# TGD and M-Theory

M. Pitkänen,

June 25, 2017

Email: matpitka@luukku.com.
http://tgdtheory.com/public_html/.
Recent postal address: Karkinkatu 3 I 3, 00360, Karkkila, Finland.

## Contents

**Abstract**

In this chapter a critical comparison of M-theory and TGD as two competing theories is carried out. Dualities and black hole physics are regarded as basic victories of M-theory.

1. The counterpart of electric magnetic duality plays an important role also in TGD and it has become clear that it might change the sign of Kähler coupling strength rather than leaving it invariant. The different signs would be related to different time orientations of the space-time sheets. This option is favored also by TGD inspired cosmology but unitarity seems to exclude it.
2. The AdS/CFT duality of Maldacena involved with the quantum gravitational holography has a direct counterpart in TGD with 3-dimensional causal determinants serving as holograms so that the construction of absolute minima of Kähler action reduces to a local problem.
3. The attempts to develop further the nebulous idea about space-time surfaces as associative (co-associative) sub-manifolds of an octonionic imbedding space led to the realization of duality which could be called number theoretical spontaneous compactification. Space-time region can be regarded equivalently as a associative (co-associative) space-time region in $M^8$ with octonionic structure or as a 4-surface in $M^4 \times CP_2$. If the map taking these surface to each other preserves associtiativity in octonionic structure of $H$ then the generalization to $H-H$ duality becomes natural and would make preferred extremals a category.
4. The notion of cotangent bundle of configuration space of 3-surfaces (WCW) suggests the interpretation of the number-theoretical compactification as a wave-particle duality in infinite-dimensional context. These ideas generalize at the formal level also to the M-theory assuming that stringy configuration space is introduced. The existence of Kähler metric very probably does not allow dynamical target space.

In TGD framework black holes are possible but putting black holes and particles in the same basket seems to be mixing of apples with oranges. The role of black hole horizons is taken in TGD by 3-D light like causal determinants, which are much more general objects. Black hole-elementary particle correspondence and p-adic length scale hypothesis have already earlier led to a formula for the entropy associated with elementary particle horizon.

In TGD framework the interior of blackhole is naturally replaced with a region of Euclidian signature of induced metric and can be seen as analog for the line of Feynman diagram. Blackholes appear only in GRT limit of TGD which lumps together the sheets of many-sheeted space-time to a piece of Minkowski space and provides it with an effective metric determined as sum of Minkowski metric and deviations of the metrics of space-time sheets from Minkowski metric.

The recent findings from RHIC have led to the realization that TGD predicts black hole like objects in all length scales. They are identifiable as highly tangled magnetic flux tubes in Hagedorn temperature and containing conformally confined matter with a large Planck constant and behaving like dark matter in a macroscopic quantum phase. The fact that string like structures in macroscopic quantum states are ideal for topological quantum computation modifies dramatically the traditional view about black holes as information destroyers.

The discussion of the basic weaknesses of M-theory is motivated by the fact that the few predictions of the theory are wrong which has led to the introduction of anthropic principle to save the theory. The mouse as a tailor history of M-theory, the lack of a precise problem to which M-theory would be a solution, the hard nosed reductionism, and the censorship in Los Alamos archives preventing the interaction with competing theories could be seen as the basic reasons for the recent blind alley in M-theory.

# 1 Introduction

In this chapter a critical comparison of M-theory [B8] and TGD (see [K46, K31, K26, K21, K32, K38, K37] and [K42, K5, K30, K3, K13, K18, K20, K36] ) as two competing theories is carried out. Also some comments about the sociology of Big Science are made.

The problem with this chapter is that it is almost by definition always out-of-date. I have recently (I am writing this in 2015) updated the file trying to mention the most recent steps of progress about which there is a summary [L7] at my homepage as an article with links to my blog where one can find links to books about TGD.

## 1.1 From Hadronic String Model To M-Theory

The evolution of string theories began 1968 from Veneziano formula realizing duality symmetry of hadronic interactions. It took two years to realize that Veneziano amplitude could be interpreted in terms of interacting strings: Nambu, Susskind and Nielsen made the discovery simultaneously 1970. The need to describe also fermions led to the discovery of super-symmetry [B23] and Ramond and Neveu-Schwartz type superstrings in the beginning of seventies.

Gradually it became however clear that the strings do not describe hadrons: for instance, the critical dimensions for strings *resp.* superstrings where 26 *resp.* 10, and the breakthrough of QCD at 1973 meant an end for the era of hadronic string theory. 1974 Schwartz and Scherk proposed that strings might provide a quantum theory of gravitation [B28] if one accepts that space-time has compactified dimensions.

The first superstring revolution was initiated around 1984 by the paper by Green and Schwartz demonstrating the cancellation of anomalies in certain superstring theories [B13, B14]. The proposal was that superstrings might provide a divergence-free and anomaly-free quantum theory of gravitation. A crucial boost was given by Witten's interest on superstrings. Also the highly effective use of media played a key role in establishing superstring hegemony.

It became clear that superstrings come in five basic types [B22]. There are type I strings (both open and closed) with $N = 1$ super-symmetry and gauge group $SO(32)$, type IIA and IIB closed strings with $N = 2$ super-symmetry, and heterotic strings, which are closed and possess $N = 1$ super-symmetry with gauge groups $SO(32)$ and $E^8 \times E^8$. There is an entire landscape of solutions associated with each superstring theory defined by the compactifications whose dynamics is partially determined by the vanishing of conformal anomalies. For a moment it was believed that it would be an easy task to find which of the superstrings would allow the compactification which corresponds to the observed Universe but it became clear that this was too much to hope. In particular, the number 4 for non-compact space-time dimensions is by no means in a special position.

Around 1995 came the second superstring revolution with the idea that various superstring species could be unified in terms of an 11-dimensional M-theory with M meaning membrane in the lowest approximation [B8]. M-theory allowed to see various superstrings as limiting situations when 11-D theory reduces to 10-D one so that very special kind of membranes reduce to strings. This allowed to justify heuristically the claimed dualities between various superstrings [B22]. Matrix Theory as a proposal for a non-perturbative formulation of M-theory appeared 2 years later [B11].

Now, almost a decade later, M-theory is in a deep crisis: the few predictions that the theory can make are definitely wrong and even anthropic principle is advocated as a means to save the theory [B20]. Despite this, very many people continue to work with M-theory and fill hep-th with highly speculative preprints proving that this is dual with that although the flow of papers dealing with strings and M-theory has reduced dramatically.

A reader interested in critical views about string theory can consult the article of Smolin [B19] criticizing anthropic principle, the web-lectures "Fantasy, Fashion, and Faith in Theoretical Physics" of Penrose [B26] as well as his article in New Scientist [B27] criticizing the notion of hidden space time dimensions, and the articles of Peter [C4] [B25]. Also the discussion group "Not Even Wrong" [B3] gives a critical perspective to the situation almost a decade after the birth of M-theory.

## 1.2 Evolution Of TGD Very Briefly

The first superstring revolution shattered the world at 1984, about two years after my own doctoral dissertation (1982), and four years after the Esalem conference in which the quantum consciousness movement started. Remarkably, David Finkelstein was one of the organizers of the conference besides being the chief editor of "International Journal of Theoretical Physics", in which I managed to publish first articles about TGD. The first and last contact with stars was Wheeler's review of my first article published in IJTP, and I cannot tell what my and TGD's fate had been without Wheeler's highly encouraging review.

During the 31 years after the discovery that space-times could be regarded as 4-surfaces as well as extended objects generalizing strings, I have devoted my time to the development of TGD. Without exaggeration I can say that life devoted to TGD has been much more successful project than I dared or even could dream and has led outside the very narrow realms of particle physics and quantum gravity. Indeed, without knowing anything about Finkelstein and Esalem at that time, I started to write a book about consciousness around 1995 when the second superstring revolution occurred. TGD inspired theory of consciousness has now materialized as 8 online books at my home page.

Altogether these 37 years boil down to eight online books [K46, K31, K26, K56, K55, K54, K38, K37] about TGD proper and eight online books about TGD inspired theory of consciousness and of quantum biology [K42, K5, K30, K3, K13, K18, K36, K53] plus one printed book about TGD [K45] and second printed book about TGD inspired theory of consciousness and quantum biologys [**?**].

This makes about more than 10,000 pages of TGD spanning everything between elementary particle physics and cosmology. One might expect that the sheer waste amount of material at my web site might have stirred some interest in the physics community despite the fact that it became impossible to publish anything and to get anything into Los Alamos archives after the second super-string revolution. The only visible reaction has been from my Finnish colleagues and guarantees that I will remain unemployed in the foreseeable future. I will discuss some reasons for this state of affairs after comparing string models and TGD, and considering the reasons for the failure of the theory formerly known as superstring model.

Before continuing, I hasten to admit that I am not a string specialist and I do not handle the technicalities of M-theory. On the other hand, TGD has given quite a good perspective about the real problems of TOEs and provides also solutions to them. Hence it is relatively easy to identify the heuristic and usually slippery parts of various arguments from the formula jungle. Also I want to express my deep admiration for the people living in the theory world but from my own experience I know how easy it is to fall on wishful thinking and how necessary but painful it is to lose face now and then.

My humble suggestion is that M-theorists might gain a lot by asking what "What possibly went wrong?". This chapter suggests answers to this question: see also [L11]. Perhaps M-theorists might also spend a few hours in the web to check whether M-theory is indeed the only viable approach to quantum gravity: the material at my own home page might provide a surprise in this respect.

Ironically, TGD seems to be predict more stringy physics than string model. For instance, the well-definedness of em charge localizes the modes of induced spinor fields in generic case to 2-D surfaces so that strings become genuine part of TGD. Furthermore, string like objects defined by magnetic flux tubes appear in all scales, even in nuclear physics. These two kinds of strings actually seem to accompany each other.

Also the AdS/CFT correspondence generalizes in TGD framework (the conformal symmetries in TGD are gigantic as compared to those in string models) and is made obvious by the fact that WCW Kähler metric can be expressed either in terms of Kähler function or as commutations of WCW gamma matrices identifiable as Noether super charges in the Yangian of super-symplectic algebra.

A further fascinating quite recent finding is that if one assumes that strings connecting partonic 2-surfaces serve as correlates for the formation of gravitationally bound states, string tension $T = 1/\hbar G$ allows only bound states of size of order Planck length - a fatal prediction. Even the replacement $h \rightarrow h_{eff} = n \times h$ does not help. The solution of the problem comes from the supersymmetry and generalization of AdS/CFT correspondence. By supersymmetry Kähler action must be expressible as bosonic string action defined by the total string world sheet area in the effective metric defined by the anti-commutator of Kähler-Dirac matrices at string world sheets. This predicts that string tension is proportional to $1/h_{eff}^2$ and allows to understand the formation of gravitationally bound states. Macroscopic quantum gravitational coherence even in astrophysical scales is predicted.

The most recent steps of progress relate to the formulation of scattering amplitudes based on the proposed 8-D variant of twistor approach involving octonionic representation of the imbedding space gamma matrices in an essential manner and light-like momenta in 8-D sense plus the lifting of space-time surfaces to their twistor spaces represented as surfaces in the twistor space of $M^4 \times CP_2$. The Yangian of super-symplectic algebra for which Noether charges are expressible as integrals over strings connecting partonic 2-surfaces defines the basic 3-vertices as product and co-product and the scattering amplitudes can be seen a sequences of algebraic manipulating connecting initial and final state identified as states at the opposite boundaries of CD. Universe would be doing Yangian arithmetics.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L3]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4]. The topics relevant to this chapter are given by the following list.

- Comparison with other theories [L5]
- How TGD differs from standard model [L9]
- How quantum TGD differs from standard quantum physics [L8]
- Similarities between TGD and string models [L10]
- Differences between TGD and string models [L6]

# 2 A Summary About The Evolution Of TGD

The basic idea about space-time as a 4-surface popped in my mind in autumn at 1978, I am not quite sure about the year, it might be also 1977. . The first implication was that I lost my job at Helsinki University. During the next 4 years this idea led to a thesis with the title "Topological GeometroDynamics" (TGD), which I think was suggested by David Finkelstein to distinguish TGD from Wheeler's GeometroDynamics.

## 2.1 Space-Times As 4-Surfaces

TGD can be seen as as a solution to the energy problem of General Relativity via the unification of special and general relativities by assuming that space-times are representable as 4-surfaces in certain 8-dimensional space-time with the symmetries of empty Minkowski space. An alternative interpretation is as a generalization of string models by replacing strings with 3-dimensional surfaces: depending on their size they would represent elementary particles or the space we live in and anything between these extremes. From this point of view superstring theories are unique candidates for a Theory of Everything if space-time were 2- rather than 4-dimensional.

The first superstring revolution made me happy since I was convinced that it would be a matter of few years before TGD would replace superstring models as a natural generalization allowing to understand the four-dimensionality of the space-time. After all, only a half-page argument, a simple exercise in the realization of standard model symmetries, leads to a unique identification of the higher-dimensional imbedding space as a Cartesian product of Minkowski space and complex projective space $CP_2$ unifying electro-weak and color symmetries in terms of its holonomy and isometry groups. By the 4-dimensionality of the basic objects there was no need for the imbedding space geometry to be dynamical. Theory realized the dream about the geometrization of fundamental interactions and predicted the observed quantum numbers. In particular, the horrors of spontaneous compactification to be crystallized in the notion of M-theory landscape two decades later can be circumvented completely.

## 2.2 Uniqueness Of The Imbedding Space From The Requirement Of Infinite-Dimensional Kähler Geometric Existence

Later I discovered heuristic mathematical arguments suggesting but not proving that the choice of the imbedding space is unique. The arguments relied on the uniqueness of the infinite-dimensional Kähler geometry of WCW of 3-surfaces. This uniqueness was discovered already in the context of loop spaces by Dan Freed [A4].

$CH$, the "world of the classical worlds" serves as the arena of quantum dynamics [K9], which reduces to the theory of classical spinor fields in $CH$ and geometrizes fermionic anti-commutation relations and the notion of super-symmetry in terms of the gamma matrices of $CH$ [K50]. Only quantum jump is the genuinely non-classical element of the theory in $CH$ context. The heuristic argument states that $CH$ geometry exists only for $H = M^4 \times CP_2$.

The strongest argument for the uniqueness of $H$ emerged only rather recently (2014) [K43]. $M^4$ and $CP_2$ are the only 4-D manifolds allowing twistor space with Kähler structure. This fact has been discovered by Hitchin at about same time as I discovered the basic idea of TGD [A6] but had escaped my attention. This leads to a formulation of TGD using liftings of space-time surfaces to their twistor spaces: allowed space-time surfaces are those whose twistor spaces can be induced from the product of twistor spaces of $M^4$ and $CP_2$.

Also number theoretical arguments relating to quaternions and octonions fix the dimensions of space-time and imbedding space to four and 8 respectively. The fact that the space of quaternionic sub-spaces of octonion space containing preferred plane complex plane is $CP_2$ suggest an explanation for the special role of $CP_2$.

This stimulated a development, which led to notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in $M^8$ or as surfaces in $M^4 \times CP_2$ [K41]. What makes this duality possible is that $CP_2$ parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multiplied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

The weakest form of number theoretical compactification states that light-like 3-surfaces $X_l^3 \subset HO$ are mapped to $X_l^3 \subset M^4 \times CP_2$ and requires only that one can assign preferred plane $M^2 \subset M^4$ to any connected component of $X_l^3$. This hyper-complex plane of hyper-quaternionic $M^4$ has interpretation as the plane of non-physical polarizations so that the gauge conditions of super string theories are obtained purely number theoretically. $M^2$ corresponds also to the degrees of freedom which do not contribute to the metric of WCW . The un-necessarily strong form would require that hyper-quaternionic 4-surfaces correspond to preferred extremals of Kähler action.

The requirement that $M^2$ belongs to the tangent space $T(X^4(X_l^3))$ at each point point of $X_l^3$ fixes also the boundary conditions for the preferred extremal of Kähler action. The construction of WCW spinor structure supports the conclusion that there must exist preferred coordinates of $X^4$ in which additional conditions $g_{ni} = 0$ and $J_{ni} = 0$ at $X_l^3$. The conditions state that induced metric and Kähler form are stationary at $X_l^3$. $M^2$ plays a key role also in many other constructions of quantum TGD, in particular the generalization of the imbedding space needed to realize the idea about hierarchy of Planck constant allowing to identify dark matter as matter with a non-standard value of Planck constant.

The realization of 4-D general coordinate invariance forces to assume that Kähler function assigns a unique space-time surface to a given 3-surface: by the breakdown of the strict classical determinism of Kähler action unions of 3-surfaces with time like separations must be however allowed as 3-D causal determinants and quantum classical correspondence allows to interpret them as representations of quantum jump sequences at space-time level. Space-time surface defined as a preferred extremal [K41] of Kähler action is analogous to Bohr orbit so that classical physics becomes part of the definition of configuration space geometry rather than being a result of a stationary phase approximation.

What "preferred" has been a longstanding problem. In zero energy ontology (ZEO) 3-surfaces are pairs of 3-surfaces at the opposite light-like boundaries of causal diamond (CD), whose $M^4$ projection is an intersection of future and past directed light-cones. In spirit with what I call strong form of holography, the space-time surfaces connecting these two 3-surfaces are assumed to possess vanishing Noether charges in a sub-algebra of super-symplectic algebra with conformal weights coming as $n$-multiple of the weights of the entire algebra. This condition is extremely powerful. For the sub-algebra labelled by $n$ super-symplectic generators act as conformal gauge symmetries, and one obtains infinite number of hierarchies of conformal gauge symmetry breakings. One can also interpret these conformal hierarchies in terms of gradually reduced quantum criticality. An attractive interpretation is that $n$ corresponds the value of effective Planck constant $h_{eff}/h = n$, whose values label a hierarchy of dark matter phases. Also a connection with hierarchies of hyper-finite factors emerges. There are many other partial characterizations of blockquotepreferred to be discussed later but this looks to me the most attractive one now.

## 2.3 TGD Inspired Theory Of Consciousness

During the last decade a lot has happened in TGD and it is sad that only those colleagues with mind open enough to make a visit my home page have had opportunity to be informed about this. Knowing the fact that a typical theoretical physicist reads only the articles published in respected journals about his own speciality, one can expect that the number of these physicists is not very high. Some examples of the work done during this decade are in order.

I have developed quantum TGD in a considerable detail with highly non-trivial number theoretical speculations relating to Riemann hypothesis and Riemann Zeta in riema. One outcome is a proposal for the proof of Riemann hypothesis [L1].

During the same period I have constructed TGD inspired theory of consciousness [K42]. One outcome is a theory of quantum measurement and of observer having direct implications for the quantum TGD itself. The results of the modification of the double slit experiment carried out by Afshar [D1] , [C3] provides a difficult challenge for the existing interpretations of quantum theory and a support for the TGD view about quantum measurement in which space-time provides correlates for the non-deterministic process in question. The new views about energy and time have also profound technological implications.

The hierarchy of Planck constants, quantum criticality, and the notion of magnetic body inspired by the notions of many-sheeted space-time and topological field quantization have become central concepts in TGD inspired theory of consciousness. Also p-adic physics as physics of cognition is key element.

The new view about measurement theory based on Zero Energy Ontology (ZEO) and the notion of causal diamond (CD) forces a more detailed view about state function reduction. In quantum context one has quantum superposition of CDs and each CD carries zero energy state: it is assumed that the CDs in superposition have second boundary which belongs to common light-cone boundary. State function can occur at both boundaries of CD and self as a conscious entity can be identified as the sequence of repeated state function reductions occurring at fixed boundary doing nothing for the fixed boundary. The experience about flow of time and arrow of time can be understood and the latter can correspond to both arrow of geometric time. Volitional act corresponds to the first reduction at the opposite boundary.

Negentropy Maximization Principle (NMP) serves as the basic variational principle and implies ordinary quantum measurement theory and second law for a generic entanglement. There is however a notable exception. When the density matrix decomposes into direct sum containing $n \times n$ unit matrices with $n > 1$: this happens in two-particle system when the entanglement coefficients define a unitary matrix. One can assign number theoretic variant of Shannon entropy to a state with this kind of density matrix and the state is stable with respect to NMP. One can speak of negentropic entangleement since entanglement entropy is negative. NMP predicts that the amount of negentropic entanglement increases in the Universe. Negentropic entanglement has interpretation as abstraction: the state pairs in the superposition represent instances of a rule. An obvious conjecture is that $n$ relates to $h_{eff}/h = n$ and to the hierarchy of quantum criticalities.

## 2.4 Number Theoretic Vision

Physics as infinite-D spinor geometry of WCW and physics as generalized number theory are the two basic visiona about TGD.

The number theoretic vision involves three threads.

1. The first thread involves the notion of number theoretic universality: quantum TGD should make sense in both real and p-adic number fields (and their algebraic extensions). p-Adic number fields would be needed to understand the space-time correlates of cognition and intentionality [K22, K12, K24]. .

   p-Adic number fields lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K22, K12]. One of the first applications was the calculation of elementary particle masses [K19]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra is involved. Not only the fundamental mass scales reduce to number theory but also individual masses are predicted correctly under very mild assumptions. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K19].

2. Second thread is inspired by the dimensions of the basic objects of TGD and assumes that classical number fields are in a crucial role in TGD. 8-D imbedding space would have octonionic structure and space-time surfaces would have associative (quaternionic) tangent space or normal space. String world sheets would correspond to commutative surfaces. Also the notion of $M^8-H$-duality is part of this thread and states that quaternionic 4-surfaces of $M^8$ containing preferred $M^2$ in its tangent space can be mapped to preferred extremals of Kähler action in $H$ by assigning to the tangent space $CP_2$ point parametrizing it. $M^2$ could be replaced by integrable distribution of $M^2(x)$. If the preferred extremals are also quaternionic one has also $H-H$ duality allowing to iterate the map so that preferred extremals form a category.

3. The third thread corresponds to infinite primes [K39] leading to several speculations. The construction of infinite primes is structurally analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory with free particle states characterized by primes. The many-sheeted structure of TGD space-time could reflect directly the structure of infinite prime coding it. Space-time point would become infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography. Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

### 2.4.1 p-Adic physics as physics of cognition

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics (see **Fig.** `http://tgdtheory.fi/appfigures/book.jpg` or **Fig. ??** in the appendix of this book). The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) $p$ in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range.

An ideal realization of the space-time sheet as a cognitive representation results if the $CP_2$ coordinates as functions of $M^4_+$ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes $p$ and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of WCW spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization

Principle (NMP) stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K39]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

One very interesting aspect of number theoretic vision is the possibility that scattering amplitude could be regarded as a representation for sequences of algebraic operations (product and co-product) in super-symplectic Yangian representing 3-vertices and leading from initial set of algebraic objects to to a final set of them [K43]. The construction would have a gigantic symmetry: any sequence of operations connecting initial and final state would correspond to the same scattering amplitude.

### 2.4.2 Number theoretical symmetries

TGD as a generalized number theory vision leads to a highly speculative idea that also number theoretical symmetries are important for physics. Reader can decided whether the following should be taken with any seriousness. Also I try to do so.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group $S_\infty$ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of $S_\infty$ is HFF which can be mapped to the HFF defined by WCW spinors. This picture suggest a number theoretical gauge invariance stating that $S_\infty$ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of $S_\infty$. The groups $G$ should relate closely to finite groups defining inclusions of HFFs.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space $M^8$ *resp.* $M^4 \times CP_2$.

3. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

## 2.5 Hierachy Of Planck Constants And Dark Matter

TGD has lead to two proposals for how non-standard values of Planck constants might appear in physics.

### 2.5.1 Large Planck constant from neuroscience

The strange quantal effects of ELF em fields on vertebrate brain suggest that the energies $E = hf$ of ELF photons were above thermal energy at physiological temperature. This suggests the replacement $h \rightarrow h_{eff} = n \times h$ and the leads to the vision that bio-systems are macroscopic quantum systems with ordinary quantum scales scaled up by factor $n$.

The earlier work with topological quantum computation [K47] had already led to the idea that Planck constant could relate to the quantum phase $q = exp(i\pi/n)$. The improved understanding of Jones inclusions and their role in TGD [K49] allowed to deduce then extremely simple formula $h_{eff} = n \times h$. Much later came the realization that the hierarchy of Planck constants corresponds naturally to a hierarchy of gauge symmetry breakings assignable with the super-symplectic algebra possessing conformal structure and having also interpretation as a hierarchy of improved measurement resolutions suggested to have mathematical description in terms of inclusions of hyper-finite factors of type $II_1$. Since the inclusions are accompanied by quantum groups characterized by $q$ the connection with the inclusions and $h_{eff}$ can be understood. The localization of the induced spinor fields at string world sheets is also essential: their 2-D character is what makes possible to pose a quantum version of anti-commutation relations for the induced spinor fields. Hence it seems that the $h_{eff} = n \times h$ hypothesis fits naturally to the framework of physical principles and mathematical concepts underlying TGD.

One can speculate about the most probable values of $n$. I have suggested that the values of $n$ for which the quantum phase is expressible using only iterated square root operation (corresponding polygon is obtained by ruler and compass construction) are of special interest since they correspond to the lowest evolutionary levels for cognition so that corresponding systems should be especially abundant in the Universe. One should be however extremely cautious with this kind of speculations.

