

## Appendix

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Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regard as stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of imbedding space and related spaces are discussed and the relationship of  $CP_2$  to standard model is summarized. The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure. Hierarchy of Planck constants can be now understood in terms of the non-determinism of Kähler action and the recent vision about connections to other key ideas is summarized.

## 1 Imbedding Space $M^4 \times CP_2$ And Related Notions

Space-times are regarded as 4-surfaces in  $H = M^4 \times CP_2$  the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space  $CP_2$  with size scale of order  $10^4$  Planck lengths. One can say that imbedding space is obtained by replacing each point  $m$  of empty Minkowski space with 4-D tiny  $CP_2$ . The space-time of general relativity is replaced by a 4-D surface in  $H$  which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

**Fig. 1.** Imbedding space  $H = M^4 \times CP_2$  as Cartesian product of Minkowski space  $M^4$  and compact projective space  $CP_2$ . <http://tgdtheory.fi/appfigures/Hoo.jpg>

Denote by  $M^4_+$  and  $M^4_-$  the future and past directed lightcones of  $M^4$ . Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) causal diamond (CD) is defined as cartesian product  $CD \times CP_2$ . Often I use CD to refer just to  $CD \times CP_2$  since  $CP_2$  factor is relevant from the point of view of ZEO.

**Fig. 2.** Future and past light-cones  $M^4_+$  and  $M^4_-$ . Causal diamonds (CD) are defined as their intersections. <http://tgdtheory.fi/appfigures/futurepast.jpg>

**Fig. 3.** Causal diamond (CD) is highly analogous to Penrose diagram but simpler. <http://tgdtheory.fi/appfigures/penrose.jpg>

A rather recent discovery was that  $CP_2$  is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure.  $M^4$  is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure so that  $H = M^4 \times CP_2$  is twistorially unique.

One can loosely say that quantum states in a given sector of “world of classical worlds” (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of  $CP_2$  radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

## 2 Basic Facts About $CP_2$

$CP_2$  as a four-manifold is very special. The following arguments demonstrates that it codes for the symmetries of standard models via its isometries and holonomies.

### 2.1 $CP_2$ As A Manifold

$CP_2$ , the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space  $C^3$  under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) . \quad (2.1)$$

Here  $\lambda$  is any non-zero complex number. Note that  $CP_2$  can be also regarded as the coset space  $SU(3)/U(2)$ . The pair  $z^i/z^j$  for fixed  $j$  and  $z^i \neq 0$  defines a complex coordinate chart for  $CP_2$ . As  $j$  runs from 1 to 3 one obtains an atlas of three coordinate charts covering  $CP_2$ , the charts being holomorphically related to each other (e.g.  $CP_2$  is a complex manifold). The points  $z^3 \neq 0$  form a subset of  $CP_2$  homeomorphic to  $R^4$  and the points with  $z^3 = 0$  a set homeomorphic to  $S^2$ . Therefore  $CP_2$  is obtained by “adding the 2-sphere at infinity to  $R^4$ ”.

Besides the standard complex coordinates  $\xi^i = z^i/z^3$ ,  $i = 1, 2$  the coordinates of Eguchi and Freund [A3] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it , \\ \xi^2 &= x + iy . \end{aligned} \quad (2.2)$$

These are related to the “spherical coordinates” via the equations

$$\begin{aligned} \xi^1 &= r \exp(i \frac{(\Psi + \Phi)}{2}) \cos(\frac{\Theta}{2}) , \\ \xi^2 &= r \exp(i \frac{(\Psi - \Phi)}{2}) \sin(\frac{\Theta}{2}) . \end{aligned} \quad (2.3)$$

The ranges of the variables  $r, \Theta, \Phi, \Psi$  are  $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$  respectively.

Considered as a real four-manifold  $CP_2$  is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second  $b = 1$ .

**Fig. 4.**  $CP_2$  as manifold. <http://tgdtheory.fi/appfigures/cp2.jpg>

### 2.2 Metric And Kähler Structure Of $CP_2$

In order to obtain a natural metric for  $CP_2$ , observe that  $CP_2$  can be thought of as a set of the orbits of the isometries  $z^i \rightarrow \exp(i\alpha)z^i$  on the sphere  $S^5$ :  $\sum z^i \bar{z}^i = R^2$ . The metric of  $CP_2$  is obtained by projecting the metric of  $S^5$  orthogonally to the orbits of the isometries. Therefore the distance between the points of  $CP_2$  is that between the representative orbits on  $S^5$ .

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b , \quad (2.4)$$

where the Hermitian, in fact Kähler metric  $g_{a\bar{b}}$  is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K , \quad (2.5)$$

where the function  $K$ , Kähler function, is defined as

$$\begin{aligned} K &= \log(F) , \\ F &= 1 + r^2 . \end{aligned} \quad (2.6)$$

The Kähler function for  $S^2$  has the same form. It gives the  $S^2$  metric  $dzd\bar{z}/(1+r^2)^2$  related to its standard form in spherical coordinates by the coordinate transformation  $(r, \phi) = (\tan(\theta/2), \phi)$ .

