Are fundamental entities discrete or continuous and what discretization at fundamental level could mean?

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Abstract

Are fundamental physical objects are discrete or continuous? Is it possible to have unique discretization in given measurement resolution? These questions inspired this article trying to provide overall view about number theoretical discretization provided by adelic physics in which reals and extensions of various p-adic number fields induced by given extension of rationals are fused together to form adele. Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) - cognitive representation - having interpretation in terms of finite measurement resolution.

Number theory emerges in TGD also via classical number fields and one has dimensional hierarchy with rationals and their extensions at bottom, and reals, complex numbers, quaternions, and octonions above them. $M^8 - H$ duality reduces physics to algebraic equations at the level of octonionic M^8 and associativity is the fundamental dynamical principle. Space-time surfaces in M^8 are identified as roots of octonionic polynomials (real or imaginary part of polynomial in quaternionic sense vanishes).

The hierarchy of polynomials with rational coefficients corresponds to the hierarchy for extensions of rationals and also evolutionary hierarchy in biology. The hierarchy of Planck constants labelling the dark matter as phases of ordinary matter corresponds also to this hierarchy.

Space-time surfaces can be regarded as representations of extensions of rationals in terms of polynomials. One can identify simple extensions as extensions with simple Galois group and extensions of extensions can be built from these. One can say that simple extensions are like elementary particles and that they correspond to codons of space-time genes. One can construct infinite number of hierarchies of extensions of extensions of... by functional composition of polynomials. These hierarchies could correspond to the hierarchies of inclusions of hyperfinite factors proposed also to be usable for the description of finite measurement resolution.

Also the roots of analytic functions with rational Taylor coefficients define space-time surfaces. Can one allow also these space-time surfaces analogous to transcendental numbers? This would require giving up the idea about finiteness of cognition.

Especially interesting analytic functions are Dedekind zetas [**?**]haractering extensions of rationals. One can form composites of Dedekind zetas (quite not functional composition), and this leads to the conjecture that the compositions of extensions corresponds to composition of corresponding Dedekind zeta.

Contents

1 Introduction

FB discussion about whether fundamental physical objects are discrete or continuous and about discretization inspired this article. The aim is to provide overall view about number theoretical discretization provide by adelic physics [L2, L3] in which reals and extensions of various p-adic number fields induced by given extension of rationals are fused together to form an adele. Rationals and their extensions labelling the adeles give rise to a unique discretization of space-time surface (for instance) as the set of points with imbedding space coordinates in extension: I call it cognitive representation. The interpretation for discretization is in terms of finite cognitive and sensory resolution. Remarkably, no lattice structures are needed.

Number theory emerges in TGD also via classical number fields.

1. One has dimensional hierarchy having rationals and their extensions at bottom, and reals, complex numbers, quaternions, and octonions above them. $M^8 - H$ duality [L1, L11, L7] is central. M^8 picture reduces physics to algebraic equations at the level of octonionic M^8 and associativity is the fundamental dynamical principle. Space-time surfaces in M^8 are identified as roots of octonionic polynomials (real or imaginary part of polynomial in quaternionic sense vanishes). The space-time surfaces in M^8 are mapped to H by $M^8 - H$ duality.

   This map has some resemblance to spontaneous compactification but there is no dynamics involved. There is also an analogy with the wave-mechanical p-q duality of descriptions using momentum space and configuration spaces. Space-time surfaces in M^8 *resp.* H could be seen as 4-surfaces in the tangent space M^4 of H *resp.* H respectively.

2. The hierarchy of polynomials with rational coefficients corresponds to the hierarchy for extensions of rationals and also evolutionary hierarchy in biology [L4]. The hierarchy of Planck constants labelling dark matter as phases of ordinary matter corresponds also to this hierarchy [K1, K4].

3. Space-time surfaces in M^8 and by $M^8 - H$ duality also in H can be regarded as representations of extensions of rationals in terms of polynomials. The Galois group of extension E_1 of extension E_2 has the Galois group of E_2 as normal sub-group. One can identify simple extensions as extensions with simple Galois group and extensions of extensions can be constructed from these. What is remarkable that all simple finite groups are known (see `http://tinyurl.com/y3xh4hrh`).

