive Grassmannian property and volume interpretation which emerge. In particular, the existence of twistor structure possible in Minkowskian signature only in M^4 is absolutely crucial for the beautiful outcome, which certainly can mean a revolution as far as calculational techniques are considered and certainly the new view about perturbation theory should be important also in TGD framework.

The talks inspired the consideration of the possible Grassmannian formulation in TGD framework in more detail and to ask whether positivity might have some deeper meaning in TGD framework.

1. The generalization of the BCFW recursion relation using 4-fermion vertex with fermions of internal lines massless in real sense and having unphysical helicity suggests that all loops in Feynman sense vanish and only tree diagrams remain. This would simplify enormously the analog of BCFW construction and would allow to circumvent the restrictions due to the planarity since non-planar diagrams correspond to trees and non-planar diagrams would be obtained by permutations of external particles. Unfortunately this does not work: by the argument implying cancellation of loops involving SUSY also the bosonic wormhole throat propagator should vanish!

   The problem can be circumvented by starting directly from stringy diagrams forced also by the Kähler magnetic charge of wormhole throats and localization of fermions to string world sheets (right handed neutrinos being an exception). BCFW construction generalizes to stringy objects at the formal level at least and cuts are now performed for string world sheets.

2. The generalization to gravitational sector is not a problem in sub-manifold gravity since M^4 - the only space-time geometry with Minkowski signature allowing twistor structure - appears as a Cartesian factor of the imbedding space. A further finding is that CP_2 and S^4 are the only Euclidian 4-manifolds allowing twistor space with Kähler structure. Since S^4 does not allow Kähler structure, CP_2 is completely unique just like M^4. The analog of twistorial construction in CP_2 degrees of freedom based on the notion of flag manifold and geometric quantization can be considered.

3. The triviality of coupling constant evolution could be seem as a problem in standard QFT framework but discrete p-adic coupling constant evolution with local RG invariance could resolve the problem: this would give very profound role for the p-adicity.

4. As both Arkani-Hamed and Trnka state "everything is positive". This is highly interesting since p-adicization involves canonical identification which is well defined only for non-negative reals without further assumptions! This raises the conjecture that positivity is necessary to achieve number theoretical universality.

6.1 About the definition of positive Grassmannian

The lecture of Trnka [B3, B2] and the earlier article by Arkani-Hamed et al [B7] give an excellent view about positive Grassmannians. The lectures of Postikov (http://www-math.mit.edu/~ahmorales/18.318lecs/lectures.pdf) provide a more detailed mathematical summary [B1]. Essentially convex polytopes of Grassmannian are in question.

1. The starting point is triangle in plane. Its interior points can be defined as center of mass coordinates for a system containing masses at the vertices Z^i of the triangle. As the non-negative masses vary over all possible values one obtains points of the triangle. The generalization to the case of projective plane is obvious. Definition generalizes to n-simplexes defined by $n+1$

points. In three dimensions the construction gives the interior points of tetrahedron. A further generalization of trianges is from projective spaces $G(1,n)$ to projective spaces $G(k,n)$. Now the positivity condition for masses guaranteeing interior point property generalizes to the conditions that all minors defined by the k colmes of the $k \times n$ matrix defining point of $G(k,n)$ are positive. The resulting convex polytope is called $G_+(k,n)$.

2. In the case of projective plane one can also consider convex polygons with $n > 3$ sides. Convexity requires that the minors of the $3 \times n$ matrix (3 is the number of projective coordinates) are positive. Also this construction generalizes to $G(k,n)$.

The positivity makes sense only for real Grassmannian and if the scattering amplitude is volume in strict sense it is real. This cannot make sense in the general case. I have got the impression that the positivity condition generalizes also to the complex case. In the case of $(1,1,-1,-1)$ signature twistors are real: does the positive Grassmannian makes sense in this case and allows to perform calculations and identify scattering amplitude as volume of a convex polytope and analytically continue the result to the Minkowskian signature.

The scattering amplitude would be the volume for a convex polygon defining a positive Grassmannian. Feynman diagrams and BCFW defines triangulation of this polygon and perturbative calculation of the volume by adding volumes of the parts of the triangulation. The integration measure is defined in the standard representation of scattering amplitudes linearizing the momentum conservation constraint expressed in terms of twistors using the coordinates $C_{\alpha a}$ of $G(k,n)$. Integration measure and the $k \times k$ minors of $k \times n$ matrix representing the point of $G(k,n)$ can be expressed in terms of so called face variables f_i. This allows to express integration measure as product $\prod_i dlog(f_i)$. The integration measure has logarithmic singularities at the facets of the convex polygon defined by the external momenta and helicities. The face variables associated with the loop interiors give a multiplicative factor whereas the integral over the other face variables gives what corresponds to the scattering amplitude as a function of external momenta and helicities. For other than $\mathcal{N} = 4$ theories UV singularities are in the interior of the convex polygon.

6.2 The recent TGD based view about BCFW construction of scattering amplitudes

What could be the counterpart of BCFW construction in TGD framework? The following view is the latest one and differs from the first guess in that QFT type BCFW is replaced by its stringy variant. Views are still fluctuating wildly.

1. The first task is to define precisely what on-mass-shell and off-mass-shell properties mean. On-mass-shell property for external fermion means that the line is massless and has physical helicity. Internal fermions are also massless but have non-physical helicity. Hence the line containing the inverse of the massless fermion propagator after residue integration over p^2 does not vanish. I have described earlier how pair of fermion and anti-fermion at opposite throats of wormhole contact give rise to effective boson exchanges with space-like momentum (the sign of energy of internal fermion line can be negative). Contrary to the first beliefs, the consideration of the microscopic details of propagation cannot be avoided and their consideration forces stringy variant of BCFW.

2. For 3-vertex of SYM momentum conservation forces the momenta to be parallel. All loop corrections in the sense of Feynman graphs vanish reflecting the fact that coupling constant renormalization is trivial. 4-fermion vertex can be non-vanishing for non-parallel momenta. Therefore the internal fermion lines can be massless in real sense rather than in only in complex sense as in the case of SYM.

3. Bosonic emergence suggests an additional constraint on diagrams. At least one pair of lines in 4-fermion vertex corresponds to the opposite throats of bosonic wormhole contact. This would reduce the vertex effectively to BFF or BBB vertex. This option looks realistic from QFT point of view. In BCFW construction the cuts would be for bosonic lines. However, Kähler magnetic charges of wormhole throats and localization of induced spinor fields to string world sheets force to accept strings as fundamental objects.

