Twistor lift of TGD and WCW geometry

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/.

June 20, 2019

Abstract

The recent view about geometry of "world of classical worlds" (WCW) forced by twistor lift of TGD and zero energy ontology (ZEO) is summarized. Twistor lift brings to the action a volume term but without breaking conformal invariance and without introducing cosmological constant as a fundamental dimensional dynamical coupling. The proposed construction of the gamma matrices of WCW giving rise to Kähler metric as anti-commutators is now in terms of the Noether super charges associated with the super-symplectic algebra. This I dare to regard as a very important step of progress.

1 Possible weak points of the earlier vision about WCW geometry and spinor structure

To make progress it is wise to try to identify the possible weak points of the earlier vision.

1. The huge vacuum degeneracy of Kähler action [K1] defining the Kähler function of WCW Kähler metric is analogous to gauge degeneracy of Maxwell action and coded by symplectic transformations of CP_2. It implies that the degeneracy of the metric increases as one approaches vacuum extremals and is maximal for the space-time surfaces representing canonical imbeddings of Minkowski space: Kähler action vanishes up to fourth order in deformation. The original interpretation was in terms of 4-D spin glass degeneracy assumed to be induced by quantum degeneracy.

   One could however argue that classical non-determinism of Kähler action is not acceptable and that a small term removing the vacuum degeneracy is needed to make the situation mathematically acceptable. There is an obvious candidate: a volume term having an interpretation in terms of cosmological constant. This term however seems to mean the presence of length scale as a fundamental constant and is in conflict with the basic lesson learned from gauge theories teaching that only dimensionless couplings can be allowed.

2. The construction of WCW Kähler metric relies on the hypothesis that the basic result from the construction of loop space geometries [A1] generalizes: the Kähler metric should be essentially unique from the condition that the isometry group is maximal - this guarantees the existence of Riemann connection. For $D = 3$ this condition is expected to be even stronger than for $D = 1$.

   The hypothesis is that in zero energy ontology (ZEO) the symplectic group acting at the light-like boundaries of causal diamond (CD) (one has CD= $cd \times CP_2$, where cd is the intersection of future and past directed light-cones) acts as the isometries of the Kähler metric.

   It would be enough to identify complexified WCW gamma matrices and define WCW metric in terms of their anti-commutators. The natural proposal is that gamma matrices are expressible as linear combinations of fermionic oscillator operators for second quantized induced spinor fields at space-time surface. One could even ask whether fermionic super charges and conserved fermionic Noether charges are involved with the construction.

The explicit construction of gamma matrices [K2, K3] has however been based on somewhat ad hoc formulas, and what I call effective 2-dimensionality argued to follow from quantum criticality is somewhat questionable as exact notion.

2 Twistor lift of TGD and ZEO

Twistor lift of TGD and ZEO [K5, K4, K6, K7] meant a revolution in the view about WCW geometry and spinor structure.

1. The basic idea is to replace 4-D Kähler action with dimensionally reduced 6-D Kähler for the analog of twistor space of space-time surface. The induction procedure for the spinors would be generalized so that it applies to twistor structure [L4]. The twistor structure of the imbedding space is identified as the product of twistor spaces $M^4 \times S^2$ of M^4 and $SU(3)/U(1) \times U(1)$ of CP_2. In momentum degrees of freedom the twistor space of M^4 would be the usual CP_3.

   Remarkably, M^4 and CP_2 are the only spaces allowing twistor space with Kähler structure [A2]. In the case of M^4 the Kähler structure is a generalization of that for E^4. TGD would be unique from the existence of twistor lift. This predicts CP breaking at fundamental level possibly responsible for CP breaking and matter-antimatter asymmetry.

2. One would still have Kähler coupling strength α_K as the only single dimensionless coupling strength, whose spectrum is dictated by quantum criticality meaning that it is analogous to critical temperature. All coupling constant like parameters would be determined by quantum criticality. Cosmological constant would not be fundamental constant and this makes itself visible also in the concrete expressions for conserved Noether currents. The breaking of the scale invariance removing vacuum degeneracy of 4-D Kähler action would be analogous to spontaneous symmetry breaking and would remove vacuum degeneracy and classical non-determinism.

   The volume term would emerge from dimensional reduction required to give for the 6-surface the structure of S^2 bundle having space-time surfaces as base space. Cosmological constant would be determined by dynamics and depend on p-adic length scale depending in the average on length scale of space-time sheet proportional to the cosmic time sense like $1/a^2$, a cosmic time. This would solve the problem of large cosmological constant and predict extremely small cosmological constant in cosmic scales in the recent cosmology. This suggests that in long length scales one still has spin glass degeneracy realized in terms of many-sheeted space-time.

