Could one generalize braid invariant defined by vacuum expectation of Wilson loop to an invariant of braid cobordisms and of 2-knots?

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January 18, 2012

Contents

1 Introduction

Witten has highly inspiring popular lecture about knots and quantum physics [7] mentioning also his recent work with knots related to an attempt to understand Khovanov homology. Witten manages to explain in rather comprehensible manner both the construction recipe of Jones polynomial and the idea about how Jones polynomial emerges from topological quantum field theory as a vacuum expectation of so called Wilson loop defined by path integral with weighting coming from Chern-Simons action [8]. Witten also tells that during the last year he has been working with an attempt to understand in terms of quantum theory the so called Khovanov polynomial associated with a much more abstract link invariant whose interpretation and real understanding remains still open. In particular, he mentions the approach of Gukov, Schwartz, and Vafa [15] as an attempt to understand Khovanov polynomial.

This kind of talks are extremely inspiring and lead to a series of questions unavoidably culminating to the frustrating "Why I do not have the brain of Witten making perhaps possible to answer these questions?". This one must just accept. In the following I summarize some thoughts inspired by the associations of the talk of Witten with quantum TGD and with the model of DNA as topological quantum computer. In my own childish manner I dare believe that these associations are interesting and dare also hope that some more brainy individual might take them seriously.

An idea inspired by TGD approach which also main streamer might find interesting is that the Jones invariant defined as vacuum expectation for a Wilson loop in 2+1-D space-time generalizes to a vacuum expectation for a collection of Wilson loops in 2+2-D space-time and could define an invariant for 2-D knots and for cobordisms of braids analogous to Jones polynomial. As a matter fact, it turns out that a generalization of gauge field known as gerbe is needed and that in TGD framework classical color gauge fields defined the gauge potentials of this field. Also topological string theory in 4-D space-time could define this kind of invariants. Of course, it might well be that this kind of ideas have been already discussed in literature.

Khovanov homology generalizes the Jones polynomial as knot invariant. The challenge is to find a quantum physical construction of Khovanov homology analous to the topological QFT defined by Chern-Simons action allowing to interpret Jones polynomial as vacuum expectation value of Wilson loop in non-Abelian gauge theory.

Witten's approach to Khovanov homology relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms. This comparison turns out to be extremely useful from TGD point of view.

An essentially unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach. Even more, the conjectured slicings of preferred extremals by 3-D surfaces and string world sheets central for quantum TGD can be identified uniquely. The slicing by 3-surfaces would be interpreted in gauge theory in terms of Higgs= constant surfaces with radial coordinate of CP_2 playing the role of Higgs. The slicing by string world sheets would be induced by different choices of $U(2)$ subgroup of $SU(3)$ leaving Higgs=constant surfaces invariant.

Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

2 Some TGD background

What makes quantum TGD [4] interesting concerning the description of braids and braid cobordisms is that braids and braid cobordisms emerge both at the level of generalized Feynman diagrams and in the model of DNA as a topological quantum computer [3].

2.1 Time-like and space-like braidings for generalized Feynman diagrams

1. In TGD framework space-times are 4-D surfaces in 8-D imbedding space. Basic objects are partonic 2-surfaces at the two ends of causal diamonds CD (intersections of future and past directed light-cones of 4-D Minkowski space with each point replaced with CP_2). The light-like orbits of partonic 2-surfaces define 3-D light-like 3-surfaces identifiable as lines of generalized Feynman diagrams. At the vertices of generalized Feynman diagrams incoming and outgoing

light-like 3-surfaces meet. These diagrams are not direct generalizations of string diagrams since they are singular as 4-D manifolds just like the ordinary Feynman diagrams.

By strong form of holography one can assign to the partonic 2-surfaces and their tangent space data space-time surfaces as preferred exrtremals of Kähler action. This guarantees also general coordinate invariance and allows to interpret the extremals as generalized Bohr orbits.

2. One can assign to the partonic 2-surfaces discrete sets of points carrying quantum numbers. As a matter fact, these sets of points seem to emerge from the solutions of the Chern-Simons Dirac equation rather naturally. These points define braid strands as the partonic 2-surface moves and defines a light-like 3-surface as its orbit as a surface of 4-D space-time surface. In the generic case the strands get tangled in time direction and one has linking and knotting giving rise to a time-like braiding.

3. Also space-like braidings are possible. One can imagine that the partonic 2-surfaces are connected by space-like curves defining TGD counterparts for strings and that in the initial state these curves define space-like braids whose ends belong to different partonic 2-surfaces. Quite generally, the basic conjecture is that the preferred extremals define orbits of string-like objects with their ends at the partonic 2-surfaces. One would have slicing of space-time surfaces by string world sheets one one hand and by partonic 2-surface on one hand. This string model is very special due to the fact that the string orbits define what could be called braid cobordisms representing which could represent unknotting of braids. String orbits in higher dimensional space-times do not allow this topological interpretation.

2.2 Dance metaphor

Time like braidings induces space-like braidings and one can speak of time-like or dynamical braiding and even duality of time-like and space-like braiding. What happens can be understood in terms of dance metaphor.

1. One can imagine that the points carrying quantum numbers are like dancers at parquettes defined by partonic 2-surfaces. These parquettes are somewhat special in that it is moving and changing its shape.

2. Space-like braidings means that the feet of the dancers at different parquettes are connected by threads. As the dance continues, the threads connecting the feet of different dancers at different parquettes get tangled so that the dance is coded to the braiding of the threads. Time-like braiding induce space-like braiding. One has what might be called a cobordism for space-like braiding transforming it to a new one.

2.3 DNA as topological quantum computer

The model for topological quantum computation is based on the idea that time-like braidings defining topological quantum computer programs. These programs are robust since the topology of braiding is not affected by small deformations.

1. The first key idea in the model of DNA as topological quantum computer is based on the observation that the lipids of cell membrane form a 2-D liquid whose flow defines the dance in which dancers are lipids which define a flow pattern defining a topological quantum computation. Lipid layers assignable to cellular and nuclear membranes are the parquettes. This 2-D flow pattern can be induced by the liquid flow near the cell membrane or in case of nerve pulse transmission by the nerve pulses flowing along the axon. This alone defines topological quantum computation.