One might hope that one could at QFT limit neglect the second end of string carrying only neutrino pair neutralizing weak isospin. This hope seems to be unrealistic. It turns out that bosonic wormhole contact propagator diverges in absence SUSY and vanishes if SUSY applies separately to wormhole contacts rather than only to a string like object having wormhole contacts at its "ends". Hence the stringy generalization of twistor Grassmannian approach seems unavoidable unless one is ready to assume that SUSY is broken in long scales and to eliminate the logarithmic divergences appearing already in the emergent gauge boson propagators by using CP_2 mass scale as cutoff scale.

4. The original dream about the cancellation of all loops in QFT sense turned out to be unrealistic and has reincarnated as a dream about the cancellation of stringy loops, and might be equally unrealistic. The idea about discrete p-adic coupling constant evolution with local renormalisation group invariance is however too beautiful to be thrown to the paper basket, and one can hope that stringy BCFW could realise it.

5. Whether SUSY is present or not has been a long standing open question. The argument below relying on the properties of the modified gamma matrices and the special properties of right-handed neutrinos suggests that SUSY emerges from strong gravitation in space-time regions with Euclidian signature - that is inside CP_2 type vacuum extremals defining the lines of generalized Feynman diagrams. SUSY would be broken at very high mass scale - perhaps CP_2 mass scale by a mechanism provided by p-adic thermodynamics.

6. One must consider also CP_2 degrees of freedom. In long length scales one expects that QFT type description of color as spin-like quantum number is a good approximation. In short length scales this cannot be the case. The optimistic guess would be that the construction of the scattering amplitude factorizes so that for a given tree amplitude M^4 and CP_2 degrees can be treated separately and that for a given diagram one obtains just a product of M^4 and CP_2 contributions. The Grassmannian approach following from the momentum conservation constraint is not expected to apply in CP_2 degrees of freedom. If I^3 and Y conservation corresponds to a geometric constraint in F, the question is what happens in vertices.

6.3 SUSY or no SUSY?

SUSY is the basic poorly understood aspect of TGD. Mathematically SUSY is certainly an extremely attractive idea byt naive physical arguments do not support it.

1. Do covariantly right-handed neutrino and its antiparticle assign to fermions with helicity 1/2 and -1/2 SUSY multiplet with 4 members as mathematical elegance would suggest? Naive physical intuition suggests that the decoupling of right-handed neutrino from standard model interactions implies that fermion and accompanying right-handed neutrinos behave completely independently so that it would not be possible to speak about SUSY multiplets.

   One can of course build SUSY multiplets but SUSY would be badly broken: the spartners of fermions behave just like the fermions with respect to standard model interactions so that it would not be possible to distinguish between fermion and its spartners experimentally? This would lead to contradictions with experimental facts since the number of spartners would appear as degeneracy factor in annihilation rates and in number densities in thermodynamics (say density of photons and photon energy in blackbody radiation). Something clearly goes wrong in this argument.

2. Situation is not so gloomy actually. There *is* a coupling between different right-and left handed neutrinos coming from modified gamma matrices which are superpositions of M^4 and CP_2 gamma matrices but the physical interpretation of this coupling has remained open. CP_2 parts of the modified gamma matrices couple the right-handed neutrino to left-handed one and make it possible to talk about massivation of neutrinos.

   This coupling can be classified as gravitational coupling and is extremely small for space-time sheets with Minkowskian signature unless gravitational fields are very strong and the induced metric is very near to Euclidian. For CP_2 type vacuum extremals with Euclidian signature of the induced metric and assigned with the lines of generalized Feynman diagrams the situation is totally different. This would support the idea about separate SUSY multiplets associated with different fermion helicities makes sense in shor enough length scales. The response of spartners to standard model interactions with their entire spin could follow from this coupling. Strong gravitation would generates SUSY dynamically as an ultra-short distance phenomenon. p-adic thermodynamics with different p-adic length scale for members of SUSY multiplet. It would not be terribly surprising if the p-adic length scale for spartners would be rather short so that they would be very massive having mass of order CP_2 mass.

3. The following argument provides additional support for this interpretation. Covariantly constant right-handed neutrinos are associated with entire space-time sheets whereas other fermions are localized at string world sheets. For Minkowskian space-time sheets of macroscopic size right-handed neutrinos are for all practical purposes absent since the macroscopic quantum state has just four SUSY partners having practically no interactions with the state itself! For CP_2 type vacuum externals with strong coupling between left and right-handed neutrinos situation changes and has important implications already for geneneralized Feynman diagrams identified as stringy diagrams.

6.4 Bosonic emergence

The Feynman diagrammatics involving only four-fermion vertex with constant value L^2 of the coupling constant strength but no additional assumptions (assumption about bosonic wormhole contacts) looks un-realistic.

1. Dimensional coupling of length squared allows to expect divergences and non-renormalizability. A possible manner to save the situation could be that L^2 corresponds to the square of p-adic length scale L_p determined by the momentum squared assignable to the bosonic wormhole contact.

2. Bosonic emergence requires that standard massless bosonic propagator proportional to $1/p^2$ emerges from fermion loop when combined with vertex factor depending on bosonic line only. If the dimensional coupling L^2 is constant, this is certainly not the case.

3. It is also highly questionable whether it is possible to obtain the analogs of space-like boson exchanges using only four-fermion vertex and tree diagrams even if one allows negative energies. Rather, the theory would look like that of weak interactions with very large weak boson mass.

Bosonic emergence [K4] is one of the basic ideas of TGD approach and means the identification of the basic building blocks of gauge bosons and gravitons as wormhole contacts having fermion and antifermion at their boundaries.

1. Wormhole contacts behave like particles: if the second throat is empty, one has fermion and if the throats carry fermion and antifermion, one has boson. 4-vertices would reduce effectively to 3-vertex with 2 fermionic or bosonic lines and 1 bosonic line and 2-vertex with 2 bosonic vertices. The latter would have interpretation as a mass insertion expected to lead to wave function renormalization of boson propagator.

2. This picture could also resolve the problem created by dimensional coupling constant L^2. BFF coupling would reduce naturally to a product of three factors: Kähler coupling constant, coupling matrix dictated by gauge symmetry and quantum numbers of fermions and boson, and dimensional factor $1/p^2$ replacing L^2: here p is the momentum associated with the wormhole contact corresponding to gauge boson. This identification is indeed possible since wormhole contact property distinguishes bosonic line uniquely for BFF. BBB coupling would involve the product of three bosonic propagators in vertex and BBB cases. Possible BB vertex would have $1/p^2$ factor in vertex.

