New insights about quantum criticality for twistor lift inspired by analogy with ordinary criticality

M. Pitkänen
Email: matpitka6@gmail.com.
http://tgdtheory.com/.

June 20, 2019

Abstract

Quantum criticality (QC) is one of the basic ideas of TGD. Zero energy ontology (ZEO) is second key notion and leads to a theory of consciousness as a formulation of quantum measurement theory making observer part of the quantum system in terms of notion of self identified as a generalized Zeno effect or analog for a sequence of weak measurements, and solving the basic paradox of standard quantum measurement theory, which one usually tries to avoid by introducing some "interpretation".

ZEO allows to see quantum theory could be seen as "square root" of thermodynamics. It is interesting to apply this vision in the case of quantum criticality to gain additional insights about its meaning. We have a picture about criticality in the framework of thermodynamics: what would be the analogy in ZEO based interpretation of Quantum TGD? Could it help to understand more clearly the somewhat poorly understood views about the notion of self, which as a quantum physical counterpart of observer becomes in ZEO a key concept of fundamental physics?

The correspondence rules are simple. The mixture of phases with different 3-volumes per particle in a critical region of thermodynamical system is replaced with a superposition of space-time surfaces of different 4-volumes assignable to causal diamonds (CDs) with different sizes. Energy E is replaced with action S for preferred extremals defining Kähler function in the "world of classical worlds" (WCW). S is sum of Kähler action and 4-volume term, and these terms correspond to entropy and volume in the generalization $E = TS - PV \rightarrow S$. P resp. T corresponds to the inverse of Kähler coupling strength α_K resp. cosmological constant Λ. Both have discrete spectrum of values determined by number theoretically determined discrete coupling constant evolution. Number theoretical constraints force the analog of microcanonical ensemble so that S as the analog of E is constant for all 4-surfaces appearing in the quantum superposition. This implies quantization rules for Kähler action and volume, which are very strong since α_K is complex.

One is led to a rather detailed picture about quantum criticality by using this analogy and one gains new insights about the notion of self as sequence of quantum critical systems created by a unitary process and identified as superposition of space-time surfaces with same action S but varying 4-volume so that time is not well-defined observable. Weak measurements perform a state function reduction to a space-time surface with fixed volume inside CD of fixed size and thus measure time. When there are no space-time surfaces of larger size inside causal diamond CD satisfying the quantization conditions, self dies in the first reduction to the opposite boundary of CD. This is space-time correlate for the situation in which there are no measurements that would leave the members of state pairs at the passive boundary of CD unaffected.

This picture generalizes to the twistor lift of TGD and cosmology provides an interesting application. One ends up with a precise model for the p-adic coupling constant evolution of the cosmological constant Λ explaining the positive sign and smallness of Λ in long length scales as a cancellation effect for M^4 and CP_2 parts of the Kähler action for the sphere of twistor bundle in dimensional reduction, a prediction for the radius of the sphere of M^4 twistor bundle as Compton length associated with Planck mass (2π times Planck length), and a prediction for the p-adic coupling constant evolution for Λ and coupling strength of M^4 part of Kähler action giving also insights to the CP breaking and matter antimatter asymmetry. The observed two values of Λ could correspond to two different p-adic length scales differing by a factor of $\sqrt{2}$.

Contents

1 Introduction

Quantum criticality (QC) is one of the basic ideas of TGD. Zero energy ontology (ZEO) is second key notion and leads to a theory of consciousness as a formulation of quantum measurement theory making observer part of the quantum system in terms of notion of self identified as a generalized Zeno effect or analog for a sequence of weak measurements, and solving the basic paradox of standard quantum measurement theory, which one usually tries to avoid by introducing some "interpretation".

ZEO allows to see quantum theory could be seen as "square root" of thermodynamics. It occurred to me that it would be interesting to apply this vision in the case of quantum criticality to perhaps gain additional insights about its meaning. We have a picture about criticality in the framework of thermodynamics: what would be the analogy in ZEO based interpretation of Quantum TGD? Could it help to understand more clearly the somewhat poorly understood views about the notion of self, which as a quantum physical counterpart of observer becomes in ZEO a key concept of fundamental physics?

The basic ingredients involved are discrete coupling constant evolution, zero energy ontology (ZEO) implying that quantum theory is analogous to "square root" of thermodynamics, self as generalized Zeno effect as counterpart of observer made part of the quantum physical system, $M^8 \leftrightarrow M^4 \times CP_2$ duality, and quantum criticality. A further idea is that vacuum functional is analogous to a thermodynamical partition function as exponent of energy $E = TS - PV$.

The correspondence rules are simple. The mixture of phases with different 3-volumes per particle in a critical region of thermodynamical system is replaced with a superposition of space-time surfaces of different 4-volumes assignable to causal diamonds (CDs) with different sizes. Energy E is replaced with action S for preferred extremals defining Kähler function in the "world of classical worlds" (WCW). S is sum of Kähler action and 4-volume term, and these terms correspond to entropy and volume in the generalization $E = TS - PV \to S$. P resp. T corresponds to the inverse of Kähler coupling strength α_K resp. cosmological constant Λ. Both have discrete spectrum of values determined by number theoretically determined discrete coupling constant evolution. Number theoretical constraints force the analog of micro-canonical ensemble so that S as the analog of E is constant for all 4-surfaces appearing in the quantum superposition. This implies quantization rules for Kähler action and volume, which are very strong since α_K is complex.