2. In DNA as topological quantum computer model one however makes a stronger assumption motivated by the vision that DNA is the brain of cell and that information must be communicated to DNA level wherefrom it is communicated to what I call magnetic body. It is assumed that the lipids of the cell membrane are connected to DNA nucleotides by magnetic flux tubes defining a space-like braiding. It is also possible to connect lipids of cell membrane to the lipids of other

cell membranes, to the tubulins at the surfaces of microtubules, and also to the aminoadics of proteins. The spectrum of possibilities is really wide.

The space-like braid strands would correspond to magnetic flux tubes connecting DNA nucleotides to lipids of nuclear or cell membrane. The running of the topological quantum computer program defined by the time-like braiding induced by the lipid flow would be coded to a space-like braiding of the magnetic flux tubes. The braiding of the flux tubes would define a universal memory storage mechanism and combined with 4-D view about memory provides a very simple view about how memories are stored and how they are recalled.

3 Could braid cobordisms define more general braid invariants?

Witten says that one should somehow generalize the notion of knot invariant. The above described framework indeed suggests a very natural generalization of braid invariants to those of braid cobordisms reducing to braid invariants when the braid at the other end is trivial. This description is especially natural in TGD but allows a generalization in which Wilson loops in 4-D sense describe invariants of braid cobordisms.

3.1 Difference between knotting and linking

Before my modest proposal of a more general invariant some comments about knotting and linking are in order.

1. One must distinguish between internal knotting of each braid strand and linking of 2 strands. They look the same in the 3-D case but in higher dimensions knotting and linking are not the same thing. Codimension 2 surfaces get knotted in the generic case, in particular the 2-D orbits of the braid strands can get knotted so that this gives additional topological flavor to the theory of strings in 4-D space-time. Linking occurs for two surfaces whose dimension d_1 and d_2 satisfying $d_1 + d_2 = D - 1$, where D is the dimension of the imbedding space.

2. 2-D orbits of strings do not link in 4-D space-time but do something more radical since the sum of their dimensions is $D = 4$ rather than only $D - 1 = 3$. They intersect and it is impossible to eliminate the intersection without a change of topology of the stringy 2-surfaces: a hole is generated in either string world sheet. With a slight deformation intersection can be made to occur generically at discrete points.

3.2 Topological strings in 4-D space-time define knot cobordisms

What makes the 4-D braid cobordisms interesting is following.

1. The opening of knot by using brute force by forcing the strands to go through each other induces this kind of intersection point for the corresponding 2-surfaces. From 3-D perspective this looks like a temporary cutting of second string, drawing the string ends to some distance and bringing them back and gluing together as one approaches the moment when the strings would go through each other. This surgical operation for either string produces a pair of non-intersecting 2-surfaces with the price that the second string world sheet becomes topologically non-trivial carrying a hole in the region were intersection would occur. This operation relates a given crossing of braid strands to its dual crossing in the construction of Jones polynomial in given step (string 1 above string 2 is transformed to string 2 above string 1).

2. One can also cut both strings temporarily and glue them back together in such a manner that end a/b of string 1 is glued to the end c/d of string 2. This gives two possibilities corresponding to two kinds of reconnections. Reconnections appears as the second operation in the construction of Jones invariant besides the operation putting the string above the second one below it or vice versa. Jones polynomial relates in a simple manner to Kauffman bracket allowing a recursive construction. At a given step a crossing is replaced with a weighted sum of the two reconnected terms [5, 6]. Reconnection represents the analog of trouser vertex for closed strings replaced with braid strands.

3. These observations suggest that stringy diagrams describe the braid cobordisms and a kind of topological open string model in 4-D space-time could be used to construct invariants of braid cobordisms. The dynamics of strand ends at the partonic 2-surfaces would partially induce the dynamics of the space-like braiding. This dynamics need not induce the un-knotting of space-like braids and simple string diagrams for open strings are enough to define a cobordism leading to un-knotting. The holes needed to realize the crossover for braid strands would contribute to the Wilson loop an additional factor corresponding to the rotation of the gauge potential around the boundary of the hole (non-integrable phase factor). In abelian case this gives simple commuting phase factor.

Note that braids are actually much more closer to the real world than knots since a useful strand of knotted structure must end somewhere. The abstract closed loops of mathematician floating in empty space are not very useful in real life albeit mathematically very convenient as Witten notices. Also the braid cobordisms with ends of a collection of space-like braids at the ends of causal diamond are more practical than 2-knots in 4-D space. Mathematician would see these objects as analogous to surfaces in relative homology allowed to have boundaries if they located at fixed sub-manifolds. Homology for curves with ends fixed to be on some surfaces is a good example of this. Now these fixed sub-manifolds would correspond to space-like 3-surfaces at the ends CDs and light-like wormhole throats at which the signature of the induced metric changes and which are carriers of elementary particle quantum numbers.

4 Invariants 2-knots as vacuum expectations of Wilson loops in 4-D space-time?

The interpretation of string world sheets in terms of Wilson loops in 4-dimensional space-time is very natural. This raises the question whether Witten's a original identification of the Jones polynomial as vacuum expectation for a Wilson loop in 2+1-D space might be replaced with a vacuum expectation for a collection of Wilson loops in 3+1-D space-time and would characterize in the general case (multi-)braid cobordism rather than braid. If the braid at the lower or upped boundary is trivial, braid invariant is obtained. The intersections of the Wilson loops would correspond to the violent un-knotting operations and the boundaries of the resulting holes give an additional Wilson loop. An alternative interpretation would be as the analog of Jones polynomial for 2-D knots in 4-D space-time generalizing Witten's theory. This description looks completely general and does not require TGD at all.

The following considerations suggest that Wilson loops are not enough for the description of general 2-knots and that that Wilson loops must be replaced with 2-D fluxes. This requires a generalization of gauge field concept so that it corresponds to a 3-form instead of 2-form is needed. In TGD framework this kind of generalized gauge fields exist and their gauge potentials correspond to classical color gauge fields.

4.1 What 2-knottedness means concretely?

It is easy to imagine what ordinary knottedness means. One has circle imbedded in 3-space. One projects it in some plane and looks for crossings. If there are no crossings one knows that un-knot is in question. One can modify a given crossing by forcing the strands to go through each other and this either generates or removes knottedness. One can also destroy crossing by reconnection and this always reduces knottedness. Since knotting reduces to linking in 3-D case, one can find a simple interpretation for knottedness in terms of linking of two circles. For 2-knots linking is not what gives rise to knotting.