3. The only vertices would be BFF, BBB, and BB vertices: in BCFW construction these vertices are indeed enough since B^4 vertex of gauge theories is a consequence of off-mass shell gauge invariance and does not appear for on mass shell amplitudes. In graviton scattering infinite number of higher vertices are consequences of general coordinate invariance and BCFW construction is proposed to yield planar tree diagrams at least.

The basic objection is that bosonic emergence in this form neglects the stringy character of physical particles and cannot work as such. The following arguments show that this anticipation is correct.

6.5 About BCFW construction of scattering amplitudes

In the fundamental stringy description one can identify string world sheets as loci of the induced spinor fields solving the modified Dirac equation. The condition that electric charge for the spinor modes have a well-defined electric charge - despite the fact that projection of the vielbein connection of CP_2 to space-time surface defines classical electroweak gauge fields having also charged part - forces this [K12].

In the higher level description all fundamental fermions (not the elementary particles) are assumed to be on mass shell fermions in the sense that momenta are light-like. This corresponds to on mass shell property for modified Dirac equation at the microscopic level. In internal lines the fermions must have non-physical helicity since internal line contains the inverse of the Dirac propagator. This gives dimensions correctly when integration is allowed only over light-like momenta. This form can be also interpreted as outcome of residue integration over 4-momenta with massless fermion propagator so that an ad hoc assumption is not in question. Physical fermions and bosons are bound states of massless fundamental fermions and involve pairs of wormhole contacts and a Kähler magnetic monopole flux forming a closed flux loop.

This description leads to either QFT type description or to stringy description at imbedding space level. Both could rely on twistors if both real and virtual fermionic lines have light-like momenta. Hence one would have either QFT type or stringy type generalization of BCFW recursion.

For both options the two 3-vertices of SYM corresponding to $k = 1$ and $k = 2$ negative helicity gauge bosons (black and white) are replaced at microscopic level with fermionic 4-vertex with 2 positive and 2 negative helicities. One cannot assign any color to the vertex since one has 2 positive and 2 negative helicities. For 4-vertex kinematics allows the light-like momenta to be non-parallel and the vertex is not singular. The microscopic description of 4-fermion vertex in terms of the geometry of wormhole contact and its deformations was considered already earlier. For effective 3-vertex the bosonic state represented as wormhole contact is off mass shell and the ordinary and four-momentum conservation forces all four-momenta to be parallel if they are on mass shell and real.

/Users/mattipitkanen/Desktop/tgd/figures/BCFW.png

Figure 1: BCFW recursion relation in $\mathcal{N} = 4$ Grassmannian construction of scattering amplitudes

6.5.1 Is QFT type BCFW construction possible in TGD framework?

Is QFT type BCFW construction neglecting the stringy character of physical particles possible in TGD framework? We have already developed arguments suggesting that this approach fails but the best manner to learn more is to try and see.

1. The obvious manner to proceed is just as in the case of BCFW construction. Unitary cuts would correspond naturally to bosonic wormhole contacts and the two 3-vertices (BFF and BBB) of SYM corresponding to $k = 1$ and $k = 2$ negative helicity gauge bosons (black and white) represented at microscopic level with fermionic 4-vertex with 2 positive and 2 negative helicities. One cannot assign any color (black white) to the 4-fermion vertex since one has 2 positive and 2 negative helicities. For 4-vertex kinematics allows the light-like momenta to be non-parallel and the vertex is not singular. The microscopic description of 4-fermion vertex in terms of the geometry of wormhole contact and its deformations was considered already earlier.

   Effective 3-vertices co-emerge with bosons identified wormhole contacts formed from fermion-antifermion pairs and one obtains BFF and BBB vertices as in gauge theories. Virtual bosons are in general off mass shell although the fermion and antifermion composing them are massless and on shell but with non-physical helicity. Four-fermion coupling constant is by dimensional considerations proportional to either $1/p^2$ or the p-adic length scale L_p^2 assignable to p^2. For effective 3-vertex the bosonic state represented as wormhole contact is off mass shell and the ordinary four-momentum conservation forces all three four-momenta to be parallel if they are real and on masss shell.

2. The complexification of momenta would be carried exactly as in the case of gauge theories, and would bring in complex number z as a deformation parameter. By expressing the amplitude $A(z = 0)$ as a residue integral of the integral $\oint A(z)/z$ one would obtain sum over residues at poles outside origin and identifiable in terms of massless but complex virtual momenta for the bosons at the cut lines. The bosonic propagator in the cut would be the real momentum squared which does not vanish. What is not clear whether the pole at infinity cancels as in $\mathcal{N} = 4$ SUSY. One might hope that right-handed neutrino might allow to achieve this. If so, the recursion formula generalizes also for the planar loop diagrams. How to treat the non-planar situation remains a problem unless one assumes the vanishing of loops.

3. For BCFW diagrams the notion of move is essential. There are two basic moves for BCFW diagrams of SYM. Square move replaces in BCFW square diagram black 3-vertices with white and vice versa. In TGD framework square move does not make sense. Merge expand is second move and replaces BCFW tree diagram analogous to exchange in s-channel with an exchange in t-channel: the colors of the two vertices are same. In TGD framework there BFF and BBB vertices allow the analog of this move. In SYM context moves eliminate a large number of BCFW diagrams.

4. The vision about the reduction of continuous coupling constant evolution to discrete p-adic coupling constant evolution suggests that radiative corrections cold vanish identically due to the SUSY and that the convergence of the theory requires p-adic coupling constant evolution for four-fermion coupling $L^2 \propto L_p^2$. There are some arguments in favour of the vanishing of the loop corrections.

   The breaking of conformal invariance and SUSY takes place *only for the external states* identified as bound states of fermions (via the selection of the p-adic length scale) whereas internal fermion lines remain massless. Therefore the contributions of states and their spartners could cancel each other in self-energy and vertex corrections by the analogy with $calN = 4$ theory. Indeed, if these particular loop corrections are finite they must vanish since their is no scale parameter necessary to construct dimensionless variables from momenta appearing in the correction.

   If this argument generalizes to all loop corrections, BCFW would reduce to that for tree diagrams and non-planar diagrams would not produce any troubles. The objection is that p-adic length scale defines a dimensional parameter. If it appears only in the construction of massive external states as bound states of massless fermions as p-adic thermodynamics suggests, this objection does not bite. Note also that the massivation of external states would resolve the infrared divergences by bringing in natural infrared cutoff as p-adic mass scale.

All this looks good as long as one believes that one can forget the stringy character of the physical particles requiring that the propagation of both wormhole contacts is taken into account and that the fermionic loop defining normalization of the bosonic wormhole contact propagator is finite and non-vanishing. Unfortunately this does not seem to be the case.