The representation of the  $CP_2$  metric is deducible from  $S^5$  metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} , \quad (2.7)$$

where the quantities  $\sigma_i$  are defined as

$$\begin{aligned} r^2\sigma_1 &= \text{Im}(\xi^1 d\xi^2 - \xi^2 d\xi^1) , \\ r^2\sigma_2 &= -\text{Re}(\xi^1 d\xi^2 - \xi^2 d\xi^1) , \\ r^2\sigma_3 &= -\text{Im}(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) . \end{aligned} \quad (2.8)$$

$R$  denotes the radius of the geodesic circle of  $CP_2$ . The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A , \quad (2.9)$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} , & e^1 &= \frac{r\sigma_1}{\sqrt{F}} , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} , & e^3 &= \frac{r\sigma_3}{F} . \end{aligned} \quad (2.10)$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} , & e^1 &= \frac{r(\sin\Theta \cos\Psi d\Phi + \sin\Psi d\Theta)}{2\sqrt{F}} , \\ e^2 &= \frac{r(\sin\Theta \sin\Psi d\Phi - \cos\Psi d\Theta)}{2\sqrt{F}} , & e^3 &= \frac{r(d\Psi + \cos\Theta d\Phi)}{2F} . \end{aligned} \quad (2.11)$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + \cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + \sin^2\Theta d\Phi^2) . \quad (2.12)$$

The vierbein connection satisfying the defining relation

$$de^A = -V_B^A \wedge e^B , \quad (2.13)$$

is given by

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} , & V_{23} &= \frac{e^1}{r} , \\ V_{02} &= -\frac{e^2}{r} , & V_{31} &= \frac{e^2}{r} , \\ V_{03} &= (r - \frac{1}{r})e^3 , & V_{12} &= (2r + \frac{1}{r})e^3 . \end{aligned} \quad (2.14)$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{aligned} R_{01} &= e^0 \wedge e^1 - e^2 \wedge e^3 , & R_{23} &= e^0 \wedge e^1 - e^2 \wedge e^3 , \\ R_{02} &= e^0 \wedge e^2 - e^3 \wedge e^1 , & R_{31} &= -e^0 \wedge e^2 + e^3 \wedge e^1 , \\ R_{03} &= 4e^0 \wedge e^3 + 2e^1 \wedge e^2 , & R_{12} &= 2e^0 \wedge e^3 + 4e^1 \wedge e^2 . \end{aligned} \quad (2.15)$$

Metric defines a real, covariantly constant, and therefore closed 2-form  $J$

$$J = -ig_{a\bar{b}}d\xi^a d\bar{\xi}^b , \quad (2.16)$$

the so called Kähler form. Kähler form  $J$  defines in  $CP_2$  a symplectic structure because it satisfies the condition

$$J^k{}_r J^{rl} = -s^{kl} . \quad (2.17)$$

The form  $J$  is integer valued and by its covariant constancy satisfies free Maxwell equations. Hence it can be regarded as a curvature form of a  $U(1)$  gauge potential  $B$  carrying a magnetic charge of unit  $1/2g$  ( $g$  denotes the gauge coupling). Locally one has therefore

$$J = dB , \quad (2.18)$$

where  $B$  is the so called Kähler potential, which is not defined globally since  $J$  describes homological magnetic monopole.

It should be noticed that the magnetic flux of  $J$  through a 2-surface in  $CP_2$  is proportional to its homology equivalence class, which is integer valued. The explicit representations of  $J$  and  $B$  are given by

$$\begin{aligned} B &= 2re^3 , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2} dr \wedge (d\Psi + \cos\Theta d\Phi) + \frac{r^2}{2F} \sin\Theta d\Theta d\Phi . \end{aligned} \quad (2.19)$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type  $(1, 1)$ .

Useful coordinates for  $CP_2$  are the so called canonical coordinates in which Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k . \end{aligned} \quad (2.20)$$

The relationship of the canonical coordinates to the “spherical” coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} , \\ P_2 &= \frac{r^2 \cos\Theta}{2(1+r^2)} , \\ Q_1 &= \Psi , \\ Q_2 &= \Phi . \end{aligned} \quad (2.21)$$

### 2.3 Spinors In $CP_2$

$CP_2$  doesn't allow spinor structure in the conventional sense [A2]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of  $CP_2$  play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space  $M$ . The parallel propagation around a closed curve with a base point  $x$  leads to a rotated vierbein at  $x$ :  $e^A = R_B^A e^B$  and one can associate to each closed path an element of  $SO(4)$ .

Consider now a one-parameter family of closed curves  $\gamma(v) : v \in (0, 1)$  with the same base point  $x$  and  $\gamma(0)$  and  $\gamma(1)$  trivial paths. Clearly these paths define a sphere  $S^2$  in  $M$  and the element  $R_B^A(v)$  defines a closed path in  $SO(4)$ . When the sphere  $S^2$  is contractible to a point e.g., homologically trivial, the path in  $SO(4)$  is also contractible to a point and therefore represents a trivial element of the homotopy group  $\Pi_1(SO(4)) = Z_2$ .