One might hope to find something similar in the case of 2-knots. Can one imagine some simple local operations which either increase of reduce 2-knottedness?

1. To proceed let us consider as simple situation as possible. Put sphere in 3-D time= constant section E^3 of 4-space. Add a another sphere to the same section E^3 such that the corresponding balls do not intersect. How could one build from these two spheres a knotted 2-sphere?

2. From two spheres one can build a single sphere in topological sense by connecting them with a small cylindrical tube connecting the boundaries of disks (circles) removed from the two spheres. If this is done in E^3, a trivial 2-knot results. One can however do the gluing of the cylinder in a more exotic manner by going temporarily to "hyper-space", in other words making a time travel. Let the cylinder leave the second sphere from the outer surface, let it go to future or past and return back to recent but through the interior. This is a good candidate for a knotted sphere since the attempts to deform it to self-non-intersecting sphere in E^3 are expected to fail since the cylinder starting from interior necessarily goes through the surface of sphere if wants to the exterior of the sphere.

3. One has actually 2×2 manners to perform the connected sum of 2-spheres depending on whether the cylinders leave the spheres through exterior or interior. At least one of them (exterior-exterior) gives an un-knotted sphere and intuition suggests that all the three remaining options requiring getting out from the interior of sphere give a knotted 2-sphere. One can add to the resulting knotted sphere new spheres in the same manner and obtain an infinite number of them. As a matter fact, the proposed 1+3 possibilities correspond to different versions of connected sum and one could speak of knotting and non-knotting connected sums. If the addition of knotted spheres is performed by non-knotting connected sum, one obtains composites of already existing 2-knots. Connected sum composition is analogous to the composition of integer to a product of primes. One indeed speaks of prime knots and the number of prime knots is infinite. Of course, it is far from clear whether the connected sum operation is enough to build all knots. For instance it might well be that cobordisms of 1-braids produces knots not producible in this manner. In particular, the effects of time-like braiding induce braiding of space-like strands and this looks totally different from local knotting.

4.2 Are all possible 2-knots possible for stringy world sheets?

Whether all possible 2-knots are allowed for stringy world sheets, is not clear. In particular, if they are dynamically determined it might happen that many possibilities are not realized. For instance, the condition that the signature of the induced metric is Minkowskian could be an effective killer of 2-knottedness not reducing to braid cobordism.

1. One must start from string world sheets with Minkowskian signature of the induced metric. In other words, in the previous construction one must E^3 with 3-dimensional Minkowski space M^3 with metric signature 1+2 containing the spheres used in the construction. Time travel is replaced with a travel in space-like hyper dimension. This is not a problem as such. The spheres however have at least one two special points corresponding to extrema at which the time coordinate has a local minimum or maximum. At these points the induced metric is necesssarily degenerate meaning that its determinant vanishes. If one allows this kind of singular points one can have elementary knotted spheres. This liberal attitude is encouraged by the fact that the light-like 3-surfaces defining the basic dynamical objects of quantum TGD correspond to surfaces at which 4-D induced metric is degenerate. Otherwise 2-knotting reduces to that induced by cobordisms of 1-braids. If one allows only the 2-knots assignable to the slicings of the space-time surface by string world sheets and even restricts the consideration to those suggested by the duality of 2-D generalization of Wilson loops for string world sheets and partonic 2-surfaces, it could happen that the string world sheets reduce to braidings.

2. The time=constant intersections define a representation of 2-knots as a continuous sequence of 1-braids. For critical times the character of the 1-braids changes. In the case of braiding this corresponds to the basic operations for 1-knots having interpretation as string diagrams (reconnection and analog of trouser vertex). The possibility of genuine 2-knottedness brings in also the possibility that strings pop up from vacuum as points, expand to closed strings, are fused to stringy words sheet temporarily by the analog of trouser vertex, and eventually return to the vacuum. Essentially trouser diagram but second string open and second string closed and beginning from vacuum and ending to it is in question. Vacuum bubble interacting with open string would be in question. The believer in string model might be eager to accept this picture but one must be cautious.

4.3 Are Wilson loops enough for 2-knots?

Suppose that the space-like braid strands connecting partonic 2-surfaces at given boundary of CD and light-like braids connecting partonic 2-surfaces belonging to opposite boundaries of CD form connected closed strands. The collection of closed loops can be identified as boundaries of Wilson loops and the expectation value is defined as the product of traces assignable to the loops. The definition is exactly the same as in 2+1-D case [8].

Is this generalization of Wilson loops enough to describe 2-knots? In the spirit of the proposed philosophy one could ask whether there exist two-knots not reducible to cobordisms of 1-knots whose knot invariants require cobordisms of 2-knots and therefore 2-braids in 5-D space-time. Could it be that dimension $D = 4$ is somehow very special so that there is no need to go to $D = 5$? This might be the case since for ordinary knots Jones polynomial is very faithful invariant.

Innocent novice could try to answer the question in the following manner. Let us study what happens locally as the 2-D closed surface in 4-D space gets knotted.

1. In 1-D case knotting reduces to linking and means that the first homotopy group of the knot complement is changed so that the imbedding of first circle implies that the there exists imbedding of the second circle that cannot be transformed to each other without cutting the first circle temporarily. This phenomenon occurs also for single circle as the connected sum operation for two linked circles producing single knotted circle demonstrates.

2. In 2-D case the complement of knotted 2-sphere has a non-trivial second homotopy group so that 2-balls have homotopically non- equivalent imbeddings, which cannot be transformed to each other without intersection of the 2-balls taking place during the process. Therefore the description of 2-knotting in the proposed manner would require cobordisms of 2-knots and thus 5-D space-time surfaces. However, since 3-D description for ordinary knots works so well, one could hope that the generalization the notion of Wilson loop could allow to avoid 5-D description altogether. The generalized Wilson loops would be assigned to 2-D surfaces and gauge potential A would be replaced with 2-gauge potential B defining a three-form $F = dB$ as the analog of gauge field.

3. This generalization of bundle structure known as gerbe structure has been introduced in algebraic geometry [11, 12] and studied also in theoretical physics [10]. 3-forms appear as analogs of gauge fields also in the QFT limit of string model. Algebraic geometer would see gerbe as a generalization of bundle structure in which gauge group is replaced with a gauge groupoid. Essentially a structure of structures seems to be in question. For instance, the principal bundles with given structure group for given space defines a gerbe. In the recent case the space of gauge fields in space-time could be seen as a gerbe. Gerbes have been also assigned to loop spaces and WCW can be seen as a generalization of loop space. Lie groups define a much more mundane example about gerbe. The 3-form F is given by $F(X, Y, Z) = B(X, [Y, Z])$, where B is Killing form and for $U(n)$ reduces to $(g^{-1}dg)^3$. It will be found that classical color gauge fields define gerbe gauge potentials in TGD framework in a natural manner.