6.5.2 Problem: wormhole fermionic loop diverges in absence of SUSY and vanishes in presence of SUSY!

The fermion loop assignable between two wormhole contacts is essential in the identification of the bosonic wormhole contact propagator. This loop must be by dimensional considerations proportional to p^2, where $p = p_1 + p_2$ is the total momentum of the propagating wormhole contact defined as sum of massless fermion momenta. The questions are following.

Is the resulting number is finite and non-vanishing when stringy character of elementary particles is neglected? Or is finiteness achieved only by integrating simultaneously over both virtual momenta associated with the ends of the string with vertex factor correlating the momenta at the wormhole contacts at the ends of the string?

The original naive expectation was that the light-likeness constraint could make the loop finite: unfortunately SUSY could imply its vanishing! On the other hand, the experience with QFT and string models suggests that strings are necessary and the following arguments support this expectation.

1. The loop integral is defined by performing residue integral over mass squared reducing integration to that over massless momenta for each fermion line restricted by the momentum conservation constraints in various vertices. Since integration measures $d^3p_i/2E_{p_i}$ give massdimension 4, momentum conservation delta function has mass dimension -4, and there are two inverses of massless fermion propagator, the over-all integral has mass dimension 2. A p^2 factor however factors from the fermionic trance and fermionic loop reduces to $p^2 \times \int_{S^2}(E_1/E_2)d\Omega$, where the ratio E_1/E_2 for the energies for fermions depends on angle. Since p^2 can be space-like also negative energies must be allowed. The absolute value of energy must appear in $d^3p_i/2E_{p_i}$ so that the integration measure is positive definite. Singularities of the integrand result as E_1 approaches infinite or E_2 approaches zero and this is possible when p^2 is light-like or space-like. Logarithmic singularities are expected.

2. If the loop is convergent without cutoff, the resulting integral is by dimensional considerations proportional to p^2, and one obtains the standard form of the bosonic propagator if the BFF vertex is proportional to $1/p^2$ (it could be also proportional to the square L_p^2 of the p-adic length scale assignable to p^2). Note that the invariant $p_1 \cdot p_2$ equals to $p^2/2$ so that one cannot and to the vertex dimensionless Lorentz invariants possibly guaranteeing the finiteness of the integral.

Two catastropic events could happen.

1. Formally this integral is just the ordinary diverging fermionic loop encountered in massless gauge theory. Optimistic could argue that just by the divergent character of the loop in the ordinary approach, one could achieve a finite result without posing a cutoff: the residue integral description might be seen as regularization procedure. If the integral divergences, a physical regularization involving the introduction of a p-adic cutoff momentum having interpretation in terms of measurement resolution - lower limit for the size of CDs involved- could give rise to logarithmic factors $log(p^2L_p^2/h_{eff}^2)$. This is very natural expectation in the approach based on QFT. One however want something more elegant than QFT.

2. There is also another catastrophe lurking there. The supersymmetry induced by the possibility to have spartners of fermions in the loop corresponding to 4 states constructed from covariantly constant ν_R and its charge conjugate $\overline{\nu}_R$ would most naturally imply that the loops sum up to zero! This result holds completely generally if virtual fermions are massless.

We are clearly sailing between Scylla and Charybdis!

6.5.3 Stringy variant of BCFW construction

Suppose that the doomsday scenario for QFT type BCFW is realized: the basic fermionic loop diverges without SUSY and vanishes for SUSY. How to proceed? First of all, one must remember that one is basically constructing zero energy states rather than scattering matrix between positive energy states. Hence the only rule to be obeyed is that the zero energy state is well-defined mathematically and therefore free of divergences.

1. It is physically completely natural and in harmony with the vision about finite measurement resolution that CP_2 length scale or some p-adic length scale would define a momentum cutoff. The challenge is to formulate the cutoff in an elegant manner as a restriction on the momentum squared of the wormhole contact propagator emerging spontaneously rather than being put in by hand. The natural assumption is that this cutoff applies only to positive and negative energy parts of the zero energy states and not on propagators and vertices.

2. In string theory one can avoid infinities and this suggests the introduction of the stringy description from the beginning as required also by the fact wormhole contacts carry Kähler magnetic charges.

   (a) The standard stringy approach would be stringy perturbation theory based on superconformal algebra. This would bring in CP_2 scale and perhaps also the hierarchy of p-adic length scales defining the mass scale of conformal excitations. In TGD framework however the fact that the fermionic generators of the super-conformal algebra carry fermion number seems to produce insurmountable difficulties in this approach.

   (b) The natural constraint is that p-adic length scale is associated only with the positive and negative energy parts of the states and does not affect at all the online massless propagation of fermions. This suggests a fresh approach to strings based on twistors and Grassmannians. Virtual fundamental fermions remain massless but form only basic building bricks of real and virtual particles identified as pair sof wormhole contacts. For physical particles both fermionic and bosonic propagator lines are replaced by pairs of string world sheets with wormhole contacts at their ends. Also hadronic strings result in this manner. Stringy structure implies the breaking of the generalization of the 2-D conformal invariance and the fact that covariantly constant right-handed neutrinos are associated with entire space-time sheet implies SUSY breaking. Hence stringy propagators for elementary particles can be finite and non-vanishing.

The rough vision about stringy diagrammatics and its BCFW variant would be following.

1. To avoid confusion, it should be made clear that one has three kinds of lines to consider.

   (a) Fundamental fermion lines. These are assigned to wormhole throats and accompanied by massless fermion propagators. After residue integration over p^2 they give rise to inverses $p^k\gamma_k$ of the massless Dirac propagator estimated on shell and non-vanishing only when the fundamental fermion has non-physical helicity. This micro-anatomy is not present in string models and expresses the idea that all particle states emerge from massless on mass shell fermions making in turn possible to express the momenta of wormhole contacts and of string like object itself in terms of twistors. This gives hope about BCFW recursion with cuts defined for fermionic and bosonic strings. Also Grassmannian formulation might make sense since it results from momentum conservation for states decomposing into many particle states carrying massless momenta.

   (b) Wormhole contact lines. In the bosonic case these contain inverses $p^k\gamma_k$ of massless fermion propagators at the two fundamental fermion lines. The ends of the wormhole contact line contain the generalization of bosonic propagator $1/p_i^2$ to $1/L_{0,i}$ as vertex factors at the four-fermion vertices at its ends. Fermionic wormhole contact line involves the super generator G and its hermitian conjugate $G^\dagger$ at the 4-fermion vertices at the ends of the line. This boils down to the general assumption that each fermion line in the 4-fermion vertex contains $1/G$ or $1/G^\dagger$. For bosonic wormhole contacts this gives $1/GG^\dagger = 1/L_0$ as vertex factor. These replacements bring in the dependence on CP_2 length scale defining physical UV cufoff. Note that p-adic thermodynamics is associated with external lines only. The earlier proposal [K3] is that Neveu-Schwartz *resp.* Ramond representation of Super Virasoro algebra occurs for quarks *resp.* leptons.