The general philosophy would be that when the quantum system becomes non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character. This would apply to QCD and to atoms with $Z > 137$ and to any other system. $q \neq 1$ quantum groups characterize non-perturbative phases. Macroscopic gravitation is second fundamental example: the coupling parameter $GMm/\hbar c$ exceeds unity for macroscopic systems. Here Nottale led to the hypothesis $\hbar_{gr} = GMm/v_0$ to be described in more detail below. The obvious conjecture $h_{gr} = h_{eff}$ has very interesting biological implications discussed in [K59] and [K58].

### 2.5.2 Large Planck constant from astrophysics

Another step in the rapid evolution of quantum TGD [K34], [L2] was stimulated when I learned that D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ $(\hbar = c = 1)$. $v_0$ is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.82 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of $v_0$ seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha suggest that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant assignable to the flux tubes mediating gravitational interaction so that there the dependence on both masses makes sense.

The gravitational (ordinary) Schrödinger equation - in TGD framework it is better to restrict to the Bohr orbitology version of it - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The basic objection is that astrophysical systems are extremely classical whereas TGD predicts macrotemporal quantum coherence in the scale of life time of gravitational bound states. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

TGD allows a reasonable estimate for the value of the velocity parameter $v_0$ assuming that cosmic strings and their decay remnants are responsible for the dark matter. The value of $v_0$ has interpretation as velocity of distant stars around galaxies in the gravitational field of long cosmic string like objects traversing through galactic plane. The harmonics of $v_0$ could be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes $n^2$-fold: much like the replacement of a closed orbit with an orbit closing only after $n$ turns. Sub-harmonics would result when cosmic strings decay to magnetic flux tubes: magnetic energy density per unit length is quantized by the preferred extremal property and the simplest possibility is the reduction of the energy density by a factor $1/n^2$.

That the value of $h_{gr}$ is different for inner and outer planets is of course disturbing. In this aspect quite recent progress in the understand of basic quantum TGD comes in rescue. The generalization of AdS/CFT duality to TGD framework predicts that gravitational binding is mediated by strings connecting partonic 2-surfaces. If string world sheet area is define by the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices, it is proportional to $\alpha_K^2 \propto 1/h_{eff}^2$ if one assumes $\alpha_K = g_K^2/4\pi h_{eff}$ so that $\alpha_K$ would have a spectrum of critical values coming as inverses of integers. The size scale of bound state would scale like $\hbar_{gr} = GMm/v_0$ and would be of order $GM/v_0$: this make sense. The outer planets have much larger size than inner planets and the reduction of $v_0$ by factor 1/5 helps to understand their orbits. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K58] in terms electric-magnetic duality.

As noticed, ruler and compass rule suggests a spectrum of the most plausible values of $h_{eff}/h = n$. This quantization does not depend at all on the velocity parameter $v_0$ appearing in the formula of Nottale and this gives strong additional constraints to the ratios of planetary masses and also on the masses themselves if one assumes that the gravitational Planck constant corresponds to the values allowed by ruler and compass construction. Also correct prediction for the ratio of densities of visible and dark matter emerges.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy values predicted by gravitational Bohr orbitology might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with n=1 orbit in the case of Sun is 24 hours within experimental accuracy for $v_0$. This would make sense of $h_{eff} = h_{gr}$ hypothesis holds true. Quantum gravitation would be crucial for life, as Penrose intuited, but in manner very different from what has been usually thought [K59, K58].

Needless to add, if the proposed general picture is correct, not much is left from the super-string/M-theory approach to quantum gravitation since perturbative quantum field theory as the fundamental corner stone must be given up and because the underlying physical picture about gravitational interaction is simply wrong.

### 2.5.3 Mathematical realization for the hierarchy of Planck constants

The work with hyper-finite factors of type $II_1$ (HFFs) combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter [K11]. The original proposal was that the hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional sub-manifolds to which are "quantum critical" in the sense that they are analogous to the back of a book having pages labelled by the values of Planck constant. These variants of imbedding space would be characterized by discrete subgroups of $SU(2)$ acting in $M^4$ and $CP_2$ degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

It is now clear that the coverings of imbedding space can only serve as auxiliary tools only. TGD predicts the hierarchy of Planck constants without generalization of imbedding space concept. At fundamental level $n$-coverings are realized for space-time surfaces connecting two 3-surfaces at the opposite boundaries of CD. They are analogous to singular coverings of plane defined by analytic functions $z^{1/n}$. Each sheet of covering corresponds to a gauge equivalence class of conformal symmetries defined by a sub-algebra of the symplectic algebra. What comes in mind first is that the radial light-like radial coordinate serving as the analog of complex coordinate is transformed from $r_M$ to $u = r_M^{1/n}$ so that conformal gauge symmetry is true only for the symplectic generators proportional to $u^n$ and the powers $u^k$, $k = 0, , n-1$ correspond to broken conformal symmetries and to the gauge equivalence classes -different sheets of singular covering.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

## 2.6 Von Neumann Algebras And TGD

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type $II_1$ could provide the mathematics needed to develop a more explicit view about the construction of S-matrix and its generalizations $M$-matrix and $U$-matrix suggested by ZEO.

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type $II_1$ (HFF). In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

The physical idea is following.

1. Finite measurement resolution could be represented as inclusion of HFFs - at classical level it would correspond to a discretization with some resolution defined by the algebraic extension of rationals used and by the p-adic length scale cutoffs. The included algebra would act like gauge group in the sense that its elements in zero energy ontology would generate states not distinguishable from the original one.

2. The space of physical states would be an analog of coset space but with fractal dimension given by the index of inclusion defined in terms of quantum phase. It might well be possible to act analog of gauge group with the inclusion.

3. An alternative view is that the hierarchy of inclusions is associated with the hierarchy of sub-algebras of supersymplectic algebra acting gauge transformations. The sub-algebra would be isomorphic to the entire algebra with conformal weights coming as $n$-multiples of those for the entire algebra. This subalgebra would define measurement resolution, and one would indeed have gauge group interpretation in a wide sense of the word. $n = h_{eff}/h$ identification would give a direct connection with the hierarchy of Planck constants and dark matter hierarchy.

This idea has led to speculations: two such speculations are discussed in this section. The first one is the extension of WCW Clifford algebra to a local algebra in Minkowski space. Second speculation is that Connes tensor product might help to understand interactions in TGD framework.

Unfortunately, the problem is that the understanding of Connes tensor product is for a physicist like me a tougher challenge than understanding of physics! What is obvious even for physicist like me that Connes tensor product differs from the ordinary tensor product in that it implies strong correlations between factors represented as entanglement and entanglement indeed represents interactions.

1. Quantum phase $q$ is associated also with the Yangians of super-symplectic algebra. The localization of the induced spinor fields at string world sheets makes possible to introduced quantum phase directly at the level of anti-commutators of oscillator operators. Yangian realized in terms of super-symplectic Noether charges assignable to strings connecting partonic 2-surfaces leads to a concrete proposal for the construction of scattering amplitudes utilizing product and co-product as basic vertices [K43]. This construction of vertices could relate closely to Connes tensor product.

2. The construction of zero energy states implies strong correlations between the positive and negative energy parts of zero energy state at the boundaries of CD. One cannot just construct ordinary tensor product of state spaces. These correlations are expressed classically by preferred extremal property serving as the analog of Bohr orbit and at least partially realized by the condition that 3-surfaces carry vanishing symplectic Noether charges for the sub-algebra of symplectic algebra. These strong correlations could have mathematical representation in terms of Connes tensor product.

### 2.6.1 Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken super-symplectic gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer $n$ in $h_{eff} = n \times h$ [K11] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of $n$ corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere $S^2$ with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which $n_i$ divides $n_{i+1}$ would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities: hill at the top of hill at the top....

### 2.6.2 How to localize infinite-dimensional Clifford algebra?

An interesting speculation is that one could make the WCW Clifford algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

1. Represent Minkowski coordinate of $M^d$ as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical $M^d$ is genuine quantum $M^d$ with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives $M^4$.

2. Form power series of the $M^d$ coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

3. There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of $M^8$ coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine $M^8$-localized factor of type $II_1$. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

4. Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of $M^8$. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of WCW of 3-surfaces, etc....

### 2.6.3 Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

1. A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for $II_1$ factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

2. At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

3. For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

4. The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

5. Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1, 0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

# 3 Quantum TGD In Nutshell

This section provides a very brief summary about quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of *classical* spinor fields in the "world of the classical worlds" identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). This implies a radical deviation from path integral formalism, in which one integrates over all space-time surfaces. A second important deviation is due to Zero Energy Ontology. The properties of Kähler action imply a further crucial deviation, which in fact forced the introduction of WCW , and is behind the hierarchy of Planck constants, hierarchy of quantum criticalities, and hierarchy of inclusions of hyper-finite factors.

I include also an excerpt from [K43] representing the most recent view about how scattering amplitudes could be constructed in TGD using the notion of super-symplectic Yangian and generalization of the notion of twistor structure so that it applies at the level of 8-D imbedding space.

## 3.1 Basic Physical And Geometric Ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

### 3.1.1 Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW . WCW gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of WCW forms what I have used to called super-symplectic algebra.

   WCW metric can be expressed in two manners. Either as anti-commutators of WCW gamma matrices identified as super-symplectic Noether super charges (this is highly non-trivial!) or in terms of the second derivatives of Kähler function expressible as Kähler action for the space-time regions with 4-D $CP_2$ projection and Euclidian signature of the induced metric (wormhole contacts).

   This leads to a generalization of AdS/CFT duality if one assumes that spinor modes are localized at string world sheets to guarantee well-definedness of em charge for the spinor modes following from the assumption that induced classical $W$ fields vanish at string world sheets. Also number theoretic argument requiring that octonionic spinor structure for the imbedding space is equivalent with ordinary spinor structure implies the localization. String model in space-time becomes part of TGD.

3. Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD they define TGD correlate for the degrees of freedom assignable to hadronic strings. They could be responsible for the most of the mass of hadron and resolve spin puzzle of proton.

   It has turned out that super-symplectic quanta would naturally give rise to a hierarchy of dark matters labelled by the value of effective Planck constant $h_{eff} = n \times h$. $n$ would characterize the breaking of super-symplectic symmetry as gauge symmetry and for $n = 1$ (ordinary matter) there would be no breaking.

   Besides super-symplectic symmetries there extended conformal symmetries associated with light-cone boundary and light-like orbits of partonic 2-surfaces and Super-Kac Moody symmetries assignable to light-like 3-surfaces. A further super-conformal symmetry is associated with the spinor modes at string world sheets and it corresponds to the ordinary super-conformal symmetry. The existence of quaternion conformal generalization of these symmetries is suggestive and the notion of quaternion holomorphy [A8] indeed makes sense [K60]. Together these algebras mean a gigantic extension of the conformal symmetries of string models [L11]. Some of these symmetries act as dynamical symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD.

The original proposal was that the commutator algebras of super-symplectic and super Kac-Moody algebra annihilate physical states. Recently the possibility that a sub-algebra of super-symplectic algebra (at least this algebra) with conformal weights coming as multiples of integer some integer $n$ annihilates physical states at both boundaries of CD. This would correspond to broken gauge symmetry and would predict fractal hierarchies of quantum criticalities defined by sequences of integers $n_{i+1} = \prod_{k<i+1} m_k$. The conformal algebra of string world sheet could always correspond to $n = 1$. Super Virasoro conditions could be regarded as analogs of WCW Dirac equation. These sequences would define hierarchies of inclusions of hyper finite factors of type $II_1$ and the identification $n = h_{eff}/h$ would relate this hierarchy to the hierarchy of Planck constants. $n$ would also characterize the non-determinism of Kähler action: there would be $n$ conformal gauge equivalence classes connecting members of a pair of 3-surfaces at the boundaries of CD and defining the ends of space-time.

An intriguing possibility consistent with this picture is that the conformal weights of the super-symplectic algebra charactering the exponent $h$of the power $r_M^h$ of the light-like radial coordinate $r_M$ appearing in the Hamiltonian of the symplectic transformation of $\delta M^4_\pm \times CP_2$ is not an integer but a linear combination of zeros of Riemann Zeta with integer coefficients. For physical states the weights would be real integers (if mass squared corresponds to conformal weight): one would have conformal confinement in the sense that the sum of imaginary parts of conformal weights would be zero. This is an old idea that I already gave up but seems rather attractive in the recent framework.

Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. WCW spinors define a von Neumann algebra known as hyper-finite factor of type $\mathrm{II}_1$ (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the Kähler-Dirac operator assigned to the light-like 3-surfaces.

## 3.2 The Notions Of Imbedding Space, 3-Surface, And Configuration Space

The notions of imbedding space, 3-surface (and 4-surface), and WCW (world of classical worlds ( WCW )) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in $H$. The basic idea was that the definition of Kähler metric of WCW assigns to each $X^3$ a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably.

### 3.2.1 The notion of imbedding space

Two generalizations of the notion of imbedding space were forced by number theoretical vision [K40, K41, K39].

1. p-Adicization forced to generalize the notion of imbedding space by gluing real and p-adic variants of imbedding space together along rationals and common algebraic numbers. The generalized imbedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.

2. With the discovery of zero energy ontology [K50, K8] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in $H$. If the temporal distance between upper and lower tip of CD is quantized in power-of-two multiples of $CP_2$ length, p-adic length scale hypothesis [K25] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.

3. The realization of the hierarchy of Planck constants [K11] suggests a further generalization of the notion of imbedding space, which has however turned out to be an auxiliary tool only.

   Generalized imbedding space would be obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and $CP_2$ to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized imbedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and $CP_2$ is replaced with a union of CDs and $CP_2$s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW .

   It is now clear that this generealization only provides a description for the non-determinism realized in terms of $n$ conformal equivalences of preferred extremals connecting 3-surfaces at the opposite boundaries of CD.

4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of $CP_2$. Kähler gauge potential must have what one might call pure gauge parts in $M^4$ in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in $CP_2$- play key role in the model of anyons, charge fractionization, and quantum Hall effect [K29].

### 3.2.2 The notion of 3-surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to equivalence believed to be implied by General Coordinate Invariance. There was a problem related to the realization of equivalence since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for $Y^3$ at $X^4(X^3)$ and $\mathrm{Diff}^4$ related $X^3$ should satisfy $X^4(Y^3) = X^4(X^3)$.

2. Much later it became clear that light-like 3-surfaces identified as boundaries between regions of Minkowskian and Euclidian signature (wormhole contacts and exterior) have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory.

   The condition that light-like parton orbits and space-like 3-surfaces at the ends of CD are physically equivalent allows to conclude that partonic 2-surfaces and their tangent space data should be enough for physics. One would have strong form of General Coordinate Invariance (GCI) and strong form of holography. The condition that the symplectic Noether charges for the above mentioned sub-algebra of the symplectic algebra vanish for space-like 3-surfaces at the ends of CD would be natural in this framework.

   It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

3. An important step of progress was the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. The light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams ("Feynman" could be replaced with twistor, or braid, or something else). The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

### 3.2.3 The notion of space-time surface

The basic vision has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals turned out to be far from trivial. The recent discussion of this topic can be found at [K61].

1. The obvious first guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing $X^3$. This choice would have some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If $X^3$ is light-like surface- either light-like boundary of $X^4$ or light-like 3-surface assignable to a wormhole throat at which the induced metric of $X^4$ changes its signature- this identification circumvents the obvious objections.

   This choice might well be correct for (non-negative) Kähler function identifiable as Kähler action in Euclidian space-time regions (wormhole contacts). In Minkowskian regions Kähler action is imaginary ($\sqrt{g}$ factor is imaginary) and gives a complex phase to vacuum functional and clearly serves as the analog of action in quantum field theories. The identification as preferred extremal does not look natural now.

2. The recent identification has been already described: the vanishing of symplectic Noether charges in a sub-algebra isomorphic to the entire algebra would define the conformal gauge and fix the preferred extremals in ZEO highly uniquely. For a generic pair of 3-surfaces at the boundaries of CD it is not clear whether any preferred extremal exists. The non-determinism of Kähler action makes it difficult to make any conclusions in this respect.

3. I have consider many other identifications of preferred extremals during years. In Minkowskian regions the contraction $j \cdot A$ of Kähler current and Kähler gauge potential vanishes for the known extremals. Together with the weak form of electric-magnetic duality stating $\epsilon_{ijnt}J^{nt} = kJ_{ij}$, $k$ proportionality constant, this condition would reduce Kähler action to 3-D Chern-Simons terms. This would realize TGD as almost topological QFT. Whether this condition makes sense in Euclidian regions and whether it is strong enough remains an open question.

   The construction of WCW geometry suggests also the strengthening the boundary conditions to the condition that there exists space-time coordinates in which the induced $CP_2$ Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at $X_l^3$ ($n$ denote normal direction). One could say that at $X_l^3$ situation is static both metrically and for the Maxwell field defined by the induced Kähler form. There are reasons to hope that this is the final step in a long process.

4. One possible identification of preferred extremals would be as quaternionic sub-manifolds of imbedding space with the property that quaternionic tangent space at given point contains a preferred $M^2$ identifiable as a commutative sub-space of quaternionic tangent spaces. One can also consider the possibility that $M^2$ depends on the point of space-time surface but that

one has an integrable distribution defining string world sheet in $M^4$: this leads to the notion of Hamilton-Jacobi structure [K61]. $M^8 - H$ duality allowing to map surfaces of $M^8$ with this property to surfaces in $M^8$ by mapping the local tangent space to a point of $CP_2$ relates closely to this proposal.

5. The localization of the modes of Kähler-Dirac equation to string world sheets with vanishing $W$ fields (to guarantee well-defined em charge for the modes) requires that Frobenius integrability conditions are satisfied for the 2-D tangent spaces and that the energy momentum currents as vectors of $X^4$ have no components normal to the string world sheet. I remains to be proven that these conditions can be satisfied.

   This suggests that one should construct preferred extremals as a concrete realization of holography. One would start from data given by string world sheets and partonic 2-surfaces and possibly also space-like 3-surface and the light-like orbits of partonic 2-surfaces by posing the conditions that sub-algebra of symplectic algebra acts as gauge algera. The reason for fixing of 3-surfaces apart from symplectic gauge transformation in an appropriate sub-algebra is that otherwise the possibility of strings and their orbits to get knotted and linked becomes impossible to describe. One clearly would have effective 2-dimensionality.

   According to the recent view about Kähler-Dirac action the boundaries of string world sheets are imbedding space geodesics characterizing by light-like 8-momentum. This suggests that the braiding along partonic orbits is probably possible only if one allows intermediate partonic 2-surfaces in which the direction of four-momentum changes. The particle physics interpretation would be that braiding must respect conservation of momentum and thus occurs by exchange of say bosonic quanta. So that braiding diagram would be replaced by the analog of Feynman diagram.

6. One bundle of ideas relates is inpired by basic thinking about massless fields and relies on the observation that the known extremals seems to decompose in Minkowskian regions to pieces having interpretation as classical analogs of massless field quanta allowing local polarization vector and light-like 4-momentum vector orthogonal to each other. The simplest example is provided by massless extremals for which one has linear superposition of modes in the direction of four-momentum. One has therefore very quantal behavior already classically. In particular, linear superposition fails and can be realized only for effects experienced by a particle like 3-surface topologically condensed to several space-time sheets. At GRT-QFT limit superposition of effects becomes superposition of fields when the many-sheeted space-time is approximated with slightly curved $M^4$.

   Also number theoretical vision led to a related proposal that $X^4(X^3_{l,i})$, where $X^3_{l,i}$ denotes $i^th$ connected component of the light-like 3-surface $X^3_l$, contain in their 4-D tangent space $T(X^4(X^3_{l,i}))$ a subspace $M^2_i \subset M^4$ having interpretation as the plane of non-physical polarizations. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW . This is as it must be since complexification does not make sense in $M^2$ degrees of freedom.

   In number theoretical framework $M^2_i$ has interpretation as a preferred hyper-complex subspace of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of $M^8$. A stronger condition would be that the condition holds true at all points of $X^4(X^3)$ for a global choice $M^2$ but this is un-necessary and leads to strong un-proven conjectures. The condition $M^2_i \subset T(X^4(X^3_{l,i}))$ in principle fixes the tangent space at $X^3_{l,i}$, and one has good hopes that the boundary value problem is well-defined and fixes $X^4(X^3)$ uniquely as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2_i \subset M^3$ plays also other important roles.

7. The weakest form of number theoretic compactification states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic $M^8$ can be mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal

of Kähler action associated with Kähler form of $E^4$ in the decomposition $M^8 = M^4 \times E^4$, where $M^4$ corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in $M^8$ is same as in $M^4 \times CP_2$.

#### 3.2.4 The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" ( WCW )). Should one regard $CH$ as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "$M^4_+$ or $M^4$?" had been settled in favor of $M^4_+$ by the fact that $M^4_+$ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering $M^4$ instead of $M^4_+$.

2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" ( WCW ). The spaces $CD \times CP_2$ regarded as subsets of $H$ defined the sectors of WCW .

3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW . The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the imbedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface $X^3_l$, which can be boundaries of $X^4$ and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW is a union of sub- WCW s associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

## 3.3 Could The Universe Be Doing Yangian Arithmetics?

One of the old TGD inspired really crazy ideas about scattering amplitudes is that Universe is doing some sort of arithmetics so that scattering amplitude are representations for computational sequences of minimum length. The idea is so crazy that I have even given up its original form, which led to an attempt to assimilate the basic ideas about bi-algebras, quantum groups [K1], Yangians [K43], and related exotic things. The work with twistor Grassmannian approach inspired a reconsideration of the original idea seriously with the idea that super-symplectic Yangian could define the arithmetics. I try to describe the background, motivation, and the ensuing reckless speculations in the following.

#### 3.3.1 Do scattering amplitudes represent quantal algebraic manipulations?

I seems that tensor product $\otimes$ and direct sum $\oplus$ - very much analogous to product and sum but defined between Hilbert spaces rather than numbers - are naturally associated with the basic vertices of TGD. I have written about this a highly speculative chapter - both mathematically and physically [K51]. The chapter [K1] is a remnant of earlier similar speculations.

1. In $\otimes$ vertex 3-surface splits to two 3-surfaces meaning that the 2 "incoming" 4-surfaces meet at single common 3-surface and become the outgoing 3-surface: 3 lines of Feynman diagram meeting at their ends. This has a lower-dimensional shadow realized for partonic 2-surfaces. This topological 3-particle vertex would be higher-D variant of 3-vertex for Feynman diagrams.

2. The second vertex is trouser vertex for strings generalized so that it applies to 3-surfaces. It does not represent particle decay as in string models but the branching of the particle wave function so that particle can be said to propagate along two different paths simultaneously. In double slit experiment this would occur for the photon space-time sheets.

3. The idea is that Universe is doing arithmetics of some kind in the sense that particle 3-vertex in the above topological sense represents either multiplication or its time-reversal co-multiplication.

The product, call it $\circ$, can be something very general, say algebraic operation assignable to some algebraic structure. The algebraic structure could be almost anything: a random list of structures popping into mind consists of group, Lie-algebra, super-conformal algebra quantum algebra, Yangian, etc.... The algebraic operation $\circ$ can be group multiplication, Lie-bracket, its generalization to super-algebra level, etc...). Tensor product and thus linear (Hilbert) spaces are involved always, and in product operation tensor product $\otimes$ is replaced with $\circ$.

1. The product $A_k \otimes A_l \rightarrow C = A_k \circ A_l$ is analogous to a particle reaction in which particles $A_k$ and $A_l$ fuse to particle $A_k \otimes A_l \rightarrow C = A_k \circ A_l$. One can say that $\otimes$ between reactants is transformed to $\circ$ in the particle reaction: kind of bound state is formed.

2. There are very many pairs $A_k, A_l$ giving the same product $C$ just as given integer can be divided in many manners to a product of two integers if it is not prime. This of course suggests that elementary particles are primes of the algebra if this notion is defined for it! One can use some basis for the algebra and in this basis one has $C = A_k \circ A_l = f_{klm} A_m$, $f_{klm}$ are the structure constants of the algebra and satisfy constraints. For instance, associativity $A(BC) = (AB)C$ is a constraint making the life of algebraist more tolerable and is almost routinely assumed.

   For instance, in the number theoretic approach to TGD associativity is proposed to serve as fundamental law of physics and allows to identify space-time surfaces as 4-surfaces with associative (quaternionic) tangent space or normal space at each point of octonionic imbedding space $M^4 \times CP_2$. Lie algebras are not associative but Jacobi-identities following from the associativity of Lie group product replace associativity.

3. Co-product can be said to be time reversal of the algebraic operation $\circ$. Co-product can be defined as $C = A_k \rightarrow \sum_{lm} f_k^{lm} A_l \otimes A_m$, where $f_k^{lm}$ are the structure constants of the algebra. The outcome is quantum superposition of final states, which can fuse to $C$ (the "reaction" $A_k \otimes A_l \rightarrow C = A_k \circ A_l$ is possible). One can say that $\circ$ is replaced with $\otimes$: bound state decays to a superposition of all pairs, which can form the bound states by product vertex.