4. Simple extensions are building bricks of more complex extensions and analogous to elementary particles: in particular, they correspond to polynomials with prime degree suggesting a connection with p-adicity. One can also say that simple extensions correspond to codons of space-time genes in very concrete manner. Genes as codon sequences correspond to extensions of extensions ... One can construct infinite number hierarchies of extensions of extensions of... by functional composition of polynomials, and these hierarchies could naturally correspond to the hierarchies of inclusions of hyperfinite factors proposed also to be usable for the description of finite measurement resolution [K3].

5. An objection against number theoretic discretization is that it is ultralocal and does not allow calculations of partial derivatives of imbedding space coordinates necessary for constructing metric and induced gauge fields. For polynomials however knowing the roots of the real polynomial allows to construct the polynomial and this in turn allows to calculate the partial derivatives. Also modified Dirac equation makes sense without lattice discretization forcing approximate derivatives.

   The same problem is encountered at the level of WCW. The same trick could work at this level since space-time surfaces as preferred extremals and physical states satisfy infinite number of gauge conditions. This effectively restricts WCW spinor fields to finite-dimensional manifolds of WCW and the high symmetries of these manifolds allow to deduce their metric and lattice discretization is not needed.

6. Also the roots of analytic functions with rational Taylor coefficients (by number theoretical universality) define space-time surfaces. Can one allow also their roots as space-time surfaces?

These space-time surfaces would be like transcendental numbers. This would require giving up the idea about finiteness of cognition.

Especially interesting rational functions are Dedekind zetas [L15, L12, L8] charactering extensions of rationals: for trivial extension one has ordinary Riemann zeta. One can form composites of Dedekind zetas by the formula $f_1 * f_2 = f_1 \circ f_2 - f_1(0)$ (quite not functional composition). Category theoretical thinking suggests that the composition of extensions corresponds to composition of corresponding Dedekind zetas. Riemann zeta is known to be associated with critical systems; quantum criticality is basic aspect of quantum TGD; and in zero energy ontology (ZEO) TGD can be regarded as complex square root of thermodynamics: could Dedekind zetas as generalizations of thermodynamical partition functions play a key role in TGD?

2 Discrete viz. continuous: TGD based view

There was an interesting FB discussion about discrete and continuum. I decided to write down my thoughts and emphasize those points that I see as important.

2.1 Is discretization fundamental or not?

The conversation inspired the question whether discreteness is something fundamental or not. If it is assumed to be fundamental, one encounters problems. The discrete structures are not unique. One has deep problem with the known space-time symmetries. Symmetries are reduced to discrete subgroup or totally lost. A further problem is the fact that in order to do physics, one must bring in topology and length measurements.

In discrete situation topology, in particular space-time dimension, must be put in via homology effectively already meaning use of imbedding to Euclidian space. Length measurement remains completely ad hoc. The construction of discrete metric is highly non-unique procedure and the discrete analog of of say Einstein's theory (Regge calculus) is rather clumsy. One feeds in information, which was not there by using hand weaving arguments like infrared limit. It is possible to approximate continuum by discretization but discrete to continuum won't go.

In hype physics these hand weaving arguments are general. For instance, the emergence of 3-space from discrete Hilbert space is one attempt to get continuum. One puts in what is factually a discretization of 3-space and then gets 3-space back at IR limit and shouts "Eureka!".

2.2 Can one make discretizations unique?

Then discussion went to numerics. Numerics is for mathematicians same as eating for poets. One cannot avoid it but luckily you can find people doing the necessary programming if you are a professor. Finite discretization is necessary in numerics and is highly unique.

I do not have anything personal against discretization as a numerical tool. Just the opposite, I see finite discretization as absolutely essential element of adelic physics as an attempt to describe also the correlates of cognition in terms of p-adic physics with p-adic space-time sheets as correlates of "thought bubbles" [L2, L3]. Cognition is discrete and finite and uses rational numbers: this is the basic clue.

1. Cognitive representations are discretizations of (for instance) space-time surface. One can say that physics itself builds its cognitive representation in all scales using p-adic space-time sheets. They should be unique once measurement resolution is characterized if one is really talking about fundamental physics.