5 TGD inspired theory of braid cobordisms and 2-knots

In the sequel the considerations are restricted to TGD and to a comparison of Witten's ideas with those emerging in TGD framework.

5.1 Weak form of electric-magnetic duality and duality of space-like and time-like braidings

Witten notices that much of his work in physics relies on the assumption that magnetic charges exist and that rather frustratingly, cosmic inflation implies that all traces of them disappear. In TGD Universe the non-trivial topology of CP_2 makes possible Kähler magnetic charge and inflation is replaced with quantum criticality. The recent view about elementary particles is that they correspond to string like objects with length of order electro-weak scale with Kähler magnetically charged wormhole throats at their ends. Therefore magnetic charges would be there and LHC might be able to detect their signatures if LHC would get the idea of trying to do this.

Witten mentions also electric-magnetic duality. If I understood correctly, Witten believes that it might provide interesting new insights to the knot invariants. In TGD framework one speaks about weak form of elecric magnetic duality. This duality states that Kähler electric fluxes at space-like ends of the space-time sheets inside CDs and at wormhole throats are proportional to Kähler magneic fluxes so that the quantization of Kähler electric charge quantization reduces to purely homological quantization of Kähler magnetic charge.

The weak form of electric-magnetic duality fixes the boundary conditions of field equations at the light-like and space-like 3-surfaces. Together with the conjecture that the Kähler current is proportional to the corresponding instanton current this implies that Kähler action for the preferred extremal sof Kähler action reduces to 3-D Chern-Simons term so that TGD reduces to almost topological QFT. This means an enormous mathematical simplification of the theory and gives hopes about the solvability of the theory. Since knot invariants are defined in terms of Abelian Chern-Simons action for induced Kähler gauge potential, one might hope that TGD could as a by-product define invariants of braid cobordisms in terms of the unitary U-matrix of the theory between zero energy states and having as its rows the non-unitary M-matrices analogous to thermal S-matrices.

Electric magnetic duality is 4-D phenomenon as is also the duality between space-like and time like braidings essential also for the model of topological quantum computation. Also this suggests that some kind of topological string theory for the space-time sheets inside CDs could allow to define the braid cobordism invariants.

5.2 Could Kähler magnetic fluxes define invariants of braid cobordisms?

Can one imagine of defining knot invariants or more generally, invariants of knot cobordism in this framework? As a matter fact, also Jones polynomial describes the process of unknotting and the replacement of unknotting with a general cobordism would define a more general invariant. Whether the Khovanov invariants might be understood in this more general framework is an interesting question.

1. One can assign to the 2-dimensional stringy surfaces defined by the orbits of space-like braid strands Kähler magnetic fluxes as flux integrals over these surfaces and these integrals depend only on the end points of the space-like strands so that one deform the space-like strands in an arbitrarily manner. One can in fact assign these kind of invariants to pairs of knots and these invariants define the dancing operation transforming these knots to each other. In the special case that the second knot is un-knot one obtains a knot-invariant (or link- or braid- invariant).

2. The objection is that these invariants depend on the orbits of the end points of the space-like braid strands. Does this mean that one should perform an averaging over the ends with the condition that space-like braid is not affected topologically by the allowed deformations for the positions of the end points?

3. Under what conditions on deformation the magnetic fluxes are not affect in the deformation of the braid strands at 3-D surfaces? The change of the Kähler magnetic flux is magnetic flux over the closed 2-surface defined by initial non-deformed and deformed stringy two-surfaces minus flux over the 2-surfaces defined by the original time-like and space-like braid strands connected by a thin 2-surface to their small deformations. This is the case if the deformation corresponds to a U(1) gauge transformation for a Kähler flux. That is diffeomorphism of M^4 and symplectic transformation of CP_2 inducing the U(1) gauge transformation.

   Hence a natural equivalence for braids is defined by these transformations. This is quite not a topological equivalence but quite a general one. Symplectic transformations of CP_2 at light-like and space-like 3-surfaces define isometries of the world of classical worlds so that also in this sense the equivalence is natural. Note that the deformations of space-time surfaces correspond to this kind of transformations only at space-like 3-surfaces at the ends of CDs and at the light-like wormhole throats where the signature of the induced metric changes. In fact, in quantum TGD the sub-spaces of world of classical worlds with constant values of zero modes (non-quantum fluctuating degrees of freedom) correspond to orbits of 3-surfaces under symplectic transformations so that the symplectic restriction looks rather natural also from the point of view of quantum dynamics and the vacuum expectation defined by Kähler function be defined for physical states.

4. A further possibility is that the light-like and space-like 3-surfaces carry vanishing induced Kähler fields and represent surfaces in $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 carrying vanishing Kähler form. The interior of space-time surface could in principle carry a non-vanishing Kähler form. In this case weak form of self-duality cannot hold true. This however implies that the Kähler magnetic fluxes vanish identically as circulations of Kähler gauge potential. The non-integrable phase factors defined by electroweak gauge potentials would however define non-trivial classical Wilson loops. Also electromagnetic field would do so. It would be therefore possible to imagine vacuum expectation value of Wilson loop for given quantum state. Exponent of Kähler action would define for non-vacuum extremals the weighting. For 4-D vacuum extremals this exponent is trivial and one might imagine of using imaginary exponent of electroweak Chern-Simons action. Whether the restriction to vacuum extremals in the definition of vacuum expectations of electroweak Wilson loops could define general enough invariants for braid cobordisms remains an open question.

5. The quantum expectation values for Wilson loops are non-Abelian generalizations of exponentials for the expectation values of Kähler magnetic fluxes. The classical color field is proportional to the induced Kähler form and its holonomy is Abelian which raises the question whether the non-Abelian Wilson loops for classical color gauge field could be expressible in terms of Kähler magnetic fluxes.

5.3 Classical color gauge fields and their generalizations define gerbe gauge potentials allowing to replace Wilson loops with Wilson sheets

As already noticed, the description of 2-knots seems to necessitate the generalization of gauge field to 3-form and the introduction of a gerbe structure. This seems to be possible in TGD framework.