There are motivations for representing scattering amplitudes as sequences of algebraic operations performed for the incoming set of particles leading to an outgoing set of particles with particles identified as algebraic objects acting on vacuum state. The outcome would be analogous to Feynman diagrams but only the diagram with minimal length to which a preferred extremal can be assigned is needed. Larger ones must be equivalent with it.

The question is whether it could be indeed possible to characterize particle reactions as computations involving transformation of tensor products to products in vertices and co-products to tensor products in co-vertices (time reversals of the vertices). A couple of examples gives some idea about what is involved.

1. The simplest operations would preserve particle number and to just permute the particles: the permutation generalizes to a braiding and the scattering matrix would be basically unitary braiding matrix utilized in topological quantum computation.

2. A more complex situation occurs, when the number of particles is preserved but quantum numbers for the final state are not same as for the initial state so that particles must interact. This requires both product and co-product vertices. For instance, $A_k \otimes A_l \rightarrow f_{kl}^m A_m$ followed by $A_m \rightarrow f_m^{rs} A_r \otimes A_s$ giving $A_k \rightarrow f_{kl}^m f_m^{rs} A_r \otimes A_s$ representing 2-particle scattering. State function reduction in the final state can select any pair $A_r \otimes A_s$ in the final state. This reaction is characterized by the ordinary tree diagram in which two lines fuse to single line

and defuse back to two lines. Note also that there is a non-deterministic element involved. A given final state can be achieved from a given initial state after large enough number of trials. The analogy with problem solving and mathematical theorem proving is obvious. If the interpretation is correct, Universe would be problem solver and theorem prover!

3. More complex reactions affect also the particle number. 3-vertex and its co-vertex are the simplest examples and generate more complex particle number changing vertices. For instance, on twistor Grassmann approach on can construct all diagrams using two 3-vertices. This encourages the restriction to 3-vertice (recall that fermions have only 2-vertices)

4. Intuitively it is clear that the final collection of algebraic objects can be reached by a large - maybe infinite - number of ways. It seems also clear that there is the shortest manner to end up to the final state from a given initial state. Of course, it can happen that there is no way to achieve it! For instance, if $\circ$ corresponds to group multiplication the co-vertex can lead only to a pair of particles for which the product of final state group elements equals to the initial state group element.

5. Quantum theorists of course worry about unitarity. How can avoid the situation in which the product gives zero if the outcome is element of linear space. Somehow the product should be such that this can be avoided. For instance, if product is Lie-algebra commutator, Cartan algebra would give zero as outcome.

#### 3.3.2 Generalized Feynman diagram as shortest possible algebraic manipulation connecting initial and final algebraic objects

There is a strong motivation for the interpretation of generalized Feynman diagrams as shortest possible algebraic operations connecting initial and final states. The reason is that in TGD one does not have path integral over all possible space-time surfaces connecting the 3-surfaces at the ends of CD. Rather, one has in the optimal situation a space-time surface unique apart from conformal gauge degeneracy connecting the 3-surfaces at the ends of CD (they can have disjoint components).

Path integral is replaced with integral over 3-surfaces. There is therefore only single minimal generalized Feynman diagram (or twistor diagram, or whatever is the appropriate term). It would be nice if this diagram had interpretation as the shortest possible computation leading from the initial state to the final state specified by 3-surfaces and basically fermionic states at them. This would of course simplify enormously the theory and the connection to the twistor Grassmann approach is very suggestive. A further motivation comes from the observation that the state basis created by the fermionic Clifford algebra has an interpretation in terms of Boolean quantum logic and that in ZEO the fermionic states would have interpretation as analogs of Boolean statements $A \rightarrow B$.

To see whether and how this idea could be realized in TGD framework, let us try to find counterparts for the basic operations $\otimes$ and $\circ$ and identify the algebra involved. Consider first the basic geometric objects.

1. Tensor product could correspond geometrically to two disjoint 3-surfaces representing 3-particles. Partonic 2-surfaces associated with a given 3-surface represent second possibility. The splitting of a partonic 2-surface to two could be the geometric counterpart for co-product.

2. Partonic 2-surfaces are however connected to each other and possibly even to themselves by strings. It seems that partonic 2-surface cannot be the basic unit. Indeed, elementary particles are identified as pairs of wormhole throats (partonic 2-surfaces) with magnetic monopole flux flowing from throat to another at first space-time sheet, then through throat to another sheet, then back along second sheet to the lower throat of the first contact and then back to the thirst throat. This unit seems to be the natural basic object to consider. The flux tubes at both sheets are accompanied by fermionic strings. Whether also wormhole throats contain strings so that one would have single closed string rather than two open ones, is an open question.

3. The connecting strings give rise to the formation of gravitationally bound states and the hierarchy of Planck constants is crucially involved. For elementary particle there are just two wormhole contacts each involving two wormhole throats connected by wormhole contact. Wormhole throats are connected by one or more strings, which define space-like boundaries of corresponding string world sheets at the boundaries of CD. These strings are responsible for the formation of bound states, even macroscopic gravitational bound states.

### 3.3.3 Does super-symplectic Yangian define the arithmetics?

Super-symplectic Yangian would be a reasonable guess for the algebra involved.

1. The 2-local generators of Yangian would be of form $T_1^A = f_{BC}^A T^B \otimes T^C$, where $f_{BC}^A$ are the structure constants of the super-symplectic algebra. n-local generators would be obtained by iterating this rule. Note that the generator $T_1^A$ creates an entangled state of $T^B$ and $T^C$ with $f_{BC}^A$ the entanglement coefficients. $T_n^A$ is entangled state of $T^B$ and $T_{n-1}^C$ with the same coefficients. A kind replication of $T_{n-1}^A$ is clearly involved, and the fundamental replication is that of $T^A$. Note that one can start from any irreducible representation with well defined symplectic quantum numbers and form similar hierarchy by using $T^A$ and the representation as a starting point.

   That the hierarchy $T_n^A$ and hierarchies irreducible representations would define a hierarchy of states associated with the partonic 2-surface is a highly non-trivial and powerful hypothesis about the formation of many-fermion bound states inside partonic 2-surfaces.

2. The charges $T^A$ correspond to fermionic and bosonic super-symplectic generators. The geometric counterpart for the replication at the lowest level could correspond to a fermionic/bosonic string carrying super-symplectic generator splitting to fermionic/bosonic string and a string carrying bosonic symplectic generator $T^A$. This splitting of string brings in mind the basic gauge boson-gauge boson or gauge boson-fermion vertex.

   The vision about emission of virtual particle suggests that the entire wormhole contact pair replicates. Second wormhole throat would carry the string corresponding to $T^A$ assignable to gauge boson naturally. $T^A$ should involve pairs of fermionic creation and annihilation operators as well as fermionic and anti-fermionic creation operator (and annihilation operators) as in quantum field theory.

3. Bosonic emergence suggests that bosonic generators are constructed from fermion pairs with fermion and anti-fermion at opposite wormhole throats: this would allow to avoid the problems with the singular character of purely local fermion current. Fermionic and anti-fermionic string would reside at opposite space-time sheets and the whole structure would correspond to a closed magnetic tube carrying monopole flux. Fermions would correspond to superpositions of states in which string is located at either half of the closed flux tube.

4. The basic arithmetic operation in co-vertex would be co-multiplication transforming $T_n^A$ to $T_{n+1}^A = f_{BC}^A T_n^B \otimes T^C$. In vertex the transformation of $T_{n+1}^A$ to $T_n^A$ would take place. The interpretations would be as emission/absorption of gauge boson. One must include also emission of fermion and this means replacement of $T^A$ with corresponding fermionic generators $F^A$, so that the fermion number of the second part of the state is reduced by one unit. Particle reactions would be more than mere braidings and re-grouping of fermions and anti-fermions inside partonic 2-surfaces, which can split.

5. Inside the light-like orbits of the partonic 2-surfaces there is also a braiding affecting the M-matrix. The arithmetics involved would be therefore essentially that of measuring and "co-measuring" symplectic charges.

   Generalized Feynman diagrams (preferred extremals) connecting given 3-surfaces and many-fermion states (bosons are counted as fermion-anti-fermion states) would have a minimum number of vertices and co-vertices. The splitting of string lines implies creation of pairs of fermion lines. Whether regroupings are part of the story is not quite clear. In any case, without the replication of 3-surfaces it would not be possible to understand processes like e-e scattering by photon exchange in the proposed picture.

It is easy to hear the comments of the skeptic listener in the back row.

1. The attribute "minimal" - , which could translate to minimal value of Kähler function - is dangerous. It might be very difficult to determine what the minimal diagram is - consider only travelling salesman problem or the task of finding the shortest proof of theorem. It would be much nicer to have simple calculational rules.

   The original proposal might help here. The generalization of string model duality was in question. It stated that that it is possible to move the positions of the vertices of the diagrams just as one does to transform s-channel resonances to t-channel exchange. All loops of generalized diagrams could be be eliminated by transforming the to tadpoles and snipped away so that only tree diagrams would be left. The variants of the diagram were identified as different continuation paths between different paths connecting sectors of WCW corresponding to different 3-topologies. Each step in the continuation procedure would involve product or co-product defining what continuation between two sectors means for WCW spinors. The continuations between two states require some minimal number of steps. If this is true, all computations connecting identical states are also physically equivalent. The value of the vacuum functional be same for all of them. This looks very natural.

   That the Kähler action should be same for all computational sequences connecting the same initial and final states looks strange but might be understood in terms of the vacuum degeneracy of Kähler action closed related to quantum criticality, which means infinite gauge degeneracy associated with the Yangian of a sub-algebra of super-symplectic algebra.

2. QFT perturbation theory requires that should have superposition of computations/continuations. What could the superposition of QFT diagrams correspond to in TGD framework?

   Could it correspond to a superposition of generators of the Yangian creating the physical state? After all, already quantum computer perform superpositions of computations. The fermionic state would not be the simplest one that one can imagine. Could AdS/CFT analogy allow to identify the vacuum state as a superposition of multi-string states so that single super-symplectic generator would be replaced with a superposition of its Yangian counterparts with same total quantum numbers but with a varying number of strings? The weight of a given superposition would be given by the total effective string world sheet area. The sum of diagrams would emerge from this superposition and would basically correspond to functional integration in WCW using exponent of Kähler action as weight. The stringy functional integral ("functional" if also wormhole contacts contain string portion, otherwise path integral) would give the perturbation theory around given string world sheet. One would have effective reduction of string theory.

### 3.3.4 How does this relate to the ordinary perturbation theory?

One can of course worry about how to understand the basic results of the usual perturbation theory in this picture. How does one obtain a perturbation theory in powers of coupling constant, what does running coupling constant mean, etc...? I have already discussed how the superposition of diagrams could be understood in the new picture.

1. The QFT picture with running coupling constant is expected at QFT limit, when many-sheeted space-time is replaced with a slightly curved region of $M^4$ and gravitational field and gauge potentials are identified as sums of the deviations of induced metric from $M^4$ metric and classical induced gauge potentials associated with the sheets of the many-sheeted space-time. The running coupling constant would be due to the dependence of the size scale of CD, and p-adic coupling constant evolution would be behind the continuous one.

2. The notion of running coupling constant is very physical concept and should have a description also at the fundamental level and be due to a finite computational resolution, which indeed has very concrete description in terms of Noether charges of super-symplectic Yangian creating the states at the ends of space-time surface at the boundaries of CD. The space-time surface and the diagram associated with a given pair of 3-surfaces and stringy Noether charges associated with them can be characterized by a complexity measured in terms of the number of vertices (3-surface at which three 3-surfaces meet).

For instance, 3-particle scattering can be possible only by using the simplest 3-vertex defined by product or co-product for pairs of 3-surfaces. In the generic case one has more complex diagram and what looks first 3-particle vertex has complex substructure rather than being simple product or co-product.

3. Complexity seems to have two separate aspects: the complexities of the positive and negative parts of zero energy state as many-fermion states and the complexity of associated 3-surfaces. The generalization of AdS/CFT however suggests that once the string world sheets and partonic 2-surfaces appearing in the diagram have been fixed, the space-time surface itself is fixed. The principle also suggests that the fixing partonic 2-surface and the strings connecting them at the boundaries of CD fixes the 3-surface apart from the action of sub-algebra of Yangian acting as gauge algebra (vanishing classical Noether charges). If one can determine the minimal sequence of allowed algebraic operation of Yangian connecting initial and final fermion states, one knows the minimum number of vertices and therefore the topological structure of the connecting minimal space-time surface.

4. In QFT spirit one could describe the finite measurement resolution by introducing effective 3-point vertex, which is need not be product/co-produce anymore. 3-point scattering amplitudes in general involve microscopic algebraic structure involving several vertices. One can however give up the nice algebraic interpretation and just talk about effective 3-vertex for practical purposes. Just as the QFT vertex described by running coupling constant decomposes to sum of diagrams, product/co-product in TGD could be replaced with effective product/co-product expressible as a longer computation. This would imply coupling constant evolution.

   Fermion lines could however remain as such since they are massless in 8-D sense and mass renormalization does not make sense.

   Similar practical simplification could be done the initial and final states to get rid of superposition of the Yangian generators with different numbers of strings ("cloud of virtual particles"). This would correspond to wave function renormalization.

The number of vertices and wormhole contact orbits serves as a measure for the complexity of the diagram.

1. Since fermion lines are associated with wormhole throats assignable with wormhole contacts identifiable as deformations $CP_2$ type vacuum extremals, one expects that the exponent of the Kähler function defining vacuum functional is in the first approximation the total $CP_2$ volume of wormhole contacts giving a measure for the importance of the contribution in functional integral. If it converges very rapidly only Gaussian approximation around maximum is needed.

2. Convergence depends on how large the fraction of volume of $CP_2$ is associated with a given wormhole contact. The volume is proportional to the length of the wormhole contact orbit. One expects exponential convergence with the number of fermion lines and their lengths for long lines. For short distances the exponential damping is small so that diagrams with microscopic structure of diagrams are needed and are possible. This looks like adding small scale details to the algebraic manipulations.

3. One must be of course be very cautious in making conclusions. The presence of $1/\alpha_K \propto h_{eff}$ in the exponent of Kähler function would suggest that for large values of $h_{eff}$ only the 3-surfaces with smallest possible number of wormhole contact orbits contribute. On the other hand, the generalization of AdS/CFT duality suggests that Kähler action reducible to area of string world sheet in the effective metric defined by canonical momentum currents of Kähler action behaves as $\alpha_K^2 \propto 1/h_{eff}^2$. How $1/h_{eff}^2$ proportionality might be understood is discussed in [K58] in terms electric-magnetic duality.

To sum up, the identification of vertex as a product or co-product in Yangian looks highly promising approach. The Nother charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator for each leg pair associated with two vertices. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge and essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place.

#### 3.3.5 This was not the whole story yet

The proposed amplitude represents only the value of WCW spinor field for single pair of 3-surfaces at the opposite boundaries of given CD. Hence Yangian construction does not tell the whole story.

1. Yangian algebra would give only the vertices of the scattering amplitudes. On basis of previous considerations, one expects that each fermion line carries propagator defined by 8-momentum. The structure would resemble that of super-symmetric YM theory. Fermionic propagators should emerge from summing over intermediate fermion states in various vertices and one would have integrations over virtual momenta which are carried as residue integrations in twistor Grassmann approach. 8-D counterpart of twistorialization would apply.

2. Super-symplectic Yangian would give the scattering amplitudes for single space-time surface and the purely group theoretical form of these amplitudes gives hopes about the independence of the scattering amplitude on the pair of 3-surfaces at the ends of CD near the maximum of Kähler function. This is perhaps too much to hope except approximately but if true, the integration over WCW would give only exponent of Kähler action since metric and poorly defined Gaussian and determinants would cancel by the basic properties of Kähler metric. Exponent would give a non-analytic dependence on $\alpha_K$.

   The Yangian supercharges are proportional to $1/\alpha_K$ since covariant Kähler-Dirac gamma matrices are proportional to canonical momentum currents of Kähler action and thus to $1/\alpha_K$. Perturbation theory in powers of $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is possible after factorizing out the exponent of vacuum functional at the maximum of Kähler function and the factors $1/\alpha_K$ multiplying super-symplectic charges.

   The additional complication is that the characteristics of preferred extremals contributing significantly to the scattering amplitudes are expected to depend on the value of $\alpha_K$ by quantum interference effects. Kähler action is proportional to $1/\alpha_K$. The analogy of AdS/CFT correspondence states the expressibility of Kähler function in terms of string area in the effective metric defined by the anti-commutators of K-D matrices. Interference effects eliminate string length for which the area action has a value considerably larger than one so that the string length and thus also the minimal size of CD containing it scales as $h_{eff}$. Quantum interference effects therefore give an additional dependence of Yangian super-charges on $h_{eff}$ leading to a perturbative expansion in powers of $\alpha_K$ although the basic expression for scattering amplitude would not suggest this.

# 4 Victories Of M-Theory From TGD View Point

The basic victories of the M-theory relate to conformal symmetries and dualities and black hole physics and it is useful perform comparison with TGD.

## 4.1 Super-Conformal Symmetries Of String Theory

Space-time super-symmetries are regarded as one of the basic predictions of the super string model. Typically these super-symmetries appear at the level of effective quantum field theory limit derived from spontaneous compactification and predict that massless particles possess massless super partners, sparticles. The problem has been how to generalize Higgs mechanism to break the space-time super-symmetry. That sparticles have relatively low mass scale has been seen as one of the absolute predictions of M-theory and the ability to predict at least something has been counted as a success. Since sparticles have hitherto escaped the attempts to detect them, even this belief has been now challenged, and proposals has been made that perhaps M-theory might after all predict sparticles to be very massive.

### 4.1.1 How TGD view about supersymmetries differs from the standard view?

TGD and standard views about super-symmetry differ in many respects.

1. The standard view is inspired by the mathematically awkward and formal idea of assigning to the space-time coordinates anti-commuting super part. The belief is that string world sheet super-symmetries give rise to the space-time super symmetries of the low energy effective quantum field theory assigned to the string model.

2. In TGD the super-symmetry generators of the spectrum generating super-conformal algebra act as gamma matrices of WCW (“world of classical worlds” ). Anti-commuting infinitesimals are encountered nowhere. Majorana spinors are not possible in TGD framework were B and L are conserved separately.

   The gamma matrices of WCW are identified as super-symplectic Noether super charges for induced spinor field restricted at partonic 2-surfaces and can be expressed as integral over string [K50]. This identification can be extended to the Yangian defined by multi-stringy generators of symplectic algebra. This identification implies the analog of AdS/CFT duality: one can express WCW Kähler metric either in terms of Kähler function or in terms of anti-commutators of second quantized spinor fields. The super generators of Yangian are natural candidates to the role of oscillator operators creating physical states.

3. The counterparts of the word sheet super-symmetries act as gauge super-symmetries at space-time level but it is far from clear whether they give rise to global space-time super-symmetries at the level of imbedding space. The second quantized oscillator operators of induced spinor field give rise to Clifford algebra having the structure of SUSY algebra.

   It seems that the anti-commutators of oscillator operators can be chosen so that one can have super-Poincare invariance. These super-symmetries are however broken. If conformal invariance holds true one obtains $\mathcal{N} = 4 + 4$ SUSY corresponding to quark and lepton generators. Right-handed neutrino generates the least broken $\mathcal{N} = 2$ SUSY. It is possible that sparticles have same p-adic mass scale as particles but are dark in TGD sense so that they have non-standard value of Planck constant.

### 4.1.2 Super-conformal symmetries at the space-time level

There have been a considerable progress in the understanding of super-conformal symmetries [K9, K50]. It must be however admitted that there are still several possible scenarios depending on whether these symmetries act as gauge symmetries or dynamical symmetries.

1. Super-symplectic algebra corresponds to the isometries of “world of classical worlds” ( WCW ) constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field $\Psi$ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and $\Psi$ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW .

2. Light-cone boundary is expressible as $\delta M^4_+ = R_+ \times S^2$ and is metrically 2-D . This implies that the conformal transformations of $S^2$ depending parametrically on the light-like radial coordinate $r_M$ of $R_+$ act as conformal transformations. By selecting the radial dependence so that conformal scaling factors cancel, one obtains infinite-dimensional group of isometries isomorphic to the group of conformal transformations. Similar group of conformal symmetries and isometries is associated with the light-like orbits of partonic 2-surfaces. The

interpretation of these groups has remained unclear but they are expected to play a key role. One possibility is that they act as gauge symmetries.

3. One expects also Kac-Moody type gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are $n$ gauge equivalence classes of these surfaces and that $n$ defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

4. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of $S^2$.The $S^2$ local scaling of the light-like radial coordinate $r_M$ of $R_+$ compensates the conformal scaling of the metric coming from the conformal transformation of $S^2$. Also light-like 3-surfaces allow the analogs of these isometries.

The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD $X^3_l$. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced WCW consisting of the light like 3-D CDs. The conformal symmetries in question suggests strongly a further degeneracy of the WCW metric and effective metric 2-dimensionality of 3-surfaces. These conformal symmetries could accompanied by super conformal symmetries defined by the solutions of the induced spinor fields.

Could these conformal symmetries allow a continuation to quaternion conformal super symmetries in the interior of the space-time surface realized as real analytic power series of a quaternionic space-time coordinate?

1. At first glance the answer seems to be “No”. The reason is that these symmetries involve both transversal complex coordinate and light like coordinate as independent variables whereas quaternion conformal symmetries are algebraically one-dimensional.

2. Somewhat surprisingly, it has however turned out that quaternionic analog of Riemann conditions characterizing analyticity allows also a variant, which corresponds to analyticity in two complex variables, which in Minkowskian signature would correspond to hyper-complex and complex coordinate [A8] [K43]. Also quaternion analyticity realized as powers series of quaternion is possible. The crucial trick is that the Taylor coefficients multiply powers of quaternionion from right whereas derivatives act from left. An interesting question is whether right- and left- analyticity have physical meaning and what the reflection like operation taking right-analytic to left analytic series could mean. Could it relate somehow to time reversal?

## 4.2 Dualities Of String Theories

The starting point of duality physics was the classical paper of Montonen and Olive about electric-magnetic Montonen which was generalized to what are known as S and T dualities in superstring context. The notion of duality is central also in TGD framework.

### 4.2.1 Dualities as victories of M-theory

Dualities [B22] allowing to unify various superstring models are regarded as basic victories of M-theory. The heuristic proofs for various dualities between various variants of superstring model that I have seen apply what might be called M-logic. Consider special examples defined by 11-dimensional super-gravity using a particular background and particular spontaneous compactification and demonstrate that these examples are consistent with the duality. Then generalize from special to general. For a non-specialist, it is difficult to decide, whether all this is just wishful thinking and clever choices of compactifications.

### 4.2.2 Mirror symmetry of Calabi-Yau manifolds

String theory has stimulated very general conjectures about the properties of Calabi-Yau manifolds, which have turned out to be correct. Calabi-Yau manifolds are 3-dimensional Kähler manifolds with $SU(3)$ (rather than $U(3)$) holonomy group and thus satisfy empty space Einstein equations implied by the requirement of the vanishing of conformal anomaly in closed super string models. The prediction of the mirror symmetry for Calabi-Yau manifolds [B5] emerged before the era of M-theory from the study of $N = 2$ super-conformal sigma models with Calabi-Yau manifold as a target space and closed string world sheet as the "space-time". In the 11-dimensional M-theory context Calabi-Yau manifolds are obtained only by a special compactification for which $11^{th}$ dimension corresponds to a circle. The argument taken from [B5] written in a physicist friendly manner runs as follows.

1. In conformal field theories the so called marginal operators correspond to the deformations of the original conformal field theory respecting the property of being a conformal field theory, and thus the criticality of the physical system. In particular, the deformations of complex and Kähler structures of the target space, now Calabi-Yau space, induce this kind of deformations. The basic finding was that the operators inducing these two kinds of deformations differ only by the opposite sign of their U(1) charge associated with the $U(1)$ current of $\mathcal{N} = 2$ super-symmetry algebra.

2. The mere change of the sign of $U(1)$ charge would correspond to a permutation of the spaces of complex and Kähler moduli, which means a rather drastic geometric and even a topological change. On the other hand, the physical change must be marginal since the system remains critical. Both signs of $U(1)$ charge seem highly plausible so that the hypothesis is that the Calabi-Yau manifolds appear a mirror pairs so that in a rough sense the moduli for Kähler and complex structures are permuted for the members of the mirror pair by performing a change of sign of $U(1)$ charge for the left moving modes of string. Actually a generalization of the notion of Kähler moduli is necessary. This is achieved by combining the Kähler form and antisymmetric field $B$ defining a generalization of $U(1)$ gauge potential to form a imaginary and complex parts of a more general structure for which Kähler moduli space (Kähler cone) is complexified and by introducing so called extended Kähler cone combining the Kähler moduli associated with several Calabi-Yau spaces so that single Calabi-Yau manifold can have several mirrors [B5].