This kind of quantum critical zero energy state is created in unitary evolution created in single step in the process defining self as a generalized Zeno effect. This unitary process implying time de-localization is followed by a weak measurement reducing the state to a fixed CD so that the clock time identified as the distance between its tips is well-defined. The condition that the action is same for all space-time surfaces in the superposition poses strong quantization conditions between the value of Kähler action (Kähler coupling strength is complex) and volume term proportional to

cosmological constant. The outcome is that after sufficiently large number of steps no space-time surfaces satisfying the conditions can be found, and the first reduction to the opposite boundary of CD must occur - self dies. This is the classical counterpart for the fact that eventually all state function reduction leaving the members of state pairs at the passive boundary of CD invariant are made and the first reduction to the opposite boundary remains the only option.

The generation of magnetic flux tubes provides a manner to satisfy the constancy conditions for the action so that the existing phenomenology as well as TGD counterpart of cyclic cosmology as re-incarnations of cosmic self follows as a prediction. This picture allows to add details to the understanding of the twistor lift of TGD at classical level and allows an improved understanding of the p-adic length scale evolution of cosmological constant solving the standard problem caused by the huge value of Λ. The sign of Λ is predicted correctly.

1.1   Some background

Some TGD background is needed to understand the ideas proposed in the sequel.

1.1.1   Discrete coupling constant evolution

The most obvious implication is discrete coupling constant evolution in which the set of values for coupling constants is discrete and analogous to the set of the critical values of temperature [L9] (see `http://tinyurl.com/y9hlt3rp`). Zeros of Riemann Zeta or its slight modification suggest themselves as the spectrum for the Kähler coupling strength. This discrete coupling constant evolution requires that loop corrections vanish. This vision is realized concretely in the generalization of the twistorial approach to the construction of scattering amplitudes [L9].

Non-manifest unitarity is the basic problem of the twistor Grassmann approach. A generalization of the BCFW formula without the loop corrections gives scattering amplitudes satisfying unitary constraints. The needed cuts are be replaced by sequences of massless poles in 8-D sense and cuts approximate these sequences (consider electrostatic analogy in which line charge approximates a discrete sequences of poles). The replacement cuts with sequences of poles is forced by the number theoretic discretization of momenta so that they belong to an extension of rationals defining the adele [L5] (see `http://tinyurl.com/ycbhse5c`).

Non-planar loop diagrams are a chronic problem of twistor approach since there is no general rule loop integrations allowing to combine them neatly. Also this problem disappears now.

$M^8 - H$ duality plays key role in the twistorial approach [L4] (see `http://tinyurl.com/yd43o2n2`). In the ordinary twistor approach all momenta are light-like so that it does not apply to massive particles. TGD solves this problem: at M^8 level one has quaternionic light-like 8-D momenta, which correspond to massive 4-D momenta in M^8 picture. In $H = M^4 \times CP_2$ picture ground states of super-conformal representations are constructed in terms of spinor harmonics of in $M^4 \times CP_2$, which are products plane-waves characterized by massive 4-momenta and color wave functions associated with massless Dirac equation in H. Also the analog of Dirac equation for the induced spinor fields at space-time surface is massless [K2] (see `http://tinyurl.com/yc2po5gf`).

1.1.2   ZEO and self as generalized Zeno effect

ZEO allows to see self as generalized Zeno effect [L6] (see `http://tinyurl.com/ycxm2tpd`).

1. Generalized Zeno effect can be regarded as a sequence of "small" state function reductions analogous to weak measurements performed at active boundary of causal diamond (CD). In usual Zeno effect the state is unaffected under repeated measurements: now the same is true at passive boundary of CD whereas the members of state pairs at the active boundary change. The unitary evolutions followed by these evolutions leave thus passive boundary and states at it invariant whereas active boundary shifts farther away from the passive boundary and the members of state pairs at it are affected. This gives rise to the experienced flow of time.

   The change of states is characterized unitary S-matrix. Each unitary evolution involves delocalization in the space of CDs so that one has quantum superposition of CDs with sizes not smaller than the CD to which the state was localized at previous reduction. This gives rise to

a steady increase of clock time defined as the distance between the tips of CD. Self dies and reincarnates as a self with opposite direction of clock time when the first unitary evolution at the passive boundary followed by a weak measurement at it takes place. Self dies when all observables leaving the states at passive boundary invariant are measured. There are no choices to be made anymore.

2. Quantum TGD as "square root" of thermodynamics means that the partition function of thermodynamics is replaced by its "square root" defined by the vacuum functional identified as exponent of Kähler function of "world of classical worlds" (WCW). Kähler function is analogous to energy $E = TS - PV$ in thermodynamics with T replaced with the inverse of complex Kähler coupling strength and P with cosmological constant, which have discrete spectrum of values.

   One has the analog of micro-canonical ensemble for which only states with given energy are possible. Now the action (Kähler function) is same for the space-time surfaces assignable to the zero energy states involved. This condition allows to get rid of the exponentials defining the vacuum functional otherwise appearing in the scattering amplitudes. This condition is strongly suggested by number theoretic universality for which these exponentials are extremely troublesome since both the exponent and exponential should belong to the extension of rationals used.

   This implies a huge simplification in the construction of the amplitudes [L4] (see `http://tinyurl.com/yd43o2n2`) because finite measurement resolution effectively replaces space-time surfaces with their cognitive representation defined by a discrete set of space-time points with imbedding space coordinates in the extension of rationals defining the adele.This representation codes for the space-time surface if it corresponds to zero locus of real or imaginary part (in quaterionic sense) of an octonionic polynomial with real coefficients. WCW coordinates are given by the cognitive representation and are discrete. One is led to enumerative algebraic geometry.

1.1.3 $M^8 - H$ duality

$M^8 - H$ duality [L4] (see `http://tinyurl.com/yd43o2n2`) states that the purely algebraic dynamics determined by the vanishing of real or imaginary part for octonionic polynomial is dual to the dynamics dictated by partial differential equations for an action principle.

1. There are two options for how to identify M^8 counterparts of space-time surfaces in terms roots of four polynomials defining real or imaginary part of an octonionic polynomial obtained as a continuation of real polynomial.