For a homologically nontrivial 2-surface  $S^2$  the associated path in  $SO(4)$  can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group  $Spin(4)$  (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of  $Spin(4)$  to the surface  $S^2$ . Now, however this path corresponds to a lift of the corresponding  $SO(4)$  path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed  $-1$ -factor associated with the parallel transport of the spinor around the sphere  $S^2$  by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating  $-1$ -factor. For a  $U(1)$  gauge potential this factor is given by the exponential  $exp(i2\Phi)$ , where  $\Phi$  is the magnetic flux through the surface. This factor has the value  $-1$  provided the  $U(1)$  potential carries half odd multiple of Dirac charge  $1/2g$ . In case of  $CP_2$  the required gauge potential is half odd multiple of the Kähler potential  $B$  defined previously. In the case of  $M^4 \times CP_2$  one can in addition couple the spinor components with different chiralities independently to an odd multiple of  $B/2$ .

## 2.4 Geodesic Sub-Manifolds Of $CP_2$

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the imbedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors  $h_\alpha^k$  (understood as vectors of  $H$ ) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to  $H$  and  $X^4$ .

In [A4] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space  $G/H$  is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra  $g$  of the group  $G$ . The Lie triple system  $t$  is defined as a subspace of  $g$  characterized by the closedness property with respect to double commutation

$$[X, [Y, Z]] \in t \text{ for } X, Y, Z \in t . \quad (2.22)$$

$SU(3)$  allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that  $SU(3)$  allows two nonequivalent  $SU(2)$  algebras corresponding to subgroups  $SO(3)$  (orthogonal  $3 \times 3$  matrices) and the usual isospin group  $SU(2)$ . By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of  $CP_2$ .

Standard representatives for the geodesic spheres of  $CP_2$  are given by the equations

$$S_I^2 : \xi^1 = \bar{\xi}^2 \text{ or equivalently } (\Theta = \pi/2, \Psi = 0) ,$$

$$S_{II}^2 : \xi^1 = \xi^2 \text{ or equivalently } (\Theta = \pi/2, \Phi = 0) .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in  $CP_2$ . The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for  $S_I^2$ .  $S_{II}^2$  is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

### 3 $CP_2$ Geometry And Standard Model Symmetries

#### 3.1 Identification Of The Electro-Weak Couplings

The delicacies of the spinor structure of  $CP_2$  make it a unique candidate for space  $S$ . First, the coupling of the spinors to the  $U(1)$  gauge potential defined by the Kähler structure provides the missing  $U(1)$  factor in the gauge group. Secondly, it is possible to couple different  $H$ -chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B4] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space  $H$  allows to define three different chiralities for spinors. Spinors with fixed  $H$ -chirality  $e = \pm 1$ ,  $CP_2$ -chirality  $l, r$  and  $M^4$ -chirality  $L, R$  are defined by the condition

$$\begin{aligned}\Gamma\Psi &= e\Psi, \\ e &= \pm 1,\end{aligned}\tag{3.1}$$

where  $\Gamma$  denotes the matrix  $\Gamma_9 = \gamma_5 \times \gamma_5$ ,  $1 \times \gamma_5$  and  $\gamma_5 \times 1$  respectively. Clearly, for a fixed  $H$ -chirality  $CP_2$ - and  $M^4$ -chiralities are correlated.

The spinors with  $H$ -chirality  $e = \pm 1$  can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite  $H$ -chirality one can identify the vielbein group of  $CP_2$  as the electro-weak group:  $SO(4) = SU(2)_L \times SU(2)_R$ .

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-).\tag{3.2}$$

Here  $V$  and  $B$  denote the projections of the vielbein and Kähler gauge potentials respectively and  $1_{+(-)}$  projects to the spinor  $H$ -chirality  $+(-)$ . The integers  $n_{\pm}$  are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection  $V$  and of  $B$  are given by the equations

$$\begin{aligned}V_{01} &= -\frac{e^1}{r_2}, & V_{23} &= \frac{e^1}{r_2}, \\ V_{02} &= -\frac{e^2}{r}, & V_{31} &= \frac{e^2}{r}, \\ V_{03} &= (r - \frac{1}{r})e^3, & V_{12} &= (2r + \frac{1}{r})e^3,\end{aligned}\tag{3.3}$$

and

$$B = 2re^3,\tag{3.4}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying  $\Sigma_3^0$  and  $\Sigma_2^1$  as the diagonal (neutral) Lie-algebra generators of  $SO(4)$ , one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2,\tag{3.5}$$

where one have defined

$$\begin{aligned}I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2}, \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2}.\end{aligned}\tag{3.6}$$



$A_{ch}$  is clearly left handed so that one can perform the identification

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} , \quad (3.7)$$

where  $W^\pm$  denotes the charged intermediate vector boson.