The problem in understanding stringy diagrammatics in TGD framework has been that G and $G^\dagger$ carry fermion number rather than being hermitian operators as in superstring model relying on the Majorana property of spinors. The solution of the problem emerges from the that that in the recent approach the ends of fermion number carrying wormhole contact contains $1/G$ and $1/G^\dagger$ respectively so that at the low energy limit one obtains just ordinary Dirac propagator.

(c) Stringy lines. Stringy propagator for a physical particle is obtained by integrating over light-like momenta of the fermionic lines. Correlations between the momentum integrations follow only from the momentum conserving delta function.

The first difference with respect to string models is that massless fermions are fundamental and strings are emergent, and also physical particles are string like objects. Second difference is that the super generator G carries fermion number. Third difference is on mass shell light-likeness of fundamental fermions giving hopes about the applicability of twistor Grassmannian formalism.

2. The momentum conservation constraint for the string like object makes vertices non-local in the scale of string. Stringy emergence allows only this kind of non-locality. One could of course consider also a more general non-locality. Stringy vertex could contain a dependence on the invariants constructed from the light-like fermionic momenta p_i at the ends of the string. These invariants correspond to dimensionless invariants X_{ij}/X_{kl}, $X_{ij} = p_i \cdot p_j$ and $p^2 = (\sum p_i)^2$. If the first wormhole contact carries only fermion and second wormhole contact a neutrino-antineutrino pair neutralizing weak isospin, one obtains 3 inner products and three dimensionless invariants. If both ends correspond to bosons one obtains 4 inner products affecting the stringy loop integral.

3. As explained, bosonic vertex factors $1/p_i^2$ are replaced with the Virasoro generators $1/L_{0,i}$. In the case of of fermionic lines single particle super Virasoro generator $1/G_i$ defines the analog of the inverse of the Dirac operator $p_i^k\gamma_k$ at the level of "world of classical worlds" (WCW). There is however a problem here.

   (a) If fermionic wormhole contacts carry momentum only at the second throat, they are massless and the dependence of G on four-momentum disappears completely since it reduces to the sum of CP_2 part and "vibrational" part. p-Adic mass calculations however suggest that also the second throat must carry massless four-momentum.

   (b) The most obvious manner to overcome the problem relates to the electroweak symmetry breaking requiring a pair of left- and right handed neutrinos to cancel the net weak charge in the length scale of string. One could also assume that the right-handed anti-neutrino and fermion reside at the first wormhole contact so that this state can develop mass squared by p-adic thermodynamics whereas left handed neutrino would reside at the second wormhole throat but could not develop any mass squared in this manner. The roles of ν_R and could be of course changed. Note also that the existence of ν_R modes delocalised at the entire space-time sheet of string like object does not mean the non-existence of modes localised at wormhole throats and the mixing of left- and right-handed neutrinos implied by modified gamma matrices indeed suggests this.

   (c) One can of course wonder whether this problem might be a connected with Higgs mechanism and vacuum expectation of Higgs: could fermionic wormhole contact contain Higgs or analog of coherent Higgs state? This does not seem plausible. TGD predicts Higgs like scalar particles but no Higgs vacuum expectation since p-adic thermodynamics explains massivation. The mass-proportionality of the couplings of Higgs to fermions follows form gradient coupling with same universal scalar coupling so that no problems with naturally are encountered.

4. Super conformal generators contain the dependence on CP_2 length scale so that the cutoff mass scale emerges naturally and without any ad hoc procedures. It is essential that wormhole contact propagators are correlated by momentum conservation constraint as parts of stringy propagator:

this expresses non-locality in the scale of string. At low energy limit one can replace strings with points and stringy propagators with ordinary propagators. This forces to pose artificial UV cutoff in order to obtain a finite boson propagator.

5. The expressions for the stringy propagators should remain non-vanishing as one performs sums over spartners. Here one must notice that covariantly constant right-handed neutrinos are associated with the interiors of space-time sheets, and one is led to double counting if one assumes independent super-symmetries at the ends of the stringy propagator might lead to the breaking of SUSY in the p-adic length scale in question. This is completely analogous to the reduction of rotational symmetry in two particle system to that for cm degrees of freedom. The summation over spartner combinations is possible only for stringy propagators and approximate SUSY can guarantee that they are finite and non-vanishing.

To get a more concrete view about stringy propagators it is good to look at two examples.

1. Consider a stringy diagram with bosonic wormhole contact propagator $1/L_{0,i}$ at both ends of wormhole contact orbit and reducing to $1/p_i^2$ at low energy limit. There are 4 fermionic momentum integrations d^3p_i/E_{p_i}, 4-D delta function for momentum conservation, and 4 inverse fermionic propagators: this gives contribution $\Delta D = 4\times 2-4+4 = 8$ to the mass dimension of the integral. Bosonic emergence suggests $1/p_i^2$ factor from both ends of each bosonic propagator identifiable as low energy limit of the Virasoro generator $1/L_0$: p_i^2 *cannot* be taken outside the integral sign now so that one obtains the contribution $\Delta D = -4\times 2 = -8$ to the overall mass dimension of the integrand. Bosonic propagator must have mass dimension -2 so that there must be an additional overall factor with mass dimension $D = -2$. This gives hopes about convergence of the integral. This additional factor could correspond to string tension identified in terms of CP_2 scale or p-adic scale. The presence of Super Virasoro generators bringing in dependence on CP_2 mass scale is expected to be crucial for the cancellation of UV divergences.

2. Similar consideration applies to the propagator of physical fermion with fermionic propagator at the first wormhole contact (idealization only) and bosonic propagator at second wormhole contact. There are 3 fermionic momentum integrations d^3p_i/E_{p_i}, 4-D delta function for momentum conservation, and 3 inverse fermionic propagators: this gives contribution $\Delta D = 3\times 2-4+3 = 5$ to the mass dimension of the integral. $1/p_i^2$ factor from both ends of each bosonic propagator gives a contribution $\Delta D = -4$ to the overall mass dimension of the integrand. The overall contribution to the mass dimension is $\Delta D = 1$ from these sources. Fermionic propagator must have mass dimension -1 so that there must be an additional overall factor with mass dimension $D = -2$ identifiable in terms of string tension.