1. Classical color gauge fields are proportional to the products $B_A = H_A J$ of the Hamiltonians of color isometries and of Kähler form and the closed 3-form $F_A = dB_A = dH_A \wedge J$ could serve as a colored 3-form defining the analog of U(1) gauge field. What would be interesting that color would make F non-vanishing. The "circulation" $h_A = \oint H_A J$ over a closed partonic 2-surface transforms covariantly under symplectic transformations of CP_2, whose Hamiltonians can be assigned to irreps of SU(3): just the commutator of Hamiltonians defined by Poisson bracket appears in the infinitesimal transformation. One could hope that the expectation values for the exponents of the fluxes of B_A over 2-knots could define the covariants able to catch 2-knotted-ness in TGD framework. The exponent defining Wilson loop would be replaced with $exp(iQ^A h_A)$, where Q^A denote color charges acting as operators on particles involved.

2. Since the symplectic group acting on partonic 2-surfaces at the boundary of CD replaces color group as a gauge group in TGD, one can ask whether symplectic SU(3) should be actually replaced with the entire symplectic group of $\cup_{\pm}\delta M^4_{\pm} \times CP_2$ with Hamiltonians carrying both spin and color quantum numbers. The symplectic fluxes $\oint H_A J$ are indeed used in the construction of both quantum states and of WCW geometry. This generalization is indeed possible for the gauge potentials $B_A J$ so that one would have infinite number of classical gauge fields having also interpretation as gerbe gauge potentials.

3. The objection is that symplectic transformations are not symmetries of Kähler action. Therefore the action of symplectic transformation induced on the space-time surface reduces to a symplectic transformation only at the partonic 2-surfaces. This spoils the covariant transformation law for the 2-fluxes over stringy world sheets unless there exist preferred stringy world sheets for which the action is covariant. The proposed duality between the descriptions based on partonic 2-surfaces and stringy world sheets realized in terms of slicings of space-time surface by string world sheets and partonic 2-surfaces suggests that this might be the case.

   This would mean that one can attach to a given partonic 2-surface a unique collection string world sheets. The duality suggests even stronger condition stating that the total exponents $exp(iQ^A h_A)$ of fluxes are the same irrespective whether h_A evaluated for partonic 2-surfaces or for string world sheets defining the analog of 2-knot. This would mean an immense calculational simplification! This duality would correspond very closely to the weak form of electric magnetic duality whose various forms I have pondered as a must for the geometry of WCW. Partonic 2-surfaces indeed correspond to magnetic monopoles at least for elementary particles and stringy world sheets to surfaces carrying electric flux (note that in the exponent magnetic charges do not make themselves visible so that the identity can make sense also for $H_A = 1$).

4. Quantum expectation means in TGD framework a functional integral over the symplectic orbits of partonic 2-surfaces plus 4-D tangent space data assigned to the upper and lower boundaries of CD. Suppose that holography fixes the space-like 3-surfaces at the ends of CD and light-like orbits of partonic 2-surfaces. In completely general case the braids and the stringy space-time sheets could be fixed using a representation in terms of space-time coordinates so that the representation would be always the same but the imbedding varies as also the values of the exponent of Kähler function, of the Wilson loop, and of its 2-D generalization. The functional integral over symplectic transforms of 3-surfaces implies that Wilson loop and its 2-D generalization varies.

   The proposed duality however suggests that both Wilson loop and its 2-D generalization are actually fixed by the dynamics. One can ask whether the presence of 2-D analog of Wilson loop has a direct physical meaning bringing into almost topological stringy dynamics associated with color quantum numbers and coding explicit information about space-time interior and topology of field lines so that color dynamics would also have interpretation as a theory of 2-knots. If the proposed duality suggested by holography holds true, only the data at partonic 2-surfaces would be needed to calculate the generalized Wilson loops.

This picture is very speculative and sounds too good to be true but follows if one consistently applies holography.

5.4 Summing sup the basic ideas

Let us summarize the ideas discussed above.

1. Instead of knots, links, and braids one could study knot and link cobordisms, that is their dynamical evolutions concretizable in terms of dance metaphor and in terms of interacting string world sheets. Each space-like braid strand can have purely internal knotting and braid strands can be linked. TGD could allow to identify uniquely both space-like and time-like braid strands and thus also the stringy world sheets more or less uniquely and it could be that the dynamics induces automatically the temporary cutting of braid strands when knot is opened violently so that a hole is generated. Gerbe gauge potentials defined by classical color gauge fields could make also possible to characterize 2-knottedness in symplectic invariant manner in terms of color gauge fluxes over 2-surfaces.

   The weak form of electric-magnetic duality would reduce the situation to almost topological QFT in general case with topological invariance replaced with symplectic one which corresponds to the fixing of the values of non-quantum fluctuating zero modes in quantum TGD. In the vacuum sector it would be possible to have the counterparts of Wilson loops weighted by 3-D electroweak Chern-Simons action defined bythe induced spinor connection.

2. One could also leave TGD framework and define invariants of braid cobordisms and 2-D analogs of braids as vacuum expectations of Wilson loops using Chern-Simons action assigned to 3-surfaces at which space-like and time-like braid strands end. The presence of light-like and space-like 3-surfaces assignable to causal diamonds could be assumed also now.

I checked whether the article of Gukov, Scwhartz, and Vafa entitled "Khovanov-Rozansky Homology and Topological Strings" [15] relies on the primitive topological observations made above. This does not seem to be the case. The topological strings in this case are strings in 6-D space rather than 4-D space-time.

What is interesting that twistorial considerations lead to a conjecture that 4-D space-time surfaces in 8-D imbedding space have a dual description in terms of certain 6-D homomorphic surfaces which are sphere bundles in 12-D $CP_3 \times CP_3$ and effectively 4-D. This suggests a connection between descriptions based on topological strings in 6-D space and Wilson loops in 4-D space-time. Could it really be that these completely trivial observations are not a standard part of knot theory?

There is also an article by Dror Bar-Natan with title "Khovanov's homology for tangles and cobordisms" [16]. The article states that the Khovanov homology theory for knots and links generalizes to tangles, cobordisms and 2-knots. The article does not say anything explicit about Wilson loops but talks about topological QFTs.