Consider next the identification of the neutral gauge bosons  $\gamma$  and  $Z^0$  as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 , \\ Y &= \frac{e^3}{r} , \end{aligned} \quad (3.8)$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY , \\ \bar{Z}^0 &= cX + dY , \end{aligned} \quad (3.9)$$

where the normalization condition

$$ad - bc = 1 ,$$

is satisfied. The physical fields  $\gamma$  and  $Z^0$  are related to  $\bar{\gamma}$  and  $\bar{Z}^0$  by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 . \end{aligned} \quad (3.10)$$

Identifying  $\Sigma_{12}$  and  $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$  as vectorial and axial Lie-algebra generators, respectively, the requirement that  $\gamma$  couples vectorially leads to the condition

$$c = -d . \quad (3.11)$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0 (I_L^3 - \sin^2 \theta_W Q_{em}) . \quad (3.12)$$

Here the electromagnetic charge  $Q_{em}$  and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} . \end{aligned} \quad (3.13)$$

The fields  $\gamma$  and  $Z^0$  are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) . \end{aligned} \quad (3.14)$$

The value of the Weinberg angle is given by

$$\sin^2\theta_W = \frac{3b}{2(a+b)} , \quad (3.15)$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of Weinberg angle is a dynamical problem. The angle is completely fixed once the YM action is fixed by requiring that action contains no cross term of type  $\gamma Z^0$ . Pure symmetry non-broken electro-weak YM action leads to a definite value for the Weinberg angle. One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle.

To evaluate the value of the Weinberg angle one can express the neutral part  $F_{nc}$  of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) , \quad (3.16)$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) , \end{aligned} \quad (3.17)$$

in terms of the fields  $\gamma$  and  $Z^0$  (photon and  $Z$ - boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - \sin^2\theta_W Q_{em}) . \quad (3.18)$$

Evaluating the expressions above one obtains for  $\gamma$  and  $Z^0$  the expressions

$$\begin{aligned} \gamma &= 3J - \sin^2\theta_W R_{03} , \\ Z^0 &= 2R_{03} . \end{aligned} \quad (3.19)$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + \sin^2\theta_W Z^0) . \quad (3.20)$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + f J^{\alpha\beta} J_{\alpha\beta} , \\ L_{sym} &= \frac{1}{4g^2} Tr(F^{\alpha\beta} F_{\alpha\beta}) , \end{aligned} \quad (3.21)$$

where the trace is taken in spinor representation, in terms of  $\gamma$  and  $Z^0$  one obtains for the coefficient  $X$  of the  $\gamma Z^0$  cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} , \\ K &= Tr [Q_{em}(I_L^3 - \sin^2\theta_W Q_{em})] , \end{aligned} \quad (3.22)$$

In the general case the value of the coefficient  $K$  is given by

$$K = \sum_i \left[ -\frac{(18 + 2n_i^2) \sin^2 \theta_W}{9} \right] , \quad (3.23)$$

where the sum is over the spinor chiralities, which appear as elementary fermions and  $n_i$  is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$\sin^2 \theta_W = \frac{9 \sum_i 1}{(fg^2 + 2 \sum_i (18 + n_i^2))} . \quad (3.24)$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$\sin^2 \theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} . \quad (3.25)$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is quite close to the typical value 9/24 of GUTs [B1] .

### 3.2 Discrete Symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.
2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B2] .

The action of the reflection  $P$  on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi . \quad (3.26)$$

in the representation of the gamma matrices for which  $\gamma^0$  is diagonal. It should be noticed that  $W$  and  $Z^0$  bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of  $P$ .

The guess that a complex conjugation in  $CP_2$  is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) , \\ \xi^k &\rightarrow \bar{\xi}^k , \\ \Psi &\rightarrow \gamma^1 \gamma^3 \otimes 1 \Psi . \end{aligned} \quad (3.27)$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in  $CP_2$ :

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k , \\ \Psi &\rightarrow \Psi^\dagger \gamma^2 \gamma^0 \otimes 1 . \end{aligned} \quad (3.28)$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

## 4 The Relationship Of TGD To QFT And String Models

TGD could be seen as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

**Fig. 5.** TGD replaces point-like particles with 3-surfaces. <http://tgdtheory.fi/appfigures/particletgd.jpg>

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary  $\delta M_+^4 = S^2 \times R_+$  of 4-D light-cone  $M_+^4$  is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of  $S^2$  can be compensated by  $S^2$ -local scaling of the light-like radial coordinate of  $R_+$ . These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in completely unique position as far as symmetries are considered.

String like objects obtained as deformations of cosmic strings  $X^2 \times Y^2$ , where  $X^2$  is minimal surface in  $M^4$  and  $Y^2$  a holomorphic surface of  $CP_2$  are fundamental extremals of Kähler action having string world sheet as  $M^4$  projections. Cosmic strings dominate the primordial cosmology of TGD Universe and inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D  $M^4$  projection dominate.