The overall conclusion would be that although fundamental fermions propagate as massless particles, physical particles can propagate only as string like objects as forced also by the Kähler magnetic charges of wormhole throats. Stringy propagators are finite and non-vanishing by general dimensional arguments. The QFT type BCFW works also assuming that SUSY is broken at some very high mass scale but one must introduce CP_2 scale as cutoff scale in order to obtain finite and non-vanishing bosonic propagators.

6.5.4 Twistorialization of gravitation by twistorial string diagrams

The twistorialization of gravitation is problem of the standard twistorialization approach since curved space-times do not allow twistor structure. In TGD framework this is not a problem. The above approach giving QFT type picture treats particles as wormhole contacts neglecting the fact that second wormhole contact must be present by the conservation of magnetic flux and absence of Dirac monopoles (magnetic flux lines are closed). The other wormhole contact carries weak quantum numbers neutralizing the weak quantum numbers of particles in the case of leptons. In the case of quarks the cancellation of Kähler (color-) magnetic charge might take place only at the level of the entire hadron. For gravitons second wormhole contact is necessary in order to obtain spin two states and this forces stringy picture.

The generalization of twistorial diagrams to twistorial string diagrams is forced by the replacement of wormhole contacts with pairs of them and connected by closed fermionic string (having pieces at separate space-time sheets) and also by the failure of the QFT type BCFW approach. Localization to string world sheets is implied by the modified Dirac equation and the requirement of well-defined em charge for spinor modes either than right-handed neutrino. The lines of Feynman diagrams are replaced by closed strings connecting two wormhole contacts along first space-time sheet and returning along the second one. Elementary particles correspond to pairs of string world sheets: for fermions second string world sheet is empty and for bosons the two string world sheets carry fermion and antifermion quantum numbers.

6.5.5 Could positivity be a prerequisite for number theoretical universality?

Physics as infinite-dimensional geometry of WCW ("world of classical worlds") [K5]and physics as generalized number theory [K7] are the two complementary visions about TGD. For the latter vision number theoretical universality has served as the basic guide line. It states that scattering amplitudes should make sense in both real and p-adic number fields and their algebraic extensions (and perhaps even non-algebraic but finite-dimensional extensions, say the extension obtained by adding Neper number e). This principle suggests an interpretation for the positivity of Grassmannian as a prerequisite for p-adicization [K8].

Already p-adic mass calculations [K3] forced to consider the question how to map real and p-adic numbers to each other. One can imagine two quite different manners to achieve this.

1. Direct correspondence via rationals would respect algebra and symmetries realized in terms of matrices with rational elements. It is however extremely discontinuous and not complete since p-adic integers for which the pinary expansion is infinite and not periodic do not correspond to any rational number.

2. Canonical identification- call it I - maps the pinary expansion for a positive real number to p-adic pinary expansion by just inverting the powers of p: $\sum x_n p^n \rightarrow x_n p^{-n}$. It is continuous map in both directions but maps two p-adic numbers to single real number as the p-adic generalization of $1 = .99999...$ implies. Therefore the inverse is two valued for real numbers with a finite number of pinary digits. Canonical identification respects continuity but not algebra and breaks algebraic symmetries. There is clearly a tension between symmetries and continuity.

As discussed in [K13], one can define a variant of canonical identification which is a kind of compromise between algebra and topology.

1. This variant maps positive real rationals smaller than some power p^N to itself so that symmetries are realized algebraically in finite measurement resolution. Reals with larger number of pinary digits are mapped by canonical identifying based on expansion in powers of p^N by mapping coeffiecients $0 \leq x_n < p^N$ to itself and inverting powers of p^N: $\sum x_n p^{nN} \rightarrow \sum x_n p^{-nN}$. Now continuity is respected.

2. Canonical identification in this generalized sense is used to define the notion of p-adic manifold as what might be called cognitive representation of real manifold. The inverse map define the space-time correlate for intention. The basic ideas is that chart leafs for p-adic manifold are not p-adic but real and canonical identification defines them. This allows to transfer basic notions of real topology to p-adic context. One can also define p-adic chart leafs for real manifold and they have interpretation as space-time correlates cognitive representations.

3. The condition that preferred externals of Käbler action appear at both real and p-adic sides brings in additional pinary cutoff. The preferred extremal property of Kähler action forces canonical identification since the canonical image ofa real (p-adic) space-time surface would not be differentiable in p-adic (real) sense. This requires finite pinary cutoff $M > N$ for the canonical identification. The cutoff has an interpretation as finite measurement resolution in the sense that chart maps involve only discrete set of points on both sides. The completion of discrete point set to a preferred extremal can be performed on both sides. Note that the completion need not be unique but this is of course consistent with the finite measurement resolution.

There is a problematic feature related to the canonical identification and possibly closely related to the positivity. What to do with *negative* real numbers. p-Adic -1 has representation $(p-1)(1+$

$p + p^2 + ...$) and maps by the inverse of the canonical identification to a positive real number p. Hence one cannot map real -1 to p-adic -1. Canonical identification makes sense only for non-negative reals.

1. One cannot introduce a p-adic counterpart of real -1 via algebraic extension of p-adic numbers as $-1 \equiv exp(i\pi)$ interpreted as a phase factor defining angle π and map it to real -1 since $(exp(i\pi) - 1)(exp(i\pi) + 1)$ would vanish and one would not have a number field anymore. Note that in p-adic context all angles $2\pi/N$ for prime values of N must be introduced via algebraic extensions of p-adics since the obvious candidates for the p-adic trigonometric functions are not periodic. This forces finite angular - or rather phase - - resolution.

2. A possible manner to cope with the situation would be to divide real axis to positive and negative half-axes and interpret reals as a 1-dimensional manifold with two coordinate charts and use positive coordinate for both so that p-adic counterpart could be defined by canonical identification. This construction generalizes to n-dimensional case in an obvious manner.

What makes this so interesting is that everything in the positive Grassmannian approach is positive as Nima and Jaroslav Trnka state it. The positivity of Grassmannian means positivity of all elements of $k \times n$ matrix and of all minors associated with the rows labelled by integers $< i < j < ....$ Also the scattering amplitude itself is positive as a volume as are also external data - at least in the signature (1,1,-1,-1). Could this be interpreted as guaranteeing number theoretical universality allowing to algebraically continue from real to p-adic context using some variant of canonical identification with a cutoff. Of course, an interesting question is what happens as one continues to other signatures.