An article of Witten about his physical approach to Khovanov homology has appeared in arXiv [9]. The article is more or less abracadabra for anyone not working with M-theory but the basic idea is simple. Witten reformulates 3-D Chern-Simons theory as a path integral for $\mathcal{N} = 4$ SYM in the 4-D half space $W\times; R$. This allows him to use dualities and bring in the machinery of M-theory and 6-branes. The basic structure of TGD forces a highly analogous appproach: replace 3-surfaces with 4-surfaces, consider knot cobordisms and also 2-knots, introduce gerbes, and be happy with symplectic instead of topological QFT, which might more or less be synonymous with TGD as almost topological QFT. Symplectic QFT would obviously make possible much more refined description of knots.

6 Witten's approach to Khovanov homology from TGD point of view

Witten's approach to Khovanov comohology [9] relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms.

An essentially unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach [9, 18].

Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

6.1 Intersection form and space-time topology

The violent unknotting corresponds to a sequence of steps in which braid or knot becomes trivial and this very process defines braid invariants in TGD approach in nice concordance with the basic recipe for the construction of Jones polynomial. The topological invariant characterizing this process as a dynamics of 2-D string like objects defined by braid strands becomes knot invariant or more generally, invariant depending on the initial and final braids.

The process is describable in terms of string interaction vertices and also involves crossings of braid strands identifiable as self-intersections of the string world sheet. Hence the intersection form for the 2-surfaces defining braid strand orbits becomes a braid invariant. This intersection form is also a central invariant of 4-D manifolds and Donaldson's theorem [13] says that for this invariant characterizes simply connected smooth 4-manifold completely. Rank, signature, and parity of this form in the basis defined by the generators of 2-homology (excluding torsion elements) characterize smooth closed and orientable 4-manifold. It is possible to diagonalize this form for smoothable 4-surfaces. Although the situation in the recent case differs from that in Donaldson theory in that the 4-surfaces have boundary and even fail to be manifolds, there are reasons to believe that the theory of braid cobordisms and 2-knots becomes part of the theory of topological invariants of 4-surfaces just as knot theory becomes part of the theory of 3-manifolds. The representation of 4-manifolds as space-time surfaces might also bring in physical insights.

This picture leads to ideas about string theory in 4-D space-time as a topological QFT. The string world sheets define the generators of second relative homology group. "Relative" means that closed surfaces are replaced with surfaces with boundaries at wormhole throats and ends of CD. These string world sheets, if one can fix them uniquely, would define a natural basis for homology group defining the intersection form in terms of violent unbraiding operations (note that also reconnections are involved).

Quantum classical correspondence encourages to ask whether also physical states must be restricted in such a manner that only this minimum number of strings carrying quantum numbers at their ends ending to wormhole throats should be allowed. One might hope that there exists a unique identification of the topological strings implying the same for braids and allowing to identify various symplectic invariants as Hamiltonian fluxes for the string world sheets.

6.2 Framing anomaly

In 3-D approach to knot theory the framing of links and knots represents an unavoidable technical problem [9]. Framing means a slight shift of the link so that one can define self-linking number as a linking number for the link and its shift. The problem is that this framing of the link - or trivialization of its normal bundle in more technical terms- is not topological invariant and one obtains a large number of framings. For links in S^3 the framing giving vanishing self-linking number is the unique option and Atyiah has shown that also in more general case it is possible to identify a unique framing.

For 2-D surfaces self-linking is replaced with self-intersection. This is well-defined notion even without framing and indeed a key invariant. One might hope that framing is not needed also for string world sheets. If needed, this framing would induce the framing at the space-like and light-like 3-surfaces. The restriction of the section of the normal bundle of string world sheet to the 3-surfaces must lie in the tangent space of 3-surfaces. It is not clear whether this is enough to resolve the non-uniqueness problem.

6.3 Khovanov homology briefly

Khovanov homology involves three charges Q, F, and P. Q is analogous to super charge and satisfies $Q^2 = 0$ for the elements of homology. The basic commutation relations between the charges are $[F,Q] = Q$ and $[P,Q] = 0$. One can show that the Khovanov homology $\kappa(L)$ for link can be expressed as a bi-graded direct sum of the eigen-spaces $V_{m,n}$ of F and P, which have integer valued spectra. Obviously Q increases the eigenvalue of F and maps $V_{m,n}$ to $V_{m+1,n}$ just as exterior derivative in de-Rham comology increases the degree of differential form. P acts as a symmetry allowing to label the elements of the homology by an integer valued charge n.

Jones polynomial can be expressed as an index assignable to Khovanov homology:

$$\mathcal{J}(q|L) = Tr((-1)^F q^P \quad . \tag{6.1}$$

Here q defining the argument of Jones polynomial is root of unity in Chern-Simons theory but can be extended to complex numbers by extending the positive integer valued Chern-Simons coupling k to a complex number. The coefficients of the resulting Laurent polynomial are integers: this result does not follow from Chern-Simons approach alone. Jones polynomial depends on the spectrum of F only modulo 2 so that a lot of information is lost as the homology is replaced with the polynomial.

Both the need to have a more detailed characterization of links and the need to understand why the Wilson loop expectation is Laurent polynomial with integer coefficients serve as motivations of Witten for searching a physical approach to Khovanov polynomial.

The replacement of $D = 2$ in braid group approach to Jones polynomial with $D = 3$ for Chern-Simons approach replaced by something new in $D = 4$ would naturally correspond to the dimensional hierarchy of TGD in which partonic 2-surfaces plus their 2-D tangent space data fix the physics. One cannot quite do with partonic 2-surfaces and the inclusion of 2-D tangent space-data leads to holography and unique space time surfaces and perhaps also unique string world sheets serving as duals for partonic 2-surfaces. This would realize the weak form of electric magnetic duality at the level of homology much like Poincare duality relates cohomology and homology.

6.4 Surface operators and the choice of the preferred 2-surfaces

The choice of preferred 2-surfaces and the identification of surface operators in $\mathcal{N} = 4$ YM theory is discussed in [18]. The intuitive picture is that preferred 2-surfaces- now string world sheets defining braid cobordisms and 2-knots- correspond to singularities of classical gauge fields. Surface operator can be said to create this singularity. In functional integral this means the presence of the exponent defining the analog of Wilson loop.