Also genuine string like objects emerge from TGD. The conditions that the em charge of modes of induced spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

**Fig. 6.** Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situation in Minkowskian regions of space-time surface. <http://tgdtheory.fi/appfigures/fermistring.jpg>

TGD based view about elementary particles has two aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidian signature of metric and having 4-D  $CP_2$  projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.
2. Fermion number is carried by the modes of the induced spinor field. In Minkowskian space-time regions the modes are localized at string world sheets connecting the wormhole contacts.

**Fig. 7.** TGD view about elementary particles. a) Particle corresponds 4-D generalization of world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidian signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at same sheet: the strings do not extend inside the wormhole contacts. <http://tgdtheory.fi/appfigures/elparticletgd.jpg>

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of “long” string connecting wormhole contacts and having hadronic string as physical counterpart. Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like “short” strings with length scale given by  $CP_2$  size, which is  $10^4$  times longer than Planck scale characterizing strings in string models.
2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator  $L_0$ . Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D “lines” of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particle along two different routes.

**Fig. 8.** a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. <http://tgdtheory.fi/appfigures/tgdgraphs.jpg>

## 5 Induction Procedure And Many-Sheeted Space-Time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by  $Z^0$  fields for extremals of Kähler action.

Classical em fields are always accompanied by  $Z^0$  field and some components of color gauge field. For extremals having homologically non-trivial sphere as a  $CP_2$  projection em and  $Z^0$  fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only  $W$  fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has  $U(1)$  holonomy by 2-dimensionality of the  $CP_2$  projection. Color gauge field has  $U(1)$  holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

### 5.0.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has imbedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if imbedding space at their image points. In the recent case the imbedding of space-time surface to imbedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the imbedding space to the space-time surface (see **Fig. ??**).

Induction procedure makes sense also for the spinor fields of imbedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with imbedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

**Fig. 9.** Induction of spinor connection and metric as projection to the space-time surface. <http://tgdtheory.fi/appfigures/induct.jpg>

### 5.0.2 Induced gauge fields for space-times for which $CP_2$ projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional  $CP_2$  projection, only vacuum extremals and space-time surfaces for which  $CP_2$  projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing  $W$  fields and homologically non-trivial sphere to non-vanishing  $W$  fields but vanishing  $\gamma$  and  $Z^0$ . This can be verified by explicit examples.

$r = \infty$  surface gives rise to a homologically non-trivial geodesic sphere for which  $e_0$  and  $e_3$  vanish imply the vanishing of  $W$  field. For space-time sheets for which  $CP_2$  projection is  $r = \infty$  homologically non-trivial geodesic sphere of  $CP_2$  one has

$$\gamma = \left(\frac{3}{4} - \frac{\sin^2(\theta_W)}{2}\right)Z^0 \simeq \frac{5Z^0}{8} .$$

The induced  $W$  fields vanish in this case and they vanish also for all geodesic sphere obtained by  $SU(3)$  rotation.

$Im(\xi^1) = Im(\xi^2) = 0$  corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex  $CP_2$  coordinates constant values. In this case  $e^1$  and  $e^3$  vanish so that the induced em,  $Z^0$ , and Kähler fields vanish but induced  $W$  fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D  $CP_2$  projection color rotations and weak symmetries commute.

## 5.1 Many-Sheeted Space-Time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same  $M^4$  region. Second manner to say this is that  $CP_2$  coordinates are many-valued functions of  $M^4$  coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four imbedding space coordinates.

**Fig. 10.** Illustration of many-sheeted space-time of TGD. <http://tgdtheory.fi/appfigures/manysheeted.jpg>

### 5.1.1 Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of  $M^4$  (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields. This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

### 5.1.2 Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through them so that the throats look like Kähler magnetic monopoles.

**Fig. 11.** Wormhole contact. <http://tgdtheory.fi/appfigures/wormholecontact.jpg>

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

### 5.1.3 The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in  $H$  although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of  $M^4$  and providing it with an effective metric obtained as sum of  $M^4$  metric and deviations of the induced metrics of various space-time sheets from  $M^4$  metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

**Fig. 12.** The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. <http://tgdtheory.fi/appfigures/fieldsuperpose.jpg>

Space-time surfaces of TGD are considerably simpler objects than the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of imbedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

#### 5.1.4 Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the general case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as  $M^4$  projection gives rise to magnetic flux tubes carrying monopole flux made possible by  $CP_2$  topology allowing homological Kähler magnetic monopoles.

**Fig. 13.** Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. <http://tgdtheory.fi/appfigures/field.jpg>

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominate during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

## 5.2 Imbedding Space Spinors And Induced Spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of  $M^4 \times CP_2$ .