6.5.6 What about CP_2 twistorialization?

CP_2 allows twistorialization in terms of 6-D flag manifold $F = SU(3)/U(1) \times U(1)$ having interpretation as a space for the choices of all possible quantization axes for color isospin and hypercharge defined by the Cartan algebra $u(1) \oplus u(1)$. The coordinatization for the choices of quantization axes corresponds to the complex coordinates assignable to π^+, K^+, K^0 and their complex conjugates assignable to $\pi^-, K^-, \overline{K}^0$ in the octet representation of tangent space of $SU(3)$ in terms of generators with quantum numbers of mesons.

It is of course far from obvious whether twistorialization in CP_2 degrees of freedom is useful. The original argument was that twistorialization is necessary for color symmetry but this argument need not be quite correct. One might quite well consider the possibility that one has just color conservation in vertices. If this is the case the color would be present rather passively. Hence it makes sense to ask what twistorialization in CP_2 degrees of freedom could make sense and what it could mean. In particular, one can ask whether the crucial vanishing of total momentum as a constraint generalizes to the case of color quantum numbers.

1. Should one introduce the choices of color quantization axes as a moduli space assignable to the external particles and over which it might be necessary to integrate? I_3 and Y define the counterparts of momentum components and correspond to the complement of twistorial tangent space in SU(3) Lie-algebra. One might hope that incidence relations make sense for the Hamiltonians representing I_3 and Y in F as bilinears of holomorphic coordinates and their conjugates.

2. If Lie group G acts in symplectic manifold M, the so called moment map assigns to the Lie-algebra generators of G their Hamiltonians in M as inner product of Killing vector field and 1-form defining the momentum map. F is symplectic manifold because it is Kähler manifold. Rather remarkably, only the twistor spaces associated with CP_2 and S^4 are Kähler manifolds in 4-D Eucdlian case [?] (`http://www.math.ucla.edu/~greene/YauTwister(8-9).pdf`). Furthermore, S^4 does not allow Kähler structure so that CP_2 and M^4 are completely unique! F is known to allow two non-equivalent Einstein metrics (Einstein tensor proportional to the metric tensor).

3. The vanishing of the total momentum for the diagram should have CP_2 analog and one might hope that the linearization of this constraint could lead to Grassmannian formulation. The vanishing of the sums $\sum_i Y_i$ and $\sum_i I_{3,i}$ of hyper-charge isospin Hamiltonians represent the vanishing of total quantum numbers and would select a co-dimension 2 sub-manifold in the Cartesian

product of twistor spaces associated with the external particles and in this manner correlate CP_2 twistorial degrees of freedom. If the Hamiltonians Y and I_3 are bilinear in holomorphic twistor coordinates and their conjugates and therefore analogous to harmonic oscillator Hamiltonians, the constraint is quadratic and there are hopes about the analog of Grassmannian formulation obtained by linearizing the constraints. The exterior product $\prod_k^{\wedge} \omega_k \wedge J_n^2$, $\omega_i = J_i \wedge J_i \wedge J_i \equiv \wedge J_i^3$ or the symmetrization of this form would define the symplectic volume form to be used in the integration.

4. Does the notion of positive/negative helicity have any meaning in CP_2 degrees of freedom? For M^4 spinors helicity corresponds to the eigenvalues of γ_5. The eigenvalue of γ_5 in CP_2 part of the imbedding space spinor would define the notion of helicity in CP_2 degrees of freedom. These eigenvalues are correlated since their product tells whether the imbedding space chirality of spinor corresponds to quark or lepton. They are of same sign for quarks and of opposite sign for leptons (this is of course a convention only). For antiparticles the signs are opposite. Anomalous hyper-charge could play the role of helicity since it has opposite sign for fermions and anti-fermions.

6.6 About emergence

Nima's dream is that not only gauge symmetry (that is gauge redundancy), unitary, locality, space-time and even quantum theory emerge from their approach and claim that positivity and interpretation as scattering amplitude as volume is the fundamental principle implying even quantum theory.

I cannot agree with this. For me it is much more natural to interpret the representation of scattering amplitude as a volume as emergence forced by fundamental physical principles. Even a new fundamental principle would be involved and would be number theoretical universality involving p-adicization using canonical identification: this requires positivity unless additional assumptions are made. In any case, it is interesting to consider the emergence from TGD point of view.

1. Consider first the emergence of space-time. Twistors are present and represent four-momentum. For Minkowskian signature twistors are possible only in Minkowski space so that not only space-time but also M^4 seem to be necessary. This means a severe problem for the twistorial approach to gravitation ("googly" problem). Space-time as 4-surface in $M^4 \times CP_2$ is the elegant solution allowing twistorialization also in CP_2 degrees of freedom. Also half-odd integer spin and SUSY are involved and require M^4.

2. One can say that in TGD electroweak gauge symmetries emerge from the geometrical gauge symmetry related to the freedom to choose vielbein. Electro-weak gauge group corresponds to the holonomy group of CP_2 having concrete geometric interpretation. Global gauge transformations do not mean mere gauge redundancy. Color symmetries correspond to isometries of CP_2 and color gauge symmetry is approximate and emergent at long length scales.

3. Gauge bosons and graviton emerge in TGD as bound states of massless fundamental fermions defining the fundamental particle like excitations. Even the representations of infinite-dimensional super-conformal symmetry algebras emerge and their states are expressible as bound states of massless fermions. There are also the WCW degrees of freedom represented as Super-Kac-Moody and super-symplectic algebras in WCW and one can assign color degrees of freedom to these as well as stringy geometric degrees of freedom relevant for hadron like objects. Fermionization allows to have non-singular fundamental vertices and allows real light-likeness for internal lines.

4. In zero energy ontology (ZEO) one must introduce what I have called U-matrix having as rows M-matrices, which are products of hermitian square roots of density matrices with unitary S-matrix. Each M-matrix corresponds to an analog of S-matrix in thermal QFT and S-matrix should have the standard interpretation. Therefore the notion of unitary is generalized. Locality is definitely lost since point-like particle is replaced with 3-surface - or by strong form of holography with particle 2-surface together with its 4-D tangent space data defining the basic dynamical unit. Locality emerges at the point-like limit of the theory.

5. Yangian symmetry in $\mathcal{N} = 4$ SYM extends the conformal symmetries of M^4 and should be present also in TGD framework. Besides this there is a generalization of the Yangian symmetry

with super-conformal algebras associated with partonic 2-surfaces and the integer $n = 1, 2, ..$ defining the characteristic "n-point" property of the generators of Yangian corresponds concretely to the number partonic 2-surfaces to which the Yangian generator acts. Hence the finite-dimensional conformal Lie-algebra is replaced with infinite-dimensional conformal algebras assignable with the collections of partonic 2-surfaces associated with the space-time surface.