   The idea abou tp-adic physics as physics of cognition indeed led to powerful calculational recipes. In p-adic thermodynamics the predictions come in power series of p-adic prime p and for the values of p assignable to elementary particles the two lowest terms give practically exact result [K2]. Corrections are of order 10^{-76} for electron characterized by Mersenne prime $M_{127} = 2^{127} - 1 \sim 10^{38}$.

2. Adelic physics [L2] provides the formulation of p-adic physics: it is assumed that cognition is universal. Adele is a book like structure having as pages reals and extensions of various p-adic number fields induced by given extension of rationals. Each extension of rationals defines its own extension of the rational adele by inducing extensions of p-adic number fields. Common points between pages consist of points in extension of rationals. The books associated with the adeles give rise to an infinite library.

   At space-time level the points with coordinates in extension define what I call cognitive representation. In the generic case it is discrete and has finite number of points. The loss of general coordinate invariance is the obvious objection. In TGD however the symmetries of the imbedding space fix the coordinates used highly uniquely. M^8-H duality ($H=M^4\times CP_2$) and octonionic interpretation implies that M^8 octonionic linear coordinates are highly unique [L1, L7]. Note that M^8 must be complexified. Different coordinatizations correspond to different octonionic structures- to different moduli - related by Poincare transformations of M^8. Only rational time translations as transformations of octonionic real coordinate are allowed as coordinate changes respecting octonionic structure.

3. Discretization by cognitive representation is unique for given extension of rationals defining the measurement resolution. At the limit of algebraic numbers algebraic points form a dense set of real space-time surface and p-adic space-time surfaces so that the measurement resolution is ideal. One avoids the usual infinities of quantum field theories induced by continuous delta functions, which for cognitive representations are replaced with Kronecker deltas. This seems to be the best that one can achieve with algebraic extensions of rationals. Also for transcendental extensions the situation is discrete.

   This leads to a number theoretic vision about second quantization of induced spinor fields central for the construction of gamma matrices defining the spinor structure of "world of classical worlds" (WCW) providing the arena of quantum dynamics in TGD analogous to the super-space of Wheeler [L14]. One ends up to a construction allowing to understand TGD view about SUSY as necessary aspect of second quantization of fermions and leads to the conclusions that in the simplest scenario only quarks are elementary fermions and leptons can be seen as their local composites analogous to super partners.

4. Given polynomial defining space-time surfaces in M^8 defines via its roots extension of rationals. The hierarchy of extensions defines an evolutionary hierarchy. The dimension n of extension defines kind of IQ measuring algebraic complexity and n corresponds also to effective Planck constant labelling phases of dark matter in TGD sense so that a direct connection with physics emerges.

   Imbedding space assigns to a discretization a natural metric. Distances between points of metric are geodesic distances computed at the level of imbedding space.

5. An unexpected finding was that the equations defining space-time surfaces as roots of real or imaginary parts of octonionic polynomials have also 6-D brane like entities with topology of S^6 as solutions [L5, **?**]. These entities intersect space-time surfaces at 3-D sections for which linear M^4 time is constant. 4-D roots can be glued together along these branes. These solutions turn out to have an interpretation in TGD based theory of quantum measurement extending to a theory of consciousness. The interpretation as moments of "small" state function reductions as counterparts of so called weak measurements. They could correspond to special moments in the life of conscious entity.

2.3 Can discretization be performed without lattices?

For a systems obeying dynamics defined by partial differential equations, the introduction of lattices seems to be necessary aspect of discretization. The problem is that the replacement of derivatives with discrete approximations however means that there is no hope about exact results. In the general case the discretization for partial differential equations involving derivatives forces to introduce lattice like structures. This is not needed in TGD.

1. At the level of M^8 ordinary polynomials give rise to octonionic polynomials and space-time surfaces are algebraic surfaces for which imaginary or real part of octonionic polynomial in

quaternionic sense vanishes. The equations are purely algebraic involving no partial derivatives and there is no need for lattice discretization.