1. In [18] the 2-D singular surfaces are identified as poles for the magnitude r of the Higgs field. One can assign to the 2-surface fractional magnetic charges defined for the Cartan algebra part A_C of the gauge connection as circulations $\oint A_C$ around a small circle around the axis of singularity at $r = \infty$. What happens that 3-D $r = constant$ surface reduces to a 2-D surface at $r = \infty$ whereas A_C and entire gauge potential behaves as $A = A_C = \alpha d\phi$ near singularity. Here ϕ is coordinate analogous to angle of cylindrical coordinates when t-z plane represents the singular 2-surface. α is a linear combination of Cartan algebra generators.

2. The phase factor assignable to the circulation is essentially $exp(i2\pi\alpha)$ and for non-fractional magnetic charges it differs from unity. One might perhaps say that string word sheets correspond to singularities for the slicing of space-time surface with 3-surfaces at which 3-surfaces reduce to 2-surfaces.

Consider now the situation in TGD framwork.

1. The gauge group is color gauge group and gauge color gauge potentials correspond to the quantities $H_A J$. One can also consider a generalization by allowing all Hamiltonians generating symplectic transformations of CP_2. Kähler gauge potential is in essential role since color gauge field is proportional to Kähler form.

2. The singularities of color gauge fields can be identified by studing the theory locally as a field theory from CP_2 to M^4. It is quite possible to have space-time surfaces for which M^4 coordinates are many-valued functions of CP_2 coordinates so that one has a covering of CP_2 locally. For singular 2-surfaces this covering becomes singular in the sense that separate sheets coincide. These coverings do not seem to correspond to those assignable to the hierarchy of Planck constants implied by the many-valuedness of the time derivatives of the imbedding space coordinates as functions of canonical momentum densities but one must be very cautious in making too strong conclusions here.

3. To proceed introduce the Eguchi-Hanson coordinates

$$(\xi^1, \xi^2) = [rcos(\theta/2)exp(i(\Psi + \Phi)/2), rsin(\theta/2)exp(i(-\Psi + \Phi)/2]$$

   for CP_2 with the defining property that the coordinates transform linearly under $U(2) \subset SU(3)$. In QFT context these coordinates would be identified as Higgs fields. The choice of these coordinates is unique apart from the choice of the $U(2)$ subgroup and rotation by element of $U(2)$ once this choice has been made. In TGD framework the definition of CD involves the fixing of these coordinates and the interpretation is in terms of quantum classical correspondence realizing the choice of quantization axes of color at the level of the WCW geometry.

   r has a natural identification as the magnitude of Higgs field invariant under $U(2) \subset SU(3)$. The $SU(2) \times U(1)$ invariant 3-sphere reduces to a homologically non-trivial geodesic 2-sphere at $r = \infty$ so that for this choice of coordinates this surface defines in very natural manner the counterpart of singular 2-surface in CP_2 geometry. At this sphere the second phase associated with CP_2 coordinates- Φ - becomes a redundant coordinate just like the angle Φ at the poles of sphere. There are two other similar spheres and these three spheres are completely analogous to North and South poles of 2-sphere.

4. The singular surfaces would naturally correspond to the inverse images for imbedding map for $r = \infty$ geodesic sphere of CP_2 for a CD corresponding to a given choice of quantization axes. Possibly also the inverse images of the other two geodesic spheres must be included. The inverse images of this geodesic 2-sphere under the imbedding map would naturally correspond to 2-D string world sheets for the preferred extremals.

5. The existence of dual slicings of space-time surface by 3-surfaces and lines on one hand and by string world sheets Y^2 and 2-surfaces X^2 with Euclidian signature of metric on one hand, is one of the basic conjectures about the properties of preferred extremals of Kähler action. A stronger conjecture is that partonic 2-surfaces represent particular instances of X^2. The proposed picture suggests an amazingly simple and physically attractive identification of these slicings.

(a) The slicing induced by the slicing of CP_2 by $r = constant$ surfaces defines an excellent candidate for the slicing by 3-surfaces. Physical the slices would correspond to equivalence classes of choices of the quantization axes for color group related by $U(2)$. In gauge theory context they would correspond to different breakings of $SU(3)$ symmetry labelled by the vacuum expectation of the Higgs field r which would be dynamical for CP_2 projections and play the role of time coordinate.

(b) The slicing by string world sheets would naturally correspond to the slicing induced by the 2-D space of homologically non-trivial geodesic spheres (or triplets of them) and could be called "CP_2/S^2". One has clearly bundle structure with S^2 as base space and "CP_2/S^2" as fiber. Partonic 2-surfaces could be seen locally as sections of this bundle like structure assigning a point of "CP_2/S^2" to each point of S^2. Globally this does not make sense for partonic 2-surfaces with genus larger than $g = 0$.

6. In TGD framework the Cartan algebra of color gauge group is the natural identification for the Cartan algebra involved and the fluxes defining surface operators would be the classical fluxes $\int H_A J$ over the 2-surfaces in question restricted to Cartan algebra. What would be new is the interpretation as gerbe gauge potentials so that flux becomes completely analogous to Abelian circulation.

   If one accepts the extension of the gauge algebra to a symplectic algebra, one would have the Cartan algebra of the symplectic algebra. This algebra is defined by generators which depend on the second half P_i or Q_i of Darboux coordinates. If P_i are chosen to be functions of the coordinates (r, θ) of CP_2 coordinates whose Poisson brackets with color isospin and hyper charge generators inducing rotations of phases (Ψ, Φ) of CP_2 complex coordinates vanish, the symplectic Cartan algebra would correspond to color neutral Hamiltonians. The spherical harmonics with non-vanishing angular momentum vanish at poles and one expects that same happens for CP_2 spherical harmonics at the three poles of CP_2. Therefore Cartan algebra is selected automatically for gauge fluxes.

   This subgroup leaves the ends of the points of braids at partonic 2-surfaces invariant so that symplectic transformations do not induce braiding.

If this picture -resulting as a rather straightforward translation of the picture applied in QFT context- is correct, TGD would predict uniquely the preferred 2-surfaces and therefore also the braids as inverse images of CP_2 geodesic sphere for the imbedding of space-time surface to $CD \times CP_2$. Also the conjecture slicings by 3-surfaces and string world sheets could be identified. The identification of braids and slicings has been indeed one of the basic challenges in quantum TGD since in quantum theory one does not have anymore the luxury of topological invariance and I have proposed several identifications. If one accepts only these space-time sheets then the stringy content for a given space-time surface would be uniquely fixed.