   (a) One can allow all roots $x + iy$ and project them to M^4 or M^8 from M_c^8. One can decompose these surfaces to regions with associative (quaternionic) tangent space or normal space and they are analogous to external particles of a twistor diagram entering CD and to interaction regions in which associativity does not hold true and which correspond to interiors of CD. One can criticizes the projection as somewhat adhoc process.

   (b) It became later clear that that one can also consider space-time surface as Minkowskian real regions so that the projection to a sub-space $M^4 \subset M_c^8$ of complexified octonions is invariant under the conjugation $i \to -i, I_k \to -I_k$, where I_k are quaternionic units. M_c^4 parts of space-time coordinates would be form $m = m^0 + iI_k m^k$, m^0, m^k real. This conditions need not or even cannot be posed on E_c^4 coordinates since $M^8 - H$ duality assigns to the tangent space of space-time surface a CP_2 point irrespective of whether the point is in M_c^8 or M^8.

2. At the level of H external particles correspond to minimal surfaces, which are also extremals of Kähler action and in accordance with the number theoretical universality and quantum criticality do not depend on the coupling parameters at all. They are obtained by a map taking the 4-surfaces in M^8 to those in H. These conditions should be equivalent with the condition that the 6-D surfaces X^6 in 12-D twistor space of H define twistor bundles of space-time surfaces X^4.

3. The space-time regions in the interiors of CDs are not minimal surfaces so that Kähler action and volume term couple dynamically and coupling parameters characterize the extremals. The analog is motion of point like particle in the Maxwell field defined by induced Kähler form: this is generalize to the motion of 3-D object with purely internal Kähler field and that associated with wormhole contacts and mediating interaction with larger and smaller space-time sheets.

   In these regions the map mediating $M^8 - H$ duality does not exist since one cannot label the tangent spaces of space-time surface by points of CP_2. The non-existence of this map is due to the failure of either associativity of tangent space or normal space at M^8 level. The initial values at boundaries of CD for the incoming preferred extremals however allows to fix the time evolution in the interior of CD. This is essentially due to the infinite number of gauge conditions for the super-symplectic algebra.

   It has later turned out [L9] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.

1.1.4   Quantum criticality

Quantum criticality is a further key notion of TGD and was originally motivated by the idea that Kähler coupling strength must be unique in order that the theory is unique.

1. The first implication of quantum criticality is quantization of various coupling strengths as analogs of critical temperature and of other critical parameters such as pressure. This quantization is required also by number theoretical universality in the adelic approach: coupling constant parameters must belong to the extension of rationals used.

2. Second implication of quantum criticality is a huge generalization of conformal symmetries to their 4-D analogs. The key observation is that 3-D light-like surfaces allow a generalization of conformal invariance to get the Kac-Moody algebra associated with the isometries of H (at least) as symmetries. In the case of boundary of CD this leads to what I call supersymplectic invariance: the symplectic transformations of the two components of $\delta CD \times CP_2$ act as isometries of WCW. This algebra allows a fractal hierarchy of sub-algebras isomorphic to the algebra itself and gauge conditions state that this kind of sub-algebra and its commutator with the entire algebra annihilate physical states and classical Noether charges for them vanish [L9] (see `http://tinyurl.com/y9hlt3rp`). By quantum classical correspondence (QCC) the eigenvalues of quantum charges are equal to the classical Noether charges in Cartan algebra of supersymplectic algebra.

3. The third implication is the understanding of preferred extremals in $H = M^4 \times CP_2$ and their counterparts at the level of M^8. Associativity condition at the level of M^8 satisfied by the spacetime surfaces representing external particles arriving into CD corresponds to quantum criticality posing conditions on the coefficients of octonionic polynomials. The space-time regions inside CD the space-time surfaces do not satisfy associativity conditions and are not critical.

4. TGD as "square root" of thermodynamics idea suggests a fourth application of quantum criticality. This analogy might allow a better understanding of self as Zeno effect. This application will be studied in the sequel.

   This picture generalizes to the twistor lift of TGD and cosmology provides an interesting application. One ends up with a precise model for the p-adic coupling constant evolution of the cosmological constant Λ explaining the positive sign and smallness of Λ in long length scales as a cancellation effect for M^4 and CP_2 parts of the Kähler action for the sphere of twistor bundle in dimensional reduction, a prediction for the radius of the sphere of M^4 twistor bundle as Compton

length associated with Planck mass (2π times Planck length), and a prediction for the p-adic coupling constant evolution for Λ and coupling strength of M^4 part of Kähler action giving also insights to the CP breaking and matter antimatter asymmetry. The observed two values of Λ could correspond to two different p-adic length scales differing by a factor of $\sqrt{2}$.

2 Analogy of the vacuum functional with thermodynamical partition function

Consider first the thermodynamical view about criticality. I have discussed criticality from slightly different perspective in [L7] (see `http://tinyurl.com/ydhknc2c`).

1. Thermodynamical states in critical region, where phases with different densities - say liquid and gas - are present serves as a basic example. This situation is actually a problem of the approach relying on partition function as van der Waals equation predicting 3 different densities for the density of molecules as function of pressure and temperature. Cusp catastrophe gives a view about situation: number density n is behavior variable and P and T are the control variables.

2. The experimental fact is that the density is constant as function of volume V for fixed temperature T whereas van der Waals predicts dependence on V. The phase corresponding to the middle sheet of the cusp is not at all present and the portions of liquid and gas phases vary. Maxwell's rules (area rule and lever rule) allow to solve the problem plaguing actually all approaches based on partition function. Lever rule assumes that there are actually two kinds of "elements" present. Molecules are the first element but what the second element could be? TGD identification is as magnetic tubes [L7].