For surfaces defined by polynomials the roots of polynomial are enough to fix the polynomials and therefore also the space-time surface uniquely: discretization is not an approximation but gives an exact result! This could be called number theoretical holography and generealizes the ordinary holography. Space-time surfaces are coded by the roots of polynomials with rational coefficients.

2. What about the field equations at the level of $H = M^4 \times CP_2$? $M^8 - H$ duality maps these surfaces to preferred extremals as 4-surfaces in H analogous to Bohr orbits. Twistor lift of TGD predicts that they should be minimal surfaces with 2-D singularities being also extremals of 4-D Kähler action. The field equations would reduce locally to purely algebraic conditions. In properly chosen coordinates for H they are expected to be determined in terms of polynomials coding for the same extension of rationals as their M^8 counterparts so that the degree should be same [L7]. This would allow to deduce the partial derivatives of imbedding space for the image surfaces without lattice approximation.

3. The simplest assumption is that the polynomials have rational coefficients. Number theoretic universality allows to consider also algebraic coefficients. In both cases also WCW is discretized and given point -space-time surface in QCD has coordinates given by the points of the number theoretically universal cognitive representation of the space-time surface. Even real coefficients are possible. This would allow to obtain WCW as a continuum central for the construction of WCW metric but is not consistent with number theoretical universality.

   Can one have polynomial/functions with rational coefficients and discretization of WCW without lattice but without losing WCW metric? Maybe the same trick that works at space-time level works also in WCW!

   (a) The group WCW isometries is identified as symplectic transformations of $\delta M^4_\pm \times CP_2$ ($\delta M^4_\pm$ denotes light-cone boundary) containing the boundary of causal diamond CD. The Lie algebra *Sympl* of this group is analogous half-Kac Moody algebra having symplectic transformations of $S^2 \times CP_2$ as counterpart of finite-D Lie group has fractal structure containing infinite number of sub-algebras $Sympl_n$ isomorphic to algebra itself: the conformal weights assignable to radial light-like coordinate are n-multiples of those for the entire algebra. Note that conformal weights of *Sympl* are non-negative.

   (b) One formulation for the preferred extremal property is in terms of infinite number of analogs of gauge conditions stating the vanishing of classical and also Noether charges for $Sympl_n$ and $[Sympl_n, Sympl]$. The conditions generalize to the super-counterpart of *Sympl* and apply also to quantum states rather than only space-time surfaces. In fact, while writing this I realized that - contrary to the original claim - also the vanishing of the Noether charges of higher commutators is required so that effectively $Sympl_n$ would define normal subgroup of *Sympl*. These conditions does not follow automatically.

   The Hamiltonians of $Sympl(S^2 \times CP_2)$ are also labelled by the representations of the product of the rotation group $SO(3) \subset SO(3,1)$ of S^2 and color group $SU(3)$ together forming the analog of the Lie group defining Kac-Moody group. This group does not have have the fractal hierarchy of subgroups. The strongest condition is that the algebra corresponding to Hamiltonian isometries does not annihilate the physical states.

   The space of states satisfying the gauge conditions is finite-D and that WCW becomes effectively finite-dimensional. A coset space associated with *Sympl* would be in question and it would have maximal symmetries as also WCW. The geometry of the reduced WCW, WCW$_{red}$ could be deduced from symmetry considerations alone.

   (c) Number theoretic discretization would correspond to a selection of points of this subspace with the coordinates in the extension of rationals.The metric of WCW$_{red,n}$ at the points of discretization would be known and no lattice discretization would be needed. The gauge conditions are analogous to massless Dirac equation in WCW and could be solved in the points of discretization without introducing the lattice to approximate derivatives. As a matter fact, Dirac equation can be formulated solely in terms of the generators of *Sympl*.

(d) This effectively restricts WCW to $WCW_{red,n}$ in turn reduced to its discrete subset - since infinite number of WCW coordinates are fixed. If this sub-space can be regarded as realization of infinite number of algebraic conditions by polynomials with rational coefficients one can assign to it extension of rationals defining naturally the discretization of $\text{WCW}_{red,n}$. This extension is naturally the same as for space-time surfaces involved so that the degree of polynomials defining $\text{WCW}_{red,n}$ would be naturally n and same as that for the polynomial defining the space-time surface. $\text{WCW}_{red,n}$ would decompose to union of spaces WCW_{red,E_n} labelled by extensions E_n of rationals with same dimension n.