There are two implications. First, two different Calabi-Yau geometries and even topologies give rise to the same conformally invariant physics: the physics $\leftrightarrow$ geometry identification of General Relativity is not strictly true anymore. Secondly, the continuous change of the complex moduli for the Calabi-Yau manifold corresponds to a topology change for the mirror manifold so that even topology change corresponds to a quite smooth change of physics, in fact a change respecting 2-dimensional criticality. Even the possibility that the change involves a temporary contraction of the Calabi-Yau to a point during the change cannot be excluded [B5], which looks really weird. Also singular Calabi-Yau manifolds are possible and not mere limiting cases of non-singular ones [B5].

These implications might be also seen as a failure of the theory basically due to the spontaneous compactification trick. In TGD imbedding space is fixed and similar phenomenon does not occur. The moduli space of conformal structures of the metrically 2-dimensional light like causal determinants effectively corresponding to closed string word sheets is however involved also now, and implies naturally the concept of elementary particle vacuum functional defined in the moduli space of complex structures characterizing the effectively 2-D induced metrics at causal determinants [K7]. The notion is essential for p-adic mass calculations and predicts correct ratios for electron, muon, and tau lepton masses [K19].

To conclude, the discovery of the mirror symmetry is quite beautiful and impressive but as such does not provide support for the super string theory as a physical theory. The discovery could have been made by a conformal field theorist interested in two-dimensional critical statistical systems.

The enormous mathematical significance of Calabi-Yau spaces is that the tools of algebraic geometry and complex analysis become available. Only quite recently (2014) it turned out that a generalization of twistor approach makes similar tools available in TGD framework. The lift of space-time surfaces to their twistor spaces imbedded to the product of 6-D twistor spaces of $M^4$

and $CP_2$ - the only twistor spaces that are Kähler manifolds - assumed to receive their twistor structure by induction suggests one further formulation of the preferred extremal property [K43].

Mirror symmetry means non-uniqueness of geometry-physics correspondence: several Calabi-Yau spaces describe the same physics. The analog of this phenomenon would be the existence of several lifts of a preferred extremal to its induced twistor space: this of course assuming that space-time surface code for the physics and twistor spaces are only an auxiliary tool. Note that in TGD the space-time surfaces connecting 3-surfaces at the ends of CD are non-unique if the recent view about symplectic symmetry holds true so that geometry-physics correspondence fails to be 1-1 also in this sense.

## 4.3 Dualities And Conformal Symmetries In TGD Framework

The reason for discussing the rather speculative notion of dualities before considering the definition of the Kähler-Dirac action and discussing the proposal how to define Kähler function in terms of Dirac determinants, is that the duality thinking gives the necessary overall view about the complex situation: even wrong vision is better than no vision at all.

The first candidate for a duality in TGD is electric-magnetic duality appearing in the construction of WCW geometry.

### 4.3.1 Weak form of electric-magnetic duality

$CP_2$ Kähler form is self-dual. This does not however hold for the induced Kähler form and the duality is replaced with weak form of electric magnetic duality posed only as a boundary condition stating that at the light-like orbits of partonic 2-surfaces defining the boundaries between Minkowskian and Euclidian space-time regions Kähler magnetic field is the dual of Kähler electric field. If this duality holds true, the proposed vanishing of the Coulombic contribution $j \cdot A$ to Kähler action density for preferred extremals would reduce Kähler action to mere 3-D Chern-Simons terms. This would reduce enormously difficult problem of identifying preferred extremals of Kähler action and calculating corresponding Kähler action to a local data at light-light-like 3-surfaces. A concrete realization of holography would be in question and one could speak about TGD as almost topological QFT.

### 4.3.2 Quantum gravitational holography

The so called AdS/CFT duality of Maldacena [B17] correspondence relates to quantum-gravitational holography states roughly that the gravitational theory formulated in terms string model in 10-dimensional $AdS_{10-n} \times S^n$ manifold is equivalent with the conformal field theory at the boundary of $AdS_D$ factor, which is $D-1$-dimensional Minkowski space. This duality has been seen as a manifestation of a duality between super-gravity with Kaluza-Klein quantum numbers (closed strings) and super Yang-Mills theories (open strings with quantum numbers at the ends of string).

In TGD framework this duality is not enough since the super-conformal symmetries of TGD are gigantic as compared to those in string models. One obtains an analog of this duality.

1. The condition that the value of electromagnetic charge is well-defined for the modes of the induced spinor field implies the localization of the modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - with the property that the induced $W$ field and above weak scale also the induced $Z^0$ field vanish. The fermionic sector of TGD is very similar to string model.

2. One can express WCW Kähler metric in two manners. In terms of second derivatives of Kähler function, a purely bosonic object given as Kähler action for magnetically charged wormhole contacts having Euclidian signature of the induced metric, or as anti-commutators of WCW gamma matrices identified as super-symplectic Noether super charges assignable to strings connecting partonic 2-surfaces. One must generalize the super charges to those of Yangian since multi-stringy objects are physically unavoidable.

3. This is highly non-trivial duality between purely bosonic degrees of freedom assignable to elementary particles (wormhole contacts server as their building bricks) and fermionic degrees

of freedom assignable to string world sheets. Note that it is still not clear whether string world sheets are present also inside wormhole contacts or only in Minkowskian regions. Depending on this TGD would rely on open or closed strings.

4. This duality is expected to generalize. For instance, Kähler action should be expressible as string world sheet area in the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices at string world sheet. Dirac determinant should be expressible as exponent of Kähler action and by almost topological QFT property of TGD as an exponent of Chern-Simons terms -at least in Minkowskian space-time regions.

Perhaps the most practical form of the quantum gravitational holography is implied by the generalized conformal invariance implying effective 2-dimensionality. This means that $X_l^3$ represent generalized Feynman diagrams with lines representing by light-like 3-surfaces and vertices as 2-surfaces $X^2 \subset \delta CD \times CP_2$ at which these lines meet. Vertices can be expressed as N-point functions of super-conformal field theory at these 2-surfaces. Only effective two-dimensionality is in question since one has hierarchy of CDs within CDs and improvement of measurement resolution brings into consideration CDs with smaller size. Effective 2-dimensionality obvious means quantum holography in lower dimensional sense and this sequence of holographies continues down to the level of number theoretic braids with information about M-matrix coded by a set of discrete points at partonic 2-surfaces $X^2$.

Computationally TGD would reduce to almost string model type theory since light like 3-surfaces are analogous to closed string word sheets on one hand, and to the ends of open string on the other hand. A possible sketch based on twistors and the idea about scattering amplitudes as algebraic computations was already formulated. There is also an analogy with the Wess-Zumino-Witten model: light like causal determinants would correspond to the 2-D space of WZW model and 4-surface to the associated 3-D space defining the central extension of the Kac-Moody algebra.

## 4.4 Number Theoretic Compactification And $M^8 - H$ Duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of $M^4$ and $CP_2$ are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of $M^8$ as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of $M^8$ and determined by Kähler action at the level of $H$. Situation becomes more democratic if Kähler action defines the dynamics in both $M^8$ and $H$: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of $M^8$, and motivates also the coupling of Kähler gauge potential to $M^8$ spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as surfaces of $H$ or as surfaces of $M^8$ composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with $H$ should be essentially the same as that associated with $M^8$. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere K̈hler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3) \times U(1)$ by em charge and color quantum numbers just as for $CP_2$ - at least formally.

Harmonic oscillator potential defined by self-dual em field splits $M^8$ to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that $E^4$ effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

**Remark**: The Minkowskian signatures of $M^8$ and $M^4$ produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit $j$. If complexified quaternions are used for $H$, Minkowskian signature requires the introduction of two commuting imaginary units $j$ and $i$ meaning double complexification.

2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and $jI_k$, where $I_k$ are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the subspace of $M^8$ by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

3. Ordinary quaternions $Q$ are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of $H$ correspond hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and $M^8$ duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for $M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in $M^8$ and $H$ have same induced metric and induced Kähler form? Could the WCW s associated with $M^8$ and $H$ be identical with this assumption so that duality would provide different interpretations for the same physics?

2. One can formulate associativity in $M^8$ by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of $H$ as one might expect if Kähler action is involved in both cases? The analog of this formulation in $H$ might be as quaternionic "reality" since tangent space of $H$ corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in $M^8$ tangent space. This formulation is enough to define what associativity means although one can protest. Somehow $H$ is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level.* One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of $H$ tangent space would make sense at the level of space-time surfaces.

3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of $CP_2$ projection not larger than 2.

4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \to H \to H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of $M^8$. This brings in mind the functional composition of $O_c$-real analytic

functions ($O_c$ denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in $M^8$ would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in $H$ also in terms of Kähler-Dirac gamma matrices defined by Kähler action (certainly not $M^8$).

1. All known extremals are associative or co-associative in $H$ in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the Kähler-Dirac gamma matrices need not span the entire tangent space. The space spanned by the Kähler-Dirac gammas is not necessarily tangent space. For instance for $CP_2$ type vacuum extremals the Kähler-Dirac gamma matrices are $CP_2$ gamma matrices plus an additional light-like component from $M^4$ gamma matrices.

   If the space spanned by Kähler-Dirac gammas has dimension $D$ smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

2. For Kähler-Dirac gamma matrices the notion of co-associativity can produce problems since Kähler-Dirac gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by Kähler-Dirac gamma matrices? Co-associativity option must be considered for $D = 4$ only. $CP_2$ type vacuum extremals provide a good example. In this case the Kähler-Dirac gamma matrices reduce to sums of ordinary $CP_2$ gamma matrices and ligt-like $M^4$ contribution. The orthogonal complement for the Kähler-Dirac gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of $M^4$ and trivially associative.

### 4.4.1 Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that $M^8$ has unique decomposition $M^8 = M^4 \times E^4$. This would be most naturally due to Kähler structure in $E^4$ defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say $ie_1$ in $M^4$ - defining a preferred plane $M^2$ in $M^4$. Here it is essential that the gamma matrices of $E^4$ defined in terms of octonion units commute to gamma matrices in $M^4$. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group $G_2$ respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it $e_1$. Fixed complex structure therefore corresponds to a point of $S^6$.

3. Quaternionic sub-algebras of $M^8$ are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of $S^6$) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of $CP_2$, as the automorphism sub-group of

octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by $CP_2$. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of $e_1$) are labeled by $U(2) \subset SU(3)$. The choice of $e_2$ and $e_3$ amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of $e_1$ and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having $e_2$ and $e_3$ components. Hence all possible completions of $1, e_1$ by adding $e_2, e_3$ doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is $X^4$ corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate $x$ that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each $x$.

2. Since the Kähler structure of $M^8$ implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of $CP_2$. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

3. One could also map the associative surface in $M^8$ to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether $S^6$ allows genuine complex structure and Kähler structure which is essential for TGD formulation.

4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of $H$ can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space $M^8$ of $H$ using octonionization and can formulate it also terms of induced gamma matrices.

5. The associativity defined in terms of induced gamma matrices in both in $M^8$ and $H$ has the interesting feature that one can assign to the associative surface in $H$ a new associative surface in $H$ by assigning to each point of the space-time surface its $M^4$ projection and point of $CP_2$ characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition differs from the first proposal for years ago stating that each point of $X^4$ contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of $M^2$ depends on space-time point and is not restricted to $M^4$. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

2. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K2]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

3. Co-associative Euclidian 4-surfaces, say $CP_2$ type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which $W$ boson field is pure gauge so that the modes of the modified Dirac operator [K50] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

   For octonionic spinor structure the $W$ coupling is however absent so that the condition does not make sense in $M^8$. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

   There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

4. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in $M^8$. There is no need to introduce the counterpart of Kähler action in $M^8$ since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

   The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in $H$ are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of $H$ can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in $H$ is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in $H$. One could at least hope that associativity/co-associativity in $H$ is consistent with the preferred extremal property.

5. One can also consider a variant of associativity based on modified gamma matrices - but only in $H$. This notion does not make sense in $M^8$ since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

### 4.4.2 Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of $M^8$ using gamma matrices (for background see [K48] ).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of $X^4$ in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

2. Could/should one define the analog of associativity at the level of $H$? One can identify the tangent space of $H$ as $M^8$ and can define octonionic structure in the tangent space and this

allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

Skeptic however reminds $M^4$ allows hyper-quaternionic structure and $CP_2$ quaternionic structure so that complexified quaternionic structure would look more natural for $H$. The tangent space would decompose as $M^8 = HQ + ijQ$, weher $j$ is commuting imaginary unit and $HQ$ is spanned by real unit and by units $iI_k$, where $i$ second commutating imaginary unit and $I_k$ denotes quaternionic imaginary units. There is no need to make anything associative.

There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the $CP_2$ spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both $M^8$ and $H$ and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

### 4.4.3 Are Kähler and spinor structures necessary in $M^8$?

If one introduces $M^8$ as dual of $H$, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in $H$ are also extremals of $M^8$ Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in $H$ should have full $M^8$ dual.

*1. Are also the 4-surfaces in $M^8$ preferred extremals of Kähler action?*

It would be a mathematical miracle if associative and co-associative surfaces in $M^8$ would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in $M^8$. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of $CP_2$ type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of $H$).

The strongest form of duality would be that the space-time surfaces in $M^8$ and $H$ have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in $M^8$ would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that $M^8$ picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for $M^8$. Certainly it should be equivalent with WCW for $H$: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from $H$ to $M^8$. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of $E^4$ does not pose any technical problems.

*2. Spinor connection of $M^8$*

There are strong physical constraints on $M^8$ dual and they could kill the hypothesis. The basic constraint to the spinor structure of $M^8$ is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different $H$-chiralities and parity breaking.

1. By the flatness of the metric of $E^4$ its spinor connection is trivial. $E^4$ however allows full $S^2$ of covariantly constant Kähler forms so that one can accommodate free independent Abelian

gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of $CP_2$.

2. One should be able to distinguish between quarks and leptons also in $M^8$, which suggests that one introduce spinor structure and Kähler structure in $E^4$. The Kähler structure of $E^4$ is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of $S^2$ representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of $H$.

3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and $Z^0$ contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of $CP_2$ which vanishes for $E^4$ so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.

4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where $H$ picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of $E^4$ partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

*3. Dirac equation for leptons and quarks in $M^8$*

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing $H$ spinors decompose to $1+1+3+\overline{3}$ under SU(3) representing color automorphisms but the intepretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1+kI_1$, where $I_1$ is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of $Q_{em}$ so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of $M^8$ since the gauge potential is linear in $E^4$ coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make $E^4$ effectively a compact space.

4. The square of Dirac operator gives potential term proportional to $r^2 = x^2+y^2+z^2+t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

   If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1+ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of $e_1$ under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof $CP_2$ harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for $CP_2$.

5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to $iI_1$ and complexified octonionic units can be chosen to be its eigenstates with eigen value $\pm 1$. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

*4. What about the analog of Kähler Dirac equation*

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to $O_c$-real-analyticity would be extremely nice but not necessary ($O_c$ denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in $M^8$. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in $H$ could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in $M^8$. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in $H$. The fact that only holomorphy is involved with the definition of modes could make this map possible.

### 4.4.4 How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \to H \to H$... iteration generating new solutions from existing ones.

*1. Could octonion-real analyticity be equivalent with associativity/co-associativity?*

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of $M^8$ perhaps also at the level of $H$. Signature however causes problems - at least technical. Also the compactness of $CP_2$ causes technical difficulties but they need not be insurmountable.

For $E^8$ the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at $M^4$ light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by $O_c$-real-analytic functions (I use $O_c$ for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $Re(Q_1) + iIm(Q_2)$ with signature (1, -1, -, 1-, 1) is non-vanishing. Co-associative surfaces would be surfaces for which the projections to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ so that only the projection to $iIm(O_2)$ with signature $(-1-1-1-1)$ is non-vanishing.

These sub-manifolds are excellent candidate for associative and co-associative 4-surfaces if one believes on the intuition from complex analysis (the image of real axes under the map defined by $O_c$-real-analytic function is real axes in the new coordinates defined by the map). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of $O_c$-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

The alert reader has probably observed that the inverse image of the $M^4$ or $E^4$ as sub-space of $O_c$ does not belong to $M^4 \times E^4$ sub-space of $O_c$. One can however assign to each point of this 4-surface a unique point of $M^4$ as projection and a unique point of $CP_2$ as characterization of the quaternionic tangent plane hence $O_c \rightarrow H$ correspondence holds true.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

*2. Quaternionicity condition for space-time surfaces*

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both $M^8$ and $H$ with minor modifications if one accepts that also $H$ can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a,b,c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space $M^4$ coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

   One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of $3 \times 3$ determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections $e^A_\alpha$, vielbein vectors $e^A_k$, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants $f_{ABC}$ the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned}
e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\
A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\
e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\
\Gamma_k &= e^A_k \gamma_A \ .
\end{aligned} \tag{4.1}$$

The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{4.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associativity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" $a_{ijk}$ with 2-valued indiced (see `http://en.wikipedia.org/wiki/Hyperdeterminant` ). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^By^Cz^D = 0$ of trilinear forms defined by the associators. The conditions say something only about the octonionic structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A7] (see **Fig. 1** ) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units $e_1$ and $e_2$ their product $e_1e_2$ (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections $e_1, e_2$, their product $e_3 = k(x)e_1e_2$ and real fourth "timelike" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over $i$ is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

### 4.4.5 Quaternionicity at the level of imbedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A7] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic $M^4$ algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing $CP_2$ tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred $M^2$ contained in tangent space of space-time surface (the $M^2$: s could form an integrable distribution). Four-momentum restricted to $M^2$ and $I_3$ and $Y$ interpreted as tangent vectors in $CP_2$ tangent space defined quaternionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to $M^2$. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

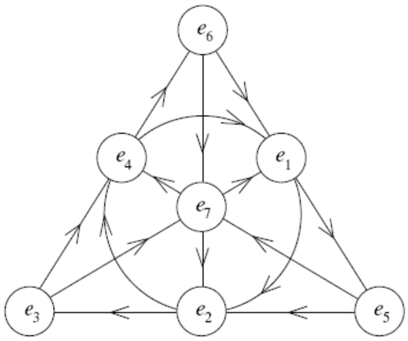


**Figure 1:** Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

### 4.4.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

*1. Could associativity condition be formulated using modified gamma matrices?*

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in $M^8$ is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of $M^8$ this option cannot work. One cannot exclude it for $H$.

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of $X^4$ a 4-D space which need not be tangent space anymore or even its sub-space.

   The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of $M^8$ the duality map to $H$ is therefore lost.

2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D $CP_2$ projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For $CP_2$ vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for $CP_2$ and the situation reduces to the quaternionicity of $CP_2$. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in $H$.

3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, $Y^2$ a Lagrange sub-manifold of $CP_2$, are trivially hyper-quaternionic surfaces. The modified definition of associativity in $H$ does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in $M^8$ allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both $M^8$ and $H$.

**Remark**: A side comment not strictly related to associativity is in order. The anticommutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

**Remark added later**: In fact, it has turned that this effective metric assignable to string world sheets plays a fundamental role in the recent formulation of TGD and allows to understand gravitational bound states in terms of string connecting partonic 2-surfaces: something impossible in string model.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler action also at the level of $M^8$? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

*2. Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?*

The 8-dimensionality of $M^8$ allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, $k$ positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as $CP_2$ type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the $CP_2$ type extremal and $CP_2$ size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

*3. Can $M^8 - H$ duality be useful?*

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in $H$, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in $M^8$ and the coupling of $M^8$ spinors to Kähler form. Note that the Kähler form in $E^4$ would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. $M^8$ description might work when $H$-description fails. For instance, perturbative QCD which corresponds to $H$-description fails at low energies whereas $M^8$ description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by

flatness of $E^4$ spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in $CP_2$. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for $Mx$ Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

*4. $M^8 - H$ duality in low energy physics and low energy hadron physics*

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using $E^4$ and $CP_2$ partial waves and low energy hadron physics corresponds to a situation in which $M^8$ picture provides the perturbative approach whereas $H$ picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in $CP_2$ degrees of freedom that can approximate $CP_2$ with a small region of its tangent space $E^4$. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of $CP_2$ can be neglected and one has effectively $E^4$. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the $E^4$ Hamiltonians in $M^8$ picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of $E^4$ valued vector field or equivalently collection of four $E^4$ Hamiltonians corresponding to spherical $E^4$ coordinates. Pion corresponds to $S^3$ valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the $E^4$ radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks $E^4$ partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be be important whereas at higher energies higher partial waves would be excited and the description based on $CP_2$ partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K23].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

### 4.4.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for $M^8$ and $H$. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \to H \to H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in $M^8$ and $H$ have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. $M^8_H$ duality might provide two descriptions of same underlying dynamics: $M^8$ description would apply in long length scales and $H$ description in short length scales.

## 4.5 Black Hole Physics

The hierarchy of Planck constants has forced to modify dramatically TGD based view about black holes. TGD black holes however have a lot of common with ordinary black holes.

### 4.5.1 M-theory and black holes

The reproduction of the formula for the black hole entropy [B29, B22] has been sold as a victory of M-theory. The first thing that has been forgotten is that GRT based formula has never been experimentally verified and could be even wrong.

One can also criticize the procedure leading to the formula.

1. First M-theory is replaced by 11-D super gravity in order to calculate something. What this effectively means that, although the aim was to replace General Relativity with something more fundamental, one ends up with 11-D classical super-gravity after all.

2. After this one finds black-hole type solutions and identifies them with M-branes. At this step one could protest by saying that the fundamental theory should replace black holes with something less singular.

3. Next quantum gravitational holography is assumed and a conformal field theory on brane identified as a black hole horizon leads to an estimate for the entropy and estimates for what are known as greyness factors. The last step is nice in the 4-D situation and also TGD would suggest something very similar.

In Matrix Theory based estimate things look even less elegant. In [B11] a matrix theory based estimate for the entropy is made producing the correct order of magnitude for the entropy estimate using conformal field theory. An essential step is the estimate for the number $N$ of 0-branes (ordinary particles) and is ad hoc (in particular one does not take the limit $N \to \infty$). I do not whether the arguments are more rigorous in other estimates but, to put it mildly, I do not find this argument is not too convincing.

### 4.5.2 Black holes in TGD framework

Black holes in the standard sense are possible in TGD framework but would be basically astrophysical objects and putting black holes and elementary particles in the same basket would be mixing apples with oranges. The vision about dark matter as a macroscopic quantum phases with large value of Planck constant (the value of gravitational Planck constant is enormous) forces to reconsider the identification of black holes. One can view TGD counterparts of black hole horizons as light-like 3-surfaces at which the signature of the induced metric changes to Euclidian.

Black holes would be gigantic elementary particle (or rather parton-) like objects containing particles in anyonic phase with fractional charges guaranteeing confinement. Dark anyonic matter at light-like 3-surfaces of astrophysical size analogous to stringy black holes thought to be tightly tangled strings has several basic characteristics of black hole and would populate TGD Universe in all length scales.

In TGD Universe the role of black hole horizons is taken by light like 3-surfaces, which are fundamental objects of the theory whereas the role of big bang is taken by the boundary $\delta M^4_+$ of causal diamond (CD). The basic difference to black hole horizons is that the signature of induced metric changes at the wormhole throat.

1. The basic example is provided by elementary particle horizons surrounding the ends of the wormhole contacts having Euclidian signature of the induced metric and connecting with each other space-time sheets with Minkowskian signature of the induced metric. The light-like wormhole throats are carriers of fermion numbers. The interpretation of wormhole contacts is in terms of gauge bosons and Higgs bosons consisting of fermion and anti-fermion at the two wormhole throats. By its spin the only possible identification of graviton is as a pair of wormhole contacts connected by a flux tube carrying various gauge fluxes. Elementary fermions correspond to wormhole throats associated with $CP_2$ type vacuum extremals (note Euclidian signature of induced metric) glued to the background space-time with Minkowskian signature of metric.

2. Second example is provided by light-like surfaces separating maximal deterministic regions of the space-time sheet. Light-like boundaries is a further example. By their metric 2-dimensionality various causal determinants indeed allow conformal field theory in an effectively 2-dimensional sense.

3. The formula for the black hole entropy generalizes to elementary particle level and involves p-adic length scale hypothesis and p-adic mass calculations [K25].

4. The new element is the hierarchy of Planck constants [K34, K27, K11] inspired by the findings that gravitational Planck constant might have gigantic value [E1]. This leads to a vision about dark matter as phases of matter with large Planck constant and hence macroscopically quantum coherent since all quantum scales are scaled up. The space-time sheets mediating gravitational interaction would have gigantic value of Planck constant: $\hbar_{gr} = GM_1M_2/v_0$, $v_0 = 2^{-11}$ gives a good example about the situation. The implication is that black hole entropy proportional to $1/\hbar$ is of order unity if $\hbar_{gr} = GM^2/v_0$, $v_0 = 1/4$ holds true for black holes. This would change completely the view about black holes as highly entropic objects. In particular, Planck length scales as $\sqrt{\hbar}$ so that Schwartschild radius represents Planck length for this kind of black hole and defines naturally kind of minimum length scales below which the signature of induced metric becomes Euclidian in TGD Universe.