3. In the more general case in which the catastrophe is more general than cusp and has more sheets, two or more phases with different volumes are present and their volumes and possibly other behavior variables analogous to volume vary at criticality.

4. If one applies criticality in stronger sense by requiring that the function which has extremum as function of n at the surface represented by cusp catastrophe has same value at different sheets of the cusp, only the boundary line of the cusp having V-shaped projection in (p, T)-plane remains.

2.1 Generalization of thermodynamical criticality to TGD context

The generalization of this picture to TGD framework replaces the mixture of thermodynamical phases with different volumes with quantum superposition of space-time surfaces with different 4-volumes assignable to CDs with different quantized sizes (by number theoretical constraints).

1. Vacuum functional, which is exponent of Kähler function of WCW expressible as Kähler action for its preferred extremal, can be regarded as a complex "square root" of thermodynamical partition function Z meaning that its real valued modulus squared is analogous to partition function [K6, K5, K4, K7].

   Action S, whose value for preferred extremal defines Kähler function of WCW serves as the analog of energy assumed to have expression $E = PV - TS$, which is not generally true but implied by the condition that E is homogenous as function of conjugate variable pairs P, V and T, S. The analogs of P and T correspond to coupling constant parameters. Pressure p is replaced with the coefficient of volume term in action - essentially cosmological constant. T is replaced with the coefficient $1/\alpha_K$ of Kähler action representing entropy (or negentropy depending on situation).

   Remark: Note that T corresponds now to $1/\alpha_K$ rather than α_K analogous to temperature when Kähler action S_K is regarded as analog of energy E rather than entropy S.

2. Quantum criticality in the sense of ZEO is the counterpart for the criticality in thermodynamics. The mixture of thermodynamical phases with different 3-volumes is replaced with

quantum superposition of zero energy states with 4-surface having same action S but different 4-volumes assignable to different CDs. Critical system consists of several phases with same values of coupling parameters α_K and Λ but different 4-volume.

There is also a number theoretic constraint identifiable as the counterpart of the constant energy condition defining micro-canonical ensemble. The exponent of action S must cancel from the scattering amplitudes to avoid serious existence problems in the p-adic sectors of adele associated with given extension of rationals. Criticality means thus that $exp(S)$ has same value for all preferred extremals involved. Real parts are same for all of them and imaginary parts of the action exponential are fixed modulo multiple of 2π. The analog in the case of van der Waals equation of state that the allowed states are associated with the boundary of the projection of the cusp catastrophe to (p, T) plane.

Critical quantum states are superpositions of space-time surfaces with different 4-volumes associated with CDs with quantized size scales (distance between tips) and are generated by unitary evolution. The value of time as size of CD (distance between its tips) is not well-defined in these states.

Remark: Quantum critical states are "timeless" as meditative practices would express it.

This kind of superposition is created by unitary evolution operator at each step in the sequence of unitary evolutions followed by a state function reduction measuring clock time as the distance between the tips of CD. Localization to single CD is the outcome and only superposition with same time-scale and same S but possibly different 4-volumes.

3. The condition that action is same is very strong and applies to both real and imaginary parts of action (α_K is complex). The proposal [L1, L9] (see `http://tinyurl.com/yas6ofhv` and `http://tinyurl.com/y9hlt3rp`) is that the coupling constant evolution as p-adic length scale $p \simeq 2^k$, k prime corresponds to zero of Riemann ζ for $1/\alpha_K$ or is proportional to it by rational multiplier q. For $q = 1$ $Re(1/\alpha_K)$ analogous to the ordinary temperature would be equal to $Re(s) = 1/2$ for the zeros at the critical line and imaginary parts would correspond to the imaginary parts $Im(s)$ of the zeros. Constancy of the action S would boil down to the conditions

$$Re(S_K) + Re(S_{vol}) = constant \quad , \quad Im(S_K) + Im(S_{vol}) = constant \ \ mod \ \ 2\pi \ . \qquad (2.1)$$

Note that the condition for imaginary part is a typical quantization condition.

4-volume can can have arbitrary large values but for S_K this is probably not the case - this already by the quantization conditions. Hence one expects that there is some maximal possible volume for preferred extremals and thus maximal distance between the tips of CDs involved.

When the zero energy state is a superposition of only space-time surfaces with this maximal volume, further unitary evolutions are not possible and the first state function reduction to the opposite boundary of CD happens (death of self and reincarnation with opposite direction of clock time). Self has finite lifetime! This would be the classical correlate for the situation in which no quantum measurements leaving invariant the members of state pairs at the passive boundary of CD are possible.

2.2 The constancy of $Re(S)$

How the cancellation of real part of $\Delta(Re(S_K)) + \Delta(Re(S_{vol}))$ could take place?

1. The physical picture is that the time evolution giving rise to self starts from flux tube dominated phase obtained in the first state function reduction to the opposite boundary of CD and that also asymptotically one obtains flux tube dominated phase again but the flux tubes are scaled up. This is the TGD view about quantum cosmology as a sequences of selves and of their time reversals [K1] [L2] (see `http://tinyurl.com/y7fmaapa`). This picture suggests that the generation of magnetic flux tubes allows to satisfy the $\Delta Re(S_K) + \Delta Re(S_{vol}) = 0$

condition: in Minkowskian regions the change magnetic part of $\Delta Re(S_K)$ tends to cancel $\Delta Re(S_{vol})$ whereas the electric part is of the same sign. Therefore magnetic flux tubes are favored.

If the sign of the volume term is negative the exponential defining the vacuum functional decreases with volume. If the relative sign of S_K and S_{vol} is negative, the magnetic part of the action is positive. The generation of flux tubes generates positive magnetic action ΔS_K helping to cancel the change ΔS_{vol}.

The additional conditions coming from the imaginary parts are analogous to semiclassical quantization conditions.