In the case of $N = 4$ SYM conformal Yangian corresponds to diffeomorphisms preserving the positive Grassmannian property of the polytope (intuitively clear since conformal invariance respects light-likeness). Whether also the huge Yangian associated with super-conformal symmetries acts as a symmetry of the polytope possibly associated with the scattering amplitudes in TGD framework is an open question. Certainly these scattering amplitudes must have additional symmetries if all loop corrections in Feynman sense vanish.

6.7 Possible problems

Consider next the possible problems of $\mathcal{N} = 4$ SYM and TGD approach assuming that the proposed conjecture makes sense.

1. Ordinary Grassmannian approach applies only to planar Feynman graphs. Stringy twistorialization and BCFW recursion is free of this problem.

2. Gravitation is problem in standard QFT approach since twistors make sense only for M^4 if Minkowskian signature is assumed. Sub-manifold gravity of TGD would resolve the problem. Twistor diagrams have a natural stringy generalization forced by internal consitsency and allowing the description of all elementary particles in similar footing.

3. The basic problem of $\mathcal{N} = 4$ SYM is that there is no coupling constant evolution. For stringy BCFW one has SUSY breaking and non-vanishing loops so that the problem is probably not encountered. p-Adic coupling constant evolution is however a highly attractive notion in TGD framework. Coupling constant evolution would discretize and mass squared scales would given by inverses of primes with primes near certain powers of two favored.

   Discretization would mean that each interval between two subsequent prime corresponds to a fixed point of renormalization group. Primes or preferred primes would label the fixed points of coupling constant evolution. Also the scales of CDs could define mass scale hierarchy. No breaking of conformal symmetry and SUSY would take for internal fermion lines and these symmetries would be broken only for the external states and characterized by p-adic mass scale defining also natural IR cutoff.

4. Nima notices in his lecture that BCF equations have exactly the same form as renormalization group equations. In TGD framework the equations would indeed state the triviality of the renormalization group flow and different solutions for the condition satisfied by 4-vertex could correspond to the hierarchy of CDs, to different p-adic primes, or subset of them allowed by p-adic length scale hypothesis.

5. The connection with the notion of finite measurement resolution is interesting. Intuitively finite length scale resolution corresponds to a minimum size scale for the causal diamonds (CDs) taken into account in the generalized Feynman diagrams. In the similar manner upper size scale for CD corresponds to IR cutoff. Does the proposed description make sense only for single CD? Or should one combine different CDs somehow in the general situation? Hyperfinite factors [K10] have been proposed to describe the finite measurement resolution and the question is whether there is a hierarchy of polytopes corresponding to the hierarchy of CDs/p-adic length scales. Does the inclusion for HFFs correspond to inclusion of corresponding CDs with sub-CD defining measurement resolution?

7 Conclusions

The conclusions of these lengthy considerations are following.

1. Twistorialization takes place naturally at the level of imbedding space and twistor space is Cartesian product of those associated with M^4 and CP_2. The twistor space has interpretation as a flag manifold characterizing the choices of quantization axes for longitudinal momentum components and spin and for isospin and hyper-charge. The integration over twistor space guarantees Lorentz invariance and color invariance.

2. The Super Virasoro conditions apply only to the entire physical states associated with particle like 3-surfaces containing in general several partonic 2-surfaces. These states can be regarded as bound states of in general non-parallelly propagating massless fermions. Virtual fermions are massless but possess wrong polarization and residue integral replaces fermion propagator with its inverse making sense mathematically. The light-likeness conditions for light-like 3-surfaces allow to deduce the general form of Virasoro conditions. Covariantly constant right-handed neutrinos could define the fermion number conserving analog of $\mathcal{N}=4$ SUSY.

3. Apart from CP_2 twistorialization the resulting formalism resembles closely the Grassmannian twistor formalism with one important exception. The 3-vertex of gauge theories is replaced with fermionic 4-vertex which is non-vanishing also for non-parallel on mass shell real momenta and thus avoids the IR singularity of gauge theory vertex.

4. At the level of WCW twistorial incidence relations have an analogy following from expressibility of Kac-Moody generators as sums of bosonic parts analogous to M^4 coordinates and fermionic parts bilinear in fermionic operators creating WCW spinors and thus analogous to spinors. The attempt to generalize four-momentum conservation to quadratic conditions for WCW spinors fails.

Twistor formalism allows to construct the analogs of Feynman rules for QFT limit of TGD. This process has been rather tortuous and has involved several unpleasant surprises and there are still many open problems.

1. The generalization of the BFCW recursion relation using 4-fermion vertex with fermions of internal lines massless in real sense and possessing unphysical helicity can be considered. Bosonic emergence is essential element of the construction and suggests a construction very similar to that in gauge theories involving only BFF and BBB vertices as fundamental vertices. This approach however encounters a serious difficulty: contrary to the original optimistic expectations, the fermionic loop defining bosonic wormhole propagator diverges without SUSY but vanishes with SUSY.

2. The only manner to circumvent the problem is to begin from stringy propagators for real elementary particles identified as pairs of wormhole contacts as required by the Kähler magnetic charges of wormhole throats. Since SUSY is associated with the entire space-time sheet, it does not apply to individual wormhole throat lines separately and does not imply the vanishing of bosonic wormhole throat propagators. As a matter of fact, one cannot even define these propagators since string is the basic object. Stringy propagators in turn remain finite. The challenge is to generalize the BFCW recursion relations. The natural guess is that BFCW cuts are performed for the string world sheets by making some momenta complex. Loops would correspond to stringy loops. In the stringy approach the problems due to non-planarity disappear. There is no spesific reason to except the vanishing of stringy loops.

3. The generalization to gravitational sector is not a problem in sub-manifold gravity since M^4 - the only space-time geometry with Minkowski signature allowing twistor structure - appears as the Cartesian factor of the imbedding space. Furthermore, CP_2 is the only Euclidian 4-D Kähler manifold allowing twistor space with Kähler structure. The analog of twistorial construction in CP_2 degrees of freedom based on the notion of flag manifold can be considered but the situation remains unclear. Graviton as stringy object is geometrically very similar with ordinary elementary particles.

4. Discrete p-adic coupling constant evolution with local RG invariance is very attractive notion giving a very profound role for the p-adicity but not required by the stringy BFCW. Positivity of Grassmannian - assuming that amplitudes reduce to something proportional to amplituhehdron volume - might be necessary in order to achieve number theoretical universality.

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