$CP_2$  does not allow spinor structure in the ordinary sense but one can couple the opposite  $H$ -chiralities of  $H$ -spinors to an  $n = 1$  ( $n = 3$ ) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of  $SU(3)$  Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.
2. Spinor harmonics of imbedding space correspond to triality  $t = 1$  ( $t = 0$ ) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of imbedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers of these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the imbedding space spinor connection carries  $W$  gauge potentials one can say that the imbedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D  $CP_2$  projection and Euclidian signature of the induced metric.
2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the  $CP_2$  projection of the regions carrying induced spinor field is such that the induced  $W$  fields and above weak scale also the induced  $Z^0$  fields vanish in order to avoid large parity breaking effects. This condition forces the  $CP_2$  projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.
3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D  $CP_2$  projection.
4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.
5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D  $CP_2$  projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

### 5.3 Space-Time Surfaces With Vanishing Em, $Z^0$ , Or Kähler Fields

In the following the induced gauge fields are studied for general space-time surface without assuming the extremal property. In fact, extremal property reduces the study to the study of vacuum extremals and surfaces having geodesic sphere as a  $CP_2$  projection and in this sense the following arguments are somewhat obsolete in their generality.

#### 5.3.1 Space-times with vanishing em, $Z^0$ , or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates  $(r, \Theta, \Psi, \Phi)$  for  $CP_2$ , the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + \cos(\Theta)d\Phi) + \frac{r^2}{2F} \sin(\Theta)d\Theta \wedge d\Phi , \\ F &= 1 + r^2 . \end{aligned} \tag{5.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3 + 2p) \frac{r}{F^2} dr \wedge (d\Psi + \cos(\Theta)d\Phi) + (3 + p) \frac{r^2}{2F} \sin(\Theta)d\Theta \wedge d\Phi , \\ p &= \sin^2(\Theta_W) , \end{aligned} \tag{5.2}$$

where  $\Theta_W$  denotes Weinberg angle.



1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi , \\ (3+2p)\frac{1}{r^2 F}(d(r^2)/d\Theta)(k+\cos(\Theta)) + (3+p)\sin(\Theta) &= 0 , \end{aligned} \quad (5.3)$$

hold true. The conditions imply that  $CP_2$  projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} , \\ X &= D \left[ \left| \frac{k+u}{C} \right| \right]^\epsilon , \\ u &\equiv \cos(\Theta) , \quad C = k + \cos(\Theta_0) , \quad D = \frac{r_0^2}{1+r_0^2} , \quad \epsilon = \frac{3+p}{3+2p} , \end{aligned} \quad (5.4)$$

where  $C$  and  $D$  are integration constants.  $0 \leq X \leq 1$  is required by the reality of  $r$ .  $r = 0$  would correspond to  $X = 0$  giving  $u = -k$  achieved only for  $|k| \leq 1$  and  $r = \infty$  to  $X = 1$  giving  $|u+k| = [(1+r_0^2)/r_0^2]^{(3+2p)/(3+p)}$  achieved only for

$$\text{sign}(u+k) \times \left[ \frac{1+r_0^2}{r_0^2} \right]^{\frac{3+2p}{3+p}} \leq k+1 ,$$

where  $\text{sign}(x)$  denotes the sign of  $x$ .

The expressions for Kähler form and  $Z^0$  field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi , \\ Z^0 &= -\frac{6}{p} J . \end{aligned} \quad (5.5)$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range  $Z^0$  vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of  $Z^0$  fields is achieved by the replacement of the parameter  $\epsilon$  with  $\epsilon = 1/2$  as becomes clear by considering the condition stating that  $Z^0$  field vanishes identically. Also the relationship  $F_{em} = 3J = -\frac{3}{4} \frac{r^2}{F} du \wedge d\Phi$  is useful.
3. The vanishing Kähler field corresponds to  $\epsilon = 1, p = 0$  in the formula for em neutral space-times. In this case classical em and  $Z^0$  fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2} (k+u) \frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi , \\ r &= \sqrt{\frac{X}{1-X}} , \quad X = D|k+u| , \\ \gamma &= -\frac{p}{2} Z^0 . \end{aligned} \quad (5.6)$$

For a vanishing value of Weinberg angle ( $p = 0$ ) em field vanishes and only  $Z^0$  field remains as a long range gauge field. Vacuum extremals for which long range  $Z^0$  field vanishes but em field is non-vanishing are not possible.

### 5.3.2 The effective form of $CP_2$ metric for surfaces with 2-dimensional $CP_2$ projection

The effective form of the  $CP_2$  metric for a space-time having vanishing em,  $Z^0$ , or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned} ds_{eff}^2 &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s_{\Theta\Theta}^{eff}d\Theta^2 + s_{\Phi\Phi}^{eff}d\Phi^2] , \\ s_{\Theta\Theta}^{eff} &= X \times \left[ \frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X \right] , \\ s_{\Phi\Phi}^{eff} &= X \times [(1-X)(k+u)^2 + 1 - u^2] , \end{aligned} \quad (5.7)$$

and is useful in the construction of vacuum imbedding of, say Schwartzchild metric.