2. The proposed picture can be realized by a proper choice of the relative signs of volume term and Kähler action term. The relative sign comes automatically correct for a positive value of cosmological constant Λ. For this choice the total action density is

$$L_{tot} = (L_K + \frac{\Lambda}{8\pi G})\sqrt{g_4} \ . \tag{2.2}$$

This choice gives positive vacuum energy density associated with the volume term.

3. The density of Kähler action associated with CP_2 degrees of freedom is

$$L_{K,CP_2} = -\frac{1}{4g^2} J^{\mu\nu} J_{\mu\nu} \ . \tag{2.3}$$

The action is proportional to $E^2 - B^2$ in Minkowskian regions and magnetic term has sign opposite to that of volume term so that these terms can compensate with the condition guaranteeing constant action. The overall sign of action in the exponent can be chosen so that the exponential vanishes for large volumes. This suggests that the volume term is negative in the vacuum functional (Kähler function as negative of the action for preferred extremal). Euclidian regions, where CP_2 part of Kähler action is of form $B^2 + E^2$ and tends to cancel the volume term.

4. There is also Kähler action in M^4 degrees of freedom. In twistor lift dimensional reduction occurs for 6-D Kähler action and M^4 part and CP_2 part contribute to Kähler action. The S^2 parts of these actions must give rise to a cosmological constant decreasing like the inverse of p-adic length scale squared. This is achieved if the Kähler contributions have opposite signs so that M^4 contribution has a non-standard sign. This is possible if M^4 Kähler form is proportional to imaginary unit and M^4 Kähler coupling strength contains additional scaling factor.

The induced Kähler form must be sum of the M^4 parts and CP_2 parts and also the action must be sum of M^4 and CP_2 parts. This is achieved if the charge matrices of these two Kähler forms are orthogonal (the trace of their product vanishes). Since CP_2 part couples to both 1 and Γ_9 giving rise to Kähler charges proportional to 1 for quarks and 3 for leptons having opposite chiralities, the corresponding charges would be proportional to 3 for quarks and -1 for leptons.

The imaginary unit multiplying M^4 Kähler form disappears in action and field equations and one obtains

$$L_K = -\frac{1}{4g_K^2}(\epsilon^2 J^2(M^4) + J^2(CP_2) \ , \tag{2.4}$$

where ϵ is purely imaginary so that one has $\epsilon^2 < 0$. Since the fields are induced, negative sign for M^4 Kähler action is not expected to lead to difficulties if M^4 term is small.

Some examples are in order.

1. For cosmic string extremals Kähler action is multiple of volume action. The condition that the two actions cancel would give a constraint between Λ and α_K. Net string tension would be reduced from the value determined by CP_2 scale to a rather small value. This need not occur generally but might be true for very short p-adic length scales, where Λ is large as required by the large value of string tension associated with Kähler action. For thickened cosmic strings (magnetic flux tubes) the value of string tension assignable to Kähler action is reduced and the condition can be satisfied for smaller values of Λ.

2. For CP_2 type extremals assignable to wormhole contacts serving as basic building bricks of elementary particles the action would be finite for all size scales of CD. Both magnetic and electric contribution to the action are of same sign. For Euclidian regions with 4-D space-time projection with so strong electric field that it changes the signature of the induced metric the same is true.

3. One can ask whether blackhole interiors as Euclidian regions correspond to these Euclidian space-time sheets or to highly tangled magnetic flux tubes with length considerably longer than Schwartschild radius for which cancellation also can occur (see `http://tinyurl.com/ydhknc2c`). Both pictures are consistent in many-sheeted space-time: magnetic flux tube tangle could topologically condense to a space-time sheet with Euclidian signature. Cancellation cannot last for ever so that also blackholes are unstable against big state function reduction changing the arrow of time. Blackhole evaporation might relate to this instability.

2.3   The constancy of $Im(S)$ modulo 2π

If cosmological constant is real, the condition for the constancy of imaginary part of ΔS modulo 2π applies only to the case of S_K and implies that ΔS_K is fixed modulo 2π in the superposition of space-time surfaces. If zeros of ζ [L1] (see `http://tinyurl.com/yas6ofhv`) or its modification) [L9]) (see `http://tinyurl.com/y9hlt3rp`) give the spectrum of $1/\alpha_K$ the value of $\Delta S_{K,red} = \int Tr(J^2)dV$ is given as multiples of $2\pi n/y$, where y is imaginary part for a zero of zeta. The constancy of $Re(S)$ implies that the 4-volume ΔV is quantized as multiples of $2\pi n/\Lambda$. These conditions bring in mind semiclassical quantization of the action in multiples of $\hbar$.

It however turns out that twistor lift forces same phase for M^4 and CP_2 parts of the Kähler action so that the quantization condition for volume is lost. The reason is that $1/\alpha_K(M^4)$ and $1/\alpha_K(CP_2)$ are proportional to

$$\frac{1}{\alpha_{K,6}} = \frac{1}{\alpha_{K,4}R^2} \quad , \tag{2.5}$$

where R^2 has dimensions of length squared.

3   Is the proposed picture consistent with twistor lift of Kähler action?

Is it possible to realize the cancellation of real parts of ΔS_{vol} and ΔS_K (modulo 2π for imaginary part) for the twistor lift of Kähler action? Does the sign of the cosmological constant Λ come out correctly (wrong sign of Λ is the probably fatal problem of M-theory)? Can one understand the p-adic evolution of the cosmological constant Λ implying that Λ becomes small in long p-adic length scales and thus solving the key problem related to Λ?

3.1   Dimensional reduction of the twistor lift

The condition that the induction of the product of twistor bundles of M^4 and CP_2 to the space-time surface gives the twistor bundle of the space-time surface is conjectured to determine the dynamics of the space-time surfaces. A generalization of 4-D Kähler action to 6-D Kähler action is proposed to give this dynamics, and to dimensionally reduce to a sum of Kähler actions associated with M^4 and CP_2 Kähler forms plus cosmological term.