5. The progress in the understanding of the realization of the hierarchy of Planck constants in terms of book like structure of imbedding space with the pages of book representing Cartesian products of singular coverings and factor spaces of causal diamond CD and $CP_2$ led to a detailed picture about identification of anyonic systems as macroscopic light-like 3-surfaces containing dark matter in anyonic form possessing fractional quantum numbers. Anyonicity means that the “partonic” 2-surface of macroscopic size system surrounds the tip of CD so that homologically non-trivial 2-surface is in question. Anyonic phase could be even responsible for the properties of living matter [K29, K10]. This also inspired the proposal that dark matter resides at light-like 3-surfaces of astrophysical and even cosmological size scale possessing very complex topology: typically spherical topologies glued together by flux tubes. Black holes in standard sense would result in gravitational collapse of this kind of systems. An open question is whether the topology actually transforms to simple spherical topology in this process or whether it is more or less conserved so that huge information about the topology of orbits of dark matter particles surrounding the object would be preserved.

More concrete ideas about black hole like structures emerged from the attempts to understand the strange events reported by RHIC (Relativistic Heavy Ion Collider) [C1, C2] during last years. This work led to a dramatic increase of understanding of TGD and allowed to fuse together separate threads of TGD [K35].

1. The scaled down TGD inspired cosmology involving (not so) big crunch followed by (not so) big bang serves as a model for the events, and predicts a new phase identifiable as color glass condensate identifiable as tightly tangled color magnetic flux tube modellable as a hadronic string in Hagedorn temperature.

   This state makes a phase transition to quark gluon plasma during a period of critical cosmology analogous to inflationary cosmology characterized completely by its duration and quark gluon plasma analogous to radiation dominated cosmology in turn hadronizes giving rise to the analog of matter dominated cosmology.

The assumption that anyonicity is responsible for the formation of the gluonic Bose-Einstein condensate explains the liquid like character of color glass condensate. Anyonicity forces the system to behave like a single particle like unit since fractionally charged particles cannot leave the light-like 2-surface surrounding the tip of CD.

2. RHIC events suggest processes analogous to the formation and evaporation of black hole. The TGD inspired description in terms of the formation of hadronic black hole and its evaporation and essentially identical with the description as a mini bang. The hadronic black hole is the same tightly tangled color magnetic flux tube that defines the initial state of the hadronic mini bang. The attribute "hadronic" means that Planck length is replaced with hadronic length so that strong gravitation is in question. Black hole temperature is identifiable as Hagedorn temperature and predicted to be 195 MeV for bosonic strings in 4-D space-time and slightly higher than the hadronization temperature measured to be about 176 MeV [K35].

3. As also the small value of black hole entropy suggests, black holes and their scaled counterparts would not be merciless information destroyers in TGD Universe. The entanglement of particles possessing different conformal weights to give states with a vanishing net conformal weight and having particle like integrity would make black hole like states ideal candidates for quantum computer like systems [K47]. One could even imagine that the galactic black hole is a highly tangled cosmic string in Hagedorn temperature performing quantum computations the complexity of which is totally out of reach of human intellect! Indeed, TGD inspired consciousness predicts that evolution leads to the increase of information and intelligence, and the evolution of stars should not form exception to this. Also the interpretation of black hole as consisting of dark matter follows from this picture [K10].

Concerning the mathematical description of dark matter - and of matter quite generally- TGD has led to amazingly simple mathematical framework, which might have something to with Matrix theory approach. The characteristic aspects of the classical dynamic determined by Kähler action is its vacuum degeneracy and this not only allows but even forces the notion of finite measurement resolution originally inspired by the inclusions of hyper-finite factors of type $II_1$ (HFFs) having WCW Clifford algebra as a canonical representative. The notion of finite measurement resolution leads to a discretization of physics in terms of string like objects carrying the modes of the spinor fields. If conformal symmetry for spinor modes is realized as gauge symmetry, there is effectively only a finite number of fermionic oscillator operators characterizing any subsystem [K50]. Even the infinite-dimensional world of classical worlds can be described with arbitrary accuracy as a finite-dimensional space and these descriptions define a hierarchy of inclusions of HFFs associated with WCW Clifford algebra.

How the TGD analogs of black holes could relate to GRT black holes?

1. GRT space-time is obtained from many-sheeted space-time of TGD by a rather violent operation: the sheets of the many-sheeted space-time are lumped together to form a region of $M^4$ with the deviation of metric from $M^4$ metric given by the sum of corresponding deviations for the sheets and gauge potentials identified as sums of the induced gauge potentials for sheets. What remains visible about many-sheeted physics are anomalies of GRT [K57].

2. This description should be good in regions, where the gravitational field is weak. Since the coordinates for the Schwarthild metric in TGD framework correspond naturally to Minkowski coordinates, one must interpret the diverging deviation of the metric from Minkowski metric at horizon as a failure of this approximation (usually one would argue that curvature is small at horizon so that there is no reason to worry: firewall debate has forced to question this assumption).

   By this argument GRT space-time in black hole length scales can be seen as a continuation of physics from regions, where TGD-GRT correspondence is a good approximation to regions where fails to be so. TGD physics could become visible at Schwartschild radius and at even longer distances. The description of the formation of gravitational bound states in terms of strings connecting partonic 2-surfaces assumes macroscopic quantum coherence and this is of course something completely new.

3. The space-time regions with Euclidian signature of the induced metric are not included at all in TGD description. One could of course consider also this kind of solutions and they have been used as a trick to make path integral well-defined. Einstein-Maxwell action with cosmological constant defined by $CP_2$ scale allows $CP_2$ as a gravitational instanton, and one might consider the possibility of an improved GRT limit with particles identified as deformations of $CP_2$ are glued along light-like 3-surfaces to Reissner-Nordström type metric. One might hope this to give an improved description of elementary particles. There is however a problem. In TGD framework work the existence of imbeddings of $CP_2$ to imbedding space with light-like curve as $M^4$ projection are essential for particle interpretation. It is difficult to see how particle interpretation could be possible in GRT framework.

## 4.6 WCW Gamma Matrices As Hyper-Octonionic Conformal Fields?

The fact that the Clifford algebra generated by WCW gamma matrices forms a canonical representation for hyper-finite factor of type $II_1$ (HFFs) and led to a breakthrough in the understanding of quantum TGD. The inclusions of hyper-finite factors of type $II_1$ led to a realization of finite quantum measurement resolution as a basic principle govering dynamics and together with zero energy ontology this approach led to the generalization of S-matrix to M-matrix identified as time like entanglement coefficients between positive and negative energy parts of zero energy state and its identification as Connes tensor product. HFFs generated also ideas about how quantum TGD might be reducible to a generalization of HFFs to its local variant which is necessarily complex-octonionic as also to a construction of quantum variant of gamma matrix algebra leading to identification of quantum counterparts of hyper-octonions and hyper-quaternions as unique structures.

### 4.6.1 Only the quantum variants of $M^4$ and $M^8$ emerge from local hyper-finite $II_1$ factors

The fantastic properties of hyperfinite factors of type $\mathrm{II}_1$ (HFFs) inspire the idea that a localized hyper-octonionic version of Clifford algebra of WCW might allow to see space-time, embedding space, and WCW as structures emerging from a hyper-octonionic version of HFF. Surprisingly, commutativity and associativity imply most of the speculative "must-be-true's" of quantum TGD.

WCW gamma matrices act only in vibrational degrees of freedom of 3-surface. One must also include center of mass degrees of freedom which appear as zero modes. The natural idea is that the resulting local gamma matrices define a local version of HFF of type $\mathrm{II}_1$ as a generalization of conformal field of gamma matrices appearing super string models obtained by replacing complex numbers with hyper-octonions identified as a subspace of complexified octonions.

As a matter fact, one can generalize octonions to quantum octonions for which quantum commutativity means restriction to a hyper-octonionic subspace of quantum octonions. Non-associativity is essential for obtaining something non-trivial: otherwise this algebra reduces to HFF of type $\mathrm{II}_1$ since matrix algebra as a tensor factor would give an algebra isomorphic with the original one. The octonionic variant of conformal invariance fixes the dependence of local gamma matrix field on the coordinate of $HO$. The coefficients of Laurent expansion of this field must commute with octonions!

Super-symmetry suggests that the representations of $CH$ Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for $M^8$ representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to $M^4$ coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2,F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of $M^D$ exist for all dimensions but only spaces $M^4$ and $M^8$ and their linear sub-spaces emerge from hyper-finite factors of type $II_1$. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary $M^4$ and $M^8$ which are thus already quantal concepts.

Consider first hyper-quaternions and the emergence of $M^4$.

1. The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array}\right) \ , \tag{4.3}$$

read as

$$\begin{array}{lll} ab = qba \ , & ac = qac \ , & bd = qdb \ , cd = qdc \ , \\ [a,d] = (q-q^{-1})bc \ , & bc = cb \ . & \end{array} \tag{4.4}$$

2. These relations could be extended by postulating complex conjugates of these relations for complex conjugates $a^{\dagger}, b^{\dagger}, c^{\dagger}, d^{\dagger}$ plus the following non-vanishing commutators of type $[x, y^{\dagger}]$:

$$[a, a^{\dagger}] = [b, b^{\dagger}] = [c, c^{\dagger}] = [d, d^{\dagger}] = 1 \ . \tag{4.5}$$

This extension is not necessary for what comes.

3. The matrices representing $M^4$ point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a-a^{\dagger}, d-d^{\dagger}, b-c^{\dagger}, c-b^{\dagger}\} \ . \end{array} \tag{4.6}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators $O$ commute. These conditions need not be consistent with the commutation relations between a, b, c, d and their Hermitian conjugates. This is easy to see by noticing that the difference of $J_+ - J_-$ acts apart from imaginary unit like $J_y$ and annihilates $j_y = 0$ state for every representation of rotation group diagonalized with respect to $J_y$.

4. What is essential is that the operators of $O$ are of form $A - A^{\dagger}$ and their commutators are also of the same form that the commutativity conditions reduce the condition that the Lie-algebra like structure generated by these operators annihilates the physical state. Hence it is possible to define quantum states for which $M^4$ coordinates have well-defined eigenvalues so that ordinary $M^4$ emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which $M^4$ coordinates are emerge naturally.

5. $M_{2,q}(C)$ matrices define the quantum analog of $C^4$ and one can wonder whether also other linear sub-spaces can be defined consistently or whether $M^4_q$ and thus Minkowski signature is unique. This seems to be not the case. For instance, the replacement $a - a^{\dagger} \rightarrow a + a^{\dagger}$ making also time variable Euclidian is impossible since $[a + a^{\dagger}, d - d^{\dagger}] = 2(q - q^{-1})(bc + b^{\dagger}c^{\dagger}$ is not proportional to a difference of operator and its hermitian conjugate and one does not obtain closed algebra.

What about $M^8$: does it have analogous description in terms of physical states annihilated by the Lie algebra generated by the differences $a_i - a_i^{\dagger}$, $i = 0, ..7$?

1. The representation of $M^4$ point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anti-commutation relations of gamma matrices of $M^8$ and would give classical representation of $M^8$. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

2. In quaternionic case basis for matrix algebra is formed by the sigma matrices and $M^4$ point is represented by a hermitian matrix expressible as linear combination of hermitian sigma matrices with coefficients which act on physical states like hermitian operators. In the hyper-octonionic case would expect that real octonion unit and octonionic imaginary units multiplied by commuting imaginary unit to define the counterparts of sigma matrices and that the physically representable sub-space of complex quantum octonions corresponds to operator valued coordinates which act like hermitian matrices. The restriction to complex quaternionic sub-space must give hyper-quaternions and $M^4$ so that the only sensible generalization is that $M^8$ holds quite generally. This is also required by $SO^7$ invariance allowing to choose the sub-space $M^4$ freely. Again the key point should be that the conditions giving rise to real eigenvalues give rise to a Lie-algebra which must annihilate the physical state. For other signatures one would not obtain Lie algebra.

3. One can also make guess for the concrete realization of the algebra. Introduce the coefficients of $E^4$ gamma matrices having interpretation as quaternionic units as

$$\begin{array}{ll} a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\ a_1 = x(ib+c) \ , & a_2 = x(ib-c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

4. The selections of complex and quaternionic sub-algebras of octonions are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of hyper-quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of hyper-octonions the selection of hyper-quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Hyper-quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned} [a_0,a_3] &= \frac{i}{2}(q-q^{-1})(a_1^2-a_2^2) \ , \\ [a_i,a_j] &= 0 \ , \ i,j\neq 0,3 \ , \\ a_0a_i &= qa_ia_0 \ , \ i\neq 0,3 \ , \\ a_3a_i &= qa_ia_3 \ , \ i\neq 0,3 \ . \end{aligned} \tag{4.7}$$

Note that there is symmetry breaking in the sense that the commutation relations for sub-algebras relating to both $M^4$ and $M^2$ are in distinguished role.

Dimensions $D=4$ and $D=8$ are indeed unique if one takes this argument seriously.

1. For dimensions other than $D=4$ and $D=8$ a representation of the point of $M^D$ as element of Clifford algebra of $M^D$ is needed. The coefficients should be real for the signatures and this requires that the elements of Clifford algebra are Hermitian. Gamma matrices are the only natural candidates and when Majorana conditions can be satisfied one obtains quantum representation of $M^D$. 10-D Minkowski space of super-string models would represent one example of this kind of situation.

2. For other dimensions $D \geq 8$ but now octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

### 4.6.2 WCW spinor fields as hyper-octonionic conformal fields

A further proposed application of this picture is to the construction of WCW spinor fields as generalizations of conformal fields. The basic problem is to treat center of mass degrees of freedom properly, and the idea that conformal invariance generalizes to hyper-octonionic - or at least hyper-quaternionic - conformal invariance is attractive. If so, the usual expansion in powers of complex coordinate $z$ would be replaced in powers of hyper-octonionic coordinate $h$ and the coefficients would be elements of Clifford algebra for sub- WCW consisting of light-like 3-surfaces with frozen center of mass degrees of freedom. This is possible if one can map the points of $H$ to those of $M^8$ and $M^8 - H$ duality allows to achieve this.

One could use Laurent expansions with coefficients multiplying powers of $h$ from right so that one could defined the notion of octonion analyticity in terms of a generalization of Riemann conditions as shown in [A8]. In the case of quaternionic analyticity one obtains also analyticity in two complex variables for one particular form of Riemann conditions and something similar might happen now.

Hyper-octonions do not define a number field but only linear sub-space of complexified octonions. This does not however matter in this case. Also the notions of quaternionic and complex sub-manifold are independent of signature.

The natural condition would be that N-point functions defined by WCW spinor fields for which $M^8$ coordinate labels the position of the tip of the causal diamond containing the zero energy state involve only those points which are mutually associative and would thus belong to a hyper-quaternionic sub-space $M^4 \subset M^8$ would be in question and the outcome would be the analog of $M^4$ quantum field theory.

Commutativity would restrict the points to $M^2 \subset M^4 \subset M^8$ and hyper-complex variant conformal field theory would result: this theory would be analogous with integrable models known as factorizing quantum field theories in $M^2$ in which particle scattering is almost trivial (interactions generate only phase lag).

## 4.7 Zero Energy Ontology And Witten's Approach To 3-D Quantum Gravitation

There is an interesting relationship of quantum TGD to the recent yet unpublished work of Witten related to 3-D quantum blackholes [B18], which - despite that it does not directly relate to M-theory - provides additional perspective.

1. The motivation of Witten is to find an exact quantum theory for blackholes in 3-D case. Witten proposes that the quantum theory for 3-D $AdS_3$ blackhole with a negative cosmological constant can be reduced by $AdS_3/CFT_2$ correspondence to a 2-D conformal field theory at the 2-D boundary of $AdS_3$ analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2) \times SO(1,2)$ of $AdS_3$. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

2. This conformal theory would have the so called monster group [B18, B2] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [B7]. In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that

gravitational radiation is not possible. Hence$AdS_3$ theory in Witten's sense might define this conformal field theory.

Witten's construction has obviously a strong structural similarity to TGD.

1. Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory.

2. Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_\pm \times CP_2$ with the partonic 2-surface.

3. For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD, gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.

4. Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

There are also interesting differences.

1. According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.

2. Also the fermionic counterpart of Chern-Simons action for the induced spinors whose form is dictated by the super-conformal symmetry is present. Furthermore, partonic 3-surfaces are dynamical unlike $AdS_3$ and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.

3. The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.

4. In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of $AdS_3$. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

   The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.

5. Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type $II_1$ [K11]. Compact groups and their quantum counterparts would closely relate to a hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined by inclusion as well as to the generalization of the imbedding space related to the hierarchy of Planck constants [K11]. Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [K15].

6. To make it clear, I am not suggesting that $AdS_3/CFT_2$ correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   (a) Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   (b) In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   (c) The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D light-like surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about $M$-matrix implied by the zero energy ontology.

1. Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

2. Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized $S$-matrices: **Matrix** or $M$-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrodinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the category of Feyman cobordisms and matrices have groupoid like structure (see discussion below). Without this generalization theory would reduce to a theory for 2-D fundamental objects.

3. Theory becomes genuinely 4-D because $M$-matrix is not universal anymore but characterizes zero energy states.

4. 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D light-like surfaces as sub-manifolds (analogs of blackhole horizons and light-like boundaries) [K9]. Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the Kähler-Dirac action associated

with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [K50].

# 5 What Went Wrong With String Models?

As will be found, the few physical predictions of M-theory are wrong. It is instructive to try to understand what went wrong with M-theories and string models by comparing it with earlier successful theories and with TGD.

## 5.1 Problems Of M-Theory

At the physical side the situation in M-theory can be regarded as a catastrophe and without the association of the attribute "the only known candidate for the quantum theory of gravitation..." to the letter M bringing in mind Pavlov dogs, no-one could take it seriously. The various problems of M-theory have been discussed in the article of Smolin [B19] as also by Penrose in his lecture series "Fashion, Faith and Fantasy in Theoretical Physics" [B26]. The discussions of "Not Even Wrong" [B3] group provide a vivid critical view about the situation.

1. M-theory has not been able to explain why the dimension of the space-time is four and has even failed to reproduce the standard model. Unless one assumes that the small dimensions form a singular manifold (something so ugly that it turns my stomach around), M-theory predicts chiral symmetry just like Kaluza-Klein theories: the symmetry is inconsistent with the standard model. Ironically, just this was the reason why superstrings replaced Kaluza-Klein theories in the first superstring revolution. This full $\pi$ twist represents a good example of M-logic.

   The predicted massless scalar fields have not been observed. The predicted low energy super-symmetry is experimentally absent, and now papers have begun to appear suggesting that M-theory after all might predict only high energy super-symmetry. One of the first findings after the second superstring revolution was that the prediction for the unification scale was wrong. I remember that Witten proposed at that time a suitable compactification of the $11^{th}$ dimension to a circle to circumvent this problem.

2. Cosmological constant is now believed to be non-vanishing and positive [E2] whereas the cosmological constant predicted by M-theory is negative. M-theories provide no explanation for the accelerated expansion [E2]. There is a plethora of cosmological observations which M-theory cannot even address.

This sad state of affairs has led to the introduction of the anthropic principle [B20] but not in the sense that it would really predict something but as an M-logic proof that M-theory after all predicts among other things also the cosmological constant correctly. The premise is that M-theory is correct and the conclusion is that the observed universe must represent some distant corner of the M-landscape, and we must be ready to accept as a fact, that we will never be able to find our way to this distant part of the Theory Universe, and be happy with learning new dualities.

## 5.2 Mouse As A Tailor

The history of string models differs dramatically from that for theories which has been successful as physical theories. As a rule, new theories have started from a precise problem which earlier theories have not been able to solve, and have led to a new ontology and inspired new mathematics.

String model was born as a model of hadrons. It however became gradually clear that the constraints on space-time dimensions make it unrealistic for this purpose. The conclusion of the mouse was not so humble as in the tale: admittedly string models fail for hadrons but who knows, they might describe everything.

After a decade of tailoring the cat was told that superstrings do not seem to make a TOE after all. The mouse said that he could tailor even something more grandiose just by sewing together all the previous failures. Now it has become clear that the result is an enormous bundle of solutions of the possibly existing M-theory, which at practical level is reduced after few heuristic arguments to compactifications of 11-D super gravity. There is still however a little problem: not a single one of these solutions seems to describe the Universe we live in. Now the mouse suggests that we should give up the dream about a theory of the observable universe as unrealistic, stop complaining and be happy with all these beautiful dualities.

Is the time ripe for the story to end as its original version did or shall the cat provide still another decade of financial support for the expensive tailor?

## 5.3 The Dogma Of Reductionism

### 5.3.1 M-theory as an outcome of hard-nosed reductionism

The philosophical background of string models is hard-nosed reductionism taken down to Planck length: something taken to be so self-evident that it has not been even mentioned. Hence the theory cannot make any predictions about or utilize the rich experimental input coming from the known physics.

This means that string theorists do not pay any attention to the pressing problems of quantum measurement theory, to the problems related to the relationship between experienced and geometric time, and to the problems surrounding to the poor understanding of second law. Not to even mention the questions about the difference between animate and in-animate matter, and about what it means to be a conscious system.

The belief that the action defining functional integral summarizes the physics leads to an approach which is extremely pragmatic: start from the existing formulas of perturbative field theories and try to combine them in order to cook up a more general theory. The danger that theoreticians fall into a kind of mathematical insanity in this kind of situation is obvious, and the possible failure of reductionism means a tragic failure of the entire approach.

### 5.3.2 Giving up reductionism

TGD cannot be regarded as a success from the point of view of sociology of science but the success of TGD as a physical theory is undeniable and basically due to the facts that TGD emerged as a solution to a well-defined problem, and that the notion of many-sheeted space-time plus p-adic length scale hypothesis [K25] provide a precise quantitative formulation for how reductionism fails.

1. I ended up with TGD by starting from a very real problem of general relativity and soon found that I could end up to TGD also from string models. From the beginning the contact of TGD with experimental physics was very intimate. Later the quantum classical correspondence has become a basic guide line in the construction of the theory.

2. One cannot deny that string theories partially solved the divergence problem of perturbative quantum field theories. Unfortunately, is is highly implausible that the sum of the perturbation series would converge so that as such it is useless. This has in fact been seen as a victory of the theory since one can hope that a genuinely non-perturbative approach could lead to a unique theory.

   In TGD framework the absence of the basic divergences is highly plausible already from the basic construction involving new ontology of space-time. Vacuum functional identified as an exponent of Kähler function is not anymore a local functional of 3-surface so that basic perturbative divergences resulting from the micro-locality are absent. Also Gaussian and metric determinants cancel and the definition of Kähler function in terms of Dirac determinant is free of divergencies [K50].

3. The construction of quantum TGD was not possible without the theory of consciousness. Key element is the replacement of space-time micro-locality with classical locality in the “world of classical worlds” making possible to understand how macroscopic and macro-temporal quantum coherence are possible [K16, K4, K17]. Thanks to the notion of self [K33, K44, K6], observer ceases to be an outsider and quantum measurement theory is becomes an essential part of the theory. Completely un-expected outcomes were the already mentioned generalizations of the number concept and the identification of the space-time correlates of cognition and intentionality.

4. TGD generalizes in a dramatic manner the ontology of space-time in terms of the notion of the many-sheeted space-time involving also the new view about numbers. The identification of space-time sheets as space-time counterparts of physical objects resolves the question about the generation of structures. The ontology of quantum TGD is discussed in [K6] from the point of view of category theory. One important implication is that even quantum superposition and quantum logic can have space-time correlates at the level of many-sheeted space-time.

5. TGD resolves the paradoxes due to the conflict between the non-determinism of quantum jump and determinism of Schrödinger equation and, by the classical non-determinism, quantum-classical correspondence can be realized at the space-time level even for quantum jump sequences. TGD leads to a new view about the relationship between geometric and subjectively experienced time rather than just identifying them [K44].

6. Zero energy ontology replaces positive energy ontology. Zero energy states are superpositions of pairs of positive and negative energy states with opposite energies and other conserved quantum numbers assignable to the boundaries of causal diamond (CD). In ordinary ontology they corresponds to events consisting of initial and final state.

   Negative energies make possible what I call remote metabolism playing in key role in TGD inspired theory of consciousness and of quantum biology: the system can gain energy by sending negative energy to geometric past [K44, K16, K17]. Time mirror mechanism (see **Fig. `http://tgdtheory.fi/appfigures/timemirror.jpg`** or **Fig. ??** in the appendix of this book) makes possible communications with geometric past and future and communications with an effectively super-luminal velocity become possible.

7. The duality between theory and reality is resolved. TGD based ontology postulates only three levels of existence corresponding to existences in these sense of classical and quantum physics, and conscious existence which corresponds to the quantum jumps between the quantum states [K6]. The possibility that space-time points are infinitely structured in p-adic sense although this structure is not visible in real sense [K39], would resolve the challenge posed by the question why all those structures that we can imagine mathematically, are not realized physically. Obviously, a reincarnation of the monad idea of Leibniz is in question.

## 5.4 The Loosely Defined M

In a sharp contrast with M-theory [B8], Newton's mechanics and gravitational theory, Maxwell's electrodynamics, Special and General Relativities, and even Bohr's rules were from the beginning relatively precisely defined theories able to make testable predictions. The lack of a precise definition of what "M" means has led to a flood of speculations based on speculations based on...

"M" as "membrane" would be a rather precise definition but does not really make sense since the huge conformal invariance of string models is lost as objects become 2-dimensional. For this reason one prefers to replace "M" with Mystery, Mother, or perhaps Matrix, but still think in terms of membranes which behave like strings. It became however clear that also branes of various dimensions are needed as discovered by Polchinski [B16] and identified as non-perturbative objects at which string ends are attached to: this interpretation is the only possible one since otherwise momentum conservation would be lost for D-branes.

Needless to say, a theory using geometric structures consisting of parts possessing different dimensions does not satisfy the standards of the conventional mathematical aesthetics. An outsider could argue that the non-uniqueness of the boundary conditions (Neumann, Dirichlet and mixtures of them) is the fundamental failure of the string theory, and that a viable theory should predict the dynamics of boundaries. This is indeed the case in TGD where the criticality of the Kähler action guaranteeing general coordinate invariance in 4-D sense does this and implies that the space-time surface is a field theory counterpart of Bohr orbit.

A good example of brave new M-logic is provided by the construction of what is called Matrix Theory [B11]. One starts from M-theory "known" to have 11-D supergravity as a low energy limit, replaces it with a 11-D supergravity, restricts the consideration to $N$ 0-branes (point particles) living in an effectively 10-D space, in an ad hoc manner replaces their position coordinates in 10-D space with non-commuting $N \times N$-matrix valued coordinates assuming that eigenvalues correspond to $N$ space-time points, postulates a non-relativistic Schrödinger equation for this matrix, and by generalizing bravely the notion of holography, concludes that the original theory and even more follows from this very-very special theory at $N \to \infty$ limit. From Matrix Theory one then deduces all superstring dualitiesandblack hole physics using an argumentation with a comparable rigor.

It must be added that TGD predicts a rich variety of objects resulting as asymptotic self-organization patterns for which Kähler-Lorentz 4-force vanishes by quantum classical correspondence. The solutions are classified by the dimension of either their $M^4$ or $CP_2$ projection [K2]. This variety includes cosmic strings and magnetic flux tubes besides space-time sheets. Magnetic flux tubes and string like objects can indeed attach to the boundaries of space-time sheets and there are obvious correspondences with branes with dimensions of branes restricted to run from 0 to 4 ($p = -1, .., 3$) but only as objects obtained by idealizing 4-dimensional object with a lower-dimensional object.

Even the possibility of single space-time point or space-time curve to mimic the quantum dynamics of the quantum state of Universe is predicted but only at the level of cognition and relying on the new notion about what mathematical point is [K39]. I however do not think that this has much to do with Matrix Theory.

## 5.5 What Went Wrong With Symmetries?

Theoretical physics is in deep crisis. This is not bad at all. Crisis forces eventually to challenge the existing beliefs. Crisis gives also hopes about profound changes. In physical systems criticality means sensitivity, long range fluctuations and long range correlations, and this makes phase transition possible. In TGD framework life emerges at criticality!

The crisis of theoretical physics has many aspects. The crisis relates closely to the sociology of science and to the only game in the town attitude. The prevailing materialistic philosophy of science combined with the naive length scale reductionism form part of the sad story. The seeds of the crisis were sown in birthdays of quantum mechanics. The fathers of quantum theory were well aware that quantum measurement theory is the Achilles heel of the newborn quantum theory but later the pragmatically thinking theoreticians labelled questioning of the basic concepts as "philosophy" not meant for a respectable physicist.

The recent quantum measurement theory is just a collection of rules and observer still remains an outsider. To my view the proper formulation of quantum measurement theory requires making observer a part of systems. This means that physics must be extended to a theory of consciousness.

This raises several fundamental challenges and questions. How to define "self" as conscious entity? How to resolve the conflict between two causalities: that of field equations and that of "free will"? What is the relationship between the geometric time of physicist and the experienced time? How is the arrow of time determined and is it always the same? The evidence that living matter is macroscopic quantum system is accumulating: is a generalization of quantum theory required to describe quantum systems? What about dark matter: can we understand it in the framework of existing quantum theory? This list could be continued.

In the following I will not consider this aspect more but restrict the consideration to an important key notion of recent day theoretical physics, namely symmetries. Physical theories rely nowadays on postulates about symmetries and there are many who say that quantum theory reduces almost totally group representation theory. There are refined mathematical tools making possible to derive the implications of symmetries in quantum theory such as Noether's theorem. These technical tools are extremely useful but it seems that methodology has replaced critical thought.

By this I mean that the real nature of various symmetries has not been considered seriously enough and that this is one of the basic reasons for the recent dead end. In the following I describe what I see as the mistakes due to sloppy thinking (maybe "sloppying" might be shorthand for it) and discuss briefly the TGD based solution of the problems involved.

This sloppiness manifests itself already in general relativity, in standard model there is no unification of color and electroweak symmetries and their different character is not understood, GUT approach is based on naive extension of gauge group and makes problematic predictions, supersymmetry in its standard form predicted to become visible at LHC energies is now strongly dis-favoured experimentally, and superstring model led to landscape catastrophe what has left is AdS/CFT correspondence which has not led to victories. Could it be that also conformal invariance should be re-considered seriously: a non-trivial generalization to 4-D context is highly desirable so that 10-D bulk would be replaced by 4-D space-time in the counterpart of AdS/CFT duality.

### 5.5.1 Energy problem of GRT

Energy and momentum are not well-defined notions in General Relativity. The Poincare symmetry of flat Minkowski space is lost and one cannot apply Noether's theorem so that the identification of classical conserved charges is lost and one can talk only about local conservation guaranteed by Einstein's equations realizing Equivalence Principle in weak form.

In quantum theory this kind of situation is highly unsatisfactory since Uncertainty Principle means that momentum eigenstates are delocalized. This is sloppy thinking and the fact that quantization is to high extend representation theory for symmetry groups might well explain the failure of the attempts to quantize general relativity.

TGD was born as a reaction to the challenge of constructing Poincare invariant theory of gravitation. The identification of space-times as 4-surfaces of some higher-dimensional space of form $H = M^4 \times S$ lifts Poincare symmetries from space-time level to the level of imbedding space $H$.

In this framework GRT space-time is an approximate macroscopic description obtained by replacing the space-time sheets of many-sheeted space-time with single piece of $M^4$ which is slightly curved. Gravitational fields -deviations of induced metric from Minkowski metric- are replaced with their sum for various sheets. Same applies to gauge potentials. Einstein's equations express the remnants of Poincare symmetry for the GRT space-time obtained in this manner.

In superstring models one actually considers 10-D Minkowski space so that the lifting of symmetries is possible. Also the compactification (say Calabi-Yau) to $M^4 \times C$ still have Poincare symmetries. But after that one has 10-D gravitation and the same problems that one wanted to solve by introducing strings! School example about sloppying!

### 5.5.2 Is color symmetry really understood?

May colleagues use to think that standard model is a closed chapter of theoretical physics. This is a further example of sloppy thinking.

1. Standard model gauge group is product of color and electro-weak groups which are totally independent. The analogy with Maxwell's equations is obvious. Only after Maxwell and Einstein they could be seen as parts of single tensor representing gauge field.

2. QCD and electroweak interactions differ in crucial manner. Color symmetry is exact (no Higgs fields in QCD) whereas electroweak symmetry is broken, and QCD is asymptotically free unlike electroweak interactions. In QCD color confinement takes place at low energies and remains still poorly understood.

Again TGD approach suggests a solution to these problems in terms of induced gauge field concept and a more refined view about QCD color.

1. $S = CP_2$ has color group SU(3) as isometries and electroweak gauge group as holonomies: hence $CP_2$ unifies these symmetries just like Maxwell's theory unified electric and magnetic fields. Note that the choice of $H = M^4 \times CP_2$ is not adhoc: its factors are the only 4-D spaces allowing twistor spaces with Kähler structure.

2. One can understand also the different nature of these symmetries. Color group represents exact symmetries so that symmetry breaking should not take place. Holonomies are tangent space symmetries and broken already at the level of $CP_2$ geometry and does not therefore give rise to genuine Noether symmetries. One can however assign broken electroweak gauge symmetries to the holonomies.

   The isometry group defines Kac-Moody algebra in quantum TGD and color group acts as Kac-Moody group rather than gauge group. The differences is very delicate since only the central extension of Kac-Moody algebra distinguishes it from gauge algebra.

3. Color is not spin-like quantum number as in QCD but colored states correspond to color partial waves in $CP_2$ rather. Both leptons and quarks allow colored excitations which are however expected to be very heavy.

### 5.5.3 Is Higgs mechanism only a parameterization of particle masses?

The discovery of Higgs at LHC was very important step of progress but did not prove Higgs mechanism as a mechanism of massivation as sloppy thinkers believe. Fermion masses are not a prediction of the theory: they are put in by hand by assuming that Higgs couplings are proportional to the Higgs mass. It might well be that Higgs vacuum expectation value is the unique quantum field theoretic representation of particle massivation but that QFT approach cannot predict the masses and that the understanding of the massivation requires transcending QFT so that one describing particles as extended objects. String models were the first step to this direction but one step was not enough.

In TGD framework more radical generalization is performed. Point-like particle is replaced with a 3-surface and particle massivation is described in terms of p-adic thermodynamics, which relies on very general assumptions such as a non-trivial generalization of 2-D conformal invariance to 4-D context to be discussed later, p-adic thermodynamics, p-adic length scale hypothesis, and mapping of the predictions for p-adic mass squared to real mass squared by what I call anonical identification. In this framework Higgs vacuum expectation value parametrizes the QFT limit already described and is calculable from generalized Feynman diagrammatics.

### 5.5.4 GUT approach as more sloppy thoughts

After the successes of standard model the naive guess was that theory of everything could be constructed by a simple trick: extend the gauge group to a larger group containing standard model gauge group as sub-group. One can do this and there is a refined machinery allowing to deduce particle multiplets, effective actions, beta functions, etc.. There exists of course an infinite variety of Lie groups and endless variety of GUTs have been proposed.

The view about the Universe provided by GUTs is rather weird looking.

1. Above weak mass scale there should be a huge desert of 14 orders of magnitudes containing no new physics! This is like claiming that the world ends at my backyard.

2. Only the sum of baryon and lepton numbers would be conserved and proton would be unstable. The experimental lower limit for proton lifetime has been however steadily increasing and all GUTs derived from superstring models share a fine tuning to keep proton alive.

3. Standard model gauge group seems to be all that is needed: there are no indications for larger gauge group. Fermion families seem to be copies of each other with different mass scales. Also the mass scales of these fermions differ dramatically and forcing them to multiplets of single gauge group could also be sloppy thinking. One would expect that the masses differ by simple numerical factors but they do not.

From TGD viewpoint the GUT approach is un-necessary.

1. In TGD quarks and leptons correspond to different chiralities of imbedding space spinors. 8-D chiral invariance implies that quark and lepton numbers are separately conserved so that proton does not decay - at least in the manner predicted by GUTs. $CP_2$ mass scale is of same order of magnitude as the mass scale assigned to the super heavy additional gauge bosons mediating proton decay.

2. Family replication phenomenon does not require extension of gauge group since fermion families correspond to different topologies for partonic 2-surfaces representing fundamental particles (genus-generation correspondence) [K7] ). Note that the orbits of partonic 2-surfaces correspond to light-like 3-surface at which the induced metric changes its signature from Euclidian to Minkowskian: these surfaces or equivalently the 4-surfaces with Euclidian signature can be regarded as lines of generalized Feynman diagrams. The three lowest genera are special in the sense that they always allow $Z_2$ as global conformal symmetry whereas

higher genera allow this symmetry only in case of hyper-elliptic surfaces: this leads to an explanation for the experimental absence of higher genera. Higher genera could be more naturally many particle states with continuum mass spectrum with handles taking the role of particles.

3. p-Adic length scale hypothesis emerging naturally in TGD framework allows to understand the mass ratios of fermions which are very un-natural if different fermion families are assumed to be related by gauge symmetries.

### 5.5.5 Supersymmetry in crisis

Supersymmetry is very beautiful generalization of the ordinary symmetry concept by generalizing Lie-algebra by allowing grading such that ordinary Lie algebra generators are accompanied by super-generators transforming in some representation of the Lie algebra for which Lie-algebra commutators are replaced with anti-commutators. In the case of Poincare group the super-generators would transform like spinors. Clifford algebras are actually super-algebras. Gamma matrices anti-commute to metric tensor and transform like vectors under the vielbein group (SO(n) in Euclidian signature). In supersymmetric gauge theories one introduced super translations anti-commuting to ordinary translations.

Supersymmetry algebras defined in this manner are characterized by the number of super-generators and in the simplest situation their number is one: one speaks about $\mathcal{N} = 1$ SUSY and minimal super-symmetric extension of standard model (MSSM) in this case. These models are most studied because they are the simplest ones. They have however the strange property that the spinors generating SUSY are Majorana spinors- real in well-defined sense unlike Dirac spinors. This implies that fermion number is conserved only modulo two: this has not been observed experimentally. A second problem is that the proposed mechanisms for the breaking of SUSY do not look feasible.

LHC results suggest MSSM does not become visible at LHC energies. This does not exclude more complex scenarios hiding simplest $\mathcal{N} = 1$ to higher energies but the number of real believers is decreasing. Something is definitely wrong and one must be ready to consider more complex options or totally new view abot SUSY.

What is the situation in TGD? Here I must admit that I am still fighting to gain understanding of SUSY in TGD framework [K52]. That I can still imagine several scenarios shows that I have not yet completely understood the problem but I am working hardly to avoid falling to the sin of sloppying myself. In the following I summarize the situation as it seems just now.

1. In TGD framework $\mathcal{N} = 1$ SUSY is excluded since B and L and conserved separately and imbedding space spinors are not Majorana spinors. The possible analog of space-time SUSY should be a remnant of a much larger super-conformal symmetry in which the Clifford algebra generated by fermionic oscillator operators giving also rise to the Clifford algebra generated by the gamma matrices of the "world of classical worlds" ( WCW ) and assignable with string world sheets. This algebra is indeed part of infinite-D super-conformal algebra behind quantum TGD. One can construct explicitly the conserved super conformal charges accompanying ordinary charges and one obtains something analogous to $\mathcal{N} = \infty$ super algebra. This SUSY is however badly broken by electroweak interactions.

2. The localization of induced spinors to string world sheets emerges from the condition that electromagnetic charge is well-defined for the modes of induced spinor fields. There is however an exception: covariantly constant right handed neutrino spinor $\nu_R$: it can be de-localized along entire space-time surface. Right-handed neutrino has no couplings to electroweak fields. It couples however to left handed neutrino by induced gamma matrices except when it is covariantly constant. Note that standard model does not predict $\nu_R$ but its existence is necessary if neutrinos develop Dirac mass. $\nu_R$ is indeed something which must be considered carefully in any generalization of standard model.

Could covariantly constant right-handed spinors generate exact $\mathcal{N} = 2$ SUSY? There are two spin directions for them meaning the analog $\mathcal{N} = 2$ Poincare SUSY. Could these spin directions correspond to right-handed neutrino and antineutrino. This SUSY would not look like Poincare

SUSY for which anti-commutator of super generators would be proportional to four-momentum. The problem is that four-momentum vanishes for covariantly constant spinors! Does this mean that the sparticles generated by covariantly constant $\nu_R$ are zero norm states and represent super gauge degrees of freedom? This might well be the case although I have considered also alternative scenarios.

Both imbedding space spinor harmonics and the Kähler-Dirac equation have also right-handed neutrino spinor modes not constant in $M^4$. If these are responsible for SUSY then SUSY is broken.

1. Consider first the situation at space-time level. Both induced gamma matrices and their generalizations to Kähler-Dirac gamma matrices defined as contractions of imbedding space gamma matrices with the canonical momentum currents for Kähler action are superpositions of $M^4$ and $CP_2$ parts. This gives rise to the mixing of right-handed and left-handed neutrinos. Note that non-covariantly constant right-handed neutrinos must be localized at string world sheets.

   This in turn leads neutrino massivation and SUSY breaking. Given particle would be accompanied by sparticles containing varying number of right-handed neutrinos and antineutrinos localized at partonic 2-surfaces.

2. One an consider also the SUSY breaking at imbedding space level. The ground states of the representations of extended conformal algebras are constructed in terms of spinor harmonics of the imbedding space and form the addition of right-handed neutrino with non-vanishing four-momentum would make sense. But the non-vanishing four-momentum means that the members of the super-multiplet cannot have same masses. This is one manner to state what SUSY breaking is.

3. The simplest form of massivation would be that all members of the super-multiplet obey the same mass formula but that the p-adic length scales associated with them are different. This could allow very heavy sparticles. What fixes the p-adic mass scales of sparticles? If this scale is $CP_2$ mass scale SUSY would be experimentally unreachable.

4. One can even consider the possibility that SUSY breaking makes sparticles unstable against phase transition to their dark variants with $h_{eff} = n \times h$. Sparticles could have same mass but be non-observable as dark matter not appearing in same vertices as ordinary matter! Geometrically the addition of right-handed neutrino to the state would induce many-sheeted covering in this case with right handed neutrino perhaps associated with different space-time sheet of the covering.

   This idea need not be so outlandish at it looks first. The generation of many-sheeted covering has interpretation in terms of breaking of conformal invariance. The sub-algebra for which conformal weights are $n$-tuples of integers becomes the algebra of conformal transformations and the remaining conformal generators do note represent gauge degrees of freedom anymore. They could however represent conserved conformal charges still.

   This generalization of conformal symmetry breaking gives rise to infinite number of fractal hierarchies formed by sub-algebras of conformal algebra and is also something new and a fruit of an attempt to avoid sloppy thinking. The breaking of conformal symmetry is indeed expected in massivation related to the SUSY breaking.

### 5.5.6 Have we been thinking sloppily also about super-conformal symmetries?

Super string models were once seen as the only possible candidate for the TOE. By looking at the proceedings of string theory conferences one sees that the age of of super strings is over. Landscape problem and multiverse do not give much hopes about predictive theory and the only defence for super string models is as the only game in the town. Super string gurus do not know about competing scenarion but this is not a wonder given the fact that publishing of competing scenarios has been impossible since superstrings have indeed been the only game in the town! One of the very few almost-predictions of superstring theory was $\mathcal{N} = 1$ SUSY at LHC and it seems that it is already now excluded at LHC energies.

AdS/CFT correspondence (`http://en.wikipedia.org/wiki/AdS/CFT_correspondence` ) is a deep mathematical discovery inspired by super-string models. One of its variants states that there is duality between conformal theory in $M^4$ appearing as boundary of 5-D AdS and string theory in 10-D space $AdS_5 \times S^5$. A more general duality would be between conformal theory in $M^n$ and 10-D space $AdS_{n+1} \times S^{10-n-1}$. For $n = 2$ the CFT would give conformal theory at 2-D Minkowski space for which conformal symmetries (actually their hypercomplex variant) form an infinite-D group. Duality has interpretation in terms of holography but the notion of holography is much more general than AdS/CFT.

AdS/CFT have been applied to nuclear physics but nothing sensational have been discovered. AdS/CFT have been tried also to explain the finding that what was expected to be QCD plasma behaves very differently. The first findings came from RHIC for heavy ion collisions and LHC has found that the strange effects appear already for proton heavy ion collisions. Essentially a deviation from QCD predictions is in question and in the regime where QCD should be a good description. AdS/CFT has not been a success (`http://backreaction.blogspot.com/2011/10/adscft-confronts-data.html` ). AdS/CFT is now applied also to condensed matter physics. At least hitherto no dramatic successes have been reported.

This leads to ask whether sloppy thinking should be blamed again. AdS/CFT is mathematically rather sound and well-tested but is the notion of conformal invariance behind it really the one that applies to real world physics?

1. In TGD framework the ordinary conformal invariance is generalized so that it becomes 4-D one [K8, K50]: of course, the ordinary finite-dimensional conformal group in $M^4$ is not in question. The basic observation is that light-like 3-surfaces are metrically 2-dimensional and that this leads to a generalization of conformal transformations. One can locally express light-like 3-surfaces as $X^2 \times R$ and what happens is that the conformal transformations of $X^2$ are localized with respect to the light-like coordinate of $R$. Light-like orbits of partonic 2-surfaces carrying elementary particle quantum numbers would have this extended conformal invariance.

2. This is not all. In zero energy ontology (ZEO) the diamond like intersections of future and past directed light-cones - causal diamonds (CDs) are the basic objects. The space-time surfaces having 3-D ends at the boundaries of CD are the basic dynamical units. The boundaries of CD are pieces of $\delta M^4_\pm \times CP_2$. The boundary $\delta M^4_\pm = S^2 \times R_+$ is light-like 3-surface and thus allows a huge extension of conformal symmetries: with complex coordinate of $S^2$ and light-like radial coordinate playing the roles of complex coordinate for ordinary conformal symmetry.

   One can assign superconformal symmetry also the modes of the Kähler-Dirac operator localized at the string world sheets as the analog of super-conformal symmetry of superstring models.

   Besides this there is a further analog of conformal symmetry. The symplectic transformations of $\delta M^4_\pm \times CP_2$ can be regarded as symplectic transformations of $S^2 \times CP_2$ localized with respect to the light-like coordinate of $R_+$ defining the analog of the complex coordinate $z$. In TGD Universe a gigantic extension of the conformal symmetry of superstring models experiences applies.

3. Even these extended symmetries extend to a multi-local (loci correspond to partonic 2-surfaces at boundaries of CD) Yangian variant [K43]. Yangian symmetry is very closely related to quantum groups studied for decades but again without serious consideration of the question "Why quantum groups?". The hazy belief has been that they somehow emerge at Planck length scale, which itself is a hazy notion based solely on dimensional analysis and involving Planck constant and Newton's constant characterizing macroscopic gravitation.

   In TGD framework hyper-finite factors of type $II_1$ [K49] emerge naturally at the level of WCW since fermionic Fock space provides a canonical representation for them and their inclusions provide an elegant description for finite measurement resolution: the included algebra generates states which are not experimentally distinguishable from the original state.

4. Against this it is astonishing that AdS/CFT duality has very simple generalization in TGD framework and emerge from a generalization of General Coordinate Invariance (GCI) [K8] implying holography. Strong form of GCI postulates that either the space-like 3-surfaces at

the ends of causal diamonds or the light-like orbits of partonic 2-surfaces can be taken as 3-surfaces defining the WCW : this is just gauge fixing for general coordinate invariance. If this is true then partonic 2-surfaces and their 4-D tangent space data at the boundaries of CD must code for physics. One would have strong form of holography. This might be too much to require: string world sheets carrying induced spinor fields are present and it might be that they cannot be reduced to data at partonic 2-surfaces.

In any case, for this duality the 10-D space of AdS/CFT duality would be replaced with space-time surface. $M^n$ would be replaced with the light-like parton orbits and/or space-like ends of CD. Surprisingly, this holography would be very much like holography in its original form!

### 5.6 Los Alamos, M-Theory, And TGD

String models have been seen not only as a kind of holy grail of modern physics but also as an ideology promising an Utopia. As a rule, ideologies have tried to establish the new world order using censorship. String model hegemony has followed the tradition.

For about decade ago it became impossible for me to get anything to hep-th and other physics related archives. Interestingly, for few years ago my article about Riemann hypothesis was accepted to the math archives of Los Alamos and is also published [L1]: it was however not possible to get it cross-listed to hep-th. For a few years American Mathematical Society has had a link to my homepage [A1] as one of the few examples about new mathematics related to quantum physics.

I have learned that I am not the only victim of the string revolution (see the comments in "Not Even Wrong" discussion group [B3] ). Despite the official statement that anyone can contribute to LANL, an invisible peer system is acting. After 20 years of string revolutions it seems that physics itself has become the victim which has suffered the most severe injuries.

## 6 K-Theory, Branes, And TGD

K-theory has played important role in brane classification in super string models and M-theory. The excellent lectures by Harah Evslin with title *What doesn't K-theory classify?* [B15] make it possible to learn the basic motivations for the classification, what kind of classifications are possible, and what are the failures. Also the Wikipedia article [B1] gives a bird's eye of view about problems. As a by-product one learns something about the basic ideas of K-theory - at least I hope so - and about possible mathematical and physical problems of string theories and M-theory.

In the sequel I will discuss critically the basic assumptions of brane world scenario, sum up my meager understanding about the problems related to the topological classification of branes and also to the notion itself, ask what could go wrong with branes and demonstrate how the problems could be avoided in TGD framework, and just to irritate colleauges conclude with a proposal for a natural generalization of K-theory to include also the division of bundles inspired by the generalization of Feynman diagrammatics in quantum TGD, by zero energy ontology, and by the notion of finite measurement resolution.

### 6.1 Brane World Scenario

The brane world scenario looks attractive from the mathematical point of view one is able to get accustomed with the idea that basic geometric objects have varying dimensions. Even accepting the varying dimensions, the basic physical assumptions behind this scenario are vulnerable to criticism.

1. Branes are geometric objects of varying dimension in the 10-/11-dimensional space-time -call it $M$- of superstring theory/M-theory. In M-theory the fundamental strings are replaced with M-branes, which are 2-D membranes with 3-dimensional orbit having as its magnetic dual 6-D M5-brane. Branes are thought to emerge non-perturbatively from fundamental 2-branes but what this really means is not understood. One has D-p-branes with Dirichlet boundary conditions fixing a $p+1$-dimensional surface of $M$ as brane orbit: one of the dimensions corresponds to time. Also S-branes localized in time have been proposed.

2. In the description of the classical limit branes interact with the classical fields of the target space by the generalization of the minimal coupling of charged point-like particle to electromagnetic gauge potential. The coupling is simply the integral of the gauge potential over the world-line - the value of 1-form for the word-line. Point like particle represents 0-brane and in the case of p-brane the generalization is obtained by replacing the gauge potential represented by a 1-from with $p+1$-form. The exterior derivative of this $p+1$-form is $p+2$-form representing the analog of electromagnetic field. Complete dimensional democracy strongly suggests that string world sheets should be regarded as 1-branes.

3. From TGD point of view the introduction of branes looks a rather ad hoc trick. By generalizing the coupling of electromagnetic gauge potential to the word line of point like particle one could introduce extended objects of various dimensions also in the ordinary 4-D Maxwell theory but they would be always interpreted as idealizations for the carriers of 4- currents. Therefore the crucial step leading to branes involves classical idealization in conflict with Uncertainty Principle and the genuine quantal description in terms of fields coupled to gauge potentials.

   My view is that the most natural interpretation for what is behind branes is in terms of currents in D=10 or D= 11 space-time. In this scheme branes have role only as semi-classical idealizations making sense only above some scale. Both the reduction of string theories to quantum field theories by holography and the dynamical character of the metric of the target space conforms with super-gravity interpretation. Internal consistency requires also the identification of strings as branes so that superstring theories and M-theory would reduce to an idealization to 10-/11-dimensional quantum gravity.

In this framework the brave brane world episode would have been a very useful Odysseia. The possibility to interpret various geometric objects physically has proved to be an extremely powerful tool for building provable mathematical conjectures and has produced lots of immensely beautiful mathematics. As a fundamental theory this kind of approach does not look convincing to me.

## 6.2 The Basic Challenge: Classify The Conserved Brane Charges Associated With Branes

One can of course forget these critical arguments and look whether this general picture works. The first thing that one can do is to classify the branes topologically. I made the same question about 32 years ago in TGD framework: I thought that cobordism for 3-manifolds might give highly interesting topological conservation laws. I was disappointed. The results of Thom's classical article about manifold cobordism demonstrated that there is no hope for really interesting conservation laws. The assumption of Lorentz cobordism meaning the existence of global time-like vector field would make the situation more interesting but this condition looked too strong and I could not see a real justification for it. In generalized Feynman diagrammatics there is no need for this kind of condition.

There are many alternative approaches to the classification problem. One can use homotopy, homology, cohomology and their relative and other variants, topological or algebraic K-theory, twisted K-theory, and variants of K-theory not yet existing but to be proposed within next years. The list is probably endless unless something like motivic cohomology brings in enlightenment.

1. First of all one must decide whether one classifies p-dimensional time=constant sections of p-branes or their $p+1$-dimensional orbits. Both approaches have been applied although the first one is natural in the standard view about spontaneous compactification. For the first option topological invariants could be seen as conserved charges: homotopy invariants and homological and cohomological characteristics of branes provide this kind of invariants. For the latter option the invariants would be analogous to instanton number characterizing the change of magnetic charge.

2. Purely topological invariants come first in mind. Homotopy groups of the brane are invariants inherent to the brane (the brane topology can however change). Homological and cohomological characteristics of branes in singular homology characterize the imbedding to the target space. There are also more delicate differential topological invariants such as de

Rham cohomology defining invariants analogous to magnetic charges. Dolbeault cohomology emerges naturally for even-dimensional branes with complex structure.

3. Gauge theories - both abelian and non-Abelian - define a standard approach to the construction of brane charges for the bundle structures assigned with branes. Chern-Simons classes are fundamental invariants of this kind. Also more delicate invariants associated with gauge potentials can be considered. Chern-Simons theory with vanishing field strengths for solutions of field equations provides a basic example about this. For instance, SU(2) Chern-Simons theory provides 3-D topological invariants and knot invariants.

4. More refined approaches involve K-theory -closely related to motivic cohomology - and its twisted version. The idea is to reduce the classification of branes to the classification of the bundle structures associated with them. This approach has had remarkable successes but has also its short-comings.

The challenge is to find the mathematical classification which suits best the physical intuitions (, which might be fatally wrong as already proposed) but is universal at the same time. This challenge has turned out to be tough. The Ramond-Ramond (RR) p-form fields of type II superstring theory are rather delicate objects and a source of most of the problems. The difficulties emerge also by the presence of Neveu-Schwartz 3-form $H = dB$ defining classical background field.

K-theory has emerged as a good candidate for the classification of branes. It leaves the confines of homology and uses bundle structures associated with branes and classifies these. There are many K-theories. In topological K-theory bundles form an algebraic structure with sum, difference, and multiplication. Sum is simply the direct sum for the fibers of the bundle with common base space. Product reduces to a tensor product for the fibers. The difference of bundles represents a more abstract notion. It is obtained by replacing bundles with pairs in much the same way as rationals can be thought of as pairs of integers with equivalence $(m, n) = (km, kn)$, $k$ integer. Pairs $(n, 1)$ representing integers and pairs $(1, n)$ their inverses. In the recent case one replaces multiplication with sum and regards bundle pairs and $(E, F)$ and $(E + G, F + G)$ equivalent. Although the pair as such remains a formal notion, each pair must have also a real world representativs. Therefore the sign for the bundle must have meaning and corresponds to the sign of the charges assigned to the bundle. The charges are analogous to winding of the brane and one can call brane with negative winding antibrane. The interpretation in terms of orientation looks rather natural. Later a TGD inspired concrete interpretation for the bundle sum, difference, product and also division will be proposed.

## 6.3 Problems

The classification of brane structures has some problems and some of them could be argued to be not only technical but reflect the fact that the physical picture is wrong.

### 6.3.1 Problems related to the existence of spinor structure

Many problems in the classification of brane charges relate to the existence of spinor structure. The existence of spinor structure is a problem already in general general relativity since ordinary spinor structure exists only if the second Stiefel- Whitney class [A2] of the manifold is non-vanishing: if the third Stiefel-Whitney class vanishes one can introduce so called spin$^c$ structure. This kind of problems are encountered already in lattice QCD, where periodic boundary conditions imply non-uniqueness having interpretation in terms of 16 different spinor structures with no obvious physical interpretation. One the strengths of TGD is that the notion of induced spinor structure eliminates all problems of this kind completely. One can therefore find direct support for TGD based notion of spinor structure from the basic inconsistency of QCD lattice calculations!

1. Freed-Witten anomaly [B12] appearing in type II string theories represents one of the problems. Freed and Witten show that in the case of 2-branes for which the generalized gauge potential is 3-form so called spin$^c$ structure is needed and exists if the third Stiefel-Whitney class $w_3$ related to second Stiefel Whitney class whose vanishing guarantees the existence of ordinary spin structure (in TGD framework spin$^c$ structure for $CP_2$ is absolutely essential for obtaining standard model symmetries).

It can however happen that $w_3$ is non-vanishing. In this case it is possible to modify the spin$^c$ structure if the condition $w_3 + [H] = 0$ holds true. It can however happen that there is an obstruction for having this structure - in other words $w_3 + [H]$ does not vanish - known as Freed-Witten anomaly. In this case K-theory classification fails. Witten and Freed argue that physically the wrapping of cycle with non-vanishing $w_3 + [H]$ by a $Dp$-brane requires the presence of $D(p-2)$ brane cancelling the anomaly. If $D(p-2)$ brane ends to anti-Dp in which case charge conservation is lost. If there is not place for it to end one has semi-infinite brane with infinite mass, which is also problematic physically. Witten calls these branes baryons: these physically very dubious objects are not classified by K-theory.

2. The non-vanishing of $w_3 + [H] = 0$ forces to generalize K-theory to twisted K-theory [A3]. This means a modification of the exterior derivative to get twisted de Rham cohomology and twisted K-theory and the condition of closedness in this cohomology for certain form becomes the condition guaranteeing the existence of the modified spin$^c$ structure. D-branes act as sources of these fields and the coupling is completely analogous to that in electrodynamics. In the presence of classical Neveu-Schwartz (NS-NS) 3-form field $H$ associated with the background geometry the field strength $G^{p+1} = dC_p$ is not gauge invariant anymore. One must replace the exterior derivative with its twisted version to get twisted de Rham cohomology:

$$d \rightarrow d + H \wedge \quad .$$

   There is a coupling between p- and p+2-forms together and gauge symmetries must be modified accordingly. The fluxes of twisted field strengths are not quantized but one can return to original p-forms which are quantized. The coupling to external sources also becomes more complicated and in the case of magnetic charges one obtains magnetically charged $Dp$-branes. $Dp$-brane serves as a source for $D(p-2)$- branes.

   This kind of twisted cohomology is known by mathematicians as Deligne cohomology. At the level of homology this means that if branes with dimension of $p$ are presented then also branes with dimension $p + 2$ are there and serve as source of $Dp$-branes emanating from them or perhaps identifiable as their sub-manifolds. Ordinary homology fails in this kind of situation and the proposal is that so called twisted K-theory could allow to classify the brane charges.

3. A Lagrangian formulation of brane dynamics based on the notion of p-brane democracy [B24] due to Peter Townsend has been developed by various authors.

Ashoke Sen has proposed a grand vision for understanding the brane classification in terms of tachyon condensation in absence of NS-NS field H [B4]. The basic observation is that stacks of space-filling D- and anti D-branes are unstable against process called tachyon condensation which however means fusion of $p + 1$-D brane orbits rather than $p$-dimensional time slices of branes. These branes are however accompanied by lower-dimensional branes and the decay process cannot destroy these. Therefore the idea arises that suitable stacks of D9 branes and anti-D9-branes could code for all lower-dimensional brane configurations as the end products of the decay process.

This leads to a creation of lower-dimensional branes. All decay products of branes resulting in the decay cascade would be by definition equivalent. The basic step of the decay process is the fusion of D-branes in stack to single brane. In bundle theoretic language one can say that the D-branes and anti-D branes in the stack fuse together to single brane with bundle fiber which is direct sum of the fibers on the stack. This fusion process for the branes of stack would correspond in topological K-theory. The fusion of D-branes and anti-D branes would give rise to nothing since the fibers would have opposite sign. The classification would reduce to that for stacks of D9-branes and anti D9-branes.

### 6.3.2 Problems with Hodge duality and S-duality

The K-theory classification is plagued by problems all of which need not be only technical.

1. R-R fields are self dual and since metric is involved with the mapping taking forms to their duals one encounters a problem. Chern characters appearing in K-theory are rational valued

but the presence of metric implies that the Chern characters for the duals need not be rational valued. Hence K-theory must be replaced with something less demanding.

The geometric quantization inspired proposal of Diaconescu, Moore and Witten [B6] is based on the polarization using only one half of the forms to get rid of the proboem. This is like thinking the 10-D space-time as phase space and reducing it effectively to 5-D space: this brings strongly in mind the identification of space-time surfaces as hyper-quaternionic (associative) sub-manifolds of imbedding space with octonionic structure and one can ask whether the basic objects also in M-theory should be taken 5-dimensional if this line of thought is taken seriously. An alternative approach uses K-theory to classify the intersections of branes with 9-D space-time slice as has been porposed by Maldacena, Moore and Seiberg [B21].

2. There another problem related to classification of the brane charges. Witten, Moore and Diaconescu [B6] have shown that there are also homology cycles which are unstable against decay and this means that twisted K-theory is inconsistent with the S-duality of type IIB string theory. Also these cycles should be eliminated in an improved classification if one takes charge conservation as the basic condition and an hitherto un-known modification of cohomology theory is needed.

3. There is also the problem that K-theory for time slices classifies only the R-R field strengths. Also R-R gauge potentials carry information just as ordinary gauge potentials and this information is crucial in Chern-Simons type topological QFTs. K-theory for entire target space classifies D-branes as $p+1$-dimensional objects but in this case the classification of R-R field strengths is lost.

### 6.3.3 The existence of non-representable 7-D homology classes for targent space dimension $D > 9$

There is a further nasty problem which destroys the hopes that twisted K-theory could provide a satisfactory classification. Even worse, something might be wrong with the superstring theory itself. The problem is that not all homology classes allow a representation as non-singular manifolds. The first dimension in which this happens is $D = 10$, the dimension of super-string models! Situation is of course the same in M-theory. The existence of the non-representables was demonstrated by Thom - the creator of catastrophe theory and of cobordism theory for manifolds- for a long time ago.

What happens is that there can exist 7-D cycles which allow only singular imbeddings. A good example would be the imbedding of twistor space $CP_3$, whose orbit would have conical singularity for which $CP_3$ would contract to a point at the "moment of big bang". Therefore homological classification not only allows but demands branes which are orbifolds. Should orbifolds be excluded as unphysical? If so then homology gives too many branes and the singular branes must be excluded by replacing the homology with something else. Could twisted K-theory exclude non-representable branes as unstable ones by having non-vanishing $w_3 + [H]$? The answer to the question is negative: D6-branes with $w_3 + [H] = 0$ exist for which K-theory charges can be both vanishing or non-vanishing.

One can argue that non-representability is not a problem in superstring models (M-theory) since spontaneous compactification leads to $M \times X_6$ ($M \times X_7$). On the other hand, Cartesian product topology is an approximation which is expected to fail in high enough length scale resolution and near big bang so that one could encounter the problem. Most importantly, if M-theory is theory of everything it cannot contain this kind of beauty spots.

## 6.4 What Could Go Wrong With Super String Theory And How TGD Circumvents The Problems?

As a proponent of TGD I cannot avoid the temptation to suggest that at least two things could go wrong in the fundamental physical assumptions of superstrings and M-theory.

1. The basic failure would be the construction of quantum theory starting from semiclassical approximation assuming localization of currents of 10 - or 11-dimensional theory to lower-

dimensional sub-manifolds. What should have been a generalization of QFT by replacing point-like particles with higher-dimensional objects would reduce to an approximation of 10- or 11-dimensional supergravity.

This argument does not bite in TGD. 4-D space-time surfaces are indeed fundamental objects in TGD as also partonic 2-surfaces and braids. This role emerges purely number theoretically inspiring the conjecture that space-time surfaces are associative sub-manifolds of octonionic imbedding spaces, from the requirement of extended conformal invariance, and from the non-dynamical character of the imbedding space.

2. The condition that all homology equivalence classes are representable as manifolds excludes all dimensions $D > 9$ and thus super-strings and M-theory as a physical theory. This would be the case since branes are unavoidable in M-theory as is also the landscape of compactifications. In semiclassical supergravity interpretation this would not be catastrophe but if branes are fundamental objects this shortcoming is serious. If the condition of homological representability is accepted then target space must have dimension $D < 10$ and the arguments sequence leading to D=8 and TGD is rather short. The number theoretical vision provides the mathematical justification for TGD as the unique outcome.

3. The existence of spin structure is clearly the source of many problems related to R-R form. In TGD framework the induction of spin$^c$ structure of the imbedding space resolves all problems associated with sub-manifold spin structures. For some reason the notion of induced spinor structure has not gained attention in super string approach.

4. Conservative experimental physicist might criticize the emergence of branes of various dimensions as something rather weird. In TGD framework electric-magnetic duality can be understood in terms of general coordinate invariance and holography and branes and their duals have dimension 2, 3, and 4 organize to sub-manifolds of space-time sheets.

   The TGD counterpart for the fundamental D-2-brane is light-like 3-surface. Its magnetic dual has dimension given by the general formula $p_{dual} = D - p - 4$, where $D$ is the dimension of the target space [B10]. In TGD one has $D = 8$ giving $p_{dual} = 2$. The first interpretation is in terms of self-duality. A more plausible interpretation relies on the identification of the duals of light-like 3-surfaces as space-like 3-surfaces at the light-like boundaries of CD. General Coordinate Invariance in strong sense implies this duality. For partonic 2-surface and string world sheets carrying spinor modes one would have $p = 1$ and $p_{dual} = 3$. The identification of the dual would be as 4-D space-time surface: does this correspond to strong form of holography?. The crucial distinction to M-theory would be that branes of different dimension would be sub-manifolds of space-time surface.

5. For $p = 0$ one would have $p_{dual} = 4$ assigning five-dimensional surface to orbits of point-like particles identifiable most naturally as braid strands. One cannot assign to it any direct physical meaning in TGD framework and gauge invariance for the analogs of brane gauge potentials indeed excludes even-dimensional branes in TGD since corresponding forms are proportional to Kähler gauge potential (so that they would be analogous to odd-dimensional branes allowed by type $II_B$ superstrings).

   4-branes might be however mathematically useful by allowing to define Morse theory for the critical points of the Minkowskian part of Kähler action. While writing this I learned that Witten has proposed a 4-D gauge theory approach with $\mathcal{N} = 4$ SUSY to the classification of knots. Witten also ends up with a Morse theory using 5-D space-times in the category-theoretical formulation of the theory [A5]. For some time ago I also proposed that TGD as almost topological QFT defines a theory of knots, knot braidings, and of 2-knots in terms of string world sheets [K14]. Maybe the 4-branes could be useful for understanding of the extrema of TGD of the Minkowskian part of Kähler action which would take take the same role as Hamiltonian in Floer homology: the extrema of 5-D brane action would connect these extrema.

6. Light-like 3-surfaces could be seen as the analogs von Neuman branes for which the boundary conditions state that the ends of space-like 3-brane defined by the partonic 2-surfaces move with light-velocity. The interpretation of partonic 2-surfaces as space-like branes at the

ends of CD would in turn make them D-branes so that one would have a duality between D-branes and N-brane interpretations. T-duality exchanges von Neumann and Dirichlet boundary conditions so that strong from of general coordinate invariance would correspond to both electric-magnetic and T-duality in TGD framework. Note that T-duality exchanges type $II_A$ and type $II_B$ super-strings with each other.

7. What about causal diamonds and their 7-D light-like boundaries? Could one regard the light-like boundaries of CDs as analogs of 6-branes with light-like direction defining time-like direction so that space-time surfaces would be seen as 3-branes connecting them? This brane would not have magnetic dual since the formula for the dimensions of brane and its magnetic dual allows positive brane dimension $p$ only in the range (1, 3).

## 6.5 Can One Identify The Counterparts Of R-R And NS-NS Fields In TGD?

R-R and NS-NS 3-forms are clearly in fundamental role in M-theory. Since in TGD partonic 2-surfaces define the analogs of fundamental D-2-branes, one can wonder whether these 3-forms could have TGD counterparts.

1. In TGD framework the 3-forms $G_{3,A} = dC_{2,A}$ defined as the exterior derivatives of the two-forms $C_{2,A}$ identified as products $C_{2,A} = H_A J$ of Hamiltonians $H_A$ of $\delta M^4_\pm \times CP_2$ with Kähler forms of factors of $\delta M^4_\pm \times CP_2$ define an infinite family of closed 3-forms belonging to various irreducible representations of rotation group and color group. One can consider also the algebra generated by products $H_A A$, $H_A J$, $H_A A \wedge J$, $H_A J \wedge J$, where $A$ *resp.* $J$ denotes the Kähler gauge potential *resp.* Kähler form or either $\delta M^4_\pm$ or $CP_2$. $A$ *resp.* Also the sum of Kähler potentials *resp.* forms of $\delta M^4_\pm$ and $CP_2$ can be considered.

2. One can define the counterparts of the fluxes $\int Adx$ as fluxes of $H_A A$ over braid strands, $H_A J$ over partonic 2-surfaces and string world sheets, $H_A A \wedge J$ over 3-surfaces, and $H_A J \wedge J$ over space-time sheets.Gauge invariance however suggests that for non-constant Hamiltonians one must exclude the fluxes assigned to odd dimensional surfaces so that only odd-dimensional branes would be allowed. This would exclude 0-branes and the problematic 4-branes. These fluxes should be quantized for the critical values of the Minkowskian contributions and for the maxima with respect to zero modes for the Euclidian contributions to Kähler action. The interpretation would be in terms of Morse function and Kähler function if the proposed conjecture holds true. One could even hope that the charges in Cartan algebra are quantized for all preferred extremals and define charges in these irreducible representations for the isometry algebra of WCW . The quantization of electric fluxes for string world sheets would give rise to the familiar quantization of the rotation $\int E \cdot dl$ of electric field over a loop in time direction taking place in superconductivity.

3. Should one interpret these fluxes as the analogs of NS-NS-fluxes or R-R fluxes? The exterior derivatives of the forms $G_3$ vanish which is the analog for the vanishing of magnetic charge densities (it is however possible to have the analogs of homological magnetic charge). The self-duality of Ramond p-forms could be posed formally ($G_p =^* G_{8-p}$) but does not have any implications for $p < 4$ since the space-time projections vanish in this case identically for $p > 3$. For $p = 4$ the dual of the instanton density $J \wedge J$ is proportional to volume form if $M^4$ and is not of topological interest. The approach of Witten eliminating one half of self dual R-R-fluxes would mean that only the above discussed series of fluxes need to be considered so that one would have no troubles with non-rational values of the fluxes nor with the lack of higher dimensional objects assignable to them. An interesting question is whether the fluxes could define some kind of K-theory invariants.

4. In TGD imbedding space is non-dynamical and there seems to be no counterpart for the NS 3-form field $H = dB$. The only natural candidate would correspond to Hamiltonian $B = J$ giving $H = dB = 0$. At quantum level this might be understood in terms of bosonic emergence [K28] meaning that only Ramond representations for fermions are needed in the theory since bosons correspond to wormhole contacts with fermion and anti-fermions

at opposite throats. Therefore twisted cohomology is not needed and there is no need to introduce the analogy of brane democracy and 4-D space-time surfaces containing the analogs of lower-dimensional brains as sub-manifolds are enough. The fluxes of these forms over partonic 2-surfaces and string world sheets defined non-abelian analogs of ordinary gauge fluxes reducing to rotations of vector potentials and suggested be crucial for understanding braidings of knots and 2-knots in TGD framework. [K14]. Note also that the unique dimension D=4 for space-time makes 4-D space-time surfaces homologically self-dual so that only they are needed.

## 6.6 What About Counterparts Of $S$ And $U$ Dualities In TGD Framework?

The natural question is what could be the TGD counterparts of $S-$, $T-$ and $U$-dualities. If one accepts the identification of $U$-duality as product $U = ST$ and the proposed counterpart of $T$ duality as a strong form of general coordinate invariance, it remains to understand the TGD counterpart of $S$-duality - in other words electric-magnetic duality - relating the theories with gauge couplings $g$ and $1/g$.

Quantum criticality selects the preferred value of $g_K$: Kähler coupling strength is very near to fine structure constant at electron length scale and can be equal to it. Note that the hierarchy of Planck constants (dark matters) could be understood in terms of a spectrumfor $\alpha_K = g_K^2/4\pi h_{eff}$, $h_{eff} = n \times h$: in thermodynamical analogy one would have accumulation of critical points at zero temperature.

If there is no coupling constant evolution associated with $\alpha_K$, it does not make sense to say that $g_K$ becomes strong and is replaced with its inverse at some point. One should be able to formulate the counterpart of $S$-duality as an identity following from the weak form of electric-magnetic duality and the reduction of TGD to almost topological QFT. This might be the case.

1. For preferred extremals the interior parts of Kähler action reduces to a boundary term if the term $j^\mu A_\mu$ from them vanishes. The weak form of electric-magnetic duality requires that Kähler electric charge is proportional to Kähler magnetic charge, which implies reduction to abelian Chern-Simons term: the Kähler coupling strength does not appear at all in Chern-Simons term. The proportionality constant beween the electric and magnetic parts $J_E$ and $J_B$ of Kähler form however enters into the dynamics through the boundary conditions stating the weak form of electric-magnetic duality. At the Minkowskian side the proportionality constant must be proportional to $g_K^2$ to guarantee a correct value for the unit of Kähler electric charge - equal to that for electric charge in electron length scale- from the assumption that electric charge is proportional to the topologically quantized magnetic charge. It has been assumed that

$$J_E = \alpha_K J_B$$

holds true at *both sides* of the wormhole throat but this is an un-necessarily strong assumption at the Euclidian side. In fact, the self-duality of $CP_2$ Kähler form stating

$$J_E = J_B$$

favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for $CP_2$ type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of $CP_2$ radius and $\alpha_K$ the effective replacement $g_K^2 \to 1$ would spoil the argument.

2. Minkowskian and Euclidian regions should correspond to a strongly/weakly interacting phase in which Kähler magnetic/electric charges provide the proper description. In Euclidian regions associated with $CP_2$ type extremals there is a natural interpretation of interactions

between magnetic monopoles associated with the light-like throats: for $CP_2$ type vacuum extremal itself magnetic and electric charges are actually identical and cannot be distinguished from each other. Therefore the duality between strong and weak coupling phases seems to be trivially true in Euclidian regions if one has $J_B = J_E$ at Euclidian side of the wormhole throat. This is however an un-necessarily strong condition as the following argument shows.

3. In Minkowskian regions the interaction is via Kähler electric charges and elementary particles have vanishing total Kähler magnetic charge consisting of pairs of Kähler magnetic monopoles so that one has confinement characteristic for strongly interacting phase. Therefore Minkowskian regions naturally correspond to a weakly interacting phase for Kähler electric charges. One can write the action density at the Minkowskian side of the wormhole throat as

$$\frac{(J_E^2 - J_B^2)}{\alpha_K} = \alpha_K J_B^2 - \frac{J_B^2}{\alpha_K} \quad .$$

The exchange $J_E \leftrightarrow J_B$ accompanied by $\alpha_K \to -1/\alpha_K$ leaves the action density invariant. Since only the behavior of the vacuum functional infinitesimally near to the wormhole throat matters by almost topological QFT property, the duality is realized. Note that the argument goes through also in Euclidian regions so that it does not allow to decide which is the correct form of weak form of electric-magnetic duality.

4. $S$-duality could correspond geometrically to the duality between partonic 2-surfaces responsible for magnetic fluxes and string worlds sheets responsible for electric fluxes as rotations of Kähler gauge potentials around them and would be very closely related with the counterpart of $T$-duality implied by the strong form of general coordinate invariance and saying that space-like 3-surfaces at the ends of space-time sheets are equivalent with light-like 3-surfaces connecting them.

The boundary condition $J_E = J_B$ at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded $CP_2$ is such that in $CP_2$ coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for $CP_2$ type vacuum extremals since by the light-likeness of $M^4$ projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole throat. Self-duality is indeed an un-necessarily strong condition.

### 6.6.1 Comparison with standard view about dualities

One can compare the proposed realization of $T$, $S$ and $U$ to the more general dualities defined by the modular group $SL(2, Z)$, which in QFT framework can hold true for the path integral over all possible gauge field configurations. In the resent case the dualities hold true for every preferred extremal separately and the functional integral is only over the space-time projections of fixed Kähler form of $CP_2$. Modular invariance for Maxwell action was discussed by E. Verlinde for Maxwell action with $\theta$ term for a general 4-D compact manifold with Euclidian signature of metric in [B9]. In this case one has path integral giving sum over infinite number of extrema characterized by the cohomological equivalence class of the Maxwell field the action exponential to a high degree. Modular invariance is broken for $CP_2$: one obtains invariance only for $\tau \to \tau + 2$ whereas $S$ induces a phase factor to the path integral.

1. In the recent case these homology equivalence classes would correspond to homology equivalence classes of holomorphic partonic 2-surfaces associated with the critical points of Kähler function with respect to zero modes.

2. In the case that the Euclidian contribution to the Kähler action is expressible solely in terms of wormhole throat Chern-Simons terms, and one can neglect the measurement interaction terms fixing the values of some classical conserved quantities to be equal with their quantal

counterparts for the space-time surfaces allowed in quantum superposition, the exponent of Kähler action can be expressed in terms of Chern-Simons action density as

$$\begin{aligned} L &= \tau L_{C-S} \ , \\ L_{C-S} &= J \wedge A \ , \\ \tau &= \frac{1}{g_K^2} + i\frac{k}{4\pi} \ , \quad k = 1 \ . \end{aligned} \tag{6.1}$$

Here the parameter $\tau$ transforms under full $SL(2,Z)$ group as

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \ . \tag{6.2}$$

The generators of $SL(2,Z)$ transformations are $T : \tau \rightarrow \tau + 1$, $S : \tau \rightarrow -1/\tau$. The imaginary part in the exponents corresponds to Kac-Moody central extension $k = 1$.

This form corresponds also to the general form of Maxwell action with CP breaking $\theta$ term given by

$$L = \frac{1}{g_K^2} J \wedge^* J + i\frac{\theta}{8\pi^2} J \wedge J \ , \quad \theta = 2\pi \ . \tag{6.3}$$

Hence the Minkowskian part mimics the $\theta$ term but with a value of $\theta$ for which the term does not give rise to CP breaking in the case that the action is full action for $CP_2$ type vacuum extremal so that the phase equals to $2\pi$ and phase factor case is trivial. It would seem that the deviation from the full action for $CP_2$ due to the presence of wormhole throats reducing the value of the full Kähler action for $CP_2$ type vacuum extremal could give rise to CP breaking. One can visualize the excluded volume as homologically non-trivial geodesic spheres with some thickness in two transverse dimensions. At the limit of infinitely thin geodesic spheres CP breaking would vanish. The effect is exponentially sensitive to the volume deficit.

### 6.6.2 CP breaking and ground state degeneracy

Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define $2 \times 2$ matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full $CP_2$ type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. $K^0$ mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of $CP_2$ type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for $B^0$ mesons.

3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and short-lived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral mesons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only $K^0$ but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

*Remark:* The proportionality of Minkowskian and Euclidian contributions to the same Chern-Simons term implies that the critical points with respect to zero modes appear for both the phase and modulus of vacuum functional. The Kähler function property does not allow extrema for vacuum functional as a function of complex coordinates of WCW since this would mean Kähler metric with non-Euclidian signature. If this were not the case. the stationary values of phase factor and extrema of modulus of the vacuum functional would correspond to different configurations.

## 6.7 Could One Divide Bundles?

TGD differs from string models in one important aspects: stringy diagrams do not have interpretation as analogs of vertices of Feynman diagrams: the stringy decay of partonic 2-surface to two pieces does not represent particle decay but a propagation along different paths for incoming particle. Particle reactions in turn are described by the vertices of generalized Feynman diagrams in which the ends of incoming and outgoing particles meet along partonic 2-surface. This suggests a generalization of K-theory for bundles assignable to the partonic 2-surfaces. It is good to start with a guess for the concrete geometric realization of the sum and product of bundles in TGD framework.

1. The analogs of string diagrams could represent the analog for direct sum. Difference between bundles could be defined geometrically in terms of trouser vertex $A + B \rightarrow C$. $B$ would by definition represent $C - A$. Direct sum could make sense for single particle states and have as space-time correlate the conservation of braid strands.

2. A possible concretization in TGD framework for the tensor product is in terms of the vertices of generalized Feynman diagrams at which incoming light-like 3-D orbits of partons meet along their ends. The tensor product of incoming state spaces defined by fermionic oscillator algebras is naturally formed. Tensor product would have also now as a space-time correlate conservation of braid strands. This does not mean that the number of braid strands is conserved in reactions if also particular exchanges can carry the braid strands of particles coming to the vertex.

Why not define also division of bundles in terms of the division for tensor product? In terms of the 3-vertex for generalized Feynman diagrams $A \otimes B = C$ representing tensor product $B$ would be by definition $C/A$. Therefore TGD would extend the K-theory algebra by introducing also division as a natural operation necessitated by the presence of the join along ends vertices not present in string theory. I would be surprised if some mathematician would not have published the idea in some exotic journal. Below I represent an argument that this notion could be also applied in the mathematical description of finite measurement resolution in TGD framework using inclusions of hyper-finite factor. Division could make possible a rigorous definition for for non-commutative quantum spaces.

Tensor division could have also other natural applications in TGD framework.

1. One could assign bundles $M_+$ and $M_-$ to the upper and lower light-like boundaries of CD. The bundle $M_+/M_-$ would be obtained by formally identifying the upper and lower light-like boundaries. More generally, one could assign to the boundaries of CD positive and negative energy parts of WCW spinor fields and corresponding bundle structures in “half WCW ”. Zero energy states could be seen as sections of the unit bundle just like infinite rationals reducing to real units as real numbers would represent zero energy states.

2. Finite measurement resolution would encourage tensor division since finite measurement resolution means essentially the loss of information about everything below measurement resolution represented as a tensor product factor. The notion of coset space formed by hyper-finite factor and included factor could be understood in terms of tensor division and give rise to quantum group like space with fractional quantum dimension in the case of Jones inclusions [K49]. Finite measurement resolution would therefore define infinite hierarchy of finite dimensional non-commutative spaces characterized by fractional quantum dimension. In this case the notion of tensor product would be somewhat more delicate since complex numbers are effectively replaced by the included algebra whose action creates states not distinguishable from each other [K49]. The action of algebra elements to the state $|B\rangle$ in the inner product $\langle A|B\rangle$ must be equivalent with the action of its hermitian conjugate to the state $\langle A|$. Note that zero energy states are in question so that the included algebra generates always modifications of states which keep it as a zero energy state.

# REFERENCES

## Mathematics


[A1] Available at: http://www.ams.org/mathweb/mi-mathbyclass.html.

[A2] Stiefel-Whitney class. Available at: http://en.wikipedia.org/wiki/StiefelWhitney_class.

[A3] Twisted K-theory. Available at: http://en.wikipedia.org/wiki/Twisted_K-theory.

[A4] Freed DS. The Geometry of Loop Groups, 1985.

[A5] Witten E Gaiotto D. Knot Invariants from Four-Dimensional Gauge Theory. Available at: http://arxiv.org/abs/1106.4789v1, 2011.

[A6] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc.* Available at: http://tinyurl.com/pb8zpqo, 8(43):133–151, 1981.

[A7] Baez JC. The Octonions. *Bull Amer Math Soc.* Available at: http://math.ucr.edu/home/baez/Octonions/octonions.html, 39(2002), 2001.

[A8] Vandoren S Wit de B, Rocek M. Hypermultiplets, Hyperkähler Cones and Quaternion-Kähler Geometry. Available at: http://arxiv.org/pdf/hep-th/0101161.pdf, 2001.


## Theoretical Physics


[B1] K-theory (physics). Available at: http://en.wikipedia.org/wiki/K-theory_(physics).

[B2] Monster group. Available at: http://en.wikipedia.org/wiki/Monster_group.

[B3] Not Even Wrong. Available at: http://www.math.columbia.edu/~woit/blog/.

[B4] Sen A. Tachyon Condensation on the Brane Antibrane system). Available at: http://arxiv.org/abs/hep-th/9805170, 1998.

[B5] Morrison DR Aspinwall PS, Greene BR. Calabi-Yau Moduli Space, Mirror Manifolds, and Space-time Topology Change in String Theory. Available at: http://arxiv.org/abs/hep-th/9309097, 1993.

[B6] Witten E Diaconescu D-E, Moore G. E8 Gauge Theory, and a Derivation of K-Theory from M-Theory). Available at: http://arXiv.org/abs/hep-th/0005090v3, 2005.

[B7] Harvey J Dixon L, Ginsparg P. Beauty and the Beast; Superconformal Symmetry in a Monster Module. Available at: http://ccdb4fs.kek.jp/cgi-bin/img_index?8806247, 1988.

[B8] Minasian R Duff MJ, Liu JT. Eleven-Dimensional Origin of String/String Duality: A One-Loop Test. *Nucl Phys B*, 452:261, 1995.

[B9] Verlinde E. Global Aspects of Electric-Magnetic Duality). Available at: `http://arxiv.org/abs/hep-th/9506011v3`, 1995.

[B10] Ansoldi S et al. p-Branes ElectricMagnetic Duality and Stueckelberg/Higgs Mechanism: a PathIntegral Approach. Available at: `http://arxiv.org/abs/hep-th/0004044v2`, 2000.

[B11] Banks T et al. M-Theory as a Matrix Model: A Conjecture. *Phys Rev D*, 55:5112–5128, 1997.

[B12] Witten E Freed DS. Anomalies in string theory with D-branes. Available at: `http://arXiv.org/abs/hep-th/9907189v2`, 1999.

[B13] Schwarz JH Green MB. *Nucl Phys B*, 117(1985):54, 1984.

[B14] Witten E Green MB, Schartz JH. Cambridge University Press, Cambridge, 1987.

[B15] Evslin H. What doesn't K-theory classify? Available at: `http://arxiv.org/abs/hep-th/0610328`, 2006.

[B16] Polchinski J. Dirichlet-Branes and Ramond-Ramond Charges. Available at: `http://arxiv.org/abs/hep-th/9510017`, 1995.

[B17] Maldacena JM. The Large N Limit of Superconformal Field Theories and Supergravity. Available at: `http://arxiv.org/abs/hep-th/9711200`, 1997.

[B18] Motl L. Monstrous symmetry of black holes. Available at: `http://motls.blogspot.com/2007/05/monstrous-symmetry-of-black-holes.html`, 2007.

[B19] Smolin L. Scientific alternatives to the anthropic principle. Available at: `http://arxiv.org/abs/hep-th/0407213`, 2004.

[B20] Susskind L. The Anthropic Landscape of String Theory. Available at: `http://arxiv.org/abs/hep-th/0302219`, 2003.

[B21] Seiberg N Maldacena J, Moore G. D-Brane Instantons and K-Theory Charges). Available at: `http://arxiv.org/abs/hep-th/010810`, 2001.

[B22] Ohta N. Introduction to branes and M-theory for relativists and cosmologists. Available at: `http://arxiv.org/abs/gr-qc/0205036`, 2002.

[B23] Ramond P. *Nucl Phys B*, 3(1971):31, 1971.

[B24] Townsend P. p-Brane democracy). Available at: `http://xxx.lanl.gov/abs/hep-th/9507048`, 1995.

[B25] Woit P. String Theory: Evaluation. Available at: `http://arxiv.org/abs/hep-th/0102051`, 2001.

[B26] Penrose R. Fantasy, Fashion, and Faith in Theoretical Physics. Available at: `http://www.princeton.edu/WebMedia/lectures/`, 2004.

[B27] Penrose R. Strings with a twist. *New Scientist*, 183(2458), 2004.

[B28] Schwarz JH Scherk J. *Nucl Phys B*, 81, 1974.

[B29] Vafa C Strominger A. *Nucl Phys B* . Available at: `http://arxiv.org/abs/hep-th/9601029`, 99, 1996.

## Particle and Nuclear Physics

[C1] Reich ES. Black hole like phenomenon created by collider. *New Scientist*, 19(2491), 2005.

[C2] McLerran L Ludham T. What Have We Learned From the Relativistic Heavy Ion Collider? *Phys Today* . Available at: http://www.physicstoday.org/vol-56/iss-10/p48.html, October 2003.

[C3] Chown M. Quantum Rebel. *New Scientist*, 2004.

[C4] Woit P. A new long-lived particle by CDF experiment. Available at: http://www.math.columbia.edu/~woit/wordpress/?p=1045, 2008.

## Condensed Matter Physics

[D1] Chown M. Quantum Rebel. *New Scientist*, (2457), 2004.

## Cosmology and Astro-Physics

[E1] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity. Available at: http://arxiv.org/abs/astro-ph/0310036, 2003.

[E2] Perlmutter S et al. *Ap J*, 483, 1997.

## Books related to TGD

[K1] Pitkänen M. Appendix A: Quantum Groups and Related Structures. In *Hyper-finite Factors and Dark Matter Hierarchy*. In online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#bialgebra, 2006.

[K2] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. In online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#class, 2006.

[K3] Pitkänen M. *Bio-Systems as Conscious Holograms*. Online book. Available at: http://tgdtheory.fi/public_html/hologram/hologram.html, 2006.

[K4] Pitkänen M. Bio-Systems as Conscious Holograms. In *Bio-Systems as Conscious Holograms*. In online book. Available at: http://tgdtheory.fi/public_html/hologram/hologram.html#hologram, 2006.

[K5] Pitkänen M. *Bio-Systems as Self-Organizing Quantum Systems*. Online book. Available at: http://tgdtheory.fi/public_html/bioselforg/bioselforg.html, 2006.

[K6] Pitkänen M. Category Theory, Quantum TGD, and TGD inspired theory of consciousness. In *Mathematical Aspect of Consciousness*. In online book. Available at: http://tgdtheory.fi/public_html/mathconsc/mathconsc.html#categoryc, 2006.

[K7] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics*. In online book. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html#elvafu, 2006.

[K8] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Towards M-Matrix*. In online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#quthe, 2006.

[K9] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry*. In online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#compl1, 2006.

[K10] Pitkänen M. Dark Forces and Living Matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. In online book. Available at: `http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#darkforces`, 2006.

[K11] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. In online book. Available at: `http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#Planck`, 2006.

[K12] Pitkänen M. Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory. In *TGD as a Generalized Number Theory*. In online book. Available at: `http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#mblocks`, 2006.

[K13] Pitkänen M. *Genes and Memes*. Online book. Available at: `http://tgdtheory.fi/public_html/genememe/genememe.html`, 2006.

[K14] Pitkänen M. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry*. In online book. Available at: `http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#knotstgd`, 2006.

[K15] Pitkänen M. Langlands Program and TGD. In *TGD as a Generalized Number Theory*. In online book. Available at: `http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#Langlandia`, 2006.

[K16] Pitkänen M. Macro-Temporal Quantum Coherence and Spin Glass Degeneracy. In *Bio-Systems as Conscious Holograms*. In online book. Available at: `http://tgdtheory.fi/public_html/hologram/hologram.html#macro`, 2006.

[K17] Pitkänen M. Macroscopic Quantum Coherence and Quantum Metabolism as Different Sides of the Same Coin: Part I. In *Bio-Systems as Conscious Holograms*. In online book. Available at: `http://tgdtheory.fi/public_html/hologram/hologram.html#metab`, 2006.

[K18] Pitkänen M. *Magnetospheric Consciousness*. Online book. Available at: `http://tgdtheory.fi/public_html/magnconsc/magnconsc.html`, 2006.

[K19] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. In online book. Available at: `http://tgdtheory.fi/public_html/padphys/padphys.html#mless`, 2006.

[K20] Pitkänen M. *Mathematical Aspects of Consciousness Theory*. Online book. Available at: `http://tgdtheory.fi/public_html/mathconsc/mathconsc.html`, 2006.

[K21] Pitkänen M. *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. Online book. Available at: `http://tgdtheory.fi/public_html/paddark/paddark.html`, 2006.

[K22] Pitkänen M. p-Adic Numbers and Generalization of Number Concept. In *TGD as a Generalized Number Theory*. In online book. Available at: `http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#padmat`, 2006.

[K23] Pitkänen M. p-Adic Particle Massivation: Hadron Masses. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. In online book. Available at: `http://tgdtheory.fi/public_html/padphys/padphys.html#mass3`, 2006.

[K24] Pitkänen M. p-Adic Physics as Physics of Cognition and Intention. In *TGD Inspired Theory of Consciousness*. In online book. Available at: `http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#cognic`, 2006.

[K25] Pitkänen M. p-Adic Physics: Physical Ideas. In *TGD as a Generalized Number Theory*. In online book. Available at: `http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#phblocks`, 2006.

[K26] Pitkänen M. *Physics in Many-Sheeted Space-Time*. Online book. Available at: `http://tgdtheory.fi/public_html/tgdclass/tgdclass.html`, 2006.

[K27] Pitkänen M. Quantum Astrophysics. In *Physics in Many-Sheeted Space-Time*. In online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#qastro, 2006.

[K28] Pitkänen M. Quantum Field Theory Limit of TGD from Bosonic Emergence. In *Towards M-Matrix*. In online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#emergence, 2006.

[K29] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *Hyper-finite Factors and Dark Matter Hierarchy*. In online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.#anyontgd, 2006.

[K30] Pitkänen M. *Quantum Hardware of Living Matter*. Online book. Available at: http://tgdtheory.fi/public_html/bioware/bioware.html, 2006.

[K31] Pitkänen M. *Quantum Physics as Infinite-Dimensional Geometry*. Online book.Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html, 2006.

[K32] Pitkänen M. *Quantum TGD*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquant/tgdquant.html, 2006.

[K33] Pitkänen M. Self and Binding. In *TGD Inspired Theory of Consciousness*. In online book. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html#selfbindc, 2006.

[K34] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. In online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#astro, 2006.

[K35] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time*. In online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#cosmo, 2006.

[K36] Pitkänen M. *TGD and EEG*. Online book. Available at: http://tgdtheory.fi/public_html/tgdeeg/tgdeeg.html, 2006.

[K37] Pitkänen M. *TGD and Fringe Physics*. Online book. Available at: http://tgdtheory.fi/public_html/freenergy/freenergy.html, 2006.

[K38] Pitkänen M. *TGD as a Generalized Number Theory*. Online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html, 2006.

[K39] Pitkänen M. TGD as a Generalized Number Theory: Infinite Primes. In *TGD as a Generalized Number Theory*. In online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visionc, 2006.

[K40] Pitkänen M. TGD as a Generalized Number Theory: p-Adicization Program. In *TGD as a Generalized Number Theory*. In online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visiona, 2006.

[K41] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. In online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#visionb, 2006.

[K42] Pitkänen M. *TGD Inspired Theory of Consciousness*. Online book. Available at: http://tgdtheory.fi/public_html/tgdconsc/tgdconsc.html, 2006.

[K43] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix*. In online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistorstory, 2006.

[K44] Pitkänen M. Time, Spacetime and Consciousness. In *Bio-Systems as Conscious Holograms*. In online book. Available at: http://tgdtheory.fi/public_html/hologram/hologram.html#time, 2006.

[K45] Pitkänen M. *Topological Geometrodynamics*. Luniver Press, 2006.

[K46] Pitkänen M. *Topological Geometrodynamics: an Overview*. Online book.Available at: http://tgdtheory.fi/public_html/tgdview/tgdview.html, 2006.

[K47] Pitkänen M. Topological Quantum Computation in TGD Universe. In *Genes and Memes*. In online book. Available at: http://tgdtheory.fi/public_html/genememe/genememe.html#tqc, 2006.

[K48] Pitkänen M. Twistors, N=4 Super-Conformal Symmetry, and Quantum TGD. In *Towards M-Matrix*. In online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html#twistor, 2006.

[K49] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. In online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#vNeumann, 2006.

[K50] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry*. In online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#cspin, 2006.

[K51] Pitkänen M. Quantum Adeles. In *TGD as a Generalized Number Theory*. In online book. Available at: http://tgdtheory.fi/public_html/tgdnumber/tgdnumber.html#galois, 2012.

[K52] Pitkänen M. SUSY in TGD Universe. In *p-Adic Physics*. In online book. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html#susychap, 2012.

[K53] Pitkänen M. *TGD Based View About Living Matter and Remote Mental Interactions*. Online book. Available at: http://tgdtheory.fi/public_html/tgdlian/tgdlian.html, 2012.

[K54] Pitkänen M. *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html, 2013.

[K55] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: http://tgdtheory.fi/public_html/padphys/padphys.html, 2013.

[K56] Pitkänen M. *Quantum TGD*. Online book. Available at: http://tgdtheory.fi/public_html/tgdquantum/tgdquantum.html, 2013.

[K57] Pitkänen M. TGD and Potential Anomalies of GRT. In *Physics in Many-Sheeted Space-Time*. In online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#granomalies, 2013.

[K58] Pitkänen M. Criticality and dark matter. In *Hyper-finite Factors and Dark Matter Hierarchy*. In online book. Available at: http://tgdtheory.fi/public_html/neuplanck/neuplanck.html#qcritdark, 2014.

[K59] Pitkänen M. Quantum gravity, dark matter, and prebiotic evolution. In *Genes and Memes*. In online book. Available at: http://tgdtheory.fi/public_html/genememe/genememe.html#hgrprebio, 2014.

[K60] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry*. In online book. Available at: http://tgdtheory.fi/public_html/tgdgeom/tgdgeom.html#wcwnew, 2014.

[K61] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. In online book. Available at: http://tgdtheory.fi/public_html/tgdclass/tgdclass.html#prext, 2015.

## Articles about TGD

[L1] Pitkänen M. A Strategy for Proving Riemann Hypothesis. Available at: http://arxiv.org/abs/math/0111262, 2002.

[L2] Pitkänen M. Gravitational Schrödinger equation as a quantum model for the formation of astrophysical structures and dark matter? Available at: http://tgdtheory.fi/articles/nottale.pdf, 2004.

[L3] Pitkänen M. CMAP representations about TGD. Available at: http://www.tgdtheory.fi/cmaphtml.html, 2014.

[L4] Pitkänen M. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. Available at: http://www.tgdtheory.fi/tgdglossary.pdf, 2014.

[L5] Pitkänen M. Comparison with other theories.Available at: http://www.tgdtheory.fi/webCMAPs/Comparisonwithothertheories.html. 2014.

[L6] Pitkänen M. Differences between TGD and string models.Available at: http://www.tgdtheory.fi/webCMAPs/DifferencesbetweenTGDandstringmodels.html. 2014.

[L7] Pitkänen M. Further progress concerning the relationship between TGD and GRT and Kähler-Dirac action. Available at: http://tgdtheory.fi/public_html/articles/progress.pdf, 2014.

[L8] Pitkänen M. How quantum TGD differs from standard quantum physics.Available at: http://www.tgdtheory.fi/webCMAPs/HowquantumTGDdiffersfromstandardquantumphysics.html. 2014.

[L9] Pitkänen M. How TGD differs from standard model.Available at: http://www.tgdtheory.fi/webCMAPs/HowTGDdiffersfromstandardmodel.html. 2014.

[L10] Pitkänen M. Similarities between TGD and string models.Available at: http://www.tgdtheory.fi/webCMAPs/SimilaritiesbetweenTGDandstringmodels.html. 2014.

[L11] Pitkänen M. What went wrong with symmetries? Available at: http://tgdtheory.fi/public_html/articles/whatiswrong.pdf, 2015.