There is analogy with gauge fixing. WCW_{red,E_n} is a coset space of WCW defined by the gauge conditions. One can represent this coset space as a sub-manifold of WCW by taking one representative point from each coset. This choice is not unique but one can hope finding a gauge choice realized by an infinite number of polynomials of degree n defining same extension of rationals as the polynomial defining the space-time surfaces in question.

(e) WCW spinor fields would be always restricted to finite-D algebraic surface of WCW_{red,E_n} expressible in terms of algebraic equations. Finite measurement resolution indeed strongly suggests that WCW spinor field mode is non-vanishing only in a region parameterized in WCW by finite number of parameters. There is also a second manner to see this. WCW_{red,E_n} could be also seen as $n+4$-dimensional surface in WCW.

(f) One can make this more concrete. Cognitive representation by points of space-time surface with coordinates in the extension - possibly satisfying additional conditions such as belonging to the 2-D vertices at which space-time surfaces representing different roots meet - provides WCW coordinates of given space-time surface. Minimum number of points corresponds to the dimension of extension so that the selection of coordinate can be redundant. As the values of these coordinates vary, one obtains coordinatization for the sector of WCW_{red,E_n}. An interesting question is whether one could represent the distances of space-time surfaces in this space in terms of the data provided by the points of discretization.

An interesting question is whether one can represent the distances of space-time surfaces in this space in terms of the data provided by the points of cognitive representation. One can define distance between two disjoint surfaces as the minimum of distance between the points of 2-surfaces. Could something like this work now? The points would be restricted to the cognitive representations. Could one define the distance between two cognitive representations with same number N of points in the following manner.

Consider all bipartitions formed by the cognitive representations obtained by connecting their points together in 1-1 manner. There are N! bipartitions of this kind if the number of points is N. Calculate the sum of the squares of the imbedding space distances between paired points. Find the bipartition for which this distance squared is minimum and define the distance between cognitive representations as this distance. This definition works also when the the numbers of points are different.

(g) If there quantum states are the basic objects and there is nothing "physical" behind them one can ask how we can imagine mathematical structures which different from basic structure of TGD. Could quantum states of TGD Universe in some sense represent all mathematical objects which are internally consistent. One could indeed say that at the level of WCW all $n+4$-D manifolds can be represented concretely in terms of WCW spinor fields localized to n-D subspaces of WCW. WCW spinor fields can represent concept of 4-surface of $\text{WCW}_{red,n}$ as a quantum superposition of its instance and define at the same time $n+4$-D surfaces [L14] [L6, L10, L9, L13].

2.4 Simple extensions of rationals as codons of space-time genetic code

A fascinating idea is that extensions of rationals define the analog of genetic code for space-time surfaces, which would therefore represent number theory and also finite groups.

(a) The extensions of rationals define an infinite hierarchy: the proposal is that the dimension of extensions corresponds to the integer n characterizing subalgebra $Sympl_n$. This would give direct correspondence between the inclusions of HFFs assigned to the hierarchy of algebras $Sympl_n$ and hierarchy of extensions of rationals with dimension n.

Galois group for a extension of extension contains Galois group of extension as normal subgroup and is therefore *not simple*. Extension hierarchies correspond to inclusion hierarchies for normal subgroups. Simple Galois groups are in very special position and associated with what one might call simple extensions serving as fundamental building bricks of inclusion hierarchies. They would be like elementary particles and define fundamental space-time regions. Their Galois groups would act as groups of physical symmetries.

(b) One can therefore talk about elementary space-time surfaces in M^8 and their compositions by function composition of octonionic polynomials. Simple groups would label elementary space-time regions. They have been classified: (see `http://tinyurl.com/y3xh4hrh`). The famous Monster groups are well-known examples about simple finite groups and would have also space-time counterparts. Also the finite subgroups of Lie groups are special and those of $SU(2)$ are associated with Platonic solids and seem to play key role in TGD inspired quantum biology. In particular, vertebrate genetic code can be assigned to icosahedral group.