1. Twistor bundles are sphere bundles. For the extremals of 6-D Kähler action dimensional reduction takes place since 6-D extremals must be twistor bundle of corresponding space-time surface. Therefore S^2 degrees of freedom are frozen and become non-dynamical.

   One could say that the spheres appearing as fibers of twistor bundles of M^4 and CP_2 are identified in the imbedding map. The simplest correspondence between $S^2(M^4)$ and $S^2(CP_2)$ identifies (θ_1, ϕ_1) for $S^2(M^4)$ with (θ_2, ϕ_2) for $S^2(CP_2)$. This means that $S^2(X^6)$ is mapped in the same manner to $S^2(M^4)$ and $S^2(CP_2)$.

   One can imagine also correspondence with n-fold winding based on the identification $(\theta_1, \phi_1) = (\theta_2, n\phi_2)$. The area of $S^2(M^4)$ are becomes n-fold and the S^2 part of the Kähler action using θ_2 as coordinate transforms as $S_K(S^2(M^4)n = 1) \rightarrow S_K(S^2(M^4)n) = n^2 S_K(S^2(M^4))$. $n = 1$ is the most plausible option physically.

2. What the proposed general vision implies for cosmological constant as a sum of $S^2(M^4)$ and $S^2(CP_2)$ parts of 6-D Kähler action giving in dimensional reduction 4-D volume term responsible for the cosmological constant and 4-D Kähler action. If the charge matrices of M^4 and CP_2 parts of Kähler form are orthogonal one can induce Kähler form. If the coupling to M^4 Kähler form is imaginary, M^4 and CP_2 contributions to the total Kähler action have opposite signs. M^4 and CP_2 parts have opposite signs of magnetic terms and the sign of CP_2 magnetic part is opposite to the volume term.

3. The dimensionally reduced action is obtained by integrating the 6-D Kähler action over S^2 fiber. The integration gives the area $A(S^2)$ of the S^2 fiber, which in the metric induced from the spheres of twistor space of X^4 is given by

$$A(S^2) = (1 + r^2)4\pi R^2(S^2(CP_2)) \ , \quad r = \frac{R(S^2(CP_2))}{R(S^2(M^4))} \ . \tag{3.1}$$

   The very natural but un-checked assumption is that the radius of $S^2(CP_2)$ equals to the radius $R(CP_2)$ of the geodesic sphere of CP_2:

$$R(S^2(CP_2)) = R(CP_2) \ . \tag{3.2}$$

   One obtains

$$L = -\frac{1}{16\pi\alpha_{K,6}} \left[J^2(CP_2) + \epsilon^2 J^2(M^4) + J^2(S^2(CP_2)) + \epsilon^2 J^2(S^2(M^4)) \right] A(S^2) \ . \tag{3.3}$$

   The immediate conclusion is that the phases of Kähler action and volume term are same so that the quantization condition for imaginary part of the action is not obtained.

4. The Kähler coupling strengths $\alpha_K(CP_2)$ and $\alpha_K(M^4)$ can be read from the first term

$$\frac{1}{\alpha_K(CP_2)} = \frac{1}{\alpha_{K,4}4\pi(1+r^2)} \frac{R^2(CP_2)}{R^2} \ ,$$
$$\frac{1}{\alpha_K(M^4)} = \frac{\epsilon^2}{\alpha_K(CP_2)} \ . \tag{3.4}$$

   One can choose the factor R^2 to be the area of S^2 by suitably renormalizing $1/\alpha_K$. This would give simpler expression

$$\frac{1}{\alpha_K(CP_2)} = \frac{1}{\alpha_{K,4}} \ ,$$
$$\frac{1}{\alpha_K(M^4)} = \frac{\epsilon^2}{\alpha_K(CP_2)} \ . \tag{3.5}$$

5. One can deduce constraints on the value of the ϵ^2 from the smallness of the contributions of the corresponding $U(1)$ gauge potential to the ordinary Coulomb potential affecting the energies of atoms by a coupling proportional to mass number A rather than Z as for Coulomb potential. This allows to distinguish between isotopes. This gives very stringent bounds on ϵ^2. I have earlier derived an upper bound treating this term as a perturbation and by considering the contribution to the Coulomb energy of hydrogen atom [L3] (see `http://tinyurl.com/y8xcem2d`). One obtains $\epsilon^2 \leq 10^{-10}$. The upper bound is also the size scale of CP breaking induced by M^4 part and characterizes also matter-antimatter asymmetry.

3.2 Cosmological constant

Consider next the prediction for the cosmological constant term.

1. The S^2 parts of the actions have constant values. The natural normalization of Kähler form of $J(S^2(X))$, $X = M^4, CP_2$ is as $J^2 = -2$. This a convention is the overall scale of normalization can be chosen freely by rescaling $1/\alpha_{K,4}$. Taking into account the fact that index raising is carried out by induced metric one finds that the cosmological term given the sum of M^4 and CP_2 contributions to S^2 part of Kähler action multiplied by $A(S^2)$

$$\Lambda = \frac{1}{16\pi\alpha_K} \frac{2}{(1+r^2)R^2(CP_2)}(1 + \frac{\epsilon^2}{r^4}) \ . \tag{3.6}$$

If ϵ is imaginary one can achieve the cancellation giving rise to small cosmological constant.

2. The empirical condition on cosmological constant (see `https://en.wikipedia.org/wiki/Cosmological_constant`) can be expressed in terms of critical mass density corresponding to flat 3-space as

$$\Lambda = 3\Omega_\Lambda H^2 \ , \qquad \Omega \simeq .691 \ ,$$
$$H = \frac{da}{dt}a \qquad \frac{da}{dt} = \frac{1}{\sqrt{g_{aa}}} \ . \tag{3.7}$$

Here a corresponds to the proper time for the light-cone M_+^4 and t for the proper time for the space-time surface, which is Lorentz invariant under the Lorentz group leaving the boundary δM_+^4.