3. In ZEO 3-surface correspond to a union of 3-surfaces at the ends of space-time surfaces at boundaries of CD. There are many characterizations of quantum criticality.

   (a) Preferred extremal property and quantum criticality would mean that one has simultaneously an extremal of both 4-D Kähler action and volume term except at singular 2-surfaces identified as string world sheets and their boundaries. In accordance with the universality of quantum critical dynamics, one would have outside singularities local dynamics without dependence on Kähler coupling strength. The interpretation would be as geometric generalization of massless fields also characterizing criticality.

   (b) Another characterization of preferred extremal is as a space-time surfaces using sub-algebra S_m of symplectic algebra S for which generators have conformal weights coming as m-tuples of those for the full symplectic algebra. Both S_m and $[S, S_m]$ would have vanishing Noether charges. For the induced spinor fields analogous condition would hold true. Effectively the infinite number of radial conformal weights of the symplectic algebra associated with the light-like radial coordinate of $\delta M^4_{\pm}$ would reduce to a finite number.

   (c) A further characterization would be in terms of $M^8 - H$ duality [L1]. Preferred extremals in H would be images of of space-time surfaces in M^8 under $M^8 - H$ duality. The latter would correspond to roots of octonionic polynomials with coefficients in an extension of

rationals. Therefore space-time surfaces in H satisfying field equations plus preferred extremal conditions would correspond to surfaces described by algebraic equations in M^8. Algebraic dynamics would be dual to differential dynamics.

(d) In adelic physics [L2, L3] the hierarchy of Planck constants $h_{eff}/h_0 = n$ with n having an interpretation as dimensions of Galois group of extension of rationals would define further correlate of quantum criticality. The scaled up Compton lengths proportional to h_{eff} would characterize the long range fluctuations associated with quantum criticality.

3 The revised view about WCW metric and spinor structure

In this framework one can take a fresh approach to the construction of the spinor structure and Kähler metric of WCW. The basic vision is rather conservative. Rather than inducing ad hoc formulas for WCW gamma matrices one tries to identify Noether the elements super-algebra as Noether charges containing also the gamma matrices as Noether super charges.

1. The simplest guess is that the algebra generated by fermionic Noether charges Q^A for symplectic transformations $h^k \to h^k + \epsilon j^{Ak}$ assumed to induce isometries of WCW and Noether supercharges Q_n and their conjugates for the shifts $\Psi \to \Psi + \epsilon u_n$, where u_n is a solution of the modified Dirac equation, and ϵ is Grassmann number are enough to generate algebra containing the gamma matrix algebra.

2. The commutators $\Gamma^A_n = [Q^A, Q_n]$ are super-charges labelled by (A, n). One would like to identify them as gamma matrices of WCW. The problem is that they are labelled by (A, n) whereas isometry generators are labelled by A only just as symplectic Noether charges. Do all supercharges Γ^A_n except Γ^A_0 corresponding to $u_0 = constant$ annihilate the physical states so that one would have 1-1 correspondence? This would be analogous to what happens quite generally in super-conformal algebras.

3. The anti-commutators of Γ^A_0 would give the components of the Kähler metric. The allowance of singular surfaces having 2-D string world sheets as singularities would give to the metric also stringy component besides 3-D component and possible 0-D components at the ends of string. Metric 2-D property would not be exact as assumed originally.

This construction can be blamed for the lack of explicitness. The general tendency in the development of TGD has been replacement of explicit but somewhat ad hoc formulas with principles. Maybe this reflects to my own ageing and increasing laziness but my own view is that principles are what matter and get abstracted only very slowly. The less formulas, the better!

REFERENCES

Mathematics

[A1] Freed DS. The Geometry of Loop Groups, 1985.

[A2] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc.* Available at: `http://tinyurl.com/pb8zpqo`, 8(43):133–151, 1981.

Books related to TGD

[K1] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#kahler`, 2006.

[K2] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#cspin`, 2006.

[K3] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#wcwnew`, 2014.

[K4] Pitkänen M. About twistor lift of TGD? In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#hgrtwistor`, 2016.

[K5] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistorstory`, 2016.

[K6] Pitkänen M. Some questions related to the twistor lift of TGD. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistquestions`, 2017.

[K7] Pitkänen M. The Recent View about Twistorialization in TGD Framework. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#smatrix`, 2018.

Articles about TGD

[L1] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? Available at: `http://tgdtheory.fi/public_html/articles/ratpoints.pdf`, 2017.

[L2] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: `https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11`, 2017.

[L3] Pitkänen M. Philosophy of Adelic Physics. Available at: `http://tgdtheory.fi/public_html/articles/adelephysics.pdf`, 2017.

[L4] Pitkänen M. TGD view about coupling constant evolution. Available at: `http://tgdtheory.fi/public_html/articles/ccevolution.pdf`, 2018.