(c) There is also an analogy with genes. Extensions with simple Galois groups could be seen as codons and sequences of extension obtained by functional composition as analogs of genes. I have even conjectured that the space-time surfaces associated with genes could quite concretely correspond to extensions of extensions of ...

2.5 Are octonionic polynomials enough or are also analytic functions needed?

I already touched the question whether also analytic functions with rational coefficients (number theoretical universality) might be needed.

(a) The roots of analytic functions generate extension of rationals. If the roots involve transcendental numbers they define infinite extensions of rationals. Neper number e is very special in this sense since e^p is ordinary p-adic number for all primes p so that the induced extension is finite-dimensional. One could thus allow it without losing number theoretical universality. The addition of π gives infinite-D extension but one could do by adding only roots of unity to achieve finite-D extensions with finite accuracy of phase measurement. Phases would be number theoretically universal but not angles.

(b) One could of course consider only transcendental functions with rational roots. Trigonometric function $sin(x/2\pi)$ serves as a simple example. One can also argue that since physics involves in an essential manner trigonometric functions via Fourier analysis, the inclusion of analytic functions with algebraic roots must be allowed.

(c) What about analytic functions as limits of polynomials with rational coefficients such that the number of roots becomes infinite at the limit? Also their imaginary and real part can vanish in quaternionic sense and could define space-time surfaces - analogs of transcendentals as space-time surfaces. It is not clear whether these could be allowed or not.

Could one have a universal polynomial like function giving algebraic numbers as the extension of rationals defined by its algebraic roots? Could Riemann zeta (see `http://tinyurl.com/nfbkrsx`) code algebraic numbers as an extension via its roots. I have conjectured that roots of Riemann zeta are algebraic numbers: could they span all algebraic numbers?

It is known that the real or imaginary part of Riemann zeta along $s = 1/2$ critical line can approximate any function to arbitrary accuracy: also this would fit with universality. Could one think that the space-time surface defined as root of octonionic continuation of zeta could be universal entity analogous to a fixed point of iteration in the construction of fractals? This does not look plausible.

4. One can construct iterates of Riemann zeta having at least the same roots as zeta by the rule

$$\begin{aligned} f_0(s) &= \zeta(s) \ , \\ f_n(s) &= \zeta(f_{n-1}(s)) - \zeta(0), \ \ \zeta(0) = -1/2 \ . \end{aligned} \tag{2.1}$$

ζ is not a fixed point of this iteration as the fractal universality would suggest. The set of roots however is. Should one be happy with this.

5. Riemann zeta has also counterpart in all extensions of rationals known as Dedekind zeta (see `http://tinyurl.com/y5grktv`) [L15, L12, L8]. Riemann zeta is therefore not unique. One can ask whether Dedekind zetas associated with simple Galois groups are special and whether Dedekind zetas associated with extensions of extensions of .... can be constructed by using the Dedekind zetas of simple extensions. How do the roots of Dedekind zeta depend on the associated extension of rationals? How the roots of Dedekind zeta for extension of extension defined by composite of two polynomials depend on extensions involved? Are the roots union for the roots associated with the composites?

6. What about forming composites of Dedekind zetas? Categorical according to my primitive understanding raises the question whether a composition of extensions could correspond to a composition of functions. Could Dedekind zeta for a composite of extensions be obtained from a composite of Dedekind zetas for extensions? Requiring that roots of extension E_1 are preserved would give formula

$$\zeta_{D,E_1E_2} = \zeta_{D,E_1} \circ \zeta_{D,E_2} - \zeta_{D,E_1}(0) \ . \tag{2.2}$$

The zeta function would be obtained by an iteration of simple zeta functions labelled by simple extensions. The inverse image for the set of roots of ζ_{D,E_1} under ζ_{D,E_2} that is the set $\zeta^{-1}_{D,E_2}(roots(\zeta_{D,E_1})$ would define also roots of ζ_{D,E_1E_2}. This looks rather sensible.

But what about iteration of Riemann zeta, which corresponds to trivial extension? Riemann ζ is not invariant under iteration although its roots are. Should one accept this and say that it is the set of roots which defines the invariant. Could one say that the iterates of various Dedekind zetas define entities which are somehow universal.

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