From this one obtains a condition for allowing to get idea about the discrete evolution of Λ with p-adic length scale occurring in jumps:

$$1 + \frac{\epsilon^2}{r^4} = 24\pi\alpha_K(1+r^2)R^2(CP_2) \times \Omega_\Lambda H^2 \ . \tag{3.8}$$

In an excellent approximation one must have $\epsilon \simeq r^2$, $r = R(M^4)/(CP_2)$. One can consider two obvious guesses. One has either $R(M^4) = L_{Pl} = \sqrt{G}$ - that is Planck length - or one has the Compton length associated with Planck mass given by $R(M^4) = 2\pi l_{Pl}$. The first option gives in reasonable approximation $r = 2^{-11}$ and $\epsilon^2 = r^4 = 2^{-44} \sim .6 \times 10^{-13}$. The second option gives $\epsilon^2 \simeq .9 \times 10^{-10}$. This values corresponds roughly to the CP_2 breaking parameter and matter-antimatter asymmetry and M^4 part of the Kähler action indeed gives rise to CP_2 breaking. I have earlier derived an upper bound for ϵ by demanding that the Kähler U(1) forces does not give rise to observable effects in the energy levels of hydrogen atom. The upper bound is of the same magnitude as the estimate for ϵ^2 for the Compton scale option.

3. If one accepts p-adic length scale hypothesis $L_p \propto \sqrt{p}$, $p \simeq 2^k$ [K3], one expects $\Lambda(k) \propto 1/L(k)^2$ [K4] (see `http://tinyurl.com/ybrhguux`). How to achieve this? The only possibility is that the parameter ϵ^2 is subject to coupling constant evolution. One would have for the cosmological constant

$$\Lambda(k) \propto \frac{\epsilon^2}{r^4} - 1 \propto \frac{1}{L^2(k)} \propto 2^{-k} \quad . \tag{3.9}$$

This would suggest for the 2-adic coupling constant evolution of ϵ the expression

$$\epsilon^2 = -r^4(1 - X) \ , \quad X = 24\pi\alpha_K(1 + r^2)R^2(CP_2) \times \Omega_\Lambda H^2 = q \times 2^{-k} \quad . \tag{3.10}$$

where q is rational number. Note that from p-adic length scale hypothesis one has $2^{-k} \propto 1/L^2(k)$. One can consider also p-adic primes near powers of small prime in which case one obtains different evolution.

4. For Ω_Λ constant this would predict quantization of Hubble constant as $\Omega_\Lambda H^2 \propto 1/L(k)^2$ determined by naive scaling dimension. The ratio of Hubble constants for two subsequent scales would be $H(k)/H(k+1) = \sqrt{2}$ if Ω is constant. The observed - and poorly understood - variation of Hubble constant from cosmological studies and distance ladder studies is in the range $50 - 73.2$ km/s/Mpc. Cosmological studies correspond to longer scales so that the smaller value of H is consistent with the decrease of H. The ratio of these upper and lower bounds is $1.46 < \sqrt{2} \simeq 1.141$ (see `http://tinyurl.com/yd6m8sca` and `http://tinyurl.com/ycr4ffm4`).

   Remark: The uncertainty in the value of Hubble constant is reflected as uncertainty in the distances D deduced from cosmic redshift $z \simeq HD/c$. This is taken into account in the definition of cosmological distant unit $h^{-1}Mpc$, where h is in the range $.5 - .75$ corresponding to a scale factor 1.5 rather near to $\sqrt{2}$.

5. Piecewise constant evolution means that acceleration parameter is positive since constant value of H gives

$$\frac{d^2a}{dt^2} = \frac{(da/dt)^2}{a} = aH^2 > 0 \quad . \tag{3.11}$$

If the phase transitions reducing H by factor $1/2$ occur at $a(k) = 2^{k/2}a_0$, one has

$$\frac{d^2a}{dt^2} \propto 2^{-k/2} \quad . \tag{3.12}$$

Acceleration would be reduced gradually with rate determined by its naive scaling dimension.

3.3 Solution of Hubble constant discrepancy from the length scale dependence of cosmological constant

One can critize this proposal. The recent best values of the Hubble constant are 67.0 km/s/Mpc and and 73.5 km/s/Mpc and their ratio is about 1.1 rather than $\sqrt{2}$. Therefore the hypothesis that H satisfies p-adic length scale hypothesis might be too strong. In the following a proposal in which the variation of H could be due to the variation of cosmological constant Λ satisfying p-adic length scale hypothesis is discussed.

The discrepancy of the two determinations of Hubble constant has led to a suggestion that new physics might be involved (see `http://tinyurl.com/yabszzeg`).

1. Planck observatory deduces Hubble constant H giving the expansion rate of the Universe from CMB data something like 360,000 y after Big Bang, that is from the properties of the cosmos in long length scales. Riess's team deduces H from data in short length scales by starting from galactic length scale and identifies standard candles (Cepheid variables), and uses these to deduce a distance ladder, and deduces the recent value of $H(t)$ from the redshifts.

2. The result from short length scales is 73.5 km/s/Mpc and from long scales 67.0 km/s/Mpc deduced from CMB data. In short length scales the Universe appears to expand faster. These results differ too much from each other. Note that the ratio of the values is about 1.1. There is only 10 percent discrepancy but this leads to conjecture about new physics: cosmology has become rather precise science!

TGD could provide this new physics. I have already earlier considered this problem but have not found really satisfactory understanding. The following represents a new attempt in this respect.