The assignment of singularities to the homologically non-trivial geodesic sphere suggests that the homologically non-trivial space-time sheets could be seen as 1-dimensional idealizations of magnetic flux tubes carrying Kähler magnetic flux playing key role also in applications of TGD, in particular biological applications such as DNA as topological quantum computer and bio-control and catalysis.

6.5 The identification of charges Q, P and F of Khovanov homology

The challenge is to identify physically the three operators Q, F, and P appearing in Khovanov homology. Taking seriously the proposal of Witten [9] and looking for its direct counterpart in TGD leads to the identification and physical interpretation of these charges in TGD framework.

1. In Witten's approach P corresponds to instanton number assignable to the classical gauge field configuration in space-time. In TGD framework the instanton number would naturally correspond to that assignable to CP_2 Kähler form. One could consider the possibility of assigning this charge to the deformed CP_2 type vacuum extremals assigned to the space-like regions of space-time representing the lines of generalized Feynman diagrams having elementary particle interpretation. P would be or at least contain the sum of unit instanton numbers assignable to the lines of generalized Feynman diagrams with sign of the instanton number depending on the orientation of CP_2 type vacuum extremal and perhaps telling whether the line corresponds

to positive or negative energy state. Note that only pieces of vacuum extremals defined by the wormhole contacts are in question and it is somewhat questionable whether the rest of them in Minkowskian regions is included.

2. F corresponds to $U(1)$ charge assignable to R-symmetry of $N = 4$ SUSY in Witten's theory. The proposed generalization of twistorial approach in TGD framework suggests strongly that this identification generalizes to TGD. In TGD framework all solutions of modified Dirac equation at wormhole throats define super-symmetry generators but the supersymmetry is badly broken. The covariantly constant right handed neutrino defines the minimally broken supersymmetry since there are no direct couplings to gauge fields. This symmetry is however broken by the mixing of right and left handed M^4 chiralities present for both Dirac actions for induced gamma matrices and for modified Dirac equations defined by Kähler action and Chern-Simons action. It is caused by the fact that both the induced and modified gamma matrices are combinations of M^4 and CP_2 gamma matrices. F would therefore correspond to the net fermion number assignable to right handed neutrinos and antineutrinos. F is not conserved because of the chirality mixing and electroweak interactions respecting only the conservation of lepton number.

   Note that the mixing of M^4 chiralities in sub-manifold geometry is a phenomenon characteristic for TGD and also a direct signature of particle massivation and SUSY breaking. It would be nice if it would allow the physical realization of Q operator of Khovanov homology.

3. Witten proposes an explicit formula for Q in terms of 5-dimensional time evolutions interpolating between two 4-D instantons and involving sum of sign factors assignable to Dirac determinants. In TGD framework the operator Q should increase the right handed neutrino number by one unit and therefore transform one right-handed neutrino to a left handed one in the minimal situation. In zero energy ontology Q should relate to a time evolution either between ends of CD or between the ends of single line of generalized Feynman diagram. If instanton number can be assigned solely to the wormhole contacts, this evolution should increase the number of CP_2 type extremals by one unit. 3-particle vertex in which right handed neutrino assignable to a partonic 2-surface transforms to a left handed one is thus a natural candidate for defining the action of Q. In TGD framework Dirac determinant of 3-D Chern-Simons Dirac operator is conjectured to define exponent of Kähler function reducing to the exponent of Chern-Simons Kähler form. Maybe the sign factor could relate to this determinant.

4. Note that the almost topological QFT property of TGD together with the weak form of electric-magnetic duality implies that Kähler action reduces to Abelian Chern-Simons term. Ordinary Chern-Simons theory involves imaginary exponent of this term but in TGD the exponent would be real. Should one replace the real exponent of Kähler function with imaginary exponent? If so, TGD would be very near to topological QFT also in this respect. This would also force the quantization of the coupling parameter k in Chern-Simons action. On the other hand, the Chern-Simons theory makes sense also for purely imaginary k [9].

6.6 What does the replacement of topological invariance with symplectic invariance mean?

One interpretation for the symplectic invariance is as an analog of diffeo-invariance. This would imply color confinement. Another interpretation would be based on the identification of symplectic group as a color group. Maybe the first interpretation is the proper restriction when one calculates invariants of braids and 2-knots.

The replacement of topological symmetry with symplectic invariance means that TGD based invariants for braids carry much more refined information than topological invariants. In TGD approach M^4 diffeomorphisms act freely on partonic 2-surfaces and 4-D tangent space data but the action in CP_2 degrees of freedom reduces to symplectic transformations. One could of course consider also the restriction to symplectic transformations of the light-cone boundary and this would give additional refinements.

It is is easy to see what symplectic invariance means by looking what it means for the ends of braids at a given partonic 2-surface.

1. Symplectic transformations respect the Kähler magnetic fluxes assignable to the triangles defined by the finite number of braid points so that these fluxes defining symplectic areas define some minimum number of coordinates parametrizing the moduli space in question. For topological invariance all n-point configurations obtained by continuous or smooth transformations are equivalent braid end configurations. These finite-dimensional moduli spaces would be contracted with point in topological QFT.

2. This picture led to a proposal of what I call symplectic QFT [1] in which the associativity condition for symplectic fusion rules leads the hierarchy of algebras assigned with symplectic triangulations and forming a structures known as operad in category theory. The ends of braids at partonic 2-surfaces would would define unique triangulation of this kind if one accepts the identification of string like 2-surfaces as inverse images of homologically non-trivial geodesic sphere.

Note that both diffeomorphisms and symplectic transformations can in principle induce braiding of the braid strands connecting two partonic 2-surfaces. Should one consider the possibility that the allow transformations are restricted so that they do not induce braiding?

1. These transformations induce a transformation of the space-time surface which however is not a symplectic transformation in the interior in general. An attractive conjecture is that for the preferred extremals this is the case at the inverse images of the homologically non-trivial geodesic sphere. This would conform with the proposed duality between partonic 2-surfaces and string world sheets inspired by holography and also with quantum classical correspondence suggesting that at string world sheets the transformations induced by symplectic transformations at partonic 2-surfaces act like symplectic transformations.

2. If one allows only the symplectic transformations in Cartan algebra leaving the homologically non-trivial geodesic sphere invariant, the infinitesimal symplectic transformations would affect neither the string word sheets nor braidings but would modify the partonic 2-surfaces at all points except at the intersections with string world sheets.

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