### 5.3.3 Topological quantum numbers

Space-times for which either em,  $Z^0$ , or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers ( $\omega_1$  and  $\omega_2$ ) are frequency type parameters, two ( $k_1$  and  $k_2$ ) are wave vector like quantum numbers, two of the quantum numbers ( $n_1$  and  $n_2$ ) are integers. The parameters  $\omega_i$  and  $n_i$  will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of  $CP_2$  coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates  $\Psi$  and  $\Phi$  can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2 \phi + \text{Fourier expansion} , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1 \phi + \text{Fourier expansion} . \end{aligned} \quad (5.8)$$

$m^0, m^3$  and  $\phi$  denote the coordinate variables of the cylindrical  $M^4$  coordinates) so that one has  $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$ . The regions of the space-time surface with given values of the vacuum parameters  $\omega_i, k_i$  and  $n_i$  and  $m$  and  $C$  are bounded by the surfaces at which space-time surface becomes ill-defined, say by  $r > 0$  or  $r < \infty$  surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters  $r_0$  and  $\Theta_0$ . At  $r = \infty$  surfaces  $n_2, \omega_2$  and  $m$  can change since all values of  $\Psi$  correspond to the same point of  $CP_2$ : at  $r = 0$  surfaces also  $n_1$  and  $\omega_1$  can change since all values of  $\Phi$  correspond to same point of  $CP_2$ , too. If  $r = 0$  or  $r = \infty$  is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global imbedding for, say a constant magnetic field. Although global imbedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate  $u$  in general possesses discontinuous derivative at  $r = 0$  and  $r = \infty$  surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 , \quad (5.9)$$

is satisfied. In particular, the ratio  $\omega_2/\omega_1$  is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter  $n_1$  and  $n_2$  ( $\omega_1$  and  $\omega_2$ ) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

## 6 P-Adic Numbers And TGD

### 6.1 P-Adic Number Fields

p-Adic numbers ( $p$  is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A1]. p-Adic numbers are representable as power expansion of the prime number  $p$  of form

$$x = \sum_{k \geq k_0} x(k)p^k, \quad x(k) = 0, \dots, p-1 . \quad (6.1)$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} . \quad (6.2)$$

Here  $k_0(x)$  is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0} \varepsilon(x) , \quad (6.3)$$

where  $\varepsilon(x) = k + \dots$  with  $0 < k < p$ , is p-adic number with unit norm and analogous to the phase factor  $\exp(i\phi)$  of a complex number.

The distance function  $d(x, y) = |x - y|_p$  defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq \max\{d(x, y), d(y, z)\} . \quad (6.4)$$

The properties of the distance function make it possible to decompose  $R_p$  into a union of disjoint sets using the criterion that  $x$  and  $y$  belong to same class if the distance between  $x$  and  $y$  satisfies the condition

$$d(x, y) \leq D . \quad (6.5)$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes  $X$  and  $Y$  do not depend on the choice of points  $x$  and  $y$  inside classes. One can therefore speak about distance function between classes.
2. Distances of points  $x$  and  $y$  inside single class are smaller than distances between different classes.
3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B3]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

### 6.2 Canonical Correspondence Between P-Adic And Real Numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

### 6.2.1 Basic form of canonical identification

There exists a natural continuous map  $I : R_p \rightarrow R_+$  from p-adic numbers to non-negative real numbers given by the “pinary” expansion of the real number for  $x \in R$  and  $y \in R_p$  this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} , \\ y_k &\in \{0, 1, \dots, p-1\} . \end{aligned} \quad (6.6)$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ( $1 = 0.999\dots$ ) for the real numbers  $x$ , which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^N x_k p^{-k} , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0, \dots} p^{-k} . \end{aligned} \quad (6.7)$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^N x_k p^k , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0, \dots} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} , \end{aligned} \quad (6.8)$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

### 6.2.2 The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval  $[p^k, p^{k+1})$  (see **Fig. 6.2.2**) and is equal to the usual real norm at the points  $x = p^k$ : the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of  $p$  is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

**Fig. 14.** The real norm induced by canonical identification from 2-adic norm. <http://tgdtheory.fi/appfigures/norm.png>

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept

of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common binary digits. Furthermore, the condition  $x +_p y < \max\{x, y\}$  holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of  $p$ . Moreover one has  $x \times_p y < x \times y$  in general. The p-Adic negative  $-1_p$  associated with p-adic unit 1 is given by  $(-1)_p = \sum_k (p-1)p^k$  and defines p-adic negative for each real number  $x$ . An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x + y)_R &\leq x_R + y_R , \\ |x|_p |y|_R &\leq (xy)_R \leq x_R y_R , \end{aligned} \quad (6.9)$$

where  $|x|_p$  denotes p-adic norm. These inequalities can be generalized to the case of  $(R_p)^n$  (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x + y)_R &\leq x_R + y_R , \\ |\lambda|_p |y|_R &\leq (\lambda y)_R \leq \lambda_R y_R , \end{aligned} \quad (6.10)$$

where the norm of the vector  $x \in T_p^n$  is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = \left( \sum_n x_n^2 \right)_R . \quad (6.11)$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of  $p$ .