1. The notions of length scale are fractality are central in TGD inspired cosmology. Many-sheeted space-time forces to consider space-time always in some length scale and p-adic length scale defined the length scale hierarchy closely related to the hierarchy of Planck constants $h_{eff}/h_0 = n$ related to dark matter in TGD sense. The parameters such as Hubble constant depend on length scale and its value differ because the measurements are carried out in different length scales.

2. The new physics should relate to some deep problem of the recent day cosmology. Cosmological constant Λ certainly fits the bill. By theoretical arguments Λ should be huge making even impossible to speak about recent day cosmology. In the recent day cosmology Λ is incredibly small.

3. TGD predicts a hierarchy of space-time sheets characterized by p-adic length scales (Lk) so that cosmological constant Λ depends on p-adic length scale $L(k)$ as $\Lambda \propto 1/GL(k)^2$, where $p \simeq 2^k$ is p-adic prime characterizing the size scale of the space-time sheet defining the sub-cosmology. p-Adic length scale evolution of Universe involve as sequence of phase transitions increasing the value of $L(k)$. Long scales $L(k)$ correspond to much smaller value of Λ.

4. The vacuum energy contribution to mass density proportional to Λ goes like $1/L^2(k)$ being roughly $1/a^2$, where a is the light-cone proper time defining the "radius" $a = R(t)$ of the Universe in the Robertson-Walker metric $ds^2 = dt^2 - R^2(t)d\Omega^2$. As a consequence, at long length scales the contribution of Λ to the mass density decreases rather rapidly.

   Must however compare this contribution to the density ρ of ordinary matter. During radiation dominated phase it goes like $1/a^4$ from $T \propto 1/a$ and form small values of a radiation dominates over vacuum energy. During matter dominated phase one has $\rho \propto 1/a^3$ and also now matter dominates. During predicted cosmic string dominated asymptotic phase one has $\rho \propto 1/a^2$ and vacuum energy density gives a contribution which is due to Kähler magnetic energy and could be comparable and even larger than the dark energy due to the volume term in action.

5. The mass density is sum $\rho_m + \rho_d$ of the densities of matter and dark energy. One has $\rho_m \propto H^2$. $\Lambda \propto 1/L^2(k)$ implies that the contribution of dark energy in long length scales is considerably smaller than in the recent cosmology. In the Planck determination of H it is however assumed that cosmological constant is indeed constant. The value of H in long length scales is under-estimated so that also the standard model extrapolation from long to short length scales gives too low value of H. This is what the discrepancy of determinations of H performed in two different length scales indeed demonstrate.

A couple of remarks are in order.

1. The twistor lift of TGD [K6, K4] [L8] suggests an alternative parameterization of vacuum energy density as $\rho_{vac} = 1/L^4(k_1)$. k_1 is roughly square root of k. This gives rise to a pair of short and long p-adic length scales. The order of magnitude for $1/L(k_1)$ is roughly the same

as that of CMB temperature T: $1/L(k_1) \sim T$. Clearly, the parameters $1/T$ and R correspond to a pair of p-adic length scales. The fraction of dark energy density becomes smaller during the cosmic evolution identified as length scale evolution with largest scales corresponding to earliest times. During matter dominated era the mass density going like $1/a^3$ would to dominate over dark energy for small enough values of a. The asymptotic cosmology should be cosmic string dominated predicting $1/GT^2(k)$. This does not lead to contradiction since Kähler magnetic contribution rather than that due to cosmological constant dominates.

2. There are two kinds of cosmic strings: for the other type only volume action is non-vanishing and for the second type both Kähler and volume action are non-vanishing but the contribution of the volume action decreases as function of the length scale.

REFERENCES

Books related to TGD

[K1] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdclass.html#cosmo`, 2006.

[K2] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdgeom.html#cspin`, 2006.

[K3] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/padphys.html`, 2013.

[K4] Pitkänen M. About twistor lift of TGD? In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#hgrtwistor`, 2016.

[K5] Pitkänen M. From Principles to Diagrams. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#diagrams`, 2016.

[K6] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistorstory`, 2016.

[K7] Pitkänen M. Some questions related to the twistor lift of TGD. In *Towards M-Matrix*. Online book. Available at: `http://www.tgdtheory.fi/tgdhtml/tgdquantum.html#twistquestions`, 2017.

Articles about TGD

[L1] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? . Available at: `http://tgdtheory.fi/public_html/articles/fermizeta.pdf`, 2015.

[L2] Pitkänen M. Cyclic cosmology from TGD perspective. Available at: `http://tgdtheory.fi/public_html/articles/turok.pdf`, 2016.

[L3] Pitkänen M. Breaking of CP, P, and T in cosmological scales in TGD Universe. Available at: `http://tgdtheory.fi/public_html/articles/rotuniverse.pdf`, 2017.

[L4] Pitkänen M. Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry? Available at: `http://tgdtheory.fi/public_html/articles/ratpoints.pdf`, 2017.

[L5] Pitkänen M. Philosophy of Adelic Physics. Available at: `http://tgdtheory.fi/public_html/articles/adelephysics.pdf`, 2017.

[L6] Pitkänen M. Re-examination of the basic notions of TGD inspired theory of consciousness. Available at: `http://tgdtheory.fi/public_html/articles/conscrit.pdf`, 2017.

[L7] Pitkänen M. Maxwells lever rule and expansion of water in freezing: two poorly understood phenomena. Available at: `http://tgdtheory.fi/public_html/articles/leverule.pdf`, 2018.

[L8] Pitkänen M. Morphogenesis in TGD Universe. Available at: `http://tgdtheory.fi/public_html/articles/morphoTGD.pdf`, 2018.

[L9] Pitkänen M. The Recent View about Twistorialization in TGD Framework. Available at: `http://tgdtheory.fi/public_html/articles/smatrix.pdf`, 2018.