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

### 6.2.3 Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \quad (6.12)$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for  $0 \leq r < p$  and  $0 \leq s < p$ . It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of  $r$  and  $s$  mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for  $I$  and  $I_Q$  but  $I_Q$  is theoretically preferred since the real probabilities obtained from p-adic ones by  $I_Q$  sum up to one in p-adic thermodynamics.

### 6.2.4 Generalization of number concept and notion of imbedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic imbedding spaces. Since finite p-adic numbers correspond always to non-negative reals  $n$ -dimensional space  $R^n$  must be covered by  $2^n$  copies of the p-adic variant  $R_p^n$  of  $R^n$  each of which projects to a copy of  $R_+^n$  (four quadrants in the case of plane). The common points of p-adic and real imbedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field  $Q_p$  satisfying  $e^p \bmod p = 1$ .

**Fig. 15.** Various number fields combine to form a book like structure. <http://tgdtheory.fi/appfigures/book.jpg>

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real imbedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that  $M^4$  projections for the rational points of space-time surface  $X^4$  are related by a direct identification whereas  $CP_2$  coordinates of  $X^4$  at these points are related by  $I$ ,  $I_Q$  or some of its variants implying long range correlates for  $CP_2$  coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

## 6.3 The Notion Of P-Adic Manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

**Fig. 16.** The basic idea between p-adic manifold. <http://tgdtheory.fi/appfigures/padmanifold.jpg>

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used
2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution
3. Canonical identification vreaks general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic imbedding space with chart maps to real imbedding space and assuming preferred coordinates made possible by isometries of imbedding space: one however obtains several inequivalent

p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

## 7 Hierarchy Of Planck Constants And Dark Matter Hierarchy

Hierarchy of Planck constants was motivated by the “impossible” quantal effects of ELF em fields on vertebrate cyclotron energies  $E = hf = \hbar \times eB/m$  are above thermal energy is possible only if  $\hbar$  has value much larger than its standard value. Also Nottale’s finding that planetary orbits might be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierarchy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant:  $h_{eff} = n \times h$ . The particles at magnetic flux tubes characterized by  $h_{eff}$  would correspond to dark matter which would be invisible in the sense that only particle with same value of  $h_{eff}$  appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determinism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manifolds of any  $M^4 \times Y^2$ , where  $Y^2$  is Lagrangian sub-manifold of  $CP_2$ . For a given  $Y^2$  one obtains new manifolds  $Y^2$  by applying symplectic transformations of  $CP_2$ .

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the imbedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskian space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number  $n$  and define discrete physical degree of freedom and one would have  $h_{eff} = n \times h$ . This degeneracy would mean “second quantization” for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of  $n$ . This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional  $n \times n$  identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of imbedding space. A stronger assumption would be that they are expressible as products of  $n_1$ -fold covering of  $M^4$  and  $n_2$ -fold covering of  $CP_2$  meaning analogy with multi-sheeted Riemann surfaces and that  $M^4$  coordinates are  $n_1$ -valued functions and  $CP_2$  coordinates  $n_2$ -valued functions of space-time coordinates for  $n = n_1 \times n_2$ . These singular coverings of imbedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

**Fig. 17.** Hierarchy of Planck constants. <http://tgdtheory.fi/appfigures/planckhierarchy.jpg>

## 8 Some Notions Relevant To TGD Inspired Consciousness And Quantum Biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

### 8.1 The Notion Of Magnetic Body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness

theory. This is a crucial departure from the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicate the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

**Fig. 18.** Magnetic body associated with dipole field. <http://tgdtheory.fi/appfigures/fluxquant.jpg>

**Fig. 19.** Illustration of the reconnection by magnetic flux loops. <http://tgdtheory.fi/appfigures/reconnect1.jpg>

**Fig. 20.** Illustration of the reconnection by flux tubes connecting pairs of molecules. <http://tgdtheory.fi/appfigures/reconnect2.jpg>

**Fig. 21.** Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of  $h_{eff}$  allowing two molecules to find each other in dense molecular soup. <http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg>

## 8.2 Number Theoretic Entropy And Negentropic Entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

**Fig. 22.** Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. <http://tgdtheory.fi/appfigures/cat.jpg>

## 8.3 Life As Something Residing In The Intersection Of Reality And P-Adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred imbedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning life to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" ( WCW ) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

**Fig. 23.** The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of thought, cognitive representation. Its reversal would corre-



spond to a transformation of intention to action. <http://tgdtheory.fi/appfigures/padictoreal.jpg>

## 8.4 Sharing Of Mental Images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

**Fig. 24.** Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. <http://tgdtheory.fi/appfigures/sharing.jpg>

## 8.5 Time Mirror Mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibility of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig. <http://tgdtheory.fi/appfigures/timemirror.jpg>** or **Fig. 24** in the appendix of this book) providing mechanisms of both memory recall, realization of intentional action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

**Fig. 25.** Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially “seeing” in time direction is in question. <http://tgdtheory.fi/appfigures/timemirror.